Anomalies, the fine-structure constant and the proton radius

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Summary

This article discusses the concept of g-factors and anomalies in the context of the ring current or *Zitterbewegung* model of elementary particles. We suggest the anomalies are not anomalies at all. We think that the assumption that the pointlike *zbw* charge has no dimension or structure whatsoever is bound to yield these so-called anomalies between our measurements (mainly of the charge radius and the magnetic moment) and the nice theories we have about the structure of elementary particles. We illustrate the theory using the classical calculations for the electron.

We then discuss the results of the 2019 PRad experiment, which yielded a point estimate of about 0.831 fm. While this value differs only slightly from the 0.841 value that was measured by Pohl (2010) and Antognini (2013), we think the PRad value is very interesting, because it is very consistent with the anomalies (radius as well as magnetic moment) one can calculate. We think it confirms that the PRad measurement is very solid.

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Introduction

Our particular ring current or *Zitterbewegung* model accepts Einstein's mass-energy equivalence relation ($E = m \cdot c^2$) for what it is, interprets c as the tangential velocity of the naked charge ($c = a \cdot \omega$) and, then just uses the Planck-Einstein relation ($E = \hbar \cdot \omega$) to find the Compton radius[1]:

$$a = \frac{c}{\omega} = \frac{c\hbar}{mc^2} = \frac{\hbar}{mc} = \frac{\lambda_C}{2\pi} \approx 0.386159268 \dots \text{ fm}$$

Note that the Compton radius is inversely proportional to the mass. The Compton radius of a proton, for example, is much smaller than the Compton radius of an electron. Can the idea be applied to a proton? Of course. One can do it in a very classical way by discussing it in the context of photon scattering, which can also be done with protons. Let me quote from a very standard textbook here[2]:

> "The only difference is that the proton is heavier. We simply replace the electron mass in the Compton wavelength shift equation with the proton mass. […] The maximum shift is $\Delta\lambda_{max} = 2h/m_p c \approx 2.64$ fm. Fantastically small. This is roughly the size attributed to a small atomic nucleus, since the Compton wavelength sets the scale above which the nucleus can be localized in a particle-like sense."

This also shows even mainstream physicists do think of the Compton wavelength as effectively defining some space. To be precise, we think its reduced form – the Compton radius $a = \lambda_C/2\pi$ – effectively defines the space in which the *Zitterbewegung* charge is actually moving.

This pointlike *zbw* charge is supposed to whizz around at the speed of light but that assumption is – most likely – a mathematical idealization. This is why the anomalous magnetic moment is *not* an anomaly: the assumption that the elementary charge has no dimension or structure whatsoever is bound to result in an 'anomaly' between our measurements and the nice theories we have about the structure of electrons, photons and protons.[3] Mathematical idealizations are just what they are: we need the math and the mathematical ideas that come with it (including the ideas of nothingness and infinity) to describe reality – math was Wittgenstein's ladder to understanding – but Planck's quantum of action, and the finite speed of light, effectively tell us our mathematical ideas are what they are: idealized notions we use to describe a reality which is, in the end, quite finite. Something that has no dimension whatsoever probably exists in our mind only.

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[1] It should be noted that most *Zitterbewegung* theorists have a 1/2 factor in their model. However, the 1/2 factor is *not* consistent with the measured magnetic moment. We also think the 1/2 factor is theoretically not consistent. We should also note some like to distinguish *Zitterbewegung* from ring current models but we feel they are the same, *practically* speaking. The difference between various models is mostly in *interpretation*. We refer to our interpretation as a very *realist* interpretation. We have defined this interpretation in a previous paper (https://vixra.org/abs/2001.0453).

[2] See: Prof. Dr. Patrick LeClair, Physics 253 course (http://pleclair.ua.edu/PH253/Notes/compton.pdf), p. 10

[3] For the proton model, see: https://vixra.org/abs/2001.0685.

Let us stop the philosophy and calculate those anomalies and see how the fine-structure constant relates to it in the context of the electron model. We will then briefly talk about the recent PRad measurement of the proton radius and discuss an intriguing hypothesis: the 0.831 fm PRad measurement may be closer to the real radius of the proton than the previous 0.841 fm measurements.

Calculation of the anomalies for the electron

Because $\hbar$ and *c* have *precisely* **defined** values since the 2019 revision of SI units, we basically calculate the radius from the mass here.[4] That makes sense: it is easy to measure the *inertia* to some force, so the CODATA value should be pretty good in terms of finding something to hold onto:

$$m_{CODATA} = 9.1093837015(28)\times10^{-31} \text{ kg}$$

That is a *measured* value. Zillion experiments. No problem. We have a CODATA value for the radius too. Again: zillion experiments, no problem. Well… No… I should be precise, we do not: we have a CODATA value for the Compton wavelength.

Of course, we do recommend you double-check if the calculation works with the CODATA values for $\hbar$, *c* and m: you should get the same value for *a*—not approximately, but *exactly*:

$$a = \frac{\hbar}{\mathrm{m}c} \approx 0.386159268 \ldots \text{ fm}$$

Nice. The mass is the mass: $m = m_{CODATA}$. No weird anomaly stories about the mass. There shouldn't be: the mass (or energy) of an elementary particle is the mass (or) energy of an elementary particle—in our frame of reference, of course. Full stop.

Let us now see if our current ring model is consistent. We should, effectively, be able to calculate the radius from the magnetic moment, for which we also have a *measured* CODATA value[5]:

$$\mu_{CODATA} = 9.2847647043(28)\times10^{-24} \text{ J}\cdot\text{T}^{-1}$$

So let us use this value to calculate the radius. The logic is very straightforward. The magnetic moment is the product of the current and the area of the loop, and the current is the product of the elementary charge and the frequency. The frequency is, of course, the velocity of the charge divided by the circumference of the loop. Because we assume the velocity of our charge is equal to *c*, we get the following radius value:

$$\mu = \mathrm{I}\pi a^2 = \mathrm{q_e} f \pi a^2 = \mathrm{q_e}\frac{c}{2\pi a}\pi a^2 = \frac{\mathrm{q_e} c}{2} a \Leftrightarrow a = \frac{2\mu}{\mathrm{q_e} c} \approx 0.3866607 \ldots \text{pm}$$

This value is slightly *larger* than the value we get based on the mass or the Compton wavelength. So, yes, we do have an anomaly. We can use a lot of subscripts here, but they are all the same: we will think of the radius based on the mass or the Compton wavelength as some kind of *theoretical* radius and so we will put it in the denominator. You can write it like you want, with or without some subscript: *a* =

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[4] Note that the radius is inversely proportional to the mass. The Compton radius of a proton, for example, is much smaller than the Compton radius of an electron.

[5] We should put a minus sign as per the convention but, because we are interested in magnitudes here, we will omit it. It will, hopefully, confuse the reader *less*, rather than more.

$a_{CODATA} = a_m = a_\lambda = a_C$. In contrast, we will write the radius based on our calculation using the magnetic moment as a_μ. We can then write the anomaly as:

$$\frac{a_\mu - a}{a} \approx 0.00115965 \Leftrightarrow \frac{a_\mu}{a} = 1.00115965 \ldots$$

You can verify the obvious relations you are familiar with: the anomaly is, effectively, equal to about 99.85% of Schwinger's factor:

$$\frac{\alpha}{2\pi} = 0.00116141 \ldots$$

Let us, for good order, also recalculate the anomaly of the magnetic moment. We will follow a slightly different presentation than the usual one but you will see it amounts to the same result. We first calculate a new *theoretical* value for the magnetic moment using the Compton radius, which we will denote as μ_a. When writing it all out, we get this:

$$\mu_a = \mathrm{I}\pi a^2 = \mathrm{q_e} f \pi a^2 = \mathrm{q_e}\frac{c}{2\pi a}\pi a^2 = \frac{\mathrm{q_e} c}{2} a = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar \approx 9.27401 \ldots \times 10^{-24}\ \mathrm{J \cdot T^{-1}}$$

We can now calculate the anomaly – against the CODATA value – once more[6]:

$$\frac{\mu_a - \mu}{\mu} = 0.00115965 \ldots$$

We get the same anomaly—not approximately but *exactly*, as we would expect: in the *zbw* or ring current model, the anomaly is not only related to the *actual* magnetic moment but to the *actual* radius as well. This should not surprise us: the magnetic moment is, of course, proportional to the radius of the loop.[7] Hence, if the *actual* magnetic moment differs from the theoretical one, then the *actual* radius must also differ from the theoretical one.

What do we get if we use the g-factors themselves? The intellectually honest reply to this is that it depends on your assumption in regard to the angular momentum of the electron, which we can*not* directly measure. Let me write down the formula for the gyromagnetic ratio:

$$g = \frac{\mu}{\mathrm{L}} = \frac{\mathrm{I} \cdot \pi a^2}{\mathrm{m_\gamma} \cdot a \cdot c} = \frac{\mathrm{q_e} \cdot c \cdot \frac{a^2}{2a}}{\frac{\mathrm{m}}{2} \cdot a \cdot c} = \frac{\mathrm{q_e}}{\mathrm{m}}$$

You will say this doesn't look like a *g*-factor: we're missing a 1/2 factor, and what is the $\mathrm{m_\gamma}$ concept? It is the angular mass of our pointlike *Zitterbewegung* charge. Based on a geometric argument[8], one can show that the effective mass of the point charge is *half* of the rest mass of the electron, but the proof depends on the assumption that the velocity of the pointlike charge is, effectively, equal to *c*. If one does not want to use that assumption, one has to simply assume that, somehow, the mass of the electron is

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[6] You should watch out with the minus signs here – and you may want to think why you put what in the denominator – but it all works out!

[7] We have a squared radius in the numerator of the formula for the magnetic moment, and a non-squared radius factor in the denominator.

[8] See our paper on the electron as a harmonic electromagnetic oscillator (https://vixra.org/abs/1905.0521).

spread over a disk, so we can use the formula for the angular mass of a disk rather than a hoop.[9] It is the same 1/2 factor that we see in the *conventional* definition of the *g*-factor, which writes *g* as a multiple of $q_e/2m$ based on the assumption that the angular momentum is equal to ħ/2, which we think it is:

$$\boldsymbol{\mu} = -g\left(\frac{q_e}{2m}\right)\mathbf{L} \Leftrightarrow \frac{q_e}{2m}\hbar = g\frac{q_e}{2m}\frac{\hbar}{2} \Leftrightarrow g = 2$$

However, as mentioned above, we really think the concept of a g-factor obscures the matter[10], and so we will just stick to ratios of magnetic moments or radii. That also deals with the question of the unknown or undefined angular momentum which – the reader should note – should cancel out anyway when taking the *ratio* of two g-factors: the angular momentum is the angular momentum, right?

Let us continue our calculations. Our assumption is that the anomaly is, somehow, the result of our mathematical idealizations. We cannot *really* assume the pointlike *zbw* charge is whizzing around at the speed of light. It can be very *near c*, but not equal to *c*. Hence, its theoretical rest mass will also be very close to zero, but not *exactly* zero. Of course, because everything is related to everything in this model, the anomalies also suggest we have some *real* radius *r* that is probably *not quite* equal to the Compton radius *a* = ħ/m*c*. Let us write it all out. What should we put where? It is not easy to figure out, but the greater value – based on the greater radius – should be in the denominator, so – also using the *a* = ħ/m*c* relation – we write:

$$\frac{\mu_r}{\mu_a} = \frac{\frac{q_e}{2m}\hbar}{\frac{q_e v}{2}r} = \frac{\hbar}{m \cdot v \cdot r} = \frac{c \cdot a}{v \cdot r}$$

It is a weird formula to interpret. If there would be no anomaly – our mathematical idealization would match reality – then the formula just becomes unity. However, we know the anomaly exists, and it is *very* nearly equal to 1 + α/2π. For all practical purposes – we think a 99.85% explanation is pretty good – we will just equate it and re-write the expression above as:

$$1 + \frac{\alpha}{2\pi} = \frac{2\pi + \alpha}{2\pi} = \frac{c \cdot a}{v \cdot r} \Leftrightarrow v \cdot r = \frac{2\pi \cdot c \cdot a}{2\pi + \alpha} = \frac{2\pi \cdot c \cdot \frac{\hbar}{mc}}{2\pi + \alpha} = \frac{h}{m(2\pi + \alpha)}$$

$$\Leftrightarrow L = m \cdot v \cdot r = \frac{h}{2\pi + \alpha}$$

This is a strange but very nice result. At first, it seems to challenge the idea of an electron as a spin-1/2 particle, but it doesn't. We once again need to distinguish between the total mass of the electron and the effective mass of the electron as it whizzes around its center ($m = 2m_\gamma$). We can, likewise, distinguish between L and L_γ ($L = 2L_\gamma$). Mystery solved: the electron *is* a spin-1/2 particle! We re-write the expression above by adding the necessary subscript:

$$L_\gamma = m_\gamma \cdot v \cdot r = \frac{1}{2} \cdot \frac{h}{2\pi + \alpha}$$

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[9] Our particular flavor of *Zitterbewegung* model assumes the circular motion is equivalent to the motion of two linear oscillators working in tandem. We readily admit this 1/2 factor looks like the Achilles heel of the model. However, we think we were able to convincingly demonstrate why the assumption makes sense in our previous papers.

[10] See: Jean Louis Van Belle, *The Not-So-Anomalous Magnetic Moment*, 21 December 2018 (http://vixra.org/abs/1812.0233).

Let us now move to a discussion of the anomalies for the proton.

Calculation of the anomalies for the proton

In our previous paper, we used the ring current model to obtain a theoretical value for the proton radius.[11] It is equal to:

$$a_{\mathrm{p}} = \frac{4\hbar}{m_{\mathrm{p}}c} \approx 4 \cdot (0.21 \ldots \mathrm{fm}) \approx 0.84123564 \ldots \ \mathrm{fm}$$

This is *very* close to the new 2018 CODATA value for the proton radius, which is equal to:

$$r_{\mathrm{p}} = 0.8414 \pm 0.0019 \text{ fm}$$

The new CODATA value for the proton radius (0.8414 $\pm$ 0.0019 fm) takes all past measurements into account but gives very high weightage to the measurements of Pohl (2010) and Antognini (2013), which are both based on muonic-hydrogen spectroscopy. In contrast, the PRad experiment which is based on a proton-electron scattering – quite a different technique – established the following new value for the proton radius:

$$r_{\mathrm{p}} = 0.831 \pm 0.007_{\mathrm{stat}} \pm 0.012_{\mathrm{syst}} \text{ fm}$$

Prof. Dr. Randolf Pohl is of the opinion that the PRad measurement and the muonic-hydrogen spectroscopy measurements are basically in agreement. He writes: “There is no difference between the values. You have to take uncertainties seriously (sometimes we spend much more time on determining the uncertainty than we do for the central value), and these numbers are in perfect agreement with each other.”

There is, indeed, no significant difference *from a statistical point of view*. However, 0.831 and 0.841 are still two different point estimates, and the difference allows us to entertain an intriguing new hypothesis: the 0.841 value may be the theoretical value, while the PRad value (0.831) might be the *real* value, including the anomaly one would expect because – once again – our theoretical calculation assumes a theoretical pointlike charge with zero rest mass which is, therefore, whizzing around at the speed of light. Let us look at the anomalies. The theoretical value for the proton g-factor, based on the ring current model and the $4\hbar/mc$ value, is equal to[12]:

$$\mu_{\mathrm{p}} = \mathrm{g}_{\mathrm{p}} \frac{\mathrm{q_e}}{2\mathrm{m_p}} \frac{\hbar}{2} \Leftrightarrow \mathrm{g}_{\mathrm{p}} = \frac{4\mu_{\mathrm{p}}\mathrm{m_p}}{\hbar \mathrm{q_e}} = \frac{4}{\sqrt{2}} \cdot \frac{\mu_{\mathrm{L}}\mathrm{m_p}}{\hbar \mathrm{q_e}} = \frac{4}{\sqrt{2}} \cdot \frac{2\mathrm{q_e}\hbar}{\mathrm{m_p}} \cdot \frac{\mathrm{m_p}}{\hbar \mathrm{q_e}} = \frac{8}{\sqrt{2}} = 5.65685 \ldots$$

In contrast, the CODATA value is:

$$\mathrm{g}_{\mathrm{CODATA}} = 5.5856946893(16)$$

Hence, we can calculate the anomaly as:

[11] See: https://vixra.org/abs/2001.0685.

[12] See our paper for the detail of the calculations and, in particular, for an explanation of the √2 factor, which is based on the idea of precession and which is why we distinguish μ_{p} from μ_{L} ($\mu_{\mathrm{p}} = \mu_{\mathrm{L}}/\sqrt{2}$).

$$\frac{g_p - g_{CODATA}}{g_p} \approx 0.0125794\ldots$$

We can also illustrate the difference calculating the magnetic moment directly. The theoretical value – using the 4ħ/m*c* value and the ring current model – is equal to:

$$\mu_p = \frac{2q_e}{\sqrt{2}m_p}\hbar \approx 1.4286\ldots \times 10^{-26}\ \mathrm{J \cdot T^{-1}}$$

In contrast, the CODATA value is equal to:

$$\mu_{CODATA} = 1.41060679736(60) \times 10^{-26}\ \mathrm{J \cdot T^{-1}}$$

Unsurprisingly, we get the same anomaly – not approximately but *exactly* – when doing the calculations with the magnetic moments:

$$\frac{\mu_p - \mu_{CODATA}}{\mu_p} \approx 0.0125794\ldots$$

We can now point to a remarkable coincidence: the difference between the PRad value for the proton radius and the theoretical 4ħ/m*c* value is of the same order:

$$\frac{r_p - r_{PRad}}{r_p} = \frac{\frac{4\hbar}{m_p c} - r_{PRad}}{\frac{4\hbar}{m_p c}} \approx 0.0121674\ldots$$

Based on this rather remarkable coincidence, I like to think that the PRad value might be the actual proton radius, while the results from the muonic hydrogen spectroscopy experiments – for some reason I do not understand – may not include the anomaly.

Of course, one should note the anomaly is rather large: 0.012 to 0.013 is a much larger number than the α/2π factor that we get from calculating the anomalies for the electron—about 10 times larger, in fact. There is, therefore, plenty of scope for more precise measurements in the future.

There is, obviously, also plenty of scope for further refining the proton model itself so as to counter the perception that the 4ħ/m*c* value is just some "numerology." Indeed, we like to think our "back-of-the-envelope" calculations are simple but sound!

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