Correction to Maxwell's Equations

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Maxwell's equations are examined specifically with a segment of electric current under Biot-Savart law and a single charge under Gauss' law. The line integral of the magnetic field is verified to be different from the surface integral of the curl of magnetic field because the magnetic field of Biot-Savart law diverges. Faraday's induction law is examined by inserting a capacitor into the coil loop to measure the voltage. The electric field inside the capacitor is directly proportional to the time derivative of magnetic flux. An optional capacitor is also attached to the end of the straight segment of electric wire. The time derivative of the electric field inside the capacitor is verified to be proportional to the line integral of the magnetic field from the electric current. Multiple corrections are made to Maxwell's equations.

I. INTRODUCTION

In 1831. Michael Faraday described the electrical induction in a paper[1]: "The electromotive force around a closed path is equal to the negative of the time rate of change of the magnetic flux enclosed by the path". Unfortunately, his idea was ignored for not being formulated mathematically.

Franz Ernst Neumann in his 1845 paper[2] "General laws of induced electrical currents" interpreted the electric induction as analytical measures of Faradays electronic state. The equation attracted attention from Oliver Heaviside and James Clerk Maxwell.

Faraday focused on the electromotive force of the wire. The electrical induction produced voltage. Electric field was not a concern in Faraday's research on induction.

"A Dynamical Theory of the Electromagnetic Field" is a paper[3] by James Clerk Maxwell on electromagnetism, published in 1865. In the paper, Maxwell derives an electromagnetic wave equation with a velocity for light in close agreement with measurements made by experiment, and deduces that light is an electromagnetic wave. The idea of electromagnetic wave was rejected until Maxwell's death in 1879.

It wasn't until 1884 that Oliver Heaviside, concurrently with similar work by Josiah Willard Gibbs and Heinrich Hertz, grouped the twenty equations together into a set of only four, via vector notation. This group of four equations was known variously as the HertzHeaviside equations and the MaxwellHertz equations, but are now universally known as Maxwell's equations.

Maxwell did not realized that the magnetic field of Biot-Savart law is not a differentiable vector. He chose to make extension to Ampere's law which is invalid except for an electric wire of infinite length. Following these errors, Faraday's law was also altered to create a symmetry between electric field and magnetic field to validate Maxwell's speculation that an electromagnetic wave exists and travels as fast as the light.

II. PROOF

A. Biot-Savart Law

The relationship characterizing the magnetic field generated by a electric current was first described by Jean-Baptiste Biot and Flix Savart in 1820[4].

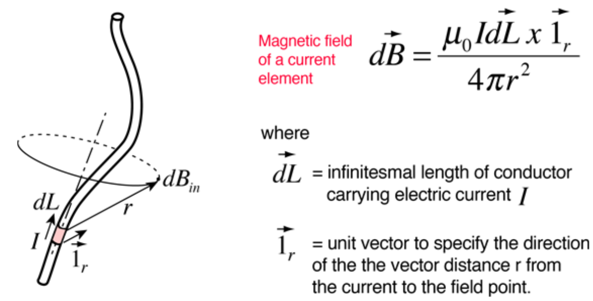

FIG. 1. Biot-Savart Law

For a straight segment of wire from (0,0,-L) to (0,0,L) in cylindrical coordinates (r, ϕ, z), the magnetic field on the x-y plane at a distance R away from the wire is

$$\vec{B} = \int_{-L}^{L} \frac{\mu_0 I}{4\pi r^3} d\vec{L} \times \vec{r} = \frac{\mu_0 I}{2\pi R} \frac{L}{\sqrt{R^2 + L^2}} (0, 1, 0) \quad (1)$$

$\vec{B}$ diverges at $R = 0$. Therefore, Stokes' theorem can not be applied to $\vec{B}$ evaluated on x-y plane.

The curl of a vector $\vec{A}$ in cylindrical coordinates is defined as

$$\vec{\nabla} \times \vec{A} = \frac{1}{r} \begin{vmatrix} \vec{a}_r & r\vec{a}_\phi & \vec{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} \quad (2)$$

From equations (1,2),

$$\vec{\nabla} \times \vec{B} = \frac{\mu_0 I}{2\pi} \frac{L}{\sqrt{R^2 + L^2}^3} (0, 0, -1) \quad (3)$$

From equation (1),

$$\int \vec{B} \bullet d\vec{r} = \mu_0 I \frac{L}{\sqrt{R^2 + L^2}} \tag{4}$$

From equation (3),

$$\iint \vec{\nabla} \times \vec{B} \bullet d\vec{S} = -\frac{\mu_0 I}{2\pi} \iint \frac{L}{\sqrt{r^2 + L^2}^3} r dr d\phi \tag{5}$$

$$= \mu_0 I (\frac{L}{\sqrt{R^2 + L^2}} - 1) \tag{6}$$

From equations (4,6),

$$\int \vec{B} \bullet d\vec{r} = \mu_0 I + \iint \vec{\nabla} \times \vec{B} \bullet d\vec{S} \tag{7}$$

Stokes' theorem is incompatible with Biot-Savart law.

B. Ampere Law

For another example that Stokes' theorem can not be applied to the magnetic field of Biot-Savart law, let R be very small compared to L.

$$R << L \tag{8}$$

From equation (1,8),

$$\vec{B} = \frac{\mu_0 I}{2\pi R}(1 - \frac{1}{2}\frac{R^2}{L^2})(0, 1, 0) \tag{9}$$

$$\int \vec{B} \bullet d\vec{r} = \mu_0 I (1 - \frac{1}{2}\frac{R^2}{L^2}) \tag{10}$$

This is universally known as Ampere's law[5] under the limitation that the magnetic field is evaluated at the immediate vicinity of the source. Ampere's research focused on the magnetic force between two electric currents, not the magnetic field from an electric current. Ampere was against the concept of magnetic field which was proposed by Biot and Savart.

From equation (3,8),

$$\vec{\nabla} \times \vec{B} = \frac{\mu_0 I}{2\pi L^2}(1 - \frac{3}{2}\frac{R^2}{L^2})(0, 0, -1) \tag{11}$$

$$\iint \vec{\nabla} \times \vec{B} \bullet d\vec{S} = -\frac{\mu_0 I}{L^2}(\frac{R^2}{2} - \frac{3}{8}\frac{R^4}{L^2}) \tag{12}$$

From euqations (10,12),

$$\int \vec{B} \bullet d\vec{r} \neq \iint \vec{\nabla} \times \vec{B} \bullet d\vec{S} \tag{13}$$

The magnetic field in the immediate vicinity of a electric wire is a good example that Stokes' theorem is incompatible with Biot-Savart law.

C. Coulomb's Law

In 1785, the French physicist Charles-Augustin de Coulomb published his first three reports of electricity and magnetism where he stated his law[6]. According to his law, the electric force from a point charge of Q to a point charge of q is represented by

$$\vec{F} = k\frac{Qq}{r^3}\vec{r} \tag{14}$$

$\vec{r}$ is the displacement vector from charge Q to charge q.

The electric field from the point charge of Q toward point charge of q is defined as

$$\vec{E} = \frac{\vec{F}}{q} = k\frac{Q}{r^3}\vec{r} \tag{15}$$

The divergence of $\vec{E}$ in spherical coordinate is

$$\vec{\nabla} \bullet \vec{E} = \vec{\nabla} \bullet \frac{kQ}{r^3}\vec{r} \tag{16}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \frac{kQ}{r^2}) \tag{17}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(kQ) \tag{18}$$

$$= \frac{0}{r^2} = 0 \tag{19}$$

The divergence of the electric field from Coulomb's law is zero. Further proof can be found in the paper, "Electric Field and Divergence Theorem".[7]

From equations (19), the volume integral of the divergence of the electric field is zero.

$$\iiint \vec{\nabla} \bullet \vec{E}\, dV = 0 \tag{20}$$

D. Gauss' Law

Gauss' law[8] was formulated by Joseph-Louis Lagrange in 1773 and Carl Friedrich Gauss in 1813. The law states that the total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity.

$$\iint \vec{E} \bullet d\vec{S} = \frac{Q}{\epsilon_0} \tag{21}$$

From equations (20,21),

$$\iint \vec{E} \bullet d\vec{S} \neq \iiint \vec{\nabla} \bullet \vec{E}\, dV \tag{22}$$

The divergence theorem is not compatible with the electric field of Coulomb's law because the divergence theorem requires a smooth vector while the electric field diverges at the origin.

E. Displacement Current

In 1864, James Clerk Maxwell speculated on the electromagnetic field in the dielectric material. Maxwell stated[3]: "In a dielectric under the action of electromotive force, we may conceive that the electricity in each molecule is so displaced that one side is rendered positively and the other negatively electrical, but that the electricity remains entirely connected with the molecule, and does not pass from one molecule to another. The effect of this action on the whole dielectric mass is to produce a general displacement of electricity in a certain direction. This displacement does not amount to a current, because when it has attained to a certain value it remains constant, but it is the commencement of a current, and its variations constitute currents in the positive or the negative direction according as the displacement is increasing or decreasing."

Let Q_d be the displacement of electricity. Maxwell speculated that Q_d should be related to a current density: "Electrical displacement consists in the opposite electrification of the sides of a molecule or particle of a body which may or may not be accompanied with transmission through the body. Let the quantity of electricity which would appear on the faces dy.dz of an element dx, dy, dz cut from the body be f.dy.dz, then f is the component of electric displacement parallel to x. We shall use f, g, h to denote the electric displacements parallel to x, y, z respectively."

$$\iint \vec{J}_d \bullet d\vec{S} = \frac{dQ_d}{dt} \tag{23}$$

Maxwell proposed to add the displacement current to the total current by stating: "The variations of the electrical displacement must be added to the currents p, q, r to get the total motion of electricity."

$$\vec{J}_{total} = \vec{J}_d + \vec{J} \tag{24}$$

"If the medium in the field is a perfect dielectric there is no true conduction, and the currents p' q', r' are only variations in the electric displacement."

$$\vec{J}_{total} = \vec{J}_d \tag{25}$$

With the new total current, Maxwell replaced the actual current density $\vec{J}$ with the imaginary current density $\vec{J}_{total}$ in Ampere's law.

$$\iint \vec{J}_{total} \bullet d\vec{S} = \frac{1}{\mu_0} \int \vec{B} \bullet d\vec{r} \tag{26}$$

However, Ampere's law is valid only for straight wire of infinite length.

From equations (7,), the dielectric theory by Maxwell for a segment of electric current should be

$$\int \vec{B} \bullet d\vec{r} = \iint (\mu_0 \vec{J}_{total} + \vec{\nabla} \times \vec{B}) \bullet d\vec{S} \tag{27}$$

F. Divergence of Magnetic Field

Let S' be a spherical surface enclosing the wire segment. The center of S' is located at (0,0,0). The radius of S' is R'.

$$R' > L \tag{28}$$

From Biot-Savart law,

$$\vec{B}(\vec{r}) = -\vec{B}(-\vec{r}) \tag{29}$$

The anti-symmetry of magnetic field vector leads to

$$\iint \vec{B} \bullet d\vec{S'} = 0 \tag{30}$$

The divergence of a vector $\vec{A}$ in cylindrical coordinates is

$$\vec{\nabla} \bullet \vec{A} = \frac{1}{r}\left(\frac{\partial(rA_r)}{\partial r} + \frac{\partial A_\phi}{\partial \phi} + r\frac{\partial A_z}{\partial z}\right) \tag{31}$$

Apply the following identity.

$$\vec{\nabla} \bullet (\vec{A} \times \vec{D}) = (\vec{\nabla} \times \vec{A}) \bullet \vec{D} - (\vec{\nabla} \times \vec{D}) \bullet \vec{A} \tag{32}$$

From equations (31,32), the divergence of the magnetic field described by Biot-Savart law is

$$\vec{\nabla} \bullet \vec{B} = 0 \tag{33}$$

G. Faraday's Law

Electromagnetic induction was discovered independently by Michael Faraday in 1831 and Joseph Henry in 1832. Any change in the magnetic environment of a coil of wire will cause a voltage/Emf to be induced in the coil.

$$Emf = -N\frac{d\Phi_B}{dt} \tag{34}$$

N is the number of turns in the coil. Φ_B is the magnetic flux through the area of the coil.

$$\Phi_B = \iint \vec{B} \bullet d\vec{S} \tag{35}$$

Emf is created by the magnetic flux, not by electric force from Coulomb's law.

$$Emf = \frac{1}{e}\int \vec{F_B} \bullet d\vec{l} \neq \int \vec{E} \bullet d\vec{l} \tag{36}$$

Insert a capacitor into the loop. Emf is equivalent to the voltage across the capacitor.

$$Emf = V = ED \tag{37}$$

E is the electric field inside the capacitor. D is the distance between two parallel plates of the capacitor.

From equations (34,37),

$$ED = -N\frac{d\Phi_B}{dt} \tag{38}$$

For a coil with static shape,

$$E = -\frac{N}{D}\iint \frac{d\vec{B}}{dt} \bullet d\vec{S} \tag{39}$$

H. Extension to Biot-Savart Law

Attach a capacitor to the end of the segment of electric wire.

$$I = \frac{dQ}{dt} = C\frac{dV}{dt} = CD\frac{dE}{dt} \tag{40}$$

C is the capacitance. D is the distance between two parallel plates of the capacitor. E is the electric field inside the capacitor.

From equations (7,40),

$$\int \vec{B} \bullet d\vec{r} = \mu_0 CD\frac{dE}{dt} + \iint \vec{\nabla} \times \vec{B} \bullet d\vec{S} \tag{41}$$

I. Integral Form

As a summary, Maxwell's equations for any charge distribution, an induction coil with capacitor, and a straight segment of electric current with a capacitor attached can be stated as:

From equation (7), for a segment of electric current,

$$\int \vec{B} \bullet d\vec{r} = \mu_0 I + \iint \vec{\nabla} \times \vec{B} \bullet d\vec{S} \tag{42}$$

From equation (41), with an extra capacitor attached at the end of a segment of electric current,

$$\int \vec{B} \bullet d\vec{r} = \mu_0 CD\frac{dE}{dt} + \iint \vec{\nabla} \times \vec{B} \bullet d\vec{S} \tag{43}$$

From equation (21), for charge described by Gauss' law,

$$\iint \vec{E} \bullet d\vec{S} = \frac{Q}{\epsilon_0} \tag{44}$$

From equation (30), for magnetic field described by Biot-Savart law,

$$\iint \vec{B} \bullet d\vec{S'} = 0 \tag{45}$$

From equation (38), a capacitor inserted into a coil.

$$E = -\frac{N}{D}\frac{d}{dt}\iint \vec{B} \bullet d\vec{S} \tag{46}$$

J. Differeantial Form

From equation (27,43),

$$\vec{\nabla} \times \vec{B} \neq \mu_0(J + \epsilon_0\frac{d\vec{E}}{dt}) \tag{47}$$

From equation (38),

$$\vec{\nabla} \bullet \vec{B} = 0 \tag{48}$$

From equations (46),

$$\vec{\nabla} \times \vec{E} \neq -N\frac{d\vec{B}}{dt} \tag{49}$$

From equation (19),

$$\vec{\nabla} \bullet \vec{E} = 0 \tag{50}$$

III. CONCLUSION

The current version of Maxwell's equations is different from the original version by Maxwell. Maxwell noticed a hidden symmetry between electric force and magnetic force. The symmetry can be used to generate electromagnetic wave. In order to establish the symmetry, Maxwell chose to ignore the fact that the magnetic field of Biot-Savart law is not compatible with Stokes' theorem. Maxwell proceeded to incorporate the curl of vector into his equations and was able to establish symmetry at a great price. His wave theory was rejected until his death.

The error continues after Maxwell's death and leads to more curl into Maxwell's equations. Faraday's law was altered to provide the curl of a virtual electric field. Ampere's law was extended to provide the time derivative of electric field by ignoring the fact that the magnetic field of Biot-Savart law may diverge.

The delta function was inserted into Gauss' law by ignoring that the divergence theorem requires a smooth vector field while the electric field of Coulomb's law diverges.

As a result, the differential form of Maxwell's equations is mostly incorrect except the divergence of the magnetic field. The incorrect differential form eventually satisfies Maxwell's original goal: electromagnetic wave formed by symmetry between electric and magnetic field.

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