CHARACTERIZATIONS OF FUNCTIONS WITH STRONGLY α-CLOSED GRAPHS *

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Abstract

In this paper, we study some properties of functions with strongly α-closed graphs by utilizing α-open sets and the α-closure operator.

1 Introduction and preliminaries

The notion of α-open sets was introduced by O. Njåstad [20] in 1965. Since then it has been widely investigated in the literature (see, [1], [2], [3], [9], [10], [11], [12], [15], [16], [17], [18], [19], [21], [23], [24], [26], [27], [28]). Functions with strongly closed graphs were introduced by Herrington and long [7] to characterize H-closed spaces. Properties of such functions were further investigated by Long and Herrington [14] and Noiri [23]. In this paper, we study some properties of functions with strongly α-closed graphs by utilizing α-open sets and the α-closure operator.

Throughout this paper, by (X,τ) and (Y,σ) (or X and Y) we always mean topological spaces. Let A be a subset of X. We denote the interior, the closure and the complement of a set A by $Int(A)$, $Cl(A)$ and $X\backslash A$ or A^c respectively. A subset A of a topological space (X,τ) is called *α-open* [20] (resp. *semi-open* [13]) if $A \subseteq Int(Cl(Int(A)))$ (resp. $A \subseteq Cl(Int(A))$). The complement of an α-open (resp. semi-open) set is called *α-closed* (resp. *semi-closed* [5]). By $\alpha\mathrm{O}(X,\tau)$ (resp. $SO(X,\tau)$, $\alpha\mathrm{C}(X,\tau)$), we denote the family of all α-open (resp. semi-open, α-closed) sets of X. We set $\alpha O(X,x) = \{U \mid x \in U \in \alpha O(X,\tau)\}$,

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$O(X,x) = \{U \mid x \in U \in \tau\}$ and $\alpha C(X,x) = \{U \mid x \in U \in \alpha C(X,\tau)\}$. The intersection of all α-closed (resp. semi-closed) sets containing A is called the *α-closure* (resp. *semi-closure* [4]) of A, denoted by $\alpha Cl(A)$ (resp. $sCl(A)$). A set U in a topological space (X,τ) is an *α-neighborhood* [16] of a point x if U contains an α-open set V such that $x \in V$.

Lemma 1.1 *The intersection of an arbitrary collection of α-closed sets in (X,τ) is α-closed*

Corollary 1.2 *[15]. Let A be a subset of X. Then, $x \in \alpha Cl(A)$ if and only if for any α-open set U in X containing x , $A \bigcap U \neq \phi$.*

Lemma 1.3 *Let A and B be subsets of a space (X,τ), then the following properties hold:*
(1) $A \subset \alpha Cl(A)$.
(2) If $A \subset B$, then $\alpha Cl(A) \subset \alpha Cl(B)$.
(3) $\alpha Cl(A)$ is α-closed.
(4) $\alpha Cl(\alpha Cl(A)) = \alpha Cl(A)$.
(5) A is α-closed if and only $A = \alpha Cl(A)$.

Corollary 1.4 *Let A_i $(i \in I)$ be a subset of a space (X,τ), then the following properties hold:*
(1) $\alpha Cl(\cap\{A_i : i \in I\}) \subset \cap\{\alpha Cl(A_i) : i \in I\}$.
(2) $\alpha Cl(\cup\{A_i : i \in I\}) \supset \cup\{\alpha Cl(A_i) : i \in I\}$.

Definition 1 *A topological space (X,τ) is said to be:*
(1) α-T_1 [17], if for any pair of distinct points x and y in X, there exist an α-open set U in X containing x but not y and an α-open set V in X containing y but not x.
(2) α-T_2 [15], if for any pair of distinct points x and y in X, there exist $U \in \alpha O(X,x)$ and $V \in \alpha O(X,y)$ such that $U \cap V = \emptyset$.

Lemma 1.5 *A topological space (X,τ) is α-T_2 if and only if it is T_2.*

Proof. This is shown in [27] and a simple proof is given in [[24], Corollary 4.7].

Definition 2 *A function $f : X \to Y$ is said to be*
(1) α-continuous [19] if $f^{-1}(V) \in \alpha O(X)$ for each open set V of Y;
(2) weakly α-continuous [23]if for each $x \in X$ and each $V \in O(Y, f(x))$, there exists $U \in \alpha O(X, x)$ such that $f(U) \subset Cl(V)$.

Lemma 1.6 *Let (X, τ) be a topological space. Then $\alpha Cl(V) = Cl(V)$ for each $V \in SO(X)$.*

Proof. For any $V \in SO(X)$, $\alpha Cl(V) = V \cup Cl(Int(Cl(V))) = V \cup Cl(Int(V)) = V \cup Cl(V) = Cl(V)$.

Lemma 1.7 *A function $f : X \to Y$ is weakly α-continuous if and only if for each $x \in X$ and each $V \in \alpha O(Y, f(x))$, there exists $U \in \alpha O(X, x)$ such that $f(U) \subset \alpha Cl(V)$.*

Proof. Necessity. Let $x \in X$ and $V \in \alpha O(Y, f(x))$. Then $f(x) \in V \subset Int(Cl(Int(V)))$ and there exists $U \in \alpha O(X, x)$ such that $f(U) \subset Cl(Int(Cl(Int(V))))$. By Lemma 1.6, we have $Cl(Int(Cl(Int(V)))) = Cl(Int(V)) = Cl(V) = \alpha Cl(V)$. Therefore, $f(U) \subset \alpha Cl(V)$.

Sufficiency. Let $x \in X$ and $V \in O(Y, f(x))$. There exists $U \in \alpha O(X, x)$ such that $f(U) \subset \alpha Cl(V)$. By Lemma 1.6, we obtain $f(U) \subset Cl(V)$.

2 Strongly α-closed graphs

If $f : (X, \tau) \to (Y, \sigma)$ is any function, then the subset $G(f) = \{(x, f(x)) : x \in X\}$ of the product space $(X \times Y, \tau \times \sigma)$ is called the graph of f [8].

Definition 3 *A function $f : X \to Y$ has a strongly α-closed (resp. strongly closed [7]) graph if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \alpha O(X, x)$ (resp. $U \in O(X, x)$) and $V \in O(Y, y)$ such that $(U \times Cl(V)) \cap G(f) = \emptyset$.*

Lemma 2.1 *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*
(1) $G(f)$ is strongly α-closed;
(2) For each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \alpha O(X, x)$ and $V \in O(Y, y)$ such that

$f(U) \cap Cl(V) = \emptyset$;
(3) For each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \alpha O(X, x)$ and $V \in \alpha O(Y, y)$ such that $(U \times \alpha Cl(V)) \cap G(f) = \emptyset$;
(4) For each $(x, y) \in (X \times Y) \setminus G(f)$, there exist $U \in \alpha O(X, x)$ and $V \in \alpha O(Y, y)$ such that $f(U) \cap \alpha Cl(V) = \emptyset$.

Proof. It is obvious that (1) ⇔ (2) and (3) ⇔ (4).
(1) ⇒ (3): Since $\tau \subset \alpha O(X) \subset SO(X)$, by Lemma 1.6 the proof is obvious.
(3) ⇒ (1): Let $(x, y) \in (X \times Y) \setminus G(f)$. There exist $U \in \alpha O(X, x)$ and $V \in \alpha O(Y, y)$ such that $(U \times \alpha Cl(V)) \cap G(f) = \emptyset$. Put $G = Int(Cl(Int(V)))$. Then $y \in V \subset G \in \sigma$ and $Cl(G) = Cl(V) = \alpha Cl(V)$. Therefore, we obtain $(U \times Cl(G)) \cap G(f) = (U \times \alpha Cl(V)) \cap G(f) = \emptyset$. This shows that $G(f)$ is strongly α-closed.

Theorem 2.2 *If $f : X \to Y$ is a function with the strongly α-closed graph, then for each $x \in X$, $f(x) = \cap\{\alpha Cl(f(U)) : U \in \alpha O(X, x)\}$.*

Proof. Suppose the theorem is false. Then there exists a $y \neq f(x)$ such that $y \in \cap\{\alpha Cl(f(U)) : U \in \alpha O(X, x)\}$. This implies that $y \in \alpha Cl(f(U))$ for every $U \in \alpha O(X, x)$. So $V \cap f(U) \neq \emptyset$ for every $V \in \alpha O(Y, y)$. This, in its turn, indicates that $\alpha Cl(V) \cap f(U) \supset V \cap f(U) \neq \emptyset$ which contradicts the hypothesis that f is a function with strongly α-closed graph. Hence the theorem holds.

Theorem 2.3 *If $f : X \to Y$ is α-continuous and Y is T_2, then $G(f)$ is strongly α-closed.*

Proof. Let $(x, y) \in (X \times Y) \backslash G(f)$. The T_2-ness of Y gives the existence of a set $V \in O(Y, y)$ such that $f(x) \notin Cl(V)$. Now $Y \backslash Cl(V) \in O(Y, f(x))$. Therefore, by the α-continuity of f there exists $U \in \alpha O(X, x)$ such that $f(U) \subset Y \backslash Cl(V)$. Consequently, $f(U) \cap Cl(V) = \emptyset$ and therefore $G(f)$ is strongly α-closed.

It is shown in ([14], Theorem 3) and ([22], Theorem 2) that if $f : X \to Y$ is surjective and $G(f)$ is strongly closed, then Y is Hausdorff. The following theorem is a slight improvement of this result.

Theorem 2.4 *If $f : X \to Y$ is surjective and has a strongly α-closed graph $G(f)$, then Y is both T_2 and α-T_1.*

Proof. Let y_1, y_2 $(y_1 \neq y_2) \in Y$. The surjectivity of f gives a $x_1 \in X$ such that $f(x_1) = y_1$. Now $(x_1, y_2) \in (X \times Y)\backslash G(f)$. The strongly α-closedness of $G(f)$ provides $U \in \alpha O(X, x_1)$, $V \in O(Y, y_2)$ such that $f(U) \cap Cl(V) = \emptyset$, whence one infers that $y_1 \notin Cl(V)$. This means that there exists $W \in O(Y, y_1)$ such that $W \cap V = \emptyset$. So, Y is T_2 and T_2-ness always guarantees α-T_1-ness. Hence Y is α-T_1.

Theorem 2.5 *A space X is T_2 if and only if the identity function $id : X \to X$ has a strongly α-closed graph $G(id)$.*

Proof. Necessity. Let X be T_2. Since the identity function $id : X \to X$ is continuous, it follows from Theorem 2.4 that $G(id)$ is strongly α-closed.
Sufficiency. Let $G(id)$ be a strongly α-closed graph. Then the surjectivity of id and strong α-closedness of $G(id)$ together imply, by Theorem 2.4, that X is T_2.

Theorem 2.6 *If $f : X \to Y$ is an injection and $G(f)$ is strongly α-closed, then X is α-T_1.*

Proof. Since f is injective, for any pair of distinct points $x_1, x_2 \in X$, $f(x_1) \neq f(x_2)$. Then $(x_1, f(x_2)) \in (X \times Y)\backslash G(f)$. Since $G(f)$ is strongly α-closed, there exist $U \in \alpha O(X, x_1)$, $V \in O(Y, f(x_2))$ such that $f(U) \cap Cl(V) = \emptyset$. Therefore $x_2 \notin U$. Pursuing the same reasoning as before we obtain a set $W \in \alpha O(X, x_2)$ such that $x_1 \notin W$. Hence Y is α-T_1.

Theorem 2.7 *If $f : X \to Y$ is a bijection with the strongly α-closed graph, then both X and Y are α-T_1.*

Proof. The proof is an immediate consequence of Theorems 2.4 and 2.6.

Theorem 2.8 *If a function $f : X \to Y$ is a weakly α-continuous injection with the strongly α-closed graph $G(f)$, then X is T_2.*

Proof. Since f is injective, for any pair of distinct points $x_1, x_2 \in X$, $f(x_1) \neq f(x_2)$. Therefore $(x_1, f(x_2)) \in (X \times Y)\backslash G(f)$. Since $G(f)$ is strongly α-closed, there exist $U \in \alpha O(X, x_1)$, $V \in O(Y, f(x_2))$ such that $f(U) \cap Cl(V) = \emptyset$; hence $U \cap f^{-1}(Cl(V)) = \emptyset$. Consequently, $f^{-1}(Cl(V)) \subset X\backslash U$. Since f is weakly α continuous, there exists $W \in \alpha O(X, x_2)$ such that $f(W) \subset Cl(V)$. From this and the foregoing it follows that $W \subset f^{-1}(Cl(V)) \subset X\backslash U$; hence $W \cap U = \emptyset$. Thus for the pair of distinct points $x_1, x_2 \in X$, there exist $U \in \alpha O(X, x_1)$, $W \in \alpha O(X, x_2)$ such that $W \cap U = \emptyset$. By Lemma 1.5, this guarantees the T_2-ness of X.

Corollary 2.9 *If a function $f : X \to Y$ is an α-continuous injection with the strongly α-closed graph, then X is T_2.*

Proof. The proof follows from Theorem 2.9 and the fact that every α-continuous is weakly α-continuous.

Remark 2.10 *If f is not T_2 in Corollary 2.9, then even α-continuity need not imply a strongly α-closed graph. For example, let X be a topological space containing more than one point with the indiscrete topology and let $id : X \to X$ the identity function. Then id is certainly α-continuous, but the graph of id is not strongly α-closed because $X \times X$ has the indiscrete topology and hence the graph of id being the diagonal set, which is different from the whole space, is not strongly α-closed.*

Theorem 2.11 *If $f : X \to Y$ is a weakly α-continuous bijection with the strongly α-closed graph, then both X and Y are T_2.*

Proof. The proof follows from Theorems 2.8 and 2.4.

Lemma 2.12 *Every clopen subset of a quasi H-closed space X is quasi H-closed relative to X.*

Proof. Let B be any clopen subset of a quasi H-closed space X. Let $\{O_\lambda : \lambda \in \Omega\}$ be any cover of B by open sets in X. Then the family $F = \{O_\lambda : \lambda \in \Omega\} \cup \{X \backslash B\}$ is a cover of X by open sets in X. Because of quasi H-closedness of X there exists a finite subfamily $F^* = \{O_{\lambda_i} : 1 \leq i \leq n\} \cup \{X \backslash B\}$ of F whose closure covers X. So, because of clopenness of B we now infer that the family $\{Cl(O_{\lambda_i}) : 1 \leq i \leq n\}$ covers B. Therefore, B is quasi H-closed relative to X.

Theorem 2.13 *If Y is a quasi H-closed extremally disconnected space, then a function $f : X \to Y$ with the strongly α-closed graph $G(f)$ is weakly α-continuous.*

Proof. Let $x \in X$ and $V \in O(Y, f(x))$. Take any $y \in Y \backslash Cl(V)$. Then $(x, y) \in (X \times Y) \backslash G(f)$. Now the strong α-closedness of $G(f)$ induces the existence of $U_y(x) \in \alpha O(X, x)$, $V_y \in O(Y, y)$ such that $f(U_y(x)) \cap Cl(V_y) = \emptyset$....(*).
Now extremal disconnectedness of Y induces the clopenness of $Cl(V)$ and hence $Y \backslash Cl(V)$ is also clopen. Now $\{V_y : y \in Y \backslash Cl(V)\}$ is a cover of $Y \backslash Cl(V)$ by open sets in Y. By Lemma 2.13, there exists a finite subfamily $\{V_{y_i} : 1 \leq i \leq n\}$ such that $Y \backslash Cl(V) \subset \bigcup\limits_{i=1}^{n} Cl(V_{y_i})$. Let $W = \bigcap\limits_{i=1}^{n} U_{y_i}(x)$, where $U_{y_i}(x)$ are α-open sets in X satisfying (*). Also, $W \in \alpha O(X, x)$. Now $f(W) \cap (Y \backslash Cl(V)) \subset f[\bigcap\limits_{i=1}^{n} U_{y_i}(x)] \cap (\bigcup\limits_{i=1}^{n} Cl(V_{y_i})) \subset \bigcup\limits_{i=1}^{n} (f[U_{y_i}(x)] \cap Cl(V_{y_i})) = \emptyset$, by (*). Therefore, $f(W) \subset Cl(V)$ and this indicates that f is weakly α-continuous.

Noiri [22] showed that if $G(f)$ is strongly closed then f has the following property:
(P) For every set B which is quasi H–closed relative to Y, $f^{-1}(B)$ is a closed set of X.

Analogously, we have the following theorem.

Theorem 2.14 *If a function $f : X \to Y$ has a strongly α-closed graph $G(f)$, then f enjoys the following property:*

(P^*) *For every set F which is quasi H-closed relative to Y, $f^{-1}(F)$ is α-closed in X.*

Proof. Let $f^{-1}(F)$ be not α-closed in X. Then there exists $x \in \alpha Cl(f^{-1}(F)) \backslash f^{-1}(F)$. Let $y \in F$. Then $(x, y) \in (X \times Y) \backslash G(f)$. Strong α-closedness of $G(f)$ gives the existence of

$U_y(x) \in \alpha O(X, x)$ and $V_y \in O(Y, y)$ such that $f(U_y(x)) \cap Cl(V_y) = \emptyset$....(*).
Clearly $\{V_y : y \in F\}$ is a cover of F by open sets in Y. Since F is quasi H-closed relative to Y, there exist a finite number of open sets $V_{y_1}, V_{y_2}, ..., V_{y_n}$ in Y such that $F \subset \bigcup_{i=1}^{n} Cl(V_{y_i})$.
Let $U = \bigcap_{i=1}^{n} U_{y_i}(x)$, where $U_{y_i}(x)$ are the α-open sets in X satisfying (*). Also $U \in \alpha O(X, x)$.
Now $f(U) \cap F \subset f[\bigcap_{i=1}^{n} U_{y_i}(x)] \cap (\bigcup_{i=1}^{n} Cl(V_{y_i})) \subset \bigcup_{i=1}^{n} (f[U_{y_i}(x)] \cap Cl(V_{y_i})) = \emptyset$. But since $x \in \alpha Cl(f^{-1}(F))$, $U \cap f^{-1}(F) \neq \emptyset$; hence $f(U) \cap F \neq \emptyset$. This is a contradiction. Hence the result holds.

3 Additional properties

Lemma 3.1 *For a topological space X, the following properties are equivalent:*
(1) X is Urysohn;
(2) For every pair of distinct points $x, y \in X$, there exist $U \in \alpha O(X, x)$, $V \in \alpha O(X, y)$ such that $Cl(U) \cap Cl(V) = \emptyset$;
(3) For every pair of distinct points $x, y \in X$, there exist $U \in \alpha O(X, x)$, $V \in \alpha O(X, y)$ such that $\alpha Cl(U) \cap \alpha Cl(V) = \emptyset$.

Proof. (1) $\Rightarrow$ (2): This is obvious.
(2) $\Rightarrow$ (3): Since $\alpha Cl(U) = Cl(U)$ for each $U \in \alpha(X)$ by Lemma 1.6, this is obvious.
(3) $\Rightarrow$ (1): Suppose that (3) holds. For every pair of distinct points x, y, there exist $U \in \alpha O(X, x)$, $V \in \alpha O(X, y)$ such that $\alpha Cl(U) \cap \alpha Cl(V) = \emptyset$. Now, put $G = Int(Cl(Int(U)))$ and $H = Int(Cl(Int(V)))$, then G and H are open sets containing x and y, respectively. Furthermore, $Cl(G) \cap Cl(H) = Cl(U) \cap Cl(V) = \alpha Cl(U) \cap \alpha Cl(V) = \emptyset$. Therefore, X is Urysohn.

Recall, that a function $f : X \to Y$ is said to be α-open [19] if $f(A) \in \alpha O(Y)$ for all open set A of Y.

Lemma 3.2 *Let a bijection $f : X \to Y$ be α-open. Then for any closed set B of X, $f(B) \in \alpha C(Y)$.*

Urysohn spaces remain invariant under certain bijective function as is shown in the next theorem.

Theorem 3.3 *If a bijection $f : X \to Y$ is α-open and X is Urysohn, then Y is Urysohn.*

Proof. Let $y_1, y_2 \in Y$ and $y_1 \neq y_2$. Since f is bijective, $f^{-1}(y_1)$, $f^{-1}(y_2) \in X$ and $f^{-1}(y_1) \neq f^{-1}(y_2)$. The Urysohn property of X gives the existence of sets $U \in O(X, f^{-1}(y_1))$, $V \in O(X, f^{-1}(y_2))$ such that $Cl(U) \cap Cl(V) = \emptyset$. As $Cl(U)$ is a closed set in X, then by the bijectivity and α-openness of f together then indicate, by Lemma 3.2 that $f(Cl(U)) \in \alpha C(Y)$. Therefore by the injectivity of f, $\alpha Cl(f(U)) \cap \alpha Cl(f(V)) \subset f(Cl(U)) \cap f(Cl(V)) = f(Cl(U) \cap Cl(V)) = \emptyset$. Thus α-openness of f gives the existence of two sets $f(U) \in \alpha O(Y, y_1)$, $f(V) \in \alpha O(Y, y_2)$, with $\alpha Cl(f(U)) \cap \alpha Cl(f(V)) = \emptyset$. By Lemma 3.1, Y is Urysohn.

Theorem 3.4 *If $f : X \to Y$ is weakly α-continuous and Y is Urysohn, then $G(f)$ is strongly α-closed.*

Proof. Let $(x, y) \in (X \times Y) \backslash G(f)$. Then $y \neq f(x)$. Since Y is Urysohn, there exist $V \in O(Y, y)$, $W \in O(Y, f(x))$ such that $Cl(V) \cap Cl(W) = \emptyset$. Since f is weakly α-continuous, there exists $U \in \alpha O(X, x)$ such that $f(U) \subset Cl(W)$. This, therefore, implies that $f(U) \cap Cl(V) = \emptyset$. So by Lemma 2.2, $G(f)$ is strongly α-closed.

Theorem 3.5 *Let X be a Urysohn space. Then any α-open bijection $f : X \to Y$ has a strongly α-closed graph.*

Proof. Let $(x, y) \in (X \times Y) \setminus G(f)$. Then $y \neq f(x)$ and $y \neq f^{-1}(y)$, where $f^{-1}(y)$ is a singleton. Since X is Urysohn, there exist open sets U_x and U_y such that $x \in U_x$, $f^{-1}(y) \in U_y$ and $Cl(U_x) \cap Cl(U_y) = \emptyset$. Since f is α-open, $f(U_x) \in \alpha O(Y, f(x))$, $f(U_y) \in \alpha O(Y, y)$ and $f(U_x) \cap \alpha Cl(f(U_y)) \subset \alpha Cl(f(U_x)) \cap \alpha Cl(f(U_y)) \subset f(Cl(U_x)) \cap f(Cl(U_y)) = \emptyset$. Therefore, by Lemma 2.2, $G(f)$ is strongly α-closed.

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