

**On various Ramanujan's equations (Hardy-Ramanujan number, taxicab numbers, etc) linked to some parameters of Standard Model Particles and String Theory: New possible mathematical connections. V**

**Michele Nardelli<sup>1</sup>, Antonio Nardelli**

**Abstract**

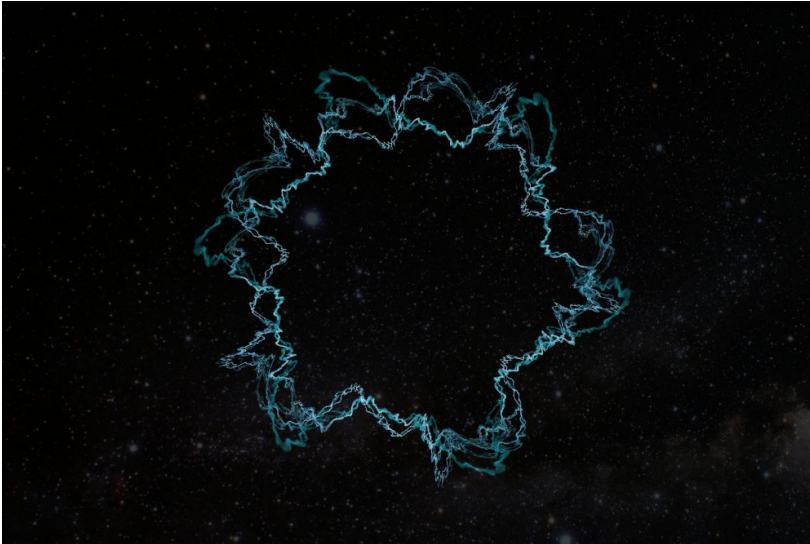
*In this research thesis, we have described and deepened further Ramanujan equations (Hardy-Ramanujan number, taxicab numbers, etc) linked to some parameters of Standard Model Particles and String Theory. We have therefore obtained further possible mathematical connections.*

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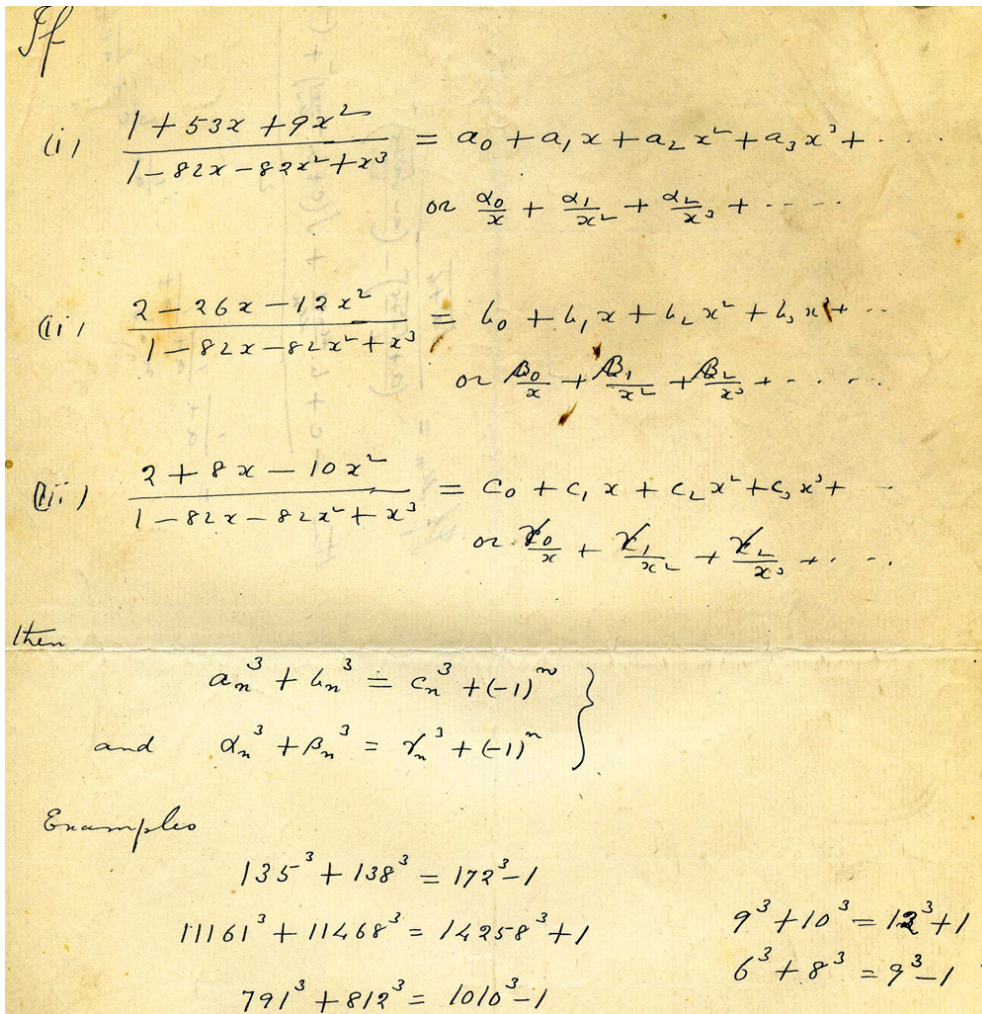
<sup>1</sup> M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy



<https://www.britannica.com/biography/Srinivasa-Ramanujan>



<https://futurism.com/brane-science-complex-notions-of-superstring-theory>



<https://plus.maths.org/content/ramanujan>

## Ramanujan's manuscript

The representations of 1729 as the sum of two cubes appear in the bottom right corner. The equation expressing the near counter examples to Fermat's last theorem appears further up:  $\alpha^3 + \beta^3 = \gamma^3 + (-1)^n$ .

From Wikipedia

The **taxicab number**, typically denoted  $Ta(n)$  or  $Taxicab(n)$ , also called the  $n$ th **Hardy–Ramanujan number**, is defined as the smallest integer that can be expressed as a sum of two positive integer cubes in  $n$  distinct ways. The most famous taxicab number is  $1729 = Ta(2) = 1^3 + 12^3 = 9^3 + 10^3$ .

From:

## Two-Field Born-Infeld with Diverse Dualities

*S. Ferrara, A. Sagnotti and A. Yeranyan* - arXiv:1602.04566v3 [hep-th] 8 Jul 2016

From:

$$\bar{\phi} = 6; \phi = 8; F = 9; \bar{F} = 10; V = 12; \bar{V} = 135; \nu = 138; \bar{\nu} = 172$$

$$9^3 + 10^3 = 12^3 + 1$$

$$6^3 + 8^3 = 9^3 - 1$$

$$135^3 + 138^3 = 172^3 - 1$$

$$F = 6; \bar{F} = 8; f = 9 \text{ and } \gamma = 10$$

$$\mathcal{L} = f^2 \left[ 1 - \sqrt{\left(1 + \frac{F^2 + \bar{F}^2}{2f^2}\right)^2 - \frac{1}{f^2} \sqrt{F^2 \bar{F}^2} \left(\frac{1}{f^2} \sqrt{F^2 \bar{F}^2} - \gamma\right)} \right. \quad (2.38)$$

$$\left. + \gamma \operatorname{ArcTanh} \left( \frac{1 + \frac{F^2 + \bar{F}^2}{2f^2} - \sqrt{\left(1 + \frac{F^2 + \bar{F}^2}{2f^2}\right)^2 - \frac{1}{f^2} \sqrt{F^2 \bar{F}^2} \left(\frac{1}{f^2} \sqrt{F^2 \bar{F}^2} - \gamma\right)}}{\frac{1}{f^2} \sqrt{F^2 \bar{F}^2} - \gamma} \right) \right],$$

$$81[1 - (((1 + (6^2 + 8^2)/(2 \cdot 9^2))^2 - 1/81 \cdot \sqrt{6^2 \cdot 8^2} \cdot (1/81 \cdot \sqrt{6^2 \cdot 8^2} - 10)))^{1/2} + 10 \operatorname{atanh} (((1 + (6^2 + 8^2)/(2 \cdot 9^2)) - (((1 + (6^2 + 8^2)/(2 \cdot 9^2))^2 - 1/81 \cdot \sqrt{6^2 \cdot 8^2} \cdot (1/81 \cdot \sqrt{6^2 \cdot 8^2} - 10)))^{1/2})) / ((1/81 \cdot \sqrt{6^2 \cdot 8^2} - 10))]$$

$$\left(\left(\left(1+\frac{6^2+8^2}{2 \times 9^2}\right)^2 - \frac{1}{81} \sqrt{6^2 \times 8^2} \left(\frac{1}{81} \sqrt{6^2 \times 8^2} - 10\right)\right)\right)^{1/2}$$

**Input:**

$$\sqrt{\left(1 + \frac{6^2 + 8^2}{2 \times 9^2}\right)^2 - \frac{1}{81} \sqrt{6^2 \times 8^2} \left(\frac{1}{81} \sqrt{6^2 \times 8^2} - 10\right)}$$

**Result:**

$$\frac{\sqrt{53737}}{81}$$

**Decimal approximation:**

2.861881779887940244147018014647189581730989623566768581840...

2.86188177988794

$$81 \left[ 1 - (2.86188177988794) + 10 \operatorname{atanh} \left( \frac{\left( \left( \left( 1 + \frac{6^2 + 8^2}{2 \times 9^2} \right) - (2.86188177988794) \right) \right)}{\left( \frac{1}{81} \sqrt{6^2 \times 8^2} - 10 \right)} \right) \right]$$

**Input interpretation:**

$$81 \left( 1 - 2.86188177988794 + 10 \operatorname{tanh}^{-1} \left( \frac{1 + \frac{6^2 + 8^2}{2 \times 9^2} - 2.86188177988794}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right)$$

$\operatorname{tanh}^{-1}(x)$  is the inverse hyperbolic tangent function

**Result:**

-43.017729954751...

-43.017729954751...

**Alternative representations:**

$$81 \left( 1 - 2.861881779887940000 + 10 \operatorname{tanh}^{-1} \left( \frac{1 + \frac{6^2 + 8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) =$$

$$81 \left( -1.861881779887940000 + 10 \operatorname{sn}^{-1} \left( \frac{-1.861881779887940000 + \frac{6^2 + 8^2}{2 \times 9^2}}{-10 + \frac{1}{81} \sqrt{6^2 \times 8^2}} \middle| 1 \right) \right)$$

$$\begin{aligned}
& 81 \left( 1 - 2.861881779887940000 + 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) = \\
& 81 \left( -1.861881779887940000 - 10 i \operatorname{sc}^{-1} \left( \frac{i(-1.861881779887940000 + \frac{6^2+8^2}{2 \times 9^2})}{-10 + \frac{1}{81} \sqrt{6^2 \times 8^2}} \right) \middle| 0 \right) \\
& 81 \left( 1 - 2.861881779887940000 + 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) = \\
& 81 \left( -1.861881779887940000 + 5 \left( -\log \left( 1 - \frac{-1.861881779887940000 + \frac{6^2+8^2}{2 \times 9^2}}{-10 + \frac{1}{81} \sqrt{6^2 \times 8^2}} \right) + \right. \right. \\
& \quad \left. \left. \log \left( 1 + \frac{-1.861881779887940000 + \frac{6^2+8^2}{2 \times 9^2}}{-10 + \frac{1}{81} \sqrt{6^2 \times 8^2}} \right) \right) \right)
\end{aligned}$$

**Series representations:**

$$\begin{aligned}
& 81 \left( 1 - 2.861881779887940000 + 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) = \\
& -150.8124241709231400 + 810.0000000000000000 \\
& \sum_{k=0}^{\infty} \frac{100.8124241709231400^{1+2k} \left( -\frac{1.0000000000000000}{-810.0000000000000000 + \sqrt{2304}} \right)^{1+2k}}{1+2k}
\end{aligned}$$

$$\begin{aligned}
& 81 \left( 1 - 2.861881779887940000 + 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) = \\
& -150.8124241709231400 - 405 \log(2) + \\
& 405 \log \left( \frac{-910.812424170923140 + \sqrt{2304}}{-810.0000000000000000 + \sqrt{2304}} \right) + \\
& 405 \sum_{k=1}^{\infty} \frac{0.50000000000000000000^k \left( \frac{-910.812424170923140 + \sqrt{2304}}{-810.0000000000000000 + \sqrt{2304}} \right)^k}{k}
\end{aligned}$$

$$\begin{aligned}
& 81 \left( 1 - 2.861881779887940000 + 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) = \\
& -150.8124241709231400 + 405.0000000000000000 \log(2) - \\
& 405.0000000000000000 \log \left( \frac{-709.187575829076860 + \sqrt{2304}}{-810.0000000000000000 + \sqrt{2304}} \right) - \\
& 405.0000000000000000 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(\frac{709.187575829076860 - 1.0000000000000000 \sqrt{2304}}{-810.0000000000000000 + \sqrt{2304}}\right)^k}{k}
\end{aligned}$$

**Integral representations:**

$$\begin{aligned}
& 81 \left( 1 - 2.861881779887940000 + 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) = \\
& \frac{122\,158.063578447743}{-810.0000000000000000 + \sqrt{2304}} - \frac{81\,658.063578447743}{-810.0000000000000000 + \sqrt{2304}} \\
& \int_0^1 \frac{1}{1 - \frac{10\,163.1448672181282\,t^2}{(-810.0000000000000000 + \sqrt{2304})^2}} dt - \frac{150.812424170923140\,\sqrt{2304}}{-810.0000000000000000 + \sqrt{2304}}
\end{aligned}$$

$$\begin{aligned}
& 81 \left( 1 - 2.861881779887940000 + 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) = \\
& -150.8124241709231400 + \\
& \frac{20\,414.5158946119359\,i}{\pi^{3/2} (-810.0000000000000000 + \sqrt{2304})} \int_{-i\infty+\gamma}^{i\infty+\gamma} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s)^2 \\
& \left( 1 - \frac{10\,163.14486721812815}{(-810.0000000000000000 + \sqrt{2304})^2} \right)^{-s} ds \text{ for } 0 < \gamma < \frac{1}{2}
\end{aligned}$$



We have that:

$$-3 \times 81 \left[ 1 - (2.86188177988794) + 10 \operatorname{atanh} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.86188177988794}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right] + 47 - 4$$

**Input interpretation:**

$$-3 \times 81 \left( 1 - 2.86188177988794 + 10 \operatorname{tanh}^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.86188177988794}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) + 47 - 4$$

$\operatorname{tanh}^{-1}(x)$  is the inverse hyperbolic tangent function

**Result:**

172.05318986425...

172.05318986425...  $\approx$  172 (Ramanujan taxicab number)

**Alternative representations:**

$$-3 \times 81 \left( 1 - 2.861881779887940000 + 10 \operatorname{tanh}^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) + 47 - 4 = 43 - 243 \left( -1.861881779887940000 + 10 \operatorname{sn}^{-1} \left( \frac{-1.861881779887940000 + \frac{6^2+8^2}{2 \times 9^2}}{-10 + \frac{1}{81} \sqrt{6^2 \times 8^2}} \middle| 1 \right) \right)$$

$$-3 \times 81 \left( 1 - 2.861881779887940000 + 10 \operatorname{tanh}^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) + 47 - 4 = 43 - 243 \left( -1.861881779887940000 - 10 i \operatorname{sc}^{-1} \left( \frac{i(-1.861881779887940000 + \frac{6^2+8^2}{2 \times 9^2})}{-10 + \frac{1}{81} \sqrt{6^2 \times 8^2}} \middle| 0 \right) \right)$$



$$\begin{aligned}
& -3 \times 81 \left( 1 - 2.861881779887940000 + \right. \\
& \quad \left. 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) + 47 - 4 = 43 - \\
& 243 \left( -1.861881779887940000 + 5 \left( -\log \left( 1 - \frac{-1.861881779887940000 + \frac{6^2+8^2}{2 \times 9^2}}{-10 + \frac{1}{81} \sqrt{6^2 \times 8^2}} \right) \right) + \right. \\
& \quad \left. \log \left( 1 + \frac{-1.861881779887940000 + \frac{6^2+8^2}{2 \times 9^2}}{-10 + \frac{1}{81} \sqrt{6^2 \times 8^2}} \right) \right)
\end{aligned}$$

### Series representations:

$$\begin{aligned}
& -3 \times 81 \left( 1 - 2.861881779887940000 + \right. \\
& \quad \left. 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) + 47 - 4 = \\
& 495.437272512769420 - 2430.0000000000000000 \\
& \sum_{k=0}^{\infty} \frac{100.8124241709231400^{1+2k} \left( -\frac{1.0000000000000000}{-810.0000000000000000 + \sqrt{2304}} \right)^{1+2k}}{1+2k}
\end{aligned}$$

$$\begin{aligned}
& -3 \times 81 \left( 1 - 2.861881779887940000 + \right. \\
& \quad \left. 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) + 47 - 4 = \\
& 495.437272512769420 + 1215.0000000000000000 \log(2) - \\
& 1215.0000000000000000 \log \left( \frac{-910.812424170923140 + \sqrt{2304}}{-810.0000000000000000 + \sqrt{2304}} \right) - \\
& 1215.0000000000000000 \\
& \sum_{k=1}^{\infty} \frac{0.50000000000000000000^k \left( \frac{-910.812424170923140 + \sqrt{2304}}{-810.0000000000000000 + \sqrt{2304}} \right)^k}{k}
\end{aligned}$$

$$\begin{aligned}
& -3 \times 81 \left( 1 - 2.861881779887940000 + \right. \\
& \quad \left. 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) + 47 - 4 = \\
& 495.437272512769420 - 1215.0000000000000000 \log(2) + \\
& 1215.0000000000000000 \log \left( \frac{-709.187575829076860 + \sqrt{2304}}{-810.0000000000000000 + \sqrt{2304}} \right) + \\
& 1215.0000000000000000 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(\frac{709.187575829076860 - 1.0000000000000000 \sqrt{2304}}{-810.0000000000000000 + \sqrt{2304}}\right)^k}{k}
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& -3 \times 81 \left( 1 - 2.861881779887940000 + \right. \\
& \quad \left. 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) + 47 - 4 = \\
& \frac{401\,304.190735343}{-810.0000000000000000 + \sqrt{2304}} + \frac{244\,974.190735343}{-810.0000000000000000 + \sqrt{2304}} \\
& \int_0^1 \frac{1}{1 - \frac{10\,163.1448672181282\,t^2}{(-810.0000000000000000 + \sqrt{2304})^2}} dt + \frac{495.437272512769\,\sqrt{2304}}{-810.0000000000000000 + \sqrt{2304}}
\end{aligned}$$

$$\begin{aligned}
& -3 \times 81 \left( 1 - 2.861881779887940000 + \right. \\
& \quad \left. 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) + 47 - 4 = \\
& \frac{401\,304.19073534323}{-810.0000000000000000 + \sqrt{2304}} - \frac{61\,243.547683835808\,i}{\pi^{3/2} (-810.0000000000000000 + \sqrt{2304})} \\
& \int_{-i\infty+\gamma}^{i\infty+\gamma} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1 - s) \Gamma(s)^2 \left(1 - \frac{10\,163.14486721812815}{(-810.0000000000000000 + \sqrt{2304})^2}\right)^{-s} ds + \\
& \frac{495.43727251276942\,\sqrt{2304}}{-810.0000000000000000 + \sqrt{2304}} \quad \text{for } 0 < \gamma < \frac{1}{2}
\end{aligned}$$

$$-1/7((((81[1-(2.86188177988794) + 10 \operatorname{atanh} ((((((1+(6^2+8^2))/(2*9^2)-(2.86188177988794)))))) / ((1/81*\sqrt{6^2*8^2}-10)))))))]))\wedge 3 + 89 + 7$$

**Input interpretation:**

$$-\frac{1}{7} \left( 81 \left( 1 - 2.86188177988794 + 10 \operatorname{tanh}^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.86188177988794}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) \right)^3 + 89 + 7$$

$\operatorname{tanh}^{-1}(x)$  is the inverse hyperbolic tangent function

**Result:**

11468.19837370...

11468.1983737...  $\approx$  11468 (Ramanujan taxicab number)

**Alternative representations:**

$$\frac{1}{7} \left( 81 \left( 1 - 2.861881779887940000 + 10 \operatorname{tanh}^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) \right)^3 (-1) + 89 + 7 = 96 - \frac{1}{7} \left( 81 \left( -1.861881779887940000 + 10 \operatorname{sn}^{-1} \left( \frac{-1.861881779887940000 + \frac{6^2+8^2}{2 \times 9^2}}{-10 + \frac{1}{81} \sqrt{6^2 \times 8^2}} \middle| 1 \right) \right) \right)^3$$

$$\frac{1}{7} \left( 81 \left( 1 - 2.861881779887940000 + \right. \right. \\ \left. \left. 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) \right)^3 (-1) + \\ 89 + 7 = 96 - \frac{1}{7} \left( 81 \left( -1.861881779887940000 - \right. \right. \\ \left. \left. 10 \operatorname{sc}^{-1} \left( \frac{i \left( -1.861881779887940000 + \frac{6^2+8^2}{2 \times 9^2} \right)}{-10 + \frac{1}{81} \sqrt{6^2 \times 8^2}} \right) \right) \right)^3 \left. \right)$$

$$\frac{1}{7} \left( 81 \left( 1 - 2.861881779887940000 + \right. \right. \\ \left. \left. 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) \right)^3 (-1) + 89 + 7 = \\ 96 - \frac{1}{7} \left( 81 \left( -1.861881779887940000 + 10 \coth^{-1} \left( \frac{1}{\frac{-1.861881779887940000 + \frac{6^2+8^2}{2 \times 9^2}}{-10 + \frac{1}{81} \sqrt{6^2 \times 8^2}}} \right) \right) \right)^3 \right)$$

### Series representations:

$$\frac{1}{7} \left( 81 \left( 1 - 2.861881779887940000 + \right. \right. \\ \left. \left. 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) \right)^3 (-1) + 89 + 7 = \\ 96 - 7.59201428571428571 \times 10^7 \left( -0.1861881779887940000 + \right. \\ \left. \sum_{k=0}^{\infty} \frac{100.8124241709231400^{1+2k} \left( -\frac{1.0000000000000000}{-810.0000000000000000 + \sqrt{2304}} \right)^{1+2k}}{1+2k} \right)^3 \right)$$

$$\frac{1}{7} \left( 81 \left( 1 - 2.861881779887940000 + \right. \right. \\ \left. \left. 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) \right)^3 \\ (-1) + 89 + 7 = 96 + 9.49001785714285714 \times 10^6 \\ \left( 0.3723763559775880000 + \log(2) - 1.0000000000000000000 \right. \\ \left. \log \left( \frac{-910.812424170923140 + \sqrt{2304}}{-810.0000000000000000 + \sqrt{2304}} \right) - 1.0000000000000000000 \right. \\ \left. \sum_{k=1}^{\infty} \frac{0.5000000000000000000^k \left( \frac{-910.812424170923140 + \sqrt{2304}}{-810.0000000000000000 + \sqrt{2304}} \right)^k}{k} \right)^3$$

$$\frac{1}{7} \left( 81 \left( 1 - 2.861881779887940000 + \right. \right. \\ \left. \left. 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) \right)^3 \\ (-1) + 89 + 7 = 96 - 9.49001785714285714 \times 10^6 \\ \left( -0.3723763559775880000 + \log(2) - 1.0000000000000000000 \right. \\ \left. \log \left( \frac{-709.187575829076860 + \sqrt{2304}}{-810.0000000000000000 + \sqrt{2304}} \right) - 1.0000000000000000000 \right. \\ \left. \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left( \frac{709.187575829076860 - 1.0000000000000000000 \sqrt{2304}}{-810.0000000000000000 + \sqrt{2304}} \right)^k}{k} \right)^3$$

**Integral representations:**

$$\frac{1}{7} \left( 81 \left( 1 - 2.861881779887940000 + \right. \right. \\ \left. \left. 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) \right)^3 (-1) + 89 + 7 = \\ 96 + \left( 7.77855972817279 \times 10^{13} \left( -1.495970614844329 + \right. \right. \\ \left. \left. 1.0000000000000000 \int_0^1 \frac{1}{1 - \frac{10 \ 163.1448672181282 t^2}{(-810.00000000000000 + \sqrt{2304})^2}} dt + \right. \right. \\ \left. \left. 0.001846877302276949 \sqrt{2304} \right) \right)^3 / \\ (-810.00000000000000 + \sqrt{2304})^3$$

$$\frac{1}{7} \left( 81 \left( 1 - 2.861881779887940000 + \right. \right. \\ \left. \left. 10 \tanh^{-1} \left( \frac{1 + \frac{6^2+8^2}{2 \times 9^2} - 2.861881779887940000}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10} \right) \right) \right)^3 (-1) + \\ 89 + 7 = 96 - \frac{531441}{7} \left( -1.861881779887940000 + \right. \\ \left. \frac{3.111494573176640123 i}{\pi^{3/2} \left( -10 + \frac{\sqrt{2304}}{81} \right)} \int_{-i \infty + \gamma}^{i \infty + \gamma} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1 - s) \Gamma(s)^2 \right. \\ \left. \left( 1 - \frac{1.549023756625229104}{\left( -10 + \frac{\sqrt{2304}}{81} \right)^2} \right)^{-s} ds \right)^3 \text{ for } 0 < \gamma < \frac{1}{2}$$

$$(-81[1-(2.86188177988794) + 10 \operatorname{atanh} (\frac{(((1+(6^2+8^2))/(2*9^2)-(2.86188177988794))))}{((1/81*\sqrt{6^2*8^2}-10))})]^{(64*2)/10^3})$$

**Input interpretation:**

$$\left( -81 \left( 1 - 2.86188177988794 + 10 \operatorname{tanh}^{-1} \left( \frac{1 + \frac{6^2+8^2}{2*9^2} - 2.86188177988794}{\frac{1}{81} \sqrt{6^2 \times 8^2} - 10}} \right) \right) \right)^{(64 \times 2) / 10^3}$$

$\operatorname{tanh}^{-1}(x)$  is the inverse hyperbolic tangent function

**Result:**

1.618478291345236343849011468058325401351447944122400678325...

1.6184782913... result that is a very good approximation to the value of the golden ratio 1,618033988749...

1.6184782913452363438490114680583254013514479441224006

**Input interpretation:**

1.6184782913452363438490114680583254013514479441224006

1.6184782913...

**Rational approximation:**

$$\frac{574081862166558393704516348}{354704703323142342218284581} = 1 + \frac{219377158843416051486231767}{354704703323142342218284581}$$

**Possible closed forms:**

$$\cosh\left(\sinh\left(\frac{8411398}{9100445}\right)\right) \approx 1.6184782913452363440579$$

$$\left(\frac{41531845}{21856069}\right)^{3/4} \approx 1.618478291345236371937$$

$$\frac{2581 - 8048e + 3077e^2}{782e} \approx 1.6184782913452363442987$$

$$\frac{\log\left(\frac{157732589}{2826767}\right)}{\log(12)} \approx 1.618478291345236343864055$$



$$\frac{4012714503 \pi}{7788991963} \approx 1.618478291345236343871470$$

$$\boxed{\text{root of } 9146 x^3 - 57908 x^2 + 55621 x + 22892 \text{ near } x = 1.61848} \approx 1.618478291345236343852718$$

$$\pi \boxed{\text{root of } 2599 x^4 + 2899 x^3 + 2620 x^2 - 2888 x + 213 \text{ near } x = 0.515178} \approx 1.618478291345236343860362$$

$$\boxed{\text{root of } 496 x^5 - 516 x^4 - 346 x^3 + 133 x^2 + 66 x - 956 \text{ near } x = 1.61848} \approx 1.618478291345236343869206$$

$$\pi \boxed{\text{root of } 51239 x^3 + 136775 x^2 + 7267 x - 47051 \text{ near } x = 0.515178} \approx 1.618478291345236343851053$$

$$\frac{1}{\boxed{\text{root of } 22892 x^3 + 55621 x^2 - 57908 x + 9146 \text{ near } x = 0.617864}} \approx 1.618478291345236343852718$$

$$\boxed{\text{root of } 3295 x^4 - 7313 x^3 + 2616 x^2 - 559 x + 2447 \text{ near } x = 1.61848} \approx 1.61847829134523634384989152$$

$$\pi \boxed{\text{root of } 1824 x^5 - 530 x^4 - 222 x^3 + 165 x^2 - 909 x + 426 \text{ near } x = 0.515178} \approx 1.6184782913452363438434703$$

$$\frac{1}{\boxed{\text{root of } 2447 x^4 - 559 x^3 + 2616 x^2 - 7313 x + 3295 \text{ near } x = 0.617864}} \approx 1.61847829134523634384989152$$

$$\frac{3 \times 3^{1219/1860} e^{(683 \gamma)/310}}{8 \times 2^{2131/2790}} \approx 1.6184782913452363423785$$

$$\frac{645 + 686 \pi - 285 \pi^2}{-602 + 142 \pi + 15 \pi^2} \approx 1.61847829134523625287$$

From:

With our choices one can now revert to the ordinary variables  $\phi^{kl}$ , solving eq. (3.49) for  $a$  with  $h_1$  as in (3.51) and substituting in the Lagrangian (3.52). The end result (with the scale  $f$  of eq. (1.1) set to one for brevity),

$$\mathcal{L} = 1 - \sqrt{(1 + \text{Re}[\phi_t])^2 - |\phi_t|^2 - \text{Det}[\phi - \bar{\phi}] + 2 \left( \text{Re}[\phi_d] - \sqrt{|\phi_d|^2} \right)}. \quad (3.53)$$

has  $U(2)$  duality and reduces to the BI theory if the two Abelian field strengths coincide.

$$\mathcal{L} = 1 - \sqrt{(1 + \text{Re}[\phi_t])^2 - |\phi_t|^2 - \text{Det}[\phi - \bar{\phi}] + 2 \left( \text{Re}[\phi_d] - \sqrt{|\phi_d|^2} \right)}$$

$$9^3 + 10^3 = 12^3 + 1$$

$$135^3 + 138^3 = 172^3 - 1$$

$$\phi_t = 9; \phi_d = 10; \phi = 138; \bar{\phi} = 135$$

$$1 - \sqrt{((1 + \text{Re}(9)))^2 - 9^2 - \text{Det}\{\{1, 138 - 135\}, \{138 - 135, 1\}\} + 2(\text{Re}(10) - \sqrt{10^2})}$$

**Input interpretation:**

$$1 - \sqrt{(1 + \text{Re}(9))^2 - 9^2 - \begin{vmatrix} 1 & 138 - 135 \\ 138 - 135 & 1 \end{vmatrix} + 2 \left( \text{Re}(10) - \sqrt{10^2} \right)}$$

$\text{Re}(z)$  is the real part of  $z$

$|m|$  is the determinant

**Result:**

$$1 - 3\sqrt{3}$$

**Decimal approximation:**

-4.19615242270663188058233902451761710082841576143114188416...

-4.1961524227...

$$-\left[\left(\left(\left(1-\sqrt{\left(\left(1+\operatorname{Re}(9)\right)^2-9^2-\operatorname{Det}\left\{\left\{1, 138-135\right\}, \left\{138-135, 1\right\}\right\}+2\left(\operatorname{Re}(10)-\sqrt{10^2}\right)\right)\right)\right)^5+(144 \times 2+3)\right]$$

**Input interpretation:**

$$-\left(\left(1-\sqrt{\left(1+\operatorname{Re}(9)\right)^2-9^2-\left|\begin{array}{cc} 1 & 138-135 \\ 138-135 & 1 \end{array}\right|+2\left(\operatorname{Re}(10)-\sqrt{10^2}\right)}\right)^5+(144 \times 2+3)\right)$$

Re(z) is the real part of z  
|m| is the determinant

**Result:**

$$-291-\left(1-3 \sqrt{3}\right)^5$$

**Decimal approximation:**

1009.937032397458408104668380615687569231729424476866451704...

1009.937... ≈ 1010 (Ramanujan taxicab number)

**Alternate form:**

$$3012 \sqrt{3}-4207$$

$$144+89+8+3+\left(\left(-2 \cdot\left[\left(\left(1-\sqrt{\left(\left(1+\operatorname{Re}(9)\right)^2-9^2-\operatorname{Det}\left\{\left\{1, 138-135\right\}, \left\{138-135, 1\right\}\right\}+2\left(\operatorname{Re}(10)-\sqrt{10^2}\right)\right)\right)\right]^6\right)\right)$$

**Input interpretation:**

$$144+89+8+3-2 \times(-1)\left(1-\sqrt{\left(1+\operatorname{Re}(9)\right)^2-9^2-\left|\begin{array}{cc} 1 & 138-135 \\ 138-135 & 1 \end{array}\right|+2\left(\operatorname{Re}(10)-\sqrt{10^2}\right)}\right)^6$$

Re(z) is the real part of z  
|m| is the determinant

**Result:**

$$244+2\left(1-3 \sqrt{3}\right)^6$$

**Decimal approximation:**

11161.86016056674229506978399874664772784838890751756385979...

11161.8601605... ≈ 11161 (Ramanujan taxicab number)

**Alternate forms:**

$$62292 - 29520\sqrt{3}$$

$$-12(2460\sqrt{3} - 5191)$$

-34(((1-sqrt[((1+Re(9))^2-9^2-Det{{1, 138-135}, {138-135, 1}})+2(Re(10)-sqrt(10^2))])))) - 18 + 1/golden ratio

**Input interpretation:**

$$-34 \left( 1 - \sqrt{(1 + \operatorname{Re}(9))^2 - 9^2 - \begin{vmatrix} 1 & 138 - 135 \\ 138 - 135 & 1 \end{vmatrix} + 2(\operatorname{Re}(10) - \sqrt{10^2})} \right) - 18 + \frac{1}{\phi}$$

Re(z) is the real part of z  
|m| is the determinant  
φ is the golden ratio

**Result:**

$$\frac{1}{\phi} - 18 - 34(1 - 3\sqrt{3})$$

**Decimal approximation:**

125.2872163607753787880041136679646195458864450684645869238...

125.28721636... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternate forms:**

$$\frac{1}{\phi} - 52 + 102\sqrt{3}$$

$$\frac{1}{\phi} - 18 + 34(3\sqrt{3} - 1)$$

$$\frac{2(51\sqrt{3} - 26)\phi + 1}{\phi}$$

$$-5 + 27 \times \frac{1}{2} \left( \left( -34 \left( \left( 1 - \sqrt{(1 + \operatorname{Re}(9))^2 - 9^2} - \left| \begin{matrix} 1 & 138 - 135 \\ 138 - 135 & 1 \end{matrix} \right| + 2 \left( \operatorname{Re}(10) - \sqrt{10^2} \right) \right) \right) \right) - 18 + \pi + \frac{1}{\phi} \right)$$

**Input interpretation:**

$$-5 + 27 \times \frac{1}{2} \left( -34 \left( 1 - \sqrt{(1 + \operatorname{Re}(9))^2 - 9^2} - \left| \begin{matrix} 1 & 138 - 135 \\ 138 - 135 & 1 \end{matrix} \right| + 2 \left( \operatorname{Re}(10) - \sqrt{10^2} \right) \right) - 18 + \pi + \frac{1}{\phi} \right)$$

Re(z) is the real part of z  
|m| is the determinant  
φ is the golden ratio

**Result:**

$$\frac{27}{2} \left( \frac{1}{\phi} - 18 - 34 \left( 1 - 3 \sqrt{3} \right) + \pi \right) - 5$$

**Decimal approximation:**

1728.788921693929822357301220191795652806128795315835852054...  
1728.78892169...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Property:**

$-5 + \frac{27}{2} \left( -18 - 34 \left( 1 - 3 \sqrt{3} \right) + \frac{1}{\phi} + \pi \right)$  is a transcendental number

**Alternate forms:**

$$\frac{27}{2} \left( \frac{1}{\phi} - 52 + 102 \sqrt{3} + \pi \right) - 5$$

$$\frac{27}{2\phi} - 707 + 1377 \sqrt{3} + \frac{27\pi}{2}$$

$$\frac{27}{2} \left( \frac{1}{\phi} - 18 + 34(3\sqrt{3} - 1) + \pi \right) - 5$$

Now, we have that:

Reverting to the field strengths, the Lagrangian takes finally the form

$$\mathcal{L} = 1 - \sqrt{(1 + \text{Re}[\phi_t])^2 - |\phi_t|^2 - \text{Det}[\phi - \bar{\phi}]} . \quad (3.63)$$

From

$$\mathcal{L} = 1 - \sqrt{(1 + \text{Re}[\phi_t])^2 - |\phi_t|^2 - \text{Det}[\phi - \bar{\phi}]} .$$

For  $\phi_t = 9$ ;  $\phi_d = 10$ ;  $\phi = 138$ ;  $\bar{\phi} = 135$ , we obtain:

$$1 - \sqrt{((1 + \text{Re}(9)))^2 - 9^2 - \text{Det}\{\{1, 138 - 135\}, \{138 - 135, 1\}\}}$$

**Input interpretation:**

$$1 - \sqrt{(1 + \text{Re}(9))^2 - 9^2 - \begin{vmatrix} 1 & 138 - 135 \\ 138 - 135 & 1 \end{vmatrix}}$$

$\text{Re}(z)$  is the real part of  $z$

$|m|$  is the determinant

**Result:**

$$1 - 3\sqrt{3}$$

**Decimal approximation:**

-4.19615242270663188058233902451761710082841576143114188416...

-4.1961524227..... the same previous result

We have also:

$$\left(-\left(\left(1-\sqrt{\left(\left(1+\operatorname{Re}(9)\right)^2-9^2-\operatorname{Det}\left\{\left\{1, 138-135\right\}, \left\{138-135, 1\right\}\right\}\right)}\right)\right)^{1/3}+5 \times \frac{1}{10^3}\right)$$

**Input interpretation:**

$$\sqrt[3]{-\left(1-\sqrt{\left(1+\operatorname{Re}(9)\right)^2-9^2-\left|\begin{array}{cc} 1 & 138-135 \\ 138-135 & 1 \end{array}\right|}\right)}+5 \times \frac{1}{10^3}$$

$\operatorname{Re}(z)$  is the real part of  $z$   
 $|m|$  is the determinant

**Result:**

$$\frac{1}{200}+\sqrt[3]{3 \sqrt{3}-1}$$

**Decimal approximation:**

1.617935813642020182463303405226893817920083356882506337493...

1.617935813642.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

**Alternate form:**

$$\frac{1}{200}\left(1+200 \sqrt[3]{3 \sqrt{3}-1}\right)$$

We have that:

In terms of the field strengths, the Lagrangian becomes

$$\mathcal{L}=1-\sqrt{\left(1+\operatorname{Re}\left[\phi_t\right]\right)^2-\left|\phi_t\right|^2} . \quad (3.67)$$

$$1-\sqrt{\left(\left(1+\operatorname{Re}(9)\right)^2-9^2\right)}$$

**Input:**

$$1-\sqrt{\left(1+\operatorname{Re}(9)\right)^2-9^2}$$

$\operatorname{Re}(z)$  is the real part of  $z$



**Exact result:**

$$1 - \sqrt{19}$$

**Decimal approximation:**

-3.35889894354067355223698198385961565913700392523244493689...

-3.3588989435...

**Alternative representations:**

$$1 - \sqrt{(1 + \operatorname{Re}(9))^2 - 9^2} = 1 - \sqrt{-9^2 + (1 + \operatorname{Im}(9i))^2}$$

$$1 - \sqrt{(1 + \operatorname{Re}(9))^2 - 9^2} = 1 - \sqrt{-9^2 + (1 - \operatorname{Im}(-9i))^2}$$

$$1 - \sqrt{(1 + \operatorname{Re}(9))^2 - 9^2} = 1 - \sqrt{-9^2 + (10 - i \operatorname{Im}(9))^2}$$

**Series representations:**

$$1 - \sqrt{(1 + \operatorname{Re}(9))^2 - 9^2} = 1 - \sqrt{-82 + (1 + \operatorname{Re}(9))^2} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (-82 + (1 + \operatorname{Re}(9))^2)^{-k}$$

$$1 - \sqrt{(1 + \operatorname{Re}(9))^2 - 9^2} = 1 - \sqrt{-82 + (1 + \operatorname{Re}(9))^2} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-82 + (1 + \operatorname{Re}(9))^2)^{-k}}{k!}$$

$$1 - \sqrt{(1 + \operatorname{Re}(9))^2 - 9^2} = 1 - \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-81 + (1 + \operatorname{Re}(9))^2 - z_0)^k z_0^{-k}}{k!}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

Multiplying the previous result by -0.481715587144498 that is equal to:

$$1/233 * -(((76-4) \pi) - (322+29+7) * 1/\pi)$$

we obtain:

$$1/233 * -(((76-4) \pi) - (322+29+7) * 1/\pi) * (((1 - \sqrt{(1 + \operatorname{Re}(9))^2 - 9^2}))^2)$$

**Input:**

$$\frac{1}{233} \times (-1) \left( (76 - 4) \pi - (322 + 29 + 7) \times \frac{1}{\pi} \right) \left( 1 - \sqrt{(1 + \operatorname{Re}(9))^2 - 9^2} \right)^2$$

$\text{Re}(z)$  is the real part of  $z$

**Exact result:**

$$-\frac{1}{233} (1 - \sqrt{19}) \left( 72\pi - \frac{358}{\pi} \right)$$

**Decimal approximation:**

1.618033976746729868559323994611158393657325290039278466390...

1.618033976746... result that is the value of the golden ratio 1,618033988749...

**Property:**

$$-\frac{1}{233} (1 - \sqrt{19}) \left( -\frac{358}{\pi} + 72\pi \right) \text{ is a transcendental number}$$

**Alternate forms:**

$$\frac{1}{233} (\sqrt{19} - 1) \left( 72\pi - \frac{358}{\pi} \right)$$

$$-\frac{2(\sqrt{19} - 1)(179 - 36\pi^2)}{233\pi}$$

$$\frac{2(\sqrt{19} - 1)(36\pi^2 - 179)}{233\pi}$$

**Alternative representations:**

$$-\frac{1}{233} \left( (76 - 4)\pi - \frac{322 + 29 + 7}{\pi} \right) \left( 1 - \sqrt{(1 + \text{Re}(9))^2 - 9^2} \right) =$$

$$-\frac{1}{233} \left( 72\pi - \frac{358}{\pi} \right) \left( 1 - \sqrt{-9^2 + (1 + \text{Im}(9i))^2} \right)$$

$$-\frac{1}{233} \left( (76 - 4)\pi - \frac{322 + 29 + 7}{\pi} \right) \left( 1 - \sqrt{(1 + \text{Re}(9))^2 - 9^2} \right) =$$

$$-\frac{1}{233} \left( 72\pi - \frac{358}{\pi} \right) \left( 1 - \sqrt{-9^2 + (1 - \text{Im}(-9i))^2} \right)$$

$$-\frac{1}{233} \left( (76 - 4)\pi - \frac{322 + 29 + 7}{\pi} \right) \left( 1 - \sqrt{(1 + \text{Re}(9))^2 - 9^2} \right) =$$

$$-\frac{1}{233} \left( 72\pi - \frac{358}{\pi} \right) \left( 1 - \sqrt{-9^2 + (10 - i \text{Im}(9))^2} \right)$$

**Series representations:**

$$\frac{-\frac{1}{233} \left( (76-4)\pi - \frac{322+29+7}{\pi} \right) \left( 1 - \sqrt{(1+\operatorname{Re}(9))^2 - 9^2} \right) + 2(-179+36\pi^2) \left( -1 + \sqrt{-82+(1+\operatorname{Re}(9))^2} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (-82+(1+\operatorname{Re}(9))^2)^{-k} \right)}{233\pi} =$$

$$\frac{-\frac{1}{233} \left( (76-4)\pi - \frac{322+29+7}{\pi} \right) \left( 1 - \sqrt{(1+\operatorname{Re}(9))^2 - 9^2} \right) + 2(-179+36\pi^2) \left( -1 + \sqrt{-82+(1+\operatorname{Re}(9))^2} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-82+(1+\operatorname{Re}(9))^2)^{-k}}{k!} \right)}{233\pi} =$$

$$\frac{-\frac{1}{233} \left( (76-4)\pi - \frac{322+29+7}{\pi} \right) \left( 1 - \sqrt{(1+\operatorname{Re}(9))^2 - 9^2} \right) + 2(-179+36\pi^2) \left( -1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (-81+(1+\operatorname{Re}(9))^2 - z_0)^k z_0^{-k}}{k!} \right)}{233\pi} =$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

Now, we have that:

$$\mathcal{L} = 1 - \sqrt{\left[ 1 + \frac{1}{4} (\mathcal{F}^+ \cdot \mathcal{F}^-) \right]^2 - \frac{1}{32} C - \frac{1}{32} \sqrt{D}}, \quad (5.8-5.9)$$

$$C = \left| (\mathcal{F}^+)^2 \right|^2 + (\mathcal{F}^+ \cdot \mathcal{F}^-)^2 + \left| \mathcal{F}^+ \cdot \tilde{\mathcal{F}}^- \right|^2 + \left| \mathcal{F}^+ \cdot \tilde{\mathcal{F}}^+ \right|^2,$$

$$(2^2)^2 + (2 \times 3)^2 + (2 \times 5)^2 + (2 \times 8)^2$$

$$(2^2)^2 + (2 \times 3)^2 + (2 \times 5)^2 + (2 \times 8)^2$$

408

C = 408

$$D = \left[ (\mathcal{F}^+ \cdot \mathcal{F}^-)^2 - (\mathcal{F}^+ \cdot \tilde{\mathcal{F}}^-)^2 + \left| \mathcal{F}^+ \cdot \tilde{\mathcal{F}}^+ \right|^2 - \left| \mathcal{F}^{+2} \right|^2 \right]^2 + \left[ (\mathcal{F}^+)^2 (\mathcal{F}^- \cdot \tilde{\mathcal{F}}^-) + (\mathcal{F}^-)^2 (\mathcal{F}^+ \cdot \tilde{\mathcal{F}}^+) - 2 (\mathcal{F}^+ \cdot \mathcal{F}^-) (\mathcal{F}^+ \cdot \tilde{\mathcal{F}}^-) \right]^2 \quad (5.10)$$

$$[(2 \cdot 3)^2 - (2 \cdot 5)^2 + (2 \cdot 8)^2 - (2^2)^2]^2$$

$$(2 \times 3)^2 - (2 \times 5)^2 + (2 \times 8)^2 - (2^2)^2$$

$$30976$$

$$[2^2 \cdot (3 \cdot 5) + 3^2 \cdot (2 \cdot 8) - 2 \cdot (2 \cdot 3) \cdot (2 \cdot 5)]^2$$

$$(2^2 (3 \times 5) + 3^2 (2 \times 8) - 2 (2 \times 3) (2 \times 5))^2$$

$$7056$$

$$((((2 \cdot 3)^2 - (2 \cdot 5)^2 + (2 \cdot 8)^2 - (2^2)^2)))^2 + (((2^2 \cdot (3 \cdot 5) + 3^2 \cdot (2 \cdot 8) - 2 \cdot (2 \cdot 3) \cdot (2 \cdot 5))))^2$$

$$((2 \times 3)^2 - (2 \times 5)^2 + (2 \times 8)^2 - (2^2)^2)^2 + (2^2 (3 \times 5) + 3^2 (2 \times 8) - 2 (2 \times 3) (2 \times 5))^2$$

$$38032$$

$$D = 38032$$

Thence:

$$\mathcal{L} = 1 - \sqrt{\left[1 + \frac{1}{4} (\mathcal{F}^+ \cdot \mathcal{F}^-)\right]^2 - \frac{1}{32} C - \frac{1}{32} \sqrt{D}},$$

$$1 - \sqrt{(((1 + 1/4(2 \cdot 3))^2 - 1/32(408) - 1/32(\sqrt{38032})))}$$

**Input:**

$$1 - \sqrt{\left(1 + \frac{1}{4} (2 \times 3)\right)^2 - \frac{1}{32} \times 408 - \frac{1}{32} \sqrt{38032}}$$

**Result:**

$$1 - i \sqrt{\frac{13}{2} + \frac{\sqrt{2377}}{8}}$$

**Decimal approximation:**

$$1 -$$

$$3.54884641420623512949851258564743971100517368738485736186... i$$

**Polar coordinates:**

$$r \approx 3.68705 \text{ (radius), } \theta \approx -74.2631^\circ \text{ (angle)}$$

$$3.68705$$

**Alternate forms:**

$$\frac{1}{4} \left( 4 - i \sqrt{2 \left( 52 + \sqrt{2377} \right)} \right)$$

$$1 - \frac{1}{2} i \sqrt{\frac{1}{2} \left( 52 + \sqrt{2377} \right)}$$

$$1 + \boxed{\text{root of } 64x^4 + 832x^2 + 327 \text{ near } x = -3.54885 i}$$

**Minimal polynomial:**

$$64x^4 - 256x^3 + 1216x^2 - 1920x + 1223$$

$\left( \left( \left( \left( 1 - \sqrt{2 \left( \left( 1 + \frac{1}{4} (2 \times 3) \right)^2 - \frac{1}{32} (408) - \frac{1}{32} \sqrt{38032} \right)} \right) \right) \right) \right)^4 - 55i + (\text{golden ratio})i$

**Input:**

$$\left( 1 - \sqrt{\left( 1 + \frac{1}{4} (2 \times 3) \right)^2 - \frac{1}{32} \times 408 - \frac{1}{32} \sqrt{38032}} \right)^4 - 55i + \phi i$$

*i* is the imaginary unit

$\phi$  is the golden ratio

**Result:**

$$i\phi + -55i + \left( 1 - i \sqrt{\frac{13}{2} + \frac{\sqrt{2377}}{8}} \right)^4$$

**Decimal approximation:**

84.0508011013711709689604386111978578060140271317272306163... +  
111.203748236577129136527174054238412070089415979349593844... *i*

**Polar coordinates:**

$r \approx 139.394$  (radius),  $\theta \approx 52.917^\circ$  (angle)

139.394 result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternate forms:**

$$\frac{1}{256} \left( -256 i \sqrt{2(52 + \sqrt{2377})} + i 128 \sqrt{5} + 224 \sqrt{2377} + \right. \\ \left. i 32 \sqrt{2(511420 + 10489 \sqrt{2377})} + 10596 - 13952 i \right)$$

$$i \phi + -55 i + \frac{1}{256} \left( \sqrt{2(52 + \sqrt{2377})} + 4 i \right)^4$$

$$-55 i + \frac{1}{2} i (1 + \sqrt{5}) + \left( 1 - i \sqrt{\frac{13}{2} + \frac{\sqrt{2377}}{8}} \right)^4$$

**Minimal polynomial:**

$$79\,228\,162\,514\,264\,337\,593\,543\,950\,336\,x^{16} - \\ 52\,468\,850\,625\,071\,557\,571\,324\,481\,110\,016\,x^{15} + \\ 25\,603\,209\,435\,281\,972\,755\,860\,841\,943\,793\,664\,x^{14} - \\ 78\,116\,591\,997\,443\,192\,924\,480\,648\,689\,116\,774\,400\,x^{13} + \\ 1\,889\,513\,057\,074\,708\,850\,625\,002\,823\,926\,260\,170\,752\,x^{12} - \\ 331\,445\,056\,901\,235\,858\,699\,716\,289\,180\,316\,137\,947\,136\,x^{11} + \\ 47\,408\,412\,254\,625\,986\,730\,031\,814\,559\,813\,076\,286\,177\,280\,x^{10} - \\ 5\,126\,006\,746\,536\,899\,430\,283\,499\,907\,416\,593\,546\,516\,365\,312\,x^9 + \\ 485\,223\,526\,076\,130\,174\,516\,112\,041\,544\,827\,864\,936\,731\,377\,664\,x^8 - \\ 35\,496\,972\,632\,655\,962\,563\,131\,854\,178\,692\,921\,904\,465\,860\,100\,096\,x^7 + \\ 254\,250\,626\,159\,616\,257\,397\,980\,117\,251\,627\,245\,182\,534\,906\,019\,840\,x^6 - \\ 122\,685\,740\,194\,384\,795\,631\,175\,853\,162\,642\,773\,133\,485\,017\,715\,965\,952\,x^5 + \\ 73\,090\,251\,012\,783\,128\,408\,838\,417\,287\,673\,007\,110\,222\,693\,629\,864\,968\,192\,x^4 - \\ 208\,213\,217\,324\,652\,462\,788\,311\,546\,797\,027\,091\,890\,904\,626\,705\,224\,171\,520\,x^3 + \\ 11\,033\,513\,561\,385\,470\,011\,447\,927\,667\,651\,262\,861\,637\,666\,903\,505\,862\,885\,376 \\ x^2 - \\ 138\,643\,452\,011\,937\,923\,815\,051\,003\,108\,090\,761\,479\,435\,795\,558\,312\,273\,421\,312 \\ x + \\ 68\,622\,390\,171\,824\,230\,171\,112\,684\,822\,140\,702\,761\,241\,186\,549\,848\,175\,935\,164\,801$$

**Expanded form:**

$$\left( \frac{2649}{64} - \frac{109 i}{2} \right) + \frac{i \sqrt{5}}{2} + \frac{7 \sqrt{2377}}{8} + \\ 22 i \sqrt{\frac{13}{2} + \frac{\sqrt{2377}}{8}} + \frac{1}{2} i \sqrt{2377 \left( \frac{13}{2} + \frac{\sqrt{2377}}{8} \right)}$$

**Series representations:**

$$\left(1 - \sqrt{\left(1 + \frac{2 \times 3}{4}\right)^2 - \frac{408}{32} - \frac{\sqrt{38032}}{32}}\right)^4 - i55 + \phi i =$$

$$-55i + \phi i + \left(-1 + \sqrt{-\frac{15}{2} - \frac{\sqrt{38032}}{32}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-\frac{15}{2} - \frac{\sqrt{38032}}{32}\right)^{-k}\right)^4$$

$$\left(1 - \sqrt{\left(1 + \frac{2 \times 3}{4}\right)^2 - \frac{408}{32} - \frac{\sqrt{38032}}{32}}\right)^4 - i55 + \phi i =$$

$$-55i + \phi i + \left(-1 + \sqrt{-\frac{15}{2} - \frac{\sqrt{38032}}{32}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{15}{2} - \frac{\sqrt{38032}}{32}\right)^{-k}}{k!}\right)^4$$

$$\left(1 - \sqrt{\left(1 + \frac{2 \times 3}{4}\right)^2 - \frac{408}{32} - \frac{\sqrt{38032}}{32}}\right)^4 - i55 + \phi i =$$

$$-55i + \phi i + \left(-1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{13}{2} - \frac{\sqrt{38032}}{32} - z_0\right)^k z_0^{-k}}{k!}\right)^4$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\left(\left(\left(\left(1 - \sqrt{\left(\left(1 + \frac{1}{4}(2 \times 3)\right)^2 - \frac{1}{32}(408) - \frac{1}{32}\sqrt{38032}\right)}\right)\right)\right)\right)^4 - 55i - 13i - \pi i$$

**Input:**

$$\left(1 - \sqrt{\left(1 + \frac{1}{4}(2 \times 3)\right)^2 - \frac{1}{32} \times 408 - \frac{1}{32} \sqrt{38032}}\right)^4 - 55i - 13i - \pi i$$

$i$  is the imaginary unit

**Result:**

$$-68i + \left(1 - i \sqrt{\frac{13}{2} + \frac{\sqrt{2377}}{8}}\right)^4 - i\pi$$

**Decimal approximation:**

84.0508011013711709689604386111978578060140271317272306163... +  
93.4441215942374410498599438365932710681719374001687251611...  $i$



**Property:**

$$-68i + \left(1 - i\sqrt{\frac{13}{2} + \frac{\sqrt{2377}}{8}}\right)^4 - i\pi \text{ is a transcendental number}$$

**Polar coordinates:**

$$r \approx 125.683 \text{ (radius), } \theta \approx 48.0294^\circ \text{ (angle)}$$

125.683 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternate forms:**

$$\frac{1}{256} \left( -256i\sqrt{2(52 + \sqrt{2377})} + 224\sqrt{2377} + \right. \\ \left. i32\sqrt{2}\sqrt{511420 + 10489\sqrt{2377}} - 256i\pi + 10596 - 17408i \right)$$

$$-68i + \frac{1}{256} \left( \sqrt{2(52 + \sqrt{2377})} + 4i \right)^4 - i\pi$$

$$\frac{1}{64} \left( (2649 - 4352i) + 56\sqrt{2377} + \right. \\ \left. 352i\sqrt{2(52 + \sqrt{2377})} + 8i\sqrt{4754(52 + \sqrt{2377})} \right) - i\pi$$

**Expanded form:**

$$\left( \frac{2649}{64} - 68i \right) + \frac{7\sqrt{2377}}{8} + 22i\sqrt{\frac{13}{2} + \frac{\sqrt{2377}}{8}} + \frac{1}{2}i\sqrt{2377\left(\frac{13}{2} + \frac{\sqrt{2377}}{8}\right)} - i\pi$$

**Series representations:**

$$\left( 1 - \sqrt{\left(1 + \frac{2 \times 3}{4}\right)^2 - \frac{408}{32} - \frac{\sqrt{38032}}{32}} \right)^4 - i55 - i13 - i\pi = \\ -68i - i\pi + \left( -1 + \sqrt{-\frac{15}{2} - \frac{\sqrt{38032}}{32}} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left( -\frac{15}{2} - \frac{\sqrt{38032}}{32} \right)^{-k} \right)^4$$

$$\left(1 - \sqrt{\left(1 + \frac{2 \times 3}{4}\right)^2 - \frac{408}{32} - \frac{\sqrt{38032}}{32}}\right)^4 - i55 - i13 - i\pi =$$

$$-68i - i\pi + \left(-1 + \sqrt{-\frac{15}{2} - \frac{\sqrt{38032}}{32}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{15}{2} - \frac{\sqrt{38032}}{32}\right)^{-k}}{k!}\right)^4$$

$$\left(1 - \sqrt{\left(1 + \frac{2 \times 3}{4}\right)^2 - \frac{408}{32} - \frac{\sqrt{38032}}{32}}\right)^4 - i55 - i13 - i\pi =$$

$$-68i - i\pi + \left(-1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-\frac{13}{2} - \frac{\sqrt{38032}}{32} - z_0\right)^k z_0^{-k}}{k!}\right)^4$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

From:

### **Integrable Scalar Cosmologies I. Foundations and links with String Theory**

*P. Fre , A. Sagnotti and A.S. Sorin - arXiv:1307.1910v3 [hep-th] 16 Oct 2013*

Depending on the choice made for the real exponent  $\gamma$ , these potentials can describe barriers or wells of different shapes, and the presence of the second term restricts in general the domain to the region  $\varphi > 0$ . For the sake of brevity and simplicity, we shall concentrate on a special but very significant case of potential wells, with  $\gamma = \frac{1}{3}$ , which affords relatively handy solutions in terms of elliptic functions. The potentials that we would like to discuss here in detail are thus

with  $\lambda > 0$ , since a relative factor between the two exponentials can clearly be absorbed into a shift of  $\varphi$ . One can also assume, without any loss of generality, that  $0 < \gamma < 1$ , so that the first

$$\mathcal{V}_{IIIa}(\varphi) = \frac{\lambda}{16} \left[ \left(1 - \frac{1}{3\sqrt{3}}\right) e^{-6\varphi/5} + \left(7 + \frac{1}{\sqrt{3}}\right) e^{-2\varphi/5} \right] \quad (5.17)$$

$$+ \left(7 - \frac{1}{\sqrt{3}}\right) e^{2\varphi/5} + \left(1 + \frac{1}{3\sqrt{3}}\right) e^{6\varphi/5} \Big]. \quad (5.18)$$

From the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

We put for  $\varphi > 0$   $\varphi = 4$  and for  $\lambda > 0$   $\lambda = 0.9991104$ , an obtain:

$$0.9991104/16[(1-1/(3\sqrt{3}))e^{(-24/5)}+(7+1/(\sqrt{3}))e^{(-8/5)}+(7-1/(\sqrt{3}))e^{(8/5)}+(1+1/(3\sqrt{3}))e^{(24/5)}]$$

**Input interpretation:**

$$\frac{0.9991104}{16} \left( \left(1 - \frac{1}{3\sqrt{3}}\right) e^{-24/5} + \left(7 + \frac{1}{\sqrt{3}}\right) e^{-8/5} + \left(7 - \frac{1}{\sqrt{3}}\right) e^{8/5} + \left(1 + \frac{1}{3\sqrt{3}}\right) e^{24/5} \right)$$

**Result:**

11.13029...

11.13029...

**Series representations:**

$$\frac{1}{16} \left( \left(1 - \frac{1}{3\sqrt{3}}\right) e^{-24/5} + \left(7 + \frac{1}{\sqrt{3}}\right) e^{-8/5} + \left(7 - \frac{1}{\sqrt{3}}\right) e^{8/5} + \left(1 + \frac{1}{3\sqrt{3}}\right) e^{24/5} \right) 0.99911 =$$

$$\frac{1}{e^{24/5}} \left( 0.0624444 + 0.437111 e^{16/5} + \right.$$

$$\left. 0.437111 e^{32/5} + 0.0624444 e^{48/5} + \frac{0.0208148 (-1 + e^{16/5})^3}{\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}} \right)$$

$$\frac{1}{16} \left( \left(1 - \frac{1}{3\sqrt{3}}\right) e^{-24/5} + \left(7 + \frac{1}{\sqrt{3}}\right) e^{-8/5} + \left(7 - \frac{1}{\sqrt{3}}\right) e^{8/5} + \left(1 + \frac{1}{3\sqrt{3}}\right) e^{24/5} \right) 0.99911 =$$

$$\frac{1}{e^{24/5}} \left( 0.0624444 + 0.437111 e^{16/5} + \right.$$

$$\left. 0.437111 e^{32/5} + 0.0624444 e^{48/5} + \frac{0.0208148 (-1 + e^{16/5})^3}{\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)_k \left(-\frac{1}{2}\right)_k}{k!}} \right)$$

$$\frac{1}{16} \left( \left(1 - \frac{1}{3\sqrt{3}}\right) e^{-24/5} + \left(7 + \frac{1}{\sqrt{3}}\right) e^{-8/5} + \left(7 - \frac{1}{\sqrt{3}}\right) e^{8/5} + \left(1 + \frac{1}{3\sqrt{3}}\right) e^{24/5} \right) 0.99911 =$$

$$\frac{1}{e^{24/5}} \left( 0.0624444 + 0.437111 e^{16/5} + 0.437111 e^{32/5} + \right.$$

$$\left. 0.0624444 e^{48/5} + \frac{0.0416296 (-1 + e^{16/5})^3 \sqrt{\pi}}{\sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)} \right)$$

$$\left( \left( \left( \left( \left( \left( \frac{0.9991104}{16} \left[ \left(1 - \frac{1}{3\sqrt{3}}\right) e^{-24/5} + \left(7 + \frac{1}{\sqrt{3}}\right) e^{-8/5} + \left(7 - \frac{1}{\sqrt{3}}\right) e^{8/5} + \left(1 + \frac{1}{3\sqrt{3}}\right) e^{24/5} \right] \right) \right)^2 + 11 + \left(\frac{1}{\sqrt{3}}\right)^3 \right) \right) \right) \right)$$

**Input interpretation:**

$$\left( \frac{0.9991104}{16} \left( \left(1 - \frac{1}{3\sqrt{3}}\right) e^{-24/5} + \left(7 + \frac{1}{\sqrt{3}}\right) e^{-8/5} + \right. \right.$$

$$\left. \left. \left(7 - \frac{1}{\sqrt{3}}\right) e^{8/5} + \left(1 + \frac{1}{3\sqrt{3}}\right) e^{24/5} \right) \right)^2 + 11 + \left(\frac{1}{\sqrt{3}}\right)^3$$

**Result:**

135.0758...

135.0758...  $\approx$  135 (Ramanujan taxicab number)

**Series representations:**

$$\left(\frac{1}{16} \left( \left(1 - \frac{1}{3\sqrt{3}}\right) e^{-24/5} + \left(7 + \frac{1}{\sqrt{3}}\right) e^{-8/5} + \left(7 - \frac{1}{\sqrt{3}}\right) e^{8/5} + \left(1 + \frac{1}{3\sqrt{3}}\right) e^{24/5} \right) \right. \\ \left. 0.99911 \right)^2 + 11 + \left(\frac{1}{\sqrt{3}}\right)^3 = 11 + \frac{8\sqrt{\pi}^3}{\left(\sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)\right)^3} + \\ \left(0.00173302 \left( (-1 + e^{16/5})^3 \sqrt{\pi} + (1.5 + 10.5 e^{16/5} + 10.5 e^{32/5} + 1.5 e^{48/5}) \right. \right. \\ \left. \left. \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right)^2 \right) / \\ \left( e^{48/5} \left( \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right)^2 \right)$$

$$\left(\frac{1}{16} \left( \left(1 - \frac{1}{3\sqrt{3}}\right) e^{-24/5} + \left(7 + \frac{1}{\sqrt{3}}\right) e^{-8/5} + \left(7 - \frac{1}{\sqrt{3}}\right) e^{8/5} + \left(1 + \frac{1}{3\sqrt{3}}\right) e^{24/5} \right) \right. \\ \left. 0.99911 \right)^2 + 11 + \left(\frac{1}{\sqrt{3}}\right)^3 = \\ \left(0.0038993 \left( 256.456 e^{48/5} + 0.111111 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} - \right. \right. \\ 0.666667 e^{16/5} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} + 1.666667 e^{32/5} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} - \\ 2.222222 e^{48/5} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} + 1.666667 e^{64/5} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} - \\ 0.666667 e^{16} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} + 0.111111 e^{96/5} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} - \\ 0.666667 \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 - 2.666667 e^{16/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 + \\ 7.333333 e^{32/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 - 7.333333 e^{64/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 + \\ 2.666667 e^{16} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 + 0.666667 e^{96/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 + \\ \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + 14 e^{16/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + \\ 63 e^{32/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + 2921.02 e^{48/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + \\ 63 e^{64/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + 14 e^{16} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + \\ \left. \left. e^{96/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 \right) / \left( e^{48/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 \right) \right)$$

$$\begin{aligned}
& \left( \frac{1}{16} \left( \left( 1 - \frac{1}{3\sqrt{3}} \right) e^{-24/5} + \left( 7 + \frac{1}{\sqrt{3}} \right) e^{-8/5} + \left( 7 - \frac{1}{\sqrt{3}} \right) e^{8/5} + \left( 1 + \frac{1}{3\sqrt{3}} \right) e^{24/5} \right) \right. \\
& \quad \left. 0.999911 \right)^2 + 11 + \left( \frac{1}{\sqrt{3}} \right)^3 = \\
& \left( 0.0038993 \left( 256.456 e^{48/5} + 0.111111 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \right. \right. \\
& \quad 0.666667 e^{16/5} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 1.66667 e^{32/5} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad 2.22222 e^{48/5} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 1.66667 e^{64/5} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad 0.666667 e^{16} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 0.111111 e^{96/5} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad 0.666667 \sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 - 2.66667 e^{16/5} \sqrt{2}^2 \\
& \quad \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 + 7.33333 e^{32/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 - 7.33333 \\
& \quad e^{64/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 + 2.66667 e^{16} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 + \\
& \quad 0.666667 e^{96/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 + \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + \\
& \quad 14 e^{16/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + 63 e^{32/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + \\
& \quad 2921.02 e^{48/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + 63 e^{64/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + \\
& \quad \left. \left. 14 e^{16} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + e^{96/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 \right) \right) / \\
& \left( e^{48/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 \right)
\end{aligned}$$

(((0.9991104/16[(1-1/(3sqrt3))\*e^(-24/5)+(7+1/(sqrt3))\*e^(-8/5)+(7-1/(sqrt3))\*e^(8/5)+(1+1/(3sqrt3))\*e^(24/5)]))^(2+13+(1/(sqrt3))^3 + golden ratio^2

**Input interpretation:**

$$\left(\frac{0.9991104}{16} \left( \left(1 - \frac{1}{3\sqrt{3}}\right) e^{-24/5} + \left(7 + \frac{1}{\sqrt{3}}\right) e^{-8/5} + \left(7 - \frac{1}{\sqrt{3}}\right) e^{8/5} + \left(1 + \frac{1}{3\sqrt{3}}\right) e^{24/5} \right)\right)^2 + 13 + \left(\frac{1}{\sqrt{3}}\right)^3 + \phi^2$$

ϕ is the golden ratio

**Result:**

139.6938...

139.6938... result practically equal to the rest mass of Pion meson 139.57 MeV

**Series representations:**

$$\left(\frac{1}{16} \left( \left(1 - \frac{1}{3\sqrt{3}}\right) e^{-24/5} + \left(7 + \frac{1}{\sqrt{3}}\right) e^{-8/5} + \left(7 - \frac{1}{\sqrt{3}}\right) e^{8/5} + \left(1 + \frac{1}{3\sqrt{3}}\right) e^{24/5} \right)\right)^2 + 13 + \left(\frac{1}{\sqrt{3}}\right)^3 + \phi^2 =$$

$$13 + \phi^2 + \frac{8\sqrt{\pi}^3}{\left(\sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)\right)^3} +$$

$$\left(0.00173302 \left( (-1 + e^{16/5})^3 \sqrt{\pi} + (1.5 + 10.5 e^{16/5} + 10.5 e^{32/5} + 1.5 e^{48/5}) \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right)^2 \right) /$$

$$\left( e^{48/5} \left( \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right)^2 \right)$$



$$\begin{aligned}
& \left( \frac{1}{16} \left( \left( 1 - \frac{1}{3\sqrt{3}} \right) e^{-24/5} + \left( 7 + \frac{1}{\sqrt{3}} \right) e^{-8/5} + \left( 7 - \frac{1}{\sqrt{3}} \right) e^{8/5} + \left( 1 + \frac{1}{3\sqrt{3}} \right) e^{24/5} \right) \right. \\
& \quad \left. 0.999911 \right)^2 + 13 + \left( \frac{1}{\sqrt{3}} \right)^3 + \phi^2 = \\
& \left( 0.0038993 \left( 256.456 e^{48/5} + 0.111111 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right) - \right. \\
& \quad 0.666667 e^{16/5} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} + 1.666667 e^{32/5} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} - \\
& \quad 2.22222 e^{48/5} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} + 1.666667 e^{64/5} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} - \\
& \quad 0.666667 e^{16} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} + 0.111111 e^{96/5} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} - \\
& \quad 0.666667 \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 - 2.666667 e^{16/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 + \\
& \quad 7.33333 e^{32/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 - \\
& \quad 7.33333 e^{64/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 + 2.666667 e^{16} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 + \\
& \quad 0.666667 e^{96/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 + \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + \\
& \quad 14 e^{16/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + 63 e^{32/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + \\
& \quad 3433.93 e^{48/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + 63 e^{64/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + \\
& \quad 14 e^{16} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + e^{96/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + \\
& \quad \left. 256.456 e^{48/5} \phi^2 \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 \right) / \left( e^{48/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{1}{16} \left( \left( 1 - \frac{1}{3\sqrt{3}} \right) e^{-24/5} + \left( 7 + \frac{1}{\sqrt{3}} \right) e^{-8/5} + \left( 7 - \frac{1}{\sqrt{3}} \right) e^{8/5} + \left( 1 + \frac{1}{3\sqrt{3}} \right) e^{24/5} \right) \right. \\
& \quad \left. 0.999911 \right)^2 + 13 + \left( \frac{1}{\sqrt{3}} \right)^3 + \phi^2 = \\
& \left( 0.0038993 \left( 256.456 e^{48/5} + 0.111111 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \right. \right. \\
& \quad 0.666667 e^{16/5} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 1.66667 e^{32/5} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad 2.22222 e^{48/5} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 1.66667 e^{64/5} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad 0.666667 e^{16} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 0.111111 e^{96/5} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad 0.666667 \sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 - 2.66667 e^{16/5} \sqrt{2}^2 \\
& \quad \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 + 7.33333 e^{32/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 - 7.33333 \\
& \quad e^{64/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 + 2.66667 e^{16} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 + \\
& \quad 0.666667 e^{96/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 + \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + \\
& \quad 14 e^{16/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + 63 e^{32/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + \\
& \quad 3433.93 e^{48/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + 63 e^{64/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + \\
& \quad 14 e^{16} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + e^{96/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + \\
& \quad \left. \left. 256.456 e^{48/5} \phi^2 \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 \right) \right) / \\
& \left( e^{48/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 \right)
\end{aligned}$$

$(((((0.9991104/16[(1-1/(3\sqrt{3}))^*e^{(-24/5)}+(7+1/(\sqrt{3}))^*e^{(-8/5)}+(7-1/(\sqrt{3}))^*e^{(8/5)}+(1+1/(3\sqrt{3}))^*e^{(24/5)})]))))^2+(1/(\sqrt{3}))^3 + \text{golden ratio}$

**Input interpretation:**

$$\left(\frac{0.9991104}{16} \left( \left(1 - \frac{1}{3\sqrt{3}}\right) e^{-24/5} + \left(7 + \frac{1}{\sqrt{3}}\right) e^{-8/5} + \left(7 - \frac{1}{\sqrt{3}}\right) e^{8/5} + \left(1 + \frac{1}{3\sqrt{3}}\right) e^{24/5} \right)^2 + \left(\frac{1}{\sqrt{3}}\right)^3 + \phi$$

$\phi$  is the golden ratio

**Result:**

125.6938...

125.6938... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Series representations:**

$$\left(\frac{1}{16} \left( \left(1 - \frac{1}{3\sqrt{3}}\right) e^{-24/5} + \left(7 + \frac{1}{\sqrt{3}}\right) e^{-8/5} + \left(7 - \frac{1}{\sqrt{3}}\right) e^{8/5} + \left(1 + \frac{1}{3\sqrt{3}}\right) e^{24/5} \right) 0.99911 \right)^2 + \left(\frac{1}{\sqrt{3}}\right)^3 + \phi = \phi + \frac{8\sqrt{\pi}^3}{\left(\sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)\right)^3} + \left(0.00173302 \left( (-1 + e^{16/5})^3 \sqrt{\pi} + (1.5 + 10.5 e^{16/5} + 10.5 e^{32/5} + 1.5 e^{48/5}) \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right)^2 \right) / \left( e^{48/5} \left( \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right)^2 \right)$$

$$\begin{aligned}
& \left( \frac{1}{16} \left( \left( 1 - \frac{1}{3\sqrt{3}} \right) e^{-24/5} + \left( 7 + \frac{1}{\sqrt{3}} \right) e^{-8/5} + \left( 7 - \frac{1}{\sqrt{3}} \right) e^{8/5} + \left( 1 + \frac{1}{3\sqrt{3}} \right) e^{24/5} \right) \right. \\
& \quad \left. 0.999911 \right)^2 + \left( \frac{1}{\sqrt{3}} \right)^3 + \phi = \\
& \left( 0.0038993 \left( 256.456 e^{48/5} + 0.111111 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right) - \right. \\
& \quad 0.666667 e^{16/5} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} + 1.666667 e^{32/5} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} - \\
& \quad 2.222222 e^{48/5} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} + 1.666667 e^{64/5} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} - \\
& \quad 0.666667 e^{16} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} + 0.111111 e^{96/5} \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} - \\
& \quad 0.666667 \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 - 2.666667 e^{16/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 + \\
& \quad 7.333333 e^{32/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 - \\
& \quad 7.333333 e^{64/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 + 2.666667 e^{16} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 + \\
& \quad 0.666667 e^{96/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^2 + \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + \\
& \quad 14 e^{16/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + 63 e^{32/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + \\
& \quad 100 e^{48/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + 63 e^{64/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + \\
& \quad 14 e^{16} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + e^{96/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 + \\
& \quad \left. 256.456 e^{48/5} \phi \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 \right) / \left( e^{48/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^3 \right)
\end{aligned}$$

$$\begin{aligned}
& \left( \frac{1}{16} \left( \left( 1 - \frac{1}{3\sqrt{3}} \right) e^{-24/5} + \left( 7 + \frac{1}{\sqrt{3}} \right) e^{-8/5} + \left( 7 - \frac{1}{\sqrt{3}} \right) e^{8/5} + \left( 1 + \frac{1}{3\sqrt{3}} \right) e^{24/5} \right) \right. \\
& \quad \left. 0.999911 \right)^2 + \left( \frac{1}{\sqrt{3}} \right)^3 + \phi = \\
& \left( 0.0038993 \left( 256.456 e^{48/5} + 0.111111 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \right. \right. \\
& \quad 0.666667 e^{16/5} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 1.66667 e^{32/5} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad 2.22222 e^{48/5} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 1.66667 e^{64/5} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad 0.666667 e^{16} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + 0.111111 e^{96/5} \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad 0.666667 \sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 - 2.66667 e^{16/5} \sqrt{2}^2 \\
& \quad \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 + 7.33333 e^{32/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 - \\
& \quad 7.33333 e^{64/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 + 2.66667 e^{16} \sqrt{2}^2 \\
& \quad \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 + 0.666667 e^{96/5} \sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2 + \\
& \quad \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + 14 e^{16/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + \\
& \quad 63 e^{32/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + 100 e^{48/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + \\
& \quad 63 e^{64/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + 14 e^{16} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + \\
& \quad \left. \left. e^{96/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 + 256.456 e^{48/5} \phi \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 \right) \right) / \\
& \left( e^{48/5} \sqrt{2}^3 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^3 \right)
\end{aligned}$$



$$\begin{aligned}
& \frac{1}{2} \sqrt{729} \left( \left( \frac{1}{16} \times 0.99911 \left( \left( 1 - \frac{1}{3\sqrt{3}} \right) e^{-24/5} + \left( 7 + \frac{1}{\sqrt{3}} \right) e^{-8/5} + \left( 7 - \frac{1}{\sqrt{3}} \right) e^{8/5} + \right. \right. \right. \\
& \quad \left. \left. \left. \left( 1 + \frac{1}{3\sqrt{3}} \right) e^{24/5} \right) \right)^2 + 4 + \left( \frac{1}{\sqrt{3}} \right)^2 \right) - 2 = -2 + \frac{1}{2} \sqrt{729} \\
& \left( 4 + 0.0038993 \left( e^{8/5} \left( 7 - \frac{1}{\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} \right) + \frac{1 - \frac{1}{3\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}}{e^{24/5}} + \right. \right. \right. \\
& \quad \left. \left. e^{24/5} \left( 1 + \frac{1}{3\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} \right) + \frac{7 + \frac{1}{\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}}{e^{8/5}} \right)^2 + \right. \\
& \quad \left. \frac{1}{\sqrt{2}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^2} \right) \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{728}\right)^k \left(-\frac{1}{2}\right)_k}{k!}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \sqrt{729} \left( \left( \frac{1}{16} \times 0.99911 \left( \left( 1 - \frac{1}{3\sqrt{3}} \right) e^{-24/5} + \left( 7 + \frac{1}{\sqrt{3}} \right) e^{-8/5} + \right. \right. \right. \\
& \quad \left. \left. \left. \left( 7 - \frac{1}{\sqrt{3}} \right) e^{8/5} + \left( 1 + \frac{1}{3\sqrt{3}} \right) e^{24/5} \right) \right)^2 + 4 + \left( \frac{1}{\sqrt{3}} \right)^2 \right) - 2 = \\
& -2 + \frac{1}{2} \sqrt{z_0} \left( 4 + 0.0038993 \left( e^{8/5} \left( 7 - \frac{1}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{\left(-1\right)^k \left(-\frac{1}{2}\right)_k \left(3-z_0\right)^k z_0^{-k}}}{k!}} \right) + \right. \\
& \quad \frac{1 - \frac{1}{3\sqrt{z_0} \sum_{k=0}^{\infty} \frac{\left(-1\right)^k \left(-\frac{1}{2}\right)_k \left(3-z_0\right)^k z_0^{-k}}}{k!}}}{e^{24/5}} + \\
& \quad \left. e^{24/5} \left( 1 + \frac{1}{3\sqrt{z_0} \sum_{k=0}^{\infty} \frac{\left(-1\right)^k \left(-\frac{1}{2}\right)_k \left(3-z_0\right)^k z_0^{-k}}}{k!}} \right) + \right. \\
& \quad \left. \frac{7 + \frac{1}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{\left(-1\right)^k \left(-\frac{1}{2}\right)_k \left(3-z_0\right)^k z_0^{-k}}}{k!}}}{e^{8/5}} \right)^2 + \\
& \quad \left. \frac{1}{\sqrt{z_0}^2 \left( \sum_{k=0}^{\infty} \frac{\left(-1\right)^k \left(-\frac{1}{2}\right)_k \left(3-z_0\right)^k z_0^{-k}}}{k!} \right)^2} \right) \sum_{k=0}^{\infty} \frac{\left(-1\right)^k \left(-\frac{1}{2}\right)_k \left(729 - z_0\right)^k z_0^{-k}}{k!}
\end{aligned}$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$(((((0.9991104/16[(1-1/(3\sqrt{3}))e^{(-24/5)}+(7+1/(\sqrt{3}))e^{(-8/5)}+(7-1/(\sqrt{3}))e^{(8/5)}+(1+1/(3\sqrt{3}))e^{(24/5)}])]))))^{1/5}$

**Input interpretation:**

$$\sqrt[5]{\frac{0.9991104}{16} \left( \left(1 - \frac{1}{3\sqrt{3}}\right) e^{-24/5} + \left(7 + \frac{1}{\sqrt{3}}\right) e^{-8/5} + \left(7 - \frac{1}{\sqrt{3}}\right) e^{8/5} + \left(1 + \frac{1}{3\sqrt{3}}\right) e^{24/5} \right)}$$

**Result:**

1.6192030...

1.6192030... result that is a good approximation to the value of the golden ratio  
1,618033988749...

Now, we have that:

$$\mathcal{V}_{IIIb}(\varphi) = \frac{\lambda}{16} \left[ \left(2 - 18\sqrt{3}\right) e^{-6\varphi/5} + \left(6 + 30\sqrt{3}\right) e^{-2\varphi/5} \right] \tag{5.23}$$

$$+ \left(6 - 30\sqrt{3}\right) e^{2\varphi/5} + \left(2 + 18\sqrt{3}\right) e^{6\varphi/5} \tag{5.24}$$

We put for  $\varphi > 0$   $\varphi = 4$  and for  $\lambda > 0$   $\lambda = 0.9991104$ , an obtain:

$0.9991104/16[(2-18(\sqrt{3}))e^{(-24/5)}+(6+30(\sqrt{3}))e^{(-8/5)}+(6-30(\sqrt{3}))e^{(8/5)}+(2+18(\sqrt{3}))e^{(24/5)}]$

**Input interpretation:**

$$\frac{0.9991104}{16} \left( \left(2 - 18\sqrt{3}\right) e^{-24/5} + \left(6 + 30\sqrt{3}\right) e^{-8/5} + \left(6 - 30\sqrt{3}\right) e^{8/5} + \left(2 + 18\sqrt{3}\right) e^{24/5} \right)$$

**Result:**

238.2350...

238.235...



**Series representations:**

$$\frac{1}{16} \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right)$$

$$0.99911 = \frac{1}{e^{24/5}} \left( 0.124889 (1 + e^{16/5})^3 + \right.$$

$$\left. (-1.124 + 1.87333 e^{16/5} - 1.87333 e^{32/5} + 1.124 e^{48/5}) \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)$$

$$\frac{1}{16} \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right)$$

$$0.99911 = \frac{1}{e^{24/5}} \left( 0.124889 (1 + e^{16/5})^3 + \right.$$

$$\left. (-1.124 + 1.87333 e^{16/5} - 1.87333 e^{32/5} + 1.124 e^{48/5}) \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)$$

$$\frac{1}{16} \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right)$$

$$0.99911 = \frac{1}{e^{24/5} \sqrt{\pi}} \left( 0.124889 (1 + e^{16/5})^3 \sqrt{\pi} + \right.$$

$$\left. (-0.562 + 0.936666 e^{16/5} - 0.936666 e^{32/5} + 0.562 e^{48/5}) \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right)$$

$$1/2 * (((0.9991104/16[(2-18(\text{sqrt}3))*e^{(-24/5)}+(6+30(\text{sqrt}3))*e^{(-8/5)}+(6-30(\text{sqrt}3))*e^{(8/5)}+(2+18(\text{sqrt}3))*e^{(24/5)}])))+11+8-\text{Pi}$$

**Input interpretation:**

$$\frac{1}{2} \left( \frac{0.9991104}{16} \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + \right. \right.$$

$$\left. (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right) + 11 + 8 - \pi$$

**Result:**

134.9759...

134.9759...  $\approx$  135 (Ramanujan taxicab number) and practically equal to the rest mass of Pion meson 134.9766 MeV

**Series representations:**

$$\frac{0.99911 \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right)}{16 \times 2}$$

$$+ 11 + 8 - \pi =$$

$$19 + \frac{0.0624444}{e^{24/5}} + \frac{0.187333}{e^{8/5}} + 0.187333 e^{8/5} + 0.0624444 e^{24/5} - \pi +$$

$$\frac{(-0.562 + 0.936666 e^{16/5} - 0.936666 e^{32/5} + 0.562 e^{48/5}) \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}{e^{24/5}}$$

$$\frac{0.99911 \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right)}{16 \times 2}$$

$$+ 11 + 8 - \pi =$$

$$19 + \frac{0.0624444}{e^{24/5}} + \frac{0.187333}{e^{8/5}} + 0.187333 e^{8/5} + 0.0624444 e^{24/5} - \pi +$$

$$\frac{(-0.562 + 0.936666 e^{16/5} - 0.936666 e^{32/5} + 0.562 e^{48/5}) \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)}{k!}}{e^{24/5}}$$

$$\frac{0.99911 \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right)}{16 \times 2}$$

$$+ 11 + 8 - \pi = \frac{1}{e^{24/5} \sqrt{\pi}}$$

$$\left( (0.0624444 + 0.187333 e^{16/5} + 0.187333 e^{32/5} + 0.0624444 e^{48/5} + e^{24/5} (19 - \pi)) \right.$$

$$\left. \sqrt{\pi} + (-0.281 + 0.468333 e^{16/5} - 0.468333 e^{32/5} + 0.281 e^{48/5}) \right.$$

$$\left. \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s) \right)$$

$$1/2 * (((0.9991104/16[(2-18(\text{sqrt}3))*e^{(-24/5)}+(6+30(\text{sqrt}3))*e^{(-8/5)}+(6-30(\text{sqrt}3))*e^{(8/5)}+(2+18(\text{sqrt}3))*e^{(24/5)}])))+8-\text{Pi}+\text{golden ratio}$$

**Input interpretation:**

$$\frac{1}{2} \left( \frac{0.9991104}{16} \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right) \right) + 8 - \pi + \phi$$

$\phi$  is the golden ratio

**Result:**

125.5939...

125.5939... result very near to the dilaton mass calculated as a type of Higgs boson:  
125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Series representations:**

$$\frac{0.99911 \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right)}{16 \times 2}$$

$$+ 8 - \pi + \phi =$$

$$8 + \frac{0.0624444}{e^{24/5}} + \frac{0.187333}{e^{8/5}} + 0.187333 e^{8/5} + 0.0624444 e^{24/5} + \phi - \pi +$$

$$\frac{(-0.562 + 0.936666 e^{16/5} - 0.936666 e^{32/5} + 0.562 e^{48/5}) \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}{e^{24/5}}$$

$$\frac{0.99911 \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right)}{16 \times 2}$$

$$+ 8 - \pi + \phi =$$

$$8 + \frac{0.0624444}{e^{24/5}} + \frac{0.187333}{e^{8/5}} + 0.187333 e^{8/5} + 0.0624444 e^{24/5} + \phi - \pi +$$

$$\frac{(-0.562 + 0.936666 e^{16/5} - 0.936666 e^{32/5} + 0.562 e^{48/5}) \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(\frac{1}{2}\right)_k}{k!}}{e^{24/5}}$$

$$\frac{0.99911 \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right)}{16 \times 2}$$

$$+ 8 - \pi + \phi =$$

$$\frac{1}{e^{24/5} \sqrt{\pi}} \left( (0.0624444 + 0.187333 e^{16/5} + 0.187333 e^{32/5} + 0.0624444 e^{48/5} + \right.$$

$$\left. e^{24/5} (8 + \phi - \pi) \sqrt{\pi} + \right.$$

$$\left. (-0.281 + 0.468333 e^{16/5} - 0.468333 e^{32/5} + 0.281 e^{48/5}) \right.$$

$$\left. \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right)$$

$1/2 * (((0.9991104/16[(2-18(\sqrt{3})) * e^{(-24/5)} + (6+30(\sqrt{3})) * e^{(-8/5)} + (6-30(\sqrt{3})) * e^{(8/5)} + (2+18(\sqrt{3})) * e^{(24/5)}])))) + 11 - e + 1/\text{golden ratio}$

**Input interpretation:**

$$\frac{1}{2} \left( \frac{0.9991104}{16} \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right) \right) + 11 - e + \frac{1}{\phi}$$

$\phi$  is the golden ratio

**Result:**

128.0173...

128.0173...

**Series representations:**

$$\frac{0.99911 \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right)}{16 \times 2} + 11 - e + \frac{1}{\phi} =$$

$$11 + \frac{0.0624444}{e^{24/5}} + \frac{0.187333}{e^{8/5}} - e + 0.187333 e^{8/5} + 0.0624444 e^{24/5} + \frac{1}{\phi} +$$

$$\sum_{k=0}^{\infty} \frac{2^{-k} (-0.562 + 0.936666 e^{16/5} - 0.936666 e^{32/5} + 0.562 e^{48/5}) \binom{1/2}{k} \sqrt{2}}{e^{24/5}}$$

$$\frac{0.99911 \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right)}{16 \times 2} + 11 - e + \frac{1}{\phi} =$$

$$11 + \frac{0.0624444}{e^{24/5}} + \frac{0.187333}{e^{8/5}} - e + 0.187333 e^{8/5} + 0.0624444 e^{24/5} + \frac{1}{\phi} +$$

$$\frac{(-0.562 + 0.936666 e^{16/5} - 0.936666 e^{32/5} + 0.562 e^{48/5}) \sqrt{2} \sum_{k=0}^{\infty} \frac{\binom{-1/2}{k} \binom{-1/2}{k}}{k!}}{e^{24/5}}$$

$$\frac{0.99911((2 - 18\sqrt{3})e^{-24/5} + (6 + 30\sqrt{3})e^{-8/5} + (6 - 30\sqrt{3})e^{8/5} + (2 + 18\sqrt{3})e^{24/5})}{16 \times 2} + 11 - e + \frac{1}{\phi} =$$

$$11 + \frac{0.0624444}{e^{24/5}} + \frac{0.187333}{e^{8/5}} - e + 0.187333 e^{8/5} + 0.0624444 e^{24/5} +$$

$$\frac{1}{\phi} + \frac{1}{e^{24/5}} (-0.562 + 0.936666 e^{16/5} - 0.936666 e^{32/5} + 0.562 e^{48/5})$$

$$\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!} \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\text{sqrt}729 * 1/2 * (((1/2 * (((0.9991104/16[(2-18(\text{sqrt}3)) * e^{(-24/5)} + (6+30(\text{sqrt}3)) * e^{(-8/5)} + (6-30(\text{sqrt}3)) * e^{(8/5)} + (2+18(\text{sqrt}3)) * e^{(24/5)})])) + 11 - e + 1/\text{golden ratio}))) + 4/5$$

**Input interpretation:**

$$\sqrt{729} \times \frac{1}{2} \left( \frac{1}{2} \left( \frac{0.9991104}{16} \left( (2 - 18\sqrt{3})e^{-24/5} + (6 + 30\sqrt{3})e^{-8/5} + (6 - 30\sqrt{3})e^{8/5} + (2 + 18\sqrt{3})e^{24/5} \right) + 11 - e + \frac{1}{\phi} \right) + \frac{4}{5} \right)$$

$\phi$  is the golden ratio

**Result:**

1729.033...

1729.033...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Series representations:**

$$\begin{aligned}
& \frac{1}{2} \sqrt{729} \left( \frac{1}{2 \times 16} 0.999911 \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + \right. \right. \\
& \quad \left. \left. (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right) + 11 - e + \frac{1}{\phi} \right) + \frac{4}{5} = \\
& \frac{1}{e^{24/5} \phi} 0.281 \left( 2.84698 e^{24/5} \phi + 1.77936 e^{24/5} \sqrt{728} \sum_{k=0}^{\infty} 728^{-k} \binom{\frac{1}{2}}{k} + \right. \\
& \quad 0.111111 \phi \sqrt{728} \sum_{k=0}^{\infty} 728^{-k} \binom{\frac{1}{2}}{k} + 0.333333 e^{16/5} \phi \sqrt{728} \sum_{k=0}^{\infty} 728^{-k} \binom{\frac{1}{2}}{k} + \\
& \quad 19.573 e^{24/5} \phi \sqrt{728} \sum_{k=0}^{\infty} 728^{-k} \binom{\frac{1}{2}}{k} - 1.77936 e^{29/5} \phi \sqrt{728} \sum_{k=0}^{\infty} 728^{-k} \binom{\frac{1}{2}}{k} + \\
& \quad 0.333333 e^{32/5} \phi \sqrt{728} \sum_{k=0}^{\infty} 728^{-k} \binom{\frac{1}{2}}{k} + \\
& \quad \left. 0.111111 e^{48/5} \phi \sqrt{728} \sum_{k=0}^{\infty} 728^{-k} \binom{\frac{1}{2}}{k} - \right. \\
& \quad \phi \left( \sqrt{2} \sqrt{728} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} 2^{-k_1-3k_2} \times 91^{-k_2} \binom{\frac{1}{2}}{k_1} \binom{\frac{1}{2}}{k_2} \right) + \\
& \quad 1.66667 e^{16/5} \phi \sqrt{2} \sqrt{728} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} 2^{-k_1-3k_2} \times 91^{-k_2} \binom{\frac{1}{2}}{k_1} \binom{\frac{1}{2}}{k_2} - \\
& \quad 1.66667 e^{32/5} \phi \sqrt{2} \sqrt{728} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} 2^{-k_1-3k_2} \times 91^{-k_2} \binom{\frac{1}{2}}{k_1} \binom{\frac{1}{2}}{k_2} + \\
& \quad \left. e^{48/5} \phi \sqrt{2} \sqrt{728} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} 2^{-k_1-3k_2} \times 91^{-k_2} \binom{\frac{1}{2}}{k_1} \binom{\frac{1}{2}}{k_2} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \sqrt{729} \left( \frac{1}{2 \times 16} 0.99911 \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + \right. \right. \\
& \quad \left. \left. (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right) + 11 - e + \frac{1}{\phi} \right) + \frac{4}{5} = \\
& \frac{1}{e^{24/5} \phi} 0.281 \left( 2.84698 e^{24/5} \phi + 1.77936 e^{24/5} \sqrt{728} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{728}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& \quad 0.111111 \phi \sqrt{728} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{728}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \\
& \quad 0.333333 e^{16/5} \phi \sqrt{728} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{728}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \\
& \quad 19.573 e^{24/5} \phi \sqrt{728} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{728}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad 1.77936 e^{29/5} \phi \sqrt{728} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{728}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \\
& \quad 0.333333 e^{32/5} \phi \sqrt{728} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{728}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \\
& \quad 0.111111 e^{48/5} \phi \sqrt{728} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{728}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad \left. \phi \left( \sqrt{2} \sqrt{728} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 2^{-k_1-3k_2} \times 91^{-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!} \right) + \right. \\
& \quad 1.66667 e^{16/5} \phi \sqrt{2} \sqrt{728} \\
& \quad \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 2^{-k_1-3k_2} \times 91^{-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!} - 1.66667 e^{32/5} \\
& \quad \phi \sqrt{2} \sqrt{728} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 2^{-k_1-3k_2} \times 91^{-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!} + \\
& \quad \left. e^{48/5} \phi \sqrt{2} \sqrt{728} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 2^{-k_1-3k_2} \times 91^{-k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2}}{k_1! k_2!} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2} \sqrt{729} \left( \frac{1}{2 \times 16} 0.99911 \left( (2 - 18\sqrt{3}) e^{-24/5} + (6 + 30\sqrt{3}) e^{-8/5} + \right. \right. \\
& \quad \left. \left. (6 - 30\sqrt{3}) e^{8/5} + (2 + 18\sqrt{3}) e^{24/5} \right) + 11 - e + \frac{1}{\phi} \right) + \frac{4}{5} = \\
& \frac{1}{e^{24/5} \phi \sqrt{\pi^2}} 0.8 \left( e^{24/5} \phi \sqrt{\pi^2} + 0.3125 e^{24/5} \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 728^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) + \right. \\
& \quad 0.0195139 \phi \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 728^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) + \\
& \quad 0.0585416 e^{16/5} \phi \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 728^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) + \\
& \quad 3.4375 e^{24/5} \phi \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 728^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) - \\
& \quad 0.3125 e^{29/5} \phi \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 728^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) + \\
& \quad 0.0585416 e^{32/5} \phi \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 728^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) + \\
& \quad 0.0195139 e^{48/5} \phi \sqrt{\pi} \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 728^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) - 0.0878124 \phi \\
& \quad \sum_{j_1=0}^{\infty} \sum_{j_2=0}^{\infty} \left( \text{Res}_{s=-\frac{1}{2}+j_1} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right) \left( \text{Res}_{s=-\frac{1}{2}+j_2} 728^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right) + \\
& \quad 0.146354 e^{16/5} \phi \\
& \quad \sum_{j_1=0}^{\infty} \sum_{j_2=0}^{\infty} \left( \text{Res}_{s=-\frac{1}{2}+j_1} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right) \left( \text{Res}_{s=-\frac{1}{2}+j_2} 728^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right) - \\
& \quad 0.146354 e^{32/5} \phi \sum_{j_1=0}^{\infty} \sum_{j_2=0}^{\infty} \left( \text{Res}_{s=-\frac{1}{2}+j_1} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right) \\
& \quad \left( \text{Res}_{s=-\frac{1}{2}+j_2} 728^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right) + 0.0878124 e^{48/5} \phi \\
& \quad \left. \sum_{j_1=0}^{\infty} \sum_{j_2=0}^{\infty} \left( \text{Res}_{s=-\frac{1}{2}+j_1} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right) \left( \text{Res}_{s=-\frac{1}{2}+j_2} 728^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right) \right)
\end{aligned}$$

Now, we have that:

$$\begin{aligned}
\mathcal{V}_{V_a}(\varphi) &= \lambda \left[ a \cosh^{\frac{4}{3}} \left( \frac{3\varphi}{5} \right) + b \frac{\sinh^2 \left( \frac{3\varphi}{5} \right)}{\cosh^{\frac{2}{3}} \left( \frac{3\varphi}{5} \right)} \right] \\
&= \frac{a - b + (a + b) \cosh \left( \frac{6\varphi}{5} \right)}{2 \cosh^{\frac{2}{3}} \left( \frac{3\varphi}{5} \right)}, \tag{5.29}
\end{aligned}$$



We put for  $\varphi > 0$   $\varphi = 4$  and for  $\lambda > 0$   $\lambda = 0.9991104$ , and  $a = 138$ ,  $b = 135$  and obtain:

$$((138-135+(138+135) \cosh(24/5))) / ((2 \cosh^{2/3}(12/5)))$$

**Input:**

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)}$$

$\cosh(x)$  is the hyperbolic cosine function

**Exact result:**

$$\frac{3 + 273 \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)}$$

**Decimal approximation:**

2644.031410843619594106656897494426919135475769955533719560...

2644.03141084...

**Alternate forms:**

$$\frac{3 \left(1 + 91 \cosh\left(\frac{24}{5}\right)\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)}$$

$$\frac{3}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} + \frac{273 \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)}$$

$$\frac{3(91 + 2e^{24/5} + 91e^{48/5})}{2\sqrt[3]{2} e^{16/5} (1 + e^{24/5})^{2/3}}$$

**Alternative representations:**

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} = \frac{3 + 273 \cos\left(\frac{24i}{5}\right)}{2 \cos^{2/3}\left(\frac{12i}{5}\right)}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} = \frac{3 + 273 \cos\left(-\frac{24i}{5}\right)}{2 \cos^{2/3}\left(-\frac{12i}{5}\right)}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} = \frac{3 + \frac{273}{\sec\left(\frac{24i}{5}\right)}}{2 \left(\frac{1}{\sec\left(\frac{12i}{5}\right)}\right)^{2/3}}$$

### Series representations:

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} = \frac{3 \left(1 + 91 \sum_{k=0}^{\infty} \frac{\left(\frac{576}{25}\right)^k}{(2k)!}\right)}{2 \left(\sum_{k=0}^{\infty} \frac{\left(\frac{144}{25}\right)^k}{(2k)!}\right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} = \frac{3 \left(1 + 91 \sum_{k=0}^{\infty} \frac{(-i)^k \cos\left(\frac{k\pi}{2} - iz_0\right) \left(\frac{24}{5} - z_0\right)^k}{k!}\right)}{2 \left(\sum_{k=0}^{\infty} \frac{(-i)^k \cos\left(\frac{k\pi}{2} - iz_0\right) \left(\frac{12}{5} - z_0\right)^k}{k!}\right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} = \frac{3 \left(1 + 91 I_0\left(\frac{24}{5}\right) + 182 \sum_{k=1}^{\infty} I_{2k}\left(\frac{24}{5}\right)\right)}{2 \left(I_0\left(\frac{12}{5}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{12}{5}\right)\right)^{2/3}}$$

### Integral representations:

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} = \frac{3 \left(1 + 91 \int_{\frac{i\pi}{2}}^{\frac{24}{5}} \sinh(t) dt\right)}{2 \left(\int_{\frac{i\pi}{2}}^{\frac{12}{5}} \sinh(t) dt\right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} = \frac{6 \left(115 + 546 \int_0^1 \sinh\left(\frac{24t}{5}\right) dt\right)}{\sqrt[3]{5} \left(5 + 12 \int_0^1 \sinh\left(\frac{12t}{5}\right) dt\right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} = \frac{3 \sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds \left(2i\sqrt{\pi} + 91 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{144/(25s)+s}}{\sqrt{s}} ds\right)}}{2 \sqrt[3]{2} \sqrt[6]{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds} \quad \text{for } \gamma > 0$$

$((138-135+(138+135) \cosh(24/5))) / ((2 \cosh^{2/3}(12/5))) + \text{golden ratio}$

**Input:**

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} + \phi$$

$\cosh(x)$  is the hyperbolic cosine function

$\phi$  is the golden ratio

**Exact result:**

$$\phi + \frac{3 + 273 \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)}$$

**Decimal approximation:**

2645.649444832369488954861484328792557253196079135339482422...

2645.649444832... result practically equal to the rest mass of charmed Xi baryon  
2645.9

**Alternate forms:**

$$\frac{1}{2} + \frac{\sqrt{5}}{2} + \frac{3}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} + \frac{273 \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)}$$

$$\frac{3 + \cosh^{2/3}\left(\frac{12}{5}\right) + \sqrt{5} \cosh^{2/3}\left(\frac{12}{5}\right) + 273 \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)}$$

$$\phi + \frac{e^{8/5} \left( \frac{3}{\sqrt[3]{2}} + \frac{273}{2 \sqrt[3]{2} e^{24/5}} + \frac{273 e^{24/5}}{2 \sqrt[3]{2}} \right)}{(1 + e^{24/5})^{2/3}}$$

**Alternative representations:**

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} + \phi = \phi + \frac{3 + 273 \cos\left(\frac{24i}{5}\right)}{2 \cos^{2/3}\left(\frac{12i}{5}\right)}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} + \phi = \phi + \frac{3 + 273 \cos\left(-\frac{24i}{5}\right)}{2 \cos^{2/3}\left(-\frac{12i}{5}\right)}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} + \phi = \phi + \frac{3 + \frac{273}{\sec\left(\frac{24i}{5}\right)}}{2 \left(\frac{1}{\sec\left(\frac{12i}{5}\right)}\right)^{2/3}}$$

### Series representations:

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} + \phi = \frac{3 + \left(\sum_{k=0}^{\infty} \frac{\left(\frac{144}{25}\right)^k}{(2k)!}\right)^{2/3} + \sqrt{5} \left(\sum_{k=0}^{\infty} \frac{\left(\frac{144}{25}\right)^k}{(2k)!}\right)^{2/3} + 273 \sum_{k=0}^{\infty} \frac{\left(\frac{576}{25}\right)^k}{(2k)!}}{2 \left(\sum_{k=0}^{\infty} \frac{\left(\frac{144}{25}\right)^k}{(2k)!}\right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} + \phi = \left(3 + 273 I_0\left(\frac{24}{5}\right) + \left(I_0\left(\frac{12}{5}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{12}{5}\right)\right)^{2/3} + \sqrt{5} \left(I_0\left(\frac{12}{5}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{12}{5}\right)\right)^{2/3} + 546 \sum_{k=1}^{\infty} I_{2k}\left(\frac{24}{5}\right)\right) / \left(2 \left(I_0\left(\frac{12}{5}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{12}{5}\right)\right)^{2/3}\right)$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} + \phi = \left(3 + \left(\sum_{k=0}^{\infty} \frac{(-i)^k \cos\left(\frac{k\pi}{2} - i z_0\right) \left(\frac{12}{5} - z_0\right)^k}{k!}\right)^{2/3} + \sqrt{5} \left(\sum_{k=0}^{\infty} \frac{(-i)^k \cos\left(\frac{k\pi}{2} - i z_0\right) \left(\frac{12}{5} - z_0\right)^k}{k!}\right)^{2/3} + 273 \sum_{k=0}^{\infty} \frac{(-i)^k \cos\left(\frac{k\pi}{2} - i z_0\right) \left(\frac{24}{5} - z_0\right)^k}{k!}\right) / \left(2 \left(\sum_{k=0}^{\infty} \frac{(-i)^k \cos\left(\frac{k\pi}{2} - i z_0\right) \left(\frac{12}{5} - z_0\right)^k}{k!}\right)^{2/3}\right)$$

**Integral representations:**

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} + \phi =$$

$$\frac{3 + \left(\int_{i\frac{\pi}{2}}^{\frac{12}{5}} \sinh(t) dt\right)^{2/3} + \sqrt{5} \left(\int_{i\frac{\pi}{2}}^{\frac{12}{5}} \sinh(t) dt\right)^{2/3} + 273 \int_{i\frac{\pi}{2}}^{\frac{24}{5}} \sinh(t) dt}{2 \left(\int_{i\frac{\pi}{2}}^{\frac{12}{5}} \sinh(t) dt\right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} + \phi =$$

$$\left(1380 \times 5^{2/3} + 5 \left(5 + 12 \int_0^1 \sinh\left(\frac{12t}{5}\right) dt\right)^{2/3} + 5 \sqrt{5} \left(5 + 12 \int_0^1 \sinh\left(\frac{12t}{5}\right) dt\right)^{2/3} + 6552 \times 5^{2/3} \int_0^1 \sinh\left(\frac{24t}{5}\right) dt\right) / \left(10 \left(5 + 12 \int_0^1 \sinh\left(\frac{12t}{5}\right) dt\right)^{2/3}\right)$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} + \phi =$$

$$\left(6 i^{2/3} \sqrt{\pi} \sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds} + 2 \sqrt[6]{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds + 2 \sqrt{5} \sqrt[6]{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds + 273 \times 2^{2/3} \sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{144/(25s)+s}}{\sqrt{s}} ds\right) / \left(4 \sqrt[6]{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds\right) \text{ for } \gamma > 0$$

$$\left(\left(\frac{1}{3} \left(138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)\right)\right) / \left(2 \cosh^{2/3}\left(\frac{12}{5}\right)\right)\right) - 76 + 7 - 34 \times \frac{1}{10^2}$$

**Input:**

$$\frac{1}{3} \times \frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - 76 + 7 - 34 \times \frac{1}{10^2}$$

cosh(x) is the hyperbolic cosine function

**Exact result:**

$$\frac{3 + 273 \cosh\left(\frac{24}{5}\right)}{6 \cosh^{2/3}\left(\frac{12}{5}\right)} - \frac{3467}{50}$$

**Decimal approximation:**

812.0038036145398647022189658314756397118252566518445731867...

812.0038036145...  $\approx$  812 (Ramanujan taxicab number)**Alternate forms:**

$$\frac{-25 + 3467 \cosh^{2/3}\left(\frac{12}{5}\right) - 2275 \cosh\left(\frac{24}{5}\right)}{50 \cosh^{2/3}\left(\frac{12}{5}\right)}$$

$$-\frac{3467}{50} + \frac{1}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} + \frac{91 \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)}$$

$$\frac{91 \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - \frac{3467 \cosh^{2/3}\left(\frac{12}{5}\right) - 25}{50 \cosh^{2/3}\left(\frac{12}{5}\right)}$$

**Alternative representations:**

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{\left(2 \cosh^{2/3}\left(\frac{12}{5}\right)\right)^3} - 76 + 7 - \frac{34}{10^2} = -69 - \frac{34}{10^2} + \frac{3 + 273 \cos\left(\frac{24i}{5}\right)}{3 \left(2 \cos^{2/3}\left(\frac{12i}{5}\right)\right)}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{\left(2 \cosh^{2/3}\left(\frac{12}{5}\right)\right)^3} - 76 + 7 - \frac{34}{10^2} = -69 - \frac{34}{10^2} + \frac{3 + 273 \cos\left(-\frac{24i}{5}\right)}{3 \left(2 \cos^{2/3}\left(-\frac{12i}{5}\right)\right)}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{\left(2 \cosh^{2/3}\left(\frac{12}{5}\right)\right)^3} - 76 + 7 - \frac{34}{10^2} = -69 - \frac{34}{10^2} + \frac{3 + \frac{273}{\sec\left(\frac{24i}{5}\right)}}{3 \left(2 \left(\frac{1}{\sec\left(\frac{12i}{5}\right)}\right)^{2/3}\right)}$$

**Series representations:**

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{\left(2 \cosh^{2/3}\left(\frac{12}{5}\right)\right)^3} - 76 + 7 - \frac{34}{10^2} =$$

$$\frac{-25 + 3467 \left(\sum_{k=0}^{\infty} \frac{\left(\frac{144}{25}\right)^k}{(2k)!}\right)^{2/3} - 2275 \sum_{k=0}^{\infty} \frac{\left(\frac{576}{25}\right)^k}{(2k)!}}{50 \left(\sum_{k=0}^{\infty} \frac{\left(\frac{144}{25}\right)^k}{(2k)!}\right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{\left(2 \cosh^{2/3}\left(\frac{12}{5}\right)\right) 3} - 76 + 7 - \frac{34}{10^2} =$$

$$\frac{-25 - 2275 I_0\left(\frac{24}{5}\right) + 3467 \left(I_0\left(\frac{12}{5}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{12}{5}\right)\right)^{2/3} - 4550 \sum_{k=1}^{\infty} I_{2k}\left(\frac{24}{5}\right)}{50 \left(I_0\left(\frac{12}{5}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{12}{5}\right)\right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{\left(2 \cosh^{2/3}\left(\frac{12}{5}\right)\right) 3} - 76 + 7 - \frac{34}{10^2} =$$

$$\frac{-25 + 3467 \left(\sum_{k=0}^{\infty} \frac{(-i)^k \cos\left(\frac{k\pi}{2} - i z_0\right) \left(\frac{12}{5} - z_0\right)^k}{k!}\right)^{2/3} - 2275 \sum_{k=0}^{\infty} \frac{(-i)^k \cos\left(\frac{k\pi}{2} - i z_0\right) \left(\frac{24}{5} - z_0\right)^k}{k!}}{50 \left(\sum_{k=0}^{\infty} \frac{(-i)^k \cos\left(\frac{k\pi}{2} - i z_0\right) \left(\frac{12}{5} - z_0\right)^k}{k!}\right)^{2/3}}$$

### Integral representations:

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{\left(2 \cosh^{2/3}\left(\frac{12}{5}\right)\right) 3} - 76 + 7 - \frac{34}{10^2} =$$

$$\frac{-25 + 3467 \left(\int_{i\frac{\pi}{2}}^{\frac{12}{5}} \sinh(t) dt\right)^{2/3} - 2275 \int_{i\frac{\pi}{2}}^{\frac{24}{5}} \sinh(t) dt}{50 \left(\int_{i\frac{\pi}{2}}^{\frac{12}{5}} \sinh(t) dt\right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{\left(2 \cosh^{2/3}\left(\frac{12}{5}\right)\right) 3} - 76 + 7 - \frac{34}{10^2} =$$

$$\frac{2300 \times 5^{2/3} - 3467 \left(5 + 12 \int_0^1 \sinh\left(\frac{12t}{5}\right) dt\right)^{2/3} + 10920 \times 5^{2/3} \int_0^1 \sinh\left(\frac{24t}{5}\right) dt}{50 \left(5 + 12 \int_0^1 \sinh\left(\frac{12t}{5}\right) dt\right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{\left(2 \cosh^{2/3}\left(\frac{12}{5}\right)\right) 3} - 76 + 7 - \frac{34}{10^2} =$$

$$\left(50 i 2^{2/3} \sqrt{\pi} \sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds} - 6934 \sqrt[6]{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds + \right.$$

$$\left. 2275 \times 2^{2/3} \sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{144/(25s)+s}}{\sqrt{s}} ds \right) /$$

$$\left(100 \sqrt[6]{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds\right) \text{ for } \gamma > 0$$

$\sinh(x)$  is the hyperbolic sine function

$$(((138-135+(138+135) \cosh(24/5))) / ((2 \cosh^{2/3}(12/5))))-843-76+4$$

**Input:**

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - 843 - 76 + 4$$

$\cosh(x)$  is the hyperbolic cosine function

**Exact result:**

$$\frac{3 + 273 \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - 915$$

**Decimal approximation:**

1729.031410843619594106656897494426919135475769955533719560...

1729.031410843...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Alternate forms:**

$$-915 + \frac{3}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} + \frac{273 \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)}$$

$$-\frac{3\left(-1 + 610 \cosh^{2/3}\left(\frac{12}{5}\right) - 91 \cosh\left(\frac{24}{5}\right)\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)}$$

$$\frac{273 \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - \frac{3\left(610 \cosh^{2/3}\left(\frac{12}{5}\right) - 1\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)}$$



### Alternative representations:

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - 843 - 76 + 4 = -915 + \frac{3 + 273 \cos\left(\frac{24i}{5}\right)}{2 \cos^{2/3}\left(\frac{12i}{5}\right)}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - 843 - 76 + 4 = -915 + \frac{3 + 273 \cos\left(-\frac{24i}{5}\right)}{2 \cos^{2/3}\left(-\frac{12i}{5}\right)}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - 843 - 76 + 4 = -915 + \frac{3 + \frac{273}{\sec\left(\frac{24i}{5}\right)}}{2 \left(\frac{1}{\sec\left(\frac{12i}{5}\right)}\right)^{2/3}}$$

### Series representations:

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - 843 - 76 + 4 = \frac{3 \left( -1 + 610 \left( \sum_{k=0}^{\infty} \frac{\left(\frac{144}{25}\right)^k}{(2k)!} \right)^{2/3} - 91 \sum_{k=0}^{\infty} \frac{\left(\frac{576}{25}\right)^k}{(2k)!} \right)}{2 \left( \sum_{k=0}^{\infty} \frac{\left(\frac{144}{25}\right)^k}{(2k)!} \right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - 843 - 76 + 4 = \frac{3 \left( -1 - 91 I_0\left(\frac{24}{5}\right) + 610 \left( I_0\left(\frac{12}{5}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{12}{5}\right) \right)^{2/3} - 182 \sum_{k=1}^{\infty} I_{2k}\left(\frac{24}{5}\right) \right)}{2 \left( I_0\left(\frac{12}{5}\right) + 2 \sum_{k=1}^{\infty} I_{2k}\left(\frac{12}{5}\right) \right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - 843 - 76 + 4 = \frac{3 \left( -1 + 610 \left( \sum_{k=0}^{\infty} \frac{(-i)^k \cos\left(\frac{k\pi}{2} - iz_0\right) \left(\frac{12}{5} - z_0\right)^k}{k!} \right)^{2/3} - 91 \sum_{k=0}^{\infty} \frac{(-i)^k \cos\left(\frac{k\pi}{2} - iz_0\right) \left(\frac{24}{5} - z_0\right)^k}{k!} \right)}{2 \left( \sum_{k=0}^{\infty} \frac{(-i)^k \cos\left(\frac{k\pi}{2} - iz_0\right) \left(\frac{12}{5} - z_0\right)^k}{k!} \right)^{2/3}}$$

**Integral representations:**

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - 843 - 76 + 4 =$$

$$\frac{3 \left( -1 + 610 \left( \int_{i\pi/2}^{12/5} \sinh(t) dt \right)^{2/3} - 91 \int_{i\pi/2}^{24/5} \sinh(t) dt \right)}{2 \left( \int_{i\pi/2}^{12/5} \sinh(t) dt \right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - 843 - 76 + 4 =$$

$$\frac{3 \left( 230 \times 5^{2/3} - 1525 \left( 5 + 12 \int_0^1 \sinh\left(\frac{12t}{5}\right) dt \right)^{2/3} + 1092 \times 5^{2/3} \int_0^1 \sinh\left(\frac{24t}{5}\right) dt \right)}{5 \left( 5 + 12 \int_0^1 \sinh\left(\frac{12t}{5}\right) dt \right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - 843 - 76 + 4 =$$

$$\left( 3 \left( 2 i 2^{2/3} \sqrt{\pi} \sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds} - 1220 \sqrt[6]{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds + \right. \right.$$

$$\left. \left. 91 \times 2^{2/3} \sqrt[3]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{144/(25s)+s}}{\sqrt{s}} ds \right) \right) /$$

$$\left( 4 \sqrt[6]{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds \right) \text{ for } \gamma > 0$$

$$(((138-135+(138+135) \cosh(24/5))) / ((2 \cosh^{2/3}(12/5))))-(2452.9-1535)$$

where 2452.9 and 1535 are the rest mass of the charmed Sigma baryon and Xi baryon

**Input interpretation:**

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - (2452.9 - 1535)$$

cosh(x) is the hyperbolic cosine function

**Result:**

1726.13...

1726.13... result very near to the mass of candidate glueball  $f_0(1710)$  meson.**Alternative representations:**

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - (2452.9 - 1535) = -917.9 + \frac{3 + 273 \cos\left(\frac{24i}{5}\right)}{2 \cos^{2/3}\left(\frac{12i}{5}\right)}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - (2452.9 - 1535) = -917.9 + \frac{3 + 273 \cos\left(-\frac{24i}{5}\right)}{2 \cos^{2/3}\left(-\frac{12i}{5}\right)}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - (2452.9 - 1535) = -917.9 + \frac{3 + \frac{273}{\sec\left(\frac{24i}{5}\right)}}{2 \left(\frac{1}{\sec\left(\frac{12i}{5}\right)}\right)^{2/3}}$$

**Series representations:**

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - (2452.9 - 1535) =$$

$$917.9 \left( -0.00163416 + \left( \sum_{k=0}^{\infty} \frac{\left(\frac{144}{25}\right)^k}{(2k)!} \right)^{2/3} - 0.148709 \sum_{k=0}^{\infty} \frac{\left(\frac{576}{25}\right)^k}{(2k)!} \right)$$

$$- \frac{\left( \sum_{k=0}^{\infty} \frac{\left(\frac{144}{25}\right)^k}{(2k)!} \right)^{2/3}}{\left( \sum_{k=0}^{\infty} \frac{\left(\frac{144}{25}\right)^k}{(2k)!} \right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - (2452.9 - 1535) =$$

$$- \left( \left( 917.9 \left( -0.00163416 + \left( \sum_{k=0}^{\infty} \frac{(-i)^k \cosh\left(\frac{ik\pi}{2} + z_0\right) \left(\frac{12}{5} - z_0\right)^k}{k!} \right)^{2/3} - \right. \right. \right.$$

$$\left. \left. 0.148709 \sum_{k=0}^{\infty} \frac{(-i)^k \cosh\left(\frac{ik\pi}{2} + z_0\right) \left(\frac{24}{5} - z_0\right)^k}{k!} \right) \right) /$$

$$\left( \sum_{k=0}^{\infty} \frac{(-i)^k \cosh\left(\frac{ik\pi}{2} + z_0\right) \left(\frac{12}{5} - z_0\right)^k}{k!} \right)^{2/3}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - (2452.9 - 1535) =$$

$$\frac{1}{i \sum_{k=0}^{\infty} \frac{\left(\frac{12-i\pi}{5}\right)^{1+2k}}{(1+2k)!}} 136.5 \left( -6.72454 i \sum_{k=0}^{\infty} \frac{\left(\frac{12-i\pi}{5}\right)^{1+2k}}{(1+2k)!} + \right.$$

$$\left. 0.010989 \sqrt[3]{i \sum_{k=0}^{\infty} \frac{\left(\frac{12-i\pi}{5}\right)^{1+2k}}{(1+2k)!}} + i \sqrt[3]{i \sum_{k=0}^{\infty} \frac{\left(\frac{12-i\pi}{5}\right)^{1+2k}}{(1+2k)!}} \sum_{k=0}^{\infty} \frac{\left(\frac{24-i\pi}{5}\right)^{1+2k}}{(1+2k)!} \right)$$

### Integral representations:

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - (2452.9 - 1535) =$$

$$\frac{917.9 \left( -0.00163416 + \left( \int_{\frac{i\pi}{2}}^{\frac{12}{5}} \sinh(t) dt \right)^{2/3} \right) - 0.148709 \int_{\frac{i\pi}{2}}^{\frac{24}{5}} \sinh(t) dt}{\left( \int_{\frac{i\pi}{2}}^{\frac{12}{5}} \sinh(t) dt \right)^{2/3}}$$

$$\frac{138 - 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \cosh^{2/3}\left(\frac{12}{5}\right)} - (2452.9 - 1535) =$$

$$\left( 1.5 \left( -611.933 \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds + 1.5874 i \pi \sqrt[3]{\frac{\sqrt{\pi}}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds} + \right. \right.$$

$$\left. 72.2267 \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{144/(25s)+s}}{\sqrt{s}} ds \right) \sqrt{\pi} \sqrt[3]{\frac{\sqrt{\pi}}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds} \right) /$$

$$\left( \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{36/(25s)+s}}{\sqrt{s}} ds \right) \text{ for } \gamma > 0$$

From

$$\mathcal{V}_{Vb}(\varphi) = \lambda \left[ a \sinh^{\frac{4}{3}}\left(\frac{3\varphi}{5}\right) + b \frac{\cosh^2\left(\frac{3\varphi}{5}\right)}{\sinh^{\frac{2}{3}}\left(\frac{3\varphi}{5}\right)} \right]$$

$$= \frac{-a + b + (a + b) \cosh\left(\frac{6\varphi}{5}\right)}{2 \sinh^{\frac{2}{3}}\left(\frac{3\varphi}{5}\right)}, \quad (5.33)$$

We obtain:

for  $\varphi > 0$   $\varphi = 4$  and for  $\lambda > 0$   $\lambda = 0.9991104$ , and  $a = 138$ ,  $b = 135$  and obtain:

$$\frac{(-138+135+(138+135) \cosh(24/5))}{((2 \sinh^{2/3}(12/5))}$$

**Input:**

$$\frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)}$$

$\cosh(x)$  is the hyperbolic cosine function

$\sinh(x)$  is the hyperbolic sine function

**Exact result:**

$$\frac{273 \cosh\left(\frac{24}{5}\right) - 3}{2 \sinh^{2/3}\left(\frac{12}{5}\right)}$$

**Decimal approximation:**

2672.237998872641217733820876740236691949476671178658401997...

2672.23799887...

**Alternate forms:**

$$\frac{3(91 \cosh\left(\frac{24}{5}\right) - 1)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)}$$

$$\frac{273 \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} - \frac{3}{2 \sinh^{2/3}\left(\frac{12}{5}\right)}$$

$$\frac{3(91 - 2 e^{24/5} + 91 e^{48/5})}{2 \sqrt[3]{2} e^{16/5} (e^{24/5} - 1)^{2/3}}$$

**Alternative representations:**

$$\frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} = \frac{-3 + 273 \cos\left(\frac{24i}{5}\right)}{2 \left(\frac{1}{2} (-e^{-12/5} + e^{12/5})\right)^{2/3}}$$

$$\frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} = \frac{-3 + 273 \cos\left(-\frac{24i}{5}\right)}{2 \left(\frac{1}{2} (-e^{-12/5} + e^{12/5})\right)^{2/3}}$$

$$\frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} = \frac{-3 + 273 \cos\left(-\frac{24i}{5}\right)}{2 \left(i \cos\left(\frac{\pi}{2} + \frac{12i}{5}\right)\right)^{2/3}}$$

### Series representations:

$$\frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} = \frac{3 \left(-1 + 91 \sum_{k=0}^{\infty} \frac{\left(\frac{576}{25}\right)^k}{(2k)!}\right)}{2 \left(\sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!}\right)^{2/3}}$$

$$\frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} = \frac{3i \left(i + 91 \sum_{k=0}^{\infty} \frac{\left(\frac{24-i\pi}{5-2}\right)^{1+2k}}{(1+2k)!}\right)}{2 \left(\sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!}\right)^{2/3}}$$

$$\frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} = -\frac{3i \left(-1 + 91 \sum_{k=0}^{\infty} \frac{\left(\frac{576}{25}\right)^k}{(2k)!}\right) \sqrt[3]{i \sum_{k=0}^{\infty} \frac{\left(\frac{12-i\pi}{5-2}\right)^{2k}}{(2k)!}}}{2 \sum_{k=0}^{\infty} \frac{\left(\frac{12-i\pi}{5-2}\right)^{2k}}{(2k)!}}$$

### Integral representations:

$$\frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} = \frac{3 \sqrt[3]{\frac{3}{10}} \left(75 + 364 \int_0^1 \sinh\left(\frac{24t}{5}\right) dt\right)}{2 \left(\int_0^1 \cosh\left(\frac{12t}{5}\right) dt\right)^{2/3}}$$

$$\frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} = \frac{\sqrt[3]{\frac{3}{2}} 5^{2/3} \left(-1 + 91 \int_{\frac{i\pi}{2}}^{\frac{24}{5}} \sinh(t) dt\right)}{4 \left(\int_0^1 \cosh\left(\frac{12t}{5}\right) dt\right)^{2/3}}$$

$$\frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} = -\frac{\sqrt[3]{\frac{3}{2}} 5^{2/3} \left(2\sqrt{\pi} + 91i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{144/(25s)+s}}{\sqrt{s}} ds\right)}{8\sqrt{\pi} \left(\int_0^1 \cosh\left(\frac{12t}{5}\right) dt\right)^{2/3}} \quad \text{for } \gamma > 0$$

$((-138+135+(138+135) \cosh(24/5))) / ((2 \sinh^{2/3}(12/5))) + 21 + \pi - 1/\text{golden ratio}$

**Input:**

$$\frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} + 21 + \pi - \frac{1}{\phi}$$

$\cosh(x)$  is the hyperbolic cosine function

$\sinh(x)$  is the hyperbolic sine function

$\phi$  is the golden ratio

**Exact result:**

$$-\frac{1}{\phi} + 21 + \pi + \frac{273 \cosh\left(\frac{24}{5}\right) - 3}{2 \sinh^{2/3}\left(\frac{12}{5}\right)}$$

**Decimal approximation:**

2695.761557537481116124078933289150556715953531398227744956...

2695.7615575... result practically equal to the rest mass of charmed Omega baryon  
2695.2

**Alternate forms:**

$$-\frac{1}{\phi} + 21 + \pi + \frac{3\left(91 \cosh\left(\frac{24}{5}\right) - 1\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)}$$

$$\frac{1}{2} \left(43 - \sqrt{5}\right) + \pi + \frac{273 \cosh\left(\frac{24}{5}\right) - 3}{2 \sinh^{2/3}\left(\frac{12}{5}\right)}$$

$$21 - \frac{2}{1 + \sqrt{5}} + \pi - \frac{3}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} + \frac{273 \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)}$$

**Alternative representations:**

$$\frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} + 21 + \pi - \frac{1}{\phi} = 21 + \pi - \frac{1}{\phi} + \frac{-3 + 273 \cos\left(\frac{24i}{5}\right)}{2 \left(\frac{1}{2} (-e^{-12/5} + e^{12/5})\right)^{2/3}}$$

$$\frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} + 21 + \pi - \frac{1}{\phi} = 21 + \pi - \frac{1}{\phi} + \frac{-3 + 273 \cos\left(-\frac{24i}{5}\right)}{2 \left(\frac{1}{2} (-e^{-12/5} + e^{12/5})\right)^{2/3}}$$

$$\frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} + 21 + \pi - \frac{1}{\phi} = 21 + \pi - \frac{1}{\phi} + \frac{-3 + 273 \cos\left(-\frac{24i}{5}\right)}{2 \left(i \cos\left(\frac{\pi}{2} + \frac{12i}{5}\right)\right)^{2/3}}$$

### Series representations:

$$\begin{aligned} & \frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} + 21 + \pi - \frac{1}{\phi} = \\ & \left( -3 - 3\sqrt{5} + 273 \sum_{k=0}^{\infty} \frac{\left(\frac{576}{25}\right)^k}{(2k)!} + 273\sqrt{5} \sum_{k=0}^{\infty} \frac{\left(\frac{576}{25}\right)^k}{(2k)!} + 38 \left( \sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!} \right)^{2/3} + \right. \\ & \quad \left. 42\sqrt{5} \left( \sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!} \right)^{2/3} + 2\pi \left( \sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!} \right)^{2/3} + 2\sqrt{5}\pi \left( \sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!} \right)^{2/3} \right) / \\ & \left( 2(1+\sqrt{5}) \left( \sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!} \right)^{2/3} \right) \end{aligned}$$

$$\begin{aligned} & \frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} + 21 + \pi - \frac{1}{\phi} = \\ & \left( -3 - 3\sqrt{5} + 38 \left( \sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!} \right)^{2/3} + 42\sqrt{5} \left( \sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!} \right)^{2/3} + \right. \\ & \quad \left. 2\pi \left( \sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!} \right)^{2/3} + 2\sqrt{5}\pi \left( \sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!} \right)^{2/3} + 273i \sum_{k=0}^{\infty} \frac{\left(\frac{24}{5} - \frac{i\pi}{2}\right)^{1+2k}}{(1+2k)!} + \right. \\ & \quad \left. 273i\sqrt{5} \sum_{k=0}^{\infty} \frac{\left(\frac{24}{5} - \frac{i\pi}{2}\right)^{1+2k}}{(1+2k)!} \right) / \left( 2(1+\sqrt{5}) \left( \sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!} \right)^{2/3} \right) \end{aligned}$$



$$\begin{aligned}
& \frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} + 21 + \pi - \frac{1}{\phi} = \\
& \left( -3 - 3\sqrt{5} + 38 \left( \sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!} \right)^{2/3} + 42\sqrt{5} \left( \sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!} \right)^{2/3} + 2\pi \left( \sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!} \right)^{2/3} + \right. \\
& \left. 2\sqrt{5} \pi \left( \sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!} \right)^{2/3} + 273\sqrt{\pi} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{\left(-\frac{144}{25}\right)^{-s} \Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)} + \right. \\
& \left. 273\sqrt{5} \pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{\left(-\frac{144}{25}\right)^{-s} \Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)} \right) / \left( 2(1+\sqrt{5}) \left( \sum_{k=0}^{\infty} \frac{\left(\frac{12}{5}\right)^{1+2k}}{(1+2k)!} \right)^{2/3} \right)
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& \frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} + 21 + \pi - \frac{1}{\phi} = \\
& \left( 1125 \times 2^{2/3} \sqrt[3]{3} \sqrt[6]{5} + 225 \sqrt[3]{3} 10^{2/3} + \right. \\
& \left. 380 \left( \int_0^1 \cosh\left(\frac{12t}{5}\right) dt \right)^{2/3} + 420\sqrt{5} \left( \int_0^1 \cosh\left(\frac{12t}{5}\right) dt \right)^{2/3} + \right. \\
& \left. 20\pi \left( \int_0^1 \cosh\left(\frac{12t}{5}\right) dt \right)^{2/3} + 20\sqrt{5} \pi \left( \int_0^1 \cosh\left(\frac{12t}{5}\right) dt \right)^{2/3} + \right. \\
& \left. 5460 \times 2^{2/3} \sqrt[3]{3} \sqrt[6]{5} \int_0^1 \sinh\left(\frac{24t}{5}\right) dt + 1092 \sqrt[3]{3} 10^{2/3} \int_0^1 \sinh\left(\frac{24t}{5}\right) dt \right) / \\
& \left( 20(1+\sqrt[6]{5})(1-\sqrt[6]{5}+\sqrt[3]{5}) \left( \int_0^1 \cosh\left(\frac{12t}{5}\right) dt \right)^{2/3} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} + 21 + \pi - \frac{1}{\phi} = \\
& \left( -5 \times 2^{2/3} \sqrt[3]{3} \sqrt[6]{5} - \sqrt[3]{3} 10^{2/3} + 152 \left( \int_0^1 \cosh\left(\frac{12t}{5}\right) dt \right)^{2/3} + \right. \\
& \left. 168\sqrt{5} \left( \int_0^1 \cosh\left(\frac{12t}{5}\right) dt \right)^{2/3} + \right. \\
& \left. 8\pi \left( \int_0^1 \cosh\left(\frac{12t}{5}\right) dt \right)^{2/3} + 8\sqrt{5} \pi \left( \int_0^1 \cosh\left(\frac{12t}{5}\right) dt \right)^{2/3} + \right. \\
& \left. 455 \times 2^{2/3} \sqrt[3]{3} \sqrt[6]{5} \int_{i\pi/2}^{24} \sinh(t) dt + 91 \sqrt[3]{3} 10^{2/3} \int_{i\pi/2}^{24} \sinh(t) dt \right) / \\
& \left( 8(1+\sqrt[6]{5})(1-\sqrt[6]{5}+\sqrt[3]{5}) \left( \int_0^1 \cosh\left(\frac{12t}{5}\right) dt \right)^{2/3} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{-138 + 135 + (138 + 135) \cosh\left(\frac{24}{5}\right)}{2 \sinh^{2/3}\left(\frac{12}{5}\right)} + 21 + \pi - \frac{1}{\phi} = \\
& - \left( \left( 10 \times 2^{2/3} \sqrt[3]{3} \sqrt[6]{5} \sqrt{\pi} + 2 \sqrt[3]{3} 10^{2/3} \sqrt{\pi} + 455 i 2^{2/3} \sqrt[3]{3} \sqrt[6]{5} \right. \right. \\
& \quad \left. \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{144/(25s)+s}}{\sqrt{s}} ds + 91 i \sqrt[3]{3} 10^{2/3} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{144/(25s)+s}}{\sqrt{s}} ds - \right. \\
& \quad \left. 304 \sqrt{\pi} \left( \int_0^1 \cosh\left(\frac{12t}{5}\right) dt \right)^{2/3} - 16 \pi^{3/2} \left( \int_0^1 \cosh\left(\frac{12t}{5}\right) dt \right)^{2/3} - \right. \\
& \quad \left. 16 \sqrt{5} \pi^{3/2} \left( \int_0^1 \cosh\left(\frac{12t}{5}\right) dt \right)^{2/3} - 336 \sqrt{5} \pi \left( \int_0^1 \cosh\left(\frac{12t}{5}\right) dt \right)^{2/3} \right) / \\
& \quad \left( 16 (1 + \sqrt[6]{5}) (1 - \sqrt[6]{5} + \sqrt[3]{5}) \sqrt{\pi} \left( \int_0^1 \cosh\left(\frac{12t}{5}\right) dt \right)^{2/3} \right) \text{ for } \gamma > 0
\end{aligned}$$

Now:

It is thus convenient to define the two fields

$$\Phi_t = \sqrt{\frac{d-2}{2(d-1)}} \left( \frac{3}{2} \phi - \frac{10-d}{d-2} \sigma \right), \quad (6.7)$$

$$\Phi_s = \sqrt{\frac{10-d}{2(d-1)}} \left( \frac{1}{2} \phi + 3\sigma \right), \quad (6.8)$$

One can add to this discussion a further degree of freedom, allowing for an off-critical bulk of dimension  $d$ . Confining our attention to the case  $d > 10$ , let us add some cursory remarks on the resulting potential after a compactification to four dimensions. For simplicity, let us confine our attention to the contributions arising from  $D9$  branes and from the conformal anomaly originally described by Polyakov in [43]. Up to shifts of the two fields  $\Phi_s$  and  $\Phi_t$ , the resulting potential

contains again two terms with identical normalizations, and assuming again that  $\Phi_s$  is somehow stabilized, one is finally confronted with

$$V = V_0 \left( e^{\sqrt{3} \gamma_9 \Phi_t} + e^{\sqrt{3} \gamma_\Lambda \Phi_t} \right), \quad (6.18)$$

where

$$\gamma_9 = \sqrt{\frac{d^2 - 14d + 184}{24(d-4)}}, \quad \gamma_\Lambda = -\frac{10}{3} \frac{(d-4)(d-10)}{\sqrt{2(d^2 - 14d + 184)}} \quad (6.19)$$

Interestingly, for  $d$  slightly larger than ten  $\gamma_\Lambda$  is *small and negative* while  $\gamma_9$  is very close to one, so that one has a potential well which combines a steep wall with a rather flat one. As a result, the scalar is essentially bound to emerge from the initial singularity with the scalar descending along the mild wall and to stabilize readily at the bottom as the Universe enters a de Sitter phase.

for  $d = 11$ ,  $\phi = 6$ ,  $\sigma = 8$  and  $V_0 > 0$ ;  $V_0 = 0.5$

$$\left( \sqrt{\frac{(11-2)}{(2(11-1))}} \right) * \left( \frac{3}{2} * 6 - \frac{(10-11)*8}{(11-2)} \right)$$

**Input:**

$$\sqrt{\frac{11-2}{2(11-1)} \left( \frac{3}{2} \times 6 - (10-11) \times \frac{8}{11-2} \right)}$$

**Result:**

$$\frac{89}{6\sqrt{5}}$$

**Decimal approximation:**

6.633668333249376099347215217236119498473834466847526315336...

$$6.6336683\dots = \Phi_t$$

**Alternate form:**

$$\frac{89\sqrt{5}}{30}$$

$$\text{sqrt}(\frac{((10-11)/(2(11-1)))}{(1/2 * 6 + 3*8)})$$

**Input:**

$$\sqrt{\frac{10-11}{2(11-1)} \left( \frac{1}{2} \times 6 + 3 \times 8 \right)}$$

**Result:**

$$\frac{27i}{2\sqrt{5}}$$

**Decimal approximation:**

6.037383539249432180304768905574445835689669570951119455531... *i*

**Polar coordinates:**

$r \approx 6.03738$  (radius),  $\theta = 90^\circ$  (angle)

$$6.03738 = \Phi_s$$

$$\gamma_9 = \sqrt{\frac{d^2 - 14d + 184}{24(d-4)}}, \quad \gamma_\Lambda = -\frac{10}{3} \frac{(d-4)(d-10)}{\sqrt{2(d^2 - 14d + 184)}}$$

$$\text{sqrt}[(((11^2-14*11+184)))/((24(11-4)))]$$

**Input:**

$$\sqrt{\frac{11^2 - 14 \times 11 + 184}{24(11 - 4)}}$$

**Result:**

$$\frac{\sqrt{\frac{151}{42}}}{2}$$

**Decimal approximation:**

0.948055654384026027535475008086838750296780006857956458452...

0.948055654 =  $\gamma_9$  - result very near to the spectral index  $n_s$ , to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 (see Appendix) and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}} \approx 0.9568666373$$

From:

Astronomy & Astrophysics manuscript no. ms c ESO 2019 - September 24, 2019  
**Planck 2018 results. VI. Cosmological parameters**

*The primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a spectral index  $n_s = 0.965 \pm 0.004$ , consistent with the predictions of slow-roll, single-field, inflation.*

**Alternate form:**

$$\frac{\sqrt{6342}}{84}$$

$$-10/3 * (((11-4)(11-10))) / ((2(11^2-14*11+184)))^{1/2}$$

**Input:**

$$-\frac{10}{3} \times \frac{(11-4)(11-10)}{\sqrt{2(11^2-14 \times 11+184)}}$$

**Result:**

$$-\frac{35 \sqrt{\frac{2}{151}}}{3}$$

**Decimal approximation:**

-1.34268245451301731435134380946037693444224131610524948089...

$$-1.342682454... = \gamma_{\Lambda}$$

**Alternate form:**

$$-\frac{35 \sqrt{302}}{453}$$

Thence:

$$V = V_0 \left( e^{\sqrt{3} \gamma_9 \Phi_t} + e^{\sqrt{3} \gamma_{\Lambda} \Phi_t} \right)$$

$$0.5(e^{(\sqrt{3} * 0.948055654 * 6.6336683)} + e^{(\sqrt{3} * -1.342682454 * 6.6336683)})$$

**Input interpretation:**

$$0.5 \left( e^{\sqrt{3} * 0.948055654 * 6.6336683} + e^{\sqrt{3} * (-1.342682454) * 6.6336683} \right)$$

**Result:**

26899.7...

26899.7...

**Series representations:**

$$0.5 \left( e^{\sqrt{3} * 0.948056 * 6.63367} + e^{(\sqrt{3} * 6.63367)(-1) * 1.34268} \right) =$$

$$0.5 e^{-8.90691 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \left( 1 + e^{15.196 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \right)$$

$$0.5 \left( e^{\sqrt{3} \cdot 0.948056 \times 6.63367} + e^{(\sqrt{3} \cdot 6.63367)(-1) \cdot 1.34268} \right) =$$

$$0.5 \exp \left( -8.90691 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \left( 1 + e^{15.196 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} \right)$$

$$0.5 \left( e^{\sqrt{3} \cdot 0.948056 \times 6.63367} + e^{(\sqrt{3} \cdot 6.63367)(-1) \cdot 1.34268} \right) =$$

$$0.5 \exp \left( -\frac{4.45346 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right)$$

$$\left( 1 + \exp \left( \frac{7.598 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) \right)$$

Now, from the formula of coefficients of the '5th order' mock theta function  $\psi_1(q)$ :  
(A053261 OEIS Sequence)

$$\sqrt{\phi} \cdot \exp(\pi \sqrt{n/15}) / (2 \cdot 5^{1/4} \cdot \sqrt{n})$$

for  $n = 294$  and subtracting 322 that is a Lucas number and adding the conjugate of the golden ratio, we obtain:

$$\left( \left( \left( \sqrt{\phi} \cdot \exp(\pi \sqrt{294/15}) / (2 \cdot 5^{1/4} \cdot \sqrt{294}) \right) \right) \right) - 322 + 1/\phi$$

**Input:**

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{294}{15}}\right)}{2 \sqrt[4]{5} \sqrt{294}} - 322 + \frac{1}{\phi}$$

$\phi$  is the golden ratio

**Exact result:**

$$\frac{e^{7\sqrt{2/5} \pi} \sqrt{\frac{\phi}{6}}}{14 \sqrt[4]{5}} + \frac{1}{\phi} - 322$$

**Decimal approximation:**

26899.31667422566335943323798656204015406864467228630180239...

26899.3166...

**Property:**

$$-322 + \frac{e^{7\sqrt{2/5}\pi} \sqrt{\frac{\phi}{6}}}{14\sqrt[4]{5}} + \frac{1}{\phi} \text{ is a transcendental number}$$

**Alternate forms:**

$$\frac{1}{2}(\sqrt{5} - 645) + \frac{1}{28} \sqrt{\frac{1}{15}(5 + \sqrt{5})} e^{7\sqrt{2/5}\pi}$$

$$-322 + \frac{2}{1 + \sqrt{5}} + \frac{\sqrt{1 + \sqrt{5}} e^{7\sqrt{2/5}\pi}}{28\sqrt{3}\sqrt[4]{5}}$$

$$\frac{14\sqrt[4]{5}\sqrt{6}(1 - 322\phi) + e^{7\sqrt{2/5}\pi}\phi^{3/2}}{14\sqrt[4]{5}\sqrt{6}\phi}$$

**Series representations:**

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{294}{15}}\right)}{2\sqrt[4]{5}\sqrt{294}} - 322 + \frac{1}{\phi} =$$

$$\left( 10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (294 - z_0)^k z_0^{-k}}{k!} - 3220\phi \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (294 - z_0)^k z_0^{-k}}{k!} + 5^{3/4}\phi \right.$$

$$\left. \exp\left[\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{98}{5} - z_0\right)^k z_0^{-k}}{k!}\right] \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!} \right) /$$

$$\left( 10\phi \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (294 - z_0)^k z_0^{-k}}{k!} \right) \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\begin{aligned}
& \frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{294}{15}}\right)}{2 \sqrt[4]{5} \sqrt{294}} - 322 + \frac{1}{\phi} = \\
& \left( 10 \exp\left(i \pi \left\lfloor \frac{\arg(294-x)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (294-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} - \right. \\
& \quad 3220 \phi \exp\left(i \pi \left\lfloor \frac{\arg(294-x)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (294-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \\
& \quad 5^{3/4} \phi \exp\left(i \pi \left\lfloor \frac{\arg(\phi-x)}{2\pi} \right\rfloor\right) \exp\left(\pi \exp\left(i \pi \left\lfloor \frac{\arg\left(\frac{98}{5}-x\right)}{2\pi} \right\rfloor\right)\right) \sqrt{x} \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{98}{5}-x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k (\phi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \Bigg/ \\
& \left( 10 \phi \exp\left(i \pi \left\lfloor \frac{\arg(294-x)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (294-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)
\end{aligned}$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$\begin{aligned}
& \frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{294}{15}}\right)}{2 \sqrt[4]{5} \sqrt{294}} - 322 + \frac{1}{\phi} = \left( \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(294-z_0)/(2\pi) \rfloor} z_0^{-1/2 \lfloor \arg(294-z_0)/(2\pi) \rfloor} \right. \\
& \left( 10 \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(294-z_0)/(2\pi) \rfloor} z_0^{1/2 \lfloor \arg(294-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (294-z_0)^k z_0^{-k}}{k!} - \right. \\
& \quad 3220 \phi \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(294-z_0)/(2\pi) \rfloor} z_0^{1/2 \lfloor \arg(294-z_0)/(2\pi) \rfloor} \\
& \quad \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (294-z_0)^k z_0^{-k}}{k!} + 5^{3/4} \phi \exp\left(\pi \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg\left(\frac{98}{5}-z_0\right)/(2\pi) \rfloor}\right) \\
& \quad \left. z_0^{1/2 (1 + \lfloor \arg\left(\frac{98}{5}-z_0\right)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{98}{5}-z_0\right)^k z_0^{-k}}{k!} \right) \\
& \left. \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(\phi-z_0)/(2\pi) \rfloor} z_0^{1/2 \lfloor \arg(\phi-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi-z_0)^k z_0^{-k}}{k!} \right) \Bigg/ \\
& \left( 10 \phi \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (294-z_0)^k z_0^{-k}}{k!} \right)
\end{aligned}$$



We have also that:

$$\left( \left( \left( 0.5 \left( e^{\sqrt{3} \cdot 0.948055654 \cdot 6.6336683} + e^{\sqrt{3} \cdot (-1.342682454) \cdot 6.6336683} \right) \right) \right)^{1/2} + 8 \right)$$

**Input interpretation:**

$$\sqrt{0.5 \left( e^{\sqrt{3} \times 0.948055654 \times 6.6336683} + e^{\sqrt{3} \times (-1.342682454) \times 6.6336683} \right) + 8}$$

**Result:**

172.011...

172.011...  $\approx$  172 (Ramanujan taxicab number)

**Series representations:**

$$\sqrt{0.5 \left( e^{\sqrt{3} \cdot 0.948056 \cdot 6.63367} + e^{(\sqrt{3} \cdot 6.63367)(-1) \cdot 1.34268} \right) + 8} =$$

$$0.707107 \left( 11.3137 + \sqrt{e^{-8.90691 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \left( 1 + e^{15.196 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \right)} \right)$$

$$\sqrt{0.5 \left( e^{\sqrt{3} \cdot 0.948056 \cdot 6.63367} + e^{(\sqrt{3} \cdot 6.63367)(-1) \cdot 1.34268} \right) + 8} = 0.707107$$

$$\left( 11.3137 + \sqrt{\exp \left[ -8.90691 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \left( 1 + e^{15.196 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} \right)} \right)$$

$$\sqrt{0.5 \left( e^{\sqrt{3} \cdot 0.948056 \cdot 6.63367} + e^{(\sqrt{3} \cdot 6.63367)(-1) \cdot 1.34268} \right) + 8} =$$

$$0.707107 \left( 11.3137 + \sqrt{\left( \exp \left[ -\frac{4.45346 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right] \right) \left( 1 + \exp \left[ \frac{7.598 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right] \right)} \right)$$

$$\left( \left( \left( 0.5 \left( e^{\sqrt{3} \cdot 0.948055654 \cdot 6.6336683} + e^{\sqrt{3} \cdot (-1.342682454) \cdot 6.6336683} \right) \right) \right)^{1/2} - 34 - 5 \right)$$

**Input interpretation:**

$$\sqrt{0.5 \left( e^{\sqrt{3} \times 0.948055654 \times 6.6336683} + e^{\sqrt{3} \times (-1.342682454) \times 6.6336683} \right) - 34 - 5}$$

**Result:**

125.011...

125.011... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Series representations:**

$$\sqrt{0.5 \left( e^{\sqrt{3} \cdot 0.948056 \cdot 6.63367} + e^{(\sqrt{3} \cdot 6.63367)(-1) \cdot 1.34268} \right) - 34 - 5 = 0.707107 \left( -55.1543 + \sqrt{e^{-8.90691 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \left( 1 + e^{15.196 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \right)} \right)}$$

$$\sqrt{0.5 \left( e^{\sqrt{3} \cdot 0.948056 \cdot 6.63367} + e^{(\sqrt{3} \cdot 6.63367)(-1) \cdot 1.34268} \right) - 34 - 5 = 0.707107 \left( -55.1543 + \sqrt{\exp \left( -8.90691 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \left( 1 + e^{15.196 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} \right)} \right)}$$

$$\sqrt{0.5 \left( e^{\sqrt{3} \cdot 0.948056 \cdot 6.63367} + e^{(\sqrt{3} \cdot 6.63367)(-1) \cdot 1.34268} \right) - 34 - 5 = 0.707107 \left( -55.1543 + \sqrt{\left( \exp \left( -\frac{4.45346 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) \right) \left( 1 + \exp \left( \frac{7.598 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) \right)} \right)}$$

And:

$$\sqrt{729} \times \frac{1}{2} \left( \left( \left( \left( 0.5 \left( e^{\sqrt{3} \times 0.948055654 \times 6.6336683} + e^{\sqrt{3} \times (-1.342682454) \times 6.6336683} \right) \right) - 34 - 2 \right) \right) + \frac{4}{5} \right)$$

**Input interpretation:**

$$\sqrt{729} \times \frac{1}{2} \left( \sqrt{0.5 \left( e^{\sqrt{3} \times 0.948055654 \times 6.6336683} + e^{\sqrt{3} \times (-1.342682454) \times 6.6336683} \right) - 34 - 2} \right) + \frac{4}{5}$$

**Result:**

1728.95...

1728.95...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Series representations:**

$$\frac{1}{2} \sqrt{729} \left( \sqrt{0.5 \left( e^{\sqrt{3} \times 0.948056 \times 6.63367} + e^{\sqrt{3} \times (-1.34268) \times 6.63367} \right) - 34 - 2} \right) + \frac{4}{5} =$$

$$0.353553 \left( 2.26274 - 50.9117 \sqrt{728} \sum_{k=0}^{\infty} 728^{-k} \binom{\frac{1}{2}}{k} + \sqrt{e^{-8.90691\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \left( 1 + e^{15.196\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \right) \sqrt{728} \sum_{k=0}^{\infty} 728^{-k} \binom{\frac{1}{2}}{k}} \right)$$

$$\frac{1}{2} \sqrt{729} \left( \sqrt{0.5 \left( e^{\sqrt{3} \cdot 0.948056 \times 6.63367} + e^{\sqrt{3} \cdot (-1.34268) \cdot 6.63367} \right) - 34 - 2} \right) + \frac{4}{5} =$$

$$0.353553 \left( 2.26274 - 50.9117 \sqrt{728} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{728}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \right.$$

$$\left. \sqrt{\exp\left[-8.90691 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right] \left(1 + e^{15.196 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}\right)} \right.$$

$$\left. \sqrt{728} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{728}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)$$

$$\frac{1}{2} \sqrt{729} \left( \sqrt{0.5 \left( e^{\sqrt{3} \cdot 0.948056 \times 6.63367} + e^{\sqrt{3} \cdot (-1.34268) \cdot 6.63367} \right) - 34 - 2} \right) + \frac{4}{5} =$$

$$0.353553 \left( 2.26274 - 50.9117 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (729 - z_0)^k z_0^{-k}}{k!} + \right.$$

$$\left. \sqrt{\left( \exp\left[-8.90691 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!}\right] \right. \right.$$

$$\left. \left. \left(1 + \exp\left[15.196 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!}\right]\right) \right) \sqrt{z_0} \right.$$

$$\left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (729 - z_0)^k z_0^{-k}}{k!} \right) \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

((((0.5(e^(sqrt3\*0.948055654\*6.6336683) + e^(sqrt3\*-1.342682454\*6.6336683))))))^(1/21) - 7\*1/10^3

### Input interpretation:

$$\sqrt[21]{0.5 \left( e^{\sqrt{3} \times 0.948055654 \times 6.6336683} + e^{\sqrt{3} \times (-1.342682454) \times 6.6336683} \right) - 7} \times \frac{1}{10^3}$$

### Result:

1.618325531898728836063509055847500751410065335542606770967...

1.6183255318.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

### Series representations:

$$\begin{aligned}
 & \sqrt[21]{0.5 \left( e^{\sqrt{3} \cdot 0.948056 \times 6.63367} + e^{(\sqrt{3} \cdot 6.63367)(-1) \cdot 1.34268} \right) - \frac{7}{10^3}} = \\
 & 0.967532 \left( -0.0072349 + \sqrt[21]{e^{-8.90691 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \left( 1 + e^{15.196 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \right)} \right) \\
 \\
 & \sqrt[21]{0.5 \left( e^{\sqrt{3} \cdot 0.948056 \times 6.63367} + e^{(\sqrt{3} \cdot 6.63367)(-1) \cdot 1.34268} \right) - \frac{7}{10^3}} = 0.967532 \left( -0.0072349 + \right. \\
 & \left. \sqrt[21]{\exp \left( -8.90691 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \left( 1 + e^{15.196 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} \right)} \right) \\
 \\
 & \sqrt[21]{0.5 \left( e^{\sqrt{3} \cdot 0.948056 \times 6.63367} + e^{(\sqrt{3} \cdot 6.63367)(-1) \cdot 1.34268} \right) - \frac{7}{10^3}} = \\
 & 0.967532 \left( -0.0072349 + \left( \exp \left( -\frac{4.45346 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right)} \right. \right. \\
 & \left. \left. \left( 1 + \exp \left( \frac{7.598 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) \right) \right)^{\wedge (1/21)} \right)
 \end{aligned}$$

### Observations

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants  $\pi$ ,  $\phi$ ,  $1/\phi$ , the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

## **Acknowledgments**

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## **References**

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### **Integrable Scalar Cosmologies I. Foundations and links with String Theory**

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