

On some Ramanujan's functions: mathematical connections with various equations concerning some sectors of Particle Physics and Black Hole Physics. II

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Abstract

The aim of this paper is to show the mathematical connections between some Ramanujan's functions and some expression of various topics of Particle Physics and Black Hole Physics

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*An equation means nothing
to me unless it expresses a
thought of God.*

Srinivasa Ramanujan (1887-1920)

(N.O.A – Pics. from the web)

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$$P = P(q) = -24E_2 = 1 - 24 \sum_{j=1}^{\infty} \frac{j q^j}{1 - q^j}, \quad (0.71)$$

$$Q = Q(q) = 240E_4 = 1 + 240 \sum_{j=1}^{\infty} \frac{j^3 q^j}{1 - q^j}, \quad (0.72)$$

$$R = R(q) = -504E_6 = 1 - 504 \sum_{j=1}^{\infty} \frac{j^5 q^j}{1 - q^j} \quad (0.73)$$

and

$$S = S(q) = 480E_8 = 1 + 480 \sum_{j=1}^{\infty} \frac{j^7 q^j}{1 - q^j}. \quad (0.74)$$

The functions P , Q , and R are called Ramanujan's Eisenstein series. Occasionally, we will write

$$P_n = P(q^n) \quad \text{and} \quad Q_n = Q(q^n), \quad (0.75)$$

where n is a positive integer.

Theorem 0.21 (Ramanujan's differential equations). Let P , Q , and R be defined by (0.71)–(0.73). Then

$$q \frac{dP}{dq} = \frac{P^2 - Q}{12}, \quad (0.78)$$

$$q \frac{dQ}{dq} = \frac{PQ - R}{3}, \quad (0.79)$$

and

$$q \frac{dR}{dq} = \frac{PR - Q^2}{2}. \quad (0.80)$$

$$1 - 24 \sum_{j=1}^{\infty} \frac{j q^j}{1 - q^j}, \quad 1 + 240 \sum_{j=1}^{\infty} \frac{j^3 q^j}{1 - q^j}, \quad 1 - 504 \sum_{j=1}^{\infty} \frac{j^5 q^j}{1 - q^j}$$

where, as usual, $q = e^{2\pi i \tau}$ and $\text{Im } \tau > 0$.

$1 - 24 \sum_{j=1}^8 \frac{j e^{(2\pi i)^j}}{1 - e^{(2\pi i)^j}}, j = 1 \text{ to } 8$

Input interpretation:

$$1 - 24 \sum_{j=1}^8 \frac{j e^{(2\pi)^j}}{1 - e^{(2\pi)^j}}$$

Result:

$$1 - 24 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} + \frac{2 e^{4\pi^2}}{1 - e^{4\pi^2}} + \frac{3 e^{8\pi^3}}{1 - e^{8\pi^3}} + \frac{4 e^{16\pi^4}}{1 - e^{16\pi^4}} + \frac{5 e^{32\pi^5}}{1 - e^{32\pi^5}} + \frac{6 e^{64\pi^6}}{1 - e^{64\pi^6}} + \frac{7 e^{128\pi^7}}{1 - e^{128\pi^7}} + \frac{8 e^{256\pi^8}}{1 - e^{256\pi^8}} \right) \approx 865.045$$

P = 865.045

$1 + 240 \sum_{j=1}^8 \frac{j^3 e^{(2\pi i)^j}}{1 - e^{(2\pi i)^j}}, j = 1 \text{ to } 8$

Input interpretation:

$$1 + 240 \sum_{j=1}^8 \frac{j^3 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}}$$

Result:

$$1 + 240 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} + \frac{8 e^{4\pi^2}}{1 - e^{4\pi^2}} + \frac{27 e^{8\pi^3}}{1 - e^{8\pi^3}} + \frac{64 e^{16\pi^4}}{1 - e^{16\pi^4}} + \frac{125 e^{32\pi^5}}{1 - e^{32\pi^5}} + \frac{216 e^{64\pi^6}}{1 - e^{64\pi^6}} + \frac{343 e^{128\pi^7}}{1 - e^{128\pi^7}} + \frac{512 e^{256\pi^8}}{1 - e^{256\pi^8}} \right) \approx -311\,039.$$

$$Q = -311039$$

Alternate forms:

$$1 - \frac{240 e^{2\pi}}{e^{2\pi} - 1} - \frac{1920 e^{4\pi^2}}{e^{4\pi^2} - 1} - \frac{6480 e^{8\pi^3}}{e^{8\pi^3} - 1} - \frac{15\,360 e^{16\pi^4}}{e^{16\pi^4} - 1} - \frac{30\,000 e^{32\pi^5}}{e^{32\pi^5} - 1} - \frac{51\,840 e^{64\pi^6}}{e^{64\pi^6} - 1} - \frac{82\,320 e^{128\pi^7}}{e^{128\pi^7} - 1} - \frac{122\,880 e^{256\pi^8}}{e^{256\pi^8} - 1}$$

$$1 + 240 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} - \frac{8 e^{4\pi^2}}{e^{4\pi^2} - 1} - \frac{27 e^{8\pi^3}}{e^{8\pi^3} - 1} - \frac{64 e^{16\pi^4}}{e^{16\pi^4} - 1} - \frac{125 e^{32\pi^5}}{e^{32\pi^5} - 1} - \frac{216 e^{64\pi^6}}{e^{64\pi^6} - 1} - \frac{343 e^{128\pi^7}}{e^{128\pi^7} - 1} - \frac{512 e^{256\pi^8}}{e^{256\pi^8} - 1} \right)$$

$$-122\,879 - \frac{240 e^{2\pi}}{(e^\pi - 1)(1 + e^\pi)} - \frac{1920 e^{4\pi^2}}{(e^{\pi^2} - 1)(1 + e^{\pi^2})(1 + e^{2\pi^2})} - \frac{6480 e^{8\pi^3}}{(e^{\pi^3} - 1)(1 + e^{\pi^3})(1 + e^{2\pi^3})(1 + e^{4\pi^3})} - \frac{15\,360 e^{16\pi^4}}{(e^{\pi^4} - 1)(1 + e^{\pi^4})(1 + e^{2\pi^4})(1 + e^{4\pi^4})(1 + e^{8\pi^4})} - \frac{30\,000 e^{32\pi^5}}{(e^{\pi^5} - 1)(1 + e^{\pi^5})(1 + e^{2\pi^5})(1 + e^{4\pi^5})(1 + e^{8\pi^5})(1 + e^{16\pi^5})} - \frac{51\,840 e^{64\pi^6}}{(e^{\pi^6} - 1)(1 + e^{\pi^6})(1 + e^{2\pi^6})(1 + e^{4\pi^6})(1 + e^{8\pi^6})(1 + e^{16\pi^6})(1 + e^{32\pi^6})} - \frac{82\,320 e^{128\pi^7}}{(e^{\pi^7} - 1)(1 + e^{\pi^7})(1 + e^{2\pi^7})(1 + e^{4\pi^7})(1 + e^{8\pi^7})(1 + e^{16\pi^7})(1 + e^{32\pi^7})(1 + e^{64\pi^7})} - \frac{480}{7680} + \frac{480}{15\,360} + \frac{960}{30\,720} + \frac{1920}{61\,440} + \frac{3840}{122\,880} + \frac{7680}{245\,760} + \frac{15\,360}{491\,520} + \frac{30\,720}{983\,040} + \frac{61\,440}{1\,966\,080} + \frac{122\,880}{3\,932\,160}$$

1-504 sum (j^5*e^((2Pi)^j))/(1-e^((2Pi)^j)), j = 1 to 8

Input interpretation:

$$1 - 504 \sum_{j=1}^8 \frac{j^5 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}}$$

Result:

$$1 - 504 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} + \frac{32 e^{4\pi^2}}{1 - e^{4\pi^2}} + \frac{243 e^{8\pi^3}}{1 - e^{8\pi^3}} + \frac{1024 e^{16\pi^4}}{1 - e^{16\pi^4}} + \frac{3125 e^{32\pi^5}}{1 - e^{32\pi^5}} + \frac{7776 e^{64\pi^6}}{1 - e^{64\pi^6}} + \frac{16807 e^{128\pi^7}}{1 - e^{128\pi^7}} + \frac{32768 e^{256\pi^8}}{1 - e^{256\pi^8}} \right) \approx 3.11351 \times 10^7$$

R = 31135100

Alternate forms:

$$1 + \frac{504 e^{2\pi}}{e^{2\pi} - 1} + \frac{16128 e^{4\pi^2}}{e^{4\pi^2} - 1} + \frac{122472 e^{8\pi^3}}{e^{8\pi^3} - 1} + \frac{516096 e^{16\pi^4}}{e^{16\pi^4} - 1} + \frac{1575000 e^{32\pi^5}}{e^{32\pi^5} - 1} + \frac{3919104 e^{64\pi^6}}{e^{64\pi^6} - 1} + \frac{8470728 e^{128\pi^7}}{e^{128\pi^7} - 1} + \frac{16515072 e^{256\pi^8}}{e^{256\pi^8} - 1}$$

$$1 - 504 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} - \frac{32 e^{4\pi^2}}{e^{4\pi^2} - 1} - \frac{243 e^{8\pi^3}}{e^{8\pi^3} - 1} - \frac{1024 e^{16\pi^4}}{e^{16\pi^4} - 1} - \frac{3125 e^{32\pi^5}}{e^{32\pi^5} - 1} - \frac{7776 e^{64\pi^6}}{e^{64\pi^6} - 1} - \frac{16807 e^{128\pi^7}}{e^{128\pi^7} - 1} - \frac{32768 e^{256\pi^8}}{e^{256\pi^8} - 1} \right)$$

$$\begin{aligned}
& 16515073 + \frac{504 e^{2\pi}}{(e^\pi - 1)(1 + e^\pi)} + \\
& \frac{16128 e^{4\pi^2}}{(e^{\pi^2} - 1)(1 + e^{\pi^2})(1 + e^{2\pi^2})} + \frac{122472 e^{8\pi^3}}{(e^{\pi^3} - 1)(1 + e^{\pi^3})(1 + e^{2\pi^3})(1 + e^{4\pi^3})} + \\
& \frac{516096 e^{16\pi^4}}{(e^{\pi^4} - 1)(1 + e^{\pi^4})(1 + e^{2\pi^4})(1 + e^{4\pi^4})(1 + e^{8\pi^4})} + \\
& \frac{1575000 e^{32\pi^5}}{(e^{\pi^5} - 1)(1 + e^{\pi^5})(1 + e^{2\pi^5})(1 + e^{4\pi^5})(1 + e^{8\pi^5})(1 + e^{16\pi^5})} + \\
& \frac{3919104 e^{64\pi^6}}{(e^{\pi^6} - 1)(1 + e^{\pi^6})(1 + e^{2\pi^6})(1 + e^{4\pi^6})(1 + e^{8\pi^6})(1 + e^{16\pi^6})(1 + e^{32\pi^6})} + \\
& \frac{8470728 e^{128\pi^7}}{(e^{\pi^7} - 1)(1 + e^{\pi^7})(1 + e^{2\pi^7})(1 + e^{4\pi^7})(1 + e^{8\pi^7})(1 + e^{16\pi^7})(1 + e^{32\pi^7})(1 + e^{64\pi^7})} + \\
& \frac{64512}{1032192} - \frac{64512}{2064384} - \frac{129024}{4128768} - \frac{258048}{8257536} - \frac{516096}{16515072} - \\
& \frac{e^{\pi^8} - 1}{1 + e^{16\pi^8}} - \frac{1 + e^{\pi^8}}{1 + e^{32\pi^8}} - \frac{1 + e^{2\pi^8}}{1 + e^{64\pi^8}} - \frac{1 + e^{4\pi^8}}{1 + e^{128\pi^8}} - \frac{1 + e^{8\pi^8}}{1 + e^{256\pi^8}} -
\end{aligned}$$

((((((((1-24 sum (j*e^((2Pi)^j))/(1-e^((2Pi)^j)), j = 1 to 8)))))))^2 - (((1+240 sum (j^3*e^((2Pi)^j))/(1-e^((2Pi)^j)), j = 1 to 8)))))))/12

Input interpretation:

$$\frac{1}{12} \left(\left(1 - 24 \sum_{j=1}^8 \frac{j e^{(2\pi)^j}}{1 - e^{(2\pi)^j}} \right)^2 - \left(1 + 240 \sum_{j=1}^8 \frac{j^3 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}} \right) \right)$$

Result:

$$\begin{aligned}
& \frac{1}{12} \left(-1 - 240 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} - \frac{8 e^{4\pi^2}}{e^{4\pi^2} - 1} - \frac{27 e^{8\pi^3}}{e^{8\pi^3} - 1} - \frac{64 e^{16\pi^4}}{e^{16\pi^4} - 1} - \right. \right. \\
& \left. \left. \frac{125 e^{32\pi^5}}{e^{32\pi^5} - 1} - \frac{216 e^{64\pi^6}}{e^{64\pi^6} - 1} - \frac{343 e^{128\pi^7}}{e^{128\pi^7} - 1} - \frac{512 e^{256\pi^8}}{e^{256\pi^8} - 1} \right) + \right. \\
& \left. \left(1 - 24 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} + \frac{2 e^{4\pi^2}}{1 - e^{4\pi^2}} + \frac{3 e^{8\pi^3}}{1 - e^{8\pi^3}} + \frac{4 e^{16\pi^4}}{1 - e^{16\pi^4}} + \frac{5 e^{32\pi^5}}{1 - e^{32\pi^5}} + \right. \right. \right. \\
& \left. \left. \left. \frac{6 e^{64\pi^6}}{1 - e^{64\pi^6}} + \frac{7 e^{128\pi^7}}{1 - e^{128\pi^7}} + \frac{8 e^{256\pi^8}}{1 - e^{256\pi^8}} \right) \right) \right)^2 \approx 88278.5
\end{aligned}$$

$$q \frac{dP}{dq} = \frac{P^2 - Q}{12},$$

$$P = 865.045$$

$$Q = -311039$$

$$R = 31135100$$

88278.48766875

$$[1 - 24 \sum_{j=1}^8 \frac{j e^{(2\pi)^j}}{1 - e^{(2\pi)^j}}] / (1 - e^{(2\pi)^8}) - [1 + 240 \sum_{j=1}^8 \frac{j^3 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}}] / (1 - e^{(2\pi)^8}) - [1 - 504 \sum_{j=1}^8 \frac{j^5 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}}] / (1 - e^{(2\pi)^8})$$

Input interpretation:

$$\left(1 - 24 \sum_{j=1}^8 \frac{j e^{(2\pi)^j}}{1 - e^{(2\pi)^j}} \right) \left(\frac{1}{3} \left(1 + 240 \sum_{j=1}^8 \frac{j^3 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}} \right) \right) - \frac{1}{3} \left(1 - 504 \sum_{j=1}^8 \frac{j^5 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}} \right)$$

Result:

$$\begin{aligned} & \frac{1}{3} \left(504 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} - \frac{32 e^{4\pi^2}}{e^{4\pi^2} - 1} - \frac{243 e^{8\pi^3}}{e^{8\pi^3} - 1} - \frac{1024 e^{16\pi^4}}{e^{16\pi^4} - 1} - \frac{3125 e^{32\pi^5}}{e^{32\pi^5} - 1} - \right. \right. \\ & \left. \left. \frac{7776 e^{64\pi^6}}{e^{64\pi^6} - 1} - \frac{16807 e^{128\pi^7}}{e^{128\pi^7} - 1} - \frac{32768 e^{256\pi^8}}{e^{256\pi^8} - 1} \right) - 1 \right) + \\ & \frac{1}{3} \left(1 - 24 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} + \frac{2 e^{4\pi^2}}{1 - e^{4\pi^2}} + \frac{3 e^{8\pi^3}}{1 - e^{8\pi^3}} + \frac{4 e^{16\pi^4}}{1 - e^{16\pi^4}} + \frac{5 e^{32\pi^5}}{1 - e^{32\pi^5}} + \right. \right. \\ & \left. \left. \frac{6 e^{64\pi^6}}{1 - e^{64\pi^6}} + \frac{7 e^{128\pi^7}}{1 - e^{128\pi^7}} + \frac{8 e^{256\pi^8}}{1 - e^{256\pi^8}} \right) \right) \\ & \left(1 + 240 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} - \frac{8 e^{4\pi^2}}{e^{4\pi^2} - 1} - \frac{27 e^{8\pi^3}}{e^{8\pi^3} - 1} - \frac{64 e^{16\pi^4}}{e^{16\pi^4} - 1} - \frac{125 e^{32\pi^5}}{e^{32\pi^5} - 1} - \right. \right. \\ & \left. \left. \frac{216 e^{64\pi^6}}{e^{64\pi^6} - 1} - \frac{343 e^{128\pi^7}}{e^{128\pi^7} - 1} - \frac{512 e^{256\pi^8}}{e^{256\pi^8} - 1} \right) \right) \approx -1.00066 \times 10^8 \end{aligned}$$

$$q \frac{dQ}{dq} = \frac{PQ - R}{3},$$

$$P = 865.045$$

$$Q = -311039$$

$$R = 31135100$$

-100065943.9183

$[1-24 \sum_{j=1}^8 (j \cdot e^{(2\pi)^j}) / (1 - e^{(2\pi)^j}), j = 1 \text{ to } 8] [1-504 \sum_{j=1}^8 (j^5 \cdot e^{(2\pi)^j}) / (1 - e^{(2\pi)^j}), j = 1 \text{ to } 8] - [1+240 \sum_{j=1}^8 (j^3 \cdot e^{(2\pi)^j}) / (1 - e^{(2\pi)^j}), j = 1 \text{ to } 8]^2$

Input interpretation:

$$\left(1 - 24 \sum_{j=1}^8 \frac{j e^{(2\pi)^j}}{1 - e^{(2\pi)^j}} \right) \left(1 - 504 \sum_{j=1}^8 \frac{j^5 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}} \right) - \left(1 + 240 \sum_{j=1}^8 \frac{j^3 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}} \right)^2$$

Result:

$$\begin{aligned} & \left(1 - 24 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} + \frac{2 e^{4\pi^2}}{1 - e^{4\pi^2}} + \frac{3 e^{8\pi^3}}{1 - e^{8\pi^3}} + \frac{4 e^{16\pi^4}}{1 - e^{16\pi^4}} + \right. \right. \\ & \quad \left. \left. \frac{5 e^{32\pi^5}}{1 - e^{32\pi^5}} + \frac{6 e^{64\pi^6}}{1 - e^{64\pi^6}} + \frac{7 e^{128\pi^7}}{1 - e^{128\pi^7}} + \frac{8 e^{256\pi^8}}{1 - e^{256\pi^8}} \right) \right) \\ & \left(1 - 504 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} - \frac{32 e^{4\pi^2}}{e^{4\pi^2} - 1} - \frac{243 e^{8\pi^3}}{e^{8\pi^3} - 1} - \frac{1024 e^{16\pi^4}}{e^{16\pi^4} - 1} - \frac{3125 e^{32\pi^5}}{e^{32\pi^5} - 1} - \right. \right. \\ & \quad \left. \left. \frac{7776 e^{64\pi^6}}{e^{64\pi^6} - 1} - \frac{16807 e^{128\pi^7}}{e^{128\pi^7} - 1} - \frac{32768 e^{256\pi^8}}{e^{256\pi^8} - 1} \right) \right) - \\ & \left(1 + 240 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} - \frac{8 e^{4\pi^2}}{e^{4\pi^2} - 1} - \frac{27 e^{8\pi^3}}{e^{8\pi^3} - 1} - \frac{64 e^{16\pi^4}}{e^{16\pi^4} - 1} - \frac{125 e^{32\pi^5}}{e^{32\pi^5} - 1} - \right. \right. \\ & \quad \left. \left. \frac{216 e^{64\pi^6}}{e^{64\pi^6} - 1} - \frac{343 e^{128\pi^7}}{e^{128\pi^7} - 1} - \frac{512 e^{256\pi^8}}{e^{256\pi^8} - 1} \right) \right)^2 \approx -6.98123 \times 10^{10} \end{aligned}$$

$[1-24 \sum_{j=1}^8 (j \cdot e^{(2\pi)^j}) / (1 - e^{(2\pi)^j}), j = 1 \text{ to } 8] [1-504 \sum_{j=1}^8 (j^5 \cdot e^{(2\pi)^j}) / (1 - e^{(2\pi)^j}), j = 1 \text{ to } 8] / 2 - (((1+240 \sum_{j=1}^8 (j^3 \cdot e^{(2\pi)^j}) / (1 - e^{(2\pi)^j}), j = 1 \text{ to } 8))^2) / 2$

Input interpretation:

$$\left(1 - 24 \sum_{j=1}^8 \frac{j e^{(2\pi)^j}}{1 - e^{(2\pi)^j}} \right) \left(\frac{1}{2} \left(1 - 504 \sum_{j=1}^8 \frac{j^5 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}} \right) \right) - \frac{1}{2} \left(1 + 240 \sum_{j=1}^8 \frac{j^3 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}} \right)^2$$

Result:

$$\frac{1}{2} \left(1 - 24 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} + \frac{2e^{4\pi^2}}{1 - e^{4\pi^2}} + \frac{3e^{8\pi^3}}{1 - e^{8\pi^3}} + \frac{4e^{16\pi^4}}{1 - e^{16\pi^4}} + \frac{5e^{32\pi^5}}{1 - e^{32\pi^5}} + \frac{6e^{64\pi^6}}{1 - e^{64\pi^6}} + \frac{7e^{128\pi^7}}{1 - e^{128\pi^7}} + \frac{8e^{256\pi^8}}{1 - e^{256\pi^8}} \right) \right. \\ \left. \left(1 - 504 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} - \frac{32e^{4\pi^2}}{e^{4\pi^2} - 1} - \frac{243e^{8\pi^3}}{e^{8\pi^3} - 1} - \frac{1024e^{16\pi^4}}{e^{16\pi^4} - 1} - \frac{3125e^{32\pi^5}}{e^{32\pi^5} - 1} - \frac{7776e^{64\pi^6}}{e^{64\pi^6} - 1} - \frac{16807e^{128\pi^7}}{e^{128\pi^7} - 1} - \frac{32768e^{256\pi^8}}{e^{256\pi^8} - 1} \right) \right) - \right. \\ \left. \frac{1}{2} \left(1 + 240 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} - \frac{8e^{4\pi^2}}{e^{4\pi^2} - 1} - \frac{27e^{8\pi^3}}{e^{8\pi^3} - 1} - \frac{64e^{16\pi^4}}{e^{16\pi^4} - 1} - \frac{125e^{32\pi^5}}{e^{32\pi^5} - 1} - \frac{216e^{64\pi^6}}{e^{64\pi^6} - 1} - \frac{343e^{128\pi^7}}{e^{128\pi^7} - 1} - \frac{512e^{256\pi^8}}{e^{256\pi^8} - 1} \right) \right)^2 \right) \approx -3.49061 \times 10^{10}$$

$$q \frac{dR}{dq} = \frac{PR - Q^2}{2}$$

$$P = 865.045$$

$$Q = -311039$$

$$R = 31135100$$

$$-34905998470.75$$

Now, we have that:

$$88278.48766875 * 1 / (-34905998470.75 / -100065943.9183)$$

Input interpretation:

$$88278.48766875 \left(-\frac{1}{\frac{-3.490599847075 \times 10^{10}}{1.000659439183 \times 10^8}} \right)$$

Result:

$$253.0702625124956899873133247321062810569395034184591646902...$$

$$253.070262512....$$

From which:

$$1/2(((88278.48766875 * 1 / (-34905998470.75 / -100065943.9183))))-1$$

Input interpretation:

$$\frac{1}{2} \left(88\,278.48766875 \left(-\frac{1}{\frac{-3.490599847075 \times 10^{10}}{1.000659439183 \times 10^8}} \right) \right) - 1$$

Result:

125.5351312562478449936566623660531405284697517092295823451...

125.535131256..... result very near to the Higgs boson mass 125.18 GeV

And:

$$27 * (((1/4(((88278.48766875 * 1 / (-34905998470.75 / -100065943.9183)))))) + 1/\text{golden ratio})) + 4$$

Input interpretation:

$$27 \left(\frac{1}{4} \left(88\,278.48766875 \left(-\frac{1}{\frac{-3.490599847075 \times 10^{10}}{1.000659439183 \times 10^8}} \right) \right) + \frac{1}{\phi} \right) + 4$$

ϕ is the golden ratio

Result:

1728.911189656...

1728.911189656... \approx 1729

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternative representations:

$$27 \left(-\frac{88\,278.487668750000}{-\frac{3.4905998470750000 \times 10^{10} \times 4}{1.0006594391830000 \times 10^8}} + \frac{1}{\phi} \right) + 4 =$$

$$4 + 27 \left(\frac{88\,278.487668750000}{-\frac{4(-3.4905998470750000 \times 10^{10})}{1.0006594391830000 \times 10^8}} + \frac{1}{2 \sin(54^\circ)} \right)$$

$$27 \left(-\frac{88\,278.487668750000}{-\frac{3.4905998470750000 \times 10^{10} \times 4}{1.0006594391830000 \times 10^8}} + \frac{1}{\phi} \right) + 4 =$$

$$4 + 27 \left(-\frac{1}{2 \cos(216^\circ)} + \frac{88\,278.487668750000}{-\frac{4(-3.4905998470750000 \times 10^{10})}{1.0006594391830000 \times 10^8}} \right)$$

$$27 \left(-\frac{88\,278.487668750000}{-\frac{3.4905998470750000 \times 10^{10} \times 4}{1.0006594391830000 \times 10^8}} + \frac{1}{\phi} \right) + 4 =$$

$$4 + 27 \left(\frac{88\,278.487668750000}{-\frac{4(-3.4905998470750000 \times 10^{10})}{1.0006594391830000 \times 10^8}} + -\frac{1}{2 \sin(666^\circ)} \right)$$

$$1/2(((88278.48766875 * 1 / (-34905998470.75 / -100065943.9183))))+13$$

Input interpretation:

$$\frac{1}{2} \left(88\,278.48766875 \left(-\frac{1}{-\frac{3.490599847075 \times 10^{10}}{1.000659439183 \times 10^8}} \right) \right) + 13$$

Result:

139.5351312562478449936566623660531405284697517092295823451...

139.535131256.... result practically equal to the rest mass of Pion meson 139.57 MeV

And also, from the following calculations:

$$((-(-34905998470.75 - 100065943.9183 + 88278.48766875)))^{1/48}$$

Input interpretation:

$$\sqrt[48]{-(-3.490599847075 \times 10^{10} - 1.000659439183 \times 10^8 + 88\,278.48766875)}$$

Result:

1.65832492758672...

1.65832492758672... result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

$$((-(-34905998470.75 - 100065943.9183 + 88278.48766875)))^{1/48} - (47-7) \times 1/10^3$$

Input interpretation:

$$\sqrt[48]{-(-3.490599847075 \times 10^{10} - 1.000659439183 \times 10^8 + 88\,278.48766875)} - (47 - 7) \times \frac{1}{10^3}$$

Result:

1.618324927586720038762470577631248480242952403678858203293...

1.61832492758672..... result that is a very good approximation to the value of the golden ratio 1,618033988749...

For $j = 1$ to 19, we obtain:

$$1 - 24 \sum_{j=1}^{19} \frac{j \cdot e^{(2\pi i)^j}}{1 - e^{(2\pi i)^j}}, j = 1 \text{ to } 19$$

Input interpretation:

$$1 - 24 \sum_{j=1}^{19} \frac{j e^{(2\pi)^j}}{1 - e^{(2\pi)^j}}$$

Result:

$$1 - 24 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} + \frac{2e^{4\pi^2}}{1 - e^{4\pi^2}} + \frac{3e^{8\pi^3}}{1 - e^{8\pi^3}} + \frac{4e^{16\pi^4}}{1 - e^{16\pi^4}} + \frac{5e^{32\pi^5}}{1 - e^{32\pi^5}} + \frac{6e^{64\pi^6}}{1 - e^{64\pi^6}} + \frac{7e^{128\pi^7}}{1 - e^{128\pi^7}} + \frac{8e^{256\pi^8}}{1 - e^{256\pi^8}} + \frac{9e^{512\pi^9}}{1 - e^{512\pi^9}} + \frac{10e^{1024\pi^{10}}}{1 - e^{1024\pi^{10}}} + \frac{11e^{2048\pi^{11}}}{1 - e^{2048\pi^{11}}} + \frac{12e^{4096\pi^{12}}}{1 - e^{4096\pi^{12}}} + \frac{13e^{8192\pi^{13}}}{1 - e^{8192\pi^{13}}} + \frac{14e^{16384\pi^{14}}}{1 - e^{16384\pi^{14}}} + \frac{15e^{32768\pi^{15}}}{1 - e^{32768\pi^{15}}} + \frac{16e^{65536\pi^{16}}}{1 - e^{65536\pi^{16}}} + \frac{17e^{131072\pi^{17}}}{1 - e^{131072\pi^{17}}} + \frac{18e^{262144\pi^{18}}}{1 - e^{262144\pi^{18}}} + \frac{19e^{524288\pi^{19}}}{1 - e^{524288\pi^{19}}} \right) \approx 4561.04$$

P = 4561.04

$1 + 240 \sum_{j=1}^{19} (j^3 * e^{((2\pi)^j)}) / (1 - e^{((2\pi)^j)})$, j = 1 to 19

Input interpretation:

$$1 + 240 \sum_{j=1}^{19} \frac{j^3 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}}$$

Result:

$$1 + 240 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} + \frac{8e^{4\pi^2}}{1 - e^{4\pi^2}} + \frac{27e^{8\pi^3}}{1 - e^{8\pi^3}} + \frac{64e^{16\pi^4}}{1 - e^{16\pi^4}} + \frac{125e^{32\pi^5}}{1 - e^{32\pi^5}} + \frac{216e^{64\pi^6}}{1 - e^{64\pi^6}} + \frac{343e^{128\pi^7}}{1 - e^{128\pi^7}} + \frac{512e^{256\pi^8}}{1 - e^{256\pi^8}} + \frac{729e^{512\pi^9}}{1 - e^{512\pi^9}} + \frac{1000e^{1024\pi^{10}}}{1 - e^{1024\pi^{10}}} + \frac{1331e^{2048\pi^{11}}}{1 - e^{2048\pi^{11}}} + \frac{1728e^{4096\pi^{12}}}{1 - e^{4096\pi^{12}}} + \frac{2197e^{8192\pi^{13}}}{1 - e^{8192\pi^{13}}} + \frac{2744e^{16384\pi^{14}}}{1 - e^{16384\pi^{14}}} + \frac{3375e^{32768\pi^{15}}}{1 - e^{32768\pi^{15}}} + \frac{4096e^{65536\pi^{16}}}{1 - e^{65536\pi^{16}}} + \frac{4913e^{131072\pi^{17}}}{1 - e^{131072\pi^{17}}} + \frac{5832e^{262144\pi^{18}}}{1 - e^{262144\pi^{18}}} + \frac{6859e^{524288\pi^{19}}}{1 - e^{524288\pi^{19}}} \right) \approx -8.664 \times 10^6$$

Q = -8.664 * 10⁶

1-504 sum $(j^5 * e^{((2\pi)^j)}) / (1 - e^{((2\pi)^j)})$, $j = 1$ to 19

Input interpretation:

$$1 - 504 \sum_{j=1}^{19} \frac{j^5 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}}$$

Result:

$$1 - 504 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} + \frac{32 e^{4\pi^2}}{1 - e^{4\pi^2}} + \frac{243 e^{8\pi^3}}{1 - e^{8\pi^3}} + \frac{1024 e^{16\pi^4}}{1 - e^{16\pi^4}} + \frac{3125 e^{32\pi^5}}{1 - e^{32\pi^5}} + \frac{7776 e^{64\pi^6}}{1 - e^{64\pi^6}} + \frac{16807 e^{128\pi^7}}{1 - e^{128\pi^7}} + \frac{32768 e^{256\pi^8}}{1 - e^{256\pi^8}} + \frac{59049 e^{512\pi^9}}{1 - e^{512\pi^9}} + \frac{100000 e^{1024\pi^{10}}}{1 - e^{1024\pi^{10}}} + \frac{161051 e^{2048\pi^{11}}}{1 - e^{2048\pi^{11}}} + \frac{248832 e^{4096\pi^{12}}}{1 - e^{4096\pi^{12}}} + \frac{371293 e^{8192\pi^{13}}}{1 - e^{8192\pi^{13}}} + \frac{537824 e^{16384\pi^{14}}}{1 - e^{16384\pi^{14}}} + \frac{759375 e^{32768\pi^{15}}}{1 - e^{32768\pi^{15}}} + \frac{1048576 e^{65536\pi^{16}}}{1 - e^{65536\pi^{16}}} + \frac{1419857 e^{131072\pi^{17}}}{1 - e^{131072\pi^{17}}} + \frac{1889568 e^{262144\pi^{18}}}{1 - e^{262144\pi^{18}}} + \frac{2476099 e^{524288\pi^{19}}}{1 - e^{524288\pi^{19}}} \right) \approx 4.60318 \times 10^9$$

$$R = 4.60318 * 10^9$$

Thence:

$$P = 4561.04; \quad Q = -8.664 * 10^6; \quad R = 4.60318 * 10^9$$

From:

$$q \frac{dP}{dq} = \frac{P^2 - Q}{12}, \quad q \frac{dQ}{dq} = \frac{PQ - R}{3}, \quad q \frac{dR}{dq} = \frac{PR - Q^2}{2}.$$

thus:

$$1/12(4561.04^2 - (-8.664e+6)); \quad 1/3(4561.04 * (-8664e+6) - (4.60318e+9));$$

$$1/2(4561.04 * (4.60318e+9) - (-8.664e+6)^2)$$

Result:

$$-2.70348039464 \times 10^{13}$$

$$q \frac{dR}{dq} = \frac{PR - Q^2}{2}$$

$$-2.70348039464 * 10^{13}$$

From the algebraic sum of the three results, we obtain:

$$(2.4555904901333e+6 - 1.317381791333e+13 - 2.70348039464e+13)$$

Input interpretation:

$$2.4555904901333 \times 10^6 - 1.317381791333 \times 10^{13} - 2.70348039464 \times 10^{13}$$

Result:

$$-4.02086194041395098667 \times 10^{13}$$

$$-4.02086194041395098667 * 10^{13}$$

From which:

$$\left(-(2.4555904901333e+6 - 1.317381791333e+13 - 2.70348039464e+13) \right)^{1/64} - 13 * 1/10^3$$

Input interpretation:

$$\sqrt[64]{ -(2.4555904901333 \times 10^6 - 1.317381791333 \times 10^{13} - 2.70348039464 \times 10^{13}) } - 13 \times \frac{1}{10^3}$$

Result:

$$1.618426403939...$$

1.618426403939... result that is a very good approximation to the value of the golden ratio 1,618033988749...

and:

$2 \log_{1.6314264039}(-2.4555904901333e+6 - 1.317381791333e+13 - 2.70348039464e+13) - \pi + 1/\text{golden ratio}$

Input interpretation:

$$2 \log_{1.6314264039} \left(- \left(2.4555904901333 \times 10^6 - 1.317381791333 \times 10^{13} - 2.70348039464 \times 10^{13} \right) \right) - \pi + \frac{1}{\phi}$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

125.47644134...

125.47644134... result very near to the Higgs boson mass 125.18 GeV

$2 \log_{1.6314264039}(-2.4555904901333e+6 - 1.317381791333e+13 - 2.70348039464e+13) + 11 + 1/\text{golden ratio}$

Input interpretation:

$$2 \log_{1.6314264039} \left(- \left(2.4555904901333 \times 10^6 - 1.317381791333 \times 10^{13} - 2.70348039464 \times 10^{13} \right) \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

139.61803399...

139.61803399... result practically equal to the rest mass of Pion meson 139.57 MeV

$27 \log_{1.6314264039}(-2.4555904901333e+6 - 1.317381791333e+13 - 2.70348039464e+13) + 1$

Input interpretation:

$$27 \log_{1.6314264039} \left(- \left(2.4555904901333 \times 10^6 - 1.317381791333 \times 10^{13} - 2.70348039464 \times 10^{13} \right) \right) + 1$$

$\log_b(x)$ is the base- b logarithm

Result:

1729.0000001...

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

“The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group $\mathbf{Z}/3\mathbf{Z}$, and its outer automorphism group is the cyclic group $\mathbf{Z}/2\mathbf{Z}$. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories”.

Now, we have that:

$$S = S(q) = 480E_8 = 1 + 480 \sum_{j=1}^{\infty} \frac{j^7 q^j}{1 - q^j}.$$

$$1 + 480 \sum_{j=1}^8 \frac{j^7 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}}, j = 1 \text{ to } 8$$

Input interpretation:

$$1 + 480 \sum_{j=1}^8 \frac{j^7 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}}$$

Result:

$$1 + 480 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} + \frac{128 e^{4\pi^2}}{1 - e^{4\pi^2}} + \frac{2187 e^{8\pi^3}}{1 - e^{8\pi^3}} + \frac{16384 e^{16\pi^4}}{1 - e^{16\pi^4}} + \frac{78125 e^{32\pi^5}}{1 - e^{32\pi^5}} + \frac{279936 e^{64\pi^6}}{1 - e^{64\pi^6}} + \frac{823543 e^{128\pi^7}}{1 - e^{128\pi^7}} + \frac{2097152 e^{256\pi^8}}{1 - e^{256\pi^8}} \right) \approx -1.58278 \times 10^9$$

$$S = -1582780000$$

Alternate forms:

$$\begin{aligned}
& 1 - \frac{480 e^{2\pi}}{e^{2\pi} - 1} - \frac{61440 e^{4\pi^2}}{e^{4\pi^2} - 1} - \frac{1049760 e^{8\pi^3}}{e^{8\pi^3} - 1} - \frac{7864320 e^{16\pi^4}}{e^{16\pi^4} - 1} - \frac{37500000 e^{32\pi^5}}{e^{32\pi^5} - 1} - \\
& \frac{134369280 e^{64\pi^6}}{e^{64\pi^6} - 1} - \frac{395300640 e^{128\pi^7}}{e^{128\pi^7} - 1} - \frac{1006632960 e^{256\pi^8}}{e^{256\pi^8} - 1} \\
& 1 + 480 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} - \frac{128 e^{4\pi^2}}{e^{4\pi^2} - 1} - \frac{2187 e^{8\pi^3}}{e^{8\pi^3} - 1} - \frac{16384 e^{16\pi^4}}{e^{16\pi^4} - 1} - \right. \\
& \left. \frac{78125 e^{32\pi^5}}{e^{32\pi^5} - 1} - \frac{279936 e^{64\pi^6}}{e^{64\pi^6} - 1} - \frac{823543 e^{128\pi^7}}{e^{128\pi^7} - 1} - \frac{2097152 e^{256\pi^8}}{e^{256\pi^8} - 1} \right) \\
& - 1006632959 - \frac{480 e^{2\pi}}{(e^\pi - 1)(1 + e^\pi)} - \\
& \frac{61440 e^{4\pi^2}}{(e^{\pi^2} - 1)(1 + e^{\pi^2})(1 + e^{2\pi^2})} - \frac{1049760 e^{8\pi^3}}{(e^{\pi^3} - 1)(1 + e^{\pi^3})(1 + e^{2\pi^3})(1 + e^{4\pi^3})} - \\
& \frac{7864320 e^{16\pi^4}}{(e^{\pi^4} - 1)(1 + e^{\pi^4})(1 + e^{2\pi^4})(1 + e^{4\pi^4})(1 + e^{8\pi^4})} - \\
& \frac{37500000 e^{32\pi^5}}{(e^{\pi^5} - 1)(1 + e^{\pi^5})(1 + e^{2\pi^5})(1 + e^{4\pi^5})(1 + e^{8\pi^5})(1 + e^{16\pi^5})} - \\
& \frac{134369280 e^{64\pi^6}}{(e^{\pi^6} - 1)(1 + e^{\pi^6})(1 + e^{2\pi^6})(1 + e^{4\pi^6})(1 + e^{8\pi^6})(1 + e^{16\pi^6})(1 + e^{32\pi^6})} - \\
& \frac{395300640 e^{128\pi^7}}{(e^{\pi^7} - 1)(1 + e^{\pi^7})(1 + e^{2\pi^7})(1 + e^{4\pi^7})(1 + e^{8\pi^7})(1 + e^{16\pi^7})(1 + e^{32\pi^7})(1 + e^{64\pi^7})} - \\
& \frac{3932160}{3932160} + \frac{3932160}{3932160} + \frac{7864320}{7864320} + \frac{15728640}{15728640} + \frac{31457280}{31457280} + \\
& \frac{e^{\pi^8} - 1}{62914560} + \frac{1 + e^{\pi^8}}{125829120} + \frac{1 + e^{2\pi^8}}{251658240} + \frac{1 + e^{4\pi^8}}{503316480} + \frac{1 + e^{8\pi^8}}{1006632960} + \\
& \frac{1}{1 + e^{16\pi^8}} + \frac{1}{1 + e^{32\pi^8}} + \frac{1}{1 + e^{64\pi^8}} + \frac{1}{1 + e^{128\pi^8}}
\end{aligned}$$

And:

$$1 + 480 \sum_{j=1}^{19} \frac{j^7 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}}, j = 1 \text{ to } 19$$

Input interpretation:

$$1 + 480 \sum_{j=1}^{19} \frac{j^7 e^{(2\pi)^j}}{1 - e^{(2\pi)^j}}$$

Result:

$$1 + 480 \left(\frac{e^{2\pi}}{1 - e^{2\pi}} + \frac{128 e^{4\pi^2}}{1 - e^{4\pi^2}} + \frac{2187 e^{8\pi^3}}{1 - e^{8\pi^3}} + \frac{16384 e^{16\pi^4}}{1 - e^{16\pi^4}} + \frac{78125 e^{32\pi^5}}{1 - e^{32\pi^5}} + \frac{279936 e^{64\pi^6}}{1 - e^{64\pi^6}} + \frac{823543 e^{128\pi^7}}{1 - e^{128\pi^7}} + \frac{2097152 e^{256\pi^8}}{1 - e^{256\pi^8}} + \frac{4782969 e^{512\pi^9}}{1 - e^{512\pi^9}} + \frac{10000000 e^{1024\pi^{10}}}{1 - e^{1024\pi^{10}}} + \frac{19487171 e^{2048\pi^{11}}}{1 - e^{2048\pi^{11}}} + \frac{35831808 e^{4096\pi^{12}}}{1 - e^{4096\pi^{12}}} + \frac{62748517 e^{8192\pi^{13}}}{1 - e^{8192\pi^{13}}} + \frac{105413504 e^{16384\pi^{14}}}{1 - e^{16384\pi^{14}}} + \frac{170859375 e^{32768\pi^{15}}}{1 - e^{32768\pi^{15}}} + \frac{268435456 e^{65536\pi^{16}}}{1 - e^{65536\pi^{16}}} + \frac{410338673 e^{131072\pi^{17}}}{1 - e^{131072\pi^{17}}} + \frac{612220032 e^{262144\pi^{18}}}{1 - e^{262144\pi^{18}}} + \frac{893871739 e^{524288\pi^{19}}}{1 - e^{524288\pi^{19}}} \right) \approx -1.2467 \times 10^{12}$$

$$S = -1246700000000$$

For $j = 1$ to 8, we have:

$$P = 865.045; \quad Q = -311039; \quad R = 31135100; \quad S = -1582780000$$

From:

Lemma 0.20. *Let U_k and V_k be defined by (0.59) and (0.60) and let P , Q , R , and S be defined by (0.71)–(0.74). Then*

$$U_2 = P,$$

$$U_4 = \frac{1}{3} (5P^2 - 2Q),$$

$$U_6 = \frac{1}{9} (35P^3 - 42PQ + 16R),$$

$$U_8 = \frac{1}{15} (175P^4 - 420P^2Q + 320PR + 84Q^2 - 144S)$$

and

$$V_2 = P,$$

$$V_4 = 3P^2 - 2Q,$$

$$V_6 = 15P^3 - 30PQ + 16R,$$

$$V_8 = 105P^4 - 420P^2Q + 448PR + 140Q^2 - 272S.$$

$\log_b(x)$ is the base- b logarithm

Result:

125.50000000...

125.5... result very near to the Higgs boson mass 125.18 GeV

$$2 \log_{1.61718355157} \left((865.045 + 1.454530753375 \times 10^6 + 3.828315972862 \times 10^9 + 1.4181416305 \times 10^{13}) \right) + 13 + \frac{1}{2}$$

Input interpretation:

$$2 \log_{1.61718355157} \left(865.045 + 1.454530753375 \times 10^6 + 3.828315972862 \times 10^9 + 1.4181416305 \times 10^{13} \right) + 13 + \frac{1}{2}$$

$\log_b(x)$ is the base- b logarithm

Result:

139.50000000...

139.5... result practically equal to the rest mass of Pion meson 139.57 MeV

$$27 \left(\log_{1.61718355157} \left((865.045 + 1.454530753375 \times 10^6 + 3.828315972862 \times 10^9 + 1.4181416305 \times 10^{13}) \right) + 1 \right) + 1$$

Input interpretation:

$$27 \left(\log_{1.61718355157} \left(865.045 + 1.454530753375 \times 10^6 + 3.828315972862 \times 10^9 + 1.4181416305 \times 10^{13} \right) + 1 \right) + 1$$

$\log_b(x)$ is the base- b logarithm

Result:

1729.00000000...

1729

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Now, we have that:

$$V_2 = P,$$

$$V_4 = 3P^2 - 2Q,$$

$$V_6 = 15P^3 - 30PQ + 16R,$$

$$V_8 = 105P^4 - 420P^2Q + 448PR + 140Q^2 - 272S.$$

For: $P = 865.045$; $Q = -311039$; $R = 31135100$; $S = -1582780000$, we obtain:

$$V_2 = 865.045$$

$$V_4 = (3 \times 865.045^2 - 2(-311039))$$

$$V_6 = (15 \times 865.045^3 - 30 \times 865.045 \times (-311039) + 16 \times 31135100)$$

$$V_8 = ((105 \times 865.045^4 - 420 \times 865.045^2 \times (-311039) + 448 \times 865.045 \times 31135100 + 140 \times (-311039)^2 - 272 \times (-1582780000))$$

From the sum, we obtain:

$$865.045 + ((3 \times 865.045^2 - 2(-311039))) + ((15 \times 865.045^3 - 30 \times 865.045 \times (-311039) + 16 \times 31135100)) + (((105 \times 865.045^4 - 420 \times 865.045^2 \times (-311039)) + 448 \times 865.045 \times 31135100 + 140 \times (-311039)^2 - 272 \times (-1582780000)))$$

Input interpretation:

$$865.045 + (3 \times 865.045^2 - 2 \times (-311039)) + (15 \times 865.045^3 + 30 \times (-311039) \times (-865.045) + 16 \times 31135100) + ((105 \times 865.045^4 - 420 \times 865.045^2 \times (-311039)) + 448 \times 865.045 \times 31135100 + 140 \times (-311039)^2 - 272 \times (-1582780000))$$

Result:

$$1.82610314133410019959940625 \times 10^{14}$$

$$1.82610314133410019959940625 \times 10^{14}$$

$$(1.82610314133410019959940625 \times 10^{14})^{1/66}$$

Input interpretation:

$$\sqrt[66]{1.82610314133410019959940625 \times 10^{14}}$$

Result:

1.6446887308743606956546514147...

$$1.6446887308... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

2log base 1.6446887308((1.8261031413341 × 10¹⁴))+7+1/golden ratio

Input interpretation:

$$2 \log_{1.6446887308}(1.8261031413341 \times 10^{14}) + 7 + \frac{1}{\phi}$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

139.61803400...

139.618034... result practically equal to the rest mass of Pion meson 139.57 MeV

2log base 1.6446887308((1.8261031413341 × 10¹⁴))-7+1/golden ratio

Input interpretation:

$$2 \log_{1.6446887308}(1.8261031413341 \times 10^{14}) - 7 + \frac{1}{\phi}$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

125.61803400...

125.618034... result very near to the Higgs boson mass 125.18 GeV

$$27((\log_{1.6446887308}((1.8261031413341 \times 10^{14}) - 2)) + 1)$$

Input interpretation:

$$27 (\log_{1.6446887308}(1.8261031413341 \times 10^{14}) - 2) + 1$$

$\log_b(x)$ is the base- b logarithm

Result:

1729.0000002...

1729

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

$$1/(2\pi) * \int_0^x ((27((\log_{1.6446887308}((1.8261031413341 \times 10^{14}) - 2)) + 1))) dx$$

Input interpretation:

$$\frac{1}{2\pi} \int (27 (\log_{1.6446887308}(1.8261031413341 \times 10^{14}) - 2) + 1) x dx$$

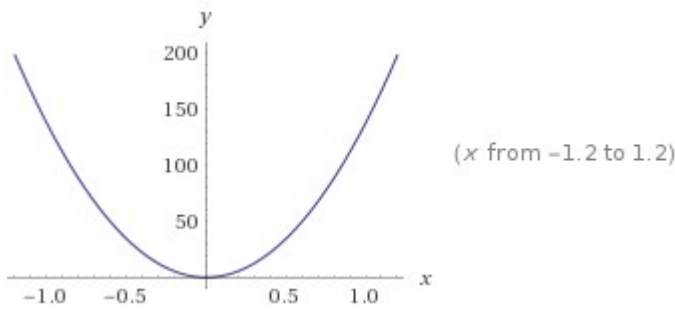
$\log_b(x)$ is the base- b logarithm

Result:

137.58944832 x^2

that for $x = 1$ give us 137.58944832, that is very near to the inverse of fine-structure constant 137,035

Plot:



From:

$$Q^3 - R^2 = 1728q \prod_{j=1}^{\infty} (1 - q^j)^{24}.$$

we obtain:

$$1728 * e^{(2\pi)} * \text{product} ((1 - (e^{(2\pi)})^j)^{24}, j = 1..0.8)$$

Input interpretation:

$$1728 e^{2\pi} \prod_{j=1}^{0.8} (1 - (e^{2\pi})^j)^{24}$$

Result:

$$1728 e^{2\pi} \approx 925\,330.$$

925330

From which:

$$13 + 1/10^3 (((1728 * e^{(2\pi)} * \text{product} ((1 - (e^{(2\pi)})^j)^{24}, j = 1..0.8))))$$

Input interpretation:

$$13 + \frac{1}{10^3} \left(1728 e^{2\pi} \prod_{j=1}^{0.8} (1 - (e^{2\pi})^j)^{24} \right)$$

Result:

$$13 + \frac{216 e^{2\pi}}{125} \approx 938.33$$

938.33 result practically equal to the proton mass in MeV

Alternate form:

$$\frac{1}{125} (1625 + 216 e^{2\pi})$$

$$Q = -311039; R = 31135100$$

$$(-311039)^3 - (31135100)^2$$

Input:

$$(-311039)^3 - 31135100^2$$

Result:

$$-31060943228162319$$

Result:

$$-3.1060943228162319 \times 10^{16}$$

$$-3.1060943228162319 * 10^{16}$$

$$((((-311039)^3 - (31135100)^2))) x = 1728 * e^{(2\pi)}$$

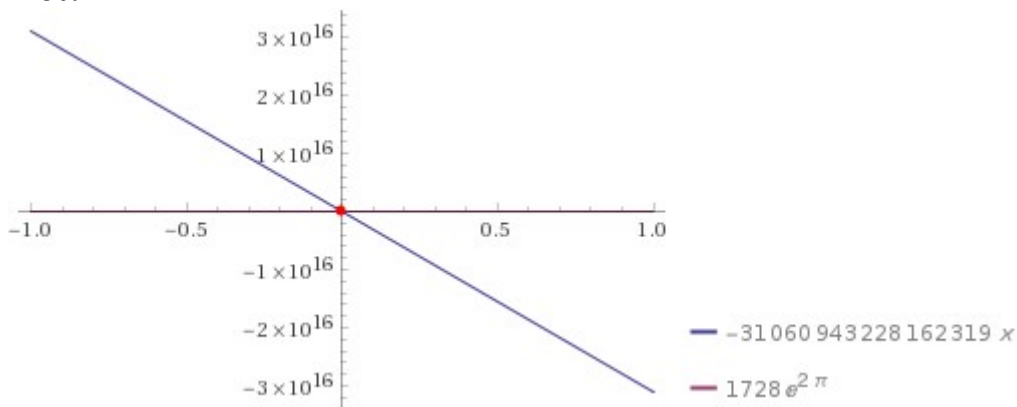
Input:

$$((-311039)^3 - 31135100^2) x = 1728 e^{2\pi}$$

Exact result:

$$-31060943228162319 x = 1728 e^{2\pi}$$

Plot:



Alternate form:

$$-31\,060\,943\,228\,162\,319\,x - 1728\,e^{2\pi} = 0$$

Number line:**Solution:**

$$x \approx -2.97907753138616 \times 10^{-11}$$

$$-2.97907753138616 * 10^{-11}$$

$$(((((-311039)^3 - (31135100)^2)))(-2.979077531e-11))$$

Input interpretation:

$$((-311\,039)^3 - 31\,135\,100^2)(-2.979077531 \times 10^{-11})$$

Result:

925329.58062684970953754389

Repeating decimal:

925 329.58062684970953754389

925329.58062...

$$1728 * e^{(2\pi)}$$

Input:

$$1728\,e^{2\pi}$$

Decimal approximation:

925329.5807467934646772692415298735295936484182584936139964...

925329.580746...

Property:

$1728\,e^{2\pi}$ is a transcendental number

Alternative representations:

$$1728 e^{2\pi} = 1728 e^{360^\circ}$$

$$1728 e^{2\pi} = 1728 e^{-2i \log(-1)}$$

$$1728 e^{2\pi} = 1728 \exp^{2\pi}(z) \text{ for } z = 1$$

Series representations:

$$1728 e^{2\pi} = 1728 e^{8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$1728 e^{2\pi} = 1728 \left(\sum_{k=0}^{\infty} \frac{1}{k!} \right)^{2\pi}$$

$$1728 e^{2\pi} = 1728 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{2\pi}$$

Integral representations:

$$1728 e^{2\pi} = 1728 e^{8 \int_0^1 \sqrt{1-t^2} dt}$$

$$1728 e^{2\pi} = 1728 e^{4 \int_0^1 1/\sqrt{1-t^2} dt}$$

$$1728 e^{2\pi} = 1728 e^{4 \int_0^{\infty} 1/(1+t^2) dt}$$

From:

(Almost) Impossible Integrals, Sums, and Series - Cornel Ioan Vălean

$$= \frac{1}{2}(-1)^n n! (\zeta(2n+2) - \eta^2(n+1))$$

$$= \frac{1}{2}(-1)^n n! (\zeta(2n+2) - (1-2^{-n})^2 \zeta^2(n+1)), \quad (1.54)$$

where ζ is the Riemann zeta function, η denotes the Dirichlet eta function, and Li_n represents the Polylogarithm function.

Examples:
 For $n = 2$,

$$\int_0^1 \frac{\log^2(x) \text{Li}_3(x)}{1+x} dx = \zeta(6) - \frac{9}{16} \zeta^2(3);$$

For $n = 3$,

$$\int_0^1 \frac{\log^3(x) \text{Li}_4(x)}{1+x} dx = -\frac{41}{128} \zeta(8);$$

For $n = 4$,

$$\int_0^1 \frac{\log^4(x) \text{Li}_5(x)}{1+x} dx = 12\zeta(10) - \frac{675}{64} \zeta^2(5).$$

((zeta (6) – 9/16 * zeta^2 (3))) - ((41/128 * zeta (8))) + ((12 zeta (10) – 675/64 * zeta^2 (5)))

Input:

$$\left(\zeta(6) - \frac{9}{16} \zeta(3)^2\right) - \frac{41}{128} \zeta(8) + \left(12 \zeta(10) - \frac{675}{64} \zeta(5)^2\right)$$

$\zeta(s)$ is the Riemann zeta function

Exact result:

$$-\frac{9 \zeta(3)^2}{16} - \frac{675 \zeta(5)^2}{64} + \frac{\pi^6}{945} - \frac{41 \pi^8}{1209600} + \frac{4 \pi^{10}}{31185}$$

Decimal approximation:

0.554678059117173137851162220710968435185487941144899455869...

0.5546780591171....

Alternate forms:

$$\frac{-22453200 \zeta(3)^2 - 420997500 \zeta(5)^2 + 42240 \pi^6 - 1353 \pi^8 + 5120 \pi^{10}}{39916800}$$

$$\frac{-22453200 \zeta(3)^2 + 42240 \pi^6 - 1353 \pi^8 + 5120 \pi^{10}}{39916800} - \frac{675 \zeta(5)^2}{64}$$

$$\frac{-5613300 (4 \zeta(3)^2 + 75 \zeta(5)^2) + 42240 \pi^6 - 1353 \pi^8 + 5120 \pi^{10}}{39916800}$$

Alternative representations:

$$\left(\zeta(6) - \frac{\zeta(3)^2 9}{16} \right) - \frac{41 \zeta(8)}{128} + \left(12 \zeta(10) - \frac{\zeta(5)^2 675}{64} \right) =$$

$$- \frac{9 \zeta(3, 1)^2}{16} - \frac{675 \zeta(5, 1)^2}{64} + \zeta(6, 1) - \frac{41 \zeta(8, 1)}{128} + 12 \zeta(10, 1)$$

$$\left(\zeta(6) - \frac{\zeta(3)^2 9}{16} \right) - \frac{41 \zeta(8)}{128} + \left(12 \zeta(10) - \frac{\zeta(5)^2 675}{64} \right) =$$

$$S_{5,1}(1) + 12 S_{9,1}(1) - \frac{41 S_{7,1}(1)}{128} - \frac{9 S_{2,1}(1)^2}{16} - \frac{675 S_{4,1}(1)^2}{64}$$

$$\left(\zeta(6) - \frac{\zeta(3)^2 9}{16} \right) - \frac{41 \zeta(8)}{128} + \left(12 \zeta(10) - \frac{\zeta(5)^2 675}{64} \right) = \frac{\psi^{(5)}(1) (-1)^6}{5!} -$$

$$\frac{41 \psi^{(7)}(1) (-1)^8}{128 \times 7!} + \frac{12 \psi^{(9)}(1) (-1)^{10}}{9!} - \frac{9}{16} \left(-\frac{\psi^{(2)}(1)}{2!} \right)^2 - \frac{675}{64} \left(\frac{\psi^{(4)}(1) (-1)^5}{4!} \right)^2$$

$\zeta(s, a)$ is the generalized Riemann zeta function

$S_{n,p}(x)$ is the Nielsen generalized polylogarithm function

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

$n!$ is the factorial function

Series representations:

$$\left(\zeta(6) - \frac{\zeta(3)^2 9}{16} \right) - \frac{41 \zeta(8)}{128} + \left(12 \zeta(10) - \frac{\zeta(5)^2 675}{64} \right) =$$

$$\frac{42240 \pi^6 - 1353 \pi^8 + 5120 \pi^{10} - 420997500 \left(\sum_{k=1}^{\infty} \frac{1}{k^5} \right)^2 - 22453200 \left(\sum_{k=1}^{\infty} \frac{1}{k^3} \right)^2}{39916800}$$

$$\left(\zeta(6) - \frac{\zeta(3)^2 9}{16}\right) - \frac{41 \zeta(8)}{128} + \left(12 \zeta(10) - \frac{\zeta(5)^2 675}{64}\right) =$$

$$\frac{1}{268520313600} \left(284148480\pi^6 - 9101631\pi^8 + 34442240\pi^{10} - \right.$$

$$\left. 3017710080000 \left(\sum_{k=0}^{\infty} \frac{1}{(1+2k)^5}\right)^2 - 197280230400 \left(\sum_{k=0}^{\infty} \frac{1}{(1+2k)^3}\right)^2 \right)$$

$$\left(\zeta(6) - \frac{\zeta(3)^2 9}{16}\right) - \frac{41 \zeta(8)}{128} + \left(12 \zeta(10) - \frac{\zeta(5)^2 675}{64}\right) =$$

$$\frac{1}{39916800} \left(-22453200 e^{2 \sum_{k=1}^{\infty} P(3k)/k} - \right.$$

$$\left. 420997500 e^{2 \sum_{k=1}^{\infty} P(5k)/k} + 42240\pi^6 - 1353\pi^8 + 5120\pi^{10} \right)$$

$P(s)$ gives the prime zeta function

$$\left(\left(\left(\left(\zeta(6) - 9/16 * \zeta^2(3)\right) - \left(41/(x+3-1/\text{golden ratio}) * \zeta(8)\right) + \left(12 \zeta(10) - 675/64 * \zeta^2(5)\right)\right)\right)\right) = 0.5546780591171$$

Input interpretation:

$$\left(\zeta(6) - \frac{9}{16} \zeta(3)^2\right) - \frac{41}{x+3-\frac{1}{\phi}} \zeta(8) + \left(12 \zeta(10) - \frac{675}{64} \zeta(5)^2\right) = 0.5546780591171$$

$\zeta(s)$ is the Riemann zeta function

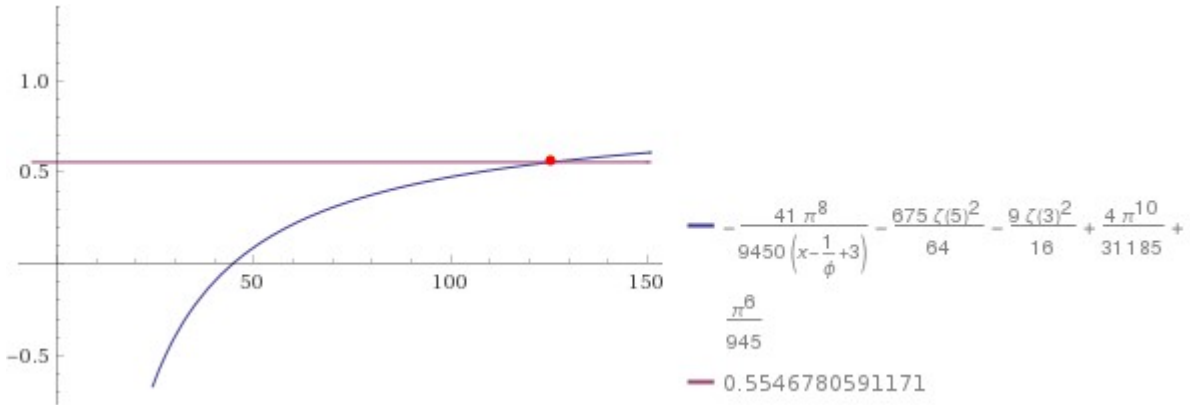
ϕ is the golden ratio

Result:

$$-\frac{41\pi^8}{9450\left(x - \frac{1}{\phi} + 3\right)} - \frac{675\zeta(5)^2}{64} - \frac{9\zeta(3)^2}{16} + \frac{4\pi^{10}}{31185} + \frac{\pi^6}{945} = 0.5546780591171$$

0.5546780591171

Plot:



Alternate form assuming x is real:

$$\frac{128.000000}{1.000000000 x + 2.381966011} = 1.000000$$

Alternate forms:

$$-\frac{41 \pi^8}{9450 \left(x + \frac{1}{2} (7 - \sqrt{5})\right)} - \frac{675 \zeta(5)^2}{64} - \frac{9 \zeta(3)^2}{16} + \frac{4 \pi^{10}}{31185} + \frac{\pi^6}{945} = 0.5546780591171$$

$$\begin{aligned} & \left(-105249375 \sqrt{5} x \zeta(5)^2 - 105249375 x \zeta(5)^2 - 5613300 \sqrt{5} x \zeta(3)^2 - \right. \\ & \quad 5613300 x \zeta(3)^2 + 1280 \sqrt{5} \pi^{10} x + 1280 \pi^{10} x + 10560 \sqrt{5} \pi^6 x + \\ & \quad 10560 \pi^6 x - 315748125 \sqrt{5} \zeta(5)^2 - 105249375 \zeta(5)^2 - \\ & \quad 16839900 \sqrt{5} \zeta(3)^2 - 5613300 \zeta(3)^2 + 3840 \sqrt{5} \pi^{10} + 1280 \pi^{10} - \\ & \quad \left. 43296 \sqrt{5} \pi^8 - 43296 \pi^8 + 31680 \sqrt{5} \pi^6 + 10560 \pi^6 \right) / \\ & \left(9979200 \left(\sqrt{5} x + x + 3 \sqrt{5} + 1 \right) \right) = 0.5546780591171 \end{aligned}$$

Alternate form assuming x is positive:

$$1.0000000000 x = 125.61803399$$

Solution:

$$x \approx 125.618033989$$

125.618033989 result very near to the Higgs boson mass 125.18 GeV

$$\left(\left(\left(\left(\zeta(6) - \frac{9}{16} \zeta(3)^2 \right) - \left(\frac{41}{x - 11 - \frac{1}{\phi}} \zeta(8) + \left(12 \zeta(10) - \frac{675}{64} \zeta(5)^2 \right) \right) + \left(12 \zeta(10) - \frac{675}{64} \zeta(5)^2 \right) \right) \right) \right) = 0.5546780591171$$

Input interpretation:

$$\left(\zeta(6) - \frac{9}{16} \zeta(3)^2 \right) - \frac{41}{x - 11 - \frac{1}{\phi}} \zeta(8) + \left(12 \zeta(10) - \frac{675}{64} \zeta(5)^2 \right) = 0.5546780591171$$

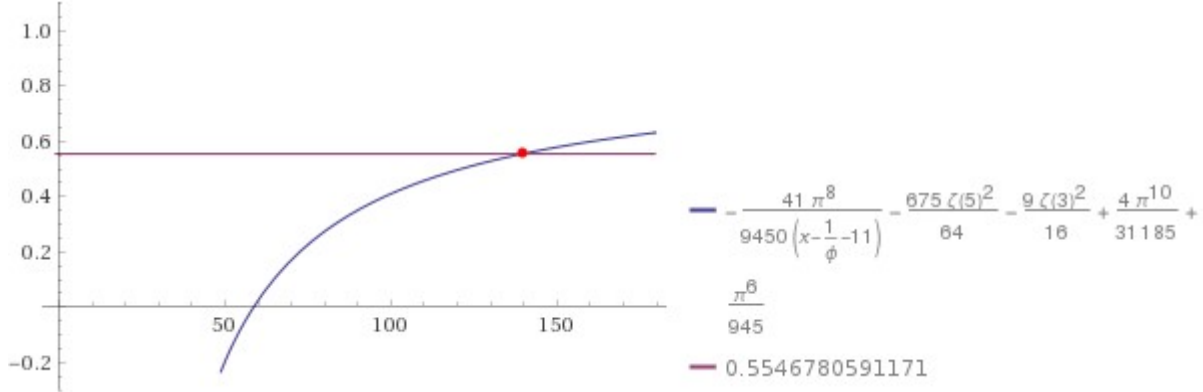
$\zeta(s)$ is the Riemann zeta function

ϕ is the golden ratio

Result:

$$-\frac{41 \pi^8}{9450 \left(x - \frac{1}{\phi} - 11\right)} - \frac{675 \zeta(5)^2}{64} - \frac{9 \zeta(3)^2}{16} + \frac{4 \pi^{10}}{31185} + \frac{\pi^6}{945} = 0.5546780591171$$

Plot:



Alternate forms:

$$-\frac{41 \pi^8}{9450 \left(x + \frac{1}{2} (-21 - \sqrt{5})\right)} - \frac{675 \zeta(5)^2}{64} - \frac{9 \zeta(3)^2}{16} + \frac{4 \pi^{10}}{31185} + \frac{\pi^6}{945} = 0.5546780591171$$

$$\begin{aligned} & \left(-105\,249\,375 \sqrt{5} x \zeta(5)^2 - 105\,249\,375 x \zeta(5)^2 - 5\,613\,300 \sqrt{5} x \zeta(3)^2 - \right. \\ & \quad 5\,613\,300 x \zeta(3)^2 + 1280 \sqrt{5} \pi^{10} x + 1280 \pi^{10} x + 10560 \sqrt{5} \pi^6 x + \\ & \quad 10560 \pi^6 x + 1\,157\,743\,125 \sqrt{5} \zeta(5)^2 + 1\,368\,241\,875 \zeta(5)^2 + \\ & \quad 61\,746\,300 \sqrt{5} \zeta(3)^2 + 72\,972\,900 \zeta(3)^2 - 14\,080 \sqrt{5} \pi^{10} - 16\,640 \pi^{10} - \\ & \quad \left. 43\,296 \sqrt{5} \pi^8 - 43\,296 \pi^8 - 116\,160 \sqrt{5} \pi^6 - 137\,280 \pi^6 \right) / \\ & \quad \left(9\,979\,200 \left(\sqrt{5} x + x - 11 \sqrt{5} - 13 \right) \right) = 0.5546780591171 \end{aligned}$$

Alternate form assuming x is positive:

$$-\frac{128.000000}{11.6180340 - 1.00000000 x} = 1.00000000$$

Solution:

$$x \approx 139.618033989$$

139.618033989 result practically equal to the rest mass of Pion meson 139.57 MeV

$$\left(\frac{\sin\left(\frac{8+\pi}{10}\right)}{\left(\zeta(6) - \frac{9}{16} \zeta(3)^2\right) - \left(\frac{41}{128} \zeta(8) + \left(12 \zeta(10) - \frac{675}{64} \zeta(5)^2\right)\right)}\right) + \left(\frac{12 \zeta(10) - \frac{675}{64} \zeta(5)^2}{\left(\zeta(6) - \frac{9}{16} \zeta(3)^2\right) - \left(\frac{41}{128} \zeta(8) + \left(12 \zeta(10) - \frac{675}{64} \zeta(5)^2\right)\right)}\right)$$

Input:

$$\frac{\sin\left(\frac{8+\pi}{10}\right)}{\left(\zeta(6) - \frac{9}{16} \zeta(3)^2\right) - \frac{41}{128} \zeta(8) + \left(12 \zeta(10) - \frac{675}{64} \zeta(5)^2\right)}$$

$\zeta(s)$ is the Riemann zeta function

Exact result:

$$\frac{\sin\left(\frac{8+\pi}{10}\right)}{-\frac{9\zeta(3)^2}{16} - \frac{675\zeta(5)^2}{64} + \frac{\pi^6}{945} - \frac{41\pi^8}{1209600} + \frac{4\pi^{10}}{31185}}$$

Decimal approximation:

1.618128540122640013726968562926579508618811247002459216601...

1.61812854012264.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

Alternate forms:

$$\frac{39916800 \sin\left(\frac{8+\pi}{10}\right)}{22453200 \zeta(3)^2 + 420997500 \zeta(5)^2 - 42240 \pi^6 + 1353 \pi^8 - 5120 \pi^{10}}$$

$$\frac{39916800 \sin\left(\frac{4}{5} + \frac{\pi}{10}\right)}{-22453200 \zeta(3)^2 - 420997500 \zeta(5)^2 + 42240 \pi^6 - 1353 \pi^8 + 5120 \pi^{10}}$$

$$\frac{39916800 \sin\left(\frac{4}{5} + \frac{\pi}{10}\right)}{-5613300 (4 \zeta(3)^2 + 75 \zeta(5)^2) + 42240 \pi^6 - 1353 \pi^8 + 5120 \pi^{10}}$$

Alternative representations:

$$\frac{\sin\left(\frac{8+\pi}{10}\right)}{\left(\zeta(6) - \frac{\zeta(3)^2}{16}\right) - \frac{41\zeta(8)}{128} + \left(12 \zeta(10) - \frac{\zeta(5)^2}{64}\right)} = \frac{\cos\left(\frac{\pi}{2} - \frac{8+\pi}{10}\right)}{-\frac{9\zeta(3,1)^2}{16} - \frac{675\zeta(5,1)^2}{64} + \zeta(6, 1) - \frac{41\zeta(8,1)}{128} + 12 \zeta(10, 1)}$$

$$\frac{\sin\left(\frac{8+\pi}{10}\right)}{\left(\zeta(6) - \frac{\zeta(3)^2}{16}\right) - \frac{41\zeta(8)}{128} + \left(12\zeta(10) - \frac{\zeta(5)^2}{64}\right)} = \frac{\cos\left(\frac{\pi}{2} + \frac{8+\pi}{10}\right)}{-\frac{9\zeta(3,1)^2}{16} - \frac{675\zeta(5,1)^2}{64} + \zeta(6, 1) - \frac{41\zeta(8,1)}{128} + 12\zeta(10, 1)}$$

$$\frac{\sin\left(\frac{8+\pi}{10}\right)}{\left(\zeta(6) - \frac{\zeta(3)^2}{16}\right) - \frac{41\zeta(8)}{128} + \left(12\zeta(10) - \frac{\zeta(5)^2}{64}\right)} = \frac{-e^{-1/10 i (8+\pi)} + e^{1/10 i (8+\pi)}}{(2i) \left(-\frac{9\zeta(3,1)^2}{16} - \frac{675\zeta(5,1)^2}{64} + \zeta(6, 1) - \frac{41\zeta(8,1)}{128} + 12\zeta(10, 1)\right)}$$

$\zeta(s, a)$ is the generalized Riemann zeta function

i is the imaginary unit

Series representations:

$$\frac{\sin\left(\frac{8+\pi}{10}\right)}{\left(\zeta(6) - \frac{\zeta(3)^2}{16}\right) - \frac{41\zeta(8)}{128} + \left(12\zeta(10) - \frac{\zeta(5)^2}{64}\right)} = \left(159667200 \sum_{k=0}^{\infty} \frac{(-1)^k 10^{-1-2k} (8+\pi)^{1+2k}}{(1+2k)!}\right) / \left(168960\pi^6 - 5412\pi^8 + 20480\pi^{10} - 105249375 \left(\sum_{n=0}^{\infty} \frac{\sum_{k=0}^n \frac{(-1)^k \binom{n}{k}}{(1+k)^4}}{1+n}\right)^2 - 22453200 \left(\sum_{n=0}^{\infty} \frac{\sum_{k=0}^n \frac{(-1)^k \binom{n}{k}}{(1+k)^2}}{1+n}\right)^2\right)$$

$$\frac{\sin\left(\frac{8+\pi}{10}\right)}{\left(\zeta(6) - \frac{\zeta(3)^2}{16}\right) - \frac{41\zeta(8)}{128} + \left(12\zeta(10) - \frac{\zeta(5)^2}{64}\right)} = \left(39916800 \sum_{k=0}^{\infty} \frac{(-1)^{3k} \left(\frac{2}{5}(-2+\pi)\right)^{2k}}{(2k)!}\right) / \left(42240\pi^6 - 1353\pi^8 + 5120\pi^{10} - 22453200 \left(\sum_{k=0}^{\infty} \frac{(3-s_0)^k \zeta^{(k)}(s_0)}{k!}\right)^2 - 420997500 \left(\sum_{k=0}^{\infty} \frac{(5-s_0)^k \zeta^{(k)}(s_0)}{k!}\right)^2\right) \text{ for } s_0 \neq 1$$

$$\frac{\sin\left(\frac{8+\pi}{10}\right)}{\left(\zeta(6) - \frac{\zeta(3)^2}{16}\right) - \frac{41\zeta(8)}{128} + \left(12\zeta(10) - \frac{\zeta(5)^2}{64}\right)} = \frac{\left(39\,916\,800 \sum_{k=0}^{\infty} \frac{(-1)^k 10^{-1-2k} (8+\pi)^{1+2k}}{(1+2k)!}\right)}{\left(42\,240\pi^6 - 1353\pi^8 + 5120\pi^{10} - 22\,453\,200 \left(\sum_{k=0}^{\infty} \frac{(3-s_0)^k \zeta^{(k)}(s_0)}{k!}\right)^2 - 420\,997\,500 \left(\sum_{k=0}^{\infty} \frac{(5-s_0)^k \zeta^{(k)}(s_0)}{k!}\right)^2\right)} \text{ for } s_0 \neq 1$$

$n!$ is the factorial function

$\binom{n}{m}$ is the binomial coefficient

Multiple-argument formulas:

$$\frac{\sin\left(\frac{8+\pi}{10}\right)}{\left(\zeta(6) - \frac{\zeta(3)^2}{16}\right) - \frac{41\zeta(8)}{128} + \left(12\zeta(10) - \frac{\zeta(5)^2}{64}\right)} = \frac{39\,916\,800 \sin\left(\frac{8+\pi}{10}\right)}{42\,240\pi^6 - 1353\pi^8 + 5120\pi^{10} - 5\,613\,300(4\zeta(3)^2 + 75\zeta(5)^2)}$$

$$\frac{\sin\left(\frac{8+\pi}{10}\right)}{\left(\zeta(6) - \frac{\zeta(3)^2}{16}\right) - \frac{41\zeta(8)}{128} + \left(12\zeta(10) - \frac{\zeta(5)^2}{64}\right)} = \frac{39\,916\,800 U_{\frac{9}{10}}(-\cos(8)) \sin(8)}{42\,240\pi^6 - 1353\pi^8 + 5120\pi^{10} - 5\,613\,300(4\zeta(3)^2 + 75\zeta(5)^2)}$$

$$\frac{\sin\left(\frac{8+\pi}{10}\right)}{\left(\zeta(6) - \frac{\zeta(3)^2}{16}\right) - \frac{41\zeta(8)}{128} + \left(12\zeta(10) - \frac{\zeta(5)^2}{64}\right)} = \frac{3 \sin\left(\frac{8+\pi}{30}\right) - 4 \sin^3\left(\frac{8+\pi}{30}\right)}{\frac{\pi^6}{945} - \frac{41\pi^8}{1209\,600} + \frac{4\pi^{10}}{31\,185} - \frac{9\zeta(3)^2}{16} - \frac{675\zeta(5)^2}{64}}$$

$U_n(x)$ is the Chebyshev polynomial of the second kind

$$= (-1)^{n-1} \frac{n!}{2^{n+1}} (1 - 2^{-n}) \zeta(n+1) \beta(n+1), \quad (1.55)$$

where ζ is the Riemann zeta function, η denotes the Dirichlet eta function, Li_n represents the Polylogarithm function, and β designates the Dirichlet beta function.

Examples:

For $n = 1$,

$$\int_0^1 \frac{\log(x) \text{Li}_2(-x)}{1+x^2} dx = \frac{1}{8} \zeta(2) G;$$

For $n = 2$,

$$\int_0^1 \frac{\log^2(x) \text{Li}_3(-x)}{1+x^2} dx = -\frac{3}{512} \pi^3 \zeta(3);$$

For $n = 3$,

$$\int_0^1 \frac{\log^3(x) \text{Li}_4(-x)}{1+x^2} dx = \frac{7}{2048} \left(\frac{1}{8} \zeta(4) \psi^{(3)}\left(\frac{1}{4}\right) - 105 \zeta(8) \right).$$

((((7/2048 (1/8* zeta(4) * digamma^3 (1/4) – 105 * zeta (8))))))

Input:

$$\frac{7}{2048} \left(\frac{1}{8} \zeta(4) \psi\left(\frac{1}{4}\right)^3 - 105 \zeta(8) \right)$$

$\zeta(s)$ is the Riemann zeta function

$\psi(x)$ is the digamma function

Exact result:

$$\frac{7 \left(\frac{1}{720} \pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 - \frac{\pi^8}{90} \right)}{2048}$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Decimal approximation:

-0.39528590153909081908260323993197653190592623144649304875...

-0.3952859015390908.....

Alternate forms:

$$\frac{7 \pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3}{1474560} - \frac{7 \pi^8}{184320}$$

$$\frac{7\pi^4 \left(8\pi^4 - \psi^{(0)}\left(\frac{1}{4}\right)^3\right)}{1474560}$$

$$\frac{7\left(8\pi^8 - \pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3\right)}{1474560}$$

Alternative representations:

$$\frac{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}{2048} = \frac{7\left(\frac{1}{8}\psi\left(\frac{1}{4}\right)^3\zeta(4,1) - 105\zeta(8,1)\right)}{2048}$$

$$\frac{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}{2048} = \frac{7\left(\frac{\psi\left(\frac{1}{4}\right)^3\zeta\left(4,\frac{1}{2}\right)}{8(-1+2^4)} - \frac{105\zeta\left(8,\frac{1}{2}\right)}{-1+2^8}\right)}{2048}$$

$$\frac{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}{2048} = \frac{7\left(\frac{1}{8}\left(\frac{\frac{\partial}{\partial x}\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}\right)}\right)^3\zeta(4,1) - 105\zeta(8,1)\right)}{2048}$$

$\zeta(s, a)$ is the generalized Riemann zeta function

$\Gamma(x)$ is the gamma function

Series representations:

$$\frac{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}{2048} = \frac{7\pi^4\left(-8\pi^4 - \left(4 + \gamma - \frac{1}{4}\sum_{k=1}^{\infty}\frac{{}_2\tilde{F}_1\left(1,2;2;-\frac{1}{4k}\right)^3}{k^2}\right)\right)}{1474560}$$

$$\frac{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}{2048} = -\frac{7\pi^8}{184320} + \frac{7\pi^4\left(\sum_{k=0}^{\infty}\frac{\psi^{(k)}(z_0)\left(\frac{1}{4}-z_0\right)^k}{k!}\right)^3}{1474560}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}{2048} = \frac{7\pi^4\left(-8\pi^4 - \frac{1}{64}\left(4(4+\gamma) - \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k(1+k)!\zeta(2+k)}{(2)_k}\right)\right)}{1474560}$$

${}_2\tilde{F}_1(a, b; c; x)$ is the regularized hypergeometric function

Alternative representations:

$$1 + \frac{1}{\sqrt{-\frac{1}{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}} = 1 + \frac{1}{\sqrt{-\frac{1}{7\left(\frac{1}{8}\psi\left(\frac{1}{4}\right)^3\zeta(4,1) - 105\zeta(8,1)\right)2048}}$$

$$1 + \frac{1}{\sqrt{-\frac{1}{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}} = 1 + \frac{1}{\sqrt{-\frac{1}{7\left(\frac{\psi\left(\frac{1}{4}\right)^3\zeta\left(4,\frac{1}{2}\right) - 105\zeta\left(8,\frac{1}{2}\right)}{8(-1+2^4)} - \frac{-1+2^8}{-1+2^8}\right)2048}}$$

$$1 + \frac{1}{\sqrt{-\frac{1}{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}} = 1 + \frac{1}{\sqrt{-\frac{1}{7\left(\frac{\left(\frac{\partial}{\partial 4}\Gamma\left(\frac{1}{4}\right)\right)^3}{\Gamma\left(\frac{1}{4}\right)}\right)\zeta(4,1) - 105\zeta(8,1)}{2048}}}$$

$\zeta(s, a)$ is the generalized Riemann zeta function

$\Gamma(x)$ is the gamma function

Series representations:

$$1 + \frac{1}{\sqrt{-\frac{1}{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}} =$$

$$1 + \frac{1}{384} \sqrt{\frac{7}{10}} \pi^2 \sqrt{8\pi^4 + \frac{1}{64} \left(4(4+\gamma) - \sum_{k=1}^{\infty} \frac{{}_2\bar{F}_1\left(1, 2; 2; -\frac{1}{4k}\right)}{k^2}\right)^3}$$

$$1 + \frac{1}{\sqrt{-\frac{1}{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}} =$$

$$1 + \frac{1}{32} \sqrt{\frac{7}{2}} \sqrt{\frac{\pi^8}{90} - \frac{1}{720} \pi^4 \left(-4 - \gamma + \frac{1}{4} \sum_{k=1}^{\infty} \frac{{}_2\bar{F}_1\left(1, 2; 2; -\frac{1}{4k}\right)}{k^2}\right)^3}$$

$$1 + \frac{1}{\sqrt{-\frac{1}{\frac{(\frac{1}{8} \zeta(4) \psi(\frac{1}{4})^3 - 105 \zeta(8)) 7}{2048}}}}} = 1 + \frac{1}{384} \sqrt{\frac{7}{10}} \pi^2 \sqrt{8 \pi^4 - \left(\sum_{k=0}^{\infty} \frac{\psi^{(k)}(z_0) \left(\frac{1}{4} - z_0\right)^k}{k!} \right)^3}$$

for not (($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

${}_2\tilde{F}_1(a, b; c; x)$ is the regularized hypergeometric function

γ is the Euler-Mascheroni constant

$n!$ is the factorial function

\mathbb{R} is the set of real numbers

$(((-55/((7/2048 (1/8 * \zeta(4) * \text{digamma}^3(1/4) - 105 * \zeta(8)))))))+1/\text{golden ratio}$

Input:

$$-\frac{55}{\frac{7}{2048} \left(\frac{1}{8} \zeta(4) \psi\left(\frac{1}{4}\right)^3 - 105 \zeta(8) \right)} + \frac{1}{\phi}$$

$\zeta(s)$ is the Riemann zeta function

$\psi(x)$ is the digamma function

ϕ is the golden ratio

Exact result:

$$\frac{1}{\phi} - \frac{112640}{7 \left(\frac{1}{720} \pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 - \frac{\pi^8}{90} \right)}$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Decimal approximation:

139.7578307430768674176687644811998004439695377182242333353...

139.75783074... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternate forms:

$$\frac{1}{\phi} + \frac{81100800}{56 \pi^8 - 7 \pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3}$$

$$\frac{1}{\phi} + \frac{81\,100\,800}{7\pi^4 \left(8\pi^4 - \psi^{(0)}\left(\frac{1}{4}\right)^3\right)}$$

$$\frac{1}{2}(\sqrt{5} - 1) - \frac{112\,640}{7\left(\frac{1}{720}\pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 - \frac{\pi^8}{90}\right)}$$

Alternative representations:

$$-\frac{\frac{55}{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}}{2048} + \frac{1}{\phi} = \frac{1}{\phi} - \frac{\frac{55}{7\left(\frac{1}{8}\psi\left(\frac{1}{4}\right)^3 \zeta(4,1) - 105\zeta(8,1)\right)}}{2048}$$

$$-\frac{\frac{55}{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}}{2048} + \frac{1}{\phi} = \frac{1}{\phi} - \frac{\frac{55}{7\left(\frac{\psi\left(\frac{1}{4}\right)^3 \zeta\left(4, \frac{1}{2}\right) - 105\zeta\left(8, \frac{1}{2}\right)}{8(-1+2^4)} - \frac{-1+2^8}{-1+2^8}\right)}}{2048}$$

$$-\frac{\frac{55}{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}}{2048} + \frac{1}{\phi} = \frac{1}{\phi} - \frac{\frac{55}{7\left(\frac{1}{8}\left(\frac{\frac{\partial}{\partial x}\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}\right)}\right)^3 \zeta(4,1) - 105\zeta(8,1)\right)}}{2048}$$

$\zeta(s, a)$ is the generalized Riemann zeta function

$\Gamma(x)$ is the gamma function

Series representations:

$$-\frac{\frac{55}{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}}{2048} + \frac{1}{\phi} = \frac{1}{\phi} - \frac{\frac{112\,640}{7\left(-\frac{\pi^8}{90} + \frac{1}{720}\pi^4\left(-4 - \gamma + \frac{1}{4}\sum_{k=1}^{\infty}\frac{{}_2F_1\left(1,2;2;-\frac{1}{4k}\right)}{k^2}\right)^3\right)}}{2048}$$

$$-\frac{\frac{55}{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7}}{2048} + \frac{1}{\phi} = \frac{1}{\phi} - \frac{\frac{112\,640}{7\left(-\frac{\pi^8}{90} + \frac{1}{720}\pi^4\left(\sum_{k=0}^{\infty}\frac{\psi^{(k)}(z_0)\left(\frac{1}{4}-z_0\right)^k}{k!}\right)^3\right)}}{2048}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$-\frac{55}{\frac{\left(\frac{1}{8} \zeta(4) \psi\left(\frac{1}{4}\right)^3 - 105 \zeta(8)\right) 7}{2048}} + \frac{1}{\phi} = \frac{1}{\phi} + \frac{81\,100\,800}{56 \pi^8 - 7 \pi^4 \left(\sum_{k=0}^{\infty} \frac{\left(\frac{1}{4} - z_0\right)^k \psi^{(0)}(0, k)(z_0)}{k!} \right)^3}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

${}_2\tilde{F}_1(a, b; c; x)$ is the regularized hypergeometric function

γ is the Euler-Mascheroni constant

$n!$ is the factorial function

\mathbb{R} is the set of real numbers

$(((-55/((7/2048 (1/8 * \zeta(4) * \text{digamma}^3(1/4) - 105 * \zeta(8)))))))-13-1/\text{golden ratio}$

Input:

$$-\frac{55}{\frac{7}{2048} \left(\frac{1}{8} \zeta(4) \psi\left(\frac{1}{4}\right)^3 - 105 \zeta(8) \right)} - 13 - \frac{1}{\phi}$$

$\zeta(s)$ is the Riemann zeta function

$\psi(x)$ is the digamma function

ϕ is the golden ratio

Exact result:

$$-\frac{1}{\phi} - 13 - \frac{112\,640}{7 \left(\frac{1}{720} \pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 - \frac{\pi^8}{90} \right)}$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Decimal approximation:

125.5217627655770777212595908124685242085289193586127076110...

125.52176276... result very near to the Higgs boson mass 125.18 GeV

Alternate forms:

$$-\frac{1}{\phi} - 13 + \frac{81\,100\,800}{56 \pi^8 - 7 \pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3}$$

$$-\frac{1}{\phi} - 13 + \frac{81\,100\,800}{7\pi^4 \left(8\pi^4 - \psi^{(0)}\left(\frac{1}{4}\right)^3\right)}$$

$$\frac{1}{2} \left(-25 - \sqrt{5}\right) - \frac{112\,640}{7\left(\frac{1}{720}\pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 - \frac{\pi^8}{90}\right)}$$

Alternative representations:

$$-\frac{55}{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7} - 13 - \frac{1}{\phi} = -13 - \frac{1}{\phi} - \frac{55}{7\left(\frac{1}{8}\psi\left(\frac{1}{4}\right)^3 \zeta(4,1) - 105\zeta(8,1)\right)}$$

$$-\frac{55}{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7} - 13 - \frac{1}{\phi} = -13 - \frac{1}{\phi} - \frac{55}{7\left(\frac{\psi\left(\frac{1}{4}\right)^3 \zeta\left(4, \frac{1}{2}\right) - 105\zeta\left(8, \frac{1}{2}\right)}{8(-1+2^4) - 1+2^8}\right)}$$

$$-\frac{55}{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7} - 13 - \frac{1}{\phi} = -13 - \frac{1}{\phi} - \frac{55}{7\left(\frac{\frac{\partial}{\partial \frac{1}{4}} \Gamma\left(\frac{1}{4}\right)^3}{\Gamma\left(\frac{1}{4}\right)} \zeta(4,1) - 105\zeta(8,1)\right)}$$

$\zeta(s, a)$ is the generalized Riemann zeta function

$\Gamma(x)$ is the gamma function

Series representations:

$$-\frac{55}{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7} - 13 - \frac{1}{\phi} = -13 - \frac{1}{\phi} - \frac{112\,640}{7\left(-\frac{\pi^8}{90} + \frac{1}{720}\pi^4 \left(-4 - \gamma + \frac{1}{4} \sum_{k=1}^{\infty} \frac{{}_2\tilde{F}_1\left(1,2;2;-\frac{1}{4k}\right)^3}{k^2}\right)\right)}$$

$$-\frac{55}{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)7} - 13 - \frac{1}{\phi} = -13 - \frac{1}{\phi} + \frac{81\,100\,800}{56\pi^8 - 7\pi^4 \left(\sum_{k=0}^{\infty} \frac{\psi^{(k)}(z_0) \left(\frac{1}{4} - z_0\right)^k}{k!}\right)^3}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$-\frac{55}{\frac{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3-105\zeta(8)\right)7}{2048}}-13-\frac{1}{\phi}=-13-\frac{1}{\phi}+\frac{81100800}{56\pi^8-7\pi^4\left(\sum_{k=0}^{\infty}\frac{\left(\frac{1}{4}-z_0\right)^k\psi^{(0)(0,k)}(z_0)}{k!}\right)^3}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

${}_2\tilde{F}_1(a, b; c; x)$ is the regularized hypergeometric function

γ is the Euler-Mascheroni constant

$n!$ is the factorial function

\mathbb{R} is the set of real numbers

$$27 \times \frac{1}{2} \times \left(\left(\left(\left(\left(\left(-55 / \left(\frac{7}{2048} \left(\frac{1}{8} \zeta(4) \psi\left(\frac{1}{4}\right)^3 - 105 \zeta(8) \right) \right) \right) \right) \right) \right) \right) \right) - 13 - \frac{1}{\text{golden ratio} + 5/2} \right) + (8 \times 2 \times 5) / 10^2$$

Input:

$$27 \times \frac{1}{2} \left(-\frac{55}{\frac{7}{2048} \left(\frac{1}{8} \zeta(4) \psi\left(\frac{1}{4}\right)^3 - 105 \zeta(8) \right)} - 13 - \frac{1}{\phi} + \frac{5}{2} \right) + (8 \times 2 \times 5) \times \frac{1}{10^2}$$

$\zeta(s)$ is the Riemann zeta function

$\psi(x)$ is the digamma function

ϕ is the golden ratio

Exact result:

$$\frac{27}{2} \left(-\frac{1}{\phi} - \frac{21}{2} - \frac{112640}{7 \left(\frac{1}{720} \pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 - \frac{\pi^8}{90} \right)} \right) + \frac{4}{5}$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Decimal approximation:

1729.093797335290549237004475968325076815140411341271552749...

1729.0937973....

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms:

$$-\frac{27}{2\phi} - \frac{2819}{20} + \frac{1094860800}{7\pi^4 \left(8\pi^4 - \psi^{(0)}\left(\frac{1}{4}\right)^3\right)}$$

$$-\frac{671}{5} - \frac{27\sqrt{5}}{4} + \frac{1094860800}{56\pi^8 - 7\pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3}$$

$$-\frac{27}{2\phi} - \frac{2819}{20} - \frac{1520640}{7\left(\frac{1}{720}\pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 - \frac{\pi^8}{90}\right)}$$

Alternative representations:

$$\frac{27}{2} \left(-\frac{55}{7 \frac{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)}{2048}} - 13 - \frac{1}{\phi} + \frac{5}{2} \right) + \frac{8(2 \times 5)}{10^2} =$$

$$\frac{80}{10^2} + \frac{27}{2} \left(-\frac{21}{2} - \frac{1}{\phi} - \frac{55}{7 \frac{\left(\frac{1}{8}\psi\left(\frac{1}{4}\right)^3 \zeta(4,1) - 105\zeta(8,1)\right)}{2048}} \right)$$

$$\frac{27}{2} \left(-\frac{55}{7 \frac{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)}{2048}} - 13 - \frac{1}{\phi} + \frac{5}{2} \right) + \frac{8(2 \times 5)}{10^2} =$$

$$\frac{80}{10^2} + \frac{27}{2} \left(-\frac{21}{2} - \frac{1}{\phi} - \frac{55}{7 \frac{\left(\frac{\psi\left(\frac{1}{4}\right)^3 \zeta\left(4, \frac{1}{2}\right) - 105\zeta\left(8, \frac{1}{2}\right)\right)}{8(-1+2^4)} - \frac{105\zeta(8, \frac{1}{2})}{-1+2^8}}{2048}} \right)$$

$$\frac{27}{2} \left(-\frac{55}{7 \frac{\left(\frac{1}{8}\zeta(4)\psi\left(\frac{1}{4}\right)^3 - 105\zeta(8)\right)}{2048}} - 13 - \frac{1}{\phi} + \frac{5}{2} \right) + \frac{8(2 \times 5)}{10^2} =$$

$$\frac{80}{10^2} + \frac{27}{2} \left(-\frac{21}{2} - \frac{1}{\phi} - \frac{55}{7 \left(\frac{\left(\frac{\frac{\partial}{\partial \frac{1}{4}} \Gamma\left(\frac{1}{4}\right)}{4} \right)^3}{\Gamma\left(\frac{1}{4}\right)} \right) \zeta(4,1) - 105\zeta(8,1)}{2048}} \right)$$

$\zeta(s, a)$ is the generalized Riemann zeta function

$\Gamma(x)$ is the gamma function

Series representations:

$$\frac{27}{2} \left(-\frac{55}{7 \left(\frac{1}{8} \zeta(4) \psi \left(\frac{1}{4} \right)^3 - 105 \zeta(8) \right)} - 13 - \frac{1}{\phi} + \frac{5}{2} \right) + \frac{8(2 \times 5)}{10^2} =$$

$$\frac{2819}{20} - \frac{27}{2\phi} - \frac{1520640}{7 \left(-\frac{\pi^8}{90} + \frac{1}{720} \pi^4 \left(-4 - \gamma + \frac{1}{4} \sum_{k=1}^{\infty} \frac{{}_2\tilde{F}_1 \left(1, 2; 2; -\frac{1}{4k} \right)^3}{k^2} \right) \right)}$$

$$\frac{27}{2} \left(-\frac{55}{7 \left(\frac{1}{8} \zeta(4) \psi \left(\frac{1}{4} \right)^3 - 105 \zeta(8) \right)} - 13 - \frac{1}{\phi} + \frac{5}{2} \right) + \frac{8(2 \times 5)}{10^2} =$$

$$\frac{2819}{20} - \frac{27}{2\phi} - \frac{1520640}{7 \left(-\frac{\pi^8}{90} + \frac{1}{720} \pi^4 \left(\sum_{k=0}^{\infty} \frac{\psi^{(k)}(z_0) \left(\frac{1}{4} - z_0 \right)^k}{k!} \right)^3 \right)}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{27}{2} \left(-\frac{55}{7 \left(\frac{1}{8} \zeta(4) \psi \left(\frac{1}{4} \right)^3 - 105 \zeta(8) \right)} - 13 - \frac{1}{\phi} + \frac{5}{2} \right) + \frac{8(2 \times 5)}{10^2} =$$

$$\frac{2819}{20} - \frac{27}{2\phi} - \frac{1520640}{7 \left(-\frac{\pi^8}{90} + \frac{1}{720} \pi^4 \left(4 \left(-1 - \frac{\gamma}{4} \right) + \frac{1}{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4} \right)^k (1+k)! \zeta(2+k)}{(2)_k} \right)^3 \right)}$$

${}_2\tilde{F}_1(a, b; c; x)$ is the regularized hypergeometric function

γ is the Euler-Mascheroni constant

$n!$ is the factorial function

\mathbb{R} is the set of real numbers

$(a)_n$ is the Pochhammer symbol (rising factorial)

Li_n denotes the Polylogarithm function, and β designates the Dirichlet beta function.

Examples:

For the integral at i), $n = 1$,

$$\int_0^1 \frac{x \log(x) \text{Li}_2(x)}{1+x^2} dx = \frac{1}{2} \left(G^2 - \frac{123}{128} \zeta(4) \right);$$

For the integral at ii), $n = 1$,

$$\int_0^1 \frac{x \log(x) \text{Li}_2(-x)}{1+x^2} dx = \frac{1}{2} \left(\frac{117}{128} \zeta(4) - G^2 \right);$$

For the integral at i), $n = 2$,

$$\int_0^1 \frac{x \log^2(x) \text{Li}_3(x)}{1+x^2} dx = \frac{1}{1024} \left(79\zeta(6) - 9\zeta^2(3) \right);$$

For the integral at ii), $n = 2$,

$$\int_0^1 \frac{x \log^2(x) \text{Li}_3(-x)}{1+x^2} dx = -\frac{1}{1024} \left(47\zeta(6) + 9\zeta^2(3) \right);$$

For the integral at i), $n = 3$,

$$\begin{aligned} & \int_0^1 \frac{x \log^3(x) \text{Li}_4(x)}{1+x^2} dx \\ &= \frac{1}{2048} \left(\frac{2839}{16} \zeta(8) - 15\zeta(4)\psi^{(3)}\left(\frac{1}{4}\right) + \frac{1}{96} \left(\psi^{(3)}\left(\frac{1}{4}\right) \right)^2 \right); \end{aligned}$$

1/2048((((2839/16) zeta(8) - 15 zeta(4)
digamma^3(1/4)+1/96(digamma^3(1/4))^2))))

Input:

$$\frac{1}{2048} \left(\frac{2839}{16} \zeta(8) - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3 \right)^2 \right)$$

$\zeta(s)$ is the Riemann zeta function

$\psi(x)$ is the digamma function

Exact result:

$$\frac{\frac{2839\pi^8}{151200} - \frac{1}{6}\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 + \frac{\psi^{(0)}\left(\frac{1}{4}\right)^6}{96}}{2048}$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Decimal approximation:

0.714925017875077243771531179526212123735404437715372679391...

0.714925017875....

Alternate forms:

$$\frac{2839 \pi^8 - 25200 \pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 + 1575 \psi^{(0)}\left(\frac{1}{4}\right)^6}{309657600}$$

$$\frac{2839 \pi^8}{309657600} - \frac{\pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3}{12288} + \frac{\psi^{(0)}\left(\frac{1}{4}\right)^6}{196608}$$

$$\frac{\frac{2839\pi^8}{151200} - \frac{1}{6} \pi^4 \left(-\gamma - \frac{\pi}{2} - \log(8)\right)^3 + \frac{1}{96} \left(-\gamma - \frac{\pi}{2} - \log(8)\right)^6}{2048}$$

$\log(x)$ is the natural logarithm

γ is the Euler-Mascheroni constant

Alternative representations:

$$\frac{\frac{2839\zeta(8)}{16} - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2}{2048} = \frac{\frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2 - 15 \psi\left(\frac{1}{4}\right)^3 \zeta(4, 1) + \frac{2839\zeta(8,1)}{16}}{2048}$$

$$\frac{\frac{2839\zeta(8)}{16} - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2}{2048} = \frac{\frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2 - \frac{15 \psi\left(\frac{1}{4}\right)^3 \zeta\left(4, \frac{1}{2}\right)}{-1+2^4} + \frac{2839\zeta\left(8, \frac{1}{2}\right)}{16(-1+2^8)}}{2048}$$

$$\frac{\frac{2839\zeta(8)}{16} - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2}{2048} = \frac{\frac{1}{96} \left(\left(\frac{\frac{\partial}{\partial \frac{1}{4}} \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} \right)^3 \right)^2 - 15 \left(\frac{\frac{\partial}{\partial \frac{1}{4}} \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} \right)^3 \zeta(4, 1) + \frac{2839\zeta(8,1)}{16}}{2048}$$

Series representations:

$$\frac{\frac{2839\zeta(8)}{16} - 15\zeta(4)\psi\left(\frac{1}{4}\right)^3 + \frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2}{2048} = \frac{\frac{2839\pi^8}{151200} - \frac{1}{6}\pi^4\left(-4 - \gamma + \frac{1}{4}\sum_{k=1}^{\infty}\frac{{}_2\tilde{F}_1\left(1,2;2;-\frac{1}{4k}\right)}{k^2}\right)^3 + \frac{1}{96}\left(-4 - \gamma + \frac{1}{4}\sum_{k=1}^{\infty}\frac{{}_2\tilde{F}_1\left(1,2;2;-\frac{1}{4k}\right)}{k^2}\right)^6}{2048}$$

$$\frac{\frac{2839\zeta(8)}{16} - 15\zeta(4)\psi\left(\frac{1}{4}\right)^3 + \frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2}{2048} = \frac{2839\pi^8 - 25200\pi^4\left(\sum_{k=0}^{\infty}\frac{\psi^{(k)}(z_0)\left(\frac{1-z_0}{4}\right)^k}{k!}\right)^3 + 1575\left(\sum_{k=0}^{\infty}\frac{\psi^{(k)}(z_0)\left(\frac{1-z_0}{4}\right)^k}{k!}\right)^6}{309657600}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{\frac{2839\zeta(8)}{16} - 15\zeta(4)\psi\left(\frac{1}{4}\right)^3 + \frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2}{2048} = \frac{1}{2048}\left(\frac{2839\pi^8}{151200} - \frac{1}{6}\pi^4\left(4\left(-1 - \frac{\gamma}{4}\right) + \frac{1}{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k(1+k)!\zeta(2+k)}{(2)_k}\right)^3 + \frac{1}{96}\left(4\left(-1 - \frac{\gamma}{4}\right) + \frac{1}{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k(1+k)!\zeta(2+k)}{(2)_k}\right)^6\right)$$

${}_2\tilde{F}_1(a, b; c; x)$ is the regularized hypergeometric function

$n!$ is the factorial function

\mathbb{R} is the set of real numbers

$(a)_n$ is the Pochhammer symbol (rising factorial)

89/((((1/2048((((2839/16) zeta(8) - 15 zeta(4)
digamma^3(1/4)+1/96(digamma^3(1/4))^2)))))))+1/golden ratio

Input:

$$\frac{89}{\frac{1}{2048} \left(\frac{2839}{16} \zeta(8) - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2 \right)} + \frac{1}{\phi}$$

$\zeta(s)$ is the Riemann zeta function

$\psi(x)$ is the digamma function

ϕ is the golden ratio

Exact result:

$$\frac{1}{\phi} + \frac{182272}{\frac{2839\pi^8}{151200} - \frac{1}{6}\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 + \frac{\psi^{(0)}\left(\frac{1}{4}\right)^6}{96}}$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Decimal approximation:

125.1066135946624361314329036493812527966777292951922813191...

125.10661359... result very near to the Higgs boson mass 125.18 GeV

Alternate forms:

$$\frac{1}{\phi} + \frac{27559526400}{2839\pi^8 - 25200\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 + 1575\psi^{(0)}\left(\frac{1}{4}\right)^6}$$

$$\frac{2}{1+\sqrt{5}} + \frac{27559526400}{2839\pi^8 - 25200\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 + 1575\psi^{(0)}\left(\frac{1}{4}\right)^6}$$

$$\frac{1}{2}(\sqrt{5}-1) + \frac{182272}{\frac{2839\pi^8}{151200} - \frac{1}{6}\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 + \frac{\psi^{(0)}\left(\frac{1}{4}\right)^6}{96}}$$

Alternative representations:

$$\frac{\frac{89}{\frac{2839\zeta(8)}{16} - 15\zeta(4)\psi\left(\frac{1}{4}\right)^3 + \frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2}}{2048} + \frac{1}{\phi} = \frac{1}{\phi} + \frac{\frac{89}{\frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2 - 15\psi\left(\frac{1}{4}\right)^3\zeta(4,1) + \frac{2839\zeta(8,1)}{16}}}{2048}$$

$$\frac{\frac{89}{\frac{2839 \zeta(8)}{16} - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2}}{2048} + \frac{1}{\phi} = \frac{1}{\phi} + \frac{89}{\frac{\frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2 - \frac{15 \psi\left(\frac{1}{4}\right)^3 \zeta\left(4, \frac{1}{2}\right) + 2839 \zeta\left(8, \frac{1}{2}\right)}{-1+2^4}}{2048}}$$

$$\frac{\frac{89}{\frac{2839 \zeta(8)}{16} - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2}}{2048} + \frac{1}{\phi} = \frac{1}{\phi} + \frac{89}{\frac{\frac{1}{96} \left(\left(\frac{\frac{\partial}{\partial \frac{1}{4}} \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} \right)^3 \right)^2 - 15 \left(\frac{\frac{\partial}{\partial \frac{1}{4}} \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} \right)^3 \zeta(4, 1) + \frac{2839 \zeta(8, 1)}{16}}{2048}}$$

Series representations:

$$\frac{\frac{89}{\frac{2839 \zeta(8)}{16} - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2}}{2048} + \frac{1}{\phi} = \frac{1}{\phi} + 182\,272 \left/ \left(\frac{2839 \pi^8}{151\,200} - \frac{1}{6} \pi^4 \left(-4 - \gamma + \frac{1}{4} \sum_{k=1}^{\infty} \frac{{}_2\tilde{F}_1\left(1, 2; 2; -\frac{1}{4k}\right)^3}{k^2} \right)^3 + \frac{1}{96} \left(-4 - \gamma + \frac{1}{4} \sum_{k=1}^{\infty} \frac{{}_2\tilde{F}_1\left(1, 2; 2; -\frac{1}{4k}\right)^6}{k^2} \right) \right)$$

$$\frac{\frac{89}{\frac{2839 \zeta(8)}{16} - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2}}{2048} + \frac{1}{\phi} = \frac{1}{\phi} + 182\,272 \left/ \left(\frac{2839 \pi^8}{151\,200} - \frac{1}{6} \pi^4 \left(4 \left(-1 - \frac{\gamma}{4} \right) + \frac{1}{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k (1+k)! \zeta(2+k)}{(2)_k} \right)^3 + \frac{1}{96} \left(4 \left(-1 - \frac{\gamma}{4} \right) + \frac{1}{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k (1+k)! \zeta(2+k)}{(2)_k} \right)^6 \right)$$

$$\frac{89}{\frac{\frac{2839}{16} \zeta(8) - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2}{2048}} + \frac{1}{\phi} =$$

$$\left(2 \left(13\,779\,763\,200 + 13\,779\,763\,200 \sqrt{5} + 2839 \pi^8 - 25\,200 \pi^4 \right. \right.$$

$$\left. \left. \left(\sum_{k=0}^{\infty} \frac{\psi^{(k)}(z_0) \left(\frac{1}{4} - z_0\right)^k}{k!} \right)^3 + 1575 \left(\sum_{k=0}^{\infty} \frac{\psi^{(k)}(z_0) \left(\frac{1}{4} - z_0\right)^k}{k!} \right)^6 \right) \right) / \left((1 + \sqrt{5}) \right.$$

$$\left. \left(2839 \pi^8 - 25\,200 \pi^4 \left(\sum_{k=0}^{\infty} \frac{\psi^{(k)}(z_0) \left(\frac{1}{4} - z_0\right)^k}{k!} \right)^3 + 1575 \left(\sum_{k=0}^{\infty} \frac{\psi^{(k)}(z_0) \left(\frac{1}{4} - z_0\right)^k}{k!} \right)^6 \right) \right)$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

${}_2\tilde{F}_1(a, b; c; x)$ is the regularized hypergeometric function

γ is the Euler-Mascheroni constant

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

\mathbb{R} is the set of real numbers

89/((((1/2048((((2839/16) zeta(8) - 15 zeta(4)
digamma^3(1/4)+1/96(digamma^3(1/4))^2))))))))+13+golden ratio

Input:

$$\frac{89}{\frac{1}{2048} \left(\frac{2839}{16} \zeta(8) - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2 \right)} + 13 + \phi$$

$\zeta(s)$ is the Riemann zeta function

$\psi(x)$ is the digamma function

ϕ is the golden ratio

Exact result:

$$\phi + 13 + \frac{182\,272}{\frac{2839 \pi^8}{151\,200} - \frac{1}{6} \pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 + \frac{\psi^{(0)}\left(\frac{1}{4}\right)^6}{96}}$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Decimal approximation:

139.1066135946624361314329036493812527966777292951922813191...

139.10661359... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternate forms:

$$\phi + 13 + \frac{27559526400}{2839\pi^8 - 25200\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 + 1575\psi^{(0)}\left(\frac{1}{4}\right)^6}$$

$$\frac{1}{2}\left(27 + \sqrt{5}\right) + \frac{27559526400}{2839\pi^8 - 25200\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 + 1575\psi^{(0)}\left(\frac{1}{4}\right)^6}$$

$$13 + \frac{1}{2}\left(1 + \sqrt{5}\right) + \frac{182272}{\frac{2839\pi^8}{151200} - \frac{1}{6}\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 + \frac{\psi^{(0)}\left(\frac{1}{4}\right)^6}{96}}$$

Alternative representations:

$$\frac{\frac{89}{\frac{2839\zeta(8)}{16} - 15\zeta(4)\psi\left(\frac{1}{4}\right)^3 + \frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2}}{2048} + 13 + \phi = 13 + \phi + \frac{\frac{89}{\frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2 - 15\psi\left(\frac{1}{4}\right)^3\zeta(4,1) + \frac{2839\zeta(8,1)}{16}}}{2048}$$

$$\frac{\frac{89}{\frac{2839\zeta(8)}{16} - 15\zeta(4)\psi\left(\frac{1}{4}\right)^3 + \frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2}}{2048} + 13 + \phi = 13 + \phi + \frac{\frac{89}{\frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2 - \frac{15\psi\left(\frac{1}{4}\right)^3\zeta\left(4, \frac{1}{2}\right) + \frac{2839\zeta\left(8, \frac{1}{2}\right)}{16(-1+2^8)}}{-1+2^4}}}{2048}$$

$$\frac{\frac{89}{\frac{2839\zeta(8)}{16} - 15\zeta(4)\psi\left(\frac{1}{4}\right)^3 + \frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2}}{2048} + 13 + \phi =$$

$$13 + \phi + \frac{\frac{89}{\frac{1}{96}\left(\left(\frac{\frac{\partial}{\partial \frac{1}{4}}\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}\right)}\right)^3\right)^2 - 15\left(\frac{\frac{\partial}{\partial \frac{1}{4}}\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}\right)}\right)^3\zeta(4,1) + \frac{2839\zeta(8,1)}{16}}}{2048}}$$

Series representations:

$$\frac{89}{\frac{2839 \zeta(8)}{16} - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2} + 13 + \phi =$$

$$\frac{2048}{13 + \phi + 182\,272} / \left(\frac{2839 \pi^8}{151\,200} - \frac{1}{6} \pi^4 \left(-4 - \gamma + \frac{1}{4} \sum_{k=1}^{\infty} \frac{{}_2\bar{F}_1\left(1, 2; 2; -\frac{1}{4k}\right)^3}{k^2} \right)^3 + \right.$$

$$\left. \frac{1}{96} \left(-4 - \gamma + \frac{1}{4} \sum_{k=1}^{\infty} \frac{{}_2\bar{F}_1\left(1, 2; 2; -\frac{1}{4k}\right)^6}{k^2} \right) \right)$$

$$\frac{89}{\frac{2839 \zeta(8)}{16} - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2} + 13 + \phi =$$

$$\frac{2048}{13 + \phi + 182\,272} / \left(\frac{2839 \pi^8}{151\,200} - \frac{1}{6} \pi^4 \left(4 \left(-1 - \frac{\gamma}{4}\right) + \frac{1}{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k (1+k)! \zeta(2+k)}{(2)_k} \right)^3 + \right.$$

$$\left. \frac{1}{96} \left(4 \left(-1 - \frac{\gamma}{4}\right) + \frac{1}{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k (1+k)! \zeta(2+k)}{(2)_k} \right)^6 \right)$$

$$\frac{89}{\frac{2839 \zeta(8)}{16} - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2} + 13 + \phi \propto 13 + \phi + 182\,272 / \left(\frac{2839 \pi^8}{151\,200} - \right.$$

$$\left. \frac{1}{6} \pi^4 \left(-\frac{1}{4} + m + \sum_{k=0}^{\infty} \frac{\left(\frac{1}{4} + m\right)^k \left((1 + (-1)^{1+k}) \psi^{(k)}(1) + (-1)^k \psi^{(k)}(1+m) \right)}{k!} \right)^3 + \right.$$

$$\left. \frac{1}{96} \left(-\frac{1}{4} + m + \sum_{k=0}^{\infty} \frac{\left(\frac{1}{4} + m\right)^k \left((1 + (-1)^{1+k}) \psi^{(k)}(1) + (-1)^k \psi^{(k)}(1+m) \right)}{k!} \right)^6 \right)$$

for $\left(-m \rightarrow \frac{1}{4} \text{ and } m \in \mathbb{Z} \text{ and } m \geq 0\right)$

${}_2\bar{F}_1(a, b; c; x)$ is the regularized hypergeometric function

γ is the Euler-Mascheroni constant

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

\mathbb{Z} is the set of integers

$27 \cdot \frac{1}{2} \cdot \left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\frac{89}{2048} \left(\frac{2839}{16} \zeta(8) - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3 \right)^2 \right) + 4 \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) - 5 - \frac{1}{\phi} \right)$
 digamma³(1/4)+1/96(digamma³(1/4))^2)))))))+4))) -5-1/golden ratio

Input:

$$27 \times \frac{1}{2} \left(\frac{89}{\frac{1}{2048} \left(\frac{2839}{16} \zeta(8) - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3 \right)^2 \right)} + 4 \right) - 5 - \frac{1}{\phi}$$

$\zeta(s)$ is the Riemann zeta function

$\psi(x)$ is the digamma function

ϕ is the golden ratio

Exact result:

$$-\frac{1}{\phi} - 5 + \frac{27}{2} \left(4 + \frac{182\,272}{\frac{2839\pi^8}{151\,200} - \frac{1}{6} \pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 + \frac{\psi^{(0)}\left(\frac{1}{4}\right)^6}{96}} \right)$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Decimal approximation:

1728.977790691069412475377690168345160048204862377912236307...

1728.97779069.... \approx 1729

This result is very near to the mass of candidate glueball **f₀(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms:

$$-\frac{1}{\phi} + 49 + \frac{372\,053\,606\,400}{2839\pi^8 - 25\,200\pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 + 1575 \psi^{(0)}\left(\frac{1}{4}\right)^6}$$

$$-\frac{1}{\phi} + 49 + \frac{2\,460\,672}{\frac{2839\pi^8}{151\,200} - \frac{1}{6} \pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 + \frac{\psi^{(0)}\left(\frac{1}{4}\right)^6}{96}}$$

$$49 - \frac{2}{1 + \sqrt{5}} + \frac{2460672}{\frac{2839\pi^8}{151200} - \frac{1}{6}\pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 + \frac{\psi^{(0)}\left(\frac{1}{4}\right)^6}{96}}$$

Alternative representations:

$$\frac{27}{2} \left(\frac{89}{\frac{\frac{2839\zeta(8)}{16} - 15\zeta(4)\psi\left(\frac{1}{4}\right)^3 + \frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2}{2048}} + 4 \right) - 5 - \frac{1}{\phi} =$$

$$-5 - \frac{1}{\phi} + \frac{27}{2} \left(4 + \frac{89}{\frac{\frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2 - 15\psi\left(\frac{1}{4}\right)^3\zeta(4,1) + \frac{2839\zeta(8,1)}{16}}{2048}} \right)$$

$$\frac{27}{2} \left(\frac{89}{\frac{\frac{2839\zeta(8)}{16} - 15\zeta(4)\psi\left(\frac{1}{4}\right)^3 + \frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2}{2048}} + 4 \right) - 5 - \frac{1}{\phi} =$$

$$-5 - \frac{1}{\phi} + \frac{27}{2} \left(4 + \frac{89}{\frac{\frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2 - \frac{15\psi\left(\frac{1}{4}\right)^3\zeta\left(4, \frac{1}{2}\right) + \frac{2839\zeta\left(8, \frac{1}{2}\right)}{16(-1+2^8)}}{-1+2^4}}{2048}} \right)$$

$$\frac{27}{2} \left(\frac{89}{\frac{\frac{2839\zeta(8)}{16} - 15\zeta(4)\psi\left(\frac{1}{4}\right)^3 + \frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2}{2048}} + 4 \right) - 5 - \frac{1}{\phi} =$$

$$-5 - \frac{1}{\phi} + \frac{27}{2} \left(4 + \frac{89}{\frac{\frac{1}{96} \left(\left(\frac{\frac{\partial}{\partial \frac{1}{4}} \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} \right)^3 \right)^2 - 15 \left(\frac{\frac{\partial}{\partial \frac{1}{4}} \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} \right)^3 \zeta(4,1) + \frac{2839\zeta(8,1)}{16}}{2048}} \right)$$

Series representations:

$$\frac{27}{2} \left(\frac{\frac{89}{\frac{2839 \zeta(8)}{16} - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2} + 4 \right) - 5 - \frac{1}{\phi} =$$

$$49 - \frac{1}{\phi} + 2460672 / \left(\frac{2839 \pi^8}{151200} - \frac{1}{6} \pi^4 \left(-4 - \gamma + \frac{1}{4} \sum_{k=1}^{\infty} \frac{{}_2\tilde{F}_1\left(1, 2; 2; -\frac{1}{4k}\right)^3}{k^2} \right)^3 + \right.$$

$$\left. \frac{1}{96} \left(-4 - \gamma + \frac{1}{4} \sum_{k=1}^{\infty} \frac{{}_2\tilde{F}_1\left(1, 2; 2; -\frac{1}{4k}\right)^6}{k^2} \right) \right)$$

$$\frac{27}{2} \left(\frac{\frac{89}{\frac{2839 \zeta(8)}{16} - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2} + 4 \right) - 5 - \frac{1}{\phi} =$$

$$49 - \frac{1}{\phi} + 2460672 / \left(\frac{2839 \pi^8}{151200} - \frac{1}{6} \pi^4 \left(4 \left(-1 - \frac{\gamma}{4} \right) + \frac{1}{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k (1+k)! \zeta(2+k)}{(2)_k} \right)^3 + \right.$$

$$\left. \frac{1}{96} \left(4 \left(-1 - \frac{\gamma}{4} \right) + \frac{1}{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k (1+k)! \zeta(2+k)}{(2)_k} \right)^6 \right)$$

$$\frac{27}{2} \left(\frac{\frac{89}{\frac{2839 \zeta(8)}{16} - 15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 + \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2} + 4 \right) - 5 - \frac{1}{\phi} \alpha$$

$$49 - \frac{1}{\phi} + 2460672 / \left(\frac{2839 \pi^8}{151200} - \frac{1}{6} \pi^4 \right.$$

$$\left. \left(-\frac{1}{\frac{1}{4} + m} + \sum_{k=0}^{\infty} \frac{\left(\frac{1}{4} + m\right)^k \left((1 + (-1)^{1+k}) \psi^{(k)}(1) + (-1)^k \psi^{(k)}(1+m) \right)}{k!} \right)^3 + \right.$$

$$\left. \frac{1}{96} \left(-\frac{1}{\frac{1}{4} + m} + \sum_{k=0}^{\infty} \frac{\left(\frac{1}{4} + m\right)^k \left((1 + (-1)^{1+k}) \psi^{(k)}(1) + (-1)^k \psi^{(k)}(1+m) \right)}{k!} \right)^6 \right)$$

for $\left(-m \rightarrow \frac{1}{4} \text{ and } m \in \mathbb{Z} \text{ and } m \geq 0\right)$

${}_2\tilde{F}_1(a, b; c; x)$ is the regularized hypergeometric function

γ is the Euler-Mascheroni constant

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

\mathbb{Z} is the set of integers

For the integral at *ii*), $n = 3$,

$$\int_0^1 \frac{x \log^3(x) \operatorname{Li}_4(-x)}{1+x^2} dx$$

$$= \frac{1}{2048} \left(15\zeta(4)\psi^{(3)}\left(\frac{1}{4}\right) - \frac{1}{96} \left(\psi^{(3)}\left(\frac{1}{4}\right)\right)^2 - \frac{2921}{16} \zeta(8) \right).$$

$$1/2048((((15 \text{ zeta}(4) \text{ digamma}^3(1/4)-1/96(\text{digamma}^3(1/4))^2-(2921/16) \text{ zeta}(8))))))$$

Input:

$$\frac{1}{2048} \left(15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)\right)^3 - \frac{2921}{16} \zeta(8) \right)$$

$\zeta(s)$ is the Riemann zeta function

$\psi(x)$ is the digamma function

Exact result:

$$\frac{-\frac{2921\pi^8}{151200} + \frac{1}{6}\pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 - \frac{\psi^{(0)}\left(\frac{1}{4}\right)^6}{96}}{2048}$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Decimal approximation:

-0.71743766262630500975752520386574183266335960082674010192...

-0.717437662626...

Alternate forms:

$$\frac{-2921\pi^8 + 25200\pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 - 1575\psi^{(0)}\left(\frac{1}{4}\right)^6}{309657600}$$

$$-\frac{2921\pi^8}{309657600} + \frac{\pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3}{12288} - \frac{\psi^{(0)}\left(\frac{1}{4}\right)^6}{196608}$$

$$\frac{-\frac{2921\pi^8}{151200} + \frac{1}{6}\pi^4 \left(-\gamma - \frac{\pi}{2} - \log(8)\right)^3 - \frac{1}{96} \left(-\gamma - \frac{\pi}{2} - \log(8)\right)^6}{2048}$$

$\log(x)$ is the natural logarithm
 γ is the Euler-Mascheroni constant

Alternative representations:

$$\frac{15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)\right)^3{}^2 - \frac{2921 \zeta(8)}{16}}{2048} = \frac{-\frac{1}{96} \left(\psi\left(\frac{1}{4}\right)\right)^3{}^2 + 15 \psi\left(\frac{1}{4}\right)^3 \zeta(4, 1) - \frac{2921 \zeta(8,1)}{16}}{2048}$$

$$\frac{15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)\right)^3{}^2 - \frac{2921 \zeta(8)}{16}}{2048} = \frac{-\frac{1}{96} \left(\psi\left(\frac{1}{4}\right)\right)^3{}^2 + \frac{15 \psi\left(\frac{1}{4}\right)^3 \zeta\left(4, \frac{1}{2}\right)}{-1+2^4} - \frac{2921 \zeta\left(8, \frac{1}{2}\right)}{16(-1+2^8)}}{2048}$$

$$\frac{15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)\right)^3{}^2 - \frac{2921 \zeta(8)}{16}}{2048} = \frac{-\frac{1}{96} \left(\left(\frac{\frac{\partial}{\partial \frac{1}{4}} \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} \right)^3 \right)^2 + 15 \left(\frac{\frac{\partial}{\partial \frac{1}{4}} \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}\right)} \right)^3 \zeta(4, 1) - \frac{2921 \zeta(8,1)}{16}}{2048}$$

Series representations:

$$\frac{15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)\right)^3{}^2 - \frac{2921 \zeta(8)}{16}}{2048} = \frac{-\frac{2921 \pi^8}{151200} + \frac{1}{6} \pi^4 \left(-4 - \gamma + \frac{1}{4} \sum_{k=1}^{\infty} \frac{{}_2\tilde{F}_1\left(1, 2; 2; -\frac{1}{4k}\right)}{k^2} \right)^3 - \frac{1}{96} \left(-4 - \gamma + \frac{1}{4} \sum_{k=1}^{\infty} \frac{{}_2\tilde{F}_1\left(1, 2; 2; -\frac{1}{4k}\right)}{k^2} \right)^6}{2048}$$

$$\frac{15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)\right)^3{}^2 - \frac{2921 \zeta(8)}{16}}{2048} = \frac{-2921 \pi^8 + 25200 \pi^4 \left(\sum_{k=0}^{\infty} \frac{\psi^{(k)}(z_0) \left(\frac{1}{4} - z_0\right)^k}{k!} \right)^3 - 1575 \left(\sum_{k=0}^{\infty} \frac{\psi^{(k)}(z_0) \left(\frac{1}{4} - z_0\right)^k}{k!} \right)^6}{309657600}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)\right)^3{}^2 - \frac{2921 \zeta(8)}{16}}{2048} = \frac{1}{2048} \left(-\frac{2921 \pi^8}{151200} + \frac{1}{6} \pi^4 \left(4 \left(-1 - \frac{\gamma}{4} \right) + \frac{1}{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k (1+k)! \zeta(2+k)}{(2)_k} \right)^3 - \frac{1}{96} \left(4 \left(-1 - \frac{\gamma}{4} \right) + \frac{1}{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k (1+k)! \zeta(2+k)}{(2)_k} \right)^6 \right)$$

$-89 \times 1 / ((((((1/2048((((((15 \zeta(4) \operatorname{digamma}^3(1/4) - 1/96(\operatorname{digamma}^3(1/4))^2 - (2921/16) \zeta(8))))))))))))) + \text{golden ratio}$

Input:

$$-89 \times \frac{1}{\frac{1}{2048} \left(15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3 \right)^2 - \frac{2921}{16} \zeta(8) \right)} + \phi$$

$\zeta(s)$ is the Riemann zeta function

$\psi(x)$ is the digamma function

ϕ is the golden ratio

Exact result:

$$\phi - \frac{182\,272}{-\frac{2921\pi^8}{151\,200} + \frac{1}{6}\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 - \frac{\psi^{(0)}\left(\frac{1}{4}\right)^6}{96}}$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Decimal approximation:

125.6706236927808516102300253361734371128826265190377662704...

125.6706236927... result very near to the Higgs boson mass 125.18 GeV

Alternate forms:

$$\phi + \frac{27559526400}{2921\pi^8 - 25200\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 + 1575\psi^{(0)}\left(\frac{1}{4}\right)^6}$$

$$\frac{1}{2}(1 + \sqrt{5}) + \frac{27559526400}{2921\pi^8 - 25200\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 + 1575\psi^{(0)}\left(\frac{1}{4}\right)^6}$$

$$\frac{1}{2}(1 + \sqrt{5}) - \frac{182\,272}{-\frac{2921\pi^8}{151\,200} + \frac{1}{6}\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 - \frac{\psi^{(0)}\left(\frac{1}{4}\right)^6}{96}}$$

Alternative representations:

$$-\frac{89}{\frac{15\zeta(4)\psi\left(\frac{1}{4}\right)^3 - \frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2 - \frac{2921\zeta(8)}{16}}}{2048} + \phi = \phi - \frac{89}{\frac{-\frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2 + 15\psi\left(\frac{1}{4}\right)^3\zeta(4,1) - \frac{2921\zeta(8,1)}{16}}}{2048}}$$

$$\begin{aligned}
& -\frac{89}{15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)\right)^2 - \frac{2921 \zeta(8)}{16}}{2048} + \phi = \phi - \frac{89}{-\frac{1}{96} \left(\psi\left(\frac{1}{4}\right)\right)^2 + \frac{15 \psi\left(\frac{1}{4}\right)^3 \zeta\left(4, \frac{1}{2}\right)}{-1+2^4} - \frac{2921 \zeta\left(8, \frac{1}{2}\right)}{16(-1+2^8)}}{2048} \\
& -\frac{89}{15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)\right)^2 - \frac{2921 \zeta(8)}{16}}{2048} + \phi = \phi - \frac{89}{-\frac{1}{96} \left(\frac{\left(\frac{\partial}{\partial \frac{1}{4}} \Gamma\left(\frac{1}{4}\right)\right)^3}{\Gamma\left(\frac{1}{4}\right)}\right)^2 + 15 \left(\frac{\frac{\partial}{\partial \frac{1}{4}} \Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}\right)}\right)^3 \zeta(4,1) - \frac{2921 \zeta(8,1)}{16}}{2048}
\end{aligned}$$

Series representations:

$$\begin{aligned}
& -\frac{89}{15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)\right)^2 - \frac{2921 \zeta(8)}{16}}{2048} + \phi = \\
& \phi - 182\,272 / \left(-\frac{2921 \pi^8}{151\,200} + \frac{1}{6} \pi^4 \left(-4 - \gamma + \frac{1}{4} \sum_{k=1}^{\infty} \frac{{}_2\tilde{F}_1\left(1, 2; 2; -\frac{1}{4k}\right)^3}{k^2} \right) - \right. \\
& \left. \frac{1}{96} \left(-4 - \gamma + \frac{1}{4} \sum_{k=1}^{\infty} \frac{{}_2\tilde{F}_1\left(1, 2; 2; -\frac{1}{4k}\right)^6}{k^2} \right) \right) \\
& -\frac{89}{15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)\right)^2 - \frac{2921 \zeta(8)}{16}}{2048} + \phi = \\
& \phi - 182\,272 / \left(-\frac{2921 \pi^8}{151\,200} + \frac{1}{6} \pi^4 \left(4 \left(-1 - \frac{\gamma}{4} \right) + \frac{1}{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k (1+k)! \zeta(2+k)}{(2)_k} \right)^3 - \right. \\
& \left. \frac{1}{96} \left(4 \left(-1 - \frac{\gamma}{4} \right) + \frac{1}{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k (1+k)! \zeta(2+k)}{(2)_k} \right)^6 \right) \\
& -\frac{89}{15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)\right)^2 - \frac{2921 \zeta(8)}{16}}{2048} + \phi \propto \phi - 182\,272 / \left(-\frac{2921 \pi^8}{151\,200} + \right. \\
& \left. \frac{1}{6} \pi^4 \left(-\frac{1}{4+m} + \sum_{k=0}^{\infty} \frac{\left(\frac{1}{4}+m\right)^k \left((1+(-1)^{1+k}) \psi^{(k)}(1) + (-1)^k \psi^{(k)}(1+m) \right)}{k!} \right)^3 - \right. \\
& \left. \frac{1}{96} \left(-\frac{1}{4+m} + \sum_{k=0}^{\infty} \frac{\left(\frac{1}{4}+m\right)^k \left((1+(-1)^{1+k}) \psi^{(k)}(1) + (-1)^k \psi^{(k)}(1+m) \right)}{k!} \right)^6 \right) \\
& \text{for } \left(-m \rightarrow \frac{1}{4} \text{ and } m \in \mathbb{Z} \text{ and } m \geq 0 \right)
\end{aligned}$$

${}_2\tilde{F}_1(a, b; c; x)$ is the regularized hypergeometric function

γ is the Euler-Mascheroni constant

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

\mathbb{Z} is the set of integers

$-89 \times \frac{1}{\left(\left(\left(\left(\frac{1}{2048} \left(15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2 - \frac{2921}{16} \zeta(8)\right)\right)\right)\right)\right)} + 13 + \pi - \frac{1}{\phi}$
 $\zeta(8)$))))) + 13 + Pi - 1/golden ratio

Input:

$$-89 \times \frac{1}{\frac{1}{2048} \left(15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2 - \frac{2921}{16} \zeta(8)\right)} + 13 + \pi - \frac{1}{\phi}$$

$\zeta(s)$ is the Riemann zeta function

$\psi(x)$ is the digamma function

ϕ is the golden ratio

Exact result:

$$-\frac{1}{\phi} + 13 + \pi - \frac{182\,272}{-\frac{2921\pi^8}{151\,200} + \frac{1}{6}\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 - \frac{\psi^{(0)}\left(\frac{1}{4}\right)^6}{96}}$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Decimal approximation:

139.5761483688708551522834950507216637616391775588013463671...

139.57614836... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternate forms:

$$-\frac{1}{\phi} + 13 + \pi + \frac{27\,559\,526\,400}{2921\pi^8 - 25\,200\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 + 1575\psi^{(0)}\left(\frac{1}{4}\right)^6}$$

$$\frac{1}{2} \left(27 - \sqrt{5}\right) + \pi - \frac{182\,272}{-\frac{2921\pi^8}{151\,200} + \frac{1}{6}\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 - \frac{\psi^{(0)}\left(\frac{1}{4}\right)^6}{96}}$$

$$\frac{11 + 13\sqrt{5}}{1 + \sqrt{5}} + \pi + \frac{27\,559\,526\,400}{2921\pi^8 - 25\,200\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 + 1575\psi^{(0)}\left(\frac{1}{4}\right)^6}$$

Alternative representations:

$$-\frac{89}{\frac{15\zeta(4)\psi\left(\frac{1}{4}\right)^3 - \frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2 - \frac{2921\zeta(8)}{16}} + 13 + \pi - \frac{1}{\phi} =$$

$$13 + \pi - \frac{1}{\phi} - \frac{89}{\frac{-\frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2 + 15\psi\left(\frac{1}{4}\right)^3\zeta(4,1) - \frac{2921\zeta(8,1)}{16}}{2048}}$$

$$\frac{1116160819200 + 476123\pi^8 - 4107600\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 + 256725\psi^{(0)}\left(\frac{1}{4}\right)^6}{3\left(2921\pi^8 - 25200\pi^4\psi^{(0)}\left(\frac{1}{4}\right)^3 + 1575\psi^{(0)}\left(\frac{1}{4}\right)^6\right)}$$

Alternative representations:

$$\frac{27}{2} \left(-\frac{89}{\frac{15\zeta(4)\psi\left(\frac{1}{4}\right)^3 - \frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2 - \frac{2921\zeta(8)}{16}}}{2048} + 4 \right) + \frac{1}{3} =$$

$$\frac{1}{3} + \frac{27}{2} \left(4 - \frac{89}{\frac{-\frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2 + 15\psi\left(\frac{1}{4}\right)^3\zeta(4,1) - \frac{2921\zeta(8,1)}{16}}}{2048} \right)$$

$$\frac{27}{2} \left(-\frac{89}{\frac{15\zeta(4)\psi\left(\frac{1}{4}\right)^3 - \frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2 - \frac{2921\zeta(8)}{16}}}{2048} + 4 \right) + \frac{1}{3} =$$

$$\frac{1}{3} + \frac{27}{2} \left(4 - \frac{89}{\frac{-\frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2 + \frac{15\psi\left(\frac{1}{4}\right)^3\zeta\left(4, \frac{1}{2}\right) - 2921\zeta\left(8, \frac{1}{2}\right)}{-1+2^4}}}{2048} \right)$$

$$\frac{27}{2} \left(-\frac{89}{\frac{15\zeta(4)\psi\left(\frac{1}{4}\right)^3 - \frac{1}{96}\left(\psi\left(\frac{1}{4}\right)^3\right)^2 - \frac{2921\zeta(8)}{16}}}{2048} + 4 \right) + \frac{1}{3} =$$

$$\frac{1}{3} + \frac{27}{2} \left(4 - \frac{89}{\frac{-\frac{1}{96}\left(\left(\frac{\frac{\partial}{\partial \frac{1}{4}}{\Gamma\left(\frac{1}{4}\right)}\right)^3\right)^2 + 15\left(\frac{\frac{\partial}{\partial \frac{1}{4}}{\Gamma\left(\frac{1}{4}\right)}\right)^3\zeta(4,1) - \frac{2921\zeta(8,1)}{16}}}{2048}} \right)$$

Series representations:

$$\frac{27}{2} \left(-\frac{89}{\frac{15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2 - \frac{2921 \zeta(8)}{16}}}{2048} + 4 \right) + \frac{1}{3} =$$

$$\frac{163}{3} - 2460672 / \left(-\frac{2921 \pi^8}{151200} + \frac{1}{6} \pi^4 \left(-4 - \gamma + \frac{1}{4} \sum_{k=1}^{\infty} \frac{{}_2\tilde{F}_1\left(1, 2; 2; -\frac{1}{4k}\right)}{k^2} \right)^3 - \right.$$

$$\left. \frac{1}{96} \left(-4 - \gamma + \frac{1}{4} \sum_{k=1}^{\infty} \frac{{}_2\tilde{F}_1\left(1, 2; 2; -\frac{1}{4k}\right)}{k^2} \right)^6 \right)$$

$$\frac{27}{2} \left(-\frac{89}{\frac{15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2 - \frac{2921 \zeta(8)}{16}}}{2048} + 4 \right) + \frac{1}{3} =$$

$$\frac{163}{3} - 2460672 / \left(-\frac{2921 \pi^8}{151200} + \frac{1}{6} \pi^4 \left(4 \left(-1 - \frac{\gamma}{4} \right) + \frac{1}{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k (1+k)! \zeta(2+k)}{(2)_k} \right)^3 - \right.$$

$$\left. \frac{1}{96} \left(4 \left(-1 - \frac{\gamma}{4} \right) + \frac{1}{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k (1+k)! \zeta(2+k)}{(2)_k} \right)^6 \right)$$

$$\frac{27}{2} \left(-\frac{89}{\frac{15 \zeta(4) \psi\left(\frac{1}{4}\right)^3 - \frac{1}{96} \left(\psi\left(\frac{1}{4}\right)^3\right)^2 - \frac{2921 \zeta(8)}{16}}}{2048} + 4 \right) + \frac{1}{3} =$$

$$\left(1116160819200 + 476123 \pi^8 - 4107600 \pi^4 \left(\sum_{k=0}^{\infty} \frac{\psi^{(k)}(z_0) \left(\frac{1}{4} - z_0\right)^k}{k!} \right)^3 + \right.$$

$$\left. 256725 \left(\sum_{k=0}^{\infty} \frac{\psi^{(k)}(z_0) \left(\frac{1}{4} - z_0\right)^k}{k!} \right)^6 \right) /$$

$$\left(3 \left(2921 \pi^8 - 25200 \pi^4 \left(\sum_{k=0}^{\infty} \frac{\psi^{(k)}(z_0) \left(\frac{1}{4} - z_0\right)^k}{k!} \right)^3 + 1575 \left(\sum_{k=0}^{\infty} \frac{\psi^{(k)}(z_0) \left(\frac{1}{4} - z_0\right)^k}{k!} \right)^6 \right) \right)$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

From:

Wormholes in generalized hybrid metric-Palatini gravity obeying the matter null energy condition everywhere - *Joao Luis Rosa, Jose P. S. Lemos, and Francisco S. N. Lobo* - arXiv:1808.08975v1 [gr-qc] 27 Aug 2018,

We have that

We impose that the wormhole solutions are described by a static and spherically symmetric metric which in the usual spherical (t, r, θ, ϕ) coordinates has components $g_{ab} = \text{diag}(g_{tt}, g_{rr}, g_{\theta\theta}, g_{\phi\phi})$, and whose corresponding line element has then the form [1]

$$ds^2 = -e^{\zeta(r)} dt^2 + \left[1 - \frac{b(r)}{r}\right]^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (15)$$

where $\zeta(r)$ is the redshift function and $b(r)$ is the shape function. The shape function $b(r)$ should obey the boundary condition $b(r_0) = r_0$, where r_0 is the radius of the wormhole throat.

IV. WORMHOLE SOLUTIONS IN THE GENERALIZED HYBRID THEORY WITH MATTER OBEYING THE NULL ENERGY CONDITION EVERYWHERE

ψ_0 and ψ_1 are integrating constants.

	$F(x)$	$F'(x)$	c
	x^2+0	$2x$	0
PRIMITIVE	x^2+1	$2x$	1
	x^2+2	$2x$	2
	x^2+3	$2x$	3
	...		

The integrating constants are $c = 0, 1, 2, 3, \dots$

$$\varphi(r) = \left[\sqrt{\psi_0} + \sqrt{\psi_1} \arctan \left(\sqrt{\frac{r^2}{r_0^2} - 1} \right) \right]^2 - \frac{r_0^2}{r^4 V_0}. \quad (38)$$

and

$$\psi(r) = \left[\sqrt{\psi_0} + \sqrt{\psi_1} \arctan \left(\sqrt{\frac{r^2}{r_0^2} - 1} \right) \right]^2, \quad (39)$$

From:

$$\psi(r) = \left[\sqrt{\psi_0} + \sqrt{\psi_1} \arctan \left(\sqrt{\frac{r^2}{r_0^2} - 1} \right) \right]^2$$

for

$$r_0 = 2\sqrt{10/11} = 1.90693 \quad V_0 = -42$$

$M = 13.12806e+39$, $r = 1.94973e+13$, $\psi_0 = 2$; $\psi_1 = 3$, we obtain:

$$[(\text{sqrt}2 + \text{sqrt}3 \text{atan}(\frac{(((((1.94973e+13)^2)/((1.90693)^2))-1))^{1/2}})))]^2$$

Input interpretation:

$$\left(\sqrt{2} + \sqrt{3} \tan^{-1} \left(\sqrt{\frac{(1.94973 \times 10^{13})^2}{1.90693^2} - 1} \right) \right)^2$$

$\tan^{-1}(x)$ is the inverse tangent function

Result:

17.0975022817868026...

(result in radians)

Result:

17.09750228178680260481033739420502562670222431958011007079...

(result in radians)

From

$$\varphi(r) = \left[\sqrt{\psi_0} + \sqrt{\psi_1} \arctan \left(\sqrt{\frac{r^2}{r_0^2} - 1} \right) \right]^2 - \frac{r_0^2}{r^4 V_0}$$

We obtain:

$$\left[(\sqrt{2} + \sqrt{3} \operatorname{atan} \left(\left(\left(\left(\left((1.94973e+13)^2 / ((1.90693)^2) - 1 \right) \right)^{1/2} \right) \right) \right) \right]^2 - \left(\frac{(1.90693)^2}{((1.94973e+13)^4 * (-42))} \right)$$

Input interpretation:

$$\left(\sqrt{2} + \sqrt{3} \tan^{-1} \left(\sqrt{\frac{(1.94973 \times 10^{13})^2}{1.90693^2} - 1} \right) \right)^2 - \frac{1.90693^2}{(1.94973 \times 10^{13})^4 \times (-42)}$$

$\tan^{-1}(x)$ is the inverse tangent function

Result:

17.0975022817868026...

(result in radians)

Result:

17.09750228178680260481033739420502562670222431958011007139...

(result in radians)

Difference between the two results is:

17.09750228178680260481033739420502562670222431958011007139-

17.09750228178680260481033739420502562670222431958011007079

Input interpretation:

17.09750228178680260481033739420502562670222431958011007139 -

17.09750228178680260481033739420502562670222431958011007079

Result:

6×10^{-55}

$6 * 10^{-55}$

Now, from the sum of $\varphi(r)$ and $\psi(r)$, we obtain:

$$\left[\left(\sqrt{2} + \sqrt{3} \operatorname{atan} \left(\left(\left(\left(\left(\left(1.94973e+13 \right)^2 \right) / \left(\left(1.90693 \right)^2 \right) - 1 \right) \right) \right)^{1/2} \right) \right) \right]^2 + \left[\left(\sqrt{2} + \sqrt{3} \operatorname{atan} \left(\left(\left(\left(\left(\left(\left(1.94973e+13 \right)^2 \right) / \left(\left(1.90693 \right)^2 \right) - 1 \right) \right) \right) \right)^{1/2} \right) \right) \right]^2 - \frac{\left(\left(\left(1.90693 \right)^2 \right) \right) / \left(\left(\left(1.94973e+13 \right)^4 \right) \times (-42) \right)}$$

Input interpretation:

$$\left(\sqrt{2} + \sqrt{3} \tan^{-1} \left(\sqrt{\frac{(1.94973 \times 10^{13})^2}{1.90693^2} - 1} \right) \right)^2 + \left(\sqrt{2} + \sqrt{3} \tan^{-1} \left(\sqrt{\frac{(1.94973 \times 10^{13})^2}{1.90693^2} - 1} \right) \right)^2 - \frac{1.90693^2}{(1.94973 \times 10^{13})^4 \times (-42)}$$

$\tan^{-1}(x)$ is the inverse tangent function

Result:

34.1950045635736052...

(result in radians)

34.1950045635736052...

$$\left[\left(\sqrt{2} + \sqrt{3} \operatorname{atan} \left(\left(\left(\left(\left(\left(1.94973e+13 \right)^2 \right) / \left(\left(1.90693 \right)^2 \right) - 1 \right) \right) \right)^{1/2} \right) \right) \right]^2 + \left[\left(\sqrt{2} + \sqrt{3} \operatorname{atan} \left(\left(\left(\left(\left(\left(\left(1.94973e+13 \right)^2 \right) / \left(\left(1.90693 \right)^2 \right) - 1 \right) \right) \right) \right)^{1/2} \right) \right) \right]^2 - \frac{\left(\left(\left(1.90693 \right)^2 \right) \right) / \left(\left(\left(1.94973e+13 \right)^4 \right) \times (-42) \right)}{21}$$

Input interpretation:

$$\frac{1}{21} \left(\left(\sqrt{2} + \sqrt{3} \tan^{-1} \left(\sqrt{\frac{(1.94973 \times 10^{13})^2}{1.90693^2} - 1} \right) \right) \right)^2 + \left(\sqrt{2} + \sqrt{3} \tan^{-1} \left(\sqrt{\frac{(1.94973 \times 10^{13})^2}{1.90693^2} - 1} \right) \right)^2 - \frac{1.90693^2}{(1.94973 \times 10^{13})^4 \times (-42)}$$

$\tan^{-1}(x)$ is the inverse tangent function

Result:

1.62833355064636215...

(result in radians)

1.62833355064636215...

Note that from previous analyzed expression, we obtain the following mathematical connection:

Input interpretation:

$$\frac{1}{21} \left(\left(\sqrt{2} + \sqrt{3} \tan^{-1} \left(\sqrt{\frac{(1.94973 \times 10^{13})^2}{1.90693^2} - 1} \right) \right)^2 + \left(\sqrt{2} + \sqrt{3} \tan^{-1} \left(\sqrt{\frac{(1.94973 \times 10^{13})^2}{1.90693^2} - 1} \right) \right)^2 - \frac{1.90693^2}{(1.94973 \times 10^{13})^4 \times (-42)} \right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Result:

1.62833355064636215...

(result in radians)

1.62833355064636215...

$$1 + 1 / (((-1 / ((7 / 2048 (1/8 * \zeta(4) * \text{digamma}^3(1/4) - 105 * \zeta(8))))))^{1/2}$$

Input:

$$1 + \frac{1}{\sqrt{-\frac{7}{2048} \left(\frac{1}{8} \zeta(4) \psi\left(\frac{1}{4}\right)^3 - 105 \zeta(8) \right)}}$$

$\zeta(s)$ is the Riemann zeta function

$\psi(x)$ is the digamma function

Exact result:

$$1 + \frac{1}{32} \sqrt{\frac{7}{2} \left(\frac{\pi^8}{90} - \frac{1}{720} \pi^4 \psi^{(0)}\left(\frac{1}{4}\right)^3 \right)}$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Decimal approximation:

1.628717664408350164403726203534362852650997318722651705780...

1.62871766440835....

$$-1/\pi^4 + \left[\left(\sqrt{2} + \sqrt{3} \operatorname{atan} \left(\frac{\sqrt{\frac{(1.94973 \times 10^{13})^2}{1.90693^2} - 1}}{1} \right) \right)^2 - \left(\sqrt{2} + \sqrt{3} \operatorname{atan} \left(\frac{\sqrt{\frac{(1.94973 \times 10^{13})^2}{1.90693^2} - 1}}{1} \right) \right)^2 - \frac{1.90693^2}{(1.94973 \times 10^{13})^4 \times (-42)} \right] / 21$$

Input interpretation:

$$-\frac{1}{\pi^4} + \frac{1}{21} \left(\left(\sqrt{2} + \sqrt{3} \tan^{-1} \left(\sqrt{\frac{(1.94973 \times 10^{13})^2}{1.90693^2} - 1} \right) \right)^2 + \left(\sqrt{2} + \sqrt{3} \tan^{-1} \left(\sqrt{\frac{(1.94973 \times 10^{13})^2}{1.90693^2} - 1} \right) \right)^2 - \frac{1.90693^2}{(1.94973 \times 10^{13})^4 \times (-42)} \right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Result:

1.61806756839167782...

(result in radians)

1.61806756839167782... result that is a very good approximation to the value of the golden ratio 1,618033988749...

From which:

$$-1/x^4 + 1.62833355064636215 = 1.61806756839167782$$

Input interpretation:

$$-\frac{1}{x^4} + 1.62833355064636215 = 1.61806756839167782$$

Result:

$$1.62833355064636215 - \frac{1}{x^4} = 1.61806756839167782$$

Alternate form assuming x is real:

$$\frac{1}{x^4} + 1.61806756839167782 = 1.62833355064636215$$

Alternate form:

$$\frac{1}{x^4} 1.6283335506463622 (1.0000000000000000 x - 0.88524637527970841) (1.0000000000000000 x + 0.88524637527970841) (1.0000000000000000 x^2 + 0.78366114494586233) = 1.61806756839167782$$

Alternate form assuming x is positive:

$$1.0000000000000000 x = 3.14159265358979 \text{ (for } x \neq 0)$$

Real solutions:

$$x \approx -3.14159265358979$$

$$x \approx 3.14159265358979$$

$$3.14159265358979 = \pi$$

Complex solutions:

$$x = -3.14159265358979 i$$

$$x = 3.14159265358979 i$$

$$(1.61806756839167782 * 100 - 21 - \text{golden ratio})$$

Input interpretation:

$$1.61806756839167782 \times 100 - 21 - \phi$$

ϕ is the golden ratio

Result:

$$139.188722850417887\dots$$

139.188722850417887... result practically equal to the rest mass of Pion meson

$$139.57 \text{ MeV}$$

$$(1.61806756839167782 * 100 - 34 - \text{golden ratio}^2)$$

Input interpretation:

$$1.61806756839167782 \times 100 - 34 - \phi^2$$

ϕ is the golden ratio

Result:

$$125.188722850417887\dots$$

125.188722850417887... result very near to the Higgs boson mass 125.18 GeV

Alternative representations:

$$1.618067568391677820000 \times 100 - 34 - \phi^2 = \\ 127.8067568391677820000 - (2 \sin(54^\circ))^2$$

$$1.618067568391677820000 \times 100 - 34 - \phi^2 = \\ 127.8067568391677820000 - (-2 \cos(216^\circ))^2$$

$$1.618067568391677820000 \times 100 - 34 - \phi^2 = \\ 127.8067568391677820000 - (-2 \sin(666^\circ))^2$$

$$27 \times \frac{1}{2} \times (((1.61806756839167782 \times 100 - 34 - \text{golden ratio}^2) + 3)) - \frac{3}{2}$$

Input interpretation:

$$27 \times \frac{1}{2} \left((1.61806756839167782 \times 100 - 34 - \phi^2) + 3 \right) - \frac{3}{2}$$

ϕ is the golden ratio

Result:

1729.04775848064148...

1729.04775848064148...

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternative representations:

$$\frac{27}{2} \left((1.618067568391677820000 \times 100 - 34 - \phi^2) + 3 \right) - \frac{3}{2} = \\ -\frac{3}{2} + \frac{27}{2} (130.8067568391677820000 - (2 \sin(54^\circ))^2)$$

$$\frac{27}{2} \left((1.618067568391677820000 \times 100 - 34 - \phi^2) + 3 \right) - \frac{3}{2} = \\ -\frac{3}{2} + \frac{27}{2} (130.8067568391677820000 - (-2 \cos(216^\circ))^2)$$

$$\frac{27}{2} \left((1.618067568391677820000 \times 100 - 34 - \phi^2) + 3 \right) - \frac{3}{2} = \\ -\frac{3}{2} + \frac{27}{2} (130.8067568391677820000 - (-2 \sin(666^\circ))^2)$$

Now, we have that:

We now find an exterior vacuum solution so that we can use the junction conditions to match the interior wormhole solution to the exterior vacuum solution. To do so, we put the stress-energy tensor to zero, $T_{ab} = 0$, as we want a vacuum solution. In addition, we chose the scalar fields to be constant in this exterior solution, i.e., $\varphi(r) = \varphi_e$ and $\psi(r) = \psi_e$ with φ_e and ψ_e constants, $\varphi_e \neq \psi_e$, and where the subscript e stands for exterior. For continuity we choose the potential to be $V = V_0(\varphi_e - \psi_e)^2$. Note that from Eq. (9) it can be

$$\zeta_0 = -10.96,$$

This class of solutions is also known as the Kottler solution, as well as Schwarzschild-dS solution if the constant cosmological term is positive and Schwarzschild-AdS solution if the constant cosmological term is negative. The metric fields $\zeta(r)$ and $b(r)$ for the exterior region outside some radius r_Σ are then

$$e^{\zeta(r)} = \left(1 - \frac{2M}{r} - \frac{V_0(\varphi_e - \psi_e)r^2}{6}\right) e^{\zeta_e}, \quad (50)$$

$$b(r) = 2M + \frac{V_0(\varphi_e - \psi_e)r^3}{6}, \quad (51)$$

Now, from the difference of $\varphi(r)$ and $\psi(r)$, we obtain: 6×10^{-55}

$$e^{(-10.96)}$$

Input:

$$\frac{1}{e^{10.96}}$$

Dividing the two results, we obtain:

$$\left(\frac{\left(\frac{1 - \left(\frac{2 \times 13.12806 \times 10^{39}}{1.94973 \times 10^{13}} - \frac{1}{6} (-42 (6e - 55) (1.94973 \times 10^{13})^2)\right)}{e^{10.96}}\right)}{2 \times 13.12806 \times 10^{39} + \frac{1}{6} (-42 (6e - 55) (1.94973 \times 10^{13})^3)}\right) \times e^{(-10.96)}$$

Input interpretation:

$$\frac{1 - \frac{2 \times 13.12806 \times 10^{39}}{1.94973 \times 10^{13}} - \frac{1}{6} (-42 (6e - 55) (1.94973 \times 10^{13})^2)}{e^{10.96}} \times \frac{1}{2 \times 13.12806 \times 10^{39} + \frac{1}{6} (-42 (6e - 55) (1.94973 \times 10^{13})^3)}$$

Result:

$$-8.91575... \times 10^{-19}$$

$$-8.91575... * 10^{-19}$$

$$\left(\frac{-8.91575 * 10^{-19}}{2e}\right) + (34+3) * 1/10^{22}$$

Input interpretation:

$$-\frac{8.91575 \times 10^{-19}}{2e} + (34 + 3) \times \frac{1}{10^{22}}$$

Result:

$$-1.60296... \times 10^{-19}$$

$-1.60296... * 10^{-19}$ result practically equal to the electron charge

$$\left(\frac{\left(\frac{1 - \left(\frac{2 \times 13.12806 \times 10^{39}}{1.94973 \times 10^{13}} - \frac{1}{6} (-42 (6e - 55) (1.94973 \times 10^{13})^2)\right)}{e^{10.96}}\right)}{2 \times 13.12806 \times 10^{39} + \frac{1}{6} (-42 (6e - 55) (1.94973 \times 10^{13})^3)}\right) \times e^{(-10.96)}$$

Input interpretation:

$$\frac{1 - \frac{2 \times 13.12806 \times 10^{39}}{1.94973 \times 10^{13}} - \frac{1}{6} (-42 (6e - 55) (1.94973 \times 10^{13})^2)}{e^{10.96}} \left(2 \times 13.12806 \times 10^{39} + \frac{1}{6} (-42 (6e - 55) (1.94973 \times 10^{13})^3)\right)$$

Result:

$$-3.68717... \times 10^{66}$$

$$-3.68717... * 10^{66}$$

Observations

Ramanujan formula for obtain the golden ratio

We have that:

Input:

$$\frac{1}{\frac{1}{32}(-1 + \sqrt{5})^5 + 5e^{(-\sqrt{5}\pi)^5}}$$

Exact result:

$$\frac{1}{\frac{1}{32}(\sqrt{5} - 1)^5 + 5e^{-25\sqrt{5}\pi^5}}$$

Decimal approximation:

11.09016994374947424102293417182819058860154589902881431067...

Input:

$$\frac{11 \times 5 e^{(-\sqrt{5}\pi)^5}}{2 \left(\frac{1}{32}(-1 + \sqrt{5})^5 + 5e^{(-\sqrt{5}\pi)^5} \right)}$$

Exact result:

$$\frac{55 e^{-25\sqrt{5}\pi^5}}{2 \left(\frac{1}{32}(\sqrt{5} - 1)^5 + 5e^{-25\sqrt{5}\pi^5} \right)}$$

Decimal approximation:

9.99290225070718723070536304129457122742436976265255... $\times 10^{-7428}$

Input:

$$\frac{5\sqrt{5} \times 5 e^{(-\sqrt{5}\pi)^5}}{2 \left(\frac{1}{32}(-1 + \sqrt{5})^5 + 5e^{(-\sqrt{5}\pi)^5} \right)}$$

Exact result:

$$\frac{25 \sqrt{5} e^{-25 \sqrt{5} \pi^5}}{2 \left(\frac{1}{32} (\sqrt{5} - 1)^5 + 5 e^{-25 \sqrt{5} \pi^5} \right)}$$

Decimal approximation:

$$1.01567312386781438874777576295646917898823529098784... \times 10^{-7427}$$

From which:

$$\left(\frac{1}{\left(\frac{1}{32} (-1 + \sqrt{5})^5 + 5 e^{(-\sqrt{5} \pi)^5} \right)} - \frac{9.99290225070718723070536304129457122742436976265255}{10^{7428}} - \frac{1.01567312386781438874777576295646917898823529098784}{10^{7427}} \right)^{(1/5)}$$

Result:

$$1.618033988749894848204586834365638117720309179805762862135...$$

Or:

$$\sqrt[5]{\frac{1}{\left(\frac{1}{32} (-1 + \sqrt{5})^5 + 5 e^{(-\sqrt{5} \pi)^5} \right)} - \frac{1.6382898797095665677239458827012056245798314722584}{10^{7429}}}$$

Result:

$$1.618033988749894848204586834365638117720309179805762862135...$$

The result, thence, is:

$$1.6180339887498948482045868343656381177203091798057628$$

This is a wonderful golden ratio, fundamental constant of various fields of mathematics and physics

Conclusions

From:

https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVqd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that $p(9) = 30$, $p(9 + 5) = 135$, $p(9 + 10) = 490$, $p(9 + 15) = 1,575$ and so on are all divisible by 5. Note that here the n 's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of $p(n)$ that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n 's separated by $5^3 = 125$ units, saying that the corresponding $p(n)$'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

*In particle physics, **Yukawa's interaction** or **Yukawa coupling**, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by **pions** (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the **Higgs field**.*

*Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of **Higgs boson**:*

125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of **Pion meson** 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and $4096 = 64^2$

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π , ϕ , $1/\phi$, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the **Fibonacci numbers**, commonly denoted F_n , form a sequence, called the **Fibonacci sequence**, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the n th Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The **Lucas numbers** or **Lucas series** are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A **Lucas prime** is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ...
(sequence A005479 in the OEIS).

In geometry, a **golden spiral** is a logarithmic spiral whose growth factor is ϕ , the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of ϕ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

References

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(Almost) Impossible Integrals, Sums, and Series - *Cornel Ioan Vălean*

Wormholes in generalized hybrid metric-Palatini gravity obeying the matter null energy condition everywhere - *Joao Luis Rosa, Jose P. S. Lemos, and Francisco S. N. Lobo* - arXiv:1808.08975v1 [gr-qc] 27 Aug 2018,