ON SOME APPLICATIONS OF b-OPEN SETS IN TOPOLOGICAL SPACES

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Abstract

The purpose of this paper is to introduce some new classes of topological spaces by utilizing b-open sets and study some of their fundamental properties.

AMS Subject Classification (2000): 54D10
Key words and phrases: b-open sets, b-closure, b-R_1 spaces, b-T_2 spaces, D-sets, b-D_1-spaces.

1 Introduction

In 1996, Andrijević [2] introduced a new class of generalized open sets called b-open sets into the field of topology. This class is a subset of the class of semi-preopen sets [3] also called β-open sets [1], i.e. a subset of a topological space which is contained in the closure of the interior of its closure. Also the class of b-open sets is a superset of the class of semi-open sets [7], i.e. a set which is contained in the closure of its interior, and the class of locally dense sets [6] or preopen sets [8], i.e. a set which is contained in the interior of its closure. Andrijević studied several fundamental and interesting properties of b-open sets. Among others, he showed that a rare b-open set is preopen [[2], Proposition 2.2]. Recall that a rare set [4] is a set with no interior points. It is well-known that for a topological space X, every rare b-open set is semi-open if and only if the interior of a dense subset is dense. Quite recently Caldas et al. [5] obtained some new generalized sets by utilizing b-open sets and investigated the topologies defined by these families of sets.

2 Preliminaries

Throughout the present paper, (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let $A \subseteq X$, then A is said to be *b-open* [2](resp. *α-open* [9]) if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$(resp. $A \subset Int(Cl(Int(A)))$), where $Cl(A)$ and $Int(A)$ denote the closure and the interior of A in (X, τ), respectively. The complement $A^c = X \setminus A$ of a b-open set A is called *b-closed* and the *b-closure* of a set A, denoted by $Cl_b(A)$, is the intersection of all b-closed sets containing A. The *b-interior* of a set A denoted by $Int_b(A)$, is the union of all b-open sets contained in A. It is obvious that if the bounday of a b-open set is nowhere dense, then it is semi-open. Moreover a rare b-open set with a nowhere dense boundary is α-open! Also a b-open set which its closure is regulary closed (or semiopen) is β-open! Recall that a subset A of a space (X, τ) is called *regular open* (resp. *regularly closed*) if $A = Int(Cl(A))$(resp. $A = Cl(Int(A))$). It is clear that if a b-open set is closed then it is semiopen.

The family of all b-open (resp. b-closed) sets in (X, τ) will be denoted by $BO(X, \tau)$ (resp. $BC(X, \tau)$).

Proposition 2.1 *(Andrijević [2])* (a) The union of any family of b-open sets is a b-open.
(b) The intersection of an open and a b-open set is a b-open set.

Lemma 2.2 The *b-closure* $Cl_b(A)$, is the set of all $x \in X$ such that $O \cap A \neq \emptyset$ for every $O \in BO(X, x)$, *where* $BO(X, x) = \{U \mid x \in U, U \in BO(X, \tau)\}$.

A subset N_x of a topological space X is said to be a b-neighbourhood of a point $x \in X$ if there exists a b-open set U such that $x \in U \subset N_x$.

Lemma 2.3 *A subset of a space X is b-open in X if and only if it is a b-neighbourhood of each of its points.*

3 b-R_1 Topological Spaces

Definition 1 *Let (X, τ) be a space and $A \subset X$. Then the b-kernel of A, denoted by $bKer(A)$ is defined to be the set*
$bKer(A) = \cap\{G \in BO(X, \tau) \mid A \subset G\}$.

It should be noticed that $bKer(A)$ is defined as B^{Λ_b} in [5].

Lemma 3.1 *Let* (X, τ) *be a space and* $x \in X$*. Then,*
$y \in bKer(\{x\})$ *if and only if* $x \in Cl_b(\{y\})$.

Proof. Assume that $y \notin bKer(\{x\})$. Then there exists a b-open set V containing x such that $y \notin V$. Therefore, we have $x \notin Cl_b(\{y\})$. The converse is similarly shown.

Lemma 3.2 *Let* (X, τ) *be a space and* A *a subset of* X*. Then,* $bKer(A) = \{x \in X \mid Cl_b(\{x\}) \cap A \neq \emptyset\}$.

Proof. Let $x \in bKer(A)$ and $Cl_b(\{x\}) \cap A = \emptyset$. Therefore, $x \notin X \setminus Cl_b(\{x\})$ which is a b-open set containing A. But this is impossible, since $x \in bKer(A)$. Consequently, $Cl_b(\{x\}) \cap A \neq \emptyset$. Now, let $x \in X$ such that $Cl_b(\{x\}) \cap A \neq \emptyset$. Suppose that $x \notin bKer(A)$. Then, there exists a b-open set U containing A and $x \notin U$. Let $y \in Cl_b(\{x\}) \cap A$. Thus, U is a b-neigbourhood of y such that $x \notin U$. By this contradiction $x \in bKer(A)$.

Lemma 3.3 *The following statements are equivalent for any points* x *and* y *in a space* (X, τ) *:*
(1) $bKer(\{x\}) \neq bKer(\{y\})$;
(2) $Cl_b(\{x\}) \neq Cl_b(\{y\})$.

Proof. (1) $\rightarrow$ (2) : Let $bKer(\{x\}) \neq bKer(\{y\})$, then there exists a point z in X such that $z \in bKer(\{x\})$ and $z \notin bKer(\{y\})$. From $z \in bKer(\{x\})$ it follows that $\{x\} \cap Cl_b(\{z\}) \neq \emptyset$ which implies $x \in Cl_b(\{z\})$. By $z \notin bKer(\{y\})$, we have $\{y\} \cap Cl_b(\{z\}) = \emptyset$. Since $x \in Cl_b(\{z\})$, $Cl_b(\{x\}) \subset Cl_b(\{z\})$ and $\{y\} \cap Cl_b(\{x\}) = \emptyset$. Therefore it follows that $Cl_b(\{x\}) \neq Cl_b(\{y\})$. Now $bKer(\{x\}) \neq bKer(\{y\})$ implies that $Cl_b(\{x\}) \neq Cl_b(\{y\})$.
(2) $\rightarrow$ (1) : Suppose that $Cl_b(\{x\}) \neq Cl_b(\{y\})$. Then there exists a point z in X such that $z \in Cl_b(\{x\})$ and $z \notin Cl_b(\{y\})$. It means that there exists a b-open set containing z. Therefore x but not y, i.e., $y \notin bKer(\{x\})$ and hence $bKer(\{x\}) \neq bKer(\{y\})$.

Recall that a space (X, τ) is called b-T_0 (resp. b-T_1 [5]) if for any distinct pair of points x and y in X, there is a b-open U in X containing x but not y or (resp. and) a b-open set V in X containing y but not x. It is worth-noticing that in a private correspondence Professor Maximilian Ganster has shown that a space is b-T_1 if and only if each singleton is either rare or regular open.

Theorem 3.4 *Every topological space* (X,τ) *is* b-T_0.

Proof. Take two points x and y in X. If $Int\{x\}$ is nonempty then $\{x\}$ is open, thus b-open and we are done. Otherwise, if $Int\{x\}$ is empty, then $\{x\}$ is preclosed , i.e. $X-\{x\}$ is a preopen (thus b-open) set containing y , and we are also done.

Theorem 3.5 *For a space* (X,τ) *each pair of distinct points* x*,* y *of* X *,* $Cl_b(\{x\}) \neq Cl_b(\{y\})$.

Proof. Let x, y be any two distinct points of X. Since every space (X,τ) is b-T_0 (Theorem 3.4), there exists a b-open set G containing x or y, say x but not y. Then G^c is a b-closed set which does not contain x but contains y. Since $Cl_b(\{y\})$ is the smallest b-closed set containing y, $Cl_b(\{y\}) \subset G^c$, and so $x \notin Cl_b(\{y\})$. Consequently $Cl_b(\{x\}) \neq Cl_b(\{y\})$.

Theorem 3.6 *A space* (X,τ) *is* b-T_1 *if and only if the singletons are* b*-closed sets.*

Proof. Suppose that (X,τ) is b-T_1 and $x \in X$. Let $y \in \{x\}^c$. Then $x \neq y$ and so there exists a b-open set U_y such that $y \in U_y$ but $x \notin U_y$. Consequently $y \in U_y \subset \{x\}^c$ i.e., $\{x\}^c = \bigcup\{U_y / y \in \{x\}^c\}$ which is b-open.

Conversely. Suppose that $\{p\}$ is b-closed for every $p \in X$. Let $x, y \in X$ with $x \neq y$. Now $x \neq y$ implies $y \in \{x\}^c$. Hence$\{x\}^c$ is a b-open set containing y but not x. Similarly $\{y\}^c$ is a b-open set containing x but not y. Accordingly X is a b-T_1 space.

Definition 2 *A space* (X,τ) *is said to be* b-R_1 *if for* x, y *in* X *with* $Cl_b(\{x\}) \neq Cl_b(\{y\})$*, there exist disjoint* b*-open sets* U *and* V *such that* $Cl_b(\{x\})$ *is a subset of* U *and* $Cl_b(\{y\})$ *is a subset of* V*.*

Theorem 3.7 *A space* (X,τ) *is* b-R_1 *if and only if for* $x, y \in X, bKer(\{x\}) \neq bKer(\{y\})$*, there exist disjoint* b*-open sets* U *and* V *such that* $Cl_b(\{x\}) \subset U$ *and* $Cl_b(\{y\}) \subset V$.

Proof. It follows from Lemma 3.3.

A space (X,τ) is called b-T_2 if for any distinct pair of points x and y in X, there exist b-open sets U and V in X containing x and y, respectively, such that $U \cap V = \emptyset$.

Theorem 3.8 *A space (X, τ) is b-T_2 if and only if (X, τ) is b-R_1.*

Proof. Necessity. Since X is b-T_2, then X is b-T_1. If $x, y \in X$ such that $Cl_b(\{x\}) \neq Cl_b(\{y\})$, then $x \neq y$. Then there exists disjoint b-open sets U and V such that $x \in U$ and $y \in V$; hence $Cl_b(\{x\}) = \{x\} \subset U$ and $Cl_b(\{y\}) = \{y\} \subset V$. Hence X is b-R_1.
Sufficiency. Let $x, y \in X$ such that $x \neq y$. By Theorem 3.4, There exists a b-open set U such that $x \in U$ and $y \notin U$. Then by Lemma 3.1 $x \notin Cl_b(\{y\})$ and hence $Cl_b(\{x\}) \neq Cl_b(\{y\})$. Therefore there exist disjoint b-open sets U_1 and U_2 such that $x \in Cl_b(\{x\}) \subset U_1$ and $y \in Cl_b(\{y\}) \subset U_2$. Thus X is b-T_2

Theorem 3.9 *A space X is b-T_2 if and only if the intersection of all b-closed b-neighourhoods of each point of X is reduced to that point.*

Proof. Necessity. Let X be b-T_2 and $x \in X$. Then for each $y \in X$ which is distinct from x, there exist b-open sets G and H such that $x \in G$, $y \in H$ and $G \cap H = \emptyset$. Since $x \in G \subset H^c$, hence H^c is a b-closed b-neighbourhood of x to which y does not belong . Consequently, the intersection of all b-closed b-neighbourhood of x is reduced to $\{x\}$.

Sufficiency. Let x, $y \in X$ and $x \neq y$. Then by hypothesis there exists a b-closed b-neighbourhood U of x such that $y \notin U$. Now there is a b-open set G such that $x \in G \subset U$. Thus G an U^c are disjoint b-open sets containing x and y respectively. Hence X is b-T_2.

Theorem 3.10 *For a space (X, τ), the following statements are equivalent :*
(1) (X, τ) is b-R_1 ;
(2) If $x, y \in X$ such that $Cl_b(\{x\}) \neq Cl_b(\{y\})$, then there exist b-closed sets F_1 and F_2 such that $x \in F_1$, $y \notin F_1$, $y \in F_2$, $x \notin F_2$ and $X = F_1 \cup F_2$.

Proof. (1) $\rightarrow$ (2) : Let $x, y \in X$ such that $Cl_b(\{x\}) \neq Cl_b(\{y\})$, and hence $x \neq y$. Therefore, there exist disjoint b-open sets U_1 and U_2 such that $x \in Cl_b(\{x\}) \subset U_1$ and $y \in Cl_b(\{y\}) \subset U_2$. Then $F_1 = X \setminus U_2$ and $F_2 = X \setminus U_1$ are b-closed sets such that $x \in F_1$, $y \notin F_1$, $y \in F_2$, $x \notin F_2$ and $X = F_1 \cup F_2$.
(2) $\rightarrow$ (1) : Suppose that x and y are distinct points of X, such that $Cl_b(\{x\}) \neq Cl_b(\{y\})$. Therefore there exist b-closed sets F_1 and F_2 such that $x \in F_1$, $y \notin F_1$, $y \in F_2$, $x \notin F_2$ and $X = F_1 \cup F_2$. Now, we set $U_1 = X \setminus F_2$ and $U_2 = X \setminus F_1$, then we obtain that $x \in U_1$, $y \in U_2$,

$U_1 \cap U_2 = \emptyset$ and U_1, U_2 are b-open. This shows that (X, τ) is b-T_2. It follows from Theorem 3.8 that (X, τ) is b-R_1.

A space (X, τ) is said to be a b-R_0 space if every b-open set contains the b-closure of each of its singletons.

Theorem 3.11 *For every space (X, τ) the following statements are equivalent:*
a) b-R_0.
b) b-T_1.

Proof. The equivalnce of b-T_1 and b-R_0 follows from the fact that b-T_1 is equivalent to b-R_0 and b-T_0.

A point x of a space (X, τ) is an b-θ-accumulation point of a subset $A \subset X$, if for each b-open U of X containing x, $Cl_b(U) \cap A \neq \emptyset$. The set $bCl_\theta(A)$ of all b-θ-accumulation points of A is called the b-θ-closure of A. The set A is said to be b-θ-closed if $bCl_\theta(A) = A$. Complement of a b-θ-closed set is said to be b-θ-open.

Lemma 3.12 *For any subset A of a space (X, τ), $Cl_b(A) \subset bCl_\theta(A)$.*

Lemma 3.13 *Let x and y are points in a space (X, τ). Then $y \in bCl_\theta(\{x\})$ if and only if $x \in bCl_\theta(\{y\})$.*

Theorem 3.14 *A space (X, τ) is b-R_1 if and only if for each $x \in X$, $Cl_b(\{x\}) = bCl_\theta(\{x\})$.*

Proof. Necessity. Assume that X is b-R_1 and $y \in bCl_\theta(\{x\}) \setminus Cl_b(\{x\})$. Then there exists a b-open set U containing y such that $Cl_b(U) \cap \{x\} \neq \emptyset$ but $U \cap \{x\} = \emptyset$. Thus $Cl_b(\{y\}) \subset U$, $Cl_b(\{x\}) \cap U = \emptyset$. Hence $Cl_b(\{x\}) \neq Cl_b(\{y\})$. Since X is b-R_1, there exist disjoint b-open sets U_1 and U_2 such that $Cl_b(\{x\}) \subset U_1$ and $Cl_b(\{y\}) \subset U_2$. Therefore $X \setminus U_1$ is a b-closed b-neigbourhood at y which does not contain x. Thus $y \notin bCl_\theta(\{x\})$. This is a contradiction.

Sufficiency. Suppose that $Cl_b(\{x\}) = bCl_\theta(\{x\})$ for each $x \in X$. We first prove that X is b-R_0. Let x belong to the b-open set U and $y \notin U$. Since $bCl_\theta(\{y\}) = Cl_b(\{y\}) \subset X \setminus U$, we have $x \notin bCl_\theta(\{y\})$ and by Lemma 3.13 $y \notin bCl_\theta(\{x\}) = Cl_b(\{x\})$. It follows that $Cl_b(\{x\}) \subset U$. Therefore (X, τ) is

b-R_0. Now, let $a, b \in X$ with $Cl_b(\{a\}) \neq Cl_b(\{b\})$. By Theorem 3.11, (X, τ) is b-T_1 and $b \notin bCl_\theta(\{a\})$ and hence there exists a b-open set U containing b such that $a \notin Cl_b(U)$. Therefore, we obtain $b \in U$, $a \in X \setminus Cl_b(U)$ and $U \cap (X \setminus Cl_b(U)) = \emptyset$. This shows that (X, τ) is b-T_2. It follows from Theorem 3.8 that (X, τ) is b-R_1.

4 Others Properties of b-open Sets

Definition 3 *A subset A of a space X is called a bD-set if there are two $U, V \in BO(X, \tau)$ such that $U \neq X$ and A=$U \setminus V$.*

One can observe that every b-open set U different from X is a bD-set if A=U and V=$\emptyset$.

Definition 4 *A space (X, τ) is called:*
(i) b-D_0 if for any distinct pair of points x and y of X there exists a bD-set of X containing x but not y or a bD-set of X containing y but not x.
(ii) b-D_1 if for any distinct pair of points x and y of X there exists a bD-set of X containing x but not y and a bD-set of X containing y but not x.
(iii) b-D_2 if for any distinct pair of points x and y of X there exist disjoint bD-sets G and E of X containing x and y, respectively.

Remark 4.1 *(i) If (X, τ) is b-T_i , then it is b-T_{i-1}, $i = 1, 2$.*
(ii) If (X, τ) is b-T_i , then (X, τ) is b-D_i , $i = 0, 1, 2$.
(iii) If (X, τ) is b-D_i , then it is b-D_{i-1} , $i = 1, 2$.

Theorem 4.2 *For a space (X, τ) the following statements are true:*
(1) (X, τ) is b-D_0 if and only if it is b-T_0.
(2) (X, τ) is b-D_1 if and only if it is b-D_2.

Proof. (1) We prove only the necessity condition since the sufficiency condition is stated in Remark 4.1(ii).

Necessity. Let (X, τ) be b-D_0. Then for each distinct pair $x, y \in X$, at least one of x, y, say x, belongs to a bD-set G but $y \notin G$. Let $G = U_1 \setminus U_2$ where $U_1 \neq X$ and U_1,$U_2 \in BO(X, \tau)$. Then $x \in U_1$, and for $y \notin G$ we have two cases: (a) $y \notin U_1$; (b) $y \in U_1$ and $y \in U_2$.

In case (a), U_1 contains x but not y ;

In case (b), U_2 contains y but not x. Hence X is b-T_0.
(2) Sufficiency. Remark 4.1(iii).

Necessity. Let X be a b-D_1 topological space. Then for each distinct pair $x, y \in X$, we have bD-sets G_1, G_2 such that $x \in G_1$, $y \notin G_1$; $y \in G_2$, $x \notin G_2$. Let $G_1 = U_1 \backslash U_2$, $G_2 = U_3 \backslash U_4$. ¿From $x \notin G_2$, we have either $x \notin U_3$ or $x \in U_3$ and $x \in U_4$. Now we consider the following two cases separately.

(1) $x \notin U_3$. From $y \notin G_1$ we have two subcases:

(a) $y \notin U_1$. From $x \in U_1 \backslash U_2$ we have $x \in U_1 \backslash (U_2 \cup U_3)$ and from $y \in U_3 \backslash U_4$ we have $y \in U_3 \backslash (U_1 \cup U_4)$. Therefore, $(U_1 \backslash (U_2 \cup U_3)) \cap (U_3 \backslash (U_1 \cup U_4) = \emptyset$.

(b) $y \in U_1$ and $y \in U_2$. We have $x \in U_1 \backslash U_2$, $y \in U_2$. $(U_1 \backslash U_2) \cap U_2 = \emptyset$.

(2) $x \in U_3$ and $x \in U_4$. We have $y \in U_3 \backslash U_4$, $x \in U_4$. $(U_3 \backslash U_4) \cap U_4 = \emptyset$.
From the discussion above we know that the space X is b-D_2.

From Theorems 4.2 and 3.4, we obtain also that every space is b-D_0.

Definition 5 *A point $x \in X$ which has X as the b-neighborhood is called a b-neat point.*

Theorem 4.3 *For a space (X, τ) the following are equivalent:*
(1) (X, τ) is b-D_1;
(2) (X, τ) has no b-neat point.

Proof. (1) $\rightarrow$ (2). Since (X, τ) is b-D_1, so each point x of X is contained in a bD-set $O = U \setminus V$ and thus in U. By definition $U \neq X$. This implies that x is not a b-neat point.

(2) $\rightarrow$ (1). By Theorem 3.4, each distinct pair of points $x, y \in X$, at least one of them, x(say) has a b-neighborhood U containing x and not y. Thus U which is different from X is a bD-set. If X has no b-neat point, then y is not a b-neat point. This means that there exists a b-neighborhood V of y such that $V \neq X$. Thus $y \in (V \setminus U)$ but not x and $V \setminus U$ is a bD-set. Hence X is b-D_1.

Remark 4.4 *It should be noted that a space (X, τ) is not b-D_1 if and only if there is a unique b-neat point in X. It is unique because if x and y are both b-neat point in X, then at least one of them say x has a b-neighborhood U containing x but not y. But this is a contradiction since $U \neq X$.*

Definition 6 *A function $f : (X, \tau) \to (Y, \sigma)$ is b-continuous if the inverse image of each b-open set is b-open.*

Theorem 4.5 *If $f : (X, \tau) \to (Y, \sigma)$ is a b-continuous surjective function and E is a bD-set in Y, then the inverse image of E is a bD-set in X.*

Proof. Let E be a bD-set in Y. Then there are b-open sets U_1 and U_2 in Y such that $S = U_1 \backslash U_2$ and $U_1 \neq Y$. By the b- continuity of f, $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are b-open in X. Since $U_1 \neq Y$, we have $f^{-1}(U_1) \neq X$. Hence $f^{-1}(E) = f^{-1}(U_1) \backslash f^{-1}(U_2)$ is a bD-set.

Theorem 4.6 *If (Y, σ) is b-D_1 and $f : (X, \tau) \to (Y, \sigma)$ is b-continuous and bijective, then (X, τ) is b-D_1.*

Proof. Suppose that Y is a b-D_1 space. Let x and y be any pair of distinct points in X. Since f is injective and Y is b-D_1, there exist bD-sets G_x and G_y of Y containing $f(x)$ and $f(y)$ respectively, such that $f(y) \notin G_x$ and $f(x) \notin G_y$. By Theorem 4.5 , $f^{-1}(G_x)$ and $f^{-1}(G_y)$ are bD-sets in X containing x and y respectively. This implies that X is a b-D_1 space.

Theorem 4.7 *A space (X, τ) is b-D_1 if and only if for each pair of distinct points $x, y \in X$, there exists a b-continuous surjective function $f : (X, \tau) \to (Y, \sigma)$, where Y is a b-D_1 space such that $f(x)$ and $f(y)$ are distinct.*

Proof. Necessity. For every pair of distinct points of X, it suffices to take the identity function on X.
Sufficiency. Let x and y be any pair of distinct points in X. By hypothesis, there exists a b-continuous, surjective function f of a space X onto a b-D_1 space Y such that $f(x) \neq f(y)$. Therefore, there exist disjoint bD-sets G_x and G_y in Y such that $f(x) \in G_x$ and $f(y) \in G_y$. Since f is b-continuous and surjective, by Theorem 4.5, $f^{-1}(G_x)$ and $f^{-1}(G_y)$ are disjoint bD-sets in X containing x and y, respectively. Hence by Theorem 4.2, X is b-D_1 space.

Acknowledgment. The authors are grateful to Professor Maximilian Ganster for his comments on an earlier version of this paper and also his observations concerning Theorems 3.4 and 3.11.

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