On the links between some Ramanujan formulas, the golden ratio and various equations of several sectors of Black Hole Physics

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Abstract

The purpose of this paper is to show the links between some Ramanujan formulas, the golden ratio and the mathematical connections with various equations of several sectors of Black Hole Physics

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Monster black hole 100,000 times more massive than the sun is found in the heart of our galaxy (SMBH Sagittarius A = $1,9891*10^{35}$)
https://www.dailymail.co.uk/sciencetech/article-4850546/Mini-black-hole-25-000-light-years-Earth.html

https://wssrmnn.net/index.php/2017/01/23/man-saw-number-pi-dreams/

From

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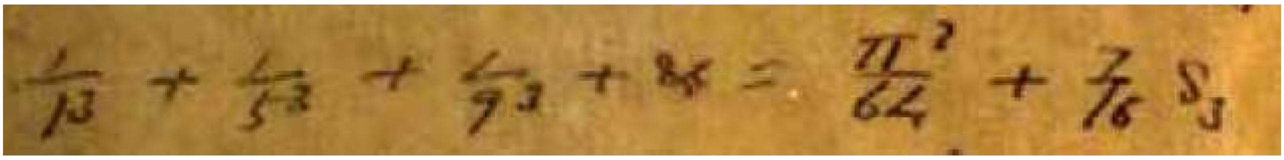


1/1^3 + 1/5^3 + 1/9^3 + …

Input interpretation:

$$\frac{1}{1^3} + \frac{1}{5^3} + \frac{1}{9^3} + \cdots$$

Infinite sum:

$$\sum_{n=1}^{\infty} \frac{1}{(4n-3)^3} = \frac{1}{64}\left(28\,\zeta(3) + \pi^3\right)$$

$\zeta(s)$ is the Riemann zeta function

Decimal approximation:

1.010372968262007190104202868584718670994451636740923068505…

1.010372968262…..

Convergence tests:

The ratio test is inconclusive.

The root test is inconclusive.

By the comparison test, the series converges.

Partial sum formula:

$$\sum_{n=1}^{m} \frac{1}{(-3+4n)^3} = \frac{1}{128}\left(\psi^{(2)}\left(m+\frac{1}{4}\right) - \psi^{(2)}\left(\frac{1}{4}\right)\right)$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Alternate form:

$$\frac{7\,\zeta(3)}{16} + \frac{\pi^3}{64}$$

Series representations:

$$\frac{1}{64}\left(\pi^3 + 28\,\zeta(3)\right) = \frac{\pi^3}{64} + \frac{7}{16}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{1}{64}\left(\pi^3 + 28\,\zeta(3)\right) = \frac{\pi^3}{64} + \frac{1}{2}\sum_{k=0}^{\infty}\frac{1}{(1+2\,k)^3}$$

$$\frac{1}{64}\left(\pi^3 + 28\,\zeta(3)\right) = \frac{7}{16}\,e^{\sum_{k=1}^{\infty} P(3\,k)/k} + \frac{\pi^3}{64}$$

$$\frac{1}{64}\left(\pi^3 + 28\,\zeta(3)\right) = \frac{1}{64}\left(\pi^3 + 14\sum_{n=0}^{\infty}\frac{\sum_{k=0}^{n}\frac{(-1)^k\binom{n}{k}}{(1+k)^2}}{1+n}\right)$$

(Pi^3)/64 + 7/16 zeta(3) (Note that S_3 is $\zeta(3)$)

Input:

$$\frac{\pi^3}{64} + \frac{7}{16}\,\zeta(3)$$

$\zeta(s)$ is the Riemann zeta function

Decimal approximation:

1.010372968262007190104202868584718670994451636740923068505...

1.010372968262….

Alternate form:

$$\frac{1}{64}\left(28\,\zeta(3) + \pi^3\right)$$

Alternative representations:

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{\pi^3}{64} + \frac{7\,\zeta(3,1)}{16}$$

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{7\,S_{2,1}(1)}{16} + \frac{\pi^3}{64}$$

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = -\frac{7\,\mathrm{Li}_3(-1)}{\frac{3\times 16}{4}} + \frac{\pi^3}{64}$$

Series representations:

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{\pi^3}{64} + \frac{7}{16}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{\pi^3}{64} + \frac{1}{2}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}$$

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{7}{16}\, e^{\sum_{k=1}^{\infty} P(3k)/k} + \frac{\pi^3}{64}$$

Integral representations:

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{\pi^3}{64} - \frac{7}{48}\int_0^1 \frac{\log^3(1-t^2)}{t^3}\,dt$$

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{\pi^3}{64} + \frac{1}{8}\int_0^{\infty} t^2\,\mathrm{csch}(t)\,dt$$

$$\frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} = \frac{\pi^3}{64} + \frac{7}{32}\int_0^{\infty}\frac{t^2}{-1+e^t}\,dt$$

Thence:

1/1^3 + 1/5^3 + 1/9^3 + … = (Pi^3)/64 + 7/16 zeta(3)

Input interpretation:

$$\frac{1}{1^3} + \frac{1}{5^3} + \frac{1}{9^3} + \cdots = \frac{\pi^3}{64} + \frac{7}{16}\zeta(3)$$

$\zeta(s)$ is the Riemann zeta function

Result:

$$\frac{1}{64}\left(28\,\zeta(3) + \pi^3\right) = \frac{7\,\zeta(3)}{16} + \frac{\pi^3}{64}$$

Alternate form:

True

From the right-hand side of the expression, we obtain:

(((1/ ((((Pi^3)/64 + 7/16 zeta(3)))))))^1/12

Input:

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{7}{16}\zeta(3)}}$$

$\zeta(s)$ is the Riemann zeta function

Exact result:

$$\frac{1}{\sqrt[12]{\frac{7\zeta(3)}{16}+\frac{\pi^3}{64}}}$$

Decimal approximation:

0.999140408144708492742501571872941269617856182995634489415...

0.999140408144…. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}}\approx 0.9991104684$$

Alternate form:

$$\frac{\sqrt{2}}{\sqrt[12]{28\,\zeta(3)+\pi^3}}$$

All 12th roots of 1/((7 ζ(3))/16 + π^3/64):

$$\frac{e^0}{\sqrt[12]{\frac{7\zeta(3)}{16}+\frac{\pi^3}{64}}}\approx 0.99914 \text{ (real, principal root)}$$

$$\frac{e^{(i\pi)/6}}{\sqrt[12]{\frac{7\zeta(3)}{16}+\frac{\pi^3}{64}}}\approx 0.8653+0.49957\,i$$

$$\frac{e^{(i\pi)/3}}{\sqrt[12]{\frac{7\zeta(3)}{16}+\frac{\pi^3}{64}}}\approx 0.49957+0.8653\,i$$

$$\frac{e^{(i\pi)/2}}{\sqrt[12]{\frac{7\zeta(3)}{16}+\frac{\pi^3}{64}}}\approx 0.99914\,i$$

$$\frac{e^{(2 i \pi)/3}}{\sqrt[12]{\frac{7\zeta(3)}{16}+\frac{\pi^3}{64}}} \approx -0.49957+0.8653\, i$$

Alternative representations:

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{7\,\zeta(3,1)}{16}}}$$

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \sqrt[12]{\frac{1}{\frac{7\,S_{2,1}(1)}{16}+\frac{\pi^3}{64}}}$$

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \sqrt[12]{\frac{1}{-\frac{7\,\mathrm{Li}_3(-1)}{\frac{3\times 16}{4}}+\frac{\pi^3}{64}}}$$

Series representations:

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \frac{\sqrt{2}}{\sqrt[12]{\pi^3+28\sum_{k=1}^{\infty}\frac{1}{k^3}}}$$

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \frac{\sqrt{2}}{\sqrt[12]{\pi^3+32\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}}}$$

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \frac{\sqrt{2}}{\sqrt[12]{28\, e^{\sum_{k=1}^{\infty} P(3k)/k}+\pi^3}}$$

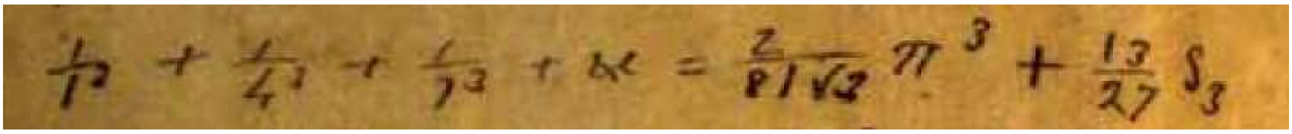
Integral representations:

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \frac{\sqrt{2}}{\sqrt[12]{\pi^3+8\int_0^{\infty} t^2\,\mathrm{csch}(t)\,dt}}$$

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \frac{\sqrt{2}}{\sqrt[12]{\pi^3+14\int_0^{\infty}\frac{t^2}{-1+e^t}\,dt}}$$

$$\sqrt[12]{\frac{1}{\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}}} = \frac{1}{\sqrt[12]{\frac{\pi^3}{64}-\frac{7}{48}\int_0^1 \frac{\log^3(1-t^2)}{t^3}\,dt}}$$

Now, we have that:



1/(1^3) + 1/(4^3) + 1/(7^3) +... = (2Pi^3)/81sqrt2 + 13/27 zeta(3)

1/(1^3) + 1/(4^3) + 1/(7^3) +...

Input interpretation:

$$\frac{1}{1^3}+\frac{1}{4^3}+\frac{1}{7^3}+\cdots$$

Infinite sum:

$$\sum_{n=1}^{\infty}\frac{1}{(3\,n-2)^3} = \frac{1}{243}\left(117\,\zeta(3)+2\sqrt{3}\,\pi^3\right)$$

$\zeta(s)$ is the Riemann zeta function

Decimal approximation:

1.020780044433363102823254739903981825353410937519069669735...

1.020780044433363...

Convergence tests:

The ratio test is inconclusive.

The root test is inconclusive.

By the comparison test, the series converges.

Partial sum formula:

$$\sum_{n=1}^{m}\frac{1}{(-2+3\,n)^3} = \frac{1}{54}\left(\psi^{(2)}\left(m+\frac{1}{3}\right)-\psi^{(2)}\left(\frac{1}{3}\right)\right)$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Alternate form:

$$\frac{13\,\zeta(3)}{27} + \frac{2\,\pi^3}{81\,\sqrt{3}}$$

Series representations:

$$\frac{1}{243}\left(2\,\sqrt{3}\,\pi^3 + 117\,\zeta(3)\right) = \frac{2\,\pi^3}{81\,\sqrt{3}} + \frac{13}{27}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{1}{243}\left(2\,\sqrt{3}\,\pi^3 + 117\,\zeta(3)\right) = \frac{2\,\pi^3}{81\,\sqrt{3}} + \frac{104}{189}\sum_{k=0}^{\infty}\frac{1}{(1+2\,k)^3}$$

$$\frac{1}{243}\left(2\,\sqrt{3}\,\pi^3 + 117\,\zeta(3)\right) = \frac{13}{27}\,e^{\sum_{k=1}^{\infty} P(3\,k)/k} + \frac{2\,\pi^3}{81\,\sqrt{3}}$$

$$\frac{1}{243}\left(2\,\sqrt{3}\,\pi^3 + 117\,\zeta(3)\right) = \frac{2}{243}\left(\sqrt{3}\,\pi^3 + 78 \times \sum_{n=0}^{\infty} 2^{-1-n}\sum_{k=0}^{n}\frac{(-1)^k \binom{n}{k}}{(1+k)^3}\right)$$

(2Pi^3)/(81sqrt2) + 13/27 zeta(3)

Input:

$$\frac{2\,\pi^3}{81\,\sqrt{2}} + \frac{13}{27}\,\zeta(3)$$

$\zeta(s)$ is the Riemann zeta function

Exact result:

$$\frac{13\,\zeta(3)}{27} + \frac{\sqrt{2}\,\pi^3}{81}$$

Decimal approximation:

1.120119953372800115556848609058141510791754061631991953629...

1.1201199533728….

Alternate form:

$$\frac{1}{81}\left(39\,\zeta(3) + \sqrt{2}\,\pi^3\right)$$

Alternative representations:

$$\frac{2\,\pi^3}{81\,\sqrt{2}} + \frac{\zeta(3)\,13}{27} = \frac{2\,\pi^3}{81\,\sqrt{2}} + \frac{13\,\zeta(3,1)}{27}$$

$$\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} = \frac{13\,S_{2,1}(1)}{27} + \frac{2\pi^3}{81\sqrt{2}}$$

$$\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} = -\frac{13\,\text{Li}_3(-1)}{\frac{3\times 27}{4}} + \frac{2\pi^3}{81\sqrt{2}}$$

Series representations:

$$\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} = \frac{\sqrt{2}\,\pi^3}{81} + \frac{13}{27}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} = \frac{\sqrt{2}\,\pi^3}{81} + \frac{104}{189}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}$$

$$\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} = \frac{13}{27}\,e^{\sum_{k=1}^{\infty} P(3k)/k} + \frac{\sqrt{2}\,\pi^3}{81}$$

Integral representations:

$$\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} = \frac{\sqrt{2}\,\pi^3}{81} - \frac{13}{81}\int_0^1 \frac{\log^3(1-t^2)}{t^3}\,dt$$

$$\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} = \frac{\sqrt{2}\,\pi^3}{81} + \frac{13}{54}\int_0^{\infty} \frac{t^2}{-1+e^t}\,dt$$

$$\frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} = \frac{\sqrt{2}\,\pi^3}{81} + \frac{26}{81}\int_0^{\infty} \frac{t^2}{1+e^t}\,dt$$

From which:

(((1/((((2Pi^3)/(81sqrt2) + 13/27 zeta(3)))))))^1/128

Input:

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}} + \frac{13}{27}\zeta(3)}}$$

$\zeta(s)$ is the Riemann zeta function

Exact result:

$$\frac{1}{\sqrt[128]{\frac{13\,\zeta(3)}{27}+\frac{\sqrt{2}\,\pi^3}{81}}}$$

Decimal approximation:

0.999114175536858768080401697435111237630999529642565743801...

0.999114175536... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

Alternate form:

$$\frac{\sqrt[32]{3}}{\sqrt[128]{39\,\zeta(3)+\sqrt{2}\,\pi^3}}$$

All 128th roots of 1/((13 ζ(3))/27 + (sqrt(2) π^3)/81):

$$\frac{e^0}{\sqrt[128]{\frac{13\,\zeta(3)}{27}+\frac{\sqrt{2}\,\pi^3}{81}}}\approx 0.999114 \text{ (real, principal root)}$$

$$\frac{e^{(i\pi)/64}}{\sqrt[128]{\frac{13\,\zeta(3)}{27}+\frac{\sqrt{2}\,\pi^3}{81}}}\approx 0.997911+0.049024\,i$$

$$\frac{e^{(i\pi)/32}}{\sqrt[128]{\frac{13\,\zeta(3)}{27}+\frac{\sqrt{2}\,\pi^3}{81}}}\approx 0.994303+0.09793\,i$$

$$\frac{e^{(3i\pi)/64}}{\sqrt[128]{\frac{13\,\zeta(3)}{27}+\frac{\sqrt{2}\,\pi^3}{81}}}\approx 0.988300+0.14660\,i$$

$$\frac{e^{(i\pi)/16}}{\sqrt[128]{\frac{13\,\zeta(3)}{27}+\frac{\sqrt{2}\,\pi^3}{81}}}\approx 0.97992+0.19492\,i$$

Alternative representations:

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}}=\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{13\,\zeta(3,1)}{27}}}$$

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}}=\sqrt[128]{\frac{1}{\frac{13\,S_{2,1}(1)}{27}+\frac{2\pi^3}{81\sqrt{2}}}}$$

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}}=\sqrt[128]{\frac{1}{-\frac{13\,\mathrm{Li}_3(-1)}{\frac{3\times 27}{4}}+\frac{2\pi^3}{81\sqrt{2}}}}$$

Series representations:

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}}=\frac{\sqrt[32]{3}}{\sqrt[128]{\sqrt{2}\,\pi^3+39\sum_{k=1}^{\infty}\frac{1}{k^3}}}$$

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}}=\frac{1}{\sqrt[128]{\frac{\sqrt{2}\,\pi^3}{81}+\frac{104}{189}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}}}$$

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}}=\frac{\sqrt[32]{3}}{\sqrt[128]{39\,e^{\sum_{k=1}^{\infty}P(3k)/k}+\sqrt{2}\,\pi^3}}$$

Integral representations:

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}}=\frac{\sqrt[32]{3}}{\sqrt[128]{\sqrt{2}\,\pi^3-13\int_0^1\frac{\log^3(1-t^2)}{t^3}\,dt}}$$

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}}=\frac{\sqrt[32]{3}}{\sqrt[128]{\sqrt{2}\,\pi^3+26\int_0^{\infty}\frac{t^2}{1+e^t}\,dt}}$$

$$\sqrt[128]{\frac{1}{\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}}}=\frac{\sqrt[32]{3}}{\sqrt[128]{\sqrt{2}\,\pi^3+26\int_0^{\infty}t^3\,\mathrm{csch}^2(t)\,dt}}$$

Now, we have that:

$$\frac{1}{1^3}+\frac{1}{7^3}+\frac{1}{13^3}+\&c = \frac{\pi^3}{36\sqrt{3}}+\frac{91}{216}S_3$$

(Pi^3)/36sqrt3 + 91/216 zeta(3)

1/(1^3) + 1/7^3 + 1/13^3 + ...

Input interpretation:

$$\frac{1}{1^3}+\frac{1}{7^3}+\frac{1}{13^3}+\cdots$$

Infinite sum:

$$\sum_{n=1}^{\infty}\frac{1}{(6n-5)^3}=\frac{1}{216}\left(91\,\zeta(3)+2\sqrt{3}\,\pi^3\right)$$

$\zeta(s)$ is the Riemann zeta function

Decimal approximation:

1.003685515347952697063230137024860573152727843593893327866...

1.00368551534….

Convergence tests:

The ratio test is inconclusive.

The root test is inconclusive.

By the comparison test, the series converges.

Partial sum formula:

$$\sum_{n=1}^{m}\frac{1}{(-5+6n)^3}=\frac{1}{432}\left(\psi^{(2)}\left(m+\frac{1}{6}\right)-\psi^{(2)}\left(\frac{1}{6}\right)\right)$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Alternate form:

$$\frac{91\,\zeta(3)}{216}+\frac{\pi^3}{36\sqrt{3}}$$

Series representations:

$$\frac{1}{216}\left(2\sqrt{3}\,\pi^3+91\,\zeta(3)\right)=\frac{\pi^3}{36\sqrt{3}}+\frac{91}{216}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{1}{216}\left(2\sqrt{3}\,\pi^3+91\,\zeta(3)\right)=\frac{\pi^3}{36\sqrt{3}}+\frac{13}{27}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}$$

$$\frac{1}{216}\left(2\sqrt{3}\,\pi^3+91\,\zeta(3)\right)=\frac{91}{216}\,e^{\sum_{k=1}^{\infty}P(3k)/k}+\frac{\pi^3}{36\sqrt{3}}$$

$$\frac{1}{216}\left(2\sqrt{3}\,\pi^3+91\,\zeta(3)\right)=\frac{1}{432}\left(4\sqrt{3}\,\pi^3+91\sum_{n=0}^{\infty}\frac{\sum_{k=0}^{n}\frac{(-1)^k\binom{n}{k}}{(1+k)^2}}{1+n}\right)$$

(Pi^3)/(36sqrt3) + 91/216 zeta(3)

Input:

$$\frac{\pi^3}{36\sqrt{3}}+\frac{91}{216}\,\zeta(3)$$

$\zeta(s)$ is the Riemann zeta function

Exact result:

$$\frac{91\,\zeta(3)}{216}+\frac{\pi^3}{36\sqrt{3}}$$

Decimal approximation:

1.003685515347952697063230137024860573152727843593893327866...

1.003685515347933333

Alternate forms:

$$\frac{1}{216}\left(91\,\zeta(3)+2\sqrt{3}\,\pi^3\right)$$

$$\frac{91\sqrt{3}\,\zeta(3)+6\,\pi^3}{216\sqrt{3}}$$

Alternative representations:

$$\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=\frac{\pi^3}{36\sqrt{3}}+\frac{91\,\zeta(3,1)}{216}$$

$$\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=\frac{91\,S_{2,1}(1)}{216}+\frac{\pi^3}{36\sqrt{3}}$$

$$\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=-\frac{91\,\mathrm{Li}_3(-1)}{\frac{3\times 216}{4}}+\frac{\pi^3}{36\sqrt{3}}$$

Series representations:

$$\frac{\pi^3}{36\sqrt{3}} + \frac{\zeta(3)\,91}{216} = \frac{\pi^3}{36\sqrt{3}} + \frac{91}{216}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{\pi^3}{36\sqrt{3}} + \frac{\zeta(3)\,91}{216} = \frac{\pi^3}{36\sqrt{3}} + \frac{13}{27}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}$$

$$\frac{\pi^3}{36\sqrt{3}} + \frac{\zeta(3)\,91}{216} = \frac{91}{216}\,e^{\sum_{k=1}^{\infty} P(3k)/k} + \frac{\pi^3}{36\sqrt{3}}$$

Integral representations:

$$\frac{\pi^3}{36\sqrt{3}} + \frac{\zeta(3)\,91}{216} = \frac{\pi^3}{36\sqrt{3}} - \frac{91}{648}\int_0^1 \frac{\log^3(1-t^2)}{t^3}\,dt$$

$$\frac{\pi^3}{36\sqrt{3}} + \frac{\zeta(3)\,91}{216} = \frac{\pi^3}{36\sqrt{3}} + \frac{91}{432}\int_0^{\infty} \frac{t^2}{-1+e^t}\,dt$$

$$\frac{\pi^3}{36\sqrt{3}} + \frac{\zeta(3)\,91}{216} = \frac{\pi^3}{36\sqrt{3}} + \frac{91}{324}\int_0^{\infty} \frac{t^2}{1+e^t}\,dt$$

1/(1^3) + 1/(3^3) + 1/(5^3) +...

Input interpretation:

$$\frac{1}{1^3} + \frac{1}{3^3} + \frac{1}{5^3} + \cdots$$

Infinite sum:

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^3} = \frac{7\,\zeta(3)}{8}$$

$\zeta(s)$ is the Riemann zeta function

Decimal approximation:

1.051799790264644999724770891322518741919363005797936521568...

1.05179979026…

Convergence tests:

The ratio test is inconclusive.

The root test is inconclusive.

By the comparison test, the series converges.

Partial sum formula:

$$\sum_{n=1}^{m} \frac{1}{(-1+2n)^3} = \frac{1}{16}\left(\psi^{(2)}\left(m+\frac{1}{2}\right) - \psi^{(2)}\left(\frac{1}{2}\right)\right)$$

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

Series representations:

$$\frac{7\zeta(3)}{8} = \frac{7}{8}\sum_{k=1}^{\infty} \frac{1}{k^3}$$

$$\frac{7\zeta(3)}{8} = \sum_{k=0}^{\infty} \frac{1}{(1+2k)^3}$$

$$\frac{7\zeta(3)}{8} = \frac{7}{8} e^{\sum_{k=1}^{\infty} P(3k)/k}$$

$$\frac{7\zeta(3)}{8} = \frac{7}{6} \times \sum_{n=0}^{\infty} 2^{-1-n} \sum_{k=0}^{n} \frac{(-1)^k \binom{n}{k}}{(1+k)^3}$$

7/8 zeta(3)

Input:

$$\frac{7}{8}\zeta(3)$$

$\zeta(s)$ is the Riemann zeta function

Exact result:

$$\frac{7\zeta(3)}{8}$$

Decimal approximation:

1.051799790264644999724770891322518741919363005797936521568...

1.0517997902646…

Alternative representations:

$$\frac{\zeta(3)\,7}{8} = \frac{7\zeta(3,1)}{8}$$

$$\frac{\zeta(3)\,7}{8} = \frac{7\,S_{2,1}(1)}{8}$$

$$\frac{\zeta(3)\,7}{8} = -\frac{7\,\text{Li}_3(-1)}{\frac{3\times 8}{4}}$$

Series representations:

$$\frac{\zeta(3)\,7}{8} = \frac{7}{8}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{\zeta(3)\,7}{8} = \sum_{k=0}^{\infty}\frac{1}{(1+2\,k)^3}$$

$$\frac{\zeta(3)\,7}{8} = \frac{7}{8}\,e^{\sum_{k=1}^{\infty} P(3\,k)/k}$$

Integral representations:

$$\frac{\zeta(3)\,7}{8} = -\frac{7}{24}\int_0^1 \frac{\log^3(1-t^2)}{t^3}\,dt$$

$$\frac{\zeta(3)\,7}{8} = \frac{1}{4}\int_0^{\infty} t^2\,\text{csch}(t)\,dt$$

$$\frac{\zeta(3)\,7}{8} = \frac{7}{16}\int_0^{\infty}\frac{t^2}{-1+e^t}\,dt$$

Now, we perform the sum of the four expressions:

7/8 zeta(3) (Note that S_3 is ζ(3))

(2Pi^3)/(81sqrt2) + 13/27 zeta(3)

(Pi^3)/64 + 7/16 zeta(3)

(Pi^3)/(36sqrt3) + 91/216 zeta(3)

We obtain:

7/8 zeta(3) + (2Pi^3)/(81sqrt2) + 13/27 zeta(3) + (Pi^3)/64 + 7/16 zeta(3) + (Pi^3)/(36sqrt3) + 91/216 zeta(3)

Input:

$$\frac{7}{8}\zeta(3)+\frac{2\pi^3}{81\sqrt{2}}+\frac{13}{27}\zeta(3)+\frac{\pi^3}{64}+\frac{7}{16}\zeta(3)+\frac{\pi^3}{36\sqrt{3}}+\frac{91}{216}\zeta(3)$$

$\zeta(s)$ is the Riemann zeta function

Exact result:

$$\frac{319\,\zeta(3)}{144}+\frac{\pi^3}{64}+\frac{\sqrt{2}\,\pi^3}{81}+\frac{\pi^3}{36\sqrt{3}}$$

Decimal approximation:

4.185978227247405002449052505990239496858296547764744871569...

4.185978227247…

Alternate forms:

$$\frac{319\,\zeta(3)}{144}+\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}$$

$$\frac{11484\,\zeta(3)+81\,\pi^3+64\sqrt{2}\,\pi^3+48\sqrt{3}\,\pi^3}{5184}$$

$$\frac{11484\sqrt{3}\,\zeta(3)+\left(144+81\sqrt{3}+64\sqrt{6}\right)\pi^3}{5184\sqrt{3}}$$

Alternative representations:

$$\frac{\zeta(3)\,7}{8}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}+\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}+\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=\frac{\pi^3}{64}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\pi^3}{36\sqrt{3}}+\frac{7\,\zeta(3,1)}{8}+\frac{7\,\zeta(3,1)}{16}+\frac{13\,\zeta(3,1)}{27}+\frac{91\,\zeta(3,1)}{216}$$

$$\frac{\zeta(3)\,7}{8}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}+\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}+\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=\frac{7\,S_{2,1}(1)}{8}+\frac{7\,S_{2,1}(1)}{16}+\frac{13\,S_{2,1}(1)}{27}+\frac{91\,S_{2,1}(1)}{216}+\frac{\pi^3}{64}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\pi^3}{36\sqrt{3}}$$

$$\frac{\zeta(3)\,7}{8}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}+\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}+\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=-\frac{7\,\mathrm{Li}_3(-1)}{\frac{3\times 8}{4}}-\frac{7\,\mathrm{Li}_3(-1)}{\frac{3\times 16}{4}}-\frac{13\,\mathrm{Li}_3(-1)}{\frac{3\times 27}{4}}-\frac{91\,\mathrm{Li}_3(-1)}{\frac{3\times 216}{4}}+\frac{\pi^3}{64}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\pi^3}{36\sqrt{3}}$$

Series representations:

$$\frac{\zeta(3)\,7}{8}+\frac{2\pi^3}{81\sqrt{2}}+\frac{\zeta(3)\,13}{27}+\frac{\pi^3}{64}+\frac{\zeta(3)\,7}{16}+\frac{\pi^3}{36\sqrt{3}}+\frac{\zeta(3)\,91}{216}=\frac{\pi^3}{64}+\frac{\sqrt{2}\,\pi^3}{81}+\frac{\pi^3}{36\sqrt{3}}+\frac{319}{144}\sum_{k=1}^{\infty}\frac{1}{k^3}$$

$$\frac{\zeta(3)\,7}{8} + \frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} + \frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} + \frac{\pi^3}{36\sqrt{3}} + \frac{\zeta(3)\,91}{216} = \frac{\pi^3}{64} + \frac{\sqrt{2}\,\pi^3}{81} + \frac{\pi^3}{36\sqrt{3}} + \frac{319}{126}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}$$

$$\frac{\zeta(3)\,7}{8} + \frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} + \frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} + \frac{\pi^3}{36\sqrt{3}} + \frac{\zeta(3)\,91}{216} = \frac{81\pi^3 + 64\sqrt{2}\,\pi^3 + 48\sqrt{3}\,\pi^3 + 5742\sum_{n=0}^{\infty}\frac{\sum_{k=0}^{n}\frac{(-1)^k\binom{n}{k}}{(1+k)^2}}{1+n}}{5184}$$

Integral representations:

$$\frac{\zeta(3)\,7}{8} + \frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} + \frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} + \frac{\pi^3}{36\sqrt{3}} + \frac{\zeta(3)\,91}{216} = \frac{\pi^3}{64} + \frac{\sqrt{2}\,\pi^3}{81} + \frac{\pi^3}{36\sqrt{3}} - \frac{319}{432}\int_0^1 \frac{\log^3(1-t^2)}{t^3}\,dt$$

$$\frac{\zeta(3)\,7}{8} + \frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} + \frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} + \frac{\pi^3}{36\sqrt{3}} + \frac{\zeta(3)\,91}{216} = \frac{\pi^3}{64} + \frac{\sqrt{2}\,\pi^3}{81} + \frac{\pi^3}{36\sqrt{3}} + \frac{319}{288}\int_0^\infty \frac{t^2}{-1+e^t}\,dt$$

$$\frac{\zeta(3)\,7}{8} + \frac{2\pi^3}{81\sqrt{2}} + \frac{\zeta(3)\,13}{27} + \frac{\pi^3}{64} + \frac{\zeta(3)\,7}{16} + \frac{\pi^3}{36\sqrt{3}} + \frac{\zeta(3)\,91}{216} = \frac{\pi^3}{64} + \frac{\sqrt{2}\,\pi^3}{81} + \frac{\pi^3}{36\sqrt{3}} + \frac{319}{216}\int_0^\infty \frac{t^2}{1+e^t}\,dt$$

From which:

((81 + 64 sqrt(2) + 48 sqrt(3)) x^3)/5184 + (319 ζ(3))/144 = 4.1859782272474

Input interpretation:

$$\frac{(81+64\sqrt{2}+48\sqrt{3})\,x^3}{5184} + \frac{319\,\zeta(3)}{144} = 4.1859782272474$$

$\zeta(s)$ is the Riemann zeta function

Result:

$$\frac{(81+64\sqrt{2}+48\sqrt{3})\,x^3}{5184} + \frac{319\,\zeta(3)}{144} = 4.1859782272474$$

Alternate forms:

$$\frac{(81 + 64\sqrt{2} + 48\sqrt{3})x^3}{5184} - 1.5230882820536 = 0$$

$$\frac{x^3}{36\sqrt{3}} + \frac{\sqrt{2}\,x^3}{81} + \frac{x^3}{64} - 1.5230882820536 = 0$$

$$\frac{\left(81 + 16\sqrt{59 + 24\sqrt{6}}\right)x^3}{5184} + \frac{319\,\zeta(3)}{144} = 4.1859782272474$$

Expanded form:

$$\frac{x^3}{36\sqrt{3}} + \frac{\sqrt{2}\,x^3}{81} + \frac{x^3}{64} + \frac{319\,\zeta(3)}{144} = 4.1859782272474$$

Real solution:

$x \approx 3.14159265359$

3.14159265359 ≈ π

Complex solutions:

$x \approx -1.57079632679 - 2.72069904635\,i$

$x \approx -1.57079632679 + 2.72069904635\,i$

((81 + 64 sqrt(2) + 48 sqrt(3)) π^3)/5184 + (319 ζ(3))/((x-1)/12) = 4.1859782272474

Input interpretation:

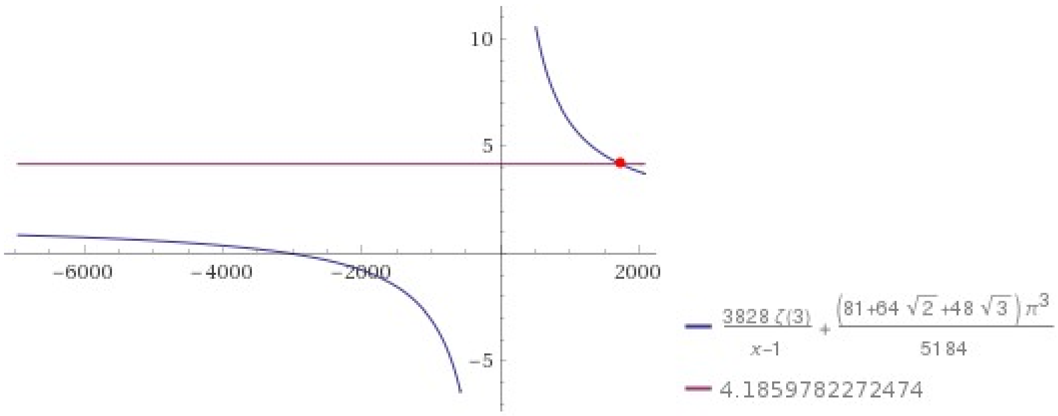
$$\frac{(81 + 64\sqrt{2} + 48\sqrt{3})\pi^3}{5184} + \frac{319\,\zeta(3)}{\frac{x-1}{12}} = 4.1859782272474$$

$\zeta(s)$ is the Riemann zeta function

Result:

$$\frac{3828\,\zeta(3)}{x-1} + \frac{(81 + 64\sqrt{2} + 48\sqrt{3})\pi^3}{5184} = 4.1859782272474$$

Plot:



Alternate form assuming x is real:

$$-\frac{1728.0000000}{1.000000000000 - 1.00000000000\, x} = 1.0000000000$$

Alternate form:

$$\frac{48\sqrt{3}\,\pi^3 x + 64\sqrt{2}\,\pi^3 x + 81\pi^3 x + 19\,844\,352\,\zeta(3) - 48\sqrt{3}\,\pi^3 - 64\sqrt{2}\,\pi^3 - 81\pi^3}{5184\,(x-1)} = 4.1859782272474$$

Solution:

$x \approx 1729.000000000$

1729

We note that, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(((((81 + 64 sqrt(2) + 48 sqrt(3)) π^3)/5184 + (319 ζ(3))/144)))^1/3

Input:

$$\sqrt[3]{\frac{(81 + 64\sqrt{2} + 48\sqrt{3})\,\pi^3}{5184} + \frac{319\,\zeta(3)}{144}}$$

$\zeta(s)$ is the Riemann zeta function

Decimal approximation:

1.611631157728558233010611244286714690400108716561115072185...

1.6116311577…. result that is near to the value of the golden ratio 1,618033988749...

Alternate forms:

$$\sqrt[3]{\frac{319\,\zeta(3)}{144}+\frac{\left(81+16\sqrt{59+24\sqrt{6}}\right)\pi^3}{5184}}$$

$$\frac{1}{12}\sqrt[3]{\frac{1}{3}\left(11\,484\,\zeta(3)+\left(81+64\sqrt{2}+48\sqrt{3}\right)\pi^3\right)}$$

$$\frac{1}{12\sqrt[3]{\frac{3}{11\,484\,\zeta(3)+81\,\pi^3+64\sqrt{2}\,\pi^3+48\sqrt{3}\,\pi^3}}}$$

All 3rd roots of (319 ζ(3))/144 + ((81 + 64 sqrt(2) + 48 sqrt(3)) π^3)/5184:

$$e^0\sqrt[3]{\frac{319\,\zeta(3)}{144}+\frac{\left(81+64\sqrt{2}+48\sqrt{3}\right)\pi^3}{5184}}\approx 1.6116\ \text{(real, principal root)}$$

$$e^{(2i\pi)/3}\sqrt[3]{\frac{319\,\zeta(3)}{144}+\frac{\left(81+64\sqrt{2}+48\sqrt{3}\right)\pi^3}{5184}}\approx -0.8058+1.3957\,i$$

$$e^{-(2i\pi)/3}\sqrt[3]{\frac{319\,\zeta(3)}{144}+\frac{\left(81+64\sqrt{2}+48\sqrt{3}\right)\pi^3}{5184}}\approx -0.8058-1.3957\,i$$

Alternative representations:

$$\sqrt[3]{\frac{\left(81+64\sqrt{2}+48\sqrt{3}\right)\pi^3}{5184}+\frac{319\,\zeta(3)}{144}}=\sqrt[3]{\frac{\pi^3\left(81+64\sqrt{2}+48\sqrt{3}\right)}{5184}+\frac{319\,\zeta(3,1)}{144}}$$

$$\sqrt[3]{\frac{\left(81+64\sqrt{2}+48\sqrt{3}\right)\pi^3}{5184}+\frac{319\,\zeta(3)}{144}}=\sqrt[3]{\frac{319\,S_{2,1}(1)}{144}+\frac{\pi^3\left(81+64\sqrt{2}+48\sqrt{3}\right)}{5184}}$$

$$\sqrt[3]{\frac{\left(81+64\sqrt{2}+48\sqrt{3}\right)\pi^3}{5184}+\frac{319\,\zeta(3)}{144}}=\sqrt[3]{-\frac{319\,\mathrm{Li}_3(-1)}{\frac{3\times 144}{4}}+\frac{\pi^3\left(81+64\sqrt{2}+48\sqrt{3}\right)}{5184}}$$

Series representations:

$$\sqrt[3]{\frac{\left(81+64\sqrt{2}+48\sqrt{3}\right)\pi^3}{5184}+\frac{319\,\zeta(3)}{144}}=\sqrt[3]{\frac{\left(81+64\sqrt{2}+48\sqrt{3}\right)\pi^3}{5184}+\frac{319}{144}\sum_{k=1}^{\infty}\frac{1}{k^3}}$$

$$\sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319\zeta(3)}{144}} = \sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319}{126}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^3}}$$

$$\sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319\zeta(3)}{144}} = \sqrt[3]{\frac{319}{144}e^{\sum_{k=1}^{\infty}P(3k)/k}+\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}}$$

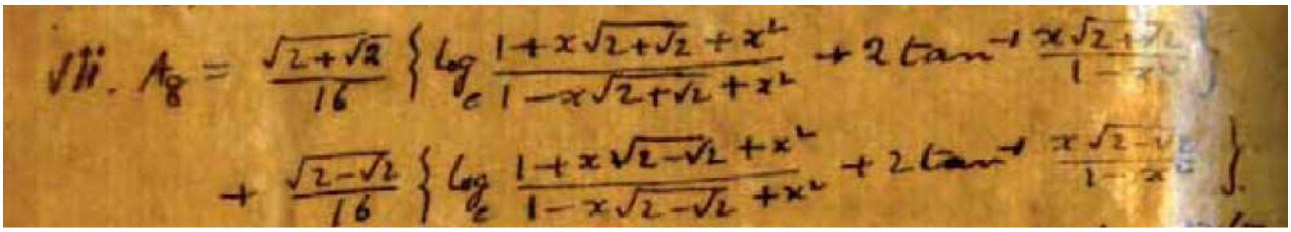
Integral representations:

$$\sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319\zeta(3)}{144}} = \frac{1}{12}\sqrt[3]{\frac{1}{3}\left(81+64\sqrt{2}+48\sqrt{3}\right)\pi^3+1914\int_0^{\infty}\frac{t^2}{-1+e^t}\,dt}$$

$$\sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319\zeta(3)}{144}} = \sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319}{504}\int_0^{\infty}t^2\operatorname{csch}(t)\,dt}$$

$$\sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}+\frac{319\zeta(3)}{144}} = \sqrt[3]{\frac{(81+64\sqrt{2}+48\sqrt{3})\pi^3}{5184}-\frac{319}{432}\int_0^{1}\frac{\log^3(1-t^2)}{t^3}\,dt}$$

Now, we have that:



1/16*(2+sqrt2)^(1/2) [ln(((((1+2(2+sqrt2)^(1/2)+4)))/(((1-2(2+sqrt2)^(1/2)+4))))) + 2 tan^-1 ((2(2+sqrt2)^(1/2)/(1-4)))]

Input:

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(2\times\frac{\sqrt{2+\sqrt{2}}}{1-4}\right)\right)$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)-2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{3}\right)\right)$$

(result in radians)

Decimal approximation:

0.013764838311382013868966278430595886004523852083036857721...

(result in radians)

0.013764838311…

Alternate forms:

$$\frac{1}{8}\sqrt{2+\sqrt{2}}\left(\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{5}\right)-\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{3}\right)\right)$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1}{514}\left(1186+400\sqrt{2}+257\sqrt{\frac{1462400}{66049}+\frac{948800\sqrt{2}}{66049}}\right)\right)-2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{3}\right)\right)$$

$$-\frac{\left(\sqrt{1-i}+\sqrt{1+i}\right)\left(2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{3}\right)-\log\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)\right)}{16\sqrt[4]{2}}$$

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

Alternative representations:

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$
$$\frac{1}{16}\left(2\tan^{-1}\left(1,-\frac{2}{3}\sqrt{2+\sqrt{2}}\right)+\log\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)\right)\sqrt{2+\sqrt{2}}$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$
$$\frac{1}{16}\left(2\tan^{-1}\left(-\frac{2}{3}\sqrt{2+\sqrt{2}}\right)+\log_e\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)\right)\sqrt{2+\sqrt{2}}$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$
$$\frac{1}{16}\left(2\tan^{-1}\left(-\frac{2}{3}\sqrt{2+\sqrt{2}}\right)+\log(a)\log_a\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)\right)\sqrt{2+\sqrt{2}}$$

Series representations:

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$
$$-\frac{1}{8}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{3}\right)+$$
$$\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(-1+\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)-\frac{1}{16}\sqrt{2+\sqrt{2}}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{2}-\frac{5}{4\sqrt{2+\sqrt{2}}}\right)^k}{k}$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$
$$-\frac{1}{8}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{3}\right)+$$
$$\frac{1}{32}\sqrt{2+\sqrt{2}}\log\left(2+\sqrt{2}\right)+\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(\frac{4}{5-2\sqrt{2+\sqrt{2}}}\right)-$$
$$\frac{1}{16}\sqrt{2+\sqrt{2}}\sum_{k=1}^{\infty}\frac{4^{-k}\left(2+\sqrt{2}\right)^{-k/2}\left(-5+2\sqrt{2+\sqrt{2}}\right)^k}{k}$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$
$$-\frac{1}{8}\sqrt{2+\sqrt{2}}\tan^{-1}(z_0)+\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(-1+\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)+$$
$$\sum_{k=1}^{\infty}\left(\frac{(-1)^{-1+k}\sqrt{2+\sqrt{2}}\left(-1+\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)^{-k}}{16k}-\frac{i\sqrt{2+\sqrt{2}}\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2+\sqrt{2}}}{3}-z_0\right)^{k}}{16k}\right)$$

for $(i z_0 \notin \mathbb{R}$ or $($ not $(1 \le i z_0 < \infty)$ and not $(-\infty < i z_0 \le -1)))$

Integral representations:

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$
$$\int_0^1 -\frac{3(2+\sqrt{2})}{4(9+4(2+\sqrt{2})t^2)}\,dt+\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$
$$\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{i(2+\sqrt{2})\left(1+\frac{4}{9}(2+\sqrt{2})\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2}{48\pi^{3/2}}\,ds+$$
$$\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)\quad\text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{1}{16}\sqrt{2+\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2+\sqrt{2}}+4}{1-2\sqrt{2+\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2+\sqrt{2}}}{1-4}\right)\right)=$$
$$\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{i\,2^{-7/2-2s}\times 3^{-1+2s}(1+\sqrt{2})(2+\sqrt{2})^{-s}\,\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)}{\pi\,\Gamma\left(\frac{3}{2}-s\right)}\,ds+$$
$$\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(\frac{5+2\sqrt{2+\sqrt{2}}}{5-2\sqrt{2+\sqrt{2}}}\right)\quad\text{for } 0<\gamma<\frac{1}{2}$$

1/16*(2-sqrt2)^(1/2) [ln(((((1+2(2-sqrt2)^(1/2)+4)))/(((1-2(2-sqrt2)^(1/2)+4))))) + 2 tan^-1 ((2(2-sqrt2)^(1/2)/(1-4)))]

Input:

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(2\times\frac{\sqrt{2-\sqrt{2}}}{1-4}\right)\right)$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)-2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{3}\right)\right)$$

(result in radians)

Decimal approximation:

-0.01487888040278285650039035025666952617526559293627054867...

(result in radians)

-0.014878880402782…

Alternate forms:

$$\frac{1}{8}\sqrt{2-\sqrt{2}}\left(\tanh^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{5}\right)-\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{3}\right)\right)$$

$$-\frac{\left(\sqrt{-1-i}+\sqrt{-1+i}\right)\left(2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{3}\right)-\log\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)\right)}{16\sqrt[4]{2}}$$

$$-\frac{1}{16}i\sqrt{2-\sqrt{2}}\log\left(1-\frac{2}{3}i\sqrt{2-\sqrt{2}}\right)+\frac{1}{16}i\sqrt{2-\sqrt{2}}\log\left(1+\frac{2}{3}i\sqrt{2-\sqrt{2}}\right)+\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)$$

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

Alternative representations:

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=$$
$$\frac{1}{16}\left(2\tan^{-1}\left(1,-\frac{2}{3}\sqrt{2-\sqrt{2}}\right)+\log\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)\right)\sqrt{2-\sqrt{2}}$$

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=$$
$$\frac{1}{16}\left(2\tan^{-1}\left(-\frac{2}{3}\sqrt{2-\sqrt{2}}\right)+\log_e\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)\right)\sqrt{2-\sqrt{2}}$$

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=$$
$$\frac{1}{16}\left(2\tan^{-1}\left(-\frac{2}{3}\sqrt{2-\sqrt{2}}\right)+\log(a)\log_a\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)\right)\sqrt{2-\sqrt{2}}$$

Series representations:

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=$$
$$-\frac{1}{8}\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{3}\right)-$$
$$\frac{1}{16}\sqrt{2-\sqrt{2}}\sum_{k=1}^{\infty}\frac{4^k\left(2-\sqrt{2}\right)^{k/2}\left(\frac{1}{-5+2\sqrt{2-\sqrt{2}}}\right)^k}{k}$$

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=-\frac{1}{16}\sqrt{2-\sqrt{2}}$$
$$\left(\sum_{k=1}^{\infty}\frac{4^k\left(2-\sqrt{2}\right)^{k/2}\left(\frac{1}{-5+2\sqrt{2-\sqrt{2}}}\right)^k}{k}+2\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{1+2k}\times3^{-1-2k}\left(2-\sqrt{2}\right)^{1/2+k}}{1+2k}\right)$$

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=$$

$$-\frac{1}{8}\sqrt{2-\sqrt{2}}\tan^{-1}(z_0)+\sum_{k=1}^{\infty}\left(\frac{(-1)^{1+k}4^{-2+k}(2-\sqrt{2})^{1/2+k/2}\left(5-2\sqrt{2-\sqrt{2}}\right)^{-k}}{k}-\frac{i\sqrt{2-\sqrt{2}}\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2-\sqrt{2}}}{3}-z_0\right)^k}{16k}\right)$$

for $(i z_0 \notin \mathbb{R}$ or (not $(1 \leq i z_0 < \infty)$ and not $(-\infty < i z_0 \leq -1)))$

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=$$

$$-\frac{1}{8}\sqrt{2-\sqrt{2}}\tan^{-1}(z_0)+\sum_{k=1}^{\infty}\left(\frac{(-1)^{-1+k}\sqrt{2-\sqrt{2}}\left(-1+\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)^k}{16k}-\frac{i\sqrt{2-\sqrt{2}}\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2-\sqrt{2}}}{3}-z_0\right)^k}{16k}\right)$$

for $(i z_0 \notin \mathbb{R}$ or (not $(1 \leq i z_0 < \infty)$ and not $(-\infty < i z_0 \leq -1)))$

Integral representations:

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=$$

$$\int_0^1 \frac{6-3\sqrt{2}}{4\left(-9+4\left(-2+\sqrt{2}\right)t^2\right)}\,dt+\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)$$

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{i\left(\frac{17}{9}-\frac{4\sqrt{2}}{9}\right)^{-s}\left(-2+\sqrt{2}\right)\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2}{48\pi^{3/2}}\,ds+\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right) \quad \text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\left(\log\left(\frac{1+2\sqrt{2-\sqrt{2}}+4}{1-2\sqrt{2-\sqrt{2}}+4}\right)+2\tan^{-1}\left(\frac{2\sqrt{2-\sqrt{2}}}{1-4}\right)\right)=$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{i\,2^{-7/2-3s}\times 3^{-1+2s}(-1+\sqrt{2})(2+\sqrt{2})^{s}\,\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)}{\pi\,\Gamma\left(\frac{3}{2}-s\right)}\,ds+$$

$$\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(\frac{5+2\sqrt{2-\sqrt{2}}}{5-2\sqrt{2-\sqrt{2}}}\right)\quad\text{for } 0<\gamma<\frac{1}{2}$$

(0.0137648383113820138-0.0148788804027828565)

Input interpretation:

0.0137648383113820138 − 0.0148788804027828565

Result:

−0.0011140420914008427

-0.0011140420914008427

Thence, we obtain:

(-(0.0137648383113820138-0.0148788804027828565))^1/1024

Input interpretation:

$$\sqrt[1024]{-(0.0137648383113820138-0.0148788804027828565)}$$

Result:

0.993381607705052362 56...

0.9933816077... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}}\approx 0.9991104684$$

1/10^52(((1+(-(0.0137648383-0.0148788804))+0.08+0.02+0.0047-0.0002)))

Input interpretation:

$$\frac{1}{10^{52}}\,(1-(0.0137648383-0.0148788804)+0.08+0.02+0.0047-0.0002)$$

Result:

$$1.1056140421\times 10^{-52}$$

1.1056140421*10^{-52} result practically equal to the value of Cosmological Constant 1.1056*10^{-52} m^{-2}

Now, we have that:

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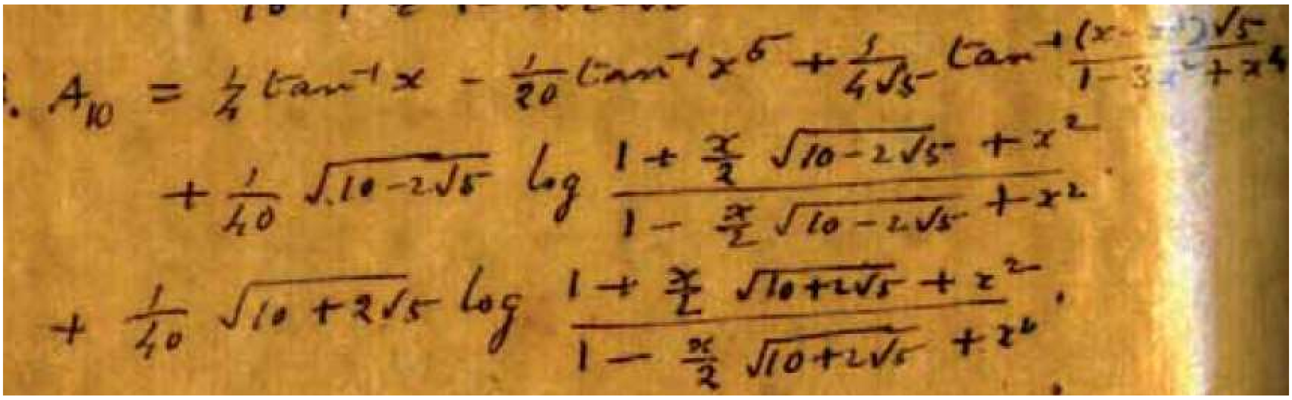


$$A_{10} = \frac{1}{4}\tan^{-1}x - \frac{1}{20}\tan^{-1}x^5 + \frac{1}{4\sqrt{5}}\tan^{-1}\frac{(x-x^3)\sqrt{5}}{1-3x^2+x^4} + \frac{1}{40}\sqrt{10-2\sqrt{5}}\log\frac{1+\frac{x}{2}\sqrt{10-2\sqrt{5}}+x^2}{1-\frac{x}{2}\sqrt{10-2\sqrt{5}}+x^2} + \frac{1}{40}\sqrt{10+2\sqrt{5}}\log\frac{1+\frac{x}{2}\sqrt{10+2\sqrt{5}}+x^2}{1-\frac{x}{2}\sqrt{10+2\sqrt{5}}+x^2}.$$

((1/4 tan^-1 (2))) – ((1/20 tan^-1 (2)^5)) + 1/(4sqrt5) tan^-1(((((2-2^3)sqrt5)) /((1-3*2^2+2^4)))) + 1/40 (10-2sqrt5)^(1/2)* ln (((1+1(10-2sqrt5)^(1/2)+4))/(((1-1(10-2sqrt5)^(1/2)+4))))

Input:

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{1}{4\sqrt{5}}\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)+\frac{1}{40}\sqrt{10-2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Exact Result:

$$\frac{1}{40}\sqrt{10-2\sqrt{5}}\,\log\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)+\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5-\frac{\tan^{-1}\left(\frac{6}{\sqrt{5}}\right)}{4\sqrt{5}}$$

(result in radians)

Decimal approximation:

0.117871277524338220859857341320591906495581624687993036863...

(result in radians)

0.1178712775243382208598...

Alternate forms:

$$\frac{1}{20}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\,\log\left(\frac{1}{41}\left(109-20\sqrt{5}+2\sqrt{10\left(305-109\sqrt{5}\right)}\right)\right)+\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5-\frac{\tan^{-1}\left(\frac{6}{\sqrt{5}}\right)}{4\sqrt{5}}$$

$$\frac{1}{8}i(\log(1-2i)-\log(1+2i))-\frac{1}{640}i(\log(1-2i)-\log(1+2i))^5-\frac{i\left(\log\left(1-\frac{6i}{\sqrt{5}}\right)-\log\left(1+\frac{6i}{\sqrt{5}}\right)\right)}{8\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\,\log\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)$$

$$\frac{1}{40}\left(\sqrt{10-2\sqrt{5}}\left(\log\left(5+\sqrt{10-2\sqrt{5}}\right)-\log\left(5-\sqrt{10-2\sqrt{5}}\right)\right)+10\tan^{-1}(2)-2\tan^{-1}(2)^5-2\sqrt{5}\tan^{-1}\left(\frac{6}{\sqrt{5}}\right)\right)$$

Alternative representations:

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)=\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{1}{40}\log_e\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}+\frac{\tan^{-1}\left(-\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}}$$

$$\frac{1}{4}\tan^{-1}(2) - \frac{1}{20}\tan^{-1}(2)^5 + \frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}} + \frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right) = \frac{1}{4}\tan^{-1}(1,2) - \frac{1}{20}\tan^{-1}(1,2)^5 + \frac{1}{40}\log\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}} + \frac{\tan^{-1}\left(1, -\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}}$$

$$\frac{1}{4}\tan^{-1}(2) - \frac{1}{20}\tan^{-1}(2)^5 + \frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}} + \frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right) = \frac{1}{4}\tan^{-1}(2) - \frac{1}{20}\tan^{-1}(2)^5 + \frac{1}{40}\log(a)\log_a\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}} + \frac{\tan^{-1}\left(-\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}}$$

Series representations:

$$\frac{1}{4}\tan^{-1}(2) - \frac{1}{20}\tan^{-1}(2)^5 + \frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}} + \frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right) = \frac{1}{4}\tan^{-1}(2) - \frac{1}{20}\tan^{-1}(2)^5 - \frac{\tan^{-1}\left(\frac{6}{\sqrt{5}}\right)}{4\sqrt{5}} + \frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right) - \frac{1}{40}\sqrt{10-2\sqrt{5}}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{2}-\frac{5}{2\sqrt{10-2\sqrt{5}}}\right)^k}{k}$$

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+$$

$$\frac{1}{40}\sqrt{10-2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)=$$

$$\frac{1}{640}\left(160\tan^{-1}(z_0)-32\sqrt{5}\tan^{-1}(z_0)-32\tan^{-1}(z_0)^5+\right.$$

$$16\sqrt{2\left(5-\sqrt{5}\right)}\,\log\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)-16\sqrt{2\left(5-\sqrt{5}\right)}$$

$$\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}}\right)^k}{k}+80\,i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{k}-$$

$$80\,i\tan^{-1}(z_0)^4\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{k}+$$

$$80\tan^{-1}(z_0)^3\left(\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{k}\right)^2+$$

$$40\,i\tan^{-1}(z_0)^2\left(\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{k}\right)^3-$$

$$10\tan^{-1}(z_0)\left(\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{k}\right)^4-$$

$$i\left(\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{k}\right)^5-$$

$$\left.16\,i\sqrt{5}\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{6}{\sqrt{5}}-z_0\right)^k}{k}\right)$$

for $(i z_0 \notin \mathbb{R}$ or (not $(1 \le i z_0 < \infty)$ and not $(-\infty < i z_0 \le -1))$)

1/40 (10+2sqrt5)^(1/2)* ln (((1+1(10+2sqrt5)^(1/2)+4))/(((1-1(10+2sqrt5)^(1/2)+4))))

Input:

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)$$

log(x) is the natural logarithm

Exact result:

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)$$

Decimal approximation:

0.18987255794011344479006186860777045433398567588140907800...

0.18987255794…

Property:

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)$$ is a transcendental number

Alternate forms:

$$\frac{1}{20}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\log\left(\frac{1}{82}\left(218+40\sqrt{5}+41\sqrt{\frac{48\,800}{1681}+\frac{17\,440\sqrt{5}}{1681}}\right)\right)$$

$$\frac{\left(\sqrt{1-2i}+\sqrt{1+2i}\right)\log\left(\frac{5+\sqrt{2\left(5+\sqrt{5}\right)}}{5-\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{8\times5^{3/4}}$$

$$\frac{1}{20}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\left(\log\left(5+\sqrt{2\left(5+\sqrt{5}\right)}\right)-\log\left(5-\sqrt{2\left(5+\sqrt{5}\right)}\right)\right)$$

Alternative representations:

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=$$
$$\frac{1}{40}\log_e\left(\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)\sqrt{10+2\sqrt{5}}$$

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=$$
$$\frac{1}{40}\log(a)\log_a\left(\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)\sqrt{10+2\sqrt{5}}$$

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=$$
$$-\frac{1}{40}\operatorname{Li}_1\left(1-\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)\sqrt{10+2\sqrt{5}}$$

Series representations:

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=$$
$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(-1+\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)-$$
$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{2}-\frac{5}{2\sqrt{2\left(5+\sqrt{5}\right)}}\right)^k}{k}$$

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=$$
$$\frac{1}{20}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\,\log\left(-\frac{2\sqrt{2\left(5+\sqrt{5}\right)}}{-5+\sqrt{2\left(5+\sqrt{5}\right)}}\right)-$$
$$\frac{1}{20}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\sum_{k=1}^{\infty}\frac{2^{-(3k)/2}\left(5+\sqrt{5}\right)^{-k/2}\left(-5+\sqrt{2\left(5+\sqrt{5}\right)}\right)^k}{k}$$

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\ \log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=$$

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\ \log\left(-1+\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)-$$

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}}\right)^k}{k}$$

Integral representations:

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\ \log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=\frac{1}{20}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\int_1^{\frac{5+\sqrt{2\left(5+\sqrt{5}\right)}}{5-\sqrt{2\left(5+\sqrt{5}\right)}}}\frac{1}{t}\,dt$$

$$\frac{1}{40}\sqrt{10+2\sqrt{5}}\ \log\left(\frac{1+1\sqrt{10+2\sqrt{5}}+4}{1-1\sqrt{10+2\sqrt{5}}+4}\right)=$$

$$-\frac{i\sqrt{10+2\sqrt{5}}}{80\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(-1+\frac{5+\sqrt{10+2\sqrt{5}}}{5-\sqrt{10+2\sqrt{5}}}\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

((1/4 tan^-1 (2))) – ((1/20 tan^-1 (2)^5)) + 1/(4sqrt5) tan^-1[(((2-2^3)sqrt5)) /((1-3*2^2+2^4))] + 1/40 (10-2sqrt5)^(1/2)* ln [((1+1(10-2sqrt5)^(1/2)+4))/((1-1(10-2sqrt5)^(1/2)+4))] + 0.18987255794

Input interpretation:

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{1}{4\sqrt{5}}\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times2^2+2^4}\right)+$$

$$\frac{1}{40}\sqrt{10-2\sqrt{5}}\ \log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)+0.18987255794$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Result:

0.30774383546...

(result in radians)

0.30774383546…

Alternative representations:

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)+0.189872557940000=0.189872557940000+\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{1}{40}\log_e\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}+\frac{\tan^{-1}\left(-\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}}$$

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)+0.189872557940000=0.189872557940000+\frac{1}{4}\tan^{-1}(1,2)-\frac{1}{20}\tan^{-1}(1,2)^5+\frac{1}{40}\log\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}+\frac{\tan^{-1}\left(1,-\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}}$$

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)+0.189872557940000=0.189872557940000+\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{1}{40}\log(a)\log_a\left(\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}+\frac{\tan^{-1}\left(-\frac{6\sqrt{5}}{-11+2^4}\right)}{4\sqrt{5}}$$

Continued fraction representations:

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)+0.189872557940000=$$

$$0.189872557940000-\frac{8}{5\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}\right)^5}+\frac{1}{2\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}\right)}-\frac{3}{10\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{36}{25}k^2\sqrt{5}^2}{1+2k}\right)}+\frac{\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}}{40\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\left\lfloor\frac{1+k}{2}\right\rfloor^2\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{1+k}\right)}=$$

$$0.189872557940000-\frac{8}{5\left(1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}\right)^5}+\frac{1}{2\left(1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}\right)}-\frac{3}{10\left(1+\frac{36}{5\left(3+\frac{144}{5\left(5+\frac{324}{5\left(7+\frac{576}{5(9+\dots)}\right)}\right)}\right)}\right)}+\frac{\sqrt{10-2\sqrt{5}}\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{40\left(1+\frac{-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}}{2+\frac{-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}}{3+\frac{4\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{4+\frac{4\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{5+\dots}}}}\right)}$$

$$\frac{1}{4}\tan^{-1}(2)-\frac{1}{20}\tan^{-1}(2)^5+\frac{\tan^{-1}\left(\frac{(2-2^3)\sqrt{5}}{1-3\times 2^2+2^4}\right)}{4\sqrt{5}}+\frac{1}{40}\sqrt{10-2\sqrt{5}}\,\log\left(\frac{1+1\sqrt{10-2\sqrt{5}}+4}{1-1\sqrt{10-2\sqrt{5}}+4}\right)+0.189872557940000=$$

$$0.189872557940000-\frac{8}{5\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}\right)^5}+\frac{1}{2\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}\right)}-\frac{3}{10\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{36}{25}k^2\sqrt{5}^2}{1+2k}\right)}+\frac{\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)\sqrt{10-2\sqrt{5}}}{40\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{\frac{1}{2}\left(3+(-1)^k(-1+k)+k\right)}\right)}=$$

$$0.189872557940000-\frac{8}{5\left(1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}\right)^5}+\frac{1}{2\left(1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}\right)}-\frac{3}{10\left(1+\frac{36}{5\left(3+\frac{144}{5\left(5+\frac{324}{5\left(7+\frac{576}{5(9+\dots)}\right)}\right)}\right)}\right)}+\frac{\sqrt{10-2\sqrt{5}}\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{40\left(1+\frac{-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}}{2+\frac{-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}}{3+\frac{2\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{2+\frac{2\left(-1+\frac{5+\sqrt{10-2\sqrt{5}}}{5-\sqrt{10-2\sqrt{5}}}\right)}{5+\dots}}}}\right)}$$

From which, we obtain:

1+1/((5(0.3077438354643382208)))

Input interpretation:

$$1+\frac{1}{5\times 0.3077438354643382208}$$

Result:

1.649891165807531749109751987002000473628420124271935712962...

1.649891165807... ≈ $\zeta(2) = \frac{\pi^2}{6} = 1.644934$ ...

1/2 tan^-1 (2) + 1/6 tan^-1 (8) + 1/(4sqrt3) ln (((1+2sqrt3+4)/(1-2sqrt3+4)))

Input:

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{1}{4\sqrt{3}}\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Exact Result:

$$\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)$$

(result in radians)

Decimal approximation:

1.040991496732833639573748611915498201204183344336196931089...

(result in radians)

1.040991496…

Alternate forms:

$$\frac{1}{12}\left(\sqrt{3}\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)+6\tan^{-1}(2)+2\tan^{-1}(8)\right)$$

$$\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)$$

$$\frac{1}{12}\left(\sqrt{3}\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)+6\tan^{-1}(2)+2\tan^{-1}(8)\right)$$

Alternative representations:

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log_e\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=\frac{1}{2}\tan^{-1}(1,2)+\frac{1}{6}\tan^{-1}(1,8)+\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$
$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log(a)\log_a\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}$$

Series representations:

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$
$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{4}{13}\left(6+5\sqrt{3}\right)\right)}{4\sqrt{3}}-\frac{\sum_{k=1}^{\infty}\frac{\left(\frac{1}{12}\left(6-5\sqrt{3}\right)\right)^k}{k}}{4\sqrt{3}}$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$
$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(-1+\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}-\frac{\sum_{k=1}^{\infty}\frac{\left(\frac{1}{12}\left(6-5\sqrt{3}\right)\right)^k}{k}}{4\sqrt{3}}$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$

$$\frac{2}{3}\tan^{-1}(z_0)+\frac{\log\left(-1+\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}+\sum_{k=1}^{\infty}\left(\frac{(-1)^{-1+k}\left(-1+\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)^{-k}}{4\sqrt{3}\,k}+\frac{i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{4k}+\frac{i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(8-z_0)^k}{12k}\right)$$

for $(i z_0 \notin \mathbb{R}$ or (not $(1 \leq i z_0 < \infty)$ and not $(-\infty < i z_0 \leq -1)))$

Integral representations:

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$

$$\int_0^1\left(\frac{1}{1+4t^2}+\frac{4}{3+192t^2}\right)dt+\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$

$$\int_1^{\frac{1}{13}\left(37+20\sqrt{3}\right)}\left(\frac{\frac{1}{1+\frac{4(1-t)^2}{\left(1+\frac{1}{13}\left(-37-20\sqrt{3}\right)\right)^2}}+\frac{4}{3\left(1+\frac{64(1-t)^2}{\left(1+\frac{1}{13}\left(-37-20\sqrt{3}\right)\right)^2}\right)}}{-1+\frac{1}{13}\left(37+20\sqrt{3}\right)}+\frac{1}{4\sqrt{3}\,t}\right)dt$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,65^{-s}\left(4+3\times 13^s\right)\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2}{12\pi^{3/2}}\,ds+\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}$$ for $0<\gamma<\frac{1}{2}$

Continued fraction representations:

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$
$$\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}}+\frac{4}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{64k^2}{1+2k}\right)}=$$
$$\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}}+\frac{4}{3\left(1+\frac{64}{3+\frac{256}{5+\frac{576}{7+\frac{1024}{9+\dots}}}}\right)}$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}=$$
$$\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}}+\frac{4}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{64k^2}{1+2k}\right)}=$$
$$\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}}+\frac{4}{3\left(1+\frac{64}{3+\frac{256}{5+\frac{576}{7+\frac{1024}{9+\dots}}}}\right)}$$

$$\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}} =$$

$$\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4(1-2k)^2}{5-6k}}+\frac{4}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{64(1-2k)^2}{65-126k}\right)} =$$

$$\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\frac{4}{-1+\frac{36}{-7+\frac{100}{-13+\frac{196}{-19+\dots}}}}}+$$

$$\frac{4}{3\left(1+\frac{64}{-61+\frac{576}{-187+\frac{1600}{-313+\frac{3136}{-439+\dots}}}}\right)}$$

((((1/2 tan^-1 (2) + 1/6 tan^-1 (8) + 1/(4sqrt3) ln (((1+2sqrt3+4)/(1-2sqrt3+4)))))))^12

Input:

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{1}{4\sqrt{3}}\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)\right)^{12}$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Exact Result:

$$\left(\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)\right)^{12}$$

(result in radians)

Decimal approximation:

1.6194449301523700387373298290094377188510163518980449164 04...

(result in radians)

1.619444930152… result that is a good approximation to the value of the golden ratio 1,618033988749...

Alternate forms:

$$\left(\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)\right)^{12}$$

$$\frac{\left(\sqrt{3}\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)+6\tan^{-1}(2)+2\tan^{-1}(8)\right)^{12}}{8916100448256}$$

$$\frac{\left(3\log\left(-\frac{5+2\sqrt{3}}{2\sqrt{3}-5}\right)+2\sqrt{3}\left(3\tan^{-1}(2)+\tan^{-1}(8)\right)\right)^{12}}{6499837226778624}$$

Alternative representations:

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12}=\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log_e\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}\right)^{12}$$

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12}=\left(\frac{1}{2}\tan^{-1}(1,2)+\frac{1}{6}\tan^{-1}(1,8)+\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}\right)^{12}$$

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12}=\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log(a)\log_a\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}\right)^{12}$$

Series representations:

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12} = \left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(-1+\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{12}\left(6-5\sqrt{3}\right)\right)^k}{k}}{4\sqrt{3}}\right)^{12}$$

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12} = \frac{1}{8\,916\,100\,448\,256}\left(8\tan^{-1}(z_0)+\sqrt{3}\log\left(-1+\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)-\sqrt{3}\sum_{k=1}^{\infty}\frac{\left(\frac{1}{12}\left(6-5\sqrt{3}\right)\right)^k}{k}+3\,i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(2-z_0)^k}{k}+i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)(8-z_0)^k}{k}\right)^{12}$$

for $(i z_0 \notin \mathbb{R}$ or $($not $(1 \le i z_0 < \infty)$ and not $(-\infty < i z_0 \le -1)))$

Continued fraction representations:

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12} = \left(\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}}+\frac{4}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{64k^2}{1+2k}\right)}\right)^{12} = \left(\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}}+\frac{4}{3\left(1+\frac{64}{3+\frac{256}{5+\frac{576}{7+\frac{1024}{9+\dots}}}}\right)}\right)^{12}$$

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12} =$$

$$\left(\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4k^2}{1+2k}}+\frac{4}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{64k^2}{1+2k}\right)}\right)^{12} =$$

$$\left(\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{1+\frac{4}{3+\frac{16}{5+\frac{36}{7+\frac{64}{9+\dots}}}}}+\frac{4}{3\left(1+\frac{64}{3+\frac{256}{5+\frac{576}{7+\frac{1024}{9+\dots}}}}\right)}\right)^{12}$$

$$\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)}{4\sqrt{3}}\right)^{12} =$$

$$\left(\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{4(1-2k)^2}{5-6k}}+\frac{4}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{64(1-2k)^2}{65-126k}\right)}\right)^{12} =$$

$$\left(\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{1+\frac{4}{-1+\frac{36}{-7+\frac{100}{-13+\frac{196}{-19+\dots}}}}}+\frac{4}{3\left(1+\frac{64}{-61+\frac{576}{-187+\frac{1600}{-313+\frac{3136}{-439+\dots}}}}\right)}\right)^{12}$$

1/10^27(((((((1/2 tan^-1 (2) + 1/6 tan^-1 (8) + 1/(4sqrt3) ln (((1+2sqrt3+4)/(1-2sqrt3+4)))))))^12 +(55-2)*1/10^3)))

Input:

$$\frac{1}{10^{27}}\left(\left(\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)+\frac{1}{4\sqrt{3}}\log\left(\frac{1+2\sqrt{3}+4}{1-2\sqrt{3}+4}\right)\right)^{12}+(55-2)\times\frac{1}{10^3}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Exact Result:

$$\frac{\frac{53}{1000}+\left(\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)\right)^{12}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

(result in radians)

Decimal approximation:

$1.6724449301523700387373298290094377188510163518980449\ldots \times 10^{-27}$

(result in radians)

$1.6724449301523\ldots*10^{-27}$ result practically equal to the proton mass

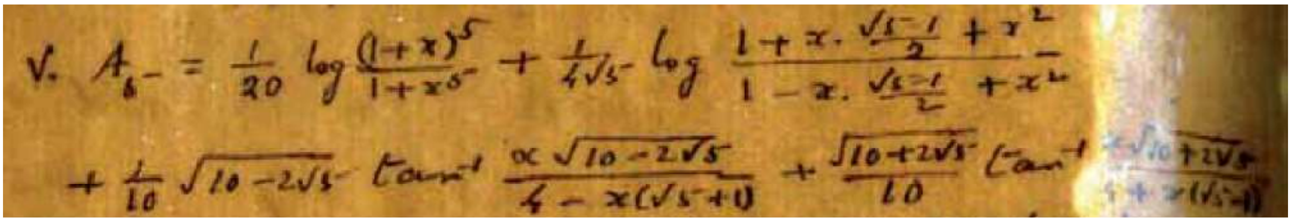
Alternate forms:

$$\frac{\frac{53}{1000}+\left(\frac{\log\left(\frac{1}{13}\left(37+20\sqrt{3}\right)\right)}{4\sqrt{3}}+\frac{1}{2}\tan^{-1}(2)+\frac{1}{6}\tan^{-1}(8)\right)^{12}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{\frac{53}{1000}+\left(\frac{\pi}{3}+\frac{\log\left(5+2\sqrt{3}\right)-\log\left(5-2\sqrt{3}\right)}{4\sqrt{3}}+\frac{1}{12}\left(\tan^{-1}\left(\frac{36}{323}\right)-\pi\right)\right)^{12}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{\frac{53}{1000}+\left(\frac{1}{4}\,i\,(\log(1-2i)-\log(1+2i))+\frac{1}{12}\,i\,(\log(1-8i)-\log(1+8i))+\frac{\log\left(\frac{5+2\sqrt{3}}{5-2\sqrt{3}}\right)}{4\sqrt{3}}\right)^{12}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

We have that:



1/20 ln((((1+2)^5)/(1+2^5))) + 1/(4sqrt5) ln ((((((1+2*((sqrt5-1)/2)+4))) / (((1-2*((sqrt5-1)/2)+4)))))) + 1/20 (10-2sqrt5)^(1/2) tan^-1 (((((2*(10-2sqrt5)^(1/2)))/((4-2(sqrt5+1))))))

Input:

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{1}{4\sqrt{5}}\log\left(\frac{1+2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)+4}{1-2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)+4}\right)+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)$$

(result in radians)

Decimal approximation:

0.0285174072317215217319787204282888130748586476772446 07539...

(result in radians)

0.0285174072…

Alternate forms:

$$\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{1}{31}\left(29+10\sqrt{5}\right)\right)}{4\sqrt{5}}-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\right)$$

$$\frac{1}{20}\left(\log\left(\frac{81}{11}\right)+\sqrt{5}\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)+\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\right)$$

$$\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}+\frac{1}{40}i\sqrt{10-2\sqrt{5}}\log\left(1-\frac{2i\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)-\frac{1}{40}i\sqrt{10-2\sqrt{5}}\log\left(1+\frac{2i\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)$$

Alternative representations:

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)=$$

$$\frac{1}{20}\log\left(\frac{3^5}{1+2^5}\right)+\frac{1}{20}\tan^{-1}\left(1,\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)=$$

$$\frac{1}{20}\log(a)\log_a\left(\frac{3^5}{1+2^5}\right)+$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}+\frac{\log(a)\log_a\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)=$$

$$\frac{1}{20}\log_e\left(\frac{3^5}{1+2^5}\right)+\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}+\frac{\log_e\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

Integral representations:

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)=$$

$$\int_0^1\frac{-5+3\sqrt{5}}{10\left(-3+\sqrt{5}+\left(-5+\sqrt{5}\right)t^2\right)}\,dt+\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)=$$

$$\int_1^{\frac{81}{11}}\left(\frac{11}{70}\left(\frac{1}{\left(4-2\left(1+\sqrt{5}\right)\right)\left(1+\frac{121\left(10-2\sqrt{5}\right)(1-t)^2}{1225\left(4-2\left(1+\sqrt{5}\right)\right)^2}\right)}-\frac{1}{\sqrt{5}\left(4-2\left(1+\sqrt{5}\right)\right)\left(1+\frac{121\left(10-2\sqrt{5}\right)(1-t)^2}{1225\left(4-2\left(1+\sqrt{5}\right)\right)^2}\right)}\right)+\frac{1}{20t}-\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\sqrt{5}\left(-\frac{81}{11}+\frac{4+\sqrt{5}}{6-\sqrt{5}}+t-\frac{\left(4+\sqrt{5}\right)t}{6-\sqrt{5}}\right)}\right)dt$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)=$$

$$-\frac{i\left(10-2\sqrt{5}\right)}{40\left(4-2\left(1+\sqrt{5}\right)\right)\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(1+\frac{4\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2\,ds+\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}\quad\text{for } 0<\gamma<\frac{1}{2}$$

(((10+2sqrt5)^(1/2)))/20 tan^-1 ((((2*(10+2sqrt5)^(1/2))/((4+2(sqrt5-1))))))

Input:

$$\left(\frac{1}{20}\sqrt{10+2\sqrt{5}}\right)\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{1}{20}\sqrt{10+2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)$$

(result in radians)

Decimal approximation:

0.16470863833823150788500444841366992125083471428369862 3665...

(result in radians)

0.164708638…

Alternate forms:

$$\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\cot^{-1}\left(\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\right)$$

$$\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\right)$$

$$\frac{\left(\sqrt{1-2i}+\sqrt{1+2i}\right)\tan^{-1}\left(\frac{\sqrt{2\left(5+\sqrt{5}\right)}}{1+\sqrt{5}}\right)}{4\times 5^{3/4}}$$

$\cot^{-1}(x)$ is the inverse cotangent function

Alternative representations:

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = \frac{1}{20}\,\mathrm{sc}^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(-1+\sqrt{5}\right)}\middle|\,0\right)\sqrt{10+2\sqrt{5}}$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = \frac{1}{20}\tan^{-1}\left(1,\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(-1+\sqrt{5}\right)}\right)\sqrt{10+2\sqrt{5}}$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = \frac{1}{20}\,i\tanh^{-1}\left(-\frac{2i\sqrt{10+2\sqrt{5}}}{4+2\left(-1+\sqrt{5}\right)}\right)\sqrt{10+2\sqrt{5}}$$

Series representations:

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = \frac{1}{40}\sqrt{10+2\sqrt{5}}\,\pi - \frac{1}{20}\sqrt{10+2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\left(10+2\sqrt{5}\right)^{1/2(-1-2k)}\left(4+2\left(-1+\sqrt{5}\right)\right)^{1+2k}}{1+2k}$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = -\frac{1}{20}\,i\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\left(\log(2)+\log\left(1+\sqrt{5}\right)-\log\left(1+\sqrt{5}-i\sqrt{2\left(5+\sqrt{5}\right)}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1+\sqrt{5}-i\sqrt{2\left(5+\sqrt{5}\right)}}{2+2\sqrt{5}}\right)^k}{k}\right)$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = -\frac{1}{40}\,i\sqrt{10+2\sqrt{5}}\,\log(2)+\frac{1}{40}\,i\sqrt{10+2\sqrt{5}}\,\log\left(-i\left(i+\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(-1+\sqrt{5}\right)}\right)\right)+\frac{1}{40}\,i\sqrt{10+2\sqrt{5}}\sum_{k=1}^{\infty}\frac{\left(\frac{1+\sqrt{5}-i\sqrt{2\left(5+\sqrt{5}\right)}}{2+2\sqrt{5}}\right)^k}{k}$$

Integral representations:

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = \frac{\left(3+\sqrt{5}\right)\left(5+\sqrt{5}\right)}{10\left(1+\sqrt{5}\right)}\int_0^1\frac{1}{3+\sqrt{5}+\left(5+\sqrt{5}\right)t^2}\,dt$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = -\frac{i\left(10+2\sqrt{5}\right)}{40\left(4+2\left(-1+\sqrt{5}\right)\right)\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(1+\frac{4\left(10+2\sqrt{5}\right)}{\left(4+2\left(-1+\sqrt{5}\right)\right)^2}\right)^{-s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = -\frac{i\left(10+2\sqrt{5}\right)}{40\left(4+2\left(-1+\sqrt{5}\right)\right)\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(4\left(10+2\sqrt{5}\right)\right)^{-s}\left(4+2\left(-1+\sqrt{5}\right)\right)^{2s}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

Continued fraction representations:

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2\left(\sqrt{5}-1\right)}\right)\sqrt{10+2\sqrt{5}} = \frac{5+\sqrt{5}}{10\left(1+\sqrt{5}\right)\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{\left(5+\sqrt{5}\right)k^2}{3+\sqrt{5}}}{1+2k}\right)} =$$

$$\frac{5+\sqrt{5}}{10\left(1+\sqrt{5}\right)\left(1+\frac{5+\sqrt{5}}{\left(3+\sqrt{5}\right)\left(3+\frac{4\left(5+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(5+\frac{9\left(5+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)\left(7+\frac{16\left(5+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)(9+\ldots)}\right)}\right)}\right)}\right)}$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2(\sqrt{5}-1)}\right)\sqrt{10+2\sqrt{5}} =$$

$$\frac{30+14\sqrt{5}+4(5+2\sqrt{5})\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{(5+\sqrt{5})(1+(-1)^{1+k}+k)^2}{3+\sqrt{5}}}{3+2k}\right)}{5(1+\sqrt{5})^3\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{(5+\sqrt{5})(1+(-1)^{1+k}+k)^2}{3+\sqrt{5}}}{3+2k}\right)} =$$

$$\frac{30+14\sqrt{5}+4(5+2\sqrt{5})\frac{9(5+\sqrt{5})}{(3+\sqrt{5})\left(5+\frac{4(5+\sqrt{5})}{(3+\sqrt{5})\left(7+\frac{25(5+\sqrt{5})}{(3+\sqrt{5})\left(9+\frac{16(5+\sqrt{5})}{(3+\sqrt{5})(11+\ldots)}\right)}\right)}\right)}}{5(1+\sqrt{5})^3\left(3+\frac{9(5+\sqrt{5})}{(3+\sqrt{5})\left(5+\frac{4(5+\sqrt{5})}{(3+\sqrt{5})\left(7+\frac{25(5+\sqrt{5})}{(3+\sqrt{5})\left(9+\frac{16(5+\sqrt{5})}{(3+\sqrt{5})(11+\ldots)}\right)}\right)}\right)}\right)}$$

$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10+2\sqrt{5}}}{4+2(\sqrt{5}-1)}\right)\sqrt{10+2\sqrt{5}} =$$

$$\frac{5+\sqrt{5}}{10(1+\sqrt{5})\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{(5+\sqrt{5})(1-2k)^2}{3+\sqrt{5}}}{\frac{4(4+\sqrt{5}-2k)}{(1+\sqrt{5})^2}}\right)} = (5+\sqrt{5}) \Bigg/ \left(10(1+\sqrt{5})\left(1+(5+\sqrt{5}) \Bigg/ \left((3+\sqrt{5})\left(\frac{4(2+\sqrt{5})}{(1+\sqrt{5})^2}+(9(5+\sqrt{5})) \Bigg/ \left((3+\sqrt{5})\left(\frac{4\sqrt{5}}{(1+\sqrt{5})^2}+\frac{25(5+\sqrt{5})}{(3+\sqrt{5})\left(\frac{4(-2+\sqrt{5})}{(1+\sqrt{5})^2}+\frac{49(5+\sqrt{5})}{(3+\sqrt{5})\left(\frac{4(-4+\sqrt{5})}{(1+\sqrt{5})^2}+\ldots\right)}\right)}\right)\right)\right)\right)\right)\right)$$

thence, we obtain:

1/20 ln((((1+2)^5)/(1+2^5))) + 1/(4sqrt5) ln ((((((1+2*((sqrt5-1)/2)+4))) / (((1-2*((sqrt5-1)/2)+4)))))) + 1/20 (10-2sqrt5)^(1/2) tan^-1 (((((2*(10-2sqrt5)^(1/2)))/((4-2(sqrt5+1)))))) + 0.164708638338

Input interpretation:

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{1}{4\sqrt{5}}\log\left(\frac{1+2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)+4}{1-2\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)+4}\right)+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.164708638338$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Result:

0.193226045570...

(result in radians)

0.19322604557…

Alternative representations:

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+0.1647086383380000=0.1647086383380000+\frac{1}{20}\log\left(\frac{3^5}{1+2^5}\right)+\frac{1}{20}\tan^{-1}\left(1,\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+$$
$$0.1647086383380000 = 0.1647086383380000+\frac{1}{20}\log(a)\log_a\left(\frac{3^5}{1+2^5}\right)+$$
$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}+\frac{\log(a)\log_a\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+$$
$$0.1647086383380000 = 0.1647086383380000+\frac{1}{20}\log_e\left(\frac{3^5}{1+2^5}\right)+$$
$$\frac{1}{20}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)\sqrt{10-2\sqrt{5}}+\frac{\log_e\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

Integral representations:

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+$$
$$0.1647086383380000 = 0.1647086383380000+$$
$$\int_0^1 -\frac{\left(-5+\sqrt{5}\right)\left(-1+\sqrt{5}\right)}{20t^2\left(-5+\sqrt{5}\right)-10\left(-1+\sqrt{5}\right)^2}\,dt+\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+$$
$$\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2(\sqrt{5}+1)}\right)+0.1647086383380000=$$
$$0.1647086383380000+\int_1^{\frac{81}{11}}\left(\frac{1}{20t}-\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\sqrt{5}\left(-\frac{81}{11}+t+\frac{4+\sqrt{5}}{6-\sqrt{5}}-\frac{t\left(4+\sqrt{5}\right)}{6-\sqrt{5}}\right)}+\right.$$
$$\frac{11}{70}\left(\frac{1}{(4-2(1+\sqrt{5}))\left(1+\frac{121(1-t)^2\left(10-2\sqrt{5}\right)}{1225\left(4-2\left(1+\sqrt{5}\right)\right)^2}\right)}-\right.$$
$$\left.\left.\frac{\sqrt{5}}{5(4-2(1+\sqrt{5}))\left(1+\frac{121(1-t)^2\left(10-2\sqrt{5}\right)}{1225\left(4-2\left(1+\sqrt{5}\right)\right)^2}\right)}\right)\right)dt$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+$$
$$\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2(\sqrt{5}+1)}\right)+0.1647086383380000=$$
$$0.1647086383380000+\frac{1}{20}\log\left(\frac{81}{11}\right)+\frac{\log\left(\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4\sqrt{5}}-\frac{i(10-2\sqrt{5})}{40\pi^{3/2}(4-2(1+\sqrt{5}))}$$
$$\int_{-i\infty+\gamma}^{i\infty+\gamma}\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2\left(1+\frac{4(10-2\sqrt{5})}{(4-2(1+\sqrt{5}))^2}\right)^{-s}ds\quad\text{for } 0<\gamma<\frac{1}{2}$$

Continued fraction representations:

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+$$

$$0.1647086383380000 = 0.1647086383380000+\frac{7}{22\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{70}{11}\left\lfloor\frac{1+k}{2}\right\rfloor^2}{1+k}\right)}+$$

$$\frac{10-2\sqrt{5}}{10\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{4k^2\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2}}{1+2k}\right)\left(4-2\left(1+\sqrt{5}\right)\right)}+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\left\lfloor\frac{1+k}{2}\right\rfloor^2\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{1+k}\right)\sqrt{5}} =$$

$$0.1647086383380000+\left(10-2\sqrt{5}\right)\Big/\left(10\left(4-2\left(1+\sqrt{5}\right)\right)\left(1+\left(4\left(10-2\sqrt{5}\right)\right)\Big/\left(\left(4-2\left(1+\sqrt{5}\right)\right)^2\left(3+\left(16\left(10-2\sqrt{5}\right)\right)\Big/\left(\left(4-2\left(1+\sqrt{5}\right)\right)^2\left(5+\frac{36\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2\left(7+\frac{64\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2(9+\ldots)}\right)}\right)\right)\right)\right)\right)\right)$$

$$+\frac{7}{22\left(1+\frac{70}{11\left(2+\frac{70}{11\left(3+\frac{280}{11\left(4+\frac{280}{11(5+\ldots)}\right)}\right)}\right)}\right)}+$$

$$\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\sqrt{5}\left(1+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{2+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{3+\frac{4\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4+\frac{4\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{5+\ldots}}}}\right)}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+$$

$$0.1647086383380000=0.1647086383380000+\frac{7}{22\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{70\left\lfloor\frac{1+k}{2}\right\rfloor}{11}}{\frac{1}{2}\left(3+(-1)^k(-1+k)+k\right)}\right)}+$$

$$\frac{10-2\sqrt{5}}{10\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{4k^2\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2}}{1+2k}\right)\left(4-2\left(1+\sqrt{5}\right)\right)}+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{\frac{1}{2}\left(3+(-1)^k(-1+k)+k\right)}\right)\sqrt{5}}=$$

$$0.1647086383380000+\left(10-2\sqrt{5}\right)\Bigg/\left(10\left(4-2\left(1+\sqrt{5}\right)\right)\left(1+\left(4\left(10-2\sqrt{5}\right)\right)\Bigg/\left(\left(4-2\left(1+\sqrt{5}\right)\right)^2\left(3+\left(16\left(10-2\sqrt{5}\right)\right)\Bigg/\left(\left(4-2\left(1+\sqrt{5}\right)\right)^2\left(5+\frac{36\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2\left(7+\frac{64\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2(9+\ldots)}\right)}\right)\right)\right)\right)\right)\right)$$

$$+\frac{7}{22\left(1+\frac{70}{11\left(2+\frac{70}{11\left(3+\frac{140}{11\left(2+\frac{140}{11(5+\ldots)}\right)}\right)}\right)}\right)}+$$

$$\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\sqrt{5}\left(1+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{2+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{3+\frac{2\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{2+\frac{2\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{5+\ldots}}}}\right)}$$

$$\frac{1}{20}\log\left(\frac{(1+2)^5}{1+2^5}\right)+\frac{\log\left(\frac{1+\frac{2}{2}\left(\sqrt{5}-1\right)+4}{1-\frac{2}{2}\left(\sqrt{5}-1\right)+4}\right)}{4\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}\tan^{-1}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(\sqrt{5}+1\right)}\right)+$$

$$0.1647086383380000 = 0.1647086383380000+\frac{7}{22\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{70}{11}\left\lfloor\frac{1+k}{2}\right\rfloor^2}{1+k}\right)}+$$

$$\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\left\lfloor\frac{1+k}{2}\right\rfloor^2\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{1+k}\right)\sqrt{5}}+\frac{1}{20}\sqrt{10-2\sqrt{5}}$$

$$\left(-\frac{8\left(10-2\sqrt{5}\right)^{3/2}}{\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{4\left(1+(-1)^{1+k}+k\right)^2\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2}}{3+2k}\right)\left(4-2\left(1+\sqrt{5}\right)\right)^3}+\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}\right)=$$

$$0.1647086383380000+\frac{1}{20}\sqrt{10-2\sqrt{5}}\left(\frac{2\sqrt{10-2\sqrt{5}}}{4-2\left(1+\sqrt{5}\right)}-\right.$$
$$\left(8\left(10-2\sqrt{5}\right)^{3/2}\right)\Big/\left(\left(4-2\left(1+\sqrt{5}\right)\right)^3\left(3+\left(36\left(10-2\sqrt{5}\right)\right)\Big/\right.\right.$$
$$\left(\left(4-2\left(1+\sqrt{5}\right)\right)^2\left(5+\left(16\left(10-2\sqrt{5}\right)\right)\Big/\left(\left(4-2\left(1+\sqrt{5}\right)\right)^2\right.\right.\right.$$
$$\left(7+\left(100\left(10-2\sqrt{5}\right)\right)\Big/\left(\left(4-2\left(1+\sqrt{5}\right)\right)^2\right.\right.$$
$$\left.\left.\left.\left.\left.\left.\left.\left.\left.\left(9+\frac{64\left(10-2\sqrt{5}\right)}{\left(4-2\left(1+\sqrt{5}\right)\right)^2(11+\ldots)}\right)\right)\right)\right)\right)\right)\right)\right)\right)+$$

$$\frac{7}{22\left(1+\frac{70}{11\left(2+\frac{70}{11\left(3+\frac{280}{11\left(4+\frac{280}{11(5+\ldots)}\right)}\right)}\right)}\right)}+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{4\sqrt{5}\left(1+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{2+\frac{-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}}{3+\frac{4\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{4+\frac{4\left(-1+\frac{4+\sqrt{5}}{6-\sqrt{5}}\right)}{5+\ldots}}}}\right)}$$

From which:

1+1/(((1/(0.193226045569721521 7319))))^1/4-(47-2)*1/10^3

Input interpretation:

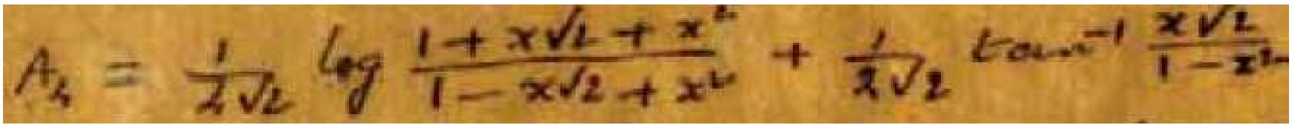
$$1+\frac{1}{\sqrt[4]{\frac{1}{0.1932260455697215217319}}}-(47-2)\times\frac{1}{10^3}$$

Result:

1.618004409019791179769 3...

1.618004409... result that is a very good approximation to the value of the golden ratio 1,618033988749...

Now, we have that:



1/(4sqrt2) ln (((1+2sqrt2+4)/(1-2sqrt2+4)))+1/(2sqrt2) tan^-1(((2sqrt2)/(1-4)))

Input:

$$\frac{1}{4\sqrt{2}}\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)+\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)$$

log(x) is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}$$

(result in radians)

Decimal approximation:

−0.04059304540290341402684888493340270092590079222787614185...

(result in radians)

-0.0405930454029034…..

Alternate forms:

$$\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)-2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{4\sqrt{2}}$$

$$\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}$$

$$\frac{\log\left(-\frac{1}{2\sqrt{2}-5}\right)+\log\left(5+2\sqrt{2}\right)-2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{4\sqrt{2}}$$

Alternative representations:

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}=\frac{\tan^{-1}\left(1,-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}=\frac{\tan^{-1}\left(-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log_e\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}=\frac{\tan^{-1}\left(-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log(a)\log_a\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}$$

Series representations:

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}=$$
$$-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log\left(\frac{4}{17}\left(4+5\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}}{4\sqrt{2}}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = \frac{\log\left(\frac{4}{17}\left(4+5\sqrt{2}\right)\right) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k} - 2\sum_{k=0}^{\infty} \frac{(-1)^k 2^{3/2+3k} \times 3^{-1-2k}}{1+2k}}{4\sqrt{2}}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = \frac{\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k} - 2\sum_{k=0}^{\infty} \frac{(-1)^k 2^{3/2+3k} \times 3^{-1-2k}}{1+2k}}{4\sqrt{2}}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = -\frac{\tan^{-1}(z_0)}{2\sqrt{2}} + \frac{\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}} + \sum_{k=1}^{\infty}\left(\frac{(-1)^{-1+k}\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)^{-k}}{4\sqrt{2}\,k} - \frac{i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2}}{3}-z_0\right)^k}{4\sqrt{2}\,k}\right)$$

for $(i z_0 \notin \mathbb{R}$ or (not $(1 \leq i z_0 < \infty)$ and not $(-\infty < i z_0 \leq -1)))$

Integral representations:

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = -3\int_0^1 \frac{1}{9+8t^2}\,dt + \frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = \int_1^{\frac{1}{17}\left(33+20\sqrt{2}\right)}\left(-\frac{3}{\left(-1+\frac{1}{17}\left(33+20\sqrt{2}\right)\right)\left(9+\frac{8(1-t)^2}{\left(1+\frac{1}{17}\left(-33-20\sqrt{2}\right)\right)^2}\right)} + \frac{1}{4\sqrt{2}\,t}\right)dt$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = \frac{i}{12\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{9}{17}\right)^s \Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2\,ds + \frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}} \quad \text{for } 0<\gamma<\frac{1}{2}$$

Continued fraction representations:

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = \frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}} - \frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8k^2}{9}}{1+2k}\right)} =$$

$$\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}} - \frac{1}{3\left(1+\frac{8}{9\left(3+\frac{32}{9\left(5+\frac{8}{7+\frac{128}{9(9+\ldots)}}\right)}\right)}\right)}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = \frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}} - \frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8k^2}{9}}{1+2k}\right)} =$$

$$\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}} - \frac{1}{3\left(1+\frac{8}{9\left(3+\frac{32}{9\left(5+\frac{8}{7+\frac{128}{9(9+\ldots)}}\right)}\right)}\right)}$$

$$\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}} = \frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}} - \frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8}{9}(1-2k)^2}{\frac{1}{9}(17+2k)}\right)} =$$

$$\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}} - \frac{1}{3\left(1+\frac{8}{9\left(\frac{19}{9}+\frac{8}{\frac{7}{3}+\frac{200}{9\left(\frac{23}{9}+\frac{392}{9\left(\frac{25}{9}+\dots\right)}\right)}}\right)}\right)}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

(64+8)* -1/(((((1/(4sqrt2) ln (((1+2sqrt2+4)/(1-2sqrt2+4)))+1/(2sqrt2) tan^-1(((2sqrt2)/(1-4)))))))-47+Pi-(2-sqrt3+1/2)

Input:

$$\frac{(64+8)\times(-1)}{\frac{1}{4\sqrt{2}}\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)+\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)} - 47 + \pi - \left(2-\sqrt{3}+\frac{1}{2}\right)$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}}$$

(result in radians)

Decimal approximation:

1729.0764855457834986270451992431707593020099622381767481 02...

(result in radians)

1729.076485545…

We know that 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms:

$$-\frac{99}{2}+\sqrt{3}+\pi+\frac{144\sqrt{2}}{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)-\tanh^{-1}\left(\frac{2\sqrt{2}}{5}\right)}$$

$$-\frac{99}{2}+\sqrt{3}+\pi+\frac{288\sqrt{2}}{\log\left(\frac{17}{33+20\sqrt{2}}\right)+2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}$$

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{288\sqrt{2}}{\log\left(-\frac{5+2\sqrt{2}}{2\sqrt{2}-5}\right)-2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}$$

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

Alternative representations:

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=-\frac{99}{2}+\pi-\frac{72}{\frac{\tan^{-1}\left(1,-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}+\sqrt{3}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=-\frac{99}{2}+\pi-\frac{72}{\frac{\tan^{-1}\left(-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log_e\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}+\sqrt{3}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$
$$-\frac{99}{2}+\pi-\frac{72}{\frac{\tan^{-1}\left(1,-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log_e\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}+\sqrt{3}$$

Series representations:

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$
$$-\frac{99}{2}+\sqrt{3}+\pi+\frac{288\sqrt{2}}{2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)+\log\left(\frac{1}{8}\left(-4+5\sqrt{2}\right)\right)+\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$
$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}}{4\sqrt{2}}}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$
$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}}{4\sqrt{2}}-\frac{\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{3/2+3k}\times 3^{-1-2k}}{1+2k}}{2\sqrt{2}}}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=-\frac{99}{2}+\sqrt{3}+\pi-$$
$$\frac{72}{\frac{\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}}{4\sqrt{2}}-\frac{\tan^{-1}(z_0)+\frac{1}{2}i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2}}{3}-z_0\right)^k}{k}}{2\sqrt{2}}}$$

for $(i z_0 \notin \mathbb{R}$ or (not $(1 \leq i z_0 < \infty)$ and not $(-\infty < i z_0 \leq -1)))$

Integral representations:

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$

$$-\frac{99}{2}+\sqrt{3}+\pi+\frac{1728}{72\int_0^1\frac{1}{9+8t^2}\,dt-3\sqrt{2}\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=-\frac{99}{2}+\sqrt{3}+\pi-$$

$$\frac{72}{\frac{i}{12\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{9}{17}\right)^s\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2\,ds+\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}\quad\text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\int_1^{\frac{5+2\sqrt{2}}{5-2\sqrt{2}}}\left(-\frac{1}{3\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)\left(1+\frac{8(1-t)^2}{9\left(1-\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)^2}\right)}+\frac{1}{4\sqrt{2}\,t}\right)dt}$$

Continued fraction representations:

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8k^2}{9}}{1+2k}\right)}}=$$

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\frac{8}{9\left(3+\frac{32}{9\left(5+\frac{8}{7+\frac{128}{9(9+\ldots)}}\right)}\right)}\right)}}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8k^2}{9}}{1+2k}\right)}}=$$

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\frac{8}{9\left(3+\frac{32}{9\left(5+\frac{8}{7+\frac{128}{9(9+\ldots)}}\right)}\right)}\right)}}$$

$$\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)=$$

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{8}{9}(1-2k)^2}{\frac{1}{9}(17+2k)}\right)}}=$$

$$-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{1}{3\left(1+\frac{8}{9\left(\frac{19}{9}+\frac{8}{\frac{7}{3}+\frac{200}{9\left(\frac{23}{9}+\frac{392}{9\left(\frac{25}{9}+\ldots\right)}\right)}}\right)}\right)}}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k/b_k$ is a continued fraction

From which:

((((64+8)* -1/((((1/(4sqrt2) ln (((1+2sqrt2+4)/(1-2sqrt2+4))+1/(2sqrt2) tan^-1(((2sqrt2)/(1-4)))))))-47+Pi-(2-sqrt3+1/2))))^1/15

Input:

$$\sqrt[15]{\frac{(64+8)\times(-1)}{\frac{1}{4\sqrt{2}}\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)+\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)}$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\sqrt[15]{-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}}}$$

(result in radians)

Decimal approximation:

1.643820076464536773658593726009304251173902735647061794707...

(result in radians)

$1.6438200764645... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934...$

Alternate forms:

$$\sqrt[15]{-\frac{99}{2}+\sqrt{3}+\pi+\frac{144\sqrt{2}}{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)-\tanh^{-1}\left(\frac{2\sqrt{2}}{5}\right)}}$$

$$\sqrt[15]{-\frac{99}{2}+\sqrt{3}+\pi+\frac{288\sqrt{2}}{\log\left(\frac{17}{33+20\sqrt{2}}\right)+2\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}}$$

$$\sqrt[15]{\frac{1}{2}\left(2\sqrt{3}-99\right)+\pi-\frac{72}{\frac{\log\left(\frac{1}{17}\left(33+20\sqrt{2}\right)\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}}}$$

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

All 15th roots of -99/2 + sqrt(3) + π - 72/(log((5 + 2 sqrt(2))/(5 - 2 sqrt(2)))/(4 sqrt(2)) - (tan^(-1)((2 sqrt(2))/3))/(2 sqrt(2))):

$$e^0\sqrt[15]{-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}}}\approx 1.6438$$ (real, principal root)

$$e^{(2i\pi)/15}\sqrt[15]{-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}}}\approx 1.5017+0.6686\,i$$

$$e^{(4 i \pi)/15} \sqrt[15]{-\frac{99}{2} + \sqrt{3} + \pi - \frac{72}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}} - \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}}} \approx 1.0999 + 1.2216 i$$

$$e^{(2 i \pi)/5} \sqrt[15]{-\frac{99}{2} + \sqrt{3} + \pi - \frac{72}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}} - \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}}} \approx 0.5080 + 1.5634 i$$

$$e^{(8 i \pi)/15} \sqrt[15]{-\frac{99}{2} + \sqrt{3} + \pi - \frac{72}{\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}} - \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}}} \approx -0.1718 + 1.6348 i$$

Alternative representations:

$$\sqrt[15]{\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}} - 47 + \pi - \left(2 - \sqrt{3} + \frac{1}{2}\right)} = \sqrt[15]{-\frac{99}{2} + \pi - \frac{72}{\frac{\tan^{-1}\left(1, -\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}} + \frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}} + \sqrt{3}}$$

$$\sqrt[15]{\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}} - 47 + \pi - \left(2 - \sqrt{3} + \frac{1}{2}\right)} = \sqrt[15]{-\frac{99}{2} + \pi - \frac{72}{\frac{\tan^{-1}\left(-\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}} + \frac{\log_e\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}} + \sqrt{3}}$$

$$\sqrt[15]{\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}} - 47 + \pi - \left(2 - \sqrt{3} + \frac{1}{2}\right)} = \sqrt[15]{-\frac{99}{2} + \pi - \frac{72}{\frac{\tan^{-1}\left(1, -\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}} + \frac{\log_e\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}} + \sqrt{3}}$$

Series representations:

$$\sqrt[15]{\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)} =$$

$$\sqrt[15]{-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{-\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{3}\right)}{2\sqrt{2}}+\frac{\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}}{4\sqrt{2}}}}$$

$$\sqrt[15]{\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)} =$$

$$\sqrt[15]{-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}}{4\sqrt{2}}-\frac{\sum_{k=0}^{\infty}\frac{(-1)^k\, 2^{3/2+3k}\times 3^{-1-2k}}{1+2k}}{2\sqrt{2}}}}$$

$$\sqrt[15]{\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)} =$$

$$\left(-\frac{99}{2}+\sqrt{3}+\pi-72\Bigg/\left(\frac{\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{8}\left(4-5\sqrt{2}\right)\right)^k}{k}}{4\sqrt{2}}-\frac{\tan^{-1}(z_0)+\frac{1}{2}\,i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2}}{3}-z_0\right)^k}{k}}{2\sqrt{2}}\right)\right)\text{^}(1/15)$$

for $(i z_0 \notin \mathbb{R}$ or (not $(1 \le i z_0 < \infty)$ and not $(-\infty < i z_0 \le -1)))$

$$\sqrt[15]{\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)} =$$

$$\left(-\frac{99}{2}+\sqrt{3}+\pi-72\Big/\left(\frac{\log\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}}\right)^k}{k}}{4\sqrt{2}}-\frac{\tan^{-1}(z_0)+\frac{1}{2}\,i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{2\sqrt{2}}{3}-z_0\right)^k}{k}}{2\sqrt{2}}\right)\right)\wedge(1/15)$$

for ($i z_0 \notin \mathbb{R}$ or (not ($1 \le i z_0 < \infty$) and not ($-\infty < i z_0 \le -1$)))

Integral representations:

$$\sqrt[15]{\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)} =$$

$$\sqrt[15]{-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{-\frac{1}{3}\int_0^1\frac{1}{1+\frac{8t^2}{9}}\,dt+\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}}$$

$$\sqrt[15]{\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)} =$$

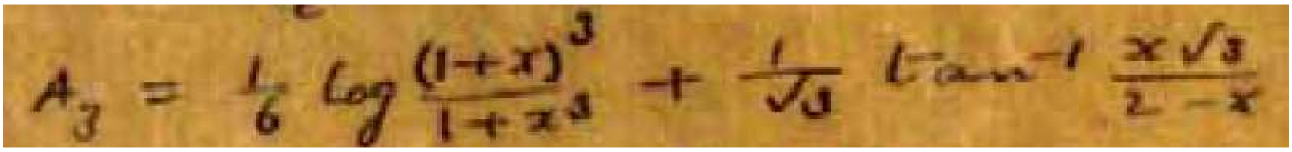
$$\sqrt[15]{-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\frac{i}{12\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{9}{17}\right)^s\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds+\frac{\log\left(\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)}{4\sqrt{2}}}}\quad\text{for}$$

$0 < \gamma < \frac{1}{2}$

$$\sqrt[15]{\frac{(64+8)(-1)}{\frac{\log\left(\frac{1+2\sqrt{2}+4}{1-2\sqrt{2}+4}\right)}{4\sqrt{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt{2}}{1-4}\right)}{2\sqrt{2}}}-47+\pi-\left(2-\sqrt{3}+\frac{1}{2}\right)} =$$

$$\sqrt[15]{-\frac{99}{2}+\sqrt{3}+\pi-\frac{72}{\int_1^{\frac{5+2\sqrt{2}}{5-2\sqrt{2}}}\left(-\frac{1}{3\left(-1+\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)\left(1+\frac{8(1-t)^2}{9\left(1-\frac{5+2\sqrt{2}}{5-2\sqrt{2}}\right)^2}\right)}+\frac{1}{4\sqrt{2}\,t}\right)dt}}$$

Now, we have that:



$$A_3 = \frac{1}{6}\log\frac{(1+x)^3}{1+x^3} + \frac{1}{\sqrt{3}}\tan^{-1}\frac{x\sqrt{3}}{2-x}$$

For x = -2 and multiplying all the expression by -1, we obtain:

-((1/6 ln (((1-2)^3)/(1-8)) + 1/sqrt3 tan^-1 (-2sqrt3/(2+2))))

Input:

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{1}{\sqrt{3}}\tan^{-1}\left(-2\times\frac{\sqrt{3}}{2+2}\right)\right)$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{\log(7)}{6}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}$$

(result in radians)

Decimal approximation:

0.736387320486844454951909129191439952702295682177676137042...

(result in radians)

0.7363873204…

Alternate forms:

$$\frac{\log(7)}{6}+\frac{\cot^{-1}\left(\frac{2}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$\frac{1}{6}\left(\log(7)+2\sqrt{3}\cot^{-1}\left(\frac{2}{\sqrt{3}}\right)\right)$$

$$\frac{1}{6}\left(\log(7)+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$$

$\cot^{-1}(x)$ is the inverse cotangent function

Alternative representations:

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=-\frac{1}{6}\log\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(1,-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=-\frac{1}{6}\log_e\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=-\frac{1}{6}\log(a)\log_a\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}$$

Series representations:

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}+\frac{\log(6)}{6}-\frac{1}{6}\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=$$
$$\frac{1}{6}\left(\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}+2\sqrt{3}\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\times 3^{1/2+k}}{1+2k}\right)$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=$$
$$\frac{\tan^{-1}(z_0)}{\sqrt{3}}+\frac{\log(6)}{6}+\sum_{k=1}^{\infty}\left(\frac{(-1)^{-1+k}\,6^{-1-k}}{k}+\frac{i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{\sqrt{3}}{2}-z_0\right)^k}{2\sqrt{3}\,k}\right)$$
for $(i z_0 \notin \mathbb{R}$ or $($ not $(1 \leq i z_0 < \infty)$ and not $(-\infty < i z_0 \leq -1)))$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=$$
$$\frac{\tan^{-1}(z_0)}{\sqrt{3}}+\frac{\log(6)}{6}+\sum_{k=1}^{\infty}\left(\frac{\left(-\frac{1}{6}\right)^{1+k}}{k}+\frac{i\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{\sqrt{3}}{2}-z_0\right)^k}{2\sqrt{3}\,k}\right)$$
for $(i z_0 \notin \mathbb{R}$ or $($ not $(1 \leq i z_0 < \infty)$ and not $(-\infty < i z_0 \leq -1)))$

Integral representations:

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=\int_1^7\left(\frac{1}{6t}+\frac{4}{49-2t+t^2}\right)dt$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=2\int_0^1\frac{1}{4+3t^2}\,dt+\frac{\log(7)}{6}$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=$$
$$-\frac{i}{8\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{4}{7}\right)^s\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds+\frac{\log(7)}{6}\quad\text{for } 0<\gamma<\frac{1}{2}$$

Continued fraction representations:

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=$$

$$\frac{1}{6}\left(\log(7)+\frac{3}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3k^2}{4}}{1+2k}}\right)=\frac{1}{6}\left(\log(7)+\frac{3}{1+\frac{3}{4\left(3+\frac{3}{5+\frac{27}{4\left(7+\frac{12}{9+\dots}\right)}}\right)}}\right)$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=\frac{1}{6}\left(\log(7)+\frac{3}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3}{4}(1-2k)^2}{\frac{1}{4}(7+2k)}}\right)=$$

$$\frac{1}{6}\left(\log(7)+\frac{3}{1+\frac{3}{4\left(\frac{9}{4}+\frac{27}{4\left(\frac{11}{4}+\frac{75}{4\left(\frac{13}{4}+\frac{147}{4\left(\frac{15}{4}+\dots\right)}\right)}\right)}\right)}}\right)$$

$$-\left(\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}\right)=\frac{1}{2}+\frac{\log(7)}{6}-\frac{3}{8\left(3+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3}{4}\left(1+(-1)^{1+k}+k\right)^2}{3+2k}\right)}=$$

$$\frac{1}{2}+\frac{\log(7)}{6}-\frac{3}{8\left(3+\frac{27}{4\left(5+\frac{3}{7+\frac{75}{4\left(9+\frac{12}{11+\dots}\right)}}\right)}\right)}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k/b_k$ is a continued fraction

27*1/2*((((((48/(((-((1/6 ln (((1-2)^3)/(1-8)) + 1/sqrt3 tan^-1 (-2sqrt3/(2+2)))))))*2-5)))+2)))+13-Pi-1/(2*golden ratio)

Input:

$$27\times\frac{1}{2}\left(\left(-\frac{48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{1}{\sqrt{3}}\tan^{-1}\left(-2\times\frac{\sqrt{3}}{2+2}\right)}\times 2-5\right)+2\right)+13-\pi-\frac{1}{2\phi}$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

ϕ is the golden ratio

Exact Result:

$$-\frac{1}{2\phi}+13-\pi+\frac{27}{2}\left(\frac{96}{\frac{\log(7)}{6}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}}-3\right)$$

(result in radians)

Decimal approximation:

1728.9927841942612738737368701751076466021633693777158131OO...

(result in radians)

1728.99278419… ≈ 1729

We know that 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms:

$$-\frac{1}{2\phi}-\frac{55}{2}-\pi+\frac{7776}{\log(7)+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}$$

$$-\frac{1}{2\phi}-\frac{55}{2}-\pi+\frac{7776\sqrt{3}}{\sqrt{3}\log(7)+6\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}$$

$$-\frac{55}{2}-\frac{1}{1+\sqrt{5}}-\pi+\frac{1296}{\frac{\log(7)}{6}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}}$$

Alternative representations:

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}=$$

$$13-\pi-\frac{1}{2\phi}+\frac{27}{2}\left(-3+\frac{96}{-\frac{1}{6}\log\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(1,-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}}\right)$$

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi} =$$
$$13-\pi-\frac{1}{2\phi}+\frac{27}{2}\left(-3+\frac{96}{-\frac{1}{6}\log_e\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}}\right)$$

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi} =$$
$$13-\pi-\frac{1}{2\phi}+\frac{27}{2}\left(-3+\frac{96}{-\frac{1}{6}\log_e\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(1,-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}}\right)$$

Series representations:

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi} =$$
$$-\frac{55}{2}-\frac{1}{1+\sqrt{5}}-\pi+\frac{1296}{\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}+\frac{1}{6}\left(\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}\right)}$$

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi} =$$
$$-\frac{55}{2}-\frac{1}{1+\sqrt{5}}-\pi+\frac{1296}{\frac{1}{6}\left(\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}\right)+\frac{\sum_{k=0}^{\infty}\frac{(-1)^k\, 2^{-1-2k}\times 3^{1/2+k}}{1+2k}}{\sqrt{3}}}$$

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi} = -\frac{55}{2}-\frac{1}{1+\sqrt{5}}-\pi+\frac{1296}{\frac{1}{6}\left(\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}\right)+\frac{\tan^{-1}(z_0)+\frac{1}{2}i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{\sqrt{3}}{2}-z_0\right)^k}{k}}{\sqrt{3}}}$$

for $(i z_0 \notin \mathbb{R}$ or (not $(1 \leq i z_0 < \infty)$ and not $(-\infty < i z_0 \leq -1)))$

Integral representations:

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi} = -\frac{55}{2}-\frac{1}{2\phi}-\pi+\frac{7776}{12\int_0^1\frac{1}{4+3t^2}\,dt+\log(7)}$$

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi} = -\frac{55}{2}-\frac{1}{2\phi}-\pi+\frac{1296}{\int_1^7\left(\frac{1}{6t}+\frac{4}{49-2t+t^2}\right)dt}$$

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi} = -\frac{55}{2}-\frac{1}{1+\sqrt{5}}-\pi+\frac{1296}{-\frac{i}{8\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{4}{7}\right)^s\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\,\Gamma(s)^2\,ds+\frac{\log(7)}{6}} \quad \text{for } 0<\gamma<\frac{1}{2}$$

Continued fraction representations:

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi} =$$

$$-\frac{55}{2}-\frac{1}{2\phi}-\pi+\frac{7776}{\log(7)+\frac{3}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3k^2}{4}}{1+2k}}} = -\frac{55}{2}-\frac{1}{2\phi}-\pi+\frac{7776}{\log(7)+\frac{3}{1+\frac{3}{4\left(3+\frac{3}{5+\frac{27}{4\left(7+\frac{12}{9+\dots}\right)}}\right)}}}$$

$$\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi} =$$

$$13-\frac{1}{2\phi}-\pi+\frac{27}{2}\left(-3+\frac{96}{\frac{\log(7)}{6}+\frac{1}{2\left(1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3k^2}{4}}{1+2k}\right)}}\right) =$$

$$13-\frac{1}{2\phi}-\pi+\frac{27}{2}\left(-3+\frac{96}{\frac{\log(7)}{6}+\frac{1}{2\left(1+\frac{3}{4\left(3+\frac{3}{5+\frac{27}{4\left(7+\frac{12}{9+\dots}\right)}}\right)}\right)}}\right)$$

$$\frac{27}{2}\left(\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}\right) =$$

$$-\frac{55}{2}-\frac{1}{2\phi}-\pi+\frac{7776}{\log(7)+\frac{3}{1+\underset{k=1}{\overset{\infty}{K}}\frac{\frac{3}{4}(1-2k)^2}{\frac{1}{4}(7+2k)}}} =$$

$$-\frac{55}{2}-\frac{1}{2\phi}-\pi+\frac{7776}{\log(7)+\frac{3}{1+\frac{3}{4\left(\frac{9}{4}+\frac{27}{4\left(\frac{11}{4}+\frac{75}{4\left(\frac{13}{4}+\frac{147}{4\left(\frac{15}{4}+\ldots\right)}\right)}\right)}\right)}}}$$

From which:

((27*1/2*((((((48/(((-((1/6 ln (((1-2)^3)/(1-8)) + 1/sqrt3 tan^-1 (-2sqrt3/(2+2)))))))*2-5)))+2)))+13-Pi-1/(2*golden ratio)))^1/15

Input:

$$\sqrt[15]{27\times\frac{1}{2}\left(\left(\left(-\frac{48}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{1}{\sqrt{3}}\tan^{-1}\left(-2\times\frac{\sqrt{3}}{2+2}\right)}\times 2-5\right)+2\right)+13-\pi-\frac{1}{2\phi}\right)}$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

ϕ is the golden ratio

Exact Result:

$$\sqrt[15]{-\frac{1}{2\phi}+13-\pi+\frac{27}{2}\left(\frac{96}{\frac{\log(7)}{6}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}}-3\right)}$$

(result in radians)

Decimal approximation:

1.643814771394787036770119180752410280641371729502784324347...

(result in radians)

$1.6438147713\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

Alternate forms:

$$\sqrt[15]{-\frac{1}{2\phi}-\frac{55}{2}-\pi+\frac{7776}{\log(7)+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}}$$

$$\sqrt[15]{-\frac{55}{2}-\frac{1}{1+\sqrt{5}}-\pi+\frac{1296}{\frac{\log(7)}{6}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}}}$$

$$\sqrt[15]{13-\frac{1}{1+\sqrt{5}}-\pi+\frac{27}{2}\left(\frac{96}{\frac{\log(7)}{6}+\frac{\cot^{-1}\left(\frac{2}{\sqrt{3}}\right)}{\sqrt{3}}}-3\right)}$$

$\cot^{-1}(x)$ is the inverse cotangent function

Expanded form:

$$\sqrt[15]{13-\frac{1}{1+\sqrt{5}}-\pi+\frac{27}{2}\left(\frac{96}{\frac{\log(7)}{6}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}}-3\right)}$$

All 15th roots of -1/(2 ϕ) + 13 - π + 27/2 (96/(log(7)/6 + (tan^(-1)(sqrt(3)/2))/sqrt(3)) - 3):

$$e^{0}\sqrt[15]{-\frac{1}{2\phi}+13-\pi+\frac{27}{2}\left(\frac{96}{\frac{\log(7)}{6}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}}-3\right)}\approx 1.64381 \text{ (real, principal root)}$$

$$e^{(2 i \pi)/15}\sqrt[15]{-\frac{1}{2\phi}+13-\pi+\frac{27}{2}\left(\frac{96}{\frac{\log(7)}{6}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}}-3\right)}\approx 1.50170+0.6686\,i$$

$$e^{(4 i \pi)/15}\sqrt[15]{-\frac{1}{2\phi}+13-\pi+\frac{27}{2}\left(\frac{96}{\frac{\log(7)}{6}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}}-3\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2 i \pi)/5}\sqrt[15]{-\frac{1}{2\phi}+13-\pi+\frac{27}{2}\left(\frac{96}{\frac{\log(7)}{6}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}}-3\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8 i \pi)/15}\sqrt[15]{-\frac{1}{2\phi}+13-\pi+\frac{27}{2}\left(\frac{96}{\frac{\log(7)}{6}+\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}}-3\right)}\approx -0.17183+1.63481\,i$$

Alternative representations:

$$\sqrt[15]{\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}} =$$

$$\sqrt[15]{13-\pi-\frac{1}{2\phi}+\frac{27}{2}\left(-3+\frac{96}{-\frac{1}{6}\log\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(1,-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}}\right)}$$

$$\sqrt[15]{\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}} = \sqrt[15]{13-\pi-\frac{1}{2\phi}+\frac{27}{2}\left(-3+\frac{96}{-\frac{1}{6}\log_e\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}}\right)}$$

$$\sqrt[15]{\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}} = \sqrt[15]{13-\pi-\frac{1}{2\phi}+\frac{27}{2}\left(-3+\frac{96}{-\frac{1}{6}\log_e\left(\frac{-1}{-7}\right)-\frac{\tan^{-1}\left(1,-\frac{2\sqrt{3}}{4}\right)}{\sqrt{3}}}\right)}$$

Series representations:

$$\sqrt[15]{\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}} = \sqrt[15]{13-\frac{1}{1+\sqrt{5}}-\pi+\frac{27}{2}\left(-3+\frac{96}{\frac{\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)}{\sqrt{3}}+\frac{1}{6}\left(\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}\right)}\right)}$$

$$\sqrt[15]{\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}} =$$

$$\sqrt[15]{13-\frac{1}{1+\sqrt{5}}-\pi+\frac{27}{2}\left(-3+\frac{96}{\frac{1}{6}\left(\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}\right)+\frac{\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\times 3^{1/2+k}}{1+2k}}{\sqrt{3}}}\right)}$$

$$\sqrt[15]{\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}} =$$

$$\left(13-\frac{1}{1+\sqrt{5}}-\pi+\frac{27}{2}\left(-3+\frac{96}{\frac{1}{6}\left(\log(6)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}\right)+\frac{\tan^{-1}(z_0)+\frac{1}{2}i\sum_{k=1}^{\infty}\frac{\left(-(-i-z_0)^{-k}+(i-z_0)^{-k}\right)\left(\frac{\sqrt{3}}{2}-z_0\right)^k}{k}}{\sqrt{3}}}\right)\right)\text{^}(1/15) \text{ for}$$

$(i z_0 \notin \mathbb{R}$ or (not $(1 \leq i z_0 < \infty)$ and not $(-\infty < i z_0 \leq -1)))$

Integral representations:

$$\sqrt[15]{\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}} = \sqrt[15]{13-\frac{1}{1+\sqrt{5}}-\pi+\frac{27}{2}\left(-3+\frac{96}{\int_1^7\left(\frac{1}{6t}+\frac{4}{49-2t+t^2}\right)dt}\right)}$$

$$\sqrt[15]{\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}} = \sqrt[15]{13-\frac{1}{1+\sqrt{5}}-\pi+\frac{27}{2}\left(-3+\frac{96}{\frac{1}{2}\int_0^1\frac{4}{4+3t^2}\,dt+\frac{\log(7)}{6}}\right)}$$

$$\sqrt[15]{\frac{27}{2}\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\phi}} = \sqrt[15]{13-\frac{1}{1+\sqrt{5}}-\pi+\frac{27}{2}\left(-3+\frac{96}{-\frac{i}{8\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\frac{4}{7}\right)^s\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)\Gamma(s)^2\,ds+\frac{\log(7)}{6}}\right)}$$

for $0<\gamma<\frac{1}{2}$

Continued fraction representations:

$$\sqrt[15]{\frac{27}{2}\left(\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)\right)+13-\pi-\frac{1}{2\phi}} =$$

$$\sqrt[15]{-\frac{55}{2}-\frac{1}{2\phi}-\pi+\frac{7776}{\log(7)+\frac{3}{1+\mathop{K}_{k=1}^{\infty}\frac{\frac{3k^2}{4}}{1+2k}}}} =$$

$$\sqrt[15]{-\frac{55}{2}-\frac{1}{2\phi}-\pi+\frac{7776}{\log(7)+\frac{3}{1+\frac{3}{4\left(3+\frac{3}{5+\frac{27}{4\left(7+\frac{12}{9+\dots}\right)}}\right)}}}}$$

$$\sqrt[15]{\frac{27}{2}\left(\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)\right)+13-\pi-\frac{1}{2\phi}} =$$

$$\sqrt[15]{13-\frac{1}{2\phi}-\pi+\frac{27}{2}\left(-3+\frac{96}{\frac{\log(7)}{6}+\frac{1}{2\left(1+\mathop{K}_{k=1}^{\infty}\frac{\frac{3k^2}{4}}{1+2k}\right)}}\right)} =$$

$$\sqrt[15]{13-\frac{1}{2\phi}-\pi+\frac{27}{2}\left(-3+\frac{96}{\frac{\log(7)}{6}+\frac{1}{2\left(1+\frac{3}{4\left(3+\frac{3}{5+\frac{27}{4\left(7+\frac{12}{9+\dots}\right)}}\right)}\right)}}\right)}$$

$$\sqrt[15]{\frac{27}{2}\left(\left(\left(-\frac{48\times 2}{\frac{1}{6}\log\left(\frac{(1-2)^3}{1-8}\right)+\frac{\tan^{-1}\left(-\frac{2\sqrt{3}}{2+2}\right)}{\sqrt{3}}}-5\right)+2\right)+13-\pi-\frac{1}{2\,\phi}\right)} =$$

$$\sqrt[15]{-\frac{55}{2}-\frac{1}{2\,\phi}-\pi+\frac{7776}{\log(7)+\frac{3}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{3}{4}(1-2k)^2}{\frac{1}{4}(7+2k)}}}} =$$

$$\sqrt[15]{-\frac{55}{2}-\frac{1}{2\,\phi}-\pi+\frac{7776}{\log(7)+\frac{3}{1+\frac{3}{4\left(\frac{9}{4}+\frac{27}{4\left(\frac{11}{4}+\frac{75}{4\left(\frac{13}{4}+\frac{147}{4\left(\frac{15}{4}+\dots\right)}\right)}\right)}\right)}}}}$$

EXAMPLE OF RAMANUJAN MATHEMATICS APPLIED TO THE COSMOLOGY

From:

A Reissner-Nordstrom+Λ black hole in the Friedman-Robertson-Walker universe- arXiv:1703.05119v1 [physics.gen-ph] 5 Mar 2017

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From:

$$r_{\pm} = M \pm \sqrt{M^2 - Q^2},$$

For MBH87 data: mass = 13.12806e+39; radius = 1.94973e+13 , we obtain:

(1.94973e+13-13.12806e+39)^2 = ((13.12806e+39)^2-x^2)

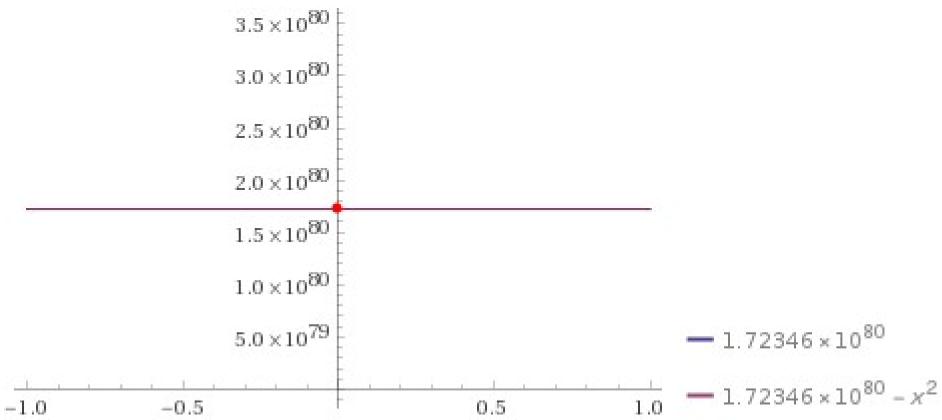
Input interpretation:

$(1.94973 \times 10^{13} - 13.12806 \times 10^{39})^2 = (13.12806 \times 10^{39})^2 - x^2$

Result:

$1.72346 \times 10^{80} = 1.72346 \times 10^{80} - x^2$

Plot:



Alternate forms:

$x^2 + 0 = 0$

$1.72346 \times 10^{80} = -(x - 1.31281 \times 10^{40})(x + 1.31281 \times 10^{40})$

Solution:

$x = 0$

Indeed:

(1.94973e+13-13.12806e+39)^2 = ((13.12806e+39)^2)

Input interpretation:

$(1.94973 \times 10^{13} - 13.12806 \times 10^{39})^2 = (13.12806 \times 10^{39})^2$

Result:

True

Thence Q = 0

Now, for

$a(v) > \frac{\sqrt{k}}{4}$. For the present universe, assuming $a(v) - 1$ and thus $k<16$. Though constant k has an upper limit, it increases with the expansion of the universe and decreases with the contraction of the universe. We should observe a peculiar change when the constant k reaches this numerical value which is the limiting value for the expansion of the universe.

For $Q = 0$ in eqn.(64),

$$2\left(2-\frac{\sqrt{1+\frac{kx^2}{4}}}{ax}\right)\left[\frac{M^2}{\left(\frac{ax}{\sqrt{1+\frac{kx^2}{4}}}\right)^3}-\frac{Q^2}{\left(\frac{ax}{\sqrt{1+\frac{kx^2}{4}}}\right)^3}\right.$$
$$\left.+\Lambda e^{-\frac{2ax}{\sqrt{1+\frac{kx^2}{4}}}}\right]+\frac{\sqrt{1+\frac{kx^2}{4}}}{ax}=0. \quad (64)$$

Hence at $x=R$ we get,

$$2\left(2-\frac{\sqrt{1+\frac{kR^2}{4}}}{aR}\right)\left[\frac{M^2}{\left(\frac{aR}{\sqrt{1+\frac{kR^2}{4}}}\right)^3}-\frac{Q^2}{\left(\frac{aR}{\sqrt{1+\frac{kR^2}{4}}}\right)^3}\right.$$
$$\left.+\Lambda e^{-\frac{2aR}{\sqrt{1+\frac{kR^2}{4}}}}\right]+\frac{\sqrt{1+\frac{kR^2}{4}}}{aR}=0. \quad (65)$$

$$\Lambda=-e^{\frac{2aR}{\sqrt{1+\frac{kR^2}{4}}}}.\left[\frac{M^2}{\left(\frac{aR}{\sqrt{1+\frac{kR^2}{4}}}\right)^3}\right.$$
$$\left.+\frac{1}{2\left(\frac{2aR}{\sqrt{1+\frac{kR^2}{4}}}-1\right)}\right], \quad (67)$$

For k = 12, and a = 1, M = 13.12806e+39; R = 1.94973e+13, we obtain:

and:

(1+((12*(1.94973e+13)^2)/4))^1/2

Input interpretation:

$$\sqrt{1+\frac{1}{4}\left(12\left(1.94973\times10^{13}\right)^2\right)}$$

Result:

$3.37703...\times10^{13}$

3.37703e+13

Substituting in the eqs. (67), we obtain:

-exp(((2*1.94973e+13)/(3.37703e+13))) * [(((13.12806e+39)^2)) / (((1.94973e+13)/(3.37703e+13)))^3 + 1/((2((((2*1.94973e+13)/(3.37703e+13)-1)))))]

Input interpretation:

$$-\exp\left(\frac{2\times 1.94973\times 10^{13}}{3.37703\times 10^{13}}\right)\left(\frac{(13.12806\times 10^{39})^2}{\left(\frac{1.94973\times 10^{13}}{3.37703\times 10^{13}}\right)^3}+\frac{1}{2\left(\frac{2\times 1.94973\times 10^{13}}{3.37703\times 10^{13}}-1\right)}\right)$$

Result:

$-2.84160\ldots \times 10^{81}$

$-2.84160\ldots*10^{81}$

which represents the Cosmological Constant inside the Schwarzschild black hole and also has a negative value.

Performing the following equation with the usual value of the Cosmological Constant 1.1056e-52, we obtain:

(1.1056e-52)x = -2.84160e+81

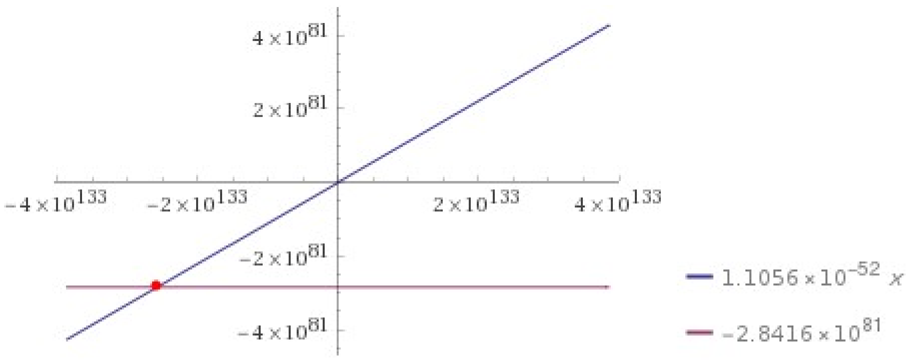
Input interpretation:

$1.1056\times 10^{-52}\, x = -2.84160\times 10^{81}$

Result:

$1.1056\times 10^{-52}\, x = -2.8416\times 10^{81}$

Plot:



Alternate form:

$1.1056\times10^{-52}\,x+2.8416\times10^{81}=0$

Alternate form assuming x is real:

$1.1056\times10^{-52}\,x+0=-2.8416\times10^{81}$

Solution:

$x =$
$-25\,701\,881\,331\,403\,766\,886\,664\,569\,715\,710\,133\,147\,602\,520\,011\,173\,198\,993\,507\,564\,120\,861\,732\,475\,370\,738\,202\,865\,312\,319\,616\,245\,712\,374\,922\,255\,343\,303\,805\,210\,672\,526\,000\,128$

Integer solution:

$x =$
$-25\,701\,881\,331\,403\,766\,886\,664\,569\,715\,710\,133\,147\,602\,520\,011\,173\,198\,993\,507\,564\,120\,861\,732\,475\,370\,738\,202\,865\,312\,319\,616\,245\,712\,374\,922\,255\,343\,303\,805\,210\,672\,526\,000\,128$

Result:

$-2.5701881331403766886664569715710133147602520011173198993507564120861732475370738202865312319616245712374922255343303805210672526000128\times10^{133}$

$-2.57018813314...*10^{133}$

Value that multiplied by 1.1056e-52, give us $-2.84160 * 10^{81}$

Multiplying this result with the usual value of the Cosmological Constant, we obtain:

(1.1056e-52) * (-2.84160e+81)

Input interpretation:

$1.1056\times10^{-52}\,(-2.84160\times10^{81})$

Result:

$-314\,167\,296\,000\,000\,000\,000\,000\,000\,000$

Result:

-3.14167296×10^{29}

$-3.14167296*10^{29}$ result that is nearly to a multiple of π with minus sign

We have also that, from the formula of coefficients of the '5th order' mock theta function $\psi_1(q)$: (A053261 OEIS Sequence)

sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n))

for n = 230 and subtracting 47, that is a Lucas number, and π, we obtain:

sqrt(golden ratio) * exp(Pi*sqrt(230/15)) / (2*5^(1/4)*sqrt(230)) -47 – Pi

Input:

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{230}{15}}\right)}{2 \sqrt[4]{5} \sqrt{230}} - 47 - \pi$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{\sqrt{46/3}\,\pi} \sqrt{\frac{\phi}{46}}}{2 \times 5^{3/4}} - 47 - \pi$$

Decimal approximation:

6122.273163239088047930830535468077939193046207568421910068...

6122.273163239…..

Alternate forms:

$$-47 + \frac{1}{20} \sqrt{\frac{1}{23}\left(5+\sqrt{5}\right)}\, e^{\sqrt{46/3}\,\pi} - \pi$$

$$-47 + \frac{\sqrt{\frac{1}{23}\left(1+\sqrt{5}\right)}\, e^{\sqrt{46/3}\,\pi}}{4 \times 5^{3/4}} - \pi$$

$$\frac{1}{460}\left(-21620 + \sqrt[4]{5} \sqrt{23\left(1+\sqrt{5}\right)}\, e^{\sqrt{46/3}\,\pi} - 460\,\pi\right)$$

Series representations:

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{230}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{230}}-47-\pi = -\left(\left(470\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(230-z_0)^k z_0^{-k}}{k!}+10\pi\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(230-z_0)^k z_0^{-k}}{k!}-5^{3/4}\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{46}{3}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\Big/\left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(230-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for not $((z_0\in\mathbb{R}$ and $-\infty<z_0\le 0))$

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{230}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{230}}-47-\pi = -\left(\left(470\exp\left(i\pi\left\lfloor\frac{\arg(230-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(230-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+10\pi\exp\left(i\pi\left\lfloor\frac{\arg(230-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(230-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}-5^{3/4}\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{46}{3}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{46}{3}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/\left(10\exp\left(i\pi\left\lfloor\frac{\arg(230-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(230-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

for $(x\in\mathbb{R}$ and $x<0)$

From which:

(-(-2.84160e+81))^(5Pi/(((sqrt(golden ratio) * exp(Pi*sqrt(230/15)) / (2*5^(1/4)*sqrt(230)) -47 - Pi))))

Input interpretation:

$$(-(-2.84160\times 10^{81}))^{5\times\pi/\left(\sqrt{\phi}\times\frac{\exp\left(\pi\sqrt{\frac{230}{15}}\right)}{2\sqrt[4]{5}\sqrt{230}}-47-\pi\right)}$$

ϕ is the golden ratio

Result:

1.618027996701560438286389221876566317933407173693842150642...

1.6180279967….. result that is a very good approximation to the value of the golden ratio 1,618033988749...

Input interpretation:

1.6180279967015604382863892218765663179334071736938421

Possible closed forms:

$$-\frac{8(45F_{FR}-1127)}{2047F_{FR}-800}\approx 1.618027996701560429601$$

$$\frac{1}{3}\sqrt{\frac{1}{55}(-200+333e+162\pi+118\log(2))}\approx 1.61802799670156043867372$$

$$-\frac{4(73-325\pi+39\pi^2)}{49-72\pi+159\pi^2}\approx 1.61802799670156043858425$$

$$\pi\ \text{root of } 522x^4+580x^3-1362x^2+919x-228 \text{ near } x=0.515034\approx$$
$$1.61802799670156043816535$$

$$\frac{\sqrt[3]{\frac{2}{51}(984-89e+1000\pi-1707\log(2))}}{5^{2/3}}\approx 1.618027996701560438265766$$

$$\frac{3709980781\pi}{7203366314}\approx 1.618027996701560438296510$$

$$\text{root of } 647x^4-350x^3-4186x^2+4220x+1179 \text{ near } x=1.61803\approx$$
$$1.618027996701560438290441$$

$$\frac{\sqrt[4]{\frac{31028619}{4409789}}\,\pi}{\sqrt{10}} \approx 1.61802799670156045 6743$$

$$\frac{1}{\text{root of } 1179x^4 + 4220x^3 - 4186x^2 - 350x + 647 \text{ near } x = 0.618036} \approx$$
1.61802799670156043829 0441

$$\text{root of } 5888x^3 - 39087x^2 + 37056x + 17431 \text{ near } x = 1.61803 \approx$$
1.6180279967015604382844533

$$\pi \; \text{root of } 29646x^3 - 33474x^2 - 52404x + 31819 \text{ near } x = 0.515034 \approx$$
1.6180279967015604382844495

$$\frac{1}{\text{root of } 17431x^3 + 37056x^2 - 39087x + 5888 \text{ near } x = 0.618036} \approx$$
1.6180279967015604382844533

$$\text{root of } 439x^5 - 1047x^4 + 217x^3 + 924x^2 - x - 1029 \text{ near } x = 1.61803 \approx$$
1.6180279967015604383 1097

$$\pi \; \text{root of } 657x^5 + 621x^4 + 647x^3 - 1476x^2 + 75x + 197 \text{ near } x = 0.515034 \approx$$
1.61802799670156043826 3743

$$\frac{e^{\frac{3}{5}-\frac{9}{10e}-\frac{3e}{10}+\frac{2}{5\pi}-\frac{3\pi}{5}}\,\pi^{(19e)/20-3/10}}{\sqrt[20]{\sin(e\pi)}\;(-\cos(e\pi))^{7/20}} \approx 1.61802799670156043862208$$

Now, we have that:

$$a = 3.2^{\frac{1}{3}}.(1 - 4Q^2\Lambda), \qquad (9)$$

$$\begin{aligned} b = [&-54 + 972M^2\Lambda - 648Q^2\Lambda \\ &+[(-54 + 972M^2\Lambda - 648Q^2\Lambda)^2 \\ &-4(9 - 36Q^2\Lambda)^3]^{\frac{1}{2}}]^{\frac{1}{3}}, \end{aligned} \qquad (10)$$

$$c = 3.2^{\frac{1}{3}}\Lambda, \qquad (11)$$

For **Q = 0.00089**, Λ = 1.1056e-52 m^{-2}**:**

convert 1.1056×10^{-52} m^{-2} (reciprocal square meters) to per kilometers squared

1.106×10^{-46}/km^2 (per kilometers squared) **Λ = -1.1056 * 10^{-46}**

Mass = 3.8 solar masses**:**

$3.8 \times 1.9891 \times 10^{30} = 7558580000000000000000000000000 = 7.55858 \times 10^{30}$

M = 7.55858e+30

We obtain:

$$a = 3.2^{\frac{1}{3}}.(1 - 4Q^2\Lambda)$$

(3.2)^1/3 (1-((4*0.00089^2*(-1.1056e-46))))

Input interpretation:

$\sqrt[3]{3.2}\left(1 - 4 \times 0.00089^2 \left(-1.1056 \times 10^{-46}\right)\right)$

Result:

1.4736125994561546423119291334319228887669032469752735839 06...

1.4736125994561546.... = a

Now, we have that:

$$b = [-54 + 972M^2\Lambda - 648Q^2\Lambda + [(-54 + 972M^2\Lambda - 648Q^2\Lambda)^2 - 4(9 - 36Q^2\Lambda)^3]^{\frac{1}{2}}]^{\frac{1}{3}},$$

sqrt[(((((-54+972*((7.55858e+30)^2*(-1.1056e-46))-648*0.00089^2*(-1.1056e-46)+(((-54+972*((7.55858e+30)^2*(-1.1056e-46))-648*0.00089^2(-1.1056e-46)))^2-4(((9-36*0.00089^2*(-1.1056e-46)^3)))))))))]^1/3

Input interpretation:

$$\left(\sqrt{\left(-54+972\left((7.55858\times10^{30})^2\left(-1.1056\times10^{-46}\right)\right)-648\times0.00089^2\left(-1.1056\times10^{-46}\right)+\left(\left(-54+972\left((7.55858\times10^{30})^2\left(-1.1056\times10^{-46}\right)\right)-648\times0.00089^2\left(-1.1056\times10^{-46}\right)\right)^2-4\left(9-36\times0.00089^2\left(-1.1056\times10^{-46}\right)^3\right)\right)\right)}\right)\hat{}\,(1/3)$$

Result:

$1.831111995417529907080402771725336322228680076788385 40\ldots\times10^6$

$1.8311119954175299\ldots*10^6 = b$

And:

$$c = 3.2^{\frac{1}{3}}\Lambda,$$

(3.2)^(1/3) * (-1.1056e-46)

Input interpretation:

$\sqrt[3]{3.2}\left(-1.1056\times10^{-46}\right)$

Result:

$-1.62923\ldots\times10^{-46}$

$-1.62923\ldots*10^{-46} = c$

From

$$r_4 = -\frac{1}{2}.\left[\frac{2}{\Lambda}+\frac{a}{\Lambda b}+\frac{b}{c}\right]^{\frac{1}{2}} +\frac{1}{2}.\left[\frac{4}{\Lambda}-\frac{a}{\Lambda b}-\frac{b}{c}+\frac{12M}{\Lambda(\frac{2}{\Lambda}+\frac{a}{\Lambda b}+\frac{b}{c})^{\frac{1}{2}}}\right]^{\frac{1}{2}}$$

We have that:

c = -1.62923e-46
b = 1.8311119954175299e+6
a = 1.4736125994561546
Λ = -1.1056e-46

-1/2((((2/(-1.1056e-46)+(1.4736125994561546) / (-1.1056e-46 * 1.8311119954175299e+6) + (1.8311119954175299e+6) / (-1.62923e-46)))))^1/2

Input interpretation:

$$-\frac{1}{2}\sqrt{\left(-\frac{2}{1.1056\times 10^{-46}}+-\frac{1.4736125994561546}{1.1056\times 10^{-46}\times 1.8311119954175299\times 10^{6}}+-\frac{1.8311119954175299\times 10^{6}}{1.62923\times 10^{-46}}\right)}$$

Result:

$-5.30074... \times 10^{25}\, i$

Polar coordinates:

$r = 5.30074\times 10^{25}$ (radius), $\theta = -90°$ (angle)

$5.30074*10^{25}$

and:

$$+\frac{1}{2}\cdot\left[\frac{4}{\Lambda}-\frac{a}{\Lambda b}-\frac{b}{c}+\frac{12M}{\Lambda(\frac{2}{\Lambda}+\frac{a}{\Lambda b}+\frac{b}{c})^{\frac{1}{2}}}\right]^{\frac{1}{2}}:$$

1/2[(4/(-1.1056e-46)-(1.4736125994561546)/(-1.1056e-46 * 1.8311119954175299e+6)-(1.8311119954175299e+6)/(-1.62923e-46)+((((12* 7.55858e+30))))/((((-1.1056e-46)(2/(-1.1056e-46)+(1.4736125994561546)/(-1.1056e-46 * 1.8311119954175299e+6)+(1.8311119954175299e+6)/(-1.62923e-46))))]^(1/2)

Input interpretation:

$$-\frac{4}{1.1056\times10^{-46}} - -\frac{1.4736125994561546}{1.1056\times10^{-46}\times1.8311119\times10^{6}} - -\frac{1.8311119\times10^{6}}{1.62923\times10^{-46}}$$

Result:

$1.123908843770763964581608573371924017283199837382128\ldots\times10^{52}$

1.1239088437707639645816085733719240172831998373821284 × 10^52

Input interpretation:

$$-\frac{12\times7.55858\times10^{30}}{1.1056\times10^{-46}\sqrt{-\frac{2}{1.1056\times10^{-46}} + -\frac{1.4736125994561546}{1.1056\times10^{-46}\times1.8311119\times10^{6}} + -\frac{1.8311119\times10^{6}}{1.62923\times10^{-46}}}}$$

Result:

$7.73850\ldots\times10^{51}\, i$

Polar coordinates:

$r = 7.7385\times10^{51}$ (radius), $\theta = 90°$ (angle)

7.7385e+51

1/2 (1.12390884377076396458l6e+52 + 7.7385e+51)^1/2

Input interpretation:

$$\frac{1}{2}\sqrt{1.123908843770763964581 6\times10^{52} + 7.7385\times10^{51}}$$

Result:

$6.887958412640794909181674504887156505331221747079637 4\ldots\times10^{25}$

$6.88795841264\ldots*10^{25}$

$5.30074*10^{25} + 6.88795841264*10^{25}$

(5.30074*10^25 + 6.88795841264*10^25)

Input interpretation:

$5.30074\times10^{25} + 6.88795841264\times10^{25}$

Result:

121 886 984 126 400 000 000 000 000

Scientific notation:

$1.218869841264 \times 10^{26}$

$r_4 = 1.218869841264 * 10^{26}$

(5.30074*10^25 - 6.88795841264*10^25)

Result:

$-1.58721841264 \times 10^{25}$

$r_3 = -1.58721841264 * 10^{25}$

Input interpretation:

$$\frac{1}{2}\sqrt{1.1239088437707639645816 \times 10^{52} - 7.7385 \times 10^{51}}$$

Result:

$2.95829... \times 10^{25}$

$2.95829... * 10^{25}$

(5.30074*10^25 + 2.9582885414153*10^25)

Input interpretation:

$5.30074 \times 10^{25} + 2.9582885414153 \times 10^{25}$

Result:

82 590 285 414 153 000 000 000 000

Scientific notation:

$8.2590285414153 \times 10^{25}$

$r_2 = 8.2590285414153 * 10^{25}$

(5.30074*10^25 - 2.9582885414153*10^25)

Input interpretation:

$5.30074 \times 10^{25} - 2.9582885414153 \times 10^{25}$

Result:

23 424 514 585 847 000 000 000 000

Scientific notation:

$2.3424514585847 \times 10^{25}$

$r_1 = 2.3424514585847*10^{25}$

From the four results (event horizons), we obtain:

$r_1 = 2.3424514585847*10^{25}$

$r_2 = 8.2590285414153*10^{25}$

$r_3 = -1.58721841264 * 10^{25}$

$r_4 = 1.218869841264 * 10^{26}$

(2.3424514585847*10^25 +8.2590285414153*10^25 -1.58721841264 * 10^25 +1.218869841264 * 10^26)

Input interpretation:

$2.3424514585847 \times 10^{25} + 8.2590285414153 \times 10^{25} + 10^{25} \times (-1.58721841264) + 1.218869841264 \times 10^{26}$

Result:

212 029 600 000 000 000 000 000 000

Scientific notation:

2.120296×10^{26}

$2.120296*10^{26}$

(2.3424514585847*10^25 +8.2590285414153*10^25 -1.58721841264 * 10^25 +1.218869841264 * 10^26)^1/126

Input interpretation:

$\left(2.3424514585847 \times 10^{25} + 8.2590285414153 \times 10^{25} + 10^{25} \times (-1.58721841264) + 1.218869841264 \times 10^{26}\right) \wedge (1/126)$

Result:

1.61785522079119...

1.61785522079119... result that is a very good approximation to the value of the golden ratio 1,618033988749...

Now, we have:

$$\left(\frac{dr}{ds}\right)^2 = 2\left[-\frac{M}{r} + \frac{Q^2}{2r^2} - \frac{\Lambda r^2}{6} + k_1^2\left(-\frac{1}{2r^2} + \frac{M}{r^3} - \frac{Q^2}{2r^4}\right)\right], \quad (44)$$

For

r = 11225.7

Λ = -1.1056e-46

Q = 0.00089

M = 7.55858e+30

2[((((-7.55858e+30) / (11225.7) + (0.00089^2) / (2*11225.7^2) – (-1.1056e-46*11225.7^2)/6+x^2((-1/(2*11225.7^2)+(7.55858e+30)/(11225.7)^3-(0.00089)^2/(2*11225.7^4))))))] = 11225.7

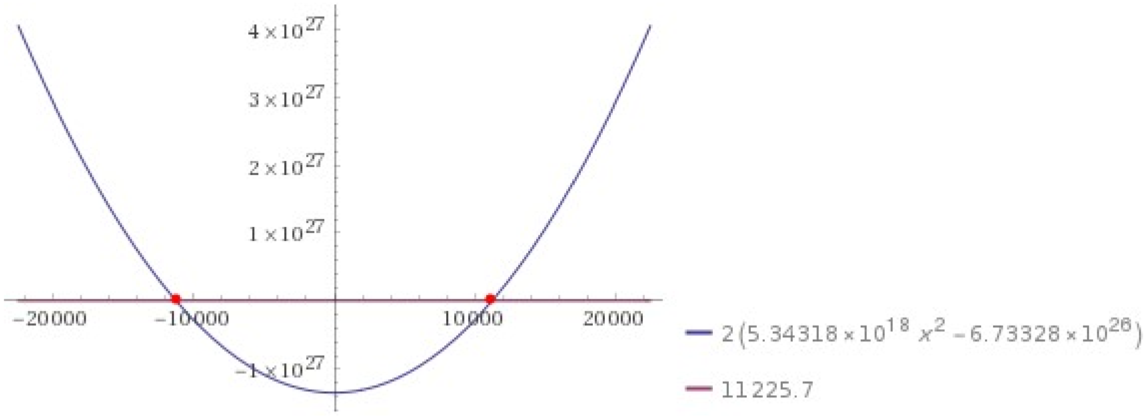
Input interpretation:

$$2\left(-\frac{7.55858\times 10^{30}}{11\,225.7} + \frac{0.00089^2}{2\times 11\,225.7^2} - \frac{1}{6}\left(-1.1056\times 10^{-46}\times 11\,225.7^2\right) + x^2\left(-\frac{1}{2\times 11\,225.7^2} + \frac{7.55858\times 10^{30}}{11\,225.7^3} - \frac{0.00089^2}{2\times 11\,225.7^4}\right)\right) = 11\,225.7$$

Result:

$$2\left(5.34318\times 10^{18}\, x^2 - 6.73328\times 10^{26}\right) = 11\,225.7$$

Plot:



Alternate forms:

$1.06864\times10^{19}\,x^2 - 1.34666\times10^{27} = 0$

$1.06864\times10^{19}\,x^2 - 1.34666\times10^{27} = 11\,225.7$

$1.06864\times10^{19}\,(x - 11\,225.7)\,(x + 11\,225.7) = 11\,225.7$

Solutions:

$x \approx -11\,225.7$

$x \approx 11\,225.7$

11225.7

Thence, we have:

2[((((-7.55858e+30) / (11225.7) + (0.00089^2) / (2*11225.7^2) – (-1.1056e-46*11225.7^2)/6+11225.7^2((-1/(2*11225.7^2)+(7.55858e+30)/(11225.7)^3-(0.00089)^2/(2*11225.7^4))))))]-11225.7

Input interpretation:

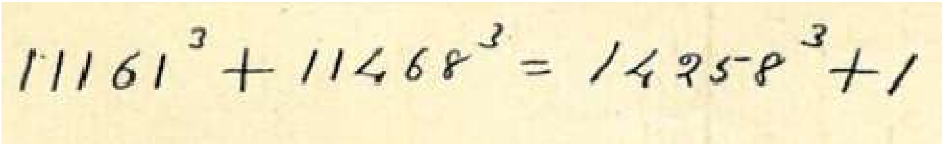
$$2\left(-\frac{7.55858\times10^{30}}{11\,225.7}+\frac{0.00089^2}{2\times11\,225.7^2}-\frac{1}{6}\left(-1.1056\times10^{-46}\times11\,225.7^2\right)+11\,225.7^2\left(-\frac{1}{2\times11\,225.7^2}+\frac{7.55858\times10^{30}}{11\,225.7^3}-\frac{0.00089^2}{2\times11\,225.7^4}\right)\right)-11\,225.7$$

Result:

−11226.69999999999999999999999999999999999953558777984752...

-11226.6999….

We note that from the Ramanujan taxicab number:


$$11161^3 + 11468^3 = 14258^3 + 1$$

11161 + 64 + ϕ = 11226.61803398..... result, with positive sign, practically equal to the above value

Furthermore:

-(13+2)/10^3+(-(2[((((-7.55858e+30)/(11225.7)+(0.00089^2)/(2*11225.7^2)–(-1.1056e-46*11225.7^2)/6+11225.7^2((-1/(2*11225.7^2)+(7.55858e+30)/(11225.7)^3-(0.00089)^2/(2*11225.7^4))))))]-11225.7))^1/19

Input interpretation:

$$-\frac{13+2}{10^3}+\left(-\left(2\left(-\frac{7.55858\times10^{30}}{11225.7}+\frac{0.00089^2}{2\times11225.7^2}-\frac{1}{6}\left(-1.1056\times10^{-46}\times11225.7^2\right)+11225.7^2\left(-\frac{1}{2\times11225.7^2}+\frac{7.55858\times10^{30}}{11225.7^3}-\frac{0.00089^2}{2\times11225.7^4}\right)\right)-11225.7\right)\right)^{\wedge}(1/19)$$

Result:

1.618695692957578160081667556270903716821925808129357404234...

1.6186956929575… result that is a very good approximation to the value of the golden ratio 1,618033988749...

Observations

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the **Fibonacci numbers**, commonly denoted F_n, form a sequence, called the **Fibonacci sequence**, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The **Lucas numbers** or **Lucas series** are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A **Lucas prime** is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a **golden spiral** is a logarithmic spiral whose growth factor is φ, the golden ratio. That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

References

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