The imaginary parity of elementary particles

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Abstract

The Dirac equation with the imaginary reflection is considered. The intrinsic imaginary parity can be generated in the proton antiproton Dalitz reaction. The violation of the intrinsic parity of a elementary particle can be explained as the oscillation of parity before the particle decay into plus and minus parity system. The explanation is the alternative form of the older explanation.

1 Introduction

The imaginary parity was postulated by Spitzer (1960, 1963), using K^0 meson. The K^0 meson has well-defined imaginary parity relative to the vacuum. Further, electromagnetic transitions could carry K^0 into $\bar{K}^0$ in the presence of an external field. A search, by analysing existing experimental results, for the implied dependence of the $K_1^0 - K_2^0$ mass difference on the external field was proposed and discussed by Natapoff (Natapoff, 1965). However, the mathematical form of the imaginary parity is not realized in the final form. We present the relation of the imaginary parity to the Dirac equation.

Bruno Pontecorvo was the first physicist who introduced the electron-neutrino, muon-neutrino and tau-neutrino oscillation. We used here his idea to define the so called parity oscillation of K_+, K_- mesons as the analogue of the Pontecorvo model for neutrino oscillations.

2 The violation of space parity

It is well known that the parity operator P acts on the wave function $\psi(x)$ as follows:

$$P\psi(x) = \lambda\psi(-x),\ P^2\psi(x) = \lambda^2\psi(x) = \psi(x), \tag{1}$$

from which follows that the eigen value of parity λ is ± 1. The wave function $\psi(x)$ corresponds to the state with positive parity and $-\psi(x)$ corresponds to the state with negative parity.

Now, let us be specific and let us consider the $\theta - \tau$ puzzle. Dalitz (1953; 1956a; 1956b) analyzed the τ into three pions and in doing so introduced the Dalitz plot into physics. The first use of the Dalitz plot revealed that the θ particle appeared to be the same as the τ, which was paradoxical. The puzzle persisted for two years: Dalitz created te hypothesis that perhaps the law of odds and evens was not true, even though all the evidence said otherwise (Dalitz, 1956b).

If the decay is according to form $\theta \to \pi + \pi$, then parity of θ is $(-1)(-1) = 1$, and if $\tau \to \pi+\pi+\pi$, then parity of τ is $(-1)(-1)(-1) = -1$. However, it was experimentally confirmed that θ and τ are the same particle called K-meson. So we have two decays:

$$K \to \pi + \pi, \tag{2}$$

and

$$K \to \pi + \pi + \pi. \tag{3}$$

The first process can be evidently described by Hamiltonian $H_1 = AB_+$, and the second process can be described by Hamiltonian $H_2 = AB_-$, while the original Hamiltonian is

$$H = A(C_1B_+ + C_2B_-), \tag{4}$$

where C_1, C_2 are some constants.

After application o the parity operator on the Hamiltonian (4), we get

$$PH = A(C_1B_+ - C_2B_-), \tag{5}$$

It is possible to show, (Lee, et al., 1956; Yang, 1964), that the angular distribution $N(\varphi)$ of electrons in beta decay of cobalt C_{60} is described by function, which follows from the Hamiltonian, which is combination of positive parity term and negative parity term (4). The resulting function is

$$N(\varphi) = A(1 + a\cos\varphi). \tag{6}$$

It means that the correct description of the process of the violation of parity is described by the Hamiltonian with the positive and negative parity term.

3 The oscillation of parity

The main idea of this section is in the following principle: If system K is described by Hamiltonian $H_1 = AB_+$, where A, B_+ are the positive parity operators, and at the same time it can be described by the Hamiltonian $H_2 = AB_-$, where B_- is the negative parity operator, then it can be in reality described by the Hamiltonian $H = A(C_1B_+ + C_2B_-)$.

The immediately statement following from the last principle is, that K system oscillates between positive and negative parity states to form the positive parity states, or the negative parity states. If the system is K-meson, then this meson oscillates between positive and negative mesons, forming θ-particle and τ-particle.

The oscillation of parity is the crucial problem of the future particle physics. It is not confirmed that the oscillation of parity is equivalent to the violation of parity. We write it as follows

$$\theta(t) = \tau(t) \to (\pi + \pi); \quad P = 1 \tag{7a}$$

$$\theta(t + \Delta t) = \tau(t + \Delta t) \to (\pi + \pi + \pi); P = -1 \tag{7b}$$

The fall of parity open the way for a reconsideration of physical theories and led to new, discoveries regarding the nature of matter and the universe. The work of Lee and Yang (1956), Wu et al. (1957), and Garwin et al. (1957) has provided definitive proof that mirror symmetry is broken in the weak interaction, and it is not a true symmetry of the Universe as a whole. This is an surprising conclusion, because the historical trend of physics up to this point was that of simplification and unification. In the case of parity violation, however, we learned that nature exhibits a very striking asymmetry for no apparent reason, and goes against our expectation of a symmetrical universe.

The matrix element describing the violation of parity during the $\beta-$ decay of neutron into the electron and electron antineutrino is of the form (Commins et al., 1983):

$$M = \frac{G_F}{\sqrt{2}} \int d\mathbf{x} \bar{\psi}_p \gamma_\mu (1 - \lambda\gamma_5)\psi_n \psi_e \gamma^\mu (1 - \gamma_5)\psi_{\bar{\nu}_e}; \quad \lambda \approx 1,25, \tag{8}$$

where the asymetry of the matrix element M is caused by therm with γ^5, where parity of $\bar{\psi}\gamma^\mu\gamma^5\psi$ is $-(-1)^\mu; \quad \gamma^5 = \gamma^1\gamma^2\gamma^3\gamma^4; \mu = 0, 1, 2, 3$ (Peskin et al., 1977). The analogical matrix element was derived for the decay of the μ meson and so on (Commins et al., 1983).

4 The imaginary parity

The imaginary parity operator Π acts on the wave function $\psi(x)$ as follows:

$$\Pi\psi(x) = \Lambda\psi(ix),\ \Pi^2\psi(x) = \Lambda^2\psi(-x),$$

$$\Pi^3\psi(x) = \Lambda^3\psi(-ix),\ \Pi^4\psi(x) = \Lambda^4\psi(x) = \psi(x) \tag{9}$$

from which follows that the eigen value of parity equation $\Lambda^4 = 1$ is ± 1 and $\pm i$ The wave function $+\psi(x)$ corresponds to the state with the positive parity and $-\psi(x)$ corresponds to the state with negative parity. The wave function $+\mathrm{i}\psi(x)$ corresponds to the state with the positive imaginary parity and $-i\psi(x)$ corresponds to the state with the negative imaginary parity. While the algebra of the states of the imaginary parity is elementary, the experimental proof of the existence of the particles in the states of the imaginary parity is sophisticated.

Let us consider the following particle reaction to establish the imaginary parity (Dalitz, 1962).

$$(\bar{p} + p) \to \pi + \pi + \pi + \pi + \pi^0, \tag{10}$$

where the particle π^0 can be detected by the Dalitz method (Dalitz, 1962). The parity of the right side of eq. (10) is $(-1)\times(-1)\times(-1)\times(-1)\times(-1) = (-1)$. So, the parity of the left side is also (-1). However the process $(\bar{p} + p)$ involves the possible channel with particles in the states with the imaginary parity leading to the conservation of parity. Namely, $(i) \times (i) \times (i) \times (i) \times (-1) = (-1)$, where only π^0-meson was not replaced by i.

The similar reactions and decay channels having the imaginary parity are involved in the Particle Data Group (Tanabashi et al., 2018) and can be verified by the well-educated experimenters.

5 The intrinsic parity and the pion

Let us consider the parity of the π^- meson (Muirhed, 1968). When π^- mesons are brought to rest in liquid hydrogen or deuterium, then the following reactions are observed (Panovsky, Aamodt and Hadley, 1951):

$$\pi^- + p \to n + \gamma, \tag{11a}$$

$$\pi^- + p \to n + \pi^0 \tag{11b}$$

and

$$\pi^- + d \to n + n, \tag{12a}$$

$$\pi^- + d \to n + n + \gamma. \tag{12b}$$

The surprising feature of these transitions is the absence of π^0 mesons from the reactions in deuterium, whereas in hydrogen one-half of the reactions produce π^0 mesons. This result may be understand if the π^- meson has odd parity.

Another demonstration of the pion parity was given in 1954, by Chinowsky and Steinberger (Chinowsky et al., 1954). They studied the decay of an "atom" made from a deuteron $({}^2_1H^+)$ and a negatively charged pion π^- in a state with zero orbital angular momentum $l = 0$ into two neutrons n.

So, we see that parity is not defined by the microscopic theory of the electron structure but only by phenomenology. Let us remark that it is well known that CP parity is violated and it is not excluded that the analogical violation is valid for $C\Pi$ parity being the crucial problem for the well educated experimenters in particle physics.

6 The parity of the wave functions

Now, let us still remember the mathematical definition of parity in quantum mechanics. So, let us consider the relative motion of two particle in quantum theory. The wave function of such two-body system is described by the wave function

$$\psi_l(r) = R(l)P_l^m(\cos\theta)e^{im\varphi}, \tag{13}$$

where $P_l^m(\cos\theta)$ is the associate Legendre function. After the operation of reflection with $(\cos\theta) \to (-\cos\theta)$, the Legendre coefficient is transformed to $(-1)^{l-m}$, where l is the quantum number of the orbital momentum with projection on the polar axis. During reflection $r \to r$, and $\theta \to \pi - \theta$, we have $\cos(\pi-\theta) \to -\cos\theta$, $\varphi \to \pi + \varphi$, and $e^{im\varphi} \to e^{im(\pi+\varphi)} \to (-1)^m e^{im\varphi}$. Then,

$$\psi_l(r) = R(l)P_l^m(\cos\theta)e^{im\varphi} \to$$

$$R(l)(-1)^{l-m}P_l^m(\cos\theta)e^{im\varphi}(-1)^m = (-1)^l\psi_l(r) \tag{14}$$

So, the parity of the system is

$$P_l = (-1)^l \tag{15}$$

7 The imaginary parity of the plane wave

If we apply the imaginary reflection $\mathbf{x} \to i\mathbf{x}$ on the particle in the state of the plane wave

$$\psi(\mathbf{x}, t) = e^{i(\mathbf{kx}-\omega t)}, \tag{16}$$

we get

$$\psi(\mathbf{x}, t) \to e^{-\mathbf{kx}-i\omega t}, \tag{17}$$

which means that the elementary particle is in the nonperiodic space state and the imaginary parity cannot be detected by the Dalitz method.

The mechanism of the imaginary reflection is not known similarly to the mechanism of the normal reflection. One possibility is the oscillation of parity of the original particle, or of the original process.

8 Parity and the Dirac equation

The transformation of the mirror reflection is

$$\mathbf{x} \to -\mathbf{x}; \quad t \to t \tag{18}$$

and the corresponding matrix of transformation is is:

$$a_{\alpha\lambda} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{19}$$

The invariance of the Dirac equation with regard to transformation (19) generates the following equation involving the transformation matrix S (Muirhead, 1968):

$$\gamma_\alpha = a_{\alpha\lambda} S \gamma_\lambda S^{-1}, \tag{20}$$

which can be rewriting in the explicit form as follows:

$$\gamma_1 = -S\gamma_1 S^{-1},\ \gamma_2 = -S\gamma_2 S^{-1},$$

$$\gamma_3 = -S\gamma_3 S^{-1},\ \gamma_4 = +S\gamma_4 S^{-1}. \tag{21}$$

with the solution

$$S = \gamma_4. \tag{22}$$

9 Imaginary parity and the Dirac equation

In this case the metric tensor is of the form

$$a_{\alpha\lambda} = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{23}$$

and we perform the operation by the analogy with the previous section. So, the invariance of the Dirac equation with regard to transformation (23) generates the following equation involving the transformation matrix S:

$$\gamma_\alpha = a_{\alpha\lambda} S \gamma_\lambda S^{-1}, \tag{24}$$

which can be rewriting in the explicit form as follows:

$$\gamma_1 = iS\gamma_1 S^{-1},\ \gamma_2 = iS\gamma_2 S^{-1},$$

$$\gamma_3 = iS\gamma_3 S^{-1}, \quad \gamma_4 = S\gamma_4 S^{-1}. \tag{25}$$

with the solution of $\gamma_4 = S\gamma_4 S^{-1}$

$$S = \gamma_4, \tag{26}$$

which is not solution for eqs. with $\gamma_1, \gamma_2, \gamma_3$ in eq. (25).

10 The space-time metric in the imaginary mirror

Let us consider the Minkowski space-time metric:

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2. \tag{27}$$

Then, this metric in the imaginary mirror is of the form:

$$ds^2 = c^2dt^2 + dx^2 + dy^2 + dz^2. \tag{28}$$

Now, let us consider relativistic spherical metric:

$$ds^2 = c^2dt^2 - dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\varphi^2. \tag{29}$$

Then, in the imaginary mirror, the metric i as follows ($r \to ir$)

$$ds^2 = c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2. \tag{30}$$

11 Discussion

To probe the spin and parity quantum numbers of the discovered particles, a systematic analysis of its production and decay processes is necessary to perform. An excess over the expected background must be used to further discriminate between signal hypotheses. These analysis are based on probing various alternative models of spin and parity.

The models can be expressed in terms of an effective Lagrangian, or, in terms of helicity amplitudes. The two approaches are equivalent. The effective Lagrangian formalism is frequently chosen to describe the models considered and a restricted number of models are discussed.

In the analysis performed by CMS a larger number of models have been investigated, however the main channels studied by both experiments are essentially the same and the main conclusions are similar and fully consistent (Tanabashi et al., 2018).

The revision of all experiments in the Tanabashi monograph (Tanabashi et al., 2018) can lead to the discovery of the imaginary parities of elementary particles and to the new deal of physics, while the parities of black hole, white hole and wormhole hole follow from the Einstein equations. On the other hand the parity of the black hole as the black body (Lee, 1988) is the parity of the black body photons with the parity of every photon (-1) (Berestetzkii, et al., 1989).

The operation of parity can also be expressed as a rotation of the decay plane, so no parity violating kinematic observables can be defined (unless they also violate rotational invariance). The use of parity-odd observables in four body decays is discussed in literature (Tanabashi et al., 2018).

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