Analyzing a Ramanujan equation: mathematical connections with various parameters of Particle Physics and Cosmology

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Abstract

The purpose of this paper is to show how using certain mathematical values and / or constants from a Ramanujan equation, some important parameters of Particle Physics and Cosmology are obtained.

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https://apod.nasa.gov/apod/ap170510.html

https://wssrmnn.net/index.php/2017/01/23/man-saw-number-pi-dreams/

From: **Manuscript Book 2 of Srinivasa Ramanujan**

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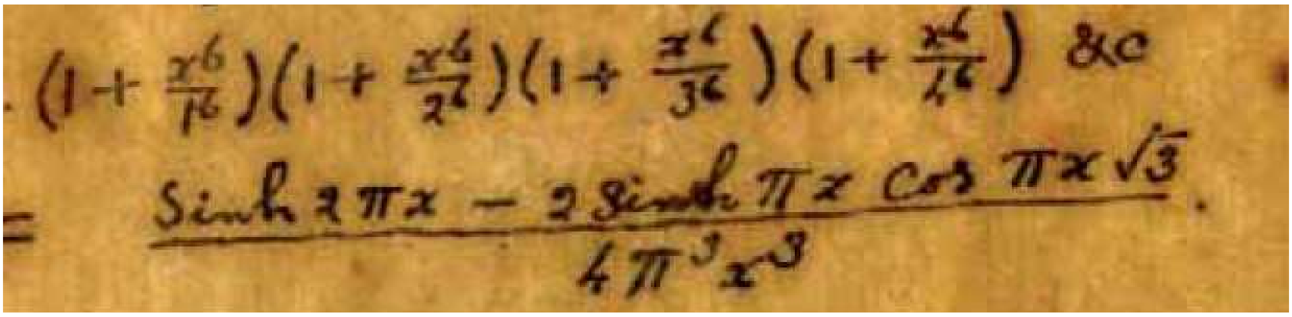

$$\left(1+\frac{x^6}{1^6}\right)\left(1+\frac{x^6}{2^6}\right)\left(1+\frac{x^6}{3^6}\right)\left(1+\frac{x^6}{4^6}\right) \text{ \&c} = \frac{\sinh 2\pi x - 2\sinh \pi x \cos \pi x\sqrt{3}}{4\pi^3 x^3}.$$

(((sinh (4Pi) – 2sinh (2Pi) cos (2Pi*sqrt3))) / ((4Pi^3*2^3))

For x = 2, we obtain, from the left hand-side:

(1+((2)^6)/(1^6))(1+((2)^6)/(2^6))(1+((2)^6)/(3^6))(1+((2)^6)/(4^6))...

Input:

$$\left(1+\frac{2^6}{1^6}\right)\left(1+\frac{2^6}{2^6}\right)\left(1+\frac{2^6}{3^6}\right)\left(1+\frac{2^6}{4^6}\right)$$

Exact result:

$$\frac{3350425}{23328}$$

Decimal approximation:

143.62247085048010973936899862825788751714677640603566652949...

143.62247085…

Always for x = 2, from the right-hand side, we obtain:

(((sinh (4Pi) – 2sinh (2Pi) cos (2Pi*sqrt3)))) / ((4Pi^3*2^3))

Input:

$$\frac{\sinh(4\pi) - (2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 \times 2^3}$$

sinh(x) is the hyperbolic sine function

Exact result:

$$\frac{\sinh(4\pi) - 2\cos\left(2\sqrt{3}\,\pi\right)\sinh(2\pi)}{32\pi^3}$$

Decimal approximation:

144.5633911784022539527052223657635096864423475588917203422...

144.56339117...

Alternate forms:

$$\frac{\sinh(2\pi)(\cosh(2\pi)-\cos(2\sqrt{3}\pi))}{16\pi^3}$$

$$-\frac{2\cos(2\sqrt{3}\pi)\sinh(2\pi)-\sinh(4\pi)}{32\pi^3}$$

$$\frac{\sinh(4\pi)}{32\pi^3}-\frac{\cos(2\sqrt{3}\pi)\sinh(2\pi)}{16\pi^3}$$

cosh(x) is the hyperbolic cosine function

Alternative representations:

$$\frac{\sinh(4\pi)-\cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}=\frac{-\cosh(-2i\pi\sqrt{3})(-e^{-2\pi}+e^{2\pi})+\frac{1}{2}(-e^{-4\pi}+e^{4\pi})}{32\pi^3}$$

$$\frac{\sinh(4\pi)-\cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}=\frac{-\cosh(2i\pi\sqrt{3})(-e^{-2\pi}+e^{2\pi})+\frac{1}{2}(-e^{-4\pi}+e^{4\pi})}{32\pi^3}$$

$$\frac{\sinh(4\pi)-\cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}=\frac{-2i\cosh(-2i\pi\sqrt{3})\cos\left(\frac{\pi}{2}+2i\pi\right)+i\cos\left(\frac{\pi}{2}+4i\pi\right)}{32\pi^3}$$

Series representations:

$$\frac{\sinh(4\pi)-\cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}=-\frac{-\sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!}+2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}(2\pi)^{1+2k_1+2k_2}}{(2k_1)!(1+2k_2)!}}{32\pi^3}$$

$$\frac{\sinh(4\pi)-\cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}=-\frac{i\left(-\sum_{k=0}^{\infty}\frac{\left(\left(4-\frac{i}{2}\right)\pi\right)^{2k}}{(2k)!}+2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}2^{2k_1-2k_2}(4-i)^{2k_2}\pi^{2k_1+2k_2}}{(2k_1)!(2k_2)!}\right)}{32\pi^3}$$

$$\frac{\sinh(4\pi)-\cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}=\sum_{k=0}^{\infty}\frac{2^{-3+2k}\pi^{-2+2k}\left(4^k-\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(1+2k)!}$$

Integral representations:

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\, 2\sinh(2\pi)}{4\pi^3\, 2^3} = \frac{\int_0^1 \cosh(4\pi t)\,dt + \int_0^1 \int_0^1 \cos\left(\frac{1}{2}\left(1 - 4\sqrt{3}\right)\pi t_2\right) \cosh(2\pi t_1)\,dt_2\,dt_1}{8\pi^2}$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\, 2\sinh(2\pi)}{4\pi^3\, 2^3} = \int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{i\, e^{\pi^2/s+s}\left(e^{(3\pi^2)/s} + \int_{\frac{\pi}{2}}^{2\sqrt{3}\pi} \sin(t)\,dt\right)}{32\pi^{5/2}\, s^{3/2}}\,ds \quad \text{for } \gamma > 0$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\, 2\sinh(2\pi)}{4\pi^3\, 2^3} = \int_0^1 \left(\frac{\cosh(4\pi t)}{8\pi^2} + \frac{i\cosh(2\pi t)}{16\pi^{5/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-(3\pi^2)/s+s}}{\sqrt{s}}\,ds\right)dt \quad \text{for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\, 2\sinh(2\pi)}{4\pi^3\, 2^3} = \frac{-4\left(-1 + 2\cos^2\left(\sqrt{3}\,\pi\right)\right)\cosh(\pi)\sinh(\pi) + 2\cosh(2\pi)\sinh(2\pi)}{32\pi^3}$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\, 2\sinh(2\pi)}{4\pi^3\, 2^3} = \frac{-4\cosh(\pi)\left(1 - 2\sin^2\left(\sqrt{3}\,\pi\right)\right)\sinh(\pi) + 2\cosh(2\pi)\sinh(2\pi)}{32\pi^3}$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\, 2\sinh(2\pi)}{4\pi^3\, 2^3} = \frac{-2\left(-1 + 2\cos^2\left(\sqrt{3}\,\pi\right)\right)\left(3\sinh\left(\frac{2\pi}{3}\right) + 4\sinh^3\left(\frac{2\pi}{3}\right)\right) + 3\sinh\left(\frac{4\pi}{3}\right) + 4\sinh^3\left(\frac{4\pi}{3}\right)}{32\pi^3}$$

Subtracting 5, that is a Fibonacci number, we obtain:

((((((sinh (4Pi) – 2sinh (2Pi) cos (2Pi*sqrt3)))) / ((4Pi^3*2^3))))) – 5

Input:

$$\frac{\sinh(4\pi) - (2\sinh(2\pi))\cos(2\pi\sqrt{3})}{4\pi^3 \times 2^3} - 5$$

sinh(x) is the hyperbolic sine function

Exact result:

$$\frac{\sinh(4\pi) - 2\cos(2\sqrt{3}\ \pi)\sinh(2\pi)}{32\pi^3} - 5$$

Decimal approximation:

139.5633911784022539527052223657635096864423475588917203422...

139.56339117… result practically equal to the rest mass of Pion meson 139.57 MeV

Alternate forms:

$$\frac{\sinh(2\pi)(\cosh(2\pi) - \cos(2\sqrt{3}\ \pi))}{16\pi^3} - 5$$

$$-5 + \frac{\sinh(4\pi)}{32\pi^3} - \frac{\cos(2\sqrt{3}\ \pi)\sinh(2\pi)}{16\pi^3}$$

$$-\frac{160\pi^3 - \sinh(4\pi) + 2\cos(2\sqrt{3}\ \pi)\sinh(2\pi)}{32\pi^3}$$

cosh(x) is the hyperbolic cosine function

Alternative representations:

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3} - 5 = -5 + \frac{-\cosh(-2i\pi\sqrt{3})(-e^{-2\pi} + e^{2\pi}) + \frac{1}{2}(-e^{-4\pi} + e^{4\pi})}{32\pi^3}$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3} - 5 = -5 + \frac{-\cosh(2i\pi\sqrt{3})(-e^{-2\pi} + e^{2\pi}) + \frac{1}{2}(-e^{-4\pi} + e^{4\pi})}{32\pi^3}$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3} - 5 = -5 + \frac{-2i\cosh(-2i\pi\sqrt{3})\cos\left(\frac{\pi}{2} + 2i\pi\right) + i\cos\left(\frac{\pi}{2} + 4i\pi\right)}{32\pi^3}$$

Series representations:

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3} - 5 = -\frac{160\pi^3 - \sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!} + 2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}(2\pi)^{1+2k_1+2k_2}}{(2k_1)!(1+2k_2)!}}{32\pi^3}$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3} - 5 = -\frac{160\pi^3 - i\sum_{k=0}^{\infty}\frac{\left(\left(4-\frac{i}{2}\right)\pi\right)^{2k}}{(2k)!} + 2i\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}\,2^{2k_1-2k_2}(4-i)^{2k_2}\pi^{2k_1+2k_2}}{(2k_1)!(2k_2)!}}{32\pi^3}$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3} - 5 = -5 + \sum_{k=0}^{\infty}\frac{2^{-3+2k}\pi^{-2+2k}\left(4^k - \sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(1+2k)!}$$

Integral representations:

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3} - 5 = -\frac{40\pi^2 - \int_0^1\cosh(4\pi t)\,dt + \int_0^1\int_0^1\cos\left(\frac{1}{2}\left(1-4\sqrt{3}\right)\pi t_2\right)\cosh(2\pi t_1)\,dt_2\,dt_1}{8\pi^2}$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3} - 5 = -5 + \int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{i\,e^{\pi^2/s+s}\left(e^{(3\pi^2)/s} + \int_{\frac{\pi}{2}}^{2\sqrt{3}\pi}\sin(t)\,dt\right)}{32\pi^{5/2}\,s^{3/2}}\,ds \quad \text{for } \gamma > 0$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3} - 5 = -5 + \int_0^1\left(\frac{\cosh(4\pi t)}{8\pi^2} + \frac{i\cosh(2\pi t)}{16\pi^{5/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-(3\pi^2)/s+s}}{\sqrt{s}}\,ds\right)dt \quad \text{for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3} - 5 = -5 + \frac{-4\left(-1 + 2\cos^2\left(\sqrt{3}\,\pi\right)\right)\cosh(\pi)\sinh(\pi) + 2\cosh(2\pi)\sinh(2\pi)}{32\pi^3}$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3} - 5 = -5 + \frac{-4\cosh(\pi)\left(1 - 2\sin^2\left(\sqrt{3}\,\pi\right)\right)\sinh(\pi) + 2\cosh(2\pi)\sinh(2\pi)}{32\pi^3}$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\, 2\sinh(2\pi)}{4\pi^3\, 2^3} - 5 = -5 + \frac{-2\left(-1 + 2\cos^2(\sqrt{3}\,\pi)\right)\left(3\sinh\left(\frac{2\pi}{3}\right) + 4\sinh^3\left(\frac{2\pi}{3}\right)\right) + 3\sinh\left(\frac{4\pi}{3}\right) + 4\sinh^3\left(\frac{4\pi}{3}\right)}{32\pi^3}$$

Now, subtracting π and 18, that is a Lucas number, and adding the golden ratio, we obtain:

((((((sinh (4Pi) – 2sinh (2Pi) cos (2Pi*sqrt3)))) / ((4Pi^3*2^3))))) - 18 - Pi+golden ratio

Input:

$$\frac{\sinh(4\pi) - (2\sinh(2\pi))\cos(2\pi\sqrt{3})}{4\pi^3 \times 2^3} - 18 - \pi + \phi$$

sinh(x) is the hyperbolic sine function

ϕ is the golden ratio

Exact result:

$$\phi - 18 - \pi + \frac{\sinh(4\pi) - 2\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}{32\pi^3}$$

Decimal approximation:

125.0398325135623555624471658168496449199654873393223773833...

125.03983251… result very near to the Higgs boson mass 125.18 GeV

Alternate forms:

$$\phi - 18 - \pi + \frac{\sinh(2\pi)\left(\cosh(2\pi) - \cos(2\sqrt{3}\,\pi)\right)}{16\pi^3}$$

$$\frac{1}{2}\left(\sqrt{5} - 35\right) - \pi + \frac{\sinh(4\pi) - 2\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}{32\pi^3}$$

$$\frac{64\pi^3(\phi - 18 - \pi) - e^{-4\pi} + e^{4\pi} - 4\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}{64\pi^3}$$

cosh(x) is the hyperbolic cosine function

Alternative representations:

$$\frac{\sinh(4\pi) - \cos\left(2\pi\sqrt{3}\right) 2\sinh(2\pi)}{4\pi^3 2^3} - 18 - \pi + \phi =$$
$$-18 + \phi - \pi + \frac{-\cosh\left(-2 i\pi\sqrt{3}\right)\left(-e^{-2\pi} + e^{2\pi}\right) + \frac{1}{2}\left(-e^{-4\pi} + e^{4\pi}\right)}{32\pi^3}$$

$$\frac{\sinh(4\pi) - \cos\left(2\pi\sqrt{3}\right) 2\sinh(2\pi)}{4\pi^3 2^3} - 18 - \pi + \phi =$$
$$-18 + \phi - \pi + \frac{-\cosh\left(2 i\pi\sqrt{3}\right)\left(-e^{-2\pi} + e^{2\pi}\right) + \frac{1}{2}\left(-e^{-4\pi} + e^{4\pi}\right)}{32\pi^3}$$

$$\frac{\sinh(4\pi) - \cos\left(2\pi\sqrt{3}\right) 2\sinh(2\pi)}{4\pi^3 2^3} - 18 - \pi + \phi =$$
$$-18 + \phi - \pi + \frac{-2 i\cosh\left(-2 i\pi\sqrt{3}\right)\cos\left(\frac{\pi}{2} + 2 i\pi\right) + i\cos\left(\frac{\pi}{2} + 4 i\pi\right)}{32\pi^3}$$

Series representations:

$$\frac{\sinh(4\pi) - \cos\left(2\pi\sqrt{3}\right) 2\sinh(2\pi)}{4\pi^3 2^3} - 18 - \pi + \phi =$$
$$\frac{-560\pi^3 + 16\sqrt{5}\pi^3 - 32\pi^4 + \sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!} - 2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}(2\pi)^{1+2k_1+2k_2}}{(2k_1)!(1+2k_2)!}}{32\pi^3}$$

$$\frac{\sinh(4\pi) - \cos\left(2\pi\sqrt{3}\right) 2\sinh(2\pi)}{4\pi^3 2^3} - 18 - \pi + \phi =$$
$$\frac{1}{32\pi^3}\left(-560\pi^3 + 16\sqrt{5}\pi^3 - 32\pi^4 + i\sum_{k=0}^{\infty}\frac{\left(\left(4 - \frac{i}{2}\right)\pi\right)^{2k}}{(2k)!} - 2 i\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1} 2^{2k_1-2k_2}(4-i)^{2k_2}\pi^{2k_1+2k_2}}{(2k_1)!(2k_2)!}\right)$$

$$\frac{\sinh(4\pi) - \cos\left(2\pi\sqrt{3}\right) 2\sinh(2\pi)}{4\pi^3 2^3} - 18 - \pi + \phi =$$
$$\frac{1}{2}\left(-35 + \sqrt{5} - 2\pi + 2\sum_{k=0}^{\infty}\frac{2^{-3+2k}\pi^{-2+2k}\left(4^k - \sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(1+2k)!}\right)$$

Integral representations:

$$\frac{\sinh(4\pi) - \cos\left(2\pi\sqrt{3}\right) 2\sinh(2\pi)}{4\pi^3 2^3} - 18 - \pi + \phi =$$
$$\frac{1}{8\pi^2}\left(-140\pi^2 + 4\sqrt{5}\pi^2 - 8\pi^3 + \int_0^1\cosh(4\pi t)\,dt + \int_0^1\int_0^1\cos\left(\frac{1}{2}\left(1 - 4\sqrt{3}\right)\pi t_2\right)\cosh(2\pi t_1)\,dt_2\,dt_1\right)$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\, 2\sinh(2\pi)}{4\pi^3\, 2^3} - 18 - \pi + \phi = \frac{1}{2}\left(-35 + \sqrt{5} - 2\pi + 2\int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{i\, e^{\pi^2/s+s}\left(e^{(3\pi^2)/s} + \int_{\frac{\pi}{2}}^{2\sqrt{3}\pi} \sin(t)\, dt\right)}{32\,\pi^{5/2}\, s^{3/2}}\, ds\right) \text{ for } \gamma > 0$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\, 2\sinh(2\pi)}{4\pi^3\, 2^3} - 18 - \pi + \phi = \frac{1}{2}\left(-35 + \sqrt{5} - 2\pi + 2\int_0^1 \left(\frac{\cosh(4\pi t)}{8\pi^2} + \frac{i\cosh(2\pi t)}{16\,\pi^{5/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-(3\pi^2)/s+s}}{\sqrt{s}}\, ds\right) dt\right)$$
$$\text{for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\, 2\sinh(2\pi)}{4\pi^3\, 2^3} - 18 - \pi + \phi = -18 + \phi - \pi + \frac{-4\left(-1 + 2\cos^2(\sqrt{3}\,\pi)\right)\cosh(\pi)\sinh(\pi) + 2\cosh(2\pi)\sinh(2\pi)}{32\,\pi^3}$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\, 2\sinh(2\pi)}{4\pi^3\, 2^3} - 18 - \pi + \phi = -18 + \phi - \pi + \frac{-4\cosh(\pi)\left(1 - 2\sin^2(\sqrt{3}\,\pi)\right)\sinh(\pi) + 2\cosh(2\pi)\sinh(2\pi)}{32\,\pi^3}$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\, 2\sinh(2\pi)}{4\pi^3\, 2^3} - 18 - \pi + \phi = -18 + \phi - \pi + \frac{-2\left(-1 + 2\cos^2(\sqrt{3}\,\pi)\right)\left(3\sinh\left(\frac{2\pi}{3}\right) + 4\sinh^3\left(\frac{2\pi}{3}\right)\right) + 3\sinh\left(\frac{4\pi}{3}\right) + 4\sinh^3\left(\frac{4\pi}{3}\right)}{32\,\pi^3}$$

Now, we subtracting 7, that is a Lucas number and 1/2 , we obtain:

((((((sinh (4Pi) – 2sinh (2Pi) cos (2Pi*sqrt3)))) / ((4Pi^3*2^3))))) - 7 – 1/2

Input:

$$\frac{\sinh(4\pi) - (2\sinh(2\pi))\cos(2\pi\sqrt{3})}{4\pi^3 \times 2^3} - 7 - \frac{1}{2}$$

sinh(x) is the hyperbolic sine function

Exact result:

$$\frac{\sinh(4\pi) - 2\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}{32\,\pi^3} - \frac{15}{2}$$

Decimal approximation:

137.0633911784022539527052223657635096864423475588917203422...

137.06339117…

This result is very near to the inverse of fine-structure constant 137,035

Alternate forms:

$$\frac{\sinh(2\pi)\left(\cosh(2\pi)-\cos\left(2\sqrt{3}\,\pi\right)\right)}{16\pi^3}-\frac{15}{2}$$

$$-\frac{15}{2}+\frac{\sinh(4\pi)}{32\pi^3}-\frac{\cos\left(2\sqrt{3}\,\pi\right)\sinh(2\pi)}{16\pi^3}$$

$$-\frac{240\pi^3-\sinh(4\pi)+2\cos\left(2\sqrt{3}\,\pi\right)\sinh(2\pi)}{32\pi^3}$$

cosh(x) is the hyperbolic cosine function

Alternative representations:

$$\frac{\sinh(4\pi)-\cos\left(2\pi\sqrt{3}\right)2\sinh(2\pi)}{4\pi^3\,2^3}-7-\frac{1}{2}=-\frac{15}{2}+\frac{-\cosh\left(-2i\pi\sqrt{3}\right)\left(-e^{-2\pi}+e^{2\pi}\right)+\frac{1}{2}\left(-e^{-4\pi}+e^{4\pi}\right)}{32\pi^3}$$

$$\frac{\sinh(4\pi)-\cos\left(2\pi\sqrt{3}\right)2\sinh(2\pi)}{4\pi^3\,2^3}-7-\frac{1}{2}=-\frac{15}{2}+\frac{-\cosh\left(2i\pi\sqrt{3}\right)\left(-e^{-2\pi}+e^{2\pi}\right)+\frac{1}{2}\left(-e^{-4\pi}+e^{4\pi}\right)}{32\pi^3}$$

$$\frac{\sinh(4\pi)-\cos\left(2\pi\sqrt{3}\right)2\sinh(2\pi)}{4\pi^3\,2^3}-7-\frac{1}{2}=-\frac{15}{2}+\frac{-2i\cosh\left(-2i\pi\sqrt{3}\right)\cos\left(\frac{\pi}{2}+2i\pi\right)+i\cos\left(\frac{\pi}{2}+4i\pi\right)}{32\pi^3}$$

Series representations:

$$\frac{\sinh(4\pi)-\cos\left(2\pi\sqrt{3}\right)2\sinh(2\pi)}{4\pi^3\,2^3}-7-\frac{1}{2}=-\frac{240\pi^3-\sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!}+2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}(2\pi)^{1+2k_1+2k_2}}{(2k_1)!(1+2k_2)!}}{32\pi^3}$$

$$\frac{\sinh(4\pi)-\cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}-7-\frac{1}{2}=$$
$$-\frac{240\pi^3-i\sum_{k=0}^{\infty}\frac{\left(\left(4-\frac{i}{2}\right)\pi\right)^{2k}}{(2k)!}+2i\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}\,2^{2k_1-2k_2}\,(4-i)^{2k_2}\,\pi^{2k_1+2k_2}}{(2k_1)!\,(2k_2)!}}{32\pi^3}$$

$$\frac{\sinh(4\pi)-\cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}-7-\frac{1}{2}=$$
$$\frac{1}{2}\left(-15+2\sum_{k=0}^{\infty}\frac{2^{-3+2k}\,\pi^{-2+2k}\left(4^k-\sqrt{\pi}\,\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(1+2k)!}\right)$$

Integral representations:

$$\frac{\sinh(4\pi)-\cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}-7-\frac{1}{2}=$$
$$-\frac{60\pi^2-\int_0^1\cosh(4\pi t)\,dt+\int_0^1\int_0^1\cos\left(\frac{1}{2}\left(1-4\sqrt{3}\right)\pi t_2\right)\cosh(2\pi t_1)\,dt_2\,dt_1}{8\pi^2}$$

$$\frac{\sinh(4\pi)-\cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}-7-\frac{1}{2}=$$
$$\frac{1}{2}\left(-15+2\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,e^{\pi^2/s+s}\left(e^{(3\pi^2)/s}+\int_{\frac{\pi}{2}}^{2\sqrt{3}\pi}\sin(t)\,dt\right)}{32\pi^{5/2}\,s^{3/2}}\,ds\right)\quad\text{for }\gamma>0$$

$$\frac{\sinh(4\pi)-\cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}-7-\frac{1}{2}=$$
$$\frac{1}{2}\left(-15+2\int_0^1\left(\frac{\cosh(4\pi t)}{8\pi^2}+\frac{i\cosh(2\pi t)}{16\pi^{5/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-(3\pi^2)/s+s}}{\sqrt{s}}\,ds\right)dt\right)\quad\text{for }\gamma>0$$

Multiple-argument formulas:

$$\frac{\sinh(4\pi)-\cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}-7-\frac{1}{2}=$$
$$-\frac{15}{2}+\frac{-4\left(-1+2\cos^2(\sqrt{3}\,\pi)\right)\cosh(\pi)\sinh(\pi)+2\cosh(2\pi)\sinh(2\pi)}{32\pi^3}$$

$$\frac{\sinh(4\pi)-\cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}-7-\frac{1}{2}=$$
$$-\frac{15}{2}+\frac{-4\cosh(\pi)\left(1-2\sin^2(\sqrt{3}\,\pi)\right)\sinh(\pi)+2\cosh(2\pi)\sinh(2\pi)}{32\pi^3}$$

$$\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3} - 7 - \frac{1}{2} =$$
$$-\frac{15}{2} + \frac{-2\left(-1 + 2\cos^2\left(\sqrt{3}\,\pi\right)\right)\left(3\sinh\left(\frac{2\pi}{3}\right) + 4\sinh^3\left(\frac{2\pi}{3}\right)\right) + 3\sinh\left(\frac{4\pi}{3}\right) + 4\sinh^3\left(\frac{4\pi}{3}\right)}{32\pi^3}$$

We observe that The Riemann hypothesis states that every nontrivial complex root of the Riemann zeta function has a real part equal to 1/2

Now, multiplying by 12, subtracting 5, 1/golden ratio and π and adding 3, where 5 and 3 are Fibonacci numbers, we obtain:

12*((((((sinh (4Pi) – 2sinh (2Pi) cos (2Pi*sqrt3)))) / ((4Pi^3*2^3))))) - 5 - 1/golden ratio - Pi + 3

Input:

$$12 \times \frac{\sinh(4\pi) - (2\sinh(2\pi))\cos(2\pi\sqrt{3})}{4\pi^3 \times 2^3} - 5 - \frac{1}{\phi} - \pi + 3$$

sinh(x) is the hyperbolic sine function

ϕ is the golden ratio

Exact result:

$$-\frac{1}{\phi} - 2 - \pi + \frac{3\left(\sinh(4\pi) - 2\cos\left(2\sqrt{3}\,\pi\right)\sinh(2\pi)\right)}{8\pi^3}$$

Decimal approximation:

1729.0010674984873593457954381715169752353906921275197754 23...

1729.0010674984…

This result is very near to **the mass of** candidate glueball **f_0(1710) meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than **the Hardy–Ramanujan number 1729 (taxicab number)**

Alternate forms:

$$-\frac{1}{\phi}-2-\pi+\frac{3\sinh(2\pi)\left(\cosh(2\pi)-\cos\left(2\sqrt{3}\ \pi\right)\right)}{4\pi^3}$$

$$\frac{1}{2}\left(-3-\sqrt{5}\right)-\pi+\frac{3\left(\sinh(4\pi)-2\cos\left(2\sqrt{3}\ \pi\right)\sinh(2\pi)\right)}{8\pi^3}$$

$$-\frac{1}{\phi}-2-\frac{3e^{-4\pi}}{16\pi^3}+\frac{3e^{4\pi}}{16\pi^3}-\pi-\frac{3\cos\left(2\sqrt{3}\ \pi\right)\sinh(2\pi)}{4\pi^3}$$

cosh(x) is the hyperbolic cosine function

Alternative representations:

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\ 2^3}-5-\frac{1}{\phi}-\pi+3=$$
$$-2-\pi-\frac{1}{\phi}+\frac{12\left(-\cosh\left(-2i\pi\sqrt{3}\right)\left(-e^{-2\pi}+e^{2\pi}\right)+\frac{1}{2}\left(-e^{-4\pi}+e^{4\pi}\right)\right)}{32\pi^3}$$

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\ 2^3}-5-\frac{1}{\phi}-\pi+3=$$
$$-2-\pi-\frac{1}{\phi}+\frac{12\left(-\cosh\left(2i\pi\sqrt{3}\right)\left(-e^{-2\pi}+e^{2\pi}\right)+\frac{1}{2}\left(-e^{-4\pi}+e^{4\pi}\right)\right)}{32\pi^3}$$

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\ 2^3}-5-\frac{1}{\phi}-\pi+3=$$
$$-2-\pi-\frac{1}{\phi}+\frac{12\left(-2i\cosh\left(-2i\pi\sqrt{3}\right)\cos\left(\frac{\pi}{2}+2i\pi\right)+i\cos\left(\frac{\pi}{2}+4i\pi\right)\right)}{32\pi^3}$$

Series representations:

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\ 2^3}-5-\frac{1}{\phi}-\pi+3=-\frac{1}{8\left(1+\sqrt{5}\right)\pi^3}$$
$$\left(32\pi^3+16\sqrt{5}\ \pi^3+8\pi^4+8\sqrt{5}\ \pi^4-3\sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!}-3\sqrt{5}\sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!}+\right.$$
$$\left.6\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}\ (2\pi)^{1+2k_1+2k_2}}{(2k_1)!\,(1+2k_2)!}+6\sqrt{5}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}\ (2\pi)^{1+2k_1+2k_2}}{(2k_1)!\,(1+2k_2)!}\right)$$

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3 2^3}-5-\frac{1}{\phi}-\pi+3=-\frac{1}{1+\sqrt{5}}$$
$$\left(4+2\sqrt{5}+\pi+\sqrt{5}\pi-\sum_{k=0}^{\infty}\frac{3\times 2^{-1+2k}\pi^{-2+2k}\left(4^k-\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(1+2k)!}-\right.$$
$$\left.\sqrt{5}\sum_{k=0}^{\infty}\frac{3\times 2^{-1+2k}\pi^{-2+2k}\left(4^k-\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(1+2k)!}\right)$$

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3 2^3}-5-\frac{1}{\phi}-\pi+3=$$
$$-\frac{1}{1+\sqrt{5}}\left(4+2\sqrt{5}+\pi+\sqrt{5}\pi-\right.$$
$$\sum_{k=0}^{\infty}\frac{3i\,2^{-3-2k}\pi^{-3+2k}\left((8-i)^{2k}-2(4-i)^{2k}\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(2k)!}-$$
$$\left.\sqrt{5}\sum_{k=0}^{\infty}\frac{3i\,2^{-3-2k}\pi^{-3+2k}\left((8-i)^{2k}-2(4-i)^{2k}\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(2k)!}\right)$$

Integral representations:

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3 2^3}-5-\frac{1}{\phi}-\pi+3=$$
$$-\frac{1}{2\left(1+\sqrt{5}\right)\pi^2}\left(8\pi^2+4\sqrt{5}\pi^2+2\pi^3+2\sqrt{5}\pi^3-3\int_0^1\cosh(4\pi t)\,dt-\right.$$
$$\left.3\sqrt{5}\int_0^1\cosh(4\pi t)\,dt+2\int_0^1\int_0^1\cos\left(\frac{1}{2}\left(1-4\sqrt{3}\right)\pi t_2\right)\cosh(2\pi t_1)\,dt_2\,dt_1\right)$$

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3 2^3}-5-\frac{1}{\phi}-\pi+3=$$
$$-\frac{1}{\left(1+\sqrt{5}\right)\pi}\left(4\pi+2\sqrt{5}\pi+\pi^2+\sqrt{5}\pi^2-\pi\int_0^1\frac{3(1+2\cosh(2\pi t))\sinh^2(\pi t)}{\pi^2}\,dt-\right.$$
$$\sqrt{5}\pi\int_0^1\frac{3(1+2\cosh(2\pi t))\sinh^2(\pi t)}{\pi^2}\,dt+$$
$$\left.2\int_0^1\int_0^1\cosh(2\pi t_1)\sin\left(2\sqrt{3}\pi t_2\right)dt_2\,dt_1\right)$$

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,2^3}-5-\frac{1}{\phi}-\pi+3=-\frac{1}{1+\sqrt{5}}$$
$$\left(4+2\sqrt{5}+\pi+\sqrt{5}\,\pi-\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}-\frac{3\,i\,e^{\pi^2/s+s}\left(e^{(3\pi^2)/s}+\int_{\frac{\pi}{2}}^{2\sqrt{3}\,\pi}\sin(t)\,dt\right)}{8\,\pi^{5/2}\,s^{3/2}}\,ds-\right.$$
$$\left.\sqrt{5}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}-\frac{3\,i\,e^{\pi^2/s+s}\left(e^{(3\pi^2)/s}+\int_{\frac{\pi}{2}}^{2\sqrt{3}\,\pi}\sin(t)\,dt\right)}{8\,\pi^{5/2}\,s^{3/2}}\,ds\right)\quad\text{for }\gamma>0$$

Multiple-argument formulas:

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,2^3}-5-\frac{1}{\phi}-\pi+3=$$
$$-2-\frac{1}{\phi}-\pi+\frac{3\left(-4\left(-1+2\cos^2\left(\sqrt{3}\,\pi\right)\right)\cosh(\pi)\sinh(\pi)+2\cosh(2\pi)\sinh(2\pi)\right)}{8\pi^3}$$

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,2^3}-5-\frac{1}{\phi}-\pi+3=$$
$$-2-\frac{1}{\phi}-\pi+\frac{3\left(-4\cosh(\pi)\left(1-2\sin^2\left(\sqrt{3}\,\pi\right)\right)\sinh(\pi)+2\cosh(2\pi)\sinh(2\pi)\right)}{8\pi^3}$$

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,2^3}-5-\frac{1}{\phi}-\pi+3=-2-\frac{1}{\phi}-\pi+$$
$$\frac{3\left(-2\left(-1+2\cos^2\left(\sqrt{3}\,\pi\right)\right)\left(3\sinh\left(\frac{2\pi}{3}\right)+4\sinh^3\left(\frac{2\pi}{3}\right)\right)+3\sinh\left(\frac{4\pi}{3}\right)+4\sinh^3\left(\frac{4\pi}{3}\right)\right)}{8\pi^3}$$

With regard the number 12, we observe that twelve is the smallest weight for which a cusp form exists. This cusp form is the discriminant $\Delta(q)$ whose Fourier coefficients are given by the Ramanujan τ-function and which is (up to a constant multiplier) the 24th power of the Dedekind eta function. This fact is related to a constellation of interesting appearances of the number twelve in mathematics ranging from the value of the Riemann zeta function at −1 i.e. $\zeta(-1) = -1/12$, the fact that the abelianization of SL(2,Z) has twelve elements, and even the properties of lattice polygons.

From the same previous formula, with the same data, adding 55, that is a Fibonacci number, we obtain:

12*((((((sinh (4Pi) – 2sinh (2Pi) cos (2Pi*sqrt3)))) / ((4Pi^3*2^3))))) - 5 - 1/golden ratio - Pi + 3 + 55

Input:

$$12\times\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos(2\pi\sqrt{3})}{4\pi^3\times 2^3}-5-\frac{1}{\phi}-\pi+3+55$$

$\sinh(x)$ is the hyperbolic sine function

ϕ is the golden ratio

Exact result:

$$-\frac{1}{\phi}+53-\pi+\frac{3(\sinh(4\pi)-2\cos(2\sqrt{3}\pi)\sinh(2\pi))}{8\pi^3}$$

Decimal approximation:

1784.0010674984873593457954381715169752353906921275197754 23...

1784.0010674984… result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV), that is a supersymmetrical particle of Gluon

Alternate forms:

$$-\frac{1}{\phi}+53-\pi+\frac{3\sinh(2\pi)(\cosh(2\pi)-\cos(2\sqrt{3}\pi))}{4\pi^3}$$

$$\frac{1}{2}\left(107-\sqrt{5}\right)-\pi+\frac{3(\sinh(4\pi)-2\cos(2\sqrt{3}\pi)\sinh(2\pi))}{8\pi^3}$$

$$-\frac{1}{\phi}+53-\frac{3e^{-4\pi}}{16\pi^3}+\frac{3e^{4\pi}}{16\pi^3}-\pi-\frac{3\cos(2\sqrt{3}\pi)\sinh(2\pi)}{4\pi^3}$$

$\cosh(x)$ is the hyperbolic cosine function

Alternative representations:

$$\frac{12(\sinh(4\pi)-(2\sinh(2\pi))\cos(2\pi\sqrt{3}))}{4\pi^3\,2^3}-5-\frac{1}{\phi}-\pi+3+55=53-\pi-\frac{1}{\phi}+\frac{12\left(-\cosh(-2i\pi\sqrt{3})(-e^{-2\pi}+e^{2\pi})+\frac{1}{2}(-e^{-4\pi}+e^{4\pi})\right)}{32\pi^3}$$

$$\frac{12(\sinh(4\pi)-(2\sinh(2\pi))\cos(2\pi\sqrt{3}))}{4\pi^3\,2^3}-5-\frac{1}{\phi}-\pi+3+55=53-\pi-\frac{1}{\phi}+\frac{12\left(-\cosh(2i\pi\sqrt{3})(-e^{-2\pi}+e^{2\pi})+\frac{1}{2}(-e^{-4\pi}+e^{4\pi})\right)}{32\pi^3}$$

$$\frac{12(\sinh(4\pi)-(2\sinh(2\pi))\cos(2\pi\sqrt{3}))}{4\pi^3\,2^3}-5-\frac{1}{\phi}-\pi+3+55=53-\pi-\frac{1}{\phi}+\frac{12\left(-2i\cosh(-2i\pi\sqrt{3})\cos\left(\frac{\pi}{2}+2i\pi\right)+i\cos\left(\frac{\pi}{2}+4i\pi\right)\right)}{32\pi^3}$$

Series representations:

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,2^3}-5-\frac{1}{\phi}-\pi+3+55=-\frac{1}{8\left(1+\sqrt{5}\right)\pi^3}\left(-408\pi^3-424\sqrt{5}\,\pi^3+8\pi^4+8\sqrt{5}\,\pi^4-3\sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!}-3\sqrt{5}\sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!}+6\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}(2\pi)^{1+2k_1+2k_2}}{(2k_1)!(1+2k_2)!}+6\sqrt{5}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}(2\pi)^{1+2k_1+2k_2}}{(2k_1)!(1+2k_2)!}\right)$$

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,2^3}-5-\frac{1}{\phi}-\pi+3+55=-\frac{1}{1+\sqrt{5}}\left(-51-53\sqrt{5}+\pi+\sqrt{5}\,\pi-\sum_{k=0}^{\infty}\frac{3\times2^{-1+2k}\pi^{-2+2k}\left(4^k-\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(1+2k)!}-\sqrt{5}\sum_{k=0}^{\infty}\frac{3\times2^{-1+2k}\pi^{-2+2k}\left(4^k-\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(1+2k)!}\right)$$

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3\,2^3}-5-\frac{1}{\phi}-\pi+3+55=-\frac{1}{1+\sqrt{5}}\left(-51-53\sqrt{5}+\pi+\sqrt{5}\,\pi-\sum_{k=0}^{\infty}\frac{3\,i\,2^{-3-2k}\pi^{-3+2k}\left((8-i)^{2k}-2(4-i)^{2k}\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(2k)!}-\sqrt{5}\sum_{k=0}^{\infty}\frac{3\,i\,2^{-3-2k}\pi^{-3+2k}\left((8-i)^{2k}-2(4-i)^{2k}\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(2k)!}\right)$$

Integral representations:

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3 2^3}-5-\frac{1}{\phi}-\pi+3+55=$$
$$-\frac{1}{2\left(1+\sqrt{5}\right)\pi^2}\left(-102\pi^2-106\sqrt{5}\pi^2+2\pi^3+2\sqrt{5}\pi^3-3\int_0^1\cosh(4\pi t)\,dt-\right.$$
$$\left.3\sqrt{5}\int_0^1\cosh(4\pi t)\,dt+2\int_0^1\int_0^1\cos\left(\frac{1}{2}\left(1-4\sqrt{3}\right)\pi t_2\right)\cosh(2\pi t_1)\,dt_2\,dt_1\right)$$

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3 2^3}-5-\frac{1}{\phi}-\pi+3+55=$$
$$-\frac{1}{\left(1+\sqrt{5}\right)\pi}\left(-51\pi-53\sqrt{5}\pi+\pi^2+\sqrt{5}\pi^2-\pi\int_0^1\frac{3(1+2\cosh(2\pi t))\sinh^2(\pi t)}{\pi^2}\,dt-\right.$$
$$\sqrt{5}\pi\int_0^1\frac{3(1+2\cosh(2\pi t))\sinh^2(\pi t)}{\pi^2}\,dt+$$
$$\left.2\int_0^1\int_0^1\cosh(2\pi t_1)\sin\left(2\sqrt{3}\pi t_2\right)dt_2\,dt_1\right)$$

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3 2^3}-5-\frac{1}{\phi}-\pi+3+55=-\frac{1}{1+\sqrt{5}}$$
$$\left(-51-53\sqrt{5}+\pi+\sqrt{5}\pi-\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{3ie^{\pi^2/s+s}\left(e^{(3\pi^2)/s}+\int_{\frac{\pi}{2}}^{2\sqrt{3}\pi}\sin(t)\,dt\right)}{8\pi^{5/2}s^{3/2}}\,ds-\right.$$
$$\left.\sqrt{5}\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{3ie^{\pi^2/s+s}\left(e^{(3\pi^2)/s}+\int_{\frac{\pi}{2}}^{2\sqrt{3}\pi}\sin(t)\,dt\right)}{8\pi^{5/2}s^{3/2}}\,ds\right)\quad\text{for }\gamma>0$$

Multiple-argument formulas:

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3 2^3}-5-\frac{1}{\phi}-\pi+3+55=$$
$$53-\frac{1}{\phi}-\pi+\frac{3\left(-4\left(-1+2\cos^2\left(\sqrt{3}\pi\right)\right)\cosh(\pi)\sinh(\pi)+2\cosh(2\pi)\sinh(2\pi)\right)}{8\pi^3}$$

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3 2^3}-5-\frac{1}{\phi}-\pi+3+55=$$
$$53-\frac{1}{\phi}-\pi+\frac{3\left(-4\cosh(\pi)\left(1-2\sin^2\left(\sqrt{3}\pi\right)\right)\sinh(\pi)+2\cosh(2\pi)\sinh(2\pi)\right)}{8\pi^3}$$

$$\frac{12\left(\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)\right)}{4\pi^3 2^3}-5-\frac{1}{\phi}-\pi+3+55=53-\frac{1}{\phi}-\pi+$$
$$\frac{3\left(-2\left(-1+2\cos^2\left(\sqrt{3}\pi\right)\right)\left(3\sinh\left(\frac{2\pi}{3}\right)+4\sinh^3\left(\frac{2\pi}{3}\right)\right)+3\sinh\left(\frac{4\pi}{3}\right)+4\sinh^3\left(\frac{4\pi}{3}\right)\right)}{8\pi^3}$$

Now, performing the 48th root of the expression and subtracting 34 + π – golden ratio (where 34 is a Fibonacci number), dividing them by 10^4, and, multiplying the whole expression by $1/10^{52}$, we obtain:

1/10^52*((((((((((sinh (4Pi) – 2sinh (2Pi) cos (2Pi*sqrt3)))) / ((4Pi^3*2^3)))))^1/48 - (34+Pi-golden ratio)*1/10^4)))

Input:

$$\frac{1}{10^{52}}\left(\sqrt[48]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\times 2^3}}-(34+\pi-\phi)\times\frac{1}{10^4}\right)$$

sinh(x) is the hyperbolic sine function

ϕ is the golden ratio

Exact result:

$$\frac{\frac{\phi-34-\pi}{10\,000}+\frac{\sqrt[48]{\sinh(4\pi)-2\cos\left(2\sqrt{3}\,\pi\right)\sinh(2\pi)}}{2^{5/48}\sqrt[16]{\pi}}}{10\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

Decimal approximation:

$1.1056255628659321573975469988253837403417099690389799\ldots\times 10^{-52}$

1.1056255628…$*10^{-52}$ result practically equal to the value of Cosmological Constant 1.1056$*10^{-52}$ m^{-2}

Alternate forms:

$$-\frac{67}{200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}+\frac{1}{40\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\sqrt{5}}-\frac{\pi}{100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}+\left(\sqrt[48]{\sinh(4\pi)-2\cos\left(2\sqrt{3}\,\pi\right)\sinh(2\pi)}\right)\Big/\left(10\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\times 2^{5/48}\sqrt[16]{\pi}\right)$$

$$\frac{\frac{\frac{1}{2}\left(\sqrt{5}-67\right)-\pi}{10\,000}+\frac{\sqrt[48]{\sinh(4\pi)-2\cos\left(2\sqrt{3}\,\pi\right)\sinh(2\pi)}}{2^{5/48}\sqrt[16]{\pi}}}{10\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\left(\sqrt[8]{2}\,\sqrt[16]{\pi}\,(\phi-34-\pi)+10\,000\sqrt[48]{-e^{-4\pi}+e^{4\pi}-4\cos\left(2\sqrt{3}\,\pi\right)\sinh(2\pi)}\right)/$$
$$\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\right.$$
$$\left.\sqrt[8]{2}\,\sqrt[16]{\pi}\right)$$

Alternative representations:

$$\frac{\sqrt[48]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\,2^3}}-\frac{34+\pi-\phi}{10^4}}{10^{52}}=$$
$$\frac{-\frac{34-\phi+\pi}{10^4}+\sqrt[48]{\frac{-\cosh\left(-2i\pi\sqrt{3}\right)\left(-e^{-2\pi}+e^{2\pi}\right)+\frac{1}{2}\left(-e^{-4\pi}+e^{4\pi}\right)}{32\pi^3}}}{10^{52}}$$

$$\frac{\sqrt[48]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\,2^3}}-\frac{34+\pi-\phi}{10^4}}{10^{52}}=$$
$$\frac{-\frac{34-\phi+\pi}{10^4}+\sqrt[48]{\frac{-\cosh\left(2i\pi\sqrt{3}\right)\left(-e^{-2\pi}+e^{2\pi}\right)+\frac{1}{2}\left(-e^{-4\pi}+e^{4\pi}\right)}{32\pi^3}}}{10^{52}}$$

$$\frac{\sqrt[48]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\,2^3}}-\frac{34+\pi-\phi}{10^4}}{10^{52}}=$$
$$\frac{-\frac{34-\phi+\pi}{10^4}+\sqrt[48]{\frac{-2i\cosh\left(-2i\pi\sqrt{3}\right)\cos\left(\frac{\pi}{2}+2i\pi\right)+i\cos\left(\frac{\pi}{2}+4i\pi\right)}{32\pi^3}}}{10^{52}}$$

Series representations:

$$\frac{\sqrt[48]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}-\frac{34+\pi-\phi}{10^4}}{10^{52}} = \left(-67\sqrt[16]{\pi}+\sqrt{5}\sqrt[16]{\pi}-2\pi^{17/16}+ 10\,000\times 2^{43/48}\sqrt[48]{\sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!}-2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}(2\pi)^{1+2k_1+2k_2}}{(2k_1)!(1+2k_2)!}}\right) \Big/ \left(200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\sqrt[16]{\pi}\right)$$

$$\frac{\sqrt[48]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}-\frac{34+\pi-\phi}{10^4}}{10^{52}} = \left(-67\sqrt[16]{\pi}+\sqrt{5}\sqrt[16]{\pi}-2\pi^{17/16}+10\,000\times 2^{43/48}\sqrt[48]{-i\left(-\sum_{k=0}^{\infty}\frac{\left(\left(4-\frac{i}{2}\right)\pi\right)^{2k}}{(2k)!}+2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}2^{2k_1-2k_2}(4-i)^{2k_2}\pi^{2k_1+2k_2}}{(2k_1)!(2k_2)!}\right)}\right) \Big/ \left(200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\sqrt[16]{\pi}\right)$$

$$\frac{\sqrt[48]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}-\frac{34+\pi-\phi}{10^4}}{10^{52}} = \left(-67\sqrt[16]{\pi}+\sqrt{5}\sqrt[16]{\pi}-2\pi^{17/16}+ 10\,000\times 2^{43/48}\sqrt[48]{\sum_{k=0}^{\infty}\frac{4^{1+k}\pi^{1+2k}\left(4^k-\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(1+2k)!}}\right) \Big/ \left(200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\sqrt[16]{\pi}\right)$$

Integral representations:

$$\frac{\sqrt[48]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}} - \frac{34+\pi-\phi}{10^4}}{10^{52}} =$$

$$\left(-67\sqrt[24]{\pi} + \sqrt{5}\sqrt[24]{\pi} - 2\pi^{25/24} + 10\,000\times 2^{15/16} \sqrt[48]{\int_0^1 \cosh(4\pi t)\,dt + \int_0^1\int_0^1 \cos\left(\frac{1}{2}\left(1-4\sqrt{3}\right)\pi t_2\right)\cosh(2\pi t_1)\,dt_2\,dt_1}\right) \Big/ \left(200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \sqrt[24]{\pi}\right)$$

$$\frac{\sqrt[48]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}} - \frac{34+\pi-\phi}{10^4}}{10^{52}} = \left(-67\sqrt[16]{\pi} + \sqrt{5}\sqrt[16]{\pi} - 2\pi^{17/16} + 10\,000\times 2^{43/48} \sqrt[48]{\int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{i\,e^{\pi^2/s+s}\sqrt{\pi}\left(e^{(3\pi^2)/s} + \int_{\frac{\pi}{2}}^{2\sqrt{3}\pi}\sin(t)\,dt\right)}{s^{3/2}}\,ds}\right) \Big/ \left(200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \sqrt[16]{\pi}\right) \text{ for } \gamma > 0$$

$$\frac{\sqrt[48]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}} - \frac{34+\pi-\phi}{10^4}}{10^{52}} =$$

$$\left(-67\sqrt[16]{\pi} + \sqrt{5}\sqrt[16]{\pi} - 2\pi^{17/16} + 10\,000\times 2^{43/48} \sqrt[48]{\int_0^1 \left(4\pi\cosh(4\pi t) + 2i\sqrt{\pi}\cosh(2\pi t)\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-(3\pi^2)/s+s}}{\sqrt{s}}\,ds\right)dt}\right) \Big/ \left(200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \sqrt[16]{\pi}\right) \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{\sqrt[48]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}-\frac{34+\pi-\phi}{10^4}}{10^{52}} = \frac{\frac{-34+\phi-\pi}{10000}+\frac{\sqrt[48]{-4\left(-1+2\cos^2\left(\sqrt{3}\pi\right)\right)\cosh(\pi)\sinh(\pi)+2\cosh(2\pi)\sinh(2\pi)}}{2^{5/48}\sqrt[16]{\pi}}}{10\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{\sqrt[48]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}-\frac{34+\pi-\phi}{10^4}}{10^{52}} = \frac{\frac{-34+\phi-\pi}{10000}+\frac{\sqrt[48]{-4\cosh(\pi)\left(1-2\sin^2\left(\sqrt{3}\pi\right)\right)\sinh(\pi)+2\cosh(2\pi)\sinh(2\pi)}}{2^{5/48}\sqrt[16]{\pi}}}{10\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{\sqrt[48]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}-\frac{34+\pi-\phi}{10^4}}{10^{52}} = \frac{\frac{-34+\phi-\pi}{10000}+\frac{\sqrt[48]{-2\left(-1+2\cos^2\left(\sqrt{3}\pi\right)\right)\left(3\sinh\left(\frac{2\pi}{3}\right)+4\sinh^3\left(\frac{2\pi}{3}\right)\right)+3\sinh\left(\frac{4\pi}{3}\right)+4\sinh^3\left(\frac{4\pi}{3}\right)}}{2^{5/48}\sqrt[16]{\pi}}}{10\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

Performing the 10th root, we obtain:

((((((sinh (4Pi) – 2sinh (2Pi) cos (2Pi*sqrt3)))) / ((4Pi^3*2^3)))))^1/10

Input:

$$\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\times 2^3}}$$

$\sinh(x)$ is the hyperbolic sine function

Exact result:

$$\frac{\sqrt[10]{\sinh(4\pi)-2\cos\left(2\sqrt{3}\pi\right)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}}$$

Decimal approximation:

1.6443938078943733653411737541283377497734383261986847080 86...

$1.64439380789\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms:

$$\frac{\sqrt[10]{-e^{-4\pi} + e^{4\pi} - 4\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}}{2^{3/5}\,\pi^{3/10}}$$

$$\frac{\sqrt[10]{\sinh(4\pi) - \sinh(2\pi - 2i\sqrt{3}\,\pi) - \sinh(2\pi + 2i\sqrt{3}\,\pi)}}{\sqrt{2}\,\pi^{3/10}}$$

$$\frac{\sqrt[10]{\frac{1}{2}\left(e^{4\pi} - e^{-4\pi}\right) - \frac{1}{2}\left(e^{2\pi} - e^{-2\pi}\right)\left(e^{-2i\sqrt{3}\,\pi} + e^{2i\sqrt{3}\,\pi}\right)}}{\sqrt{2}\,\pi^{3/10}}$$

All 10th roots of (sinh(4 π) - 2 cos(2 sqrt(3) π) sinh(2 π))/(32 π^3):

$$\frac{e^{0}\sqrt[10]{\sinh(4\pi) - 2\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}} \approx 1.6444 \quad \text{(real, principal root)}$$

$$\frac{e^{(i\pi)/5}\sqrt[10]{\sinh(4\pi) - 2\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}} \approx 1.3303 + 0.9666\,i$$

$$\frac{e^{(2i\pi)/5}\sqrt[10]{\sinh(4\pi) - 2\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}} \approx 0.5081 + 1.5639\,i$$

$$\frac{e^{(3i\pi)/5}\sqrt[10]{\sinh(4\pi) - 2\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}} \approx -0.5081 + 1.5639\,i$$

$$\frac{e^{(4i\pi)/5}\sqrt[10]{\sinh(4\pi) - 2\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}} \approx -1.3303 + 0.9666\,i$$

Alternative representations:

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}} = \sqrt[10]{\frac{-\cosh(-2i\pi\sqrt{3})\left(-e^{-2\pi}+e^{2\pi}\right)+\frac{1}{2}\left(-e^{-4\pi}+e^{4\pi}\right)}{32\pi^3}}$$

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}} = \sqrt[10]{\frac{-\cosh(2i\pi\sqrt{3})\left(-e^{-2\pi}+e^{2\pi}\right)+\frac{1}{2}\left(-e^{-4\pi}+e^{4\pi}\right)}{32\pi^3}}$$

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}} = \sqrt[10]{\frac{-2i\cosh(-2i\pi\sqrt{3})\cos\left(\frac{\pi}{2}+2i\pi\right)+i\cos\left(\frac{\pi}{2}+4i\pi\right)}{32\pi^3}}$$

Series representations:

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}} = \frac{\sqrt[10]{\sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!} - 2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}(2\pi)^{1+2k_1+2k_2}}{(2k_1)!\,(1+2k_2)!}}}{\sqrt{2}\,\pi^{3/10}}$$

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}} = \frac{\sqrt[10]{-i\left(-\sum_{k=0}^{\infty}\frac{\left(\left(4-\frac{i}{2}\right)\pi\right)^{2k}}{(2k)!} + 2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}\,2^{2k_1-2k_2}\,(4-i)^{2k_2}\,\pi^{2k_1+2k_2}}{(2k_1)!\,(2k_2)!}\right)}}{\sqrt{2}\,\pi^{3/10}}$$

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}} = \frac{\sqrt[10]{\sum_{k=0}^{\infty}\frac{4^{1+k}\,\pi^{1+2k}\left(4^k-\sqrt{\pi}\sum_{j=0}^{\infty}\text{Res}_{s=-j}\frac{3^{-s}\,\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(1+2k)!}}}{\sqrt{2}\,\pi^{3/10}}$$

Integral representations:

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos\left(2\pi\sqrt{3}\right) 2\sinh(2\pi)}{4\pi^3\, 2^3}} = \frac{\sqrt[10]{\int_0^1 \cosh(4\pi t)\, dt + \int_0^1 \int_0^1 \cos\left(\frac{1}{2}\left(1 - 4\sqrt{3}\right)\pi t_2\right) \cosh(2\pi t_1)\, dt_2\, dt_1}}{2^{3/10}\sqrt[5]{\pi}}$$

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos\left(2\pi\sqrt{3}\right) 2\sinh(2\pi)}{4\pi^3\, 2^3}} = \frac{\sqrt[10]{-i\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{\pi^2/s+s}\left(e^{(3\pi^2)/s} + \int_{\frac{\pi}{2}}^{2\sqrt{3}\pi} \sin(t)\, dt\right)}{s^{3/2}}\, ds}}{\sqrt{2}\sqrt[4]{\pi}} \quad \text{for } \gamma > 0$$

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos\left(2\pi\sqrt{3}\right) 2\sinh(2\pi)}{4\pi^3\, 2^3}} = \frac{\sqrt[10]{\int_0^1 \left(2\sqrt{\pi}\cosh(4\pi t) + i\cosh(2\pi t)\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-(3\pi^2)/s+s}}{\sqrt{s}}\, ds\right) dt}}{2^{2/5}\sqrt[4]{\pi}} \quad \text{for } \gamma > 0$$

Multiple-argument formulas:

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos\left(2\pi\sqrt{3}\right) 2\sinh(2\pi)}{4\pi^3\, 2^3}} = \frac{\sqrt[10]{\left(-\cos\left(2\sqrt{3}\pi\right) + \cosh(2\pi)\right)\sinh(2\pi)}}{2^{2/5}\pi^{3/10}}$$

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos\left(2\pi\sqrt{3}\right) 2\sinh(2\pi)}{4\pi^3\, 2^3}} = \frac{\sqrt[10]{-4\left(-1 + 2\cos^2\left(\sqrt{3}\pi\right)\right)\cosh(\pi)\sinh(\pi) + 2\cosh(2\pi)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}}$$

$$\frac{\sqrt[10]{\dfrac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\, 2\sinh(2\pi)}{4\pi^3\, 2^3}} =}{}$$

$$\frac{\sqrt[10]{i\left(2\prod_{k=0}^{3}\sinh\left(\pi + \frac{i k \pi}{4}\right) + \cos(2\sqrt{3}\,\pi)\prod_{k=0}^{1}\sinh\left(\pi + \frac{i k \pi}{2}\right)\right)}}{(2\pi)^{3/10}}$$

Now, adding $27/10^3$ to the previous expression, and multiplying all by $1/10^{27}$, we obtain:

1/10^27*((((((((((sinh (4Pi) – 2sinh (2Pi) cos (2Pi*sqrt3)))) / ((4Pi^3*2^3)))))^1/10 + 27/10^3)))

Input:

$$\frac{1}{10^{27}}\left(\sqrt[10]{\frac{\sinh(4\pi) - (2\sinh(2\pi))\cos(2\pi\sqrt{3})}{4\pi^3 \times 2^3}} + \frac{27}{10^3}\right)$$

$\sinh(x)$ is the hyperbolic sine function

Exact result:

$$\frac{\dfrac{27}{1000} + \dfrac{\sqrt[10]{\sinh(4\pi) - 2\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

Decimal approximation:

$1.6713938078943733653411737541283377497734383261986847\ldots \times 10^{-27}$

$1.671393807894\ldots * 10^{-27}$ result practically equal to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-27}\ \text{kg}$$

that is the holographic proton mass (N. Haramein)

Alternate forms:

$$\frac{27}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{\sqrt[10]{\sinh(4\pi) - 2\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\sqrt{2}\,\pi^{3/10}}$$

$$\frac{27\pi^{3/10} + 500\sqrt{2}\sqrt[10]{\sinh(4\pi) - 2\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\pi^{3/10}}$$

$$\frac{27\times 2^{3/5}\pi^{3/10} + 1000\sqrt[10]{-e^{-4\pi} + e^{4\pi} - 4\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\times 2^{3/5}\pi^{3/10}}$$

Alternative representations:

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}} + \frac{27}{10^3}}{10^{27}} = \frac{\frac{27}{10^3} + \sqrt[10]{\frac{-\cosh\left(-2i\pi\sqrt{3}\right)\left(-e^{-2\pi}+e^{2\pi}\right)+\frac{1}{2}\left(-e^{-4\pi}+e^{4\pi}\right)}{32\pi^3}}}{10^{27}}$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}} + \frac{27}{10^3}}{10^{27}} = \frac{\frac{27}{10^3} + \sqrt[10]{\frac{-\cosh\left(2i\pi\sqrt{3}\right)\left(-e^{-2\pi}+e^{2\pi}\right)+\frac{1}{2}\left(-e^{-4\pi}+e^{4\pi}\right)}{32\pi^3}}}{10^{27}}$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}} + \frac{27}{10^3}}{10^{27}} = \frac{\frac{27}{10^3} + \sqrt[10]{\frac{-2i\cosh\left(-2i\pi\sqrt{3}\right)\cos\left(\frac{\pi}{2}+2i\pi\right)+i\cos\left(\frac{\pi}{2}+4i\pi\right)}{32\pi^3}}}{10^{27}}$$

Series representations:

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}} + \frac{27}{10^3}}{10^{27}} =$$

$$\frac{27\pi^{3/10} + 500\sqrt{2}\sqrt[10]{\sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!} - 2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}(2\pi)^{1+2k_1+2k_2}}{(2k_1)!(1+2k_2)!}}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\pi^{3/10}}$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}+\frac{27}{10^3}}{10^{27}} = \left(27\pi^{3/10}+500\sqrt{2}\sqrt[10]{-i\left(-\sum_{k=0}^{\infty}\frac{\left(\left(4-\frac{i}{2}\right)\pi\right)^{2k}}{(2k)!}+2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}\,2^{2k_1-2k_2}\,(4-i)^{2k_2}\,\pi^{2k_1+2k_2}}{(2k_1)!\,(2k_2)!}\right)}\right)\Big/ \left(1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\pi^{3/10}\right)$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}+\frac{27}{10^3}}{10^{27}} = \frac{27\pi^{3/10}+500\sqrt{2}\sqrt[10]{\sum_{k=0}^{\infty}\frac{4^{1+k}\pi^{1+2k}\left(4^k-\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(1+2k)!}}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\pi^{3/10}}$$

Integral representations:

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}+\frac{27}{10^3}}{10^{27}} = \left(27\sqrt[5]{\pi}+500\times 2^{7/10}\sqrt[10]{\int_0^1\cosh(4\pi t)\,dt+\int_0^1\int_0^1\cos\left(\frac{1}{2}\left(1-4\sqrt{3}\right)\pi t_2\right)\cosh(2\pi t_1)\,dt_2\,dt_1}\right)\Big/ \left(1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\sqrt[5]{\pi}\right)$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}+\frac{27}{10^3}}{10^{27}} = \frac{27\pi^{3/10}+500\sqrt{2}\sqrt[10]{\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,e^{\pi^2/s+s}\sqrt{\pi}\left(e^{(3\pi^2)/s}+\int_{\frac{\pi}{2}}^{2\sqrt{3}\pi}\sin(t)\,dt\right)}{s^{3/2}}\,ds}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\pi^{3/10}} \quad \text{for } \gamma > 0$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}+\frac{27}{10^3}}}{10^{27}} = \left(27\pi^{3/10}+500\sqrt{2}\,\sqrt[10]{\int_0^1\left(4\pi\cosh(4\pi t)+2i\sqrt{\pi}\,\cosh(2\pi t)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-(3\pi^2)/s+s}}{\sqrt{s}}\,ds\right)dt}\right)\Big/\left(1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,\pi^{3/10}\right) \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}+\frac{27}{10^3}}}{10^{27}} = \frac{\frac{27}{1000}+\frac{\sqrt[10]{-4\left(-1+2\cos^2\left(\sqrt{3}\,\pi\right)\right)\cosh(\pi)\sinh(\pi)+2\cosh(2\pi)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}+\frac{27}{10^3}}}{10^{27}} = \frac{\frac{27}{1000}+\frac{\sqrt[10]{-4\cosh(\pi)\left(1-2\sin^2\left(\sqrt{3}\,\pi\right)\right)\sinh(\pi)+2\cosh(2\pi)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}+\frac{27}{10^3}}}{10^{27}} = \frac{\frac{27}{1000}+\frac{\sqrt[10]{-2\left(-1+2\cos^2\left(\sqrt{3}\,\pi\right)\right)\left(3\sinh\left(\frac{2\pi}{3}\right)+4\sinh^3\left(\frac{2\pi}{3}\right)\right)+3\sinh\left(\frac{4\pi}{3}\right)+4\sinh^3\left(\frac{4\pi}{3}\right)}}{\sqrt{2}\,\pi^{3/10}}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

With regard the number 27, we have that: (from Wikipedia) *"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

Subtracting (34 + 8)/10^3 (where 34 and 8 are Fibonacci numbers) and multiplying all by 1/10^{19}, we obtain:

1/10^19*((((((((((sinh (4Pi) – 2sinh (2Pi) cos (2Pi*sqrt3)))) / ((4Pi^3*2^3)))))^1/10 - (34+8)/10^3)))

Input:

$$\frac{1}{10^{19}}\left(\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos(2\pi\sqrt{3})}{4\pi^3\times 2^3}}-\frac{34+8}{10^3}\right)$$

sinh(x) is the hyperbolic sine function

Exact result:

$$\frac{\frac{\sqrt[10]{\sinh(4\pi)-2\cos\left(2\sqrt{3}\,\pi\right)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}}-\frac{21}{500}}{10\,000\,000\,000\,000\,000\,000}$$

Decimal approximation:

$1.6023938078943733653411737541283377497734383261986847... \times 10^{-19}$

1.602393807894…*10^{-19} result practically equal to the elementary charge

Alternate forms:

$$\frac{\sqrt[10]{\sinh(4\pi)-2\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}}{10\,000\,000\,000\,000\,000\,000\,\sqrt{2}\,\pi^{3/10}}-\frac{21}{5\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{250\sqrt{2}\,\sqrt[10]{\sinh(4\pi)-2\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}-21\,\pi^{3/10}}{5\,000\,000\,000\,000\,000\,000\,000\,\pi^{3/10}}$$

$$\frac{500\,\sqrt[10]{-e^{-4\pi}+e^{4\pi}-4\cos(2\sqrt{3}\,\pi)\sinh(2\pi)}-21\times 2^{3/5}\,\pi^{3/10}}{5\,000\,000\,000\,000\,000\,000\,000\times 2^{3/5}\,\pi^{3/10}}$$

Alternative representations:

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\,2^3}}-\frac{34+8}{10^3}}{10^{19}}=\frac{-\frac{42}{10^3}+\sqrt[10]{\frac{-\cosh\left(-2i\pi\sqrt{3}\right)\left(-e^{-2\pi}+e^{2\pi}\right)+\frac{1}{2}\left(-e^{-4\pi}+e^{4\pi}\right)}{32\pi^3}}}{10^{19}}$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}-\frac{34+8}{10^3}}{10^{19}}=$$

$$\frac{-\frac{42}{10^3}+\sqrt[10]{\frac{-\cosh\left(2i\pi\sqrt{3}\right)\left(-e^{-2\pi}+e^{2\pi}\right)+\frac{1}{2}\left(-e^{-4\pi}+e^{4\pi}\right)}{32\pi^3}}}{10^{19}}$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}-\frac{34+8}{10^3}}{10^{19}}=$$

$$\frac{-\frac{42}{10^3}+\sqrt[10]{\frac{-2i\cosh\left(-2i\pi\sqrt{3}\right)\cos\left(\frac{\pi}{2}+2i\pi\right)+i\cos\left(\frac{\pi}{2}+4i\pi\right)}{32\pi^3}}}{10^{19}}$$

Series representations:

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}-\frac{34+8}{10^3}}{10^{19}}=$$

$$\frac{-21\pi^{3/10}+250\sqrt{2}\sqrt[10]{\sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!}-2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}(2\pi)^{1+2k_1+2k_2}}{(2k_1)!(1+2k_2)!}}}{5\,000\,000\,000\,000\,000\,000\,000\,\pi^{3/10}}$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}-\frac{34+8}{10^3}}{10^{19}}=\left(-21\pi^{3/10}+250\sqrt{2}\right.$$

$$\left.\sqrt[10]{-i\left(-\sum_{k=0}^{\infty}\frac{\left(\left(4-\frac{i}{2}\right)\pi\right)^{2k}}{(2k)!}+2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}\,2^{2k_1-2k_2}\,(4-i)^{2k_2}\,\pi^{2k_1+2k_2}}{(2k_1)!\,(2k_2)!}\right)}\right)/$$

$$\left(5\,000\,000\,000\,000\,000\,000\,000\,\pi^{3/10}\right)$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}} - \frac{34+8}{10^3}}{10^{19}} = \frac{-21\pi^{3/10} + 250\sqrt{2}\sqrt[10]{\sum_{k=0}^{\infty}\frac{4^{1+k}\pi^{1+2k}\left(4^k-\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(1+2k)!}}}{5\,000\,000\,000\,000\,000\,000\,000\,\pi^{3/10}}$$

Integral representations:

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}} - \frac{34+8}{10^3}}{10^{19}} = \left(-21\sqrt[5]{\pi} + 250\times 2^{7/10}\sqrt[10]{\int_0^1\cosh(4\pi t)\,dt + \int_0^1\int_0^1\cos\left(\frac{1}{2}\left(1-4\sqrt{3}\right)\pi t_2\right)\cosh(2\pi t_1)\,dt_2\,dt_1}\right)\Big/\left(5\,000\,000\,000\,000\,000\,000\,000\sqrt[5]{\pi}\right)$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}} - \frac{34+8}{10^3}}{10^{19}} = \frac{-21\pi^{3/10} + 250\sqrt{2}\sqrt[10]{\int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{i e^{\pi^2/s+s}\sqrt{\pi}\left(e^{(3\pi^2)/s}+\int_{\frac{\pi}{2}}^{2\sqrt{3}\pi}\sin(t)\,dt\right)}{s^{3/2}}\,ds}}{5\,000\,000\,000\,000\,000\,000\,000\,\pi^{3/10}} \quad \text{for } \gamma > 0$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}} - \frac{34+8}{10^3}}{10^{19}} = \left(-21\pi^{3/10} + 250\sqrt{2}\sqrt[10]{\int_0^1\left(4\pi\cosh(4\pi t) + 2i\sqrt{\pi}\cosh(2\pi t)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-(3\pi^2)/s+s}}{\sqrt{s}}\,ds\right)dt}\right)\Big/\left(5\,000\,000\,000\,000\,000\,000\,000\,\pi^{3/10}\right) \quad \text{for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}-\frac{34+8}{10^3}}{10^{19}}=\frac{-\frac{21}{500}+\frac{\sqrt[10]{-4\left(-1+2\cos^2\left(\sqrt{3}\,\pi\right)\right)\cosh(\pi)\sinh(\pi)+2\cosh(2\pi)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}}}{10\,000\,000\,000\,000\,000\,000}$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}-\frac{34+8}{10^3}}{10^{19}}=\frac{-\frac{21}{500}+\frac{\sqrt[10]{-4\cosh(\pi)\left(1-2\sin^2\left(\sqrt{3}\,\pi\right)\right)\sinh(\pi)+2\cosh(2\pi)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}}}{10\,000\,000\,000\,000\,000\,000}$$

$$\frac{\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3 2^3}}-\frac{34+8}{10^3}}{10^{19}}=\frac{-\frac{21}{500}+\frac{\sqrt[10]{-2\left(-1+2\cos^2\left(\sqrt{3}\,\pi\right)\right)\left(3\sinh\left(\frac{2\pi}{3}\right)+4\sinh^3\left(\frac{2\pi}{3}\right)\right)+3\sinh\left(\frac{4\pi}{3}\right)+4\sinh^3\left(\frac{4\pi}{3}\right)}}{\sqrt{2}\,\pi^{3/10}}}{10\,000\,000\,000\,000\,000\,000}$$

In conclusion, subtracting $26/10^3$, where 26 is the dimensions number of a bosonic string, we obtain:

((((((sinh (4Pi) – 2sinh (2Pi) cos (2Pi*sqrt3)))) / ((4Pi^3*2^3)))))^1/10 - 26*1/10^3

Input:

$$\sqrt[10]{\frac{\sinh(4\pi)-(2\sinh(2\pi))\cos\left(2\pi\sqrt{3}\right)}{4\pi^3\times 2^3}}-26\times\frac{1}{10^3}$$

sinh(x) is the hyperbolic sine function

Exact result:

$$\frac{\sqrt[10]{\sinh(4\pi)-2\cos\left(2\sqrt{3}\,\pi\right)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}}-\frac{13}{500}$$

Decimal approximation:

1.618393807894373365341173754128337749773438326198684708086...

1.618393807894… result that is a very good approximation to the value of the golden ratio 1,618033988749...

Alternate forms:

$$\frac{250\sqrt{2}\sqrt[10]{\sinh(4\pi) - 2\cos(2\sqrt{3}\,\pi)\sinh(2\pi)} - 13\pi^{3/10}}{500\,\pi^{3/10}}$$

$$-\frac{13}{500} + \frac{\sqrt[10]{\sinh(4\pi) - \sinh(2\pi - 2i\sqrt{3}\,\pi) - \sinh(2\pi + 2i\sqrt{3}\,\pi)}}{\sqrt{2}\,\pi^{3/10}}$$

$$\frac{500\sqrt[10]{-e^{-4\pi} + e^{4\pi} - 4\cos(2\sqrt{3}\,\pi)\sinh(2\pi)} - 13\times 2^{3/5}\pi^{3/10}}{500\times 2^{3/5}\pi^{3/10}}$$

Alternative representations:

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}} - \frac{26}{10^3} = -\frac{26}{10^3} + \sqrt[10]{\frac{-\cosh(-2i\pi\sqrt{3})(-e^{-2\pi} + e^{2\pi}) + \frac{1}{2}(-e^{-4\pi} + e^{4\pi})}{32\pi^3}}$$

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}} - \frac{26}{10^3} = -\frac{26}{10^3} + \sqrt[10]{\frac{-\cosh(2i\pi\sqrt{3})(-e^{-2\pi} + e^{2\pi}) + \frac{1}{2}(-e^{-4\pi} + e^{4\pi})}{32\pi^3}}$$

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}} - \frac{26}{10^3} = -\frac{26}{10^3} + \sqrt[10]{\frac{-2i\cosh(-2i\pi\sqrt{3})\cos\left(\frac{\pi}{2} + 2i\pi\right) + i\cos\left(\frac{\pi}{2} + 4i\pi\right)}{32\pi^3}}$$

Series representations:

$$\sqrt[10]{\frac{\sinh(4\pi)-\cos\left(2\pi\sqrt{3}\right)2\sinh(2\pi)}{4\pi^3\,2^3}}-\frac{26}{10^3}=$$

$$\frac{-13\pi^{3/10}+250\sqrt{2}\sqrt[10]{\sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!}-2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-3)^{k_1}(2\pi)^{1+2k_1+2k_2}}{(2k_1)!(1+2k_2)!}}}{500\pi^{3/10}}$$

$$\sqrt[10]{\frac{\sinh(4\pi)-\cos\left(2\pi\sqrt{3}\right)2\sinh(2\pi)}{4\pi^3\,2^3}}-\frac{26}{10^3}=$$

$$\frac{-13\pi^{3/10}+250\sqrt{2}\sqrt[10]{\sum_{k=0}^{\infty}\frac{4^{1+k}\pi^{1+2k}\left(4^k-\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{3^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}{(1+2k)!}}}{500\pi^{3/10}}$$

$$\sqrt[10]{\frac{\sinh(4\pi)-\cos\left(2\pi\sqrt{3}\right)2\sinh(2\pi)}{4\pi^3\,2^3}}-\frac{26}{10^3}=$$

$$\frac{-13\pi^{3/10}+250\sqrt{2}\sqrt[10]{\sum_{k=0}^{\infty}\frac{(4\pi)^{1+2k}}{(1+2k)!}+2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_2}(2\pi)^{1+2k_1}\left(-\frac{\pi}{2}+2\sqrt{3}\pi\right)^{1+2k_2}}{(1+2k_1)!(1+2k_2)!}}}{500\pi^{3/10}}$$

Integral representations:

$$\sqrt[10]{\frac{\sinh(4\pi)-\cos\left(2\pi\sqrt{3}\right)2\sinh(2\pi)}{4\pi^3\,2^3}}-\frac{26}{10^3}=\frac{1}{500\sqrt[5]{\pi}}\left(-13\sqrt[5]{\pi}+250\times2^{7/10}\right.$$
$$\left.\sqrt[10]{\int_0^1\cosh(4\pi t)\,dt+\int_0^1\int_0^1\cos\left(\frac{1}{2}\left(1-4\sqrt{3}\right)\pi t_2\right)\cosh(2\pi t_1)\,dt_2\,dt_1}\right)$$

$$\sqrt[10]{\frac{\sinh(4\pi)-\cos\left(2\pi\sqrt{3}\right)2\sinh(2\pi)}{4\pi^3\,2^3}}-\frac{26}{10^3}=$$

$$\frac{-13\pi^{3/10}+250\sqrt{2}\sqrt[10]{\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,e^{\pi^2/s+s}\sqrt{\pi}\left(e^{(3\pi^2)/s}+\int_{\frac{\pi}{2}}^{2\sqrt{3}\pi}\sin(t)\,dt\right)}{s^{3/2}}\,ds}}{500\pi^{3/10}}\quad\text{for }\gamma>0$$

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}} - \frac{26}{10^3} = \frac{1}{500\,\pi^{3/10}}\left(-13\,\pi^{3/10} + 250\sqrt{2}\,\sqrt[10]{\int_0^1\left(4\pi\cosh(4\pi t) + 2\,i\sqrt{\pi}\,\cosh(2\pi t)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-(3\pi^2)/s+s}}{\sqrt{s}}\,ds\right)dt}\right)$$

for $\gamma > 0$

Multiple-argument formulas:

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}} - \frac{26}{10^3} = -\frac{13}{500} + \frac{\sqrt[10]{-4\left(-1 + 2\cos^2\left(\sqrt{3}\,\pi\right)\right)\cosh(\pi)\sinh(\pi) + 2\cosh(2\pi)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}}$$

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}} - \frac{26}{10^3} = -\frac{13}{500} + \frac{\sqrt[10]{-4\cosh(\pi)\left(1 - 2\sin^2\left(\sqrt{3}\,\pi\right)\right)\sinh(\pi) + 2\cosh(2\pi)\sinh(2\pi)}}{\sqrt{2}\,\pi^{3/10}}$$

$$\sqrt[10]{\frac{\sinh(4\pi) - \cos(2\pi\sqrt{3})\,2\sinh(2\pi)}{4\pi^3\,2^3}} - \frac{26}{10^3} = -\frac{13}{500} + \frac{\sqrt[10]{-2\left(-1 + 2\cos^2\left(\sqrt{3}\,\pi\right)\right)\left(3\sinh\left(\frac{2\pi}{3}\right) + 4\sinh^3\left(\frac{2\pi}{3}\right)\right) + 3\sinh\left(\frac{4\pi}{3}\right) + 4\sinh^3\left(\frac{4\pi}{3}\right)}}{\sqrt{2}\,\pi^{3/10}}$$

Conclusions

We highlight as in the development of this equation we have always utilized the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role to obtain the final results of the analyzed expression.

Furthermore, the Fibonacci and Lucas numbers are fundamental values that can be considered "constants", such as π and the golden ratio , that is, recurring numbers in various contexts: in the spiral arms of galaxies, as well as in Nature in general. This means that in the universe there is a mathematical order that has such constants as its foundation. Mathematics is therefore language, that is, as it was defined by philosophers, the "Logos" of the universe and all its laws that govern it. In other words, the universe, in addition to an observable physical reality, is also a mathematical and geometric entity.

References