Analyzing two Ramanujan equations: mathematical connections with various parameters of Particle Physics and Cosmology II

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Abstract

The purpose of this paper is to show how using certain mathematical values and / or constants from two Ramanujan equations, some important parameters of Particle Physics and Cosmology are obtained.

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https://apod.nasa.gov/apod/ap170510.html

https://wssrmnn.net/index.php/2017/01/23/man-saw-number-pi-dreams/

From: **Manuscript Book 2 of Srinivasa Ramanujan**

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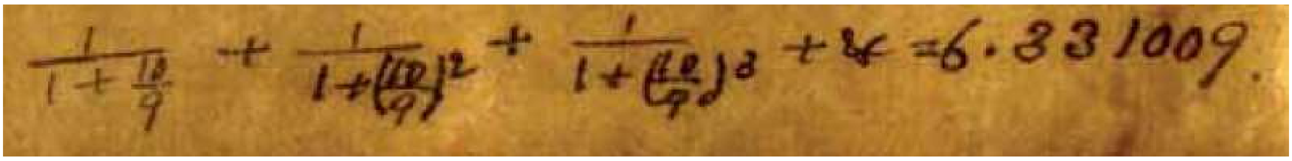


$$\frac{1}{1+\frac{10}{9}} + \frac{1}{1+\left(\frac{10}{9}\right)^2} + \frac{1}{1+\left(\frac{10}{9}\right)^3} + \&c = 6.331009.$$

1/(1+10/9) +1/(1+(10/9)^2)+1/(1+(10/9)^3)+...

Input interpretation:

$$\frac{1}{1+\frac{10}{9}} + \frac{1}{1+\left(\frac{10}{9}\right)^2} + \frac{1}{1+\left(\frac{10}{9}\right)^3} + \cdots$$

Infinite sum:

$$\sum_{n=1}^{\infty} \frac{1}{\left(\frac{10}{9}\right)^n + 1} = \frac{i \operatorname{Im}\left(\psi^{(0)}_{\frac{9}{10}}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)}{\log\left(\frac{10}{9}\right)} + \frac{\operatorname{Re}\left(\psi^{(0)}_{\frac{9}{10}}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)}{\log\left(\frac{10}{9}\right)} - \frac{\log(10)}{\log\left(\frac{10}{9}\right)}$$

$\log(x)$ is the natural logarithm

$\psi_q(z)$ gives the q-digamma function

$\operatorname{Im}(z)$ is the imaginary part of z

$\operatorname{Re}(z)$ is the real part of z

Decimal approximation:

6.331008692864745537718386879838180649341260412564743295777...

6.331008692…

Convergence tests:

By the ratio test, the series converges.

Partial sum formula:

$$\sum_{n=1}^{m} \frac{1}{1+\left(\frac{10}{9}\right)^n} = \frac{\psi_{\frac{9}{10}}^{(0)}\left(-\frac{i\pi-\log\left(\frac{10}{9}\right)}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)} - \frac{\psi_{\frac{9}{10}}^{(0)}\left(-\frac{i\pi-(m+1)\log\left(\frac{10}{9}\right)}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)}$$

Alternate forms:

$$-\frac{\log(10) - \psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)}$$

$$-\frac{\log(10)}{\log\left(\frac{10}{9}\right)} + \frac{\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)}$$

$$\frac{-\log(10) + \psi_{\frac{9}{10}}^{(0)}\left(\frac{-i\pi-2\log(3)+\log(10)}{-2\log(3)+\log(10)}\right)}{\log(10)-2\log(3)}$$

Series representations:

$$\frac{i\,\mathrm{Im}\left(\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)}{\log\left(\frac{10}{9}\right)} - \frac{\log(10)}{\log\left(\frac{10}{9}\right)} + \frac{\mathrm{Re}\left(\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)}{\log\left(\frac{10}{9}\right)} =$$

$$-\left(\left(2\pi\left\lfloor\frac{\arg(10-x)}{2\pi}\right\rfloor - \mathrm{Im}\left(\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{2i\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}}\right)\right) - i\log(x) + i\,\mathrm{Re}\left(\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{2i\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}}\right)\right) + i\sum_{k=1}^{\infty}\frac{(-1)^k(10-x)^k x^{-k}}{k}\right) \Big/ \left(2\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor - i\log(x) + i\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}\right)\right) \quad \text{for } x<0$$

$$
\frac{i\,\mathrm{Im}\left(\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\,\pi}{\log\left(\frac{10}{9}\right)}\right)\right)}{\log\left(\frac{10}{9}\right)}-\frac{\log(10)}{\log\left(\frac{10}{9}\right)}+\frac{\mathrm{Re}\left(\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\,\pi}{\log\left(\frac{10}{9}\right)}\right)\right)}{\log\left(\frac{10}{9}\right)}=
$$

$$
-\left(\left(\left(2\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor-\mathrm{Im}\left(\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\,\pi}{2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right)\right)-i\log(z_0)+\right.\right.\right.
$$

$$
i\,\mathrm{Re}\left(\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\,\pi}{2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right)\right)+
$$

$$
\left.i\sum_{k=1}^{\infty}\frac{(-1)^k\,(10-z_0)^k\,z_0^{-k}}{k}\right)\Bigg/
$$

$$
\left.\left(2\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor-i\log(z_0)+i\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)\right)
$$

$$\frac{i\,\mathrm{Im}\left(\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)}{\log\left(\frac{10}{9}\right)}-\frac{\log(10)}{\log\left(\frac{10}{9}\right)}+\frac{\mathrm{Re}\left(\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)}{\log\left(\frac{10}{9}\right)}=$$

$$\left(i\,\mathrm{Im}\left(\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right)\right)-\right.$$
$$\left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)-\log(z_0)-\left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\log(z_0)+$$
$$\mathrm{Re}\left(\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right)\right)+$$
$$\left.\sum_{k=1}^{\infty}\frac{(-1)^k(10-z_0)^k z_0^{-k}}{k}\right)\Bigg/$$
$$\left(\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)$$

From the left-hand side of the above infinite sum:

$$\sum_{n=1}^{\infty}\frac{1}{\left(\frac{10}{9}\right)^n+1}=\frac{i\,\mathrm{Im}\left(\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)}{\log\left(\frac{10}{9}\right)}+\frac{\mathrm{Re}\left(\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)}{\log\left(\frac{10}{9}\right)}-\frac{\log(10)}{\log\left(\frac{10}{9}\right)}$$

We obtain:

(((sum_(n=1)^∞ 1/((10/9)^n + 1))))

Infinite sum:

$$\sum_{n=1}^{\infty}\frac{1}{\left(\frac{10}{9}\right)^n+1}=\frac{-\log(10)+\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)}$$

$\log(x)$ is the natural logarithm

$\psi_q(z)$ gives the q-digamma function

Decimal approximation:

6.3310086928647455377183868798381806493412604125647432957770...

6.331008692…

Convergence tests:

By the ratio test, the series converges.

Partial sum formula:

$$\sum_{n=1}^{m} \frac{1}{\left(\frac{10}{9}\right)^n + 1} = \frac{\psi^{(0)}_{\frac{9}{10}}\left(-\frac{i\pi - \log\left(\frac{10}{9}\right)}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)} - \frac{\psi^{(0)}_{\frac{9}{10}}\left(-\frac{i\pi - (m+1)\log\left(\frac{10}{9}\right)}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)}$$

Alternate forms:

$$-\frac{\log(10) - \psi^{(0)}_{\frac{9}{10}}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)}$$

$$-\frac{\log(10)}{\log\left(\frac{10}{9}\right)} + \frac{\psi^{(0)}_{\frac{9}{10}}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)}$$

$$\frac{-\log(10) + \psi^{(0)}_{\frac{9}{10}}\left(\frac{-i\pi - 2\log(3) + \log(10)}{-2\log(3) + \log(10)}\right)}{\log(10) - 2\log(3)}$$

Series representations:

$$\frac{-\log(10)+\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)} = -\left(\left(2\pi\left\lfloor\frac{\arg(10-x)}{2\pi}\right\rfloor - i\log(x) + i\,\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{2i\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}}\right) + i\sum_{k=1}^{\infty}\frac{(-1)^k(10-x)^k x^{-k}}{k}\right) \Big/ \left(2\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor - i\log(x) + i\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}\right)\right) \quad \text{for } x<0$$

$$\frac{-\log(10)+\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)} = -\left(\left(2\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor - i\log(z_0) + i\,\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right) + i\sum_{k=1}^{\infty}\frac{(-1)^k(10-z_0)^k z_0^{-k}}{k}\right) \Big/ \left(2\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor - i\log(z_0) + i\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)\right)$$

$$\frac{-\log(10)+\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)} =$$

$$-\left(\left(\left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\log(z_0)-\right.\right.$$

$$\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right)-$$

$$\left.\sum_{k=1}^{\infty}\frac{(-1)^k\,(10-z_0)^k\,z_0^{-k}}{k}\right)\Bigg/\left(\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\right.$$

$$\left.\left.\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)\right)$$

from which, raising to the cube, we obtain:

(((sum_(n=1)^∞ 1/((10/9)^n + 1))))^3

Input interpretation:

$$\left(\sum_{n=1}^{\infty}\frac{1}{\left(\frac{10}{9}\right)^n+1}\right)^3$$

Result:

$$\frac{\left(-\log(10)+\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)} \approx 253.757-1.05499\times10^{-12}\,i$$

Input interpretation:

$253.757-1.05499\times10^{-12}\,i$

i is the imaginary unit

Result:

253.757... −
1.05499... × $10^{-12}\,i$

Polar coordinates:

$r = 253.757$ (radius), $\theta = -2.38206\times10^{-13}\,°$ (angle)

253.757

log(x) is the natural logarithm

$\psi_q(z)$ gives the q-digamma function

Alternate forms:

$$-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)}$$

$$\frac{\left(-\log(10)+\psi_{\frac{9}{10}}^{(0)}\left(\frac{-i\pi-2\log(3)+\log(10)}{-2\log(3)+\log(10)}\right)\right)^3}{(\log(10)-2\log(3))^3}$$

$$-\frac{3\log(10)\,\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^2}{\log^3\left(\frac{10}{9}\right)}+\frac{\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^3}{\log^3\left(\frac{10}{9}\right)}+\frac{3\log^2(10)\,\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log^3\left(\frac{10}{9}\right)}-\frac{\log^3(10)}{\log^3\left(\frac{10}{9}\right)}$$

Also from the following alternate form

$$-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)}$$

We obtain:

-(log(10) - QPolyGamma(0, 1 - (i π)/log(10/9), 9/10))^3/(log^3(10/9))

Input:

$$-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)}$$

log(x) is the natural logarithm

$\psi_q(z)$ gives the q-digamma function

i is the imaginary unit

Decimal approximation:

253.7574079632009128064137486425258441728755422819488870181...

253.7574079...

Alternate forms:

$$\frac{\left(-\log(10)+\psi^{(0)}_{\frac{9}{10}}\left(\frac{-i\pi-2\log(3)+\log(10)}{-2\log(3)+\log(10)}\right)\right)^3}{(\log(10)-2\log(3))^3}$$

$$-\frac{3\log(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^2}{\log^3\left(\frac{10}{9}\right)}+\frac{\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^3}{\log^3\left(\frac{10}{9}\right)}+\frac{3\log^2(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log^3\left(\frac{10}{9}\right)}-\frac{\log^3(10)}{\log^3\left(\frac{10}{9}\right)}$$

$$-\frac{\left(-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)+\log(2)+\log(5)\right)^3}{\log^3\left(\frac{10}{9}\right)}$$

Alternative representations:

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)}=-\frac{\left(\log_e(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log_e\left(\frac{10}{9}\right)}\right)\right)^3}{\log_e^3\left(\frac{10}{9}\right)}$$

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)}=-\frac{\left(\log(a)\log_a(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log(a)\log_a\left(\frac{10}{9}\right)}\right)\right)^3}{\left(\log(a)\log_a\left(\frac{10}{9}\right)\right)^3}$$

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)}=-\frac{\left(-\text{Li}_1(-9)-\psi^{(0)}_{\frac{9}{10}}\left(1--\frac{i\pi}{\text{Li}_1\left(1-\frac{10}{9}\right)}\right)\right)^3}{\left(-\text{Li}_1\left(1-\frac{10}{9}\right)\right)^3}$$

Series representations:

$$-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)} = -\left(\left(\left(2\pi\left\lfloor\frac{\arg(10-x)}{2\pi}\right\rfloor - i\log(x) + i\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{2i\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}}\right) + i\sum_{k=1}^{\infty}\frac{(-1)^k(10-x)^k x^{-k}}{k}\right)^3 \middle/ \left(2\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor - i\log(x) + i\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}\right)^3\right)\right) \quad \text{for } x<0$$

$$-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)} = -\left(\left(\left(2\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor - i\log(z_0) + i\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right) + i\sum_{k=1}^{\infty}\frac{(-1)^k(10-z_0)^k z_0^{-k}}{k}\right)^3 \middle/ \left(2\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor - i\log(z_0) + i\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)^3\right)\right)$$

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)} =$$

$$-\left(\left(\left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\log(z_0)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(10-z_0)^k\,z_0^{-k}}{k}\right)^3\Bigg/\left(\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)^3\right)$$

Integral representations:

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)} = -\frac{\left(\int_1^{10}\frac{1}{t}\,dt-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\int_1^{\frac{10}{9}}\frac{1}{t}\,dt}\right)\right)^3}{\left(\int_1^{\frac{10}{9}}\frac{1}{t}\,dt\right)^3}$$

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)} =$$

$$-\frac{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{9^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds-2\,i\,\pi\,\psi^{(0)}_{\frac{9}{10}}\left(1+\frac{2\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{9^{s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\right)^3}{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{9^{s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^3} \quad \text{for } -1<\gamma<0$$

Multiplying by 1/2 and subtracting the value of the golden ratio, we obtain:

1/2((-(log(10) - QPolyGamma(0, 1 - (i π)/log(10/9), 9/10))^3/(log^3(10/9))))-golden ratio

Input:

$$\frac{1}{2}\left(-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)}\right)-\phi$$

$\log(x)$ is the natural logarithm

$\psi_q(z)$ gives the q-digamma function

i is the imaginary unit

ϕ is the golden ratio

Exact result:

$$-\phi-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}$$

Decimal approximation:

125.2606699928505615550022874868972839687174619611686806469...

125.26066999.... result very near to the Higgs boson mass 125.18 GeV

Alternate forms:

$$-\phi+\frac{\left(-\log(10)+\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}$$

$$\frac{1}{2}\left(-1-\sqrt{5}\right)-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}$$

$$-\phi+\frac{\left(-\log(10)+\psi_{\frac{9}{10}}^{(0)}\left(\frac{-i\pi-2\log(3)+\log(10)}{-2\log(3)+\log(10)}\right)\right)^3}{2(\log(10)-2\log(3))^3}$$

Alternative representations:

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}-\phi=-\phi-\frac{\left(\log_e(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log_e\left(\frac{10}{9}\right)}\right)\right)^3}{2\log_e^3\left(\frac{10}{9}\right)}$$

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}-\phi=-\phi-\frac{\left(\log(a)\log_a(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log(a)\log_a\left(\frac{10}{9}\right)}\right)\right)^3}{2\left(\log(a)\log_a\left(\frac{10}{9}\right)\right)^3}$$

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}-\phi=-\phi-\frac{\left(-\text{Li}_1(-9)-\psi^{(0)}_{\frac{9}{10}}\left(1--\frac{i\pi}{\text{Li}_1\left(1-\frac{10}{9}\right)}\right)\right)^3}{2\left(-\text{Li}_1\left(1-\frac{10}{9}\right)\right)^3}$$

Series representations:

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}-\phi=-\phi-\left(2i\pi\left\lfloor\frac{\arg(10-x)}{2\pi}\right\rfloor+\log(x)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{2i\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(10-x)^k x^{-k}}{k}\right)^3\Bigg/\left(2\left(2i\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}\right)^3\right)\quad\text{for } x<0$$

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\,\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}-\phi=$$

$$-\phi-\left(\log(z_0)+\left\lfloor\frac{\arg(10-z_0)}{2\,\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\,\pi}{\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\,\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(10-z_0)^k\,z_0^{-k}}{k}\right)^3\Bigg/\left(2\left(\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\,\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)^3\right)$$

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\,\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}-\phi=$$

$$-\phi-\left(2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor+\log(z_0)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\,\pi}{2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(10-z_0)^k\,z_0^{-k}}{k}\right)^3\Bigg/\left(2\left(2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)^3\right)$$

Multiplying the previous exspression by 1/2 and adding 11, that is a Lucas number (and also the dimensions number of the M-theory) and the value of the golden ratio, we obtain:

1/2((-(log(10) - QPolyGamma(0, 1 - (i π)/log(10/9), 9/10))^3/(log^3(10/9))))+11+golden ratio

Input:

$$\frac{1}{2}\left(-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)}\right)+11+\phi$$

$\log(x)$ is the natural logarithm

$\psi_q(z)$ gives the q-digamma function

i is the imaginary unit

ϕ is the golden ratio

Exact result:

$$-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+\phi+11$$

Decimal approximation:

139.4967379703503512514114611556285602041580803207802063712...

139.49673797.... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternate forms:

$$\frac{\left(-\log(10)+\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+\phi+11$$

$$\frac{1}{2}\left(23+\sqrt{5}\right)-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}$$

$$\frac{1}{2\log^3\left(\frac{10}{9}\right)}\left(3\log^2(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)-3\log(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^2+\right.$$
$$\left.\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^3+23\log^3\left(\frac{10}{9}\right)+\sqrt{5}\,\log^3\left(\frac{10}{9}\right)-\log^3(10)\right)$$

Alternative representations:

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}+11+\phi=11+\phi-\frac{\left(\log_e(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log_e\left(\frac{10}{9}\right)}\right)\right)^3}{2\log_e^3\left(\frac{10}{9}\right)}$$

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}+11+\phi=$$
$$11+\phi-\frac{\left(\log(a)\log_a(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log(a)\log_a\left(\frac{10}{9}\right)}\right)\right)^3}{2\left(\log(a)\log_a\left(\frac{10}{9}\right)\right)^3}$$

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}+11+\phi=11+\phi-\frac{\left(-\text{Li}_1(-9)-\psi^{(0)}_{\frac{9}{10}}\left(1--\frac{i\pi}{\text{Li}_1\left(1-\frac{10}{9}\right)}\right)\right)^3}{2\left(-\text{Li}_1\left(1-\frac{10}{9}\right)\right)^3}$$

Series representations:

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}+11+\phi=11+\phi-\left(2i\pi\left\lfloor\frac{\arg(10-x)}{2\pi}\right\rfloor+\log(x)-\right.$$
$$\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{2i\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}}\right)-$$
$$\left.\sum_{k=1}^{\infty}\frac{(-1)^k(10-x)^k x^{-k}}{k}\right)^3\Bigg/$$
$$\left(2\left(2i\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}\right)^3\right)\quad\text{for } x<0$$

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}+11+\phi=$$

$$11+\phi-\left(\log(z_0)+\left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(10-z_0)^k z_0^{-k}}{k}\right)^3 \Big/ \left(2\left(\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)^3\right)$$

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}+11+\phi=$$

$$11+\phi-\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(10-z_0)^k z_0^{-k}}{k}\right)^3 \Big/ \left(2\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)^3\right)$$

Adding 11, that is a Lucas number and subtracting the value of the conjugate of the golden ratio, we obtain from the previous expression:

1/2((-(log(10) - QPolyGamma(0, 1 - (i π)/log(10/9), 9/10))^3/(log^3(10/9))))+11-1/golden ratio

Input:

$$\frac{1}{2}\left(-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)}\right)+11-\frac{1}{\phi}$$

$\log(x)$ is the natural logarithm

$\psi_q(z)$ gives the q-digamma function

i is the imaginary unit

ϕ is the golden ratio

Exact result:

$$-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}-\frac{1}{\phi}+11$$

Decimal approximation:

137.2606699928505615550022874868972839687174619611686806469...

137.2606699928....

This result is very near to the inverse of fine-structure constant 137,035

Alternate forms:

$$\frac{1}{2}\left(23-\sqrt{5}\right)-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}$$

$$-\frac{3\log(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^2}{2\log^3\left(\frac{10}{9}\right)}+\frac{\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^3}{2\log^3\left(\frac{10}{9}\right)}+\frac{3\log^2(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{2\log^3\left(\frac{10}{9}\right)}+11-\frac{2}{1+\sqrt{5}}-\frac{\log^3(10)}{2\log^3\left(\frac{10}{9}\right)}$$

$$\frac{1}{2\left(1+\sqrt{5}\right)\log^3\left(\frac{10}{9}\right)}\left(3\log^2(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)+3\sqrt{5}\,\log^2(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)-3\log(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^2-3\sqrt{5}\,\log(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^2+\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^3+\sqrt{5}\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^3+18\log^3\left(\frac{10}{9}\right)+22\sqrt{5}\,\log^3\left(\frac{10}{9}\right)-\log^3(10)-\sqrt{5}\,\log^3(10)\right)$$

Alternative representations:

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}+11-\frac{1}{\phi}=11-\frac{1}{\phi}-\frac{\left(\log_e(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log_e\left(\frac{10}{9}\right)}\right)\right)^3}{2\log_e^3\left(\frac{10}{9}\right)}$$

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}+11-\frac{1}{\phi}=11-\frac{1}{\phi}-\frac{\left(\log(a)\log_a(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log(a)\log_a\left(\frac{10}{9}\right)}\right)\right)^3}{2\left(\log(a)\log_a\left(\frac{10}{9}\right)\right)^3}$$

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}+11-\frac{1}{\phi}=11-\frac{1}{\phi}-\frac{\left(-\mathrm{Li}_1(-9)-\psi^{(0)}_{\frac{9}{10}}\left(1--\frac{i\pi}{\mathrm{Li}_1\left(1-\frac{10}{9}\right)}\right)\right)^3}{2\left(-\mathrm{Li}_1\left(1-\frac{10}{9}\right)\right)^3}$$

Series representations:

$$-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}+11-\frac{1}{\phi}=11-\frac{1}{\phi}-\left(2\,i\,\pi\left\lfloor\frac{\arg(10-x)}{2\,\pi}\right\rfloor+\log(x)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\,\pi}{2\,i\,\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(10-x)^k\,x^{-k}}{k}\right)^3\Bigg/\left(2\left(2\,i\,\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}\right)^3\right)\quad \text{for } x<0$$

$$-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}+11-\frac{1}{\phi}=$$

$$11-\frac{1}{\phi}-\left(\log(z_0)+\left\lfloor\frac{\arg(10-z_0)}{2\,\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\,\pi}{\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\,\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(10-z_0)^k\,z_0^{-k}}{k}\right)^3\Bigg/\left(2\left(\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\,\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)^3\right)$$

$$-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)2}+11-\frac{1}{\phi}=$$

$$11-\frac{1}{\phi}-\left(2 i \pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\right.$$

$$\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{2 i \pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right)-$$

$$\left.\sum_{k=1}^{\infty}\frac{(-1)^k(10-z_0)^k z_0^{-k}}{k}\right)^3 \Bigg/$$

$$\left(2\left(2 i \pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)^3\right)$$

From the previous expression, (multiplying by 1/2, adding 2 and subtracting the value of the golden ratio) multiplying for 27*1/2 ad adding 11, we obtain:

27*1/2*(((1/2((-(log(10) - QPolyGamma(0, 1 - (i π)/log(10/9), 9/10))^3/(log^3(10/9))))+2-golden ratio)))+11

Input:

$$27\times\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)}\right)+2-\phi\right)+11$$

$\log(x)$ is the natural logarithm

$\psi_q(z)$ gives the q-digamma function

i is the imaginary unit

ϕ is the golden ratio

Exact result:

$$11+\frac{27}{2}\left(-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}-\phi+2\right)$$

Decimal approximation:

1729.019044903482580992530881073113333577685736475777188733...

1729.0190449....

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms:

$$11+\frac{27}{2}\left(\frac{\left(-\log(10)+\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}-\phi+2\right)$$

$$11+\frac{27}{2}\left(\frac{1}{2}\left(3-\sqrt{5}\right)-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}\right)$$

$$-\frac{1}{4\log^3\left(\frac{10}{9}\right)}\left(-81\log^2(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)+81\log(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^2-27\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^3-125\log^3\left(\frac{10}{9}\right)+27\sqrt{5}\,\log^3\left(\frac{10}{9}\right)+27\log^3(10)\right)$$

Alternative representations:

$$\frac{27}{2}\left(-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2-\phi\right)+11=$$

$$11+\frac{27}{2}\left(2-\phi-\frac{\left(\log_e(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log_e\left(\frac{10}{9}\right)}\right)\right)^3}{2\log_e^3\left(\frac{10}{9}\right)}\right)$$

$$\frac{27}{2}\left(-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2-\phi\right)+11=$$

$$11+\frac{27}{2}\left(2-\phi-\frac{\left(\log(a)\log_a(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log(a)\log_a\left(\frac{10}{9}\right)}\right)\right)^3}{2\left(\log(a)\log_a\left(\frac{10}{9}\right)\right)^3}\right)$$

$$\frac{27}{2}\left(-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2-\phi\right)+11=$$

$$11+\frac{27}{2}\left(2-\phi-\frac{\left(-\text{Li}_1(-9)-\psi_{\frac{9}{10}}^{(0)}\left(1--\frac{i\pi}{\text{Li}_1\left(1-\frac{10}{9}\right)}\right)\right)^3}{2\left(-\text{Li}_1\left(1-\frac{10}{9}\right)\right)^3}\right)$$

Series representations:

$$\frac{27}{2}\left(-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2-\phi\right)+11=$$

$$11+\frac{27}{2}\left(2-\phi-\left(2 i\pi\left\lfloor\frac{\arg(10-x)}{2\pi}\right\rfloor+\log(x)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{2 i\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k (10-x)^k x^{-k}}{k}\right)^3 \Big/ \left(2\left(2 i\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}\right)^3\right)\right) \quad \text{for } x<0$$

$$\frac{27}{2}\left(-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2-\phi\right)+11=$$

$$11+\frac{27}{2}\left(2-\phi-\left(\log(z_0)+\left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\psi^{(0)}_{\frac{9}{10}}\left(1-(i\pi)\Big/\left(\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k (10-z_0)^k z_0^{-k}}{k}\right)^3 \Big/ \left(2\left(\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)^3\right)\right)$$

$$\frac{27}{2}\left(-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2-\phi\right)+11=$$

$$11+\frac{27}{2}\left(2-\phi-\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(10-z_0)^k z_0^{-k}}{k}\right)^3\middle/\left(2\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)^3\right)\right)$$

Performing the 15th root, we obtain, from the above expression:

((((27*1/2*(((1/2((-(log(10) - QPolyGamma(0, 1 - (i π)/log(10/9), 9/10))^3/(log^3(10/9))))+2-golden ratio)))+11))))^1/15

Input:

$$\sqrt[15]{27\times\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)}\right)+2-\phi\right)+11}$$

$\log(x)$ is the natural logarithm

$\psi_q(z)$ gives the q-digamma function

i is the imaginary unit

ϕ is the golden ratio

Exact result:

$$\sqrt[15]{11+\frac{27}{2}\left(-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}-\phi+2\right)}$$

Decimal approximation:

1.643816435848841926428094398783167607786769365141734047364...

1.643816435... ≈ $\zeta(2) = \frac{\pi^2}{6}$ = 1.644934 ...

Alternate forms:

$$\frac{1}{2^{2/15}\sqrt[5]{\log\left(\frac{10}{9}\right)}}\left(\left(81\log^2(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)-81\log(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^2+27\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^3+125\log^3\left(\frac{10}{9}\right)-27\sqrt{5}\log^3\left(\frac{10}{9}\right)-27\log^3(10)\right)\hat{}\,(1/15)\right)$$

$$\left(-\frac{81\log(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^2}{4\log^3\left(\frac{10}{9}\right)}+\frac{27\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^3}{4\log^3\left(\frac{10}{9}\right)}+\frac{81\log^2(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{4\log^3\left(\frac{10}{9}\right)}+11-\frac{27\left(-3\log^3\left(\frac{10}{9}\right)+\sqrt{5}\log^3\left(\frac{10}{9}\right)+\log^3(10)\right)}{4\log^3\left(\frac{10}{9}\right)}\right)\hat{}\,(1/15)$$

$$\left(\left(81\log^2(10)\,\psi^{(0)}_{\frac{9}{10}}\left(\frac{-i\pi-2\log(3)+\log(10)}{-2\log(3)+\log(10)}\right)-81\log(10)\,\psi^{(0)}_{\frac{9}{10}}\left(\frac{-i\pi-2\log(3)+\log(10)}{-2\log(3)+\log(10)}\right)^2+27\,\psi^{(0)}_{\frac{9}{10}}\left(\frac{-i\pi-2\log(3)+\log(10)}{-2\log(3)+\log(10)}\right)^3-54\,\phi\,(\log(10)-2\log(3))^3+125\log^3(2)-1216\log^3(3)+125\log^3(5)-3\log^2(2)\,(304\log(3)-125\log(5))+1824\log^2(3)\log(5)-912\log(3)\log^2(5)+3\log(2)\,(608\log^2(3)+125\log^2(5)-608\log(3)\log(5))\right)\hat{}\,(1/15)\right)\Big/\left(2^{2/15}\sqrt[5]{\log(10)-2\log(3)}\right)$$

Expanded form:

$$\sqrt[15]{11+\frac{27}{2}\left(-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2+\frac{1}{2}\left(-1-\sqrt{5}\right)\right)}$$

and subtracting $(29-4+1/2)/10^3$, we obtain:

((((27*1/2*(((1/2((-(log(10) - QPolyGamma(0, 1 - (i π)/log(10/9), 9/10))^3/(log^3(10/9))))+2-golden ratio)))+11))))^1/15 - (29-4+1/2)*1/10^3

Input:

$$\sqrt[15]{27\times\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)}\right)+2-\phi\right)+11}-\left(29-4+\frac{1}{2}\right)\times\frac{1}{10^3}$$

$\log(x)$ is the natural logarithm

$\psi_q(z)$ gives the q-digamma function

i is the imaginary unit

ϕ is the golden ratio

Exact result:

$$-\frac{51}{2000}+\sqrt[15]{11+\frac{27}{2}\left(-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}-\phi+2\right)}$$

Decimal approximation:

1.618316435848841926428094398783167607786769365141734047364...

1.618316435.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

Alternate forms:

$$-\frac{51}{2000}+\frac{1}{2^{2/15}\sqrt[5]{\log\left(\frac{10}{9}\right)}}\left(\left(81\log^2(10)\,\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)-81\log(10)\,\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^2+27\,\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^3+125\log^3\left(\frac{10}{9}\right)-27\sqrt{5}\,\log^3\left(\frac{10}{9}\right)-27\log^3(10)\right)\hat{}\,(1/15)\right)$$

$$\frac{1}{2000\sqrt[5]{\log\left(\frac{10}{9}\right)}}\left(-51\sqrt[5]{\log\left(\frac{10}{9}\right)}+1000\times 2^{13/15}\left(81\log^2(10)\,\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)-81\log(10)\,\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^2+27\,\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^3+125\log^3\left(\frac{10}{9}\right)-27\sqrt{5}\,\log^3\left(\frac{10}{9}\right)-27\log^3(10)\right)\hat{}\,(1/15)\right)$$

$$-\frac{51}{2000}+\left(\left(81\log^2(10)\,\psi_{\frac{9}{10}}^{(0)}\left(\frac{-i\pi-2\log(3)+\log(10)}{-2\log(3)+\log(10)}\right)-81\log(10)\,\psi_{\frac{9}{10}}^{(0)}\left(\frac{-i\pi-2\log(3)+\log(10)}{-2\log(3)+\log(10)}\right)^2+27\,\psi_{\frac{9}{10}}^{(0)}\left(\frac{-i\pi-2\log(3)+\log(10)}{-2\log(3)+\log(10)}\right)^3-54\,\phi\,(\log(10)-2\log(3))^3+125\log^3(2)-1216\log^3(3)+125\log^3(5)-3\log^2(2)\,(304\log(3)-125\log(5))+1824\log^2(3)\log(5)-912\log(3)\log^2(5)+3\log(2)\,(608\log^2(3)+125\log^2(5)-608\log(3)\log(5))\right)\hat{}\,(1/15)\right)\Big/\left(2^{2/15}\sqrt[5]{\log(10)-2\log(3)}\right)$$

Expanded form:

$$-\frac{51}{2000}+\sqrt[15]{11+\frac{27}{2}\left(-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2+\frac{1}{2}\left(-1-\sqrt{5}\right)\right)}$$

Alternative representations:

$$\sqrt[15]{\frac{27}{2}\left(-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2-\phi\right)+11}-\frac{29-4+\frac{1}{2}}{10^3}=$$

$$-\frac{51}{2\times 10^3}+\sqrt[15]{11+\frac{27}{2}\left(2-\phi-\frac{\left(\log_e(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log_e\left(\frac{10}{9}\right)}\right)\right)^3}{2\log_e^3\left(\frac{10}{9}\right)}\right)}$$

$$\sqrt[15]{\frac{27}{2}\left(-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2-\phi\right)+11}-\frac{29-4+\frac{1}{2}}{10^3}=$$

$$-\frac{51}{2\times 10^3}+\sqrt[15]{11+\frac{27}{2}\left(2-\phi-\frac{\left(\log(a)\log_a(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log(a)\log_a\left(\frac{10}{9}\right)}\right)\right)^3}{2\left(\log(a)\log_a\left(\frac{10}{9}\right)\right)^3}\right)}$$

$$\sqrt[15]{\frac{27}{2}\left(-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2-\phi\right)+11}-\frac{29-4+\frac{1}{2}}{10^3}=$$

$$-\frac{51}{2\times 10^3}+\sqrt[15]{11+\frac{27}{2}\left(2-\phi-\frac{\left(-\mathrm{Li}_1(-9)-\psi_{\frac{9}{10}}^{(0)}\left(1--\frac{i\pi}{\mathrm{Li}_1\left(1-\frac{10}{9}\right)}\right)\right)^3}{2\left(-\mathrm{Li}_1\left(1-\frac{10}{9}\right)\right)^3}\right)}$$

Series representations:

$$\sqrt[15]{\frac{27}{2}\left(-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2-\phi\right)+11}-\frac{29-4+\frac{1}{2}}{10^3}=$$

$$-\frac{51}{2000}+\left(11+\frac{27}{2}\left(2+\frac{1}{2}\left(-1-\sqrt{5}\right)-\left(2i\pi\left\lfloor\frac{\arg(10-x)}{2\pi}\right\rfloor+\log(x)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{2i\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(10-x)^k x^{-k}}{k}\right)^3\Big/\left(2\left(2i\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}\right)^3\right)\right)\right)\text{^}(1/15)\quad\text{for } x<0$$

$$\sqrt[15]{\frac{27}{2}\left(-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2-\phi\right)+11}-\frac{29-4+\frac{1}{2}}{10^3}=$$

$$-\frac{51}{2000}+\Bigg(11+\frac{27}{2}\Bigg(2+\frac{1}{2}\left(-1-\sqrt{5}\right)-\Bigg(\log(z_0)+\left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\psi^{(0)}_{\frac{9}{10}}\Bigg(1-(i\pi)\Big/\Bigg(\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\Bigg)\Bigg)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(10-z_0)^k\, z_0^{-k}}{k}\Bigg)^3\Big/\Bigg(2\Bigg(\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\Bigg)^3\Bigg)\Bigg)\Bigg)\wedge(1/15)$$

$$\sqrt[15]{\frac{27}{2}\left(-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2-\phi\right)+11}-\frac{29-4+\frac{1}{2}}{10^3}=$$

$$-\frac{51}{2000}+\left(11+\frac{27}{2}\left(2+\frac{1}{2}\left(-1-\sqrt{5}\right)-\left(2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor+\log(z_0)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\,\pi}{2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(10-z_0)^k\,z_0^{-k}}{k}\right)^3\Big/\left(2\left(2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)^3\right)\right)\right)\hat{}\,(1/15)$$

From the previous expression:

$$\sqrt[15]{27\times\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\left(\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)}\right)+2-\phi\right)+11}$$

Adding (29-2+golden ratio)/10^3, we obtain:

1/10^27((((((((27*1/2*(((1/2((-(log(10) - QPolyGamma(0, 1 - (i π)/log(10/9), 9/10))^3/(log^3(10/9))))+2-golden ratio)))+11))))^1/15 + (29-2+golden ratio)*1/10^3))))

Input:

$$\frac{1}{10^{27}}\left(\sqrt[15]{27\times\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{\log^3\left(\frac{10}{9}\right)}\right)+2-\phi\right)+11}+(29-2+\phi)\times\frac{1}{10^3}\right)$$

$\log(x)$ is the natural logarithm

$\psi_q(z)$ gives the q-digamma function

i is the imaginary unit

ϕ is the golden ratio

Exact result:

$$\frac{\frac{\phi+27}{1000}+\sqrt[15]{11+\frac{27}{2}\left(-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}-\phi+2\right)}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

Decimal approximation:

$1.6724344698375918212762989856175332459044896743215398\ldots\times 10^{-27}$

1.6724344698...*10^{-27} result practically equal to the proton mass

Alternate forms:

$$\frac{\frac{55+\sqrt{5}}{2000}+\sqrt[15]{11+\frac{27}{2}\left(\frac{1}{2}\left(3-\sqrt{5}\right)-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}\right)}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{27+\frac{1}{2}\left(1+\sqrt{5}\right)}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}+\left(\left(\left(81\log^2(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)-81\log(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^2+27\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^3+125\log^3\left(\frac{10}{9}\right)-27\sqrt{5}\log^3\left(\frac{10}{9}\right)-27\log^3(10)\right)\text{^}(1/15)\right)\Big/\left(1\,000\,000\,000\,000\,000\,000\,000\,000\,000\times 2^{2/15}\sqrt[5]{\log\left(\frac{10}{9}\right)}\right)$$

$$\left(1000\times 2^{13/15}\left(81\log^2(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)-81\log(10)\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^2+27\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^3+125\log^3\left(\frac{10}{9}\right)-27\sqrt{5}\log^3\left(\frac{10}{9}\right)-27\log^3(10)\right)\hat{}\,(1/15)+55\sqrt[5]{\log\left(\frac{10}{9}\right)}+\sqrt{5}\sqrt[5]{\log\left(\frac{10}{9}\right)}\right)\Big/\left(2\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\sqrt[5]{\log\left(\frac{10}{9}\right)}\right)$$

Expanded form:

$$\frac{\sqrt[15]{11+\frac{27}{2}\left(-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2+\frac{1}{2}\left(-1-\sqrt{5}\right)\right)}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}+\frac{11}{400\,000\,000\,000\,000\,000\,000\,000\,000\,000}+\frac{1}{400\,000\,000\,000\,000\,000\,000\,000\,000\,000\sqrt{5}}$$

Alternative representations:

$$\frac{\sqrt[15]{\frac{27}{2}\left(-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2-\phi\right)+11}+\frac{29-2+\phi}{10^3}}{10^{27}}=$$

$$\frac{\frac{27+\phi}{10^3}+\sqrt[15]{11+\frac{27}{2}\left(2-\phi-\frac{\left(\log_e(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log_e\left(\frac{10}{9}\right)}\right)\right)^3}{2\log_e^3\left(\frac{10}{9}\right)}\right)}}{10^{27}}$$

$$\frac{\sqrt[15]{\frac{27}{2}\left(-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2-\phi\right)+11}+\frac{29-2+\phi}{10^3}}{10^{27}} =$$

$$\frac{\frac{27+\phi}{10^3}+\sqrt[15]{11+\frac{27}{2}\left(2-\phi-\frac{\left(\log(a)\log_a(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log(a)\log_a\left(\frac{10}{9}\right)}\right)\right)^3}{2\left(\log(a)\log_a\left(\frac{10}{9}\right)\right)^3}\right)}}{10^{27}}$$

$$\frac{\sqrt[15]{\frac{27}{2}\left(-\frac{\left(\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^3}{2\log^3\left(\frac{10}{9}\right)}+2-\phi\right)+11}+\frac{29-2+\phi}{10^3}}{10^{27}} =$$

$$\frac{\frac{27+\phi}{10^3}+\sqrt[15]{11+\frac{27}{2}\left(2-\phi-\frac{\left(-\text{Li}_1(-9)-\psi^{(0)}_{\frac{9}{10}}\left(1--\frac{i\pi}{\text{Li}_1\left(1-\frac{10}{9}\right)}\right)\right)^3}{2\left(-\text{Li}_1\left(1-\frac{10}{9}\right)\right)^3}\right)}}{10^{27}}$$

From the first expression, multiplying by 1/6 and adding $5/10^2$ and $5/10^4$, and again multiplying all the expression by $1/10^{52}$, we obtain:

1/10^52(((1/6(((-(log(10) - QPolyGamma(0, 1 - (i π)/log(10/9), 9/10))/log(10/9))))+5/10^2+5/10^4))))

Input:

$$\frac{1}{10^{52}}\left(\frac{1}{6}\left(-\frac{\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)}\right)+\frac{5}{10^2}+\frac{5}{10^4}\right)$$

log(x) is the natural logarithm

$\psi_q(z)$ gives the q-digamma function

i is the imaginary unit

Exact result:

$$\frac{\frac{101}{2000} + \frac{-\log(10) + \psi^{(0)}_{\frac{9}{10}}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{6\log\left(\frac{10}{9}\right)}}{10\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

Decimal approximation:

$1.1056681154774575896197311466396967748902100687607905\ldots \times 10^{-52}$

$1.1056681154\ldots*10^{-52}$ result practically equal to the value of Cosmological Constant $1.1056*10^{-52}$ m^{-2}

Alternate forms:

$$\frac{1000\,\psi^{(0)}_{\frac{9}{10}}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right) + 303\log\left(\frac{10}{9}\right) - 1000\log(10)}{60\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\log\left(\frac{10}{9}\right)}$$

$$\frac{\psi^{(0)}_{\frac{9}{10}}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{60\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\log\left(\frac{10}{9}\right)} + \frac{101}{20\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} - \frac{\log(10)}{60\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\log\left(\frac{10}{9}\right)}$$

$$\frac{303\log\left(\frac{10}{9}\right) - 1000\log(10)}{60\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\log\left(\frac{10}{9}\right)} + \frac{\psi^{(0)}_{\frac{9}{10}}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{60\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\log\left(\frac{10}{9}\right)}$$

Alternative representations:

$$\frac{-\frac{\log(10) - \psi^{(0)}_{\frac{9}{10}}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{6\log\left(\frac{10}{9}\right)} + \frac{5}{10^2} + \frac{5}{10^4}}{10^{52}} = \frac{\frac{5}{10^2} + \frac{5}{10^4} + \frac{-\log(a)\log_a(10) + \psi^{(0)}_{\frac{9}{10}}\left(1 - \frac{i\pi}{\log(a)\log_a\left(\frac{10}{9}\right)}\right)}{6\left(\log(a)\log_a\left(\frac{10}{9}\right)\right)}}{10^{52}}$$

$$\frac{-\frac{\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{6\log\left(\frac{10}{9}\right)}+\frac{5}{10^2}+\frac{5}{10^4}}{10^{52}} = \frac{\frac{5}{10^2}+\frac{5}{10^4}+\frac{-\log_e(10)+\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log_e\left(\frac{10}{9}\right)}\right)}{6\log_e\left(\frac{10}{9}\right)}}{10^{52}}$$

$$\frac{-\frac{\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{6\log\left(\frac{10}{9}\right)}+\frac{5}{10^2}+\frac{5}{10^4}}{10^{52}} = \frac{\frac{5}{10^2}+\frac{5}{10^4}+\frac{\mathrm{Li}_1(-9)+\psi^{(0)}_{\frac{9}{10}}\left(1--\frac{i\pi}{\mathrm{Li}_1\left(1-\frac{10}{9}\right)}\right)}{6\left(-\mathrm{Li}_1\left(1-\frac{10}{9}\right)\right)}}{10^{52}}$$

Series representations:

$$\frac{-\frac{\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{6\log\left(\frac{10}{9}\right)}+\frac{5}{10^2}+\frac{5}{10^4}}{10^{52}} =$$

$$-\left(\left(-606\,\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor+2000\,\pi\left\lfloor\frac{\arg(10-x)}{2\pi}\right\rfloor-697\,i\log(x)+\right.\right.$$

$$1000\,i\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{2\,i\,\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}}\right)-$$

$$\left.303\,i\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}+1000\,i\sum_{k=1}^{\infty}\frac{(-1)^k(10-x)^k x^{-k}}{k}\right)\Bigg/$$

$$\left(60\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\right.$$

$$\left.\left.000\left(2\pi\left\lfloor\frac{\arg\left(\frac{10}{9}-x\right)}{2\pi}\right\rfloor-i\log(x)+i\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-x\right)^k x^{-k}}{k}\right)\right)\right)$$

for $x < 0$

$$\frac{-\frac{\log(10)-\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{6\log\left(\frac{10}{9}\right)}+\frac{5}{10^2}+\frac{5}{10^4}}{10^{52}} =$$

$$-\left(\left(1394\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-697\,i\log(z_0)+1000\,i\right.\right.$$

$$\psi_{\frac{9}{10}}^{(0)}\left(1-\frac{i\pi}{2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\right)-$$

$$\left.303\,i\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}+1000\,i\sum_{k=1}^{\infty}\frac{(-1)^k\,(10-z_0)^k\,z_0^{-k}}{k}\right)\Bigg/$$

$$\left(60\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\right.$$

$$\left(2\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-\right.$$

$$\left.\left.\left.i\log(z_0)+i\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\right)\right)\right)$$

$$\frac{-\frac{\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{6\log\left(\frac{10}{9}\right)}+\frac{5}{10^2}+\frac{5}{10^4}}{10^{52}} =$$

$$-\Bigg(\Bigg(-303\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+1000\left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+697\log(z_0)-$$

$$303\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\log(z_0)+1000\left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\log(z_0)-1000\,\psi^{(0)}_{\frac{9}{10}}\Bigg($$

$$1-\frac{i\pi}{\log(z_0)+\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}}\Bigg)+$$

$$303\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}-1000\sum_{k=1}^{\infty}\frac{(-1)^k(10-z_0)^k z_0^{-k}}{k}\Bigg)\Bigg/$$

$$\Bigg(60\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000$$

$$\Bigg(\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+$$

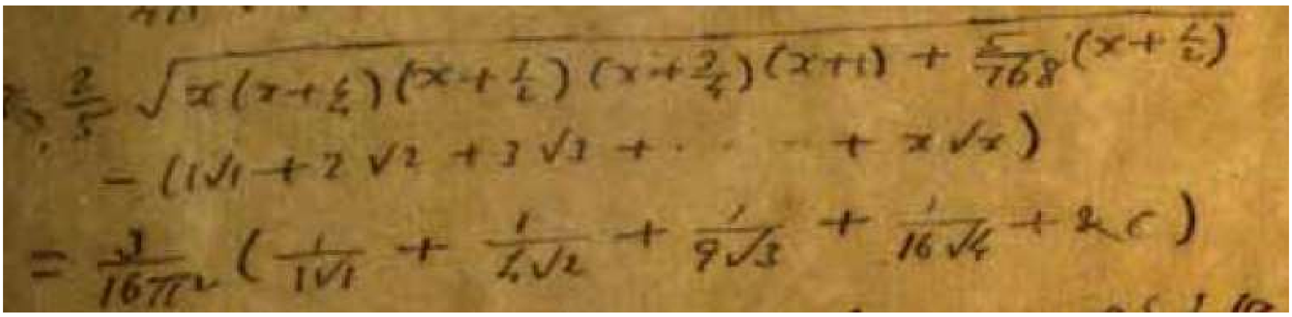
$$\left\lfloor\frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}\Bigg)\Bigg)\Bigg)$$

Integral representations:

$$\frac{-\frac{\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{6\log\left(\frac{10}{9}\right)}+\frac{5}{10^2}+\frac{5}{10^4}}{10^{52}} = \left(303\int_1^{\frac{10}{9}}\frac{1}{t}\,dt-1000\int_1^{10}\frac{1}{t}\,dt+1000\,\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\int_1^{\frac{10}{9}}\frac{1}{t}\,dt}\right)\right)\Big/\left(60\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\int_1^{\frac{10}{9}}\frac{1}{t}\,dt\right)$$

$$\frac{-\frac{\log(10)-\psi^{(0)}_{\frac{9}{10}}\left(1-\frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{6\log\left(\frac{10}{9}\right)}+\frac{5}{10^2}+\frac{5}{10^4}}{10^{52}} = -\left(\left(1000\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{9^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds-303\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{9^{s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds-2000\,i\pi\,\psi^{(0)}_{\frac{9}{10}}\left(1+\frac{2\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{9^{s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)\right)\Big/\left(60\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{9^{s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\right) \quad \text{for } -1<\gamma<0$$

Now, we have that:


$$\frac{2}{5}\sqrt{x\left(x+\tfrac{1}{4}\right)\left(x+\tfrac{1}{2}\right)\left(x+\tfrac{3}{4}\right)(x+1)}+\frac{5}{768}\left(x+\tfrac{1}{2}\right)$$
$$-\left(1\sqrt{1}+2\sqrt{2}+3\sqrt{3}+\cdots+x\sqrt{x}\right)$$
$$=\frac{3}{16\pi^2}\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\&c\right)$$

From the right-hand side, we obtain:

3/(16Pi^2) * (((1/(1sqrt1) + 1/(4sqrt2) + 1/(9sqrt3) + 1/(16sqrt4)+1/(25sqrt5) + 1/(36sqrt6)+ 1/(49sqrt7) + 1/(64sqrt8))))

Input:

$$\frac{3}{16\pi^2}\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)$$

Result:

$$\frac{3\left(\frac{33}{32}+\frac{33}{128\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}\right)}{16\pi^2}$$

Decimal approximation:

0.024975228456987917321331718174344522231879005710200746494...

0.0249752284…

Property:

$$\frac{3\left(\frac{33}{32}+\frac{33}{128\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}\right)}{16\pi^2}$$ is a transcendental number

Alternate forms:

$$\frac{1}{1580544000\pi^2}\left(305613000+38201625\sqrt{2}+10976000\sqrt{3}+2370816\sqrt{5}+1372000\sqrt{6}+864000\sqrt{7}\right)$$

$$\frac{3}{400\sqrt{5}\,\pi^2}+\frac{3}{784\sqrt{7}\,\pi^2}+\frac{(8+\sqrt{2})(891+32\sqrt{3})}{36864\pi^2}$$

$$\frac{\frac{99}{512}+\frac{99}{2048\sqrt{2}}+\frac{1}{48\sqrt{3}}+\frac{3}{400\sqrt{5}}+\frac{1}{192\sqrt{6}}+\frac{3}{784\sqrt{7}}}{\pi^2}$$

Series representations:

$$\frac{\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)3}{16\pi^2}=$$

$$\frac{3}{16\pi^2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1-z_0)^k z_0^{-k}}{k!}}+\frac{3}{64\pi^2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}+$$
$$\frac{1}{48\pi^2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}+\frac{3}{256\pi^2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(4-z_0)^k z_0^{-k}}{k!}}+$$
$$\frac{3}{400\pi^2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+\frac{1}{192\pi^2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6-z_0)^k z_0^{-k}}{k!}}+$$
$$\frac{3}{784\pi^2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k z_0^{-k}}{k!}}+\frac{3}{1024\pi^2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(8-z_0)^k z_0^{-k}}{k!}}$$

for not $((z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0))$

$$\frac{\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)3}{16\pi^2}=$$

$$\frac{3}{16\pi^2\exp\left(i\pi\left\lfloor\frac{\arg(1-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(1-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$
$$\frac{3}{64\pi^2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$
$$\frac{1}{48\pi^2\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$
$$\frac{3}{256\pi^2\exp\left(i\pi\left\lfloor\frac{\arg(4-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(4-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$
$$\frac{3}{400\pi^2\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$
$$\frac{1}{192\pi^2\exp\left(i\pi\left\lfloor\frac{\arg(6-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(6-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$
$$\frac{3}{784\pi^2\exp\left(i\pi\left\lfloor\frac{\arg(7-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(7-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$
$$\frac{3}{1024\pi^2\exp\left(i\pi\left\lfloor\frac{\arg(8-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(8-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

for $(x\in\mathbb{R}$ and $x<0)$

$$\frac{\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)3}{16\pi^2}=$$

$$\frac{3\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(1-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(1-z_0)/(2\pi)\rfloor)}}{16\pi^2\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1-z_0)^k z_0^{-k}}{k!}}+$$

$$\frac{3\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor)}}{64\pi^2\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}+$$

$$\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(3-z_0)/(2\pi)\rfloor)}}{48\pi^2\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}+$$

$$\frac{3\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(4-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(4-z_0)/(2\pi)\rfloor)}}{256\pi^2\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(4-z_0)^k z_0^{-k}}{k!}}+$$

$$\frac{3\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(5-z_0)/(2\pi)\rfloor)}}{400\pi^2\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+$$

$$\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(6-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(6-z_0)/(2\pi)\rfloor)}}{192\pi^2\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6-z_0)^k z_0^{-k}}{k!}}+$$

$$\frac{3\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(7-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(7-z_0)/(2\pi)\rfloor)}}{784\pi^2\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k z_0^{-k}}{k!}}+$$

$$\frac{3\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(8-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(8-z_0)/(2\pi)\rfloor)}}{1024\pi^2\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(8-z_0)^k z_0^{-k}}{k!}}$$

From which, performing the inversion of the formula, we obtain:

1/((((3/(16Pi^2) * (((1/(1sqrt1) + 1/(4sqrt2) + 1/(9sqrt3) + 1/(16sqrt4)+1/(25sqrt5) + 1/(36sqrt6)+ 1/(49sqrt7) + 1/(64sqrt8))))))))

Input:

$$\frac{1}{\frac{3}{16\pi^2}\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)}$$

Exact result:

$$\frac{16\pi^2}{3\left(\frac{33}{32}+\frac{33}{128\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}\right)}$$

Decimal approximation:

40.039673780049290001666516675083075901821930455844264811 80...

40.03967378…

Property:

$$\frac{16\pi^2}{3\left(\frac{33}{32}+\frac{33}{128\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}\right)}$$ is a transcendental number

Alternate forms:

$$(1580544000\pi^2)/(305613000+38201625\sqrt{2}+10976000\sqrt{3}+2370816\sqrt{5}+1372000\sqrt{6}+864000\sqrt{7})$$

$$\frac{1580544000\pi^2}{864000\sqrt{7}+343(6912\sqrt{5}+125(8+\sqrt{2})(891+32\sqrt{3}))}$$

$$\left(45158400\sqrt{35}\,\pi^2\right)/\left(1091475\sqrt{70}+4\sqrt{2}\left(39200\sqrt{210}+\sqrt{3}\left(14112\sqrt{42}+25\sqrt{5}\left(392\sqrt{7}+9\sqrt{6}\left(32+1617\sqrt{7}\right)\right)\right)\right)\right)$$

Series representations:

$$\frac{1}{\frac{\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)^3}{16\pi^2}} =$$

$$(16\pi^2)\Bigg/\left(3\left(\frac{1}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1-z_0)^k z_0^{-k}}{k!}}+\frac{1}{4\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}+\frac{1}{9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}+\frac{1}{16\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(4-z_0)^k z_0^{-k}}{k!}}+\frac{1}{25\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+\frac{1}{36\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6-z_0)^k z_0^{-k}}{k!}}+\frac{1}{49\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k z_0^{-k}}{k!}}+\frac{1}{64\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(8-z_0)^k z_0^{-k}}{k!}}\right)\right)$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{1}{\frac{\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)^3}{16\pi^2}} =$$

$$(16\pi^2)\Bigg/\left(3\left(\frac{1}{\exp\left(i\pi\left\lfloor\frac{\arg(1-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(1-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\right.\right.$$

$$\frac{1}{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$

$$\frac{1}{9\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$

$$\frac{1}{16\exp\left(i\pi\left\lfloor\frac{\arg(4-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(4-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$

$$\frac{1}{25\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$

$$\frac{1}{36\exp\left(i\pi\left\lfloor\frac{\arg(6-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(6-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$

$$\frac{1}{49\exp\left(i\pi\left\lfloor\frac{\arg(7-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(7-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$

$$\left.\left.\frac{1}{64\exp\left(i\pi\left\lfloor\frac{\arg(8-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(8-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{1}{\frac{\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)3}{16\pi^2}} =$$

$$(16\pi^2)\Bigg/\left(3\left(\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(1-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(1-z_0)/(2\pi)\rfloor)}}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor)}}{4\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(3-z_0)/(2\pi)\rfloor)}}{9\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(4-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(4-z_0)/(2\pi)\rfloor)}}{16\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(4-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(5-z_0)/(2\pi)\rfloor)}}{25\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(6-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(6-z_0)/(2\pi)\rfloor)}}{36\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(7-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(7-z_0)/(2\pi)\rfloor)}}{49\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(8-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(8-z_0)/(2\pi)\rfloor)}}{64\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(8-z_0)^k z_0^{-k}}{k!}}\right)\right)$$

and multiplying for π, and subtracting for the golden ratio conjugate, we obtain:

Pi * 1/((((3/(16Pi^2) * (((1/(1sqrt1) + 1/(4sqrt2) + 1/(9sqrt3) + 1/(16sqrt4)+1/(25sqrt5) + 1/(36sqrt6)+ 1/(49sqrt7) + 1/(64sqrt8))))))))-1/golden ratio

Input:

$$\pi\times\frac{1}{\frac{3}{16\pi^2}\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)}-\frac{1}{\phi}$$

ϕ is the golden ratio

Exact result:

$$\frac{16\pi^3}{3\left(\frac{33}{32}+\frac{33}{128\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}\right)}-\frac{1}{\phi}$$

Decimal approximation:

125.17031101078482146462036040087086730301968767007884050 88...

125.170311… result very near to the Higgs boson mass 125.18 GeV

Property:

$$-\frac{1}{\phi}+\frac{16\pi^3}{3\left(\frac{33}{32}+\frac{33}{128\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}\right)}$$ is a transcendental number

Alternate forms:

$$\frac{1580544000\pi^3}{864000\sqrt{7}+343(6912\sqrt{5}+125(8+\sqrt{2})(891+32\sqrt{3}))}-\frac{1}{\phi}$$

$$\left(45158400\sqrt{35}\pi^3\right)\Big/\left(1091475\sqrt{70}+4\sqrt{2}\left(39200\sqrt{210}+\sqrt{3}\left(14112\sqrt{42}+25\sqrt{5}\left(392\sqrt{7}+9\sqrt{6}\left(32+1617\sqrt{7}\right)\right)\right)\right)\right)-\frac{1}{\phi}$$

$$\left(2\left(-305613000-38201625\sqrt{2}-10976000\sqrt{3}-2370816\sqrt{5}-1372000\sqrt{6}-864000\sqrt{7}+790272000\pi^3+790272000\sqrt{5}\pi^3\right)\right)\Big/\left(\left(1+\sqrt{5}\right)\left(305613000+38201625\sqrt{2}+10976000\sqrt{3}+2370816\sqrt{5}+1372000\sqrt{6}+864000\sqrt{7}\right)\right)$$

Series representations:

$$\frac{\pi}{\frac{3\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)}{16\pi^2}}-\frac{1}{\phi}=-\frac{1}{\phi}+$$

$$(16\pi^3)\Bigg/\left(3\left(\frac{1}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1-z_0)^k z_0^{-k}}{k!}}+\frac{1}{4\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}+\frac{1}{9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}+\frac{1}{16\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(4-z_0)^k z_0^{-k}}{k!}}+\frac{1}{25\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+\frac{1}{36\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6-z_0)^k z_0^{-k}}{k!}}+\frac{1}{49\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k z_0^{-k}}{k!}}+\frac{1}{64\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(8-z_0)^k z_0^{-k}}{k!}}\right)\right)$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{\frac{\pi}{3\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)}}{16\pi^2}-\frac{1}{\phi}=$$

$$-\frac{1}{\phi}+(16\pi^3)\Bigg/\left(3\left(\frac{1}{\exp\left(i\pi\left\lfloor\frac{\arg(1-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(1-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{9\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{16\exp\left(i\pi\left\lfloor\frac{\arg(4-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(4-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{25\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{36\exp\left(i\pi\left\lfloor\frac{\arg(6-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(6-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{49\exp\left(i\pi\left\lfloor\frac{\arg(7-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(7-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{64\exp\left(i\pi\left\lfloor\frac{\arg(8-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(8-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\pi}{\frac{3\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)}{16\pi^2}} - \frac{1}{\phi} =$$

$$-\frac{1}{\phi} + (16\pi^3) \Big/ \left(3\left(\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(1-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(1-z_0)/(2\pi)\rfloor)}}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (1-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor)}}{4\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(3-z_0)/(2\pi)\rfloor)}}{9\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(4-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(4-z_0)/(2\pi)\rfloor)}}{16\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (4-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(5-z_0)/(2\pi)\rfloor)}}{25\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(6-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(6-z_0)/(2\pi)\rfloor)}}{36\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (6-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(7-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(7-z_0)/(2\pi)\rfloor)}}{49\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (7-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(8-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(8-z_0)/(2\pi)\rfloor)}}{64\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (8-z_0)^k z_0^{-k}}{k!}}\right)\right)$$

Multiplying for π and adding 13+1/golden ratio, we obtain:

Pi * 1/((((3/(16Pi^2) * (((1/(1sqrt1) + 1/(4sqrt2) + 1/(9sqrt3) + 1/(16sqrt4)+1/(25sqrt5) + 1/(36sqrt6)+ 1/(49sqrt7) + 1/(64sqrt8))))))))+13+1/golden ratio

Input:

$$\pi \times \frac{1}{\frac{3}{16\pi^2}\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)} + 13 + \frac{1}{\phi}$$

ϕ is the golden ratio

Exact result:

$$\frac{1}{\phi} + 13 + \frac{16\pi^3}{3\left(\frac{33}{32} + \frac{33}{128\sqrt{2}} + \frac{1}{9\sqrt{3}} + \frac{1}{25\sqrt{5}} + \frac{1}{36\sqrt{6}} + \frac{1}{49\sqrt{7}}\right)}$$

Decimal approximation:

139.4063789882846111610295340696021435384603060296903662330...

139.406378988… result practically equal to the rest mass of Pion meson 139.57 MeV

Property:

$$13 + \frac{1}{\phi} + \frac{16\pi^3}{3\left(\frac{33}{32} + \frac{33}{128\sqrt{2}} + \frac{1}{9\sqrt{3}} + \frac{1}{25\sqrt{5}} + \frac{1}{36\sqrt{6}} + \frac{1}{49\sqrt{7}}\right)}$$ is a transcendental number

Alternate forms:

$$\frac{1}{\phi} + 13 + \frac{1580544000\,\pi^3}{864000\sqrt{7} + 343\left(6912\sqrt{5} + 125\left(8+\sqrt{2}\right)\left(891 + 32\sqrt{3}\right)\right)}$$

$$\frac{1}{\phi} + 13 + \left(45158400\sqrt{35}\,\pi^3\right)\Big/\left(1091475\sqrt{70} + 4\sqrt{2}\left(39200\sqrt{210} + \sqrt{3}\left(14112\sqrt{42} + 25\sqrt{5}\left(392\sqrt{7} + 9\sqrt{6}\left(32 + 1617\sqrt{7}\right)\right)\right)\right)\right)$$

$$\left(5\left(947659608 + 114604875\sqrt{2} + 32928000\sqrt{3} + 801706248\sqrt{5} + 4116000\sqrt{6} + 2592000\sqrt{7} + 99324225\sqrt{10} + 28537600\sqrt{15} + 3567200\sqrt{30} + 2246400\sqrt{35} + 316108800\,\pi^3 + 316108800\sqrt{5}\,\pi^3\right)\right)\Big/\left(\left(1+\sqrt{5}\right)\left(305613000 + 38201625\sqrt{2} + 10976000\sqrt{3} + 2370816\sqrt{5} + 1372000\sqrt{6} + 864000\sqrt{7}\right)\right)$$

Series representations:

$$\frac{\pi}{\frac{3\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)}{16\pi^2}}+13+\frac{1}{\phi}=13+\frac{1}{\phi}+$$

$$(16\pi^3)\Big/\left(3\left(\frac{1}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1-z_0)^k z_0^{-k}}{k!}}+\frac{1}{4\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}+\frac{1}{9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}+\frac{1}{16\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(4-z_0)^k z_0^{-k}}{k!}}+\frac{1}{25\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+\frac{1}{36\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6-z_0)^k z_0^{-k}}{k!}}+\frac{1}{49\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k z_0^{-k}}{k!}}+\frac{1}{64\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(8-z_0)^k z_0^{-k}}{k!}}\right)\right)$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{\pi}{\frac{3\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)}{16\pi^2}}+13+\frac{1}{\phi}=$$

$$13+\frac{1}{\phi}+(16\pi^3)\Bigg/\left(3\left(\frac{1}{\exp\left(i\pi\left\lfloor\frac{\arg(1-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(1-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\right.\right.$$

$$\frac{1}{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$

$$\frac{1}{9\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$

$$\frac{1}{16\exp\left(i\pi\left\lfloor\frac{\arg(4-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(4-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$

$$\frac{1}{25\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$

$$\frac{1}{36\exp\left(i\pi\left\lfloor\frac{\arg(6-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(6-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$

$$\frac{1}{49\exp\left(i\pi\left\lfloor\frac{\arg(7-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(7-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$

$$\left.\left.\frac{1}{64\exp\left(i\pi\left\lfloor\frac{\arg(8-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(8-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\pi}{\frac{3\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)}{16\pi^2}}+13+\frac{1}{\phi}=$$

$$13+\frac{1}{\phi}+\left(16\pi^3\right)\Big/\left(3\left(\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(1-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(1-z_0)/(2\pi)\rfloor)}}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (1-z_0)^k z_0^{-k}}{k!}}+\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor)}}{4\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}+\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(3-z_0)/(2\pi)\rfloor)}}{9\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}}+\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(4-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(4-z_0)/(2\pi)\rfloor)}}{16\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (4-z_0)^k z_0^{-k}}{k!}}+\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(5-z_0)/(2\pi)\rfloor)}}{25\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}}+\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(6-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(6-z_0)/(2\pi)\rfloor)}}{36\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (6-z_0)^k z_0^{-k}}{k!}}+\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(7-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(7-z_0)/(2\pi)\rfloor)}}{49\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (7-z_0)^k z_0^{-k}}{k!}}+\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(8-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(8-z_0)/(2\pi)\rfloor)}}{64\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (8-z_0)^k z_0^{-k}}{k!}}\right)\right)$$

Multiplying the expression by 27*1/2, and adding 29+2, we obtain

29+2+27*1/2(((Pi* 1/((((3/(16Pi^2) * (((1/(1sqrt1) + 1/(4sqrt2) + 1/(9sqrt3) + 1/(16sqrt4)+1/(25sqrt5) + 1/(36sqrt6)+ 1/(49sqrt7) + 1/(64sqrt8)))))))))))

Input:

$$29+2+27\times\frac{1}{2}\left(\pi\times\frac{1}{\frac{3}{16\pi^2}\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)}\right)$$

Exact result:

$$31+\frac{72\pi^3}{\frac{33}{32}+\frac{33}{128\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}}$$

Decimal approximation:

1729.1426574937186702231367876756928231799899574734421455O7...

1729.14265749…

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Property:

$$31+\frac{72\pi^3}{\frac{33}{32}+\frac{33}{128\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}}$$ is a transcendental number

Alternate forms:

$$31+\frac{21337344000\pi^3}{864000\sqrt{7}+343\left(6912\sqrt{5}+125\left(8+\sqrt{2}\right)\left(891+32\sqrt{3}\right)\right)}$$

$$\left(9474003000+1184250375\sqrt{2}+340256000\sqrt{3}+73495296\sqrt{5}+42532000\sqrt{6}+26784000\sqrt{7}+21337344000\pi^3\right)/\left(305613000+38201625\sqrt{2}+10976000\sqrt{3}+2370816\sqrt{5}+1372000\sqrt{6}+864000\sqrt{7}\right)$$

$$31+\left(609638400\sqrt{35}\,\pi^3\right)/\left(1091475\sqrt{70}+4\sqrt{2}\left(39200\sqrt{210}+\sqrt{3}\left(14112\sqrt{42}+25\sqrt{5}\left(392\sqrt{7}+9\sqrt{6}\left(32+1617\sqrt{7}\right)\right)\right)\right)\right)$$

Series representations:

$$29+2+\frac{27\pi}{\frac{\left(3\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)\right)^2}{16\pi^2}} =$$

$$31+(72\pi^3)\Bigg/\left(\frac{1}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1-z_0)^k z_0^{-k}}{k!}}+\frac{1}{4\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}+\frac{1}{9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}+\frac{1}{16\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(4-z_0)^k z_0^{-k}}{k!}}+\frac{1}{25\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+\frac{1}{36\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6-z_0)^k z_0^{-k}}{k!}}+\frac{1}{49\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k z_0^{-k}}{k!}}+\frac{1}{64\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(8-z_0)^k z_0^{-k}}{k!}}\right)$$

for not $((z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0))$

$$29+2+\frac{27\pi}{\frac{\left(3\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)\right)^2}{16\pi^2}} =$$

$$31+(72\pi^3)\Bigg/\left(\frac{1}{\exp\left(i\pi\left\lfloor\frac{\arg(1-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(1-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{9\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{16\exp\left(i\pi\left\lfloor\frac{\arg(4-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(4-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{25\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{36\exp\left(i\pi\left\lfloor\frac{\arg(6-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(6-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{49\exp\left(i\pi\left\lfloor\frac{\arg(7-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(7-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{64\exp\left(i\pi\left\lfloor\frac{\arg(8-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(8-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$29+2+\frac{27\pi}{\frac{\left(3\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)\right)^2}{16\pi^2}} =$$

$$31+(72\pi^3)\Bigg/\left(\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(1-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(1-z_0)/(2\pi)\rfloor)}}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1-z_0)^k z_0^{-k}}{k!}}+\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor)}}{4\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}+\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(3-z_0)/(2\pi)\rfloor)}}{9\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}+\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(4-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(4-z_0)/(2\pi)\rfloor)}}{16\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(4-z_0)^k z_0^{-k}}{k!}}+\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(5-z_0)/(2\pi)\rfloor)}}{25\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(6-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(6-z_0)/(2\pi)\rfloor)}}{36\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6-z_0)^k z_0^{-k}}{k!}}+\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(7-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(7-z_0)/(2\pi)\rfloor)}}{49\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k z_0^{-k}}{k!}}+\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(8-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(8-z_0)/(2\pi)\rfloor)}}{64\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(8-z_0)^k z_0^{-k}}{k!}}\right)$$

Multiplying the expression by 27*1/2, and adding 29+47+11 we obtain:

29+47+11+27*1/2(((Pi* 1/((((3/(16Pi^2) * (((1/(1sqrt1) + 1/(4sqrt2) + 1/(9sqrt3) + 1/(16sqrt4)+1/(25sqrt5) + 1/(36sqrt6)+ 1/(49sqrt7) + 1/(64sqrt8))))))))))

Input:

$$29+47+11+27\times\frac{1}{2}\left(\pi\times\frac{1}{\frac{3}{16\pi^2}\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)}\right)$$

Exact result:

$$87+\frac{72\pi^3}{\frac{33}{32}+\frac{33}{128\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}}$$

Decimal approximation:

1785.142657493718670223136787675692823179989957473442145507...

1785.14265749… result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

Property:

$$87+\frac{72\pi^3}{\frac{33}{32}+\frac{33}{128\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}}$$ is a transcendental number

Alternate forms:

$$87+\frac{21337344000\pi^3}{864000\sqrt{7}+343(6912\sqrt{5}+125(8+\sqrt{2})(891+32\sqrt{3}))}$$

$$\left(3\left(8862777000+1107847125\sqrt{2}+318304000\sqrt{3}+68753664\sqrt{5}+39788000\sqrt{6}+25056000\sqrt{7}+7112448000\pi^3\right)\right)/\left(305613000+38201625\sqrt{2}+10976000\sqrt{3}+2370816\sqrt{5}+1372000\sqrt{6}+864000\sqrt{7}\right)$$

$$87+\left(609638400\sqrt{35}\,\pi^3\right)/\left(1091475\sqrt{70}+4\sqrt{2}\left(39200\sqrt{210}+\sqrt{3}\left(14112\sqrt{42}+25\sqrt{5}\left(392\sqrt{7}+9\sqrt{6}\left(32+1617\sqrt{7}\right)\right)\right)\right)\right)$$

Series representations:

$$29+47+11+\frac{27\pi}{\frac{\left(3\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)\right)^2}{16\pi^2}}=$$

$$87+(72\pi^3)\Bigg/\left(\frac{1}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1-z_0)^k z_0^{-k}}{k!}}+\frac{1}{4\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}+\frac{1}{9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}+\frac{1}{16\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(4-z_0)^k z_0^{-k}}{k!}}+\frac{1}{25\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+\frac{1}{36\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6-z_0)^k z_0^{-k}}{k!}}+\frac{1}{49\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k z_0^{-k}}{k!}}+\frac{1}{64\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(8-z_0)^k z_0^{-k}}{k!}}\right)$$

for not $((z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0))$

$$29+47+11+\frac{27\pi}{\frac{\left(3\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)\right)^2}{16\pi^2}} =$$

$$87+(72\pi^3)\Bigg/\left(\frac{1}{\exp\left(i\pi\left\lfloor\frac{\arg(1-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(1-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{9\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{16\exp\left(i\pi\left\lfloor\frac{\arg(4-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(4-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{25\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{36\exp\left(i\pi\left\lfloor\frac{\arg(6-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(6-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{49\exp\left(i\pi\left\lfloor\frac{\arg(7-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(7-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\frac{1}{64\exp\left(i\pi\left\lfloor\frac{\arg(8-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(8-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$29 + 47 + 11 + \frac{27\pi}{\frac{\left(3\left(\frac{1}{1\sqrt{1}} + \frac{1}{4\sqrt{2}} + \frac{1}{9\sqrt{3}} + \frac{1}{16\sqrt{4}} + \frac{1}{25\sqrt{5}} + \frac{1}{36\sqrt{6}} + \frac{1}{49\sqrt{7}} + \frac{1}{64\sqrt{8}}\right)\right)^2}{16\pi^2}} =$$

$$87 + (72\pi^3) \Big/ \left(\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(1-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(1-z_0)/(2\pi)\rfloor)}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor)}}{4\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(3-z_0)/(2\pi)\rfloor)}}{9\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(4-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(4-z_0)/(2\pi)\rfloor)}}{16\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (4-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(5-z_0)/(2\pi)\rfloor)}}{25\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(6-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(6-z_0)/(2\pi)\rfloor)}}{36\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (6-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(7-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(7-z_0)/(2\pi)\rfloor)}}{49\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (7-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(8-z_0)/(2\pi)\rfloor} z_0^{1/2(-1-\lfloor\arg(8-z_0)/(2\pi)\rfloor)}}{64\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (8-z_0)^k z_0^{-k}}{k!}} \right)$$

Performing the 8^{th} root of the first expression, adding 1, $(34+8)/10^3$ and multiplying all by $1/10^{27}$, we obtain:

1/10^27(((1+((((3/(16Pi^2) * (((1/(1sqrt1) + 1/(4sqrt2) + 1/(9sqrt3) + 1/(16sqrt4)+1/(25sqrt5) + 1/(36sqrt6)+ 1/(49sqrt7) + 1/(64sqrt8))))))))^1/8+(34+8)*1/10^3)))

Input:

$$\frac{1}{10^{27}}\left(1+\left(\frac{3}{16\pi^2}\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)\right)\text{^}(1/8)+(34+8)\times\frac{1}{10^3}\right)$$

Exact result:

$$\frac{\frac{521}{500}+\frac{\sqrt[8]{3\left(\frac{33}{32}+\frac{33}{128\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}\right)}}{\sqrt{2}\sqrt[4]{\pi}}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

Decimal approximation:

$1.6725052159553696438702675233907258281335553068508382\ldots\times 10^{-27}$

$1.6725052159\ldots*10^{-27}$ result practically equal to the proton mass

Property:

$$\frac{\frac{521}{500}+\frac{\sqrt[8]{3\left(\frac{33}{32}+\frac{33}{128\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}\right)}}{\sqrt{2}\sqrt[4]{\pi}}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{521}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000}+\left(\left(305\,613\,000+38\,201\,625\sqrt{2}+10\,976\,000\sqrt{3}+2\,370\,816\sqrt{5}+1\,372\,000\sqrt{6}+864\,000\sqrt{7}\right)\text{^}(1/8)\right)\Big/\left(2\,000\,000\,000\,000\,000\,000\,000\,000\,000\sqrt{2}\;35^{3/8}\sqrt[4]{3\pi}\right)$$

$$\frac{\frac{521}{500}+\frac{\sqrt[8]{\frac{99}{32}+\frac{99}{128\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{3}{25\sqrt{5}}+\frac{1}{12\sqrt{6}}+\frac{3}{49\sqrt{7}}}}{\sqrt{2}\sqrt[4]{\pi}}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{521}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000}+\frac{\sqrt[8]{3\left(\frac{33}{32}+\frac{33}{128\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}\right)}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\sqrt{2}\sqrt[4]{\pi}}$$

Series representations:

$$\frac{1+\sqrt[8]{\frac{3\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)}{16\pi^2}}+\frac{34+8}{10^3}}{10^{27}} =$$

$$\frac{521}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \left(\sqrt[8]{3}\left(\frac{1}{\pi^2}\left(\frac{1}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1-z_0)^k z_0^{-k}}{k!}} + \frac{1}{4\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}} + \frac{1}{9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}} + \frac{1}{16\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(4-z_0)^k z_0^{-k}}{k!}} + \frac{1}{25\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}} + \frac{1}{36\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6-z_0)^k z_0^{-k}}{k!}} + \frac{1}{49\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k z_0^{-k}}{k!}} + \frac{1}{64\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(8-z_0)^k z_0^{-k}}{k!}}\right)\right)^{\wedge}(1/8)\right) \Big/ \left(1\,000\,000\,000\,000\,000\,000\,000\,000\,000\sqrt{2}\right)$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{1+\sqrt[8]{\frac{3\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)}{16\pi^2}}+\frac{34+8}{10^3}}{10^{27}} =$$

$$\frac{521}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \left(\sqrt[8]{3}\left(\frac{1}{\pi^2}\left(\frac{1}{\exp\left(i\pi\left\lfloor\frac{\arg(1-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(1-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}} + \frac{1}{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}} + \frac{1}{9\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}} + \frac{1}{16\exp\left(i\pi\left\lfloor\frac{\arg(4-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(4-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}} + \frac{1}{25\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}} + \frac{1}{36\exp\left(i\pi\left\lfloor\frac{\arg(6-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(6-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}} + \frac{1}{49\exp\left(i\pi\left\lfloor\frac{\arg(7-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(7-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}} + \frac{1}{64\exp\left(i\pi\left\lfloor\frac{\arg(8-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(8-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)\right)^{\wedge}(1/8)\right) \Big/ \left(1\,000\,000\,000\,000\,000\,000\,000\,000\,000\sqrt{2}\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{1+\sqrt[8]{\frac{3\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)}{16\pi^2}}+\frac{34+8}{10^3}}{10^{27}} =$$

$$\frac{521}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000} +$$

$$\left(\sqrt[8]{3}\left(\frac{1}{\pi^2}\left(\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(1-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(1-z_0)/(2\pi)\rfloor}}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}}{4\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}}{9\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(4-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(4-z_0)/(2\pi)\rfloor}}{16\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(4-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}}{25\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(6-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(6-z_0)/(2\pi)\rfloor}}{36\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(7-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(7-z_0)/(2\pi)\rfloor}}{49\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k z_0^{-k}}{k!}} + \frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(8-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(8-z_0)/(2\pi)\rfloor}}{64\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(8-z_0)^k z_0^{-k}}{k!}}\right)\right)^{\wedge}(1/8)\right) \Big/$$

$$\left(1\,000\,000\,000\,000\,000\,000\,000\,000\,000\sqrt{2}\right)$$

Performing the 8th root of the first expression, adding 1 and subtracting (21+3)/2*1/10^3, we obtain:

(((1+((((3/(16Pi^2) * (((1/(1sqrt1) + 1/(4sqrt2) + 1/(9sqrt3) + 1/(16sqrt4)+1/(25sqrt5) + 1/(36sqrt6)+ 1/(49sqrt7) + 1/(64sqrt8)))))))^1/8-((21+3)/2)*1/10^3)))

Input:

$$1+\sqrt[8]{\frac{3}{16\pi^2}\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)}-\frac{21+3}{2}\times\frac{1}{10^3}$$

Exact result:

$$\frac{247}{250}+\frac{\sqrt[8]{3\left(\frac{33}{32}+\frac{33}{128\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}\right)}}{\sqrt{2}\sqrt[4]{\pi}}$$

Decimal approximation:

1.618505215955369643870267523390725828133555306850838234565...

1.618505215955… result that is a very good approximation to the value of the golden ratio 1,618033988749...

Property:

$$\frac{247}{250}+\frac{\sqrt[8]{3\left(\frac{33}{32}+\frac{33}{128\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}\right)}}{\sqrt{2}\sqrt[4]{\pi}}$$ is a transcendental number

Alternate forms:

$$\frac{247}{250}+\frac{\sqrt[8]{\frac{99}{32}+\frac{99}{128\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{3}{25\sqrt{5}}+\frac{1}{12\sqrt{6}}+\frac{3}{49\sqrt{7}}}}{\sqrt{2}\sqrt[4]{\pi}}$$

$$\frac{247}{250}+\frac{1}{2\sqrt{2}\,35^{3/8}\sqrt[4]{3\pi}}\left(\left(305613000+38201625\sqrt{2}+10976000\sqrt{3}+2370816\sqrt{5}+1372000\sqrt{6}+864000\sqrt{7}\right)^{\wedge}(1/8)\right)$$

$$\frac{1}{10500\sqrt[4]{\pi}}\left(25\sqrt{2}\; 3^{3/4}\times 35^{5/8}\left(305613000+38201625\sqrt{2}+10976000\sqrt{3}+2370816\sqrt{5}+1372000\sqrt{6}+864000\sqrt{7}\right)\hat{}\,(1/8)+10374\sqrt[4]{\pi}\right)$$

Series representations:

$$1+\sqrt[8]{\frac{\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)3}{16\pi^2}}-\frac{21+3}{10^3\times 2}=$$

$$\frac{247}{250}+\frac{1}{\sqrt{2}}\sqrt[8]{3}\left(\frac{1}{\pi^2}\left(\frac{1}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1-z_0)^k z_0^{-k}}{k!}}+\frac{1}{4\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}+\frac{1}{9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}+\frac{1}{16\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(4-z_0)^k z_0^{-k}}{k!}}+\frac{1}{25\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+\frac{1}{36\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6-z_0)^k z_0^{-k}}{k!}}+\frac{1}{49\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k z_0^{-k}}{k!}}+\frac{1}{64\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(8-z_0)^k z_0^{-k}}{k!}}\right)\right)\hat{}\,(1/8)$$

for not $((z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0))$

$$1+\sqrt[8]{\frac{\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)3}{16\pi^2}}-\frac{21+3}{10^3\times 2}=$$

$$\frac{247}{250}+\frac{1}{\sqrt{2}}\sqrt[8]{3}\left(\frac{1}{\pi^2}\left(\frac{1}{\exp\left(i\pi\left\lfloor\frac{\arg(1-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(1-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\right.\right.$$
$$\frac{1}{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$
$$\frac{1}{9\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$
$$\frac{1}{16\exp\left(i\pi\left\lfloor\frac{\arg(4-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(4-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$
$$\frac{1}{25\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$
$$\frac{1}{36\exp\left(i\pi\left\lfloor\frac{\arg(6-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(6-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$
$$\frac{1}{49\exp\left(i\pi\left\lfloor\frac{\arg(7-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(7-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+$$
$$\left.\left.\frac{1}{64\exp\left(i\pi\left\lfloor\frac{\arg(8-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(8-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)\right)^{\wedge}$$
$$(1/8) \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$1+\sqrt[8]{\frac{\left(\frac{1}{1\sqrt{1}}+\frac{1}{4\sqrt{2}}+\frac{1}{9\sqrt{3}}+\frac{1}{16\sqrt{4}}+\frac{1}{25\sqrt{5}}+\frac{1}{36\sqrt{6}}+\frac{1}{49\sqrt{7}}+\frac{1}{64\sqrt{8}}\right)3}{16\pi^2}}-\frac{21+3}{10^3\times 2}=$$

$$\frac{247}{250}+\frac{1}{\sqrt{2}}\sqrt[8]{3}\left(\frac{1}{\pi^2}\left(\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(1-z_0)/(2\pi)\rfloor}z_0^{-1/2-1/2\lfloor\arg(1-z_0)/(2\pi)\rfloor}}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(1-z_0)^k z_0^{-k}}{k!}}+\right.\right.$$

$$\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}}{4\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}+$$

$$\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}z_0^{-1/2-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}}{9\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}+$$

$$\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(4-z_0)/(2\pi)\rfloor}z_0^{-1/2-1/2\lfloor\arg(4-z_0)/(2\pi)\rfloor}}{16\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(4-z_0)^k z_0^{-k}}{k!}}+$$

$$\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}z_0^{-1/2-1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}}{25\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}+$$

$$\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(6-z_0)/(2\pi)\rfloor}z_0^{-1/2-1/2\lfloor\arg(6-z_0)/(2\pi)\rfloor}}{36\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6-z_0)^k z_0^{-k}}{k!}}+$$

$$\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(7-z_0)/(2\pi)\rfloor}z_0^{-1/2-1/2\lfloor\arg(7-z_0)/(2\pi)\rfloor}}{49\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k z_0^{-k}}{k!}}+$$

$$\left.\left.\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(8-z_0)/(2\pi)\rfloor}z_0^{-1/2-1/2\lfloor\arg(8-z_0)/(2\pi)\rfloor}}{64\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(8-z_0)^k z_0^{-k}}{k!}}\right)\right)^\wedge(1/8)$$

Conclusions

We highlight as in the development of this equation we have always utilized the constants π, ϕ, $1/\phi$, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role to obtain the final results of the analyzed expression.

Furthermore, the Fibonacci and Lucas numbers are fundamental values that can be considered "constants", such as π and the golden ratio , that is, recurring numbers in various contexts: in the spiral arms of galaxies, as well as in Nature in general. This means that in the universe there is a mathematical order that has such constants as its foundation. Mathematics is therefore language, that is, as it was defined by philosophers, the "Logos" of the universe and all its laws that govern it. In other words, the universe, in addition to an observable physical reality, is also a mathematical and geometric entity.

References