

On some Ramanujan's Class Invariants: mathematical connections with the Golden Ratio linked to the various equations concerning some sectors of Cosmology.

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Abstract

The aim of this paper is to show the mathematical connections between the Ramanujan's Class Invariants, the Golden Ratio and some expression of various topics of Cosmology

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An equation means nothing to me unless it expresses a thought of God.

Srinivasa Ramanujan (1887-1920)

(N.O.A – Figures from the web)

<https://silvanodonofrio.wordpress.com/2014/04/29/la-teoria-del-tutto-parte-seconda-stringhe-e-modello-standard/>



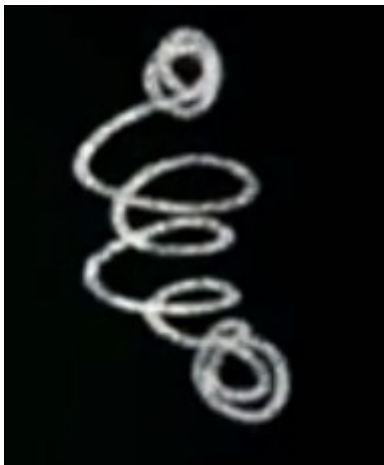
In fig. A particle with a vibrant internal structure. Not static, but a kind of elastic string. The elastic not only stretches and retracts, but sways

In 1968 a young Italian physicist Gabriele Veneziano was looking for equations that could explain the strong nuclear force, that is the powerful glue that holds the nucleus of each atom together by

binding protons to neutrons. One day in an old book on the history of mathematics he found a formula two hundred years earlier developed by Swiss mathematician Leonhard Euler. Veneziano realized that Euler's equation could describe the strong nuclear force. This accidental discovery made him immediately famous. But what does this formula have to do with strings, you will think? Now let's see it. Thanks to word of mouth between colleagues, the equation reached up to a young American physicist, Leonard Susskind who, in studying it, noticed that something new was hidden behind the abstract symbols. It described a particular type of particle with a vibrant internal structure. Not static, but a kind of elastic string. The elastic not only stretched and retracted but swayed and magically corresponded to the formula.

The leaves grow in a spiral such that the number of turns formed by rotating in one direction and the other are two consecutive Fibonacci numbers. The crossed leaves, joined to the starting one, constitute a third Fibonacci number, consecutive to the first two. (B)

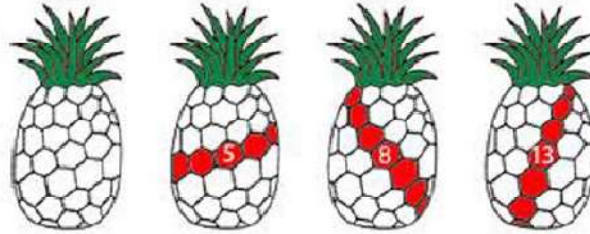
<http://web.math.unifi.it/users/mathesis/sezione/natu/page08.html>



B)

A) The string not only stretches and retracts, but sways

(see comparison with phyllotaxis)



C)

The pineapple is a magnificent example, each of the scales that cover it belongs to three different spirals which, in most of these fruits, are 5, 8 and 13 in number (just Fibonacci numbers)

D)



D) The center of sunflowers where it is possible to notice two series of golden spirals that screw each other clockwise the other counterclockwise.

Fibonacci's succession is found in an incredible variety of phenomena that are not connected to each other, but perhaps it is in the natural world that it appears with great spectacularity. The most documented case concerns PHYLLOTAXIS. It studies how leaves and branches are distributed around the stem. That the arrangement is such that the leaves do not cover each other, but that each one receives the maximum possible amount of light and rain is understandable, but one is appalled when one discovers that these schemes are expressible in mathematical terms and have a link with the Fibonacci series. In fact, the number of turns made to find the leaf aligned with the first one is

generally a number of Fibonacci. The ratio of the number of turns and the number of leaves between two symmetrical leaves, is called phyllotaxis quotient and is almost always the ratio between two consecutive or alternating numbers of the Fibonacci sequence. For example, it takes 3 full turns and go through 8 leaves to return to the leaf aligned with the first: the phyllotaxis quotient is $3/8$. Other examples. In lime trees the leaves are arranged around the branch with a phyllotaxis quotient equal to $1/2$. In the hazel, beech and bramble it is $1/3$. The apple tree, apricot and some species of oak have leaves every $2/5$ of a turn and in the pear tree and in the weeping willow every $3/8$ turn. In addition to the leaves, in the plants also other elements are arranged according to schemes based on numbers belonging to the Fibonacci series. The pineapple is a magnificent example, each of the scales that cover it belongs to three different spirals which, in most of these fruits, are 5, 8 and 13 in number (just Fibonacci numbers). No less spectacular is the center of sunflowers where it is possible to notice two series of golden spirals that screw each other clockwise the other counterclockwise.

<https://www.astronomiamo.it/Strument Astronomici/Scheda-Dati-DeepSkyObject/Galassia/1232>

E) Spiral-shaped objects are found from the "infinitely small" world to the "infinitely large" universe, for example in the arms of spiral galaxies





<https://www.eso.org/public/italy/news/eso1042/>

From:

Modular equations and approximations to π

S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

$$G_{505}^2 = (2 + \sqrt{5}) \sqrt{\left\{ \left(\frac{1 + \sqrt{5}}{2} \right) (10 + \sqrt{101}) \right\}} \\ \times \left\{ \left(\frac{5\sqrt{5} + \sqrt{101}}{4} \right) + \sqrt{\left(\frac{105 + \sqrt{505}}{8} \right)} \right\},$$

(a)

$$(2+\sqrt{5}) \left(\left(\left(\left(\left(\frac{1+\sqrt{5}}{2} \right) (10+\sqrt{101}) \right) \right) \right) \right)^{1/2} * \left(\left(\left(\left(\frac{5\sqrt{5}+\sqrt{101}}{4} \right) + \sqrt{\frac{105+\sqrt{505}}{8}} \right) \right) \right)$$

Input:

$$(2+\sqrt{5}) \sqrt{\left(\frac{1}{2}(1+\sqrt{5})\right)(10+\sqrt{101})} \left(\frac{1}{4}(5\sqrt{5}+\sqrt{101}) + \sqrt{\frac{1}{8}(105+\sqrt{505})}\right)$$

Result:

$$(2+\sqrt{5}) \sqrt{\frac{1}{2}(1+\sqrt{5})(10+\sqrt{101})} \left(\frac{1}{4}(5\sqrt{5}+\sqrt{101}) + \frac{1}{2} \sqrt{\frac{1}{2}(105+\sqrt{505})}\right)$$

Decimal approximation:

224.3689593513276391839941363576172939146443280007364930381...

224.36895935...

Alternate forms:

$$\frac{1}{8} \left(2 \sqrt{2(105+\sqrt{505})} + 10\sqrt{5} + 2\sqrt{101} + \sqrt{505} + \sqrt{10(105+\sqrt{505})} + 25 \right) \sqrt{2(1+\sqrt{5})(10+\sqrt{101})}$$

$$\sqrt{\text{root of } 256x^8 - 13134080x^7 + 12406662784x^6 + 566469885440x^5 + 8970692383216x^4 + 59000758979200x^3 + 133454526025384x^2 - 21580568998020x + 63001502001 \text{ near } x = 50341.4}$$

$$\frac{1}{4} (2+\sqrt{5}) \sqrt{\frac{1}{2}(1+\sqrt{5})(10+\sqrt{101})} (5\sqrt{5}+\sqrt{101}) + \frac{1}{4} (2+\sqrt{5}) \sqrt{(1+\sqrt{5})(10+\sqrt{101})(105+\sqrt{505})}$$

Minimal polynomial:

$$256x^{16} - 13134080x^{14} + 12406662784x^{12} + 566469885440x^{10} + 8970692383216x^8 + 59000758979200x^6 + 133454526025384x^4 - 21580568998020x^2 + 63001502001$$

From which:

$$[(2+\sqrt{5}) \left(\left(\left(\left(\left(\frac{1+\sqrt{5}}{2} \right) (10+\sqrt{101}) \right) \right) \right) \right)^{1/2} * \left(\left(\left(\left(\frac{5\sqrt{5}+\sqrt{101}}{4} \right) + \sqrt{\frac{105+\sqrt{505}}{8}} \right) \right) \right)]^{1/11}$$

Alternate form:

$$\sqrt{x} \sqrt{\begin{array}{l} \text{root of } 256x^8 - 8125440x^7 + 4996878336x^6 - 19944328960x^5 - \\ 1088503336224x^4 + 2048528617920x^3 + 55033901841536x^2 + \\ 13356686893680x + 63001502001 \text{ near } x = 31112.7 \end{array}} = 224.36895935$$

Alternate form assuming x is positive:

$$1.0000000000 \sqrt{x} = 1.2720196495$$

Expanded form:

$$\begin{aligned} & \frac{1}{2} \sqrt{\frac{5}{2} (10 + \sqrt{101})(105 + \sqrt{505})} \sqrt{x} + \sqrt{\frac{1}{2} (10 + \sqrt{101})(105 + \sqrt{505})} \sqrt{x} + \\ & \frac{1}{4} \sqrt{505 (10 + \sqrt{101})} \sqrt{x} + \frac{1}{2} \sqrt{101 (10 + \sqrt{101})} \sqrt{x} + \\ & \frac{5}{2} \sqrt{5 (10 + \sqrt{101})} \sqrt{x} + \frac{25}{4} \sqrt{10 + \sqrt{101}} \sqrt{x} = 224.36895935 \end{aligned}$$

Alternate form assuming x>0:

$$\begin{aligned} & \frac{1}{4} (2 + \sqrt{5}) \left(5 \sqrt{5 (10 + \sqrt{101})} + \sqrt{101 (10 + \sqrt{101})} + \right. \\ & \left. \sqrt{2100 + 202\sqrt{5} + 210\sqrt{101} + 20\sqrt{505}} \right) \sqrt{x} = 224.36895935 \end{aligned}$$

Solution:

$$x = 1.6180339887$$

1.6180339887 result that is equal to the value of the golden ratio

From:

ACTA ARITHMETICA LXXIII.1 (1995)

Ramanujan's class invariants and cubic continued fraction

By Bruce C. Berndt (Urbana, Ill.), Heng Huat Chan (Princeton, N.J.) and Liang-Cheng Zhang (Springfield, Mo.)

we have:

$$G(e^{-\pi\sqrt{58}}) = \frac{\sqrt{729 + 297\sqrt{6}} - \sqrt{727 + 297\sqrt{6}}}{(5 + \sqrt{29})\sqrt{11\sqrt{6} + 5\sqrt{29}}}$$

$$\left(\left((729+297\sqrt{6})^{1/2} \right) - \left((727+297\sqrt{6})^{1/2} \right) \right) * \frac{1}{\left((5+\sqrt{29}) * (11\sqrt{6}+5\sqrt{29})^{1/2} \right)}$$

Input:

$$\sqrt{729 + 297\sqrt{6}} - \sqrt{727 + 297\sqrt{6}} \times \frac{1}{(5 + \sqrt{29})\sqrt{11\sqrt{6} + 5\sqrt{29}}}$$

Result:

$$\sqrt{729 + 297\sqrt{6}} - \frac{\sqrt{\frac{727+297\sqrt{6}}{11\sqrt{6}+5\sqrt{29}}}}{5 + \sqrt{29}}$$

Decimal approximation:

37.66375478012757636101128839866973898374259238297109528655...

37.66375478...

Alternate forms:

$$\frac{1}{4} \left(4\sqrt{729 + 297\sqrt{6}} + (5 - \sqrt{29}) \sqrt{\frac{727 + 297\sqrt{6}}{11\sqrt{6} + 5\sqrt{29}}} \right)$$

$$3\sqrt{81 + 33\sqrt{6}} + \frac{1}{4} \left(-727 - 297\sqrt{6} + 5\sqrt{87(485 + 198\sqrt{6})} \right)$$

$\text{root of } 256x^8 + 372224x^7 + 134650240x^6 - 475330816x^5 - 196583295632x^4 + 99282661376x^3 + 626330118856x^2 - 159893138680x - 353967028175 \text{ near } x = 37.6638$
--

Minimal polynomial:

$$256x^8 + 372224x^7 + 134650240x^6 - 475330816x^5 - 196583295632x^4 + 99282661376x^3 + 626330118856x^2 - 159893138680x - 353967028175$$

From which:

$$4\left(\left(\left(\left(\left(\left(729+297\sqrt{6}\right)^{1/2}\right) - \left(\left(727+297\sqrt{6}\right)^{1/2}\right)\right) * \frac{1}{\left(\left(5+\sqrt{29}\right) * \left(11\sqrt{6}+5\sqrt{29}\right)^{1/2}\right)}\right)\right)\right) - 13$$

where 4 is a Lucas number and 13 is a Fibonacci number

Input:

$$4 \left(\sqrt{729 + 297\sqrt{6}} - \sqrt{727 + 297\sqrt{6}} \times \frac{1}{(5 + \sqrt{29})\sqrt{11\sqrt{6} + 5\sqrt{29}}} \right) - 13$$

Result:

$$4 \left(\sqrt{729 + 297\sqrt{6}} - \frac{\sqrt{\frac{727+297\sqrt{6}}{11\sqrt{6}+5\sqrt{29}}}}{5 + \sqrt{29}} \right) - 13$$

Decimal approximation:

137.6550191205103054440451535946789559349703695318843811462...

137.65501912... result practically equal to the golden angle value 137.5

Alternate forms:

$$4\sqrt{729 + 297\sqrt{6}} + (5 - \sqrt{29})\sqrt{\frac{727 + 297\sqrt{6}}{11\sqrt{6} + 5\sqrt{29}}} - 13$$

root of $x^8 + 5920x^7 + 8949628x^6 + 558351232x^5 - 182524553402x^4 - 9650410105952x^3 - 172785969398276x^2 - 1273933480098368x - 3275304790749119$ near $x = 137.655$

$$-13 + 12\sqrt{3(27 + 11\sqrt{6})} - \frac{4\sqrt{\frac{727+297\sqrt{6}}{11\sqrt{6}+5\sqrt{29}}}}{5 + \sqrt{29}}$$

Minimal polynomial:

$x^8 + 5920x^7 + 8949628x^6 + 558351232x^5 - 182524553402x^4 - 9650410105952x^3 - 172785969398276x^2 - 1273933480098368x - 3275304790749119$

$$4(((((((729+297\sqrt{6})^{1/2}) - ((727+297\sqrt{6})^{1/2})) * 1/(((5+\sqrt{29})*(11\sqrt{6}+5\sqrt{29})^{1/2})))))))-11$$

where 11 is a Lucas number

Input:

$$4 \left(\sqrt{729 + 297\sqrt{6}} - \sqrt{727 + 297\sqrt{6}} \times \frac{1}{(5 + \sqrt{29})\sqrt{11\sqrt{6} + 5\sqrt{29}}} \right) - 11$$

Result:

$$4 \left(\sqrt{729 + 297\sqrt{6}} - \frac{\sqrt{\frac{727+297\sqrt{6}}{11\sqrt{6}+5\sqrt{29}}}}{5 + \sqrt{29}} \right) - 11$$

Decimal approximation:

139.6550191205103054440451535946789559349703695318843811462...

139.6550191205... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternate forms:

$$4\sqrt{729 + 297\sqrt{6}} + (5 - \sqrt{29})\sqrt{\frac{727 + 297\sqrt{6}}{11\sqrt{6} + 5\sqrt{29}}} - 11$$

root of $x^8 + 5904x^7 + 8866860x^6 + 451452528x^5 - 187572744522x^4 - 8169308256528x^3 - 119306622208500x^2 - 692710785646704x - 1344316115373039$ near $x = 139.655$

$$-11 + 12\sqrt{3(27 + 11\sqrt{6})} - \frac{4\sqrt{\frac{727+297\sqrt{6}}{11\sqrt{6}+5\sqrt{29}}}}{5 + \sqrt{29}}$$

Minimal polynomial:

$$x^8 + 5904x^7 + 8866860x^6 + 451452528x^5 - 187572744522x^4 - 8169308256528x^3 - 119306622208500x^2 - 692710785646704x - 1344316115373039$$

$$4(((((((729+297\sqrt{6})^{1/2}) - ((727+297\sqrt{6})^{1/2}) * 1/((5+\sqrt{29})*(11\sqrt{6}+5\sqrt{29})^{1/2})))))))-18-7$$

Where 18 and 7 are Lucas numbers

Input:

$$4 \left(\sqrt{729 + 297\sqrt{6}} - \sqrt{727 + 297\sqrt{6}} \times \frac{1}{(5 + \sqrt{29}) \sqrt{11\sqrt{6} + 5\sqrt{29}}} \right) - 18 - 7$$

Result:

$$4 \left(\sqrt{729 + 297\sqrt{6}} - \frac{\sqrt{\frac{727+297\sqrt{6}}{11\sqrt{6}+5\sqrt{29}}}}{5 + \sqrt{29}} \right) - 25$$

Decimal approximation:

125.6550191205103054440451535946789559349703695318843811462...

125.6550191205... result very near to the Higgs boson mass 125.18 GeV

Alternate forms:

$$4\sqrt{729 + 297\sqrt{6}} + (5 - \sqrt{29}) \sqrt{\frac{727 + 297\sqrt{6}}{11\sqrt{6} + 5\sqrt{29}}} - 25$$

root of $x^8 + 6016x^7 + 9450940x^6 + 1220723296x^5 - 129332789882x^4 - 17293953317696x^3 - 665438927368004x^2 - 10780008019970720x - 63738552293650175$ near $x = 125.655$

$$-25 + 12\sqrt{3(27 + 11\sqrt{6})} - \frac{4\sqrt{\frac{727+297\sqrt{6}}{11\sqrt{6}+5\sqrt{29}}}}{5 + \sqrt{29}}$$

Minimal polynomial:

$$x^8 + 6016x^7 + 9450940x^6 + 1220723296x^5 - 129332789882x^4 - 17293953317696x^3 - 665438927368004x^2 - 10780008019970720x - 63738552293650175$$

$$-\frac{1}{\sqrt{2}} \left(\sqrt{1279 + 355\sqrt{13} + 6\sqrt{45466 - 12i} + 6\sqrt{45466 + 12i}} - \sqrt{1281 + 355\sqrt{13} + 6\sqrt{45466 - 12i} + 6\sqrt{45466 + 12i}} \right)$$

Minimal polynomial:

$$x^8 + 100x^7 - 120x^6 - 60x^5 + 94x^4 + 60x^3 - 120x^2 - 100x + 1$$

$$\left(\left(\left(\left(\frac{1}{2} \left(\left(1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right) \right) \right) \right) \right)^{0.5} + \left(\left(\left(\left(\frac{1}{2} \left(\left(1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right) \right) \right) \right) \right)^{0.5} \right)$$

Input:

$$\sqrt{\frac{1}{2} \left(1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right) + \sqrt{\frac{1}{2} \left(1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)}}$$

Decimal approximation:

101.1800534855957098546733135864456977523922232456688420658...

101.1800534855...

Alternate forms:

root of $x^8 - 100x^7 - 120x^6 + 60x^5 + 94x^4 - 60x^3 - 120x^2 + 100x + 1$
near $x = 101.18$

$$\frac{1}{\sqrt{2}} \left(\sqrt{1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} + \sqrt{1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}}} \right)$$

$$\frac{1}{\sqrt{2}} \left(\sqrt{1279 + 355\sqrt{13} + 6\sqrt{45466 - 12i} + 6\sqrt{45466 + 12i}} + \sqrt{1281 + 355\sqrt{13} + 6\sqrt{45466 - 12i} + 6\sqrt{45466 + 12i}} \right)$$

Minimal polynomial:

$$x^8 - 100x^7 - 120x^6 + 60x^5 + 94x^4 - 60x^3 - 120x^2 + 100x + 1$$

$$\left(\left(\left(\left(\frac{1}{2} \left(\left(1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right) \right) \right) \right) \right) \right)^{0.5} + \left(\left(\left(\left(\frac{1}{2} \left(\left(1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right) \right) \right) \right) \right) \right)^{0.5} + 0.0098833709$$

Input interpretation:

$$\sqrt{\frac{1}{2} \left(1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} + \sqrt{\frac{1}{2} \left(1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} + 0.0098833709$$

Result:

101.189936856...

101.189936856...

$$\left(\left(\left(\left(\frac{1}{2} \left(\left(1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right) \right) \right) \right) \right) \right)^{0.5} + \left(\left(\left(\left(\frac{1}{2} \left(\left(1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right) \right) \right) \right) \right) \right)^{0.5} + 0.0098833709 + 34 + 2 + \frac{2}{5}$$

where 34, 2 and 5 are Fibonacci numbers

Input interpretation:

$$\sqrt{\frac{1}{2} \left(1279 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} + \sqrt{\frac{1}{2} \left(1281 + 355\sqrt{13} + 12\sqrt{22733 + 6305\sqrt{13}} \right)} + 0.0098833709 + 34 + 2 + \frac{2}{5}$$

Result:

137.589936856...

137.58993685... result practically equal to the golden angle value 137.5

Series representations:

$$\begin{aligned} & \frac{27}{2} \left(\sqrt{\frac{1}{2} \left(1279 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right)} + \right. \\ & \quad \left. \sqrt{\frac{1}{2} \left(1281 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right)} + \right. \\ & \quad \left. 0.00988337 + 21 + \pi + e \right) + \frac{1}{3} = \\ & 9.54594 \left(29.7474 + 1.41421 e + 1.41421 \pi + \sqrt{\left(1279 + \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(355 \times 12^{-k} \sqrt{12} + \right. \right. \right. \\ & \quad \left. \left. \left. 12 \left(22732 + 6305 \sqrt{13} \right)^{-k} \sqrt{22732 + 6305 \sqrt{13}} \right) \right)} + \right. \\ & \quad \left. \sqrt{\left(1281 + \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(355 \times 12^{-k} \sqrt{12} + 12 \left(22732 + 6305 \sqrt{13} \right)^{-k} \right. \right. \right. \\ & \quad \left. \left. \left. \sqrt{22732 + 6305 \sqrt{13}} \right) \right)} \right) \end{aligned}$$

$$\begin{aligned} & \frac{27}{2} \left(\sqrt{\frac{1}{2} \left(1279 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right)} + \right. \\ & \quad \left. \sqrt{\frac{1}{2} \left(1281 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right)} + \right. \\ & \quad \left. 0.00988337 + 21 + \pi + e \right) + \frac{1}{3} = 9.54594 \\ & \left(29.7474 + 1.41421 e + 1.41421 \pi + \sqrt{\left(1279 + \sum_{k=0}^{\infty} \frac{1}{k!} \left(-\frac{1}{2}\right)_k \left(355 \left(-\frac{1}{12}\right)^k \sqrt{12} + \right. \right. \right. \right. \\ & \quad \left. \left. \left. 12 (-1)^k \left(22732 + 6305 \sqrt{13} \right)^{-k} \sqrt{22732 + 6305 \sqrt{13}} \right) \right)} + \right. \\ & \quad \left. \sqrt{\left(1281 + \sum_{k=0}^{\infty} \frac{1}{k!} \left(-\frac{1}{2}\right)_k \left(355 \left(-\frac{1}{12}\right)^k \sqrt{12} + 12 (-1)^k \right. \right. \right. \\ & \quad \left. \left. \left. \left(22732 + 6305 \sqrt{13} \right)^{-k} \sqrt{22732 + 6305 \sqrt{13}} \right) \right)} \right) \end{aligned}$$

$$\begin{aligned}
& \frac{27}{2} \left(\sqrt{\frac{1}{2} \left(1279 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right)} + \right. \\
& \quad \left. \sqrt{\frac{1}{2} \left(1281 + 355 \sqrt{13} + 12 \sqrt{22733 + 6305 \sqrt{13}} \right)} + \right. \\
& \quad \left. 0.00988337 + 21 + \pi + e \right) + \frac{1}{3} = \\
& 6.75 \left(42.0691 + 2e + 2\pi + 1.41421 \sqrt{\left(1279 + \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k \left(-\frac{1}{2}\right)_k \sqrt{z_0} \right.} \right. \\
& \quad \left. \left. \left(355 (13 - z_0)^k + 12 \left(22733 + 6305 \sqrt{13} - z_0 \right)^k \right) z_0^{-k} \right) + \right. \\
& \quad \left. 1.41421 \sqrt{\left(1281 + \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k \left(-\frac{1}{2}\right)_k \sqrt{z_0} \right.} \right. \\
& \quad \left. \left. \left(355 (13 - z_0)^k + 12 \left(22733 + 6305 \sqrt{13} - z_0 \right)^k \right) z_0^{-k} \right) \right) \\
& \text{for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

From:

University of Arkansas, Fayetteville - ScholarWorks@UARK
Theses and Dissertations / 5-2015

Logarithmic Spiral Arm Pitch Angle of Spiral Galaxies: Measurement and Relationship to Galactic Structure and Nuclear Supermassive Black Hole Mass
Benjamin Lee Davis

3.11.1 The Golden Spiral

The pitch angle for the Golden Spiral (P_φ) is determined by starting with the definition of a logarithmic spiral in polar coordinates

$$r = r_0 e^{b\theta}, \quad (3.16)$$

where r is the radius, θ is the central angle, r_0 is the initial radius when $\theta = 0^\circ$, and b is a growth factor such that

$$b = \tan(P). \quad (3.17)$$

The golden ratio (φ) of two quantities applies when the ratio of the sum of the quantities to the larger quantity (A) is equal to the ratio of the larger quantity to the smaller one (B), that is

$$\frac{A+B}{A} = \frac{A}{B} \equiv \varphi. \quad (3.18)$$

Solving algebraically, the golden ratio can be found via the only positive root of the quadratic equation with

$$\varphi = \frac{1+\sqrt{5}}{2} = 1.6180339887... \quad (3.19)$$

The Golden Spiral is a unique logarithmic spiral, defined such that its radius grows every quarter turn in either direction ($\pm\pi/2$) by a factor of φ . A simple solution of the pitch angle of the Golden Spiral can be yielded first by application of Equation (3.16)

$$\varphi = e^{b(\pm\pi/2)}, \quad (3.20)$$

rearranging and taking the natural logarithm

$$|b| = \frac{\ln \varphi}{\pm\pi/2} = 0.3063489625..., \quad (3.21)$$

and finally application of Equation (3.17) yields

$$|P_\varphi| = \arctan |b| \approx 17.03239113^\circ... \quad (3.22)$$

Within the errors associated with the M - P relation (Equation (3.6)), the associated mass of a SMBH residing in a spiral galaxy with pitch angle equal to that of the Golden Spiral and the

most probable pitch angle from the PDF in Figure 3.6 are equivalent with $\log(M/M_{\odot}) = 7.15 \pm 0.22$ and $\log(M/M_{\odot}) = 7.06 \pm 0.23$, respectively. Perhaps the most famous spiral galaxy, M51a or the "Whirlpool" galaxy, also exhibits spiral arms close to the Golden Spiral with a pitch angle of $P = 16.26^{\circ} \pm 2.36^{\circ}$ (Davis et al., 2012) and an implied SMBH mass of $\log(M/M_{\odot}) = 7.20 \pm 0.26$.

The Golden Spiral plays a significant role in both the history and lore of mathematics and art. It is closely approximated by the Fibonacci Spiral, which is not a true logarithmic spiral. Rather, it consists of a series of quarter-circle arcs whose radii are the consecutively increasing numbers of the Fibonacci Sequence. Both the Golden Ratio and Fibonacci Sequence are manifested in the geometry and growth rates of many structures in nature; both physical and biological. It is not surprising, therefore, that spiral galaxies should also have morphologies clustering about this aesthetically appealing case. Another situation similar in superficial appearance occurs in cyclogenesis in planetary atmospheres (e.g., hurricanes). This rate of radial growth is most familiar in the anatomical geometry of organisms. Well-known examples of roughly Golden Spirals are found in the horns of some animals (e.g., rams) and belonging to the shells of mollusks such as the nautilus, snail, and a rare squid which retains its shell, *Spirula spirula*. Of course, spiral density waves are not required to have pitch angles close to the Golden Spiral. Their pitch angle depends on the ratio of mass density in the disk (where the waves propagate) to the central mass. In the case of Saturn's rings, where this ratio is far smaller than it is in disk galaxies, pitch angles are measured in tenths of degrees. The fact that spiral arms in galaxies happen to cluster about the aesthetically appealing example of the Golden Spiral may help explain the enduring fascination that images of spiral galaxies have had on the public for decades.

3.11.2 The Milky Way

Our own Milky Way has $m = 4$ and $|P| = 22.5^\circ \pm 2.5^\circ$, as measured from neutral hydrogen observations (Levine et al., 2006). This implies a SMBH mass of $\log(M/M_\odot) = 6.82 \pm 0.30$ from the M - P relation, compared to a direct measurement mass estimate from stellar orbits around *Sgr A** (Gillessen et al., 2009) of $\log(M/M_\odot) = 6.63 \pm 0.04$. Although our Milky Way does not have a pitch angle close to the most probable pitch angle from our distribution, it is very similar to the mean pitch angle from Figure 3.6 ($\mu = 21.44^\circ$), with an associated SMBH mass of $\log(M/M_\odot) = 6.88 \pm 0.25$. However, the mean of the black hole mass distribution from Figure 3.7 is even closer with $\log(M/M_\odot) = 6.72$. Our Milky Way is somewhat atypical in that it has four spiral arms, which is only the third most probable harmonic mode for a galaxy (see Figure 3.5).

From:

rearranging and taking the natural logarithm

$$|b| = \frac{\ln \varphi}{\pm \pi/2} = 0.3063489625\dots, \quad (3.21)$$

and finally application of Equation (3.17) yields

$$|P_\varphi| = \arctan |b| \approx 17.03239113^\circ \dots \quad (3.22)$$

$$|b| = \frac{\ln \varphi}{\pm \pi/2} = 0.3063489625\dots,$$

$$|P_\varphi| = \arctan |b| \approx 17.03239113^\circ \dots$$

we obtain:

atan 0.3063489625

Input interpretation:

$\tan^{-1}(0.3063489625)$

$\tan^{-1}(x)$ is the inverse tangent function

Result:

0.2972713047...

(result in radians)

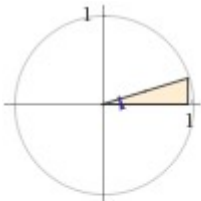
0.2972713047...

Conversion from radians to degrees:

17.03°

17.03°

Reference triangle for angle 0.2973 radians:



width	$\cos(0.297271) = 0.956139$
height	$\sin(0.297271) = 0.292912$

Alternative representations:

$$\tan^{-1}(0.306349) = \text{sc}^{-1}(0.306349 \mid 0)$$

$$\tan^{-1}(0.306349) = \cot^{-1}\left(\frac{1}{0.306349}\right)$$

$$\tan^{-1}(0.306349) = \tan^{-1}(1, 0.306349)$$

Series representations:

$$\tan^{-1}(0.306349) = \sum_{k=0}^{\infty} \frac{(-1)^k 0.306349^{1+2k}}{1+2k}$$

$$\tan^{-1}(0.306349) = \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{5}\right)^k 0.612698^{1+2k} F_{1+2k} \left(\frac{1}{1+\sqrt{1.07508}}\right)^{1+2k}}{1+2k}$$

$$\begin{aligned} \tan^{-1}(0.306349) = & \\ & \frac{1}{2} i \log(2) - \frac{1}{2} i \log(-i(-0.306349+i)) - \frac{1}{2} i \sum_{k=1}^{\infty} \frac{0.5^k (-i(-0.306349+i))^k}{k} \end{aligned}$$

Integral representations:

$$\tan^{-1}(0.306349) = 0.306349 \int_0^1 \frac{1}{1+0.0938497 t^2} dt$$

$$\begin{aligned} \tan^{-1}(0.306349) = & -\frac{0.0765872 i}{\pi^{3/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} e^{-0.0897033 s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)^2 ds \\ & \text{for } 0 < \gamma < \frac{1}{2} \end{aligned}$$

$$\tan^{-1}(0.306349) = \frac{0.0765872}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{2.36606 s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} ds \quad \text{for } 0 < \gamma < \frac{1}{2}$$

Continued fraction representations:

$$\tan^{-1}(0.306349) = \frac{0.306349}{1 + \mathbf{K}_{k=1}^{\infty} \frac{0.0938497 k^2}{1+2k}} = \frac{0.306349}{1 + \frac{0.0938497}{3 + \frac{0.375399}{5 + \frac{0.844647}{7 + \frac{1.50159}{9 + \dots}}}}}$$

$$\begin{aligned} \tan^{-1}(0.306349) = & \frac{0.306349}{1 + \mathbf{K}_{k=1}^{\infty} \frac{0.0938497(1-2k)^2}{1.09385+1.8123k}} = \\ & \frac{0.306349}{1 + \frac{0.0938497}{2.90615 + \frac{0.844647}{4.71845 + \frac{2.34624}{6.53075 + \frac{4.59863}{8.34305 + \dots}}}}} \end{aligned}$$

$$\begin{aligned} \tan^{-1}(0.306349) = & 0.306349 - \frac{0.0287508}{3 + \mathbf{K}_{k=1}^{\infty} \frac{0.0938497(1+(-1)^{1+k}k)^2}{3+2k}} = \\ & 0.306349 - \frac{0.0287508}{3 + \frac{0.844647}{5 + \frac{0.375399}{7 + \frac{2.34624}{9 + \frac{1.50159}{11 + \dots}}}}} \end{aligned}$$

$$\tan^{-1}(0.306349) = \frac{0.306349}{1.09385 + \mathop{\text{K}}_{k=1}^{\infty} \frac{0.187699 \left(1 - 2 \left\lfloor \frac{1+k}{2} \right\rfloor \left\lfloor \frac{1+k}{2} \right\rfloor\right)}{(1.04692 + 0.0469248 (-1)^k)(1+2k)}} =$$

$$\frac{0.306349}{1.09385 + \cfrac{0.187699}{3 - \cfrac{0.187699}{5.46925 - \cfrac{1.1262}{7 - \cfrac{1.1262}{9.84465 + \dots}}}}}$$

$\mathop{\text{K}}_{k=k_1}^{k_2} a_k / b_k$ is a continued fraction

$\lfloor x \rfloor$ is the floor function

From:

STRUCTURE AND DYNAMICS OF GALAXIES

1. Distribution of stars in the Milky Way Galaxy

Piet van der Kruit - Kapteyn Astronomical Institute, University of Groningen, the Netherlands - www.astro.rug.nl/~vdkruit
Beijing, September 2011

We have that (page 912):

Now apply this to our Galaxy, which has $h = 5 \text{ kpc}$, $V_m = 220 \text{ km s}^{-1}$, $\sigma_o = 400 M_\odot \text{ pc}^{-2}$.

$$\Gamma = 0.06 \quad R_m = 115 \text{ kpc} \quad R_{II} = 90 \text{ kpc} = 18h$$

$$M = 1.0 \times 10^{12} M_\odot \quad \rho_o = 2 \times 10^{-4} M_\odot \text{ pc}^{-3}$$

For other galaxies we find $\Gamma = 0.04 - 0.11$ and $\rho_o \approx 10^{-4} M_\odot \text{ pc}^{-3}$.

We note that $h = 5$ (page 220)

The tidal radius then is the solution for r of this equation:

$$r_{\text{tidal}} \sim R \left(\frac{m}{3M} \right)^{1/3}$$

For $M = 10^{12} M_{\odot}$, $m = 10^5 M_{\odot}$ and $R = 10 \text{ kpc}$ we get
 $r_{\text{tidal}} \approx 30 \text{ pc}$.

We have $R = 10$

Thence,

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$$Y = 0.615 \left\{ \frac{Q R V_{\text{rot}}}{h \langle V_{\text{R}}^2 \rangle^{1/2}} \right\}^{1/2} \exp \left(\frac{R}{2h} \right)$$

and this gives

$$\frac{Q V_{\text{rot}}}{\langle V_{\text{R}}^2 \rangle^{1/2}} \gtrsim 7.91$$

Comparing this to the equation for swing amplification we see that for spirals that are stable against global modes, swing amplification is possible for all modes with $m \geq 2$, at least at those radii where the rotation curve is flat.

From

$$Y = 0.615 \left\{ \frac{Q R V_{\text{rot}}}{h \langle V_{\text{R}}^2 \rangle^{1/2}} \right\}^{1/2} \exp \left(\frac{R}{2h} \right)$$

we obtain:

$$0.615 (7.91 \cdot 10 \cdot 1/5)^{1/2} * \exp(10/10)$$

Input:

$$0.615 \sqrt{7.91 \times 10 \times \frac{1}{5}} \exp\left(\frac{10}{10}\right)$$

Result:

6.64925...

6.64925...

From the previous expression

$$|b| = \frac{\ln \varphi}{\pm \pi/2} = 0.3063489625\dots,$$

$$|P_\varphi| = \arctan |b| \approx 17.03239113^\circ \dots$$

$$\tan^{-1}(0.3063489625)$$

0.2972713047...

(result in radians)

Conversion from radians to degrees:

17.03°

17.03°

We obtain:

$$17.03 + 0.615 (7.91 \cdot 10 \cdot 1/5)^{1/2} * \exp(10/10)$$

Input:

$$17.03 + 0.615 \sqrt{7.91 \times 10 \times \frac{1}{5}} \exp\left(\frac{10}{10}\right)$$

Result:

23.6793...

23.6793...

From which:

$$(13 \times 3) \times 1 / (((17.03 + 0.615 (7.91 \times 10 \times 1/5)^{1/2} * \exp(10/10)))) - (29) \times 1/10^3$$

where 13 and 3 are Fibonacci numbers, while 29 is a Lucas number

Input:

$$(13 \times 3) \times \frac{1}{17.03 + 0.615 \sqrt{7.91 \times 10 \times \frac{1}{5} \exp\left(\frac{10}{10}\right)}} - 29 \times \frac{1}{10^3}$$

Result:

1.618011437421453858747053512410135570414848366728117225025...

1.61801143742... result that is a very good approximation to the value of the golden ratio 1,618033988749...

From:

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It follows that the Galaxy is NOT maximum disk.

With $\kappa \sim 31 \text{ km sec}^{-1} \text{ kpc}^{-1}$ and $\sigma_{RR} \sim 40 \text{ km sec}^{-1}$ the Toomre parameter can be determined as

$$Q \sim 2.1.$$

For $Q = 2.1$, we obtain:

$$(7.91 - 2.1) / (((17.03 + 0.615 (7.91 \times 10 \times 1/5)^{1/2} * \exp(10/10))))$$

Input:

$$(7.91 - 2.1) \left(17.03 + 0.615 \sqrt{7.91 \times 10 \times \frac{1}{5} \exp\left(\frac{10}{10}\right)} \right)$$

Result:

137.576...

137.576.... result practically equal to the golden angle value 137.5

Definition:

The golden angle is the angle that divides a full angle in a golden ratio (but measured in the opposite direction so that it measures less than 180°), i.e.,

$$\begin{aligned}
 GA &= 2\pi(1 - 1/\phi) \\
 &= 2\pi/(1 + \phi) \\
 &= 2\pi(2 - \phi) \\
 &= \frac{2\pi}{\phi^2} \\
 &= \pi(3 - \sqrt{5}) \\
 &= 2.399963 \dots \\
 &= 137.507 \dots^\circ
 \end{aligned}$$

(OEIS A131988 and A096627; Livio 2002, p. 112).

It is implemented in the Wolfram Language as `GoldenAngle`.

van Iterson showed in 1907 that points separated by 137.5° on a tightly bound spiral tends to produce interlocked spirals winding in opposite directions, and that the number of spirals in these two families tend to be consecutive Fibonacci numbers.

Input:

$$\pi(3 - \sqrt{5})$$

Decimal approximation:

2.399963229728653322231555506633613853124999011058115042935...

2.39996322972...

Property:

$(3 - \sqrt{5})\pi$ is a transcendental number

Alternate forms:

$$\begin{aligned}
 &\pi(3 + \pi(-1)\sqrt{5}) \\
 &-(\sqrt{5} - 3)\pi
 \end{aligned}$$

Constant name:

golden angle

Series representations:

$$\pi(3 - \sqrt{5}) = 3\pi - \pi\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}$$

$$\pi(3 - \sqrt{5}) = 3\pi - \pi\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\pi(3 - \sqrt{5}) = 3\pi - \frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}$$

In degree, we have:

$$180(3 - \sqrt{5})$$

Input:

$$180(3 - \sqrt{5})$$

Decimal approximation:

137.5077640500378546463487396283702776206886952699253696312...

137.50776405... = Golden angle

Alternate forms:

$$540 + 180(-1)\sqrt{5}$$

$$540 - 180\sqrt{5}$$

$$-180(\sqrt{5} - 3)$$

Minimal polynomial:

$$x^2 - 1080x + 129600$$

Possible closed forms:

$$540 - 180\sqrt{5} \approx$$

137.507764050037854646348739628370277620688695269925369631238495

$$360\Phi^2 \approx$$

137.507764050037854646348739628370277620688695269925369631238495

Indeed:

Input:

$$360\left(\frac{1}{2}(\sqrt{5} - 1)\right)^2$$

Result:

$$90(\sqrt{5} - 1)^2$$

Decimal approximation:

137.5077640500378546463487396283702776206886952699253696312...

137.507764... = Golden angle

Alternate forms:

$$180(3 - \sqrt{5})$$

$$540 - 180\sqrt{5}$$

$$-180(\sqrt{5} - 3)$$

Minimal polynomial:

$$x^2 - 1080x + 129600$$

Possible closed forms:

$$540 - 180\sqrt{5} \approx$$

137.507764050037854646348739628370277620688695269925369631238495

$$360\Phi^2 \approx$$

137.507764050037854646348739628370277620688695269925369631238495

$$360 - 360\Phi \approx$$

137.507764050037854646348739628370277620688695269925369631238495

$$\frac{360\Phi}{\Phi + 1} \approx$$

137.507764050037854646348739628370277620688695269925369631238495

$$\sqrt{-19122 - 20125e + 27858\pi + 7527\log(2)} \approx 137.507764050037854651415$$

$$\pi \left[\text{root of } 921x^3 - 39605x^2 - 31851x + 39168 \text{ near } x = 43.7701 \right] \approx$$

137.5077640500378546456901

$$-24 - 26e + \frac{25e^2}{2} - \frac{87\sqrt{1+e}}{2} - 17\sqrt{1+e^2} + 50\pi + \pi^2 - 3\sqrt{1+\pi} + 34\sqrt{1+\pi^2} \approx$$

137.507764050037854646390588

$$\frac{-187 + 388\sqrt{\pi} - 3575\pi + 1947\pi^{3/2} + 1083\pi^2}{25\pi} \approx 137.50776405003785464651468$$

$$-\frac{104}{3} + \frac{274}{15\log(2)} - \frac{1591\log(2)}{30} + \frac{707}{30\log(3)} + \frac{440\log(3)}{3} \approx$$

137.5077640500378546458256

Φ is the golden ratio conjugate
 $\log(x)$ is the natural logarithm

We note that adding 2 (that is a prime number and a Fibonacci/Lucas number) we obtain:

$$(7.91-2.1)((17.03 + 0.615 (7.91 \times 10 \times \frac{1}{5})^{1/2} * \exp(10/10))) + 2$$

Input:

$$(7.91 - 2.1) \left(17.03 + 0.615 \sqrt{7.91 \times 10 \times \frac{1}{5}} \exp\left(\frac{10}{10}\right) \right) + 2$$

Result:

139.576...

139.576... result practically equal to the rest mass of Pion meson 139.57 MeV

And from:

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- ▶ Most (at least 50%) ellipticals have a small ψ_{int} ($\lesssim 10^\circ$), but some ($\approx 10\%$) rotate along their major axis.
- ▶ $\langle T \rangle \approx 0.3$ and T has a wide distribution with possibly as much as 40% of the galaxies prolate.
- ▶ The ratio c/a has a peak at about 0.6-0.7.

For the ratio $c/a = 0.604$, we obtain:

$$(7.91-2.1)((17.03 + 0.615 (7.91 \times 10 \times \frac{1}{5})^{1/2} * \exp(10/10))) - 13 + 0.604$$

where 13 is a Fibonacci number

Input:

$$(7.91 - 2.1) \left(17.03 + 0.615 \sqrt{7.91 \times 10 \times \frac{1}{5}} \exp\left(\frac{10}{10}\right) \right) - 13 + 0.604$$

Result:

125.180...

125.18... result equal to the Higgs boson mass 125.18 GeV

Performing the following calculations where 11 is a Lucas number, $Q = 2.2$ and 1.663, we obtain:

$$27 \times \frac{1}{2} \left(\left(\left(\left((7.91 - 2.1) \left(\left(17.03 + 0.615 (7.91 \times 10^{1/5})^{1/2} * \exp(10/10) \right) \right) - 11 + 1.663 \right) \right) \right) - 2.2$$

Input:

$$27 \times \frac{1}{2} \left((7.91 - 2.1) \left(17.03 + 0.615 \sqrt{7.91 \times 10 \times \frac{1}{5} \exp\left(\frac{10}{10}\right)} \right) - 11 + 1.663 \right) - 2.2$$

Result:

1729.032684262841434452206182292308681212175951461417728921...

1729.032684.....

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From:

TRANSACTIONS OF THE AMERICAN MATHEMATICAL SOCIETY Volume 349, **RAMANUJAN'S CLASS INVARIANTS, KRONECKER'S LIMIT FORMULA, AND MODULAR EQUATIONS** Number 6, June 1997, Pages 2125{2173 S 0002-9947(97)01738-8 - BRUCE C. BERNDT, HENG HUAT CHAN, AND LIANG - CHENG ZHANG

Let $Q = (G_{505}/G_{101/5})^3$. Then, by Lemma 3.4 and (4.35),

$$(4.36) \quad \begin{aligned} Q &= (P^{-1} - P) + \sqrt{(P^{-1} - P)^2 - 1} \\ &= (130\sqrt{5} + 29\sqrt{101}) + \sqrt{169440 + 7540\sqrt{505}}. \end{aligned}$$

Therefore, by (4.35) and (4.36),

$$\begin{aligned} G_{505} &= P^{-1/4} Q^{1/6} = (\sqrt{5} + 2)^{1/2} \left(\frac{\sqrt{5} + 1}{2} \right)^{1/4} (\sqrt{101} + 10)^{1/4} \\ &\quad \times \left((130\sqrt{5} + 29\sqrt{101}) + \sqrt{169440 + 7540\sqrt{505}} \right)^{1/6}. \end{aligned}$$

Thus, it remains to show that

$$(130\sqrt{5} + 29\sqrt{101}) + \sqrt{169440 + 7540\sqrt{505}} = \left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}} \right)^3,$$

which is straightforward. \square

Or, from the following Ramanujan expression:

$$\left(\left(\left(\left(\left(\sqrt{\frac{113+5\sqrt{505}}{8}}\right) + \sqrt{\frac{105+5\sqrt{505}}{8}}\right)\right)^3\right)\right)^{1/14}$$

Input:

$$\sqrt[14]{\left(\sqrt{\frac{1}{8}(113+5\sqrt{505})} + \sqrt{\frac{1}{8}(105+5\sqrt{505})}\right)^3}$$

Exact result:

$$\left(\frac{1}{2}\sqrt{\frac{1}{2}(105+5\sqrt{505})} + \frac{1}{2}\sqrt{\frac{1}{2}(113+5\sqrt{505})}\right)^{3/14}$$

Decimal approximation:

1.655784548804744724619349561761107639558068114480697960239...

1.6557845488...

Alternate forms:

$$\sqrt[28]{338881 + 15080\sqrt{505} + 4\sqrt{5(2871007052 + 127758137\sqrt{505})}}$$

$$\frac{(5\sqrt{5} + \sqrt{101} + \sqrt{105 - 40i} + \sqrt{105 + 40i})^{3/14}}{2^{3/7}}$$

$$\frac{\left(\sqrt{5(21 + \sqrt{505})} + \sqrt{113 + 5\sqrt{505}}\right)^{3/14}}{2^{9/28}}$$

Minimal polynomial:

$$x^{112} - 135524x^{84} + 400646x^{56} - 135524x^{28} + 1$$

We obtain, from the following previous formula

$$27 \times \frac{1}{2} \left((7.91 - 2.1) \left(17.03 + 0.615 \sqrt{7.91 \times 10 \times \frac{1}{5} \exp\left(\frac{10}{10}\right)} - 11 + 1.663 \right) - 2.2 \right)$$

the following elegant expression:

$$27 * 1/2 (((((7.91 - 2.1) (((17.03 + 0.615 (7.91 * 10 * 1/5)^{1/2} * \exp(10/10)))) - 11 + ((((((((((\sqrt{((113 + 5\sqrt{505}))/8)}) + \sqrt{((105 + 5\sqrt{505}))/8)})))))^3))))^{(1/14)))))) - 2.2$$

Input:

$$27 \times \frac{1}{2} \left((7.91 - 2.1) \left(17.03 + 0.615 \sqrt{7.91 \times 10 \times \frac{1}{5} \exp\left(\frac{10}{10}\right)} - 11 + \sqrt[14]{\left(\sqrt{\frac{1}{8} (113 + 5 \sqrt{505})} + \sqrt{\frac{1}{8} (105 + 5 \sqrt{505})} \right)^3} \right) - 2.2 \right)$$

Result:

1728.94...

1728.94... \approx 1729

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Series representations:

$$\begin{aligned} & \frac{27}{2} \left((7.91 - 2.1) \left(17.03 + 0.615 \sqrt{\frac{7.91 \times 10}{5}} \exp\left(\frac{10}{10}\right) \right) - 11 + \right. \\ & \quad \left. {}^{14}\sqrt{\left(\sqrt{\frac{1}{8} (113 + 5\sqrt{505})} + \sqrt{\frac{1}{8} (105 + 5\sqrt{505})} \right)^3} \right) - 2.2 = \\ & 1185.05 + 191.862 \exp(1) + 13.5 \left(\sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(8^k (97 + 5\sqrt{505})^{-k} \sqrt{\frac{97}{8} + \frac{5\sqrt{505}}{8}} + \right. \right. \\ & \quad \left. \left. \left(\frac{8}{5} \right)^k (21 + \sqrt{505})^{-k} \sqrt{\frac{5}{8} (21 + \sqrt{505})} \right) \right)^3 \wedge (1/14) \end{aligned}$$

$$\begin{aligned} & \frac{27}{2} \left((7.91 - 2.1) \left(17.03 + 0.615 \sqrt{\frac{7.91 \times 10}{5}} \exp\left(\frac{10}{10}\right) \right) - \right. \\ & \quad \left. 11 + {}^{14}\sqrt{\left(\sqrt{\frac{1}{8} (113 + 5\sqrt{505})} + \sqrt{\frac{1}{8} (105 + 5\sqrt{505})} \right)^3} \right) - \\ & 2.2 = 1185.05 + 191.862 \exp(1) + \\ & \frac{27}{2} \left(\sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-1 + \frac{1}{8} (105 + 5\sqrt{505}) \right)^{-k} \sqrt{-1 + \frac{1}{8} (105 + 5\sqrt{505})} + \right. \\ & \quad \left. \binom{\frac{1}{2}}{k} \left(-1 + \frac{1}{8} (113 + 5\sqrt{505}) \right)^{-k} \sqrt{-1 + \frac{1}{8} (113 + 5\sqrt{505})} \right)^3 \wedge (1/14) \end{aligned}$$

$$\begin{aligned} & \frac{27}{2} \left((7.91 - 2.1) \left(17.03 + 0.615 \sqrt{\frac{7.91 \times 10}{5}} \exp\left(\frac{10}{10}\right) \right) - 11 + \right. \\ & \quad \left. {}^{14}\sqrt{\left(\sqrt{\frac{1}{8} (113 + 5\sqrt{505})} + \sqrt{\frac{1}{8} (105 + 5\sqrt{505})} \right)^3} \right) - 2.2 = 1185.05 + \\ & 191.862 \exp(1) + 13.5 \left(\sum_{k=0}^{\infty} \frac{1}{k!} \left(-\frac{1}{2}\right)_k \left((-8)^k (97 + 5\sqrt{505})^{-k} \sqrt{\frac{97}{8} + \frac{5\sqrt{505}}{8}} + \right. \right. \\ & \quad \left. \left. \left(-\frac{8}{5} \right)^k (21 + \sqrt{505})^{-k} \sqrt{\frac{5}{8} (21 + \sqrt{505})} \right) \right)^3 \wedge (1/14) \end{aligned}$$

From the previous expression, we obtain also:

$$27 \cdot \frac{1}{2} \left(\left((7.91 - 2.1) \left(17.03 + \frac{1}{x} \sqrt{7.91 \cdot 10 \cdot \frac{1}{5}} \exp\left(\frac{10}{10}\right) \right) \right) - 11 + \left(\left(\left(\left(\left(\sqrt{\frac{1}{8} (113 + 5\sqrt{505})} \right) + \sqrt{\frac{1}{8} (105 + 5\sqrt{505})} \right)^3 \right) \right)^{\frac{1}{14}} \right) \right) - 2.2 = 1728.94$$

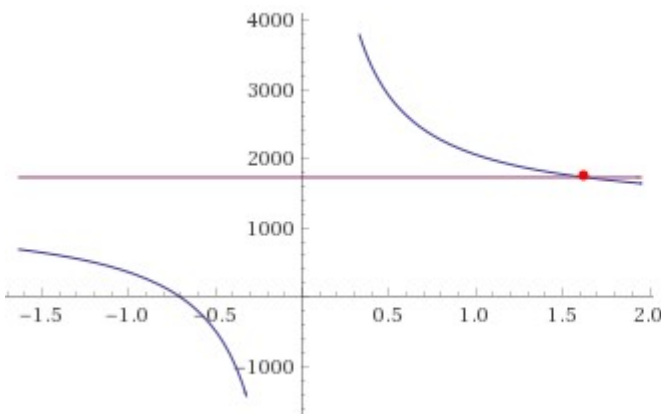
Input interpretation:

$$27 \times \frac{1}{2} \left((7.91 - 2.1) \left(17.03 + \frac{1}{x} \sqrt{7.91 \times 10 \times \frac{1}{5}} \exp\left(\frac{10}{10}\right) \right) - 11 + \sqrt[14]{\left(\sqrt{\frac{1}{8} (113 + 5\sqrt{505})} + \sqrt{\frac{1}{8} (105 + 5\sqrt{505})} \right)^3} \right) - 2.2 = 1728.94$$

Result:

$$\frac{27}{2} \left(5.81 \left(17.03 + \frac{10.8118}{x} \right) + \left(\frac{1}{2} \sqrt{\frac{1}{2} (105 + 5\sqrt{505})} + \frac{1}{2} \sqrt{\frac{1}{2} (113 + 5\sqrt{505})} \right)^{\frac{3}{14}} - 11 \right) - 2.2 = 1728.94$$

Plot:



$$\begin{aligned} & - \frac{27}{2} \left(5.81 \left(17.03 + \frac{10.8118}{x} \right) + \left(\frac{1}{2} \sqrt{\frac{1}{2} (105 + 5\sqrt{505})} + \frac{1}{2} \sqrt{\frac{1}{2} (113 + 5\sqrt{505})} \right)^{\frac{3}{14}} - 11 \right) - 2.2 \\ & - 1728.94 \end{aligned}$$

Alternate form assuming x is real:

$$\frac{1.626}{x} = 1$$

Alternate forms:

$$1207.4 + \frac{848.023}{x} = 1728.94$$

$$\frac{6.75 (178.874 x + 125.633)}{x} = 1728.94$$

Alternate form assuming x is positive:

$$x = 1.626 \text{ (for } x \neq 0)$$

Solution:

$$x \approx 1.626$$

1.626 result near to the golden ratio

Or, with the previous Ramanujan expression **(a)**:

$$27 \times \frac{1}{2} [(7.91 - 2.1) (((17.03 + 0.615 (7.91 \times 10^{1/5})^{1/2} * \exp(10/10)))) - 11 + ((2 + \sqrt{5}) ((((((1 + \sqrt{5})/2)(10 + \sqrt{101}))))))^{1/2} * ((((((5\sqrt{5} + \sqrt{101})/4) + \sqrt{((105 + \sqrt{505})/8)}))))^{1/11}] - 2.2$$

Input:

$$27 \times \frac{1}{2} \left((7.91 - 2.1) \left(17.03 + 0.615 \sqrt{7.91 \times 10 \times \frac{1}{5} \exp\left(\frac{10}{10}\right)} \right) - 11 + \left((2 + \sqrt{5}) \sqrt{\left(\frac{1}{2}(1 + \sqrt{5})\right)(10 + \sqrt{101})} \right) \left(\frac{1}{4} (5\sqrt{5} + \sqrt{101}) + \sqrt{\frac{1}{8}(105 + \sqrt{505})} \right) \right)^{(1/11)} - 2.2$$

Result:

$$1728.67\dots$$

$$1728.67\dots$$

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Series representations:

$$\begin{aligned} & \frac{27}{2} \left((7.91 - 2.1) \left(17.03 + 0.615 \sqrt{\frac{7.91 \times 10}{5}} \exp\left(\frac{10}{10}\right) \right) - \right. \\ & \quad 11 + \left((2 + \sqrt{5}) \sqrt{\frac{1}{2} (1 + \sqrt{5}) (10 + \sqrt{101})} \right. \\ & \quad \left. \left. \left(\frac{1}{4} (5\sqrt{5} + \sqrt{101}) + \sqrt{\frac{1}{8} (105 + \sqrt{505})} \right) \right)^{\wedge (1/11)} \right) - \\ & 2.2 = 6.75 \left[175.563 + 28.424 \exp(1) + 1.93797 \right. \\ & \quad \left. \left(\left(2 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right) \sqrt{\left(1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right) \left(10 + \sqrt{100} \sum_{k=0}^{\infty} 100^{-k} \binom{\frac{1}{2}}{k} \right)} \right. \right. \\ & \quad \left. \sum_{k=0}^{\infty} 4^{-1-k} \binom{\frac{1}{2}}{k} \left(5\sqrt{4} + 5^{-2k} \sqrt{100} + \right. \right. \\ & \quad \left. \left. 2^{2+5k} (97 + \sqrt{505})^{-k} \sqrt{\frac{1}{8} (97 + \sqrt{505})} \right) \right)^{\wedge (1/11)} \right] \end{aligned}$$

$$\begin{aligned} & \frac{27}{2} \left((7.91 - 2.1) \left(17.03 + 0.615 \sqrt{\frac{7.91 \times 10}{5}} \exp\left(\frac{10}{10}\right) \right) - \right. \\ & \quad 11 + \left((2 + \sqrt{5}) \sqrt{\frac{1}{2} (1 + \sqrt{5}) (10 + \sqrt{101})} \right. \\ & \quad \left. \left. \left(\frac{1}{4} (5\sqrt{5} + \sqrt{101}) + \sqrt{\frac{1}{8} (105 + \sqrt{505})} \right) \right)^{\wedge (1/11)} \right) - \\ & 2.2 = 6.75 \left[175.563 + 28.424 \exp(1) + 1.93797 \right. \\ & \quad \left. \left(\left(2 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right) \sqrt{\left(1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right) \left(10 + \sqrt{100} \sum_{k=0}^{\infty} 100^{-k} \binom{\frac{1}{2}}{k} \right)} \right. \right. \\ & \quad \left. \sum_{k=0}^{\infty} \left(\frac{1}{4} \left(5 \times 4^{-k} \binom{\frac{1}{2}}{k} \sqrt{4} + 100^{-k} \binom{\frac{1}{2}}{k} \sqrt{100} \right) + \right. \right. \\ & \quad \left. \left. \binom{\frac{1}{2}}{k} \left(-1 + \frac{1}{8} (105 + \sqrt{505}) \right)^{-k} \right. \right. \\ & \quad \left. \left. \sqrt{-1 + \frac{1}{8} (105 + \sqrt{505})} \right) \right)^{\wedge (1/11)} \right] \end{aligned}$$

$$\begin{aligned}
& \frac{27}{2} \left((7.91 - 2.1) \left(17.03 + 0.615 \sqrt{\frac{7.91 \times 10}{5}} \exp\left(\frac{10}{10}\right) \right) - \right. \\
& \quad \left. 11 + \left((2 + \sqrt{5}) \sqrt{\frac{1}{2} (1 + \sqrt{5}) (10 + \sqrt{101})} \right) \right. \\
& \quad \left. \left(\frac{1}{4} (5\sqrt{5} + \sqrt{101}) + \sqrt{\frac{1}{8} (105 + \sqrt{505})} \right) \right)^{(1/11)} - 2.2 = \\
& 6.75 \left(175.563 + 28.424 \exp(1) + 1.93797 \left(\left(2 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right. \right. \\
& \quad \left. \sqrt{\left(1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \left(10 + \sqrt{100} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{100}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)} \right. \\
& \quad \left. \sum_{k=0}^{\infty} \left[\frac{1}{4} \left(\frac{5 \left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k \sqrt{4}}{k!} + \frac{\left(-\frac{1}{100}\right)^k \left(-\frac{1}{2}\right)_k \sqrt{100}}{k!} \right) + \right. \\
& \quad \left. \left. \frac{\left(-1\right)^k \left(-\frac{1}{2}\right)_k \left(-1 + \frac{1}{8} (105 + \sqrt{505})\right)^{-k} \sqrt{-1 + \frac{1}{8} (105 + \sqrt{505})}}{k!} \right] \right) \right) \\
& \quad \left. \right)^{(1/11)}
\end{aligned}$$

From:

3.3.2 The M - P Relation

The pitch angle of a spiral galaxy has been shown to correlate well with the mass of the central SMBH residing in that galaxy (Berrier et al., 2013). Thus, using the linear best-fit M - P relation established by Berrier et al. (2013) for local spiral galaxies,

$$\log(M/M_{\odot}) = (b \pm \delta b) - (k \pm \delta k) |P|, \quad (3.6)$$

with $b = 8.21$, $\delta b = 0.16$, $k = 0.062$, and $\delta k = 0.009$, we can estimate the SMBH masses for a sample of local spiral galaxies merely by measuring their pitch angles using the method of Davis et al. (2012). The linear fit of Berrier et al. (2013) has a reduced $\chi^2 = 4.68$ with a scatter of 0.38 dex, which is lower than the intrinsic scatter ($\Delta = 0.53 \pm 0.10$ dex) of the M - σ relation for late-type galaxies (Gültekin et al., 2009) and the rms residual (0.90 dex) for the SMBH mass-spheroid stellar mass relation for Sérsic galaxies (Scott et al., 2013) in the $\log M$ direction. Ultimately, by determining the product of the mass distribution and the pitch angle distribution of a sample with a given volume, we may construct a BHMF for local late-type galaxies.

From (3.6), we obtain:

$$\log\left(\frac{0.89 \times 10^{10} \times 1.989 \times 10^{30}}{1.989 \times 10^{30}}\right)$$

Input interpretation:

$$\log_{10}\left(\frac{0.89 \times 10^{10} \times 1.989 \times 10^{30}}{1.989 \times 10^{30}}\right)$$

(note that log represent the log base 10)

Result:

9.9493900066...

9.9493900066...

$$(((\log(((0.89e+10*1.989e+30)/(1.989e+30)))))))/6$$

Input interpretation:

$$\frac{1}{6} \log_{10} \left(\frac{0.89 \times 10^{10} \times 1.989 \times 10^{30}}{1.989 \times 10^{30}} \right)$$

Result:

1.6582316678...

1.6582316678... result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

Or, with the natural logarithm and for $M_{\text{bulge}} = 2e+10$, we obtain:

$$\log(((2e+10*1.9891e+30)/(1.9891e+30)))$$

Input interpretation:

$$\log \left(\frac{2 \times 10^{10} \times 1.9891 \times 10^{30}}{1.9891 \times 10^{30}} \right)$$

$\log(x)$ is the natural logarithm

Result:

23.7190...

23.7190...

From which:

$$(((\log(((2e+10*1.9891e+30)/(1.9891e+30))))))^{1/(2\pi)}$$

Input interpretation:

$$\sqrt[2\pi]{\log \left(\frac{2 \times 10^{10} \times 1.9891 \times 10^{30}}{1.9891 \times 10^{30}} \right)}$$

$\log(x)$ is the natural logarithm

Result:

1.65521...

1.65521... result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

And:

$$(13*3)*1/(((\log(((2e+10*1.9891e+30)/(1.9891e+30))))))$$

where 13 and 3 are Fibonacci numbers

Input interpretation:

$$(13 \times 3) \times \frac{1}{\log\left(\frac{2 \times 10^{10} \times 1.9891 \times 10^{30}}{1.9891 \times 10^{30}}\right)}$$

log(x) is the natural logarithm

Result:

1.64425...

$$1.64425... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

$$((((13*3)*1/(((\log(((2e+10*1.9891e+30)/(1.9891e+30)))))))))-(21+5)*1/10^3$$

where 13, 3, 21 and 5 are Fibonacci numbers

Input interpretation:

$$(13 \times 3) \times \frac{1}{\log\left(\frac{2 \times 10^{10} \times 1.9891 \times 10^{30}}{1.9891 \times 10^{30}}\right)} - (21 + 5) \times \frac{1}{10^3}$$

log(x) is the natural logarithm

Result:

1.618251574974184817176682339560916791780304216194175350749...

1.618251574974... result that is a very good approximation to the value of the golden ratio 1,618033988749...

From:

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$$\frac{\langle V_z^2 \rangle^{1/2}}{\langle V_R^2 \rangle^{1/2}} = \sqrt{\frac{(7.2 \pm 2.5) z_e}{Q h}}$$

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In the solar neighborhood this axis ratio of the velocity ellipsoid is $\sim 0.5^7$ and for the Galaxy we have $z_e \sim 0.35$ kpc and $h \sim 4$ kpc, so that

$$Q \sim 2.5.$$

Taking all data and methods together it is found that this applies in all galaxies; disks are locally stable according to the Toomre criterion.

Numerical studies give such values for Q when disks are marginally stable.

From:

$$\frac{\langle V_z^2 \rangle^{1/2}}{\langle V_R^2 \rangle^{1/2}} = \sqrt{\frac{(7.2 \pm 2.5) z_e}{Q h}}$$

$\text{sqrt}(\frac{(7.2+2.5)}{2.5} * \frac{0.35}{4})$

Input:

$$\sqrt{\frac{7.2 + 2.5}{2.5} \times \frac{0.35}{4}}$$

Result:

0.582666285278288604704968859225621389873516975743022834999...

0.5826662852782886.... result very near to the value of the following 5th order Ramanujan mock theta function:

Mock \mathfrak{J} -functions (of 5th order).

$$\begin{aligned}
 f(q) &= 1 + \frac{q}{1+q} + \frac{q^4}{(1+q)(1+q^3)} + \dots, \\
 \phi(q) &= 1 + q(1+q) + q^4(1+q)(1+q^3) + q^9(1+q)(1+q^3)(1+q^5) + \dots, \\
 \psi(q) &= q + q^3(1+q) + q^6(1+q)(1+q^2) + q^{10}(1+q)(1+q^3)(1+q^5) + \dots, \\
 \chi(q) &= 1 + \frac{q}{1-q^2} + \frac{q^2}{(1-q^3)(1-q^4)} + \frac{q^3}{(1-q^4)(1-q^5)(1-q^6)} + \dots \\
 &= 1 + \frac{q}{1-q} + \frac{q^3}{(1-q^2)(1-q^3)} + \frac{q^6}{(1-q^3)(1-q^4)(1-q^5)} + \dots,
 \end{aligned}$$

Input interpretation:

$$\begin{aligned}
 &0.449329 + 0.449329^3 (1 + 0.449329) + 0.449329^6 (1 + 0.449329)(1 + 0.449329^2) + \\
 &0.449329^{10} (1 + 0.449329)(1 + 0.449329^2)(1 + 0.449329^3)
 \end{aligned}$$

Result:

0.595782322619129485824526179594205622329408540297077428912...

$\psi(q) = 0.5957823226...$

Or:

$\text{sqrt}(\frac{7.2-2.5}{2.5} * \frac{0.35}{4})$

Input:

$$\sqrt{\frac{7.2 - 2.5}{2.5} \times \frac{0.35}{4}}$$

Result:

0.405585995813464940201244856917933185838172616981467425162...

0.40558599581346....

From which, multiplying by 4, that is a Lucas number:

$$4 * \text{sqrt}(\left(\frac{7.2-2.5}{2.5} * \left(\frac{0.35}{4}\right)\right))$$

Input:

$$4 \sqrt{\frac{7.2 - 2.5}{2.5} \times \frac{0.35}{4}}$$

Result:

1.622343983253859760804979427671732743352690467925869700649...

1.62234398325....

$$4 * \text{sqrt}(\left(\frac{7.2-2.5}{2.5} * \left(\frac{0.35}{4}\right)\right)) - 4 * 1/10^3$$

Input:

$$4 \sqrt{\frac{7.2 - 2.5}{2.5} \times \frac{0.35}{4}} - 4 \times \frac{1}{10^3}$$

Result:

1.618343983253859760804979427671732743352690467925869700649...

1.61834398325.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

We have also:

$$(89+5) / \left(10^2 \text{sqrt}(\left(\frac{7.2+2.5}{2.5} * \left(\frac{0.35}{4}\right)\right))\right)$$

where 89 and 5 are Fibonacci numbers

Input:

$$\frac{89 + 5}{10^2 \sqrt{\frac{7.2+2.5}{2.5} \times \frac{0.35}{4}}}$$

Result:

1.613273367191726917297999197855917839414155985856970441531...
1.6132733671917.....

$$(89+5) / (((10^2 \text{ sqrt}(\frac{((7.2+2.5)/2.5 * (0.35/4))))) + 5 * 1/10^3$$

Input:

$$\frac{89+5}{10^2 \sqrt{\frac{7.2+2.5}{2.5} \times \frac{0.35}{4}}} + 5 \times \frac{1}{10^3}$$

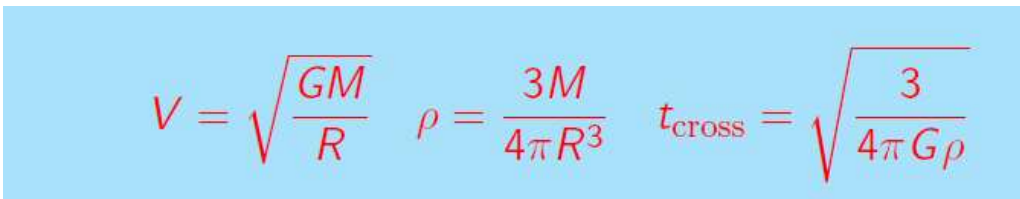
Result:

1.618273367191726917297999197855917839414155985856970441531...

1.61827336719.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

From:

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$$V = \sqrt{\frac{GM}{R}} \quad \rho = \frac{3M}{4\pi R^3} \quad t_{\text{cross}} = \sqrt{\frac{3}{4\pi G \rho}}$$

For M = 13.12806e+39, R = 1.94973e+13 , G = 6.67408e-11 ,

(M and R are the mass and the radius of SMBH 87) we obtain:

$$\text{sqrt}(\frac{((6.67408e-11 * 13.12806e+39)/(1.94973e+13))$$

Input interpretation:

$$\sqrt{\frac{6.67408 \times 10^{-11} \times 13.12806 \times 10^{39}}{1.94973 \times 10^{13}}}$$

Result:

$$2.11987... \times 10^8$$

$$2.11987... * 10^8 = V$$

$$(3 * 13.12806e+39) / (4\pi * (1.94973e+13)^3)$$

Input interpretation:

$$\frac{3 \times 13.12806 \times 10^{39}}{4 \pi (1.94973 \times 10^{13})^3}$$

Result:

$$0.422852128743236200554361005727927805152243006293719417118...$$

$$0.42285212874... = \rho$$

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$$\frac{d}{dR} \left(R^2 \frac{d\Phi}{dR} \right) = -4\pi G \rho_0 R^2 \left[e^{\Phi(R)/\langle V^2 \rangle} \operatorname{erf} \left(\sqrt{\frac{\Phi}{\langle V^2 \rangle}} \right) - \sqrt{\frac{4\Phi}{\pi \langle V^2 \rangle}} \left(1 + \frac{2\Phi}{3 \langle V^2 \rangle} \right) \right]$$

Where we have placed $\Phi = 224.368$ and the coordinate $\phi = 1.61803398$

$$\begin{aligned} & -4\pi * 6.67e-11 * 0.4228 * (1.94973e+13)^2 \\ & (((\exp(((224.368 * 1.94973e+13) / (2.11987e+8)^2))) \\ & \operatorname{erf}(\operatorname{sqrt}((224.368) / (2.11987e+8)^2)) - \\ & \operatorname{sqrt}(((4 * 1.61803398) / (\pi * (2.11987e+8)^2))))(1 + (2 * 224.368) / (3 * (2.11987e+8)^2)))))) \end{aligned}$$

Input interpretation:

$$\begin{aligned} & -4\pi \times 6.67 \times 10^{-11} \times 0.4228 (1.94973 \times 10^{13})^2 \\ & \left(\exp \left(\frac{224.368 \times 1.94973 \times 10^{13}}{(2.11987 \times 10^8)^2} \right) \operatorname{erf} \left(\sqrt{\frac{224.368}{(2.11987 \times 10^8)^2}} \right) - \right. \\ & \left. \sqrt{\frac{4 \times 1.61803398}{\pi (2.11987 \times 10^8)^2}} \left(1 + \frac{2 \times 224.368}{3 (2.11987 \times 10^8)^2} \right) \right) \end{aligned}$$

erf(x) is the error function

Result:

$$-1.09271... \times 10^{10}$$

$$-1.09271... * 10^{10}$$

From which:

$$8^2 [4\pi * 6.67e-11 * 0.4228 * (1.9497e+13)^2$$

$$(((\exp(((224.368 * 1.9497e+13)/(2.1198e+8)^2)) \operatorname{erf}(\sqrt{(224.368)/(2.1198e+8)^2}) -$$

$$\sqrt{((4 * 1.618)/(\pi * (2.1198e+8)^2))) (1 + (2 * 224.368)/(3 * (2.1198e+8)^2)))]^{1/7 - 10}$$

where 8 is a Fibonacci number and 10 = 7+3 (Lucas numbers)

Input interpretation:

$$8^2 \left(4\pi \times 6.67 \times 10^{-11} \times 0.4228 (1.9497 \times 10^{13})^2 \right.$$

$$\left. \left(\exp \left(\frac{224.368 \times 1.9497 \times 10^{13}}{(2.1198 \times 10^8)^2} \right) \operatorname{erf} \left(\sqrt{\frac{224.368}{(2.1198 \times 10^8)^2}} \right) - \right.$$

$$\left. \left. \sqrt{\frac{4 \times 1.618}{\pi (2.1198 \times 10^8)^2}} \left(1 + \frac{2 \times 224.368}{3 (2.1198 \times 10^8)^2} \right) \right) \right)^{(1/7) - 10}$$

erf(x) is the error function

Result:

$$1728.81...$$

$$1728.81... \approx 1729$$

This result is very near to the mass of candidate glueball **f₀(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

For

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$$W_{HI} = 1.663p^{-1/2}z_0 \text{ for } p \gg 1$$

$$W_{HI} = 1.763p^{-1/2}z_0 \text{ for } p = 1$$

$$5[4\pi \times 6.67 \times 10^{-11} \times 0.4228 \times (1.9497 \times 10^{13})^2 \left(\frac{\exp\left(\frac{224.368 \times 1.9497 \times 10^{13}}{(2.1198 \times 10^8)^2}\right) \operatorname{erf}\left(\sqrt{\frac{224.368}{(2.1198 \times 10^8)^2}}\right) - \sqrt{\frac{4 \times 1.618}{\pi (2.1198 \times 10^8)^2}} \left(1 + \frac{2 \times 224.368}{3(2.1198 \times 10^8)^2}\right) \right)]^{1/7+1.663}$$

where 5 is a Fibonacci number

Input interpretation:

$$5 \left(4\pi \times 6.67 \times 10^{-11} \times 0.4228 (1.9497 \times 10^{13})^2 \left(\exp\left(\frac{224.368 \times 1.9497 \times 10^{13}}{(2.1198 \times 10^8)^2}\right) \operatorname{erf}\left(\sqrt{\frac{224.368}{(2.1198 \times 10^8)^2}}\right) - \sqrt{\frac{4 \times 1.618}{\pi (2.1198 \times 10^8)^2}} \left(1 + \frac{2 \times 224.368}{3(2.1198 \times 10^8)^2}\right) \right) \right)^{(1/7) + 1.663}$$

erf(x) is the error function

Result:

137.508...

137.508.... result practically equal to the golden angle 137.507764..

And:

$$5[4\pi \times 6.67 \times 10^{-11} \times 0.4228 \times (1.9497 \times 10^{13})^2 \left(\frac{\exp\left(\frac{224.368 \times 1.9497 \times 10^{13}}{(2.1198 \times 10^8)^2}\right) \operatorname{erf}\left(\sqrt{\frac{224.368}{(2.1198 \times 10^8)^2}}\right) - \sqrt{\frac{4 \times 1.618}{\pi (2.1198 \times 10^8)^2}} \left(1 + \frac{2 \times 224.368}{3(2.1198 \times 10^8)^2}\right) \right)]^{1/7+1.656+2}$$

Where 1.656 is practically equal (excess approximation) to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578... and 2 is a Prime and Fibonacci/Lucas number

Input interpretation:

$$5 \left[4\pi \times 6.67 \times 10^{-11} \times 0.4228 (1.9497 \times 10^{13})^2 \right. \\ \left. \left(\exp \left(\frac{224.368 \times 1.9497 \times 10^{13}}{(2.1198 \times 10^8)^2} \right) \operatorname{erf} \left(\sqrt{\frac{224.368}{(2.1198 \times 10^8)^2}} \right) - \right. \right. \\ \left. \left. \sqrt{\frac{4 \times 1.618}{\pi (2.1198 \times 10^8)^2}} \left(1 + \frac{2 \times 224.368}{3 (2.1198 \times 10^8)^2} \right) \right) \right]^{\wedge (1/7) + 1.656 + 2}$$

erf(x) is the error function

Result:

139.501...

139.501... result practically equal to the rest mass of Pion meson 139.57 MeV

From

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$$W_{HI} = 1.7 \langle V_z^2 \rangle_{HI}^{1/2} \left[\frac{\pi G (M/L) \mu_0}{z_0} \right]^{-1/2} \exp(R/2h)$$

$M/L = 3, z_0 = 1, R = 10, h = 5 \quad \mu_0 = 21.67 \pm 0.30 \text{ B}$

$V = 2.11987e+8$

$1.7 * 211987000 * ((([\text{Pi} * 6.67408e-11 * 3 * 21.67]^{-1/2}))) * \exp(10/(2*5))$

Input interpretation:

$$1.7 \times 211987000 (\pi \times 6.67408 \times 10^{-11} \times 3 \times 21.67)^{-1/2} \exp\left(\frac{10}{2 \times 5}\right)$$

Result:

$8.39058... \times 10^{12}$

$8.39058... * 10^{12} = W_{HI}$

From which, we obtain:

$$\left(\left(\left(1.7 * 211987000 * \left(\left(\left[\text{Pi} * 6.67408\text{e-}11 * 3 * 21.67\right]^{-1/2}\right)\right) * \exp\left(\frac{10}{2 * 5}\right)\right)\right)\right)^{1/62 + 2 * 1/10^3}$$

where 2 is a prime and Fibonacci/Lucas number, while 62 = 55 (Fibonacci number) + 7 (Lucas number)

Input interpretation:

$$\sqrt[62]{1.7 \times 211987000 (\pi \times 6.67408 \times 10^{-11} \times 3 \times 21.67)^{-1/2} \exp\left(\frac{10}{2 \times 5}\right) + 2 \times \frac{1}{10^3}}$$

Result:

1.618025675520310429672750171670697255691084189956597987666...

1.61802567552031... result that is a very good approximation to the value of the golden ratio 1,618033988749...

And:

$$\left(\left(\left(1.7 * 211987000 * \left(\left(\left[\text{Pi} * 6.67408\text{e-}11 * 3 * 21.67\right]^{-1/2}\right)\right) * \exp\left(\frac{10}{2 * 5}\right)\right)\right)\right)^{1/6 - 5}$$

where 5 is a Fibonacci number

Input interpretation:

$$\sqrt[6]{1.7 \times 211987000 (\pi \times 6.67408 \times 10^{-11} \times 3 \times 21.67)^{-1/2} \exp\left(\frac{10}{2 \times 5}\right) - 5}$$

Result:

137.549...

137.549....result practically equal to the golden angle 137.5

Observations

From:

https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that $p(9) = 30$, $p(9 + 5) = 135$, $p(9 + 10) = 490$, $p(9 + 15) = 1,575$ and so on are all divisible by 5. Note that here the n 's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of $p(n)$ that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n 's separated by $5^3 = 125$ units, saying that the corresponding $p(n)$'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, **Yukawa's interaction** or **Yukawa coupling**, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by **pions** (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the **Higgs field**.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of **Higgs boson: 125 GeV** for $T = 0$ and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of **Pion meson** 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and $4096 = 64^2$

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π , ϕ , $1/\phi$, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

*In mathematics, the **Fibonacci numbers**, commonly denoted F_n , form a sequence, called the **Fibonacci sequence**, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses*

the n th Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

*The **Lucas numbers** or **Lucas series** are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.*

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.^[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

*A **Lucas prime** is a Lucas number that is prime. The first few Lucas primes are:*

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

*In geometry, a **golden spiral** is a logarithmic spiral whose growth factor is ϕ , the golden ratio.^[1] That is, a golden spiral gets wider (or further from its origin) by a factor of ϕ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies^[3] - golden spirals are one special case of these logarithmic spirals*

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

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Ramanujan's class invariants and cubic continued fraction

*By Bruce C. Berndt (Urbana, Ill.), Heng Huat Chan (Princeton, N.J.)
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MODULAR EQUATIONS IN THE SPIRIT OF RAMANUJAN

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**RAMANUJAN'S CLASS INVARIANTS, KRONECKER'S LIMIT FORMULA,
AND MODULAR EQUATIONS** - *BRUCE C. BERNDT, HENG HUAT CHAN,
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