

On the Ramanujan’s mathematics (Hardy-Ramanujan number and mock theta functions) applied to various parameters of Particle Physics and Black Hole Physics: New possible mathematical connections.

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Abstract

In this research thesis, we have analyzed and deepened further Ramanujan expressions (Hardy-Ramanujan number and mock theta functions) applied to various parameters of Particle Physics and Black Hole Physics. We have therefore described new possible mathematical connections.

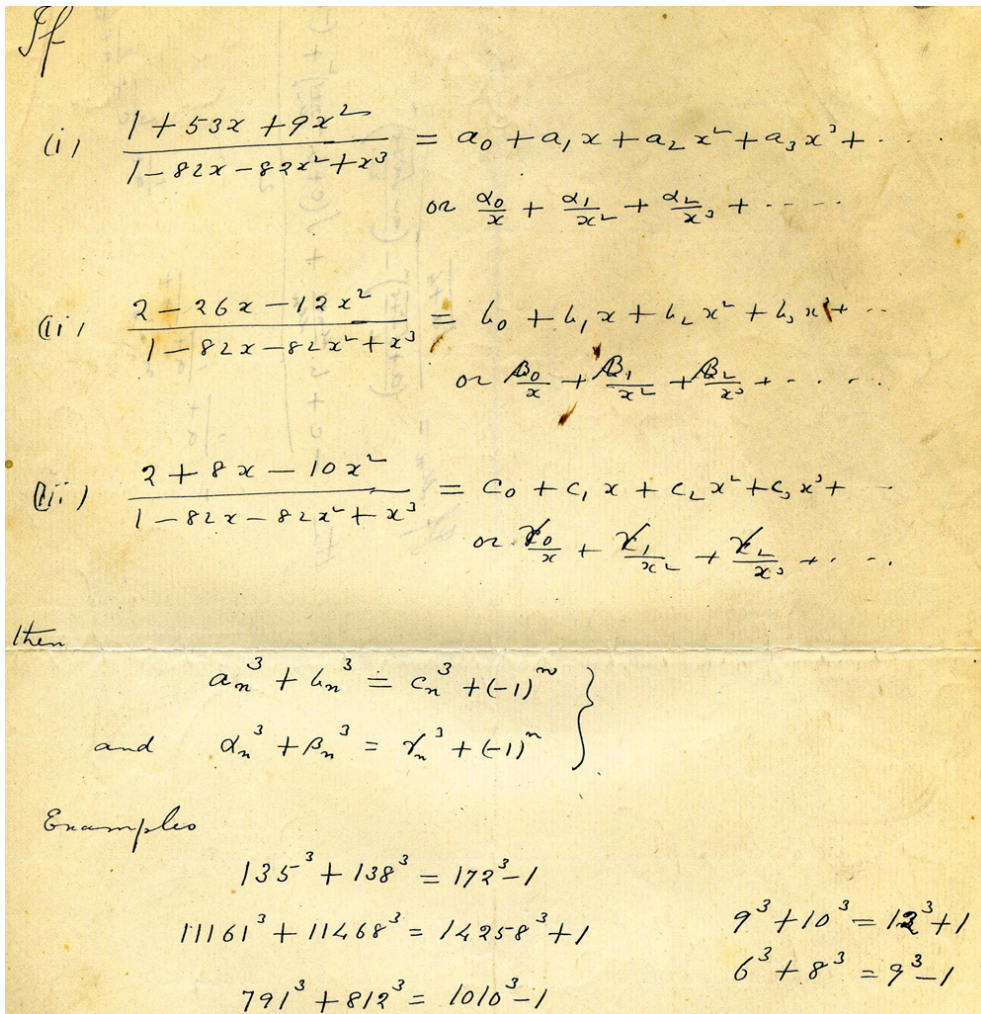
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<https://www.britannica.com/biography/Srinivasa-Ramanujan>



<http://www.meteoweb.eu/2019/10/wormhole-varchi-spazio-tempo/1332405/>



<https://plus.maths.org/content/ramanujan>

Ramanujan's manuscript

The representations of 1729 as the sum of two cubes appear in the bottom right corner. The equation expressing the near counter examples to Fermat's last theorem appears further up: $\alpha^3 + \beta^3 = \gamma^3 + (-1)^n$.

From Wikipedia

The **taxicab number**, typically denoted $Ta(n)$ or $Taxicab(n)$, also called the n th **Hardy–Ramanujan number**, is defined as the smallest integer that can be expressed as a sum of two positive integer cubes in n distinct ways. The most famous taxicab number is $1729 = Ta(2) = 1^3 + 12^3 = 9^3 + 10^3$.

From:

Ken Ono - The Last Words of a Genius December 2010 Notices of the AMS - Volume 57, Number 11

Now, we have that:

$$f_0(q) + 2\Phi(q^2) = \prod_{n=1}^{\infty} \frac{(1 - q^{5n})(1 - q^{10n-5})}{(1 - q^{5n-4})(1 - q^{5n-1})}$$

For $q = 0.5$, that is $q = e^{2\pi i\tau} = 0.5$ for $i\tau = x = -0.110318$, we obtain:

product $((1-0.5^{(5n)})(1-0.5^{(10n-5)}))/(((1-0.5^{(5n-4)}))(1-0.5^{(5n-1)}))$, $n=1$ to infinity

Input interpretation:

$$\prod_{n=1}^{\infty} \frac{(1 - 0.5^{5n})(1 - 0.5^{10n-5})}{(1 - 0.5^{5n-4})(1 - 0.5^{5n-1})}$$

Infinite product:

$$\prod_{n=1}^{\infty} \frac{(1 - 0.5^{5n})(1 - 0.5^{10n-5})}{(1 - 0.5^{5n-4})(1 - 0.5^{5n-1})} = 2.03688$$

$2 \times (2.03688)^6 - 18 + 1/\text{golden ratio}$

Input interpretation:

$$2 \times 2.03688^6 - 18 + \frac{1}{\phi}$$

ϕ is the golden ratio

Result:

125.449...

125.449... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV

Alternative representations:

$$2 \times 2.03688^6 - 18 + \frac{1}{\phi} = -18 + 2 \times 2.03688^6 + \frac{1}{2 \sin(54^\circ)}$$

$$2 \times 2.03688^6 - 18 + \frac{1}{\phi} = -18 + 2 \times 2.03688^6 + \frac{1}{2 \cos(216^\circ)}$$

$$2 \times 2.03688^6 - 18 + \frac{1}{\phi} = -18 + 2 \times 2.03688^6 + \frac{1}{2 \sin(666^\circ)}$$

From

$$\prod_{n=1}^{\infty} \frac{(1 - 0.5^{5n})(1 - 0.5^{10n-5})}{(1 - 0.5^{5n-4})(1 - 0.5^{5n-1})} = 2.03688$$

we obtain:

$$2 \times (2.03688)^6 - \pi$$

Input interpretation:

$$2 \times 2.03688^6 - \pi$$

Result:

139.689...

139.689... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternative representations:

$$2 \times 2.03688^6 - \pi = -180^\circ + 2 \times 2.03688^6$$

$$2 \times 2.03688^6 - \pi = i \log(-1) + 2 \times 2.03688^6$$

$$2 \times 2.03688^6 - \pi = -\cos^{-1}(-1) + 2 \times 2.03688^6$$

Series representations:

$$2 \times 2.03688^6 - \pi = 142.831 - 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}$$

$$2 \times 2.03688^6 - \pi = 144.831 - 2 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$2 \times 2.03688^6 - \pi = 142.831 - \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}}$$

Integral representations:

$$2 \times 2.03688^6 - \pi = 142.831 - 2 \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$2 \times 2.03688^6 - \pi = 142.831 - 4 \int_0^1 \sqrt{1-t^2} dt$$

$$2 \times 2.03688^6 - \pi = 142.831 - 2 \int_0^{\infty} \frac{\sin(t)}{t} dt$$

From

$$\prod_{n=1}^{\infty} \frac{(1 - 0.5^{5n})(1 - 0.5^{10n-5})}{(1 - 0.5^{5n-4})(1 - 0.5^{5n-1})} = 2.03688$$

we obtain:

$$24(2.03688)^6 + 11 + 4$$

Input interpretation:

$$24 \times 2.03688^6 + 11 + 4$$

Result:

1728.972718949751729653316914774016

1728.972718...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Now, we have that:

$$\omega(q) = \sum_{n=0}^{\infty} a_{\omega}(n)q^n$$

$$:= \sum_{n=0}^{\infty} \frac{q^{2n(n+1)}}{(1-q)^2(1-q^3)^2 \dots (1-q^{2n+1})^2},$$

sum $(0.5^{2n(n+1)}) / (((1-0.5)^2 (1-0.5^3)^2 (1-0.5^{2n+1})^2))$, $n=0$ to infinity

Input interpretation:

$$\sum_{n=0}^{\infty} \frac{0.5^{2n(n+1)}}{(1-0.5)^2 (1-0.5^3)^2 (1-0.5^{2n+1})^2}$$

Approximated sum:

$$\sum_{n=0}^{\infty} \frac{0.5^{2n(n+1)}}{(1-0.5)^2 (1-0.5^3)^2 (1-0.5^{2n+1})^2} \approx 21.3258$$

$6(((\text{sum } (0.5^{2n(n+1)}) / (((1-0.5)^2 (1-0.5^3)^2 (1-0.5^{2n+1})^2))))$, $n=0$ to infinity)) - $\text{Pi} + 1/\text{golden ratio}$

Input interpretation:

$$6 \sum_{n=0}^{\infty} \frac{0.5^{2n(n+1)}}{(1-0.5)^2 (1-0.5^3)^2 (1-0.5^{2n+1})^2} - \pi + \frac{1}{\phi}$$

ϕ is the golden ratio

Result:

125.431

125.431 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

$6 \left(\left(\left(\left(\sum_{n=0}^{\infty} (0.5^{2n(n+1)}) / \left((1-0.5)^2 (1-0.5^3)^2 (1-0.5^{2n+1})^2 \right) \right) \right) \right) \right) + 11 + 1/\text{golden ratio}$

Input interpretation:

$$6 \sum_{n=0}^{\infty} \frac{0.5^{2n(n+1)}}{(1-0.5)^2 (1-0.5^3)^2 (1-0.5^{2n+1})^2} + 11 + \frac{1}{\phi}$$

ϕ is the golden ratio

Result:

139.573

139.573 result practically equal to the rest mass of Pion meson 139.57 MeV

$27 * 3 \left(\left(\left(\left(\sum_{n=0}^{\infty} (0.5^{2n(n+1)}) / \left((1-0.5)^2 (1-0.5^3)^2 (1-0.5^{2n+1})^2 \right) \right) \right) \right) \right) + \text{golden ratio}$

Input interpretation:

$$27 * 3 \sum_{n=0}^{\infty} \frac{0.5^{2n(n+1)}}{(1-0.5)^2 (1-0.5^3)^2 (1-0.5^{2n+1})^2} + \phi$$

ϕ is the golden ratio

Result:

1729.01

1729.01

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From Wikipedia:

“The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group $\mathbf{Z}/3\mathbf{Z}$, and its outer automorphism group is the cyclic group $\mathbf{Z}/2\mathbf{Z}$. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories”.

Now, for $q = 535.49165$, that is $q = e^{2\pi i\tau}$, for $i\tau = 1$, that is:

Input:

$$e^{2\pi}$$

Decimal approximation:

535.4916555247647365030493295890471814778057976032949155072...

535.4916555...

Property:

$e^{2\pi}$ is a transcendental number

Alternative representations:

$$e^{2\pi} = e^{360^\circ}$$

$$e^{2\pi} = e^{-2i \log(-1)}$$

$$e^{2\pi} = \exp^{2\pi}(z) \text{ for } z = 1$$

Series representations:

$$e^{2\pi} = e^{8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$e^{2\pi} = \left(\sum_{k=0}^{\infty} \frac{1}{k!} \right)^{2\pi}$$

$$e^{2\pi} = \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{2\pi}$$

Integral representations:

$$e^{2\pi} = e^8 \int_0^1 \sqrt{1-t^2} dt$$

$$e^{2\pi} = e^4 \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$e^{2\pi} = e^4 \int_0^{\infty} \frac{1}{(1+t^2)} dt$$

From

$$f_0(q) + 2\Phi(q^2) = \prod_{n=1}^{\infty} \frac{(1 - q^{5n})(1 - q^{10n-5})}{(1 - q^{5n-4})(1 - q^{5n-1})}$$

We obtain:

product $((1-535.49165^{(5n)})(1-535.49165^{(10n-5)})) / (((1-535.49165^{(5n-4)}))(1-535.49165^{(5n-1)}))$, $n=1$ to $\sqrt{3}$

Product:

$$\prod_{n=1}^{\sqrt{3}} \frac{(1 - 535.492^{5n})(1 - 535.492^{10n-5})}{(1 - 535.492^{5n-4})(1 - 535.492^{5n-1})} = 4.41139 \times 10^{13}$$

$$4.41139 \times 10^{13}$$

From which:

$$4 \ln(4.41139 \times 10^{13})$$

Input interpretation:

$$4 \log(4.41139 \times 10^{13})$$

$\log(x)$ is the natural logarithm

Result:

125.6712...

125.6712... result very near to the dilaton mass calculated as a type of Higgs boson:
125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

From

$$\prod_{n=1}^{\sqrt{3}} \frac{(1 - 535.492^{5n})(1 - 535.492^{10n-5})}{(1 - 535.492^{5n-4})(1 - 535.492^{5n-1})} = 4.41139 \times 10^{13}$$

we have also:

$$4 \ln(4.41139 \times 10^{13}) + 13 + 1/\text{golden ratio}$$

Input interpretation:

$$4 \log(4.41139 \times 10^{13}) + 13 + \frac{1}{\phi}$$

log(x) is the natural logarithm

φ is the golden ratio

Result:

139.2892...

139.2892... result practically equal to the rest mass of Pion meson 139.57 MeV

and performing the 8th root, we obtain:

$$3 \cdot (4.41139 \times 10^{13})^{1/8} - 29 + \pi - 1/\text{golden ratio}$$

Input interpretation:

$$3 \sqrt[8]{4.41139 \times 10^{13}} - 29 + \pi - \frac{1}{\phi}$$

φ is the golden ratio

Result:

125.821...

125.821... result very near to the dilaton mass calculated as a type of Higgs boson:
125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

$$3 \cdot (4.41139 \times 10^{13})^{1/8} - 18 + \pi + \text{golden ratio}$$

Input interpretation:

$$3 \sqrt[8]{4.41139 \times 10^{13}} - 18 + \pi + \phi$$

ϕ is the golden ratio

Result:

139.0573...

139.0573... result practically equal to the rest mass of Pion meson 139.57 MeV

From

$$\begin{aligned} \omega(q) &= \sum_{n=0}^{\infty} a_{\omega}(n) q^n \\ &:= \sum_{n=0}^{\infty} \frac{q^{2n(n+1)}}{(1-q)^2 (1-q^3)^2 \dots (1-q^{2n+1})^2} \end{aligned}$$

We obtain:

$$\text{sum } (535.49165^{(2n(n+1))} / (((1-535.49165)^2 (1-535.49165^3)^2 (1-535.49165^{(2n+1)})^2))), n=0 \text{ to } \pi$$

Sum:

$$\sum_{n=0}^{\pi} \frac{535.492^{2n(n+1)}}{(1 - 535.492)^2 (1 - 535.492^3)^2 (1 - 535.492^{2n+1})^2} =$$

11 896 638 547 417 206 529 486 983 135 955 053 328 045 296 078 459 550 414 292 ∙
 839 075 407 350 543 985 375 802 513 403 824 406 584 652 312 959 764 668 226 ∙
 326 786 558 268 495 598 426 489 422 843 578 637 927 034 931 637 346 076 544 ∙
 098 378 053 174 596 265 452 292 348 376 634 531 611 938 705 466 789 126 264 ∙
 396 069 813 930 648 635 445 281 /
 41 332 844 870 979 727 420 469 946 693 704 700 186 275 564 666 417 540 891 ∙
 779 335 461 560 502 542 882 222 963 894 259 654 834 479 571 835 916 550 444 ∙
 674 884 876 190 882 450 170 411 824 123 340 107 619 557 130 031 583 295 265 ∙
 592 126 669 921 150 899 830 186 119 425 628 245 445 601 433 275 285 753 760 ∙
 153 378 099 411 600 000 000

Decimal approximation:

287825.3017558434561499794148530051227870566572698558351434...

287825.30175584.....

From which:

$$2((((\text{sum}(535.49165^{(2n(n+1)))} / (((1-535.49165)^2 (1-535.49165^3)^2 (1-535.49165^{(2n+1)})^2))), n=0 \text{ to } \pi))))^{1/3} + 7 + 1/\text{golden ratio}$$

Input interpretation:

$$2 \sqrt[3]{\sum_{n=0}^{\pi} \frac{535.49165^{2n(n+1)}}{(1 - 535.49165)^2 (1 - 535.49165^3)^2 (1 - 535.49165^{2n+1})^2}} + 7 + \frac{1}{\phi}$$

φ is the golden ratio

Result:

139.668

139.668 result practically equal to the rest mass of Pion meson 139.57 MeV

$$2((((\text{sum}(535.49165^{(2n(n+1)))} / (((1-535.49165)^2 (1-535.49165^3)^2 (1-535.49165^{(2n+1)})^2))), n=0 \text{ to } \pi))))^{1/3} - 7 + 1/\text{golden ratio}$$

Input interpretation:

$$2 \sqrt[3]{\sum_{n=0}^{\pi} \frac{535.49165^{2n(n+1)}}{(1 - 535.49165)^2 (1 - 535.49165^3)^2 (1 - 535.49165^{2n+1})^2}} - 7 + \frac{1}{\phi}$$

Result:

125.668

125.668 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

$$27 * (((((((((\sum_{n=0}^{\infty} (535.49165^{2n(n+1)}) / (((1-535.49165)^2 (1-535.49165^3)^2 (1-535.49165^{2n+1}))^2)))))))))^{1/3} - 2)) + 1/3$$

Input interpretation:

$$27 \left(\sqrt[3]{ \sum_{n=0}^{\infty} \frac{535.49165^{2n(n+1)}}{(1-535.49165)^2 (1-535.49165^3)^2 (1-535.49165^{2n+1})^2} } - 2 \right) + \frac{1}{3}$$

Result:

1729.01

1729.01

This result is very near to the mass of candidate glueball f₀(1710) meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Now, we have that:

$$E_4(\tau) = 1 + 240 \sum_{n=1}^{\infty} \sum_{d|n} d^3 q^n$$

$$= (1 - q)^{-240} (1 - q^2)^{26760} \dots = \prod_{n=1}^{\infty} (1 - q^n)^{c(n)},$$

For q = 0.5, we obtain.

$$(1-0.5)^{-240} (1-0.5^2)^{26760}$$

Input:

$$\frac{(1 - 0.5^2)^{26760}}{(1 - 0.5)^{240}}$$

Result:

$$7.70206751490355097591686814474658054348335295763413... \times 10^{-3272}$$
$$7.7020675149... * 10^{-3272}$$

From which:

$$((((1-0.5)^{-240} (1-0.5^2)^{26760}))* (10^{(3400)}))$$

Input:

$$\frac{(1 - 0.5^2)^{26760}}{(1 - 0.5)^{240}} \times 10^{3400}$$

Result:

$$7.7020675149035509759168681447465805434833529576341342... \times 10^{128}$$
$$7.7020675149... * 10^{128}$$

From which:

$$1/2 \log((((1-0.5)^{-240} (1-0.5^2)^{26760}))* (10^{(3400)})) - 11$$

Input:

$$\frac{1}{2} \log \left(\frac{(1 - 0.5^2)^{26760}}{(1 - 0.5)^{240}} \times 10^{3400} \right) - 11$$

$\log(x)$ is the natural logarithm

Result:

$$137.386...$$

$$137.386...$$

This result is very near to the inverse of fine-structure constant 137,035

Alternative representations:

$$\frac{1}{2} \log\left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}}\right) - 11 = -11 + \frac{1}{2} \log_e\left(\frac{10^{3400} (1-0.5^2)^{26760}}{0.5^{240}}\right)$$

$$\frac{1}{2} \log\left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}}\right) - 11 = -11 + \frac{1}{2} \log(a) \log_a\left(\frac{10^{3400} (1-0.5^2)^{26760}}{0.5^{240}}\right)$$

$$\frac{1}{2} \log\left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}}\right) - 11 = -11 - \frac{1}{2} \text{Li}_1\left(1 - \frac{10^{3400} (1-0.5^2)^{26760}}{0.5^{240}}\right)$$

Series representations:

$$\begin{aligned} \frac{1}{2} \log\left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}}\right) - 11 = \\ -11 + \frac{\log(7.702067514903312 \times 10^{128})}{2} - \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k e^{-296.772380704484551k}}{k} \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \log\left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}}\right) - 11 = -11 + i\pi \left[\frac{\arg(7.702067514903312 \times 10^{128} - x)}{2\pi} \right] + \\ \frac{\log(x)}{2} - \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (7.702067514903312 \times 10^{128} - x)^k x^{-k}}{k} \quad \text{for } x < 0 \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \log\left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}}\right) - 11 = \\ -11 + \frac{1}{2} \left[\frac{\arg(7.702067514903312 \times 10^{128} - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \\ \frac{\log(z_0)}{2} + \frac{1}{2} \left[\frac{\arg(7.702067514903312 \times 10^{128} - z_0)}{2\pi} \right] \log(z_0) - \\ \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (7.702067514903312 \times 10^{128} - z_0)^k z_0^{-k}}{k} \end{aligned}$$

Integral representations:

$$\frac{1}{2} \log\left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}}\right) - 11 = -11 + \frac{1}{2} \int_1^{7.702067514903312 \times 10^{128}} \frac{1}{t} dt$$

$$\begin{aligned} \frac{1}{2} \log\left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}}\right) - 11 = \\ -11 + \frac{1}{4i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-296.772380704484551s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0 \end{aligned}$$

And:

$$\frac{1}{2} \log((((((1-0.5)^{-240} (1-0.5^2)^{26760})) * (10^{3400}))) - 21 - 1 - 1 / \text{golden ratio}$$

Input:

$$\frac{1}{2} \log \left(\frac{(1-0.5^2)^{26760}}{(1-0.5)^{240}} \times 10^{3400} \right) - 21 - 1 - \frac{1}{\phi}$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Result:

125.768...

125.768... result very near to the dilaton mass calculated as a type of Higgs boson:
125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV

Alternative representations:

$$\frac{1}{2} \log \left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}} \right) - 21 - 1 - \frac{1}{\phi} = -22 + \frac{1}{2} \log_e \left(\frac{10^{3400} (1-0.5^2)^{26760}}{0.5^{240}} \right) - \frac{1}{\phi}$$

$$\begin{aligned} \frac{1}{2} \log \left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}} \right) - 21 - 1 - \frac{1}{\phi} = \\ -22 + \frac{1}{2} \log(a) \log_a \left(\frac{10^{3400} (1-0.5^2)^{26760}}{0.5^{240}} \right) - \frac{1}{\phi} \end{aligned}$$

$$\begin{aligned} \frac{1}{2} \log \left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}} \right) - 21 - 1 - \frac{1}{\phi} = \\ -22 - \frac{1}{2} \text{Li}_1 \left(1 - \frac{10^{3400} (1-0.5^2)^{26760}}{0.5^{240}} \right) - \frac{1}{\phi} \end{aligned}$$

Series representations:

$$\frac{1}{2} \log\left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}}\right) - 21 - 1 - \frac{1}{\phi} =$$

$$-22 - \frac{1}{\phi} + \frac{\log(7.702067514903312 \times 10^{128})}{2} - \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k e^{-296.772380704484551k}}{k}$$

$$\frac{1}{2} \log\left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}}\right) - 21 - 1 - \frac{1}{\phi} =$$

$$-22 - \frac{1}{\phi} + i\pi \left[\frac{\arg(7.702067514903312 \times 10^{128} - x)}{2\pi} \right] + \frac{\log(x)}{2} -$$

$$\frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (7.702067514903312 \times 10^{128} - x)^k x^{-k}}{k} \text{ for } x < 0$$

$$\frac{1}{2} \log\left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}}\right) - 21 - 1 - \frac{1}{\phi} =$$

$$-22 - \frac{1}{\phi} + \frac{1}{2} \left[\frac{\arg(7.702067514903312 \times 10^{128} - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) +$$

$$\frac{\log(z_0)}{2} + \frac{1}{2} \left[\frac{\arg(7.702067514903312 \times 10^{128} - z_0)}{2\pi} \right] \log(z_0) -$$

$$\frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (7.702067514903312 \times 10^{128} - z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$\frac{1}{2} \log\left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}}\right) - 21 - 1 - \frac{1}{\phi} = -22 - \frac{1}{\phi} + \frac{1}{2} \int_1^{7.702067514903312 \times 10^{128}} \frac{1}{t} dt$$

$$\frac{1}{2} \log\left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}}\right) - 21 - 1 - \frac{1}{\phi} =$$

$$-22 - \frac{1}{\phi} + \frac{1}{4i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-296.772380704484551s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$27 * \frac{1}{2} \left(\left(\frac{1}{2} \log \left(\left(\left(\left(\left(1 - 0.5 \right)^{-240} \left(1 - 0.5^2 \right)^{26760} \right) \right) \right) \right) \right) \right) * \left(10^{3400} \right) \right) - 21 + 1 / \text{golden ratio} \right) + 1$

Input:

$$27 \times \frac{1}{2} \left(\frac{1}{2} \log\left(\frac{(1-0.5^2)^{26760}}{(1-0.5)^{240}} \times 10^{3400}\right) - 21 + \frac{1}{\phi} \right) + 1$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Result:

1729.06...

1729.06...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternative representations:

$$\frac{27}{2} \left(\frac{1}{2} \log \left(\frac{(1 - 0.5^2)^{26760} 10^{3400}}{(1 - 0.5)^{240}} \right) - 21 + \frac{1}{\phi} \right) + 1 =$$

$$1 + \frac{27}{2} \left(-21 + \frac{1}{2} \log_e \left(\frac{10^{3400} (1 - 0.5^2)^{26760}}{0.5^{240}} \right) + \frac{1}{\phi} \right)$$

$$\frac{27}{2} \left(\frac{1}{2} \log \left(\frac{(1 - 0.5^2)^{26760} 10^{3400}}{(1 - 0.5)^{240}} \right) - 21 + \frac{1}{\phi} \right) + 1 =$$

$$1 + \frac{27}{2} \left(-21 + \frac{1}{2} \log(a) \log_a \left(\frac{10^{3400} (1 - 0.5^2)^{26760}}{0.5^{240}} \right) + \frac{1}{\phi} \right)$$

$$\frac{27}{2} \left(\frac{1}{2} \log \left(\frac{(1 - 0.5^2)^{26760} 10^{3400}}{(1 - 0.5)^{240}} \right) - 21 + \frac{1}{\phi} \right) + 1 =$$

$$1 + \frac{27}{2} \left(-21 - \frac{1}{2} \text{Li}_1 \left(1 - \frac{10^{3400} (1 - 0.5^2)^{26760}}{0.5^{240}} \right) + \frac{1}{\phi} \right)$$

Series representations:

$$\frac{27}{2} \left(\frac{1}{2} \log \left(\frac{(1 - 0.5^2)^{26760} 10^{3400}}{(1 - 0.5)^{240}} \right) - 21 + \frac{1}{\phi} \right) + 1 = -\frac{565}{2} + \frac{27}{2\phi} +$$

$$\frac{27 \log(7.702067514903312 \times 10^{128})}{4} - \frac{27}{4} \sum_{k=1}^{\infty} \frac{(-1)^k e^{-296.772380704484551k}}{k}$$

$$\begin{aligned} & \frac{27}{2} \left(\frac{1}{2} \log \left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}} \right) - 21 + \frac{1}{\phi} \right) + 1 = \\ & -\frac{565}{2} + \frac{27}{2\phi} + \frac{27}{2} i\pi \left[\frac{\arg(7.702067514903312 \times 10^{128} - x)}{2\pi} \right] + \frac{27 \log(x)}{4} - \\ & \frac{27}{4} \sum_{k=1}^{\infty} \frac{(-1)^k (7.702067514903312 \times 10^{128} - x)^k x^{-k}}{k} \quad \text{for } x < 0 \end{aligned}$$

$$\begin{aligned} & \frac{27}{2} \left(\frac{1}{2} \log \left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}} \right) - 21 + \frac{1}{\phi} \right) + 1 = \\ & -\frac{565}{2} + \frac{27}{2\phi} + \frac{27}{4} \left[\frac{\arg(7.702067514903312 \times 10^{128} - z_0)}{2\pi} \right] \log \left(\frac{1}{z_0} \right) + \\ & \frac{27 \log(z_0)}{4} + \frac{27}{4} \left[\frac{\arg(7.702067514903312 \times 10^{128} - z_0)}{2\pi} \right] \log(z_0) - \\ & \frac{27}{4} \sum_{k=1}^{\infty} \frac{(-1)^k (7.702067514903312 \times 10^{128} - z_0)^k z_0^{-k}}{k} \end{aligned}$$

Integral representations:

$$\begin{aligned} & \frac{27}{2} \left(\frac{1}{2} \log \left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}} \right) - 21 + \frac{1}{\phi} \right) + 1 = \\ & -\frac{565}{2} + \frac{27}{2\phi} + \frac{27}{4} \int_1^{7.702067514903312 \times 10^{128}} \frac{1}{t} dt \end{aligned}$$

$$\begin{aligned} & \frac{27}{2} \left(\frac{1}{2} \log \left(\frac{(1-0.5^2)^{26760} 10^{3400}}{(1-0.5)^{240}} \right) - 21 + \frac{1}{\phi} \right) + 1 = \\ & -\frac{565}{2} + \frac{27}{2\phi} + \frac{27}{8i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-296.772380704484551s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0 \end{aligned}$$

Now, we have that:

$$\begin{aligned} M(q) & := q^{-\frac{1}{8}} \\ & \times \sum_{n=0}^{\infty} \frac{(-1)^{n+1} q^{(n+1)^2} (1-q)(1-q^3) \cdots (1-q^{2n-1})}{(1+q)^2 (1+q^3)^2 \cdots (1+q^{2n+1})^2} \end{aligned}$$

$535.49165^{(-1/8)} \sum (-1)^{(n+1)} (((535.49165^{((n+1)^2)}) (1-535.49165)(1-535.49165^3)(1-535.49165^{(2n-1)})) / (((1+535.49165)^2 (1+535.49165^3)^2 (1+535.49165^{(2n+1)})^2))))$, n=0 to Pi

Input interpretation:

$$535.49165^{-1/8}$$

$$\sum_{n=0}^{\pi} (-1)^{n+1} \times \frac{535.49165^{(n+1)^2} (1 - 535.49165) (1 - 535.49165^3) (1 - 535.49165^{2n-1})}{(1 + 535.49165)^2 (1 + 535.49165^3)^2 (1 + 535.49165^{2n+1})^2}$$

Result:

$$-6.96196 \times 10^7$$

$$-6.96196 * 10^7$$

$$(1/\text{Pi}) [-535.4916^{(-1/8)} \sum (-1)^{(n+1)} (((535.4916^{((n+1)^2)}) (1-535.4916)(1-535.4916^3)(1-535.4916^{(2n-1)})))/(((1+535.4916)^2 (1+535.4916^3)^2 (1+535.4916^{(2n+1)})^2))], n=0 \text{ to } \text{Pi}]^{1/3} + (11-2)-0.618$$

Input interpretation:

$$\frac{1}{\pi} \left(-535.4916^{-1/8} \sum_{n=0}^{\pi} (-1)^{n+1} \times \frac{535.4916^{(n+1)^2} (1 - 535.4916) (1 - 535.4916^3) (1 - 535.4916^{2n-1})}{(1 + 535.4916)^2 (1 + 535.4916^3)^2 (1 + 535.4916^{2n+1})^2} \right)^{(1/3) + (11 - 2) - 0.618}$$

Result:

$$139.329$$

139.329 result practically equal to the rest mass of Pion meson 139.57 MeV

$$(1/\text{Pi}) [-535.4916^{(-1/8)} \sum (-1)^{(n+1)} (((535.4916^{((n+1)^2)}) (1-535.4916)(1-535.4916^3)(1-535.4916^{(2n-1)})))/(((1+535.4916)^2 (1+535.4916^3)^2 (1+535.4916^{(2n+1)})^2))], n=0 \text{ to } \text{Pi}]^{1/3} - 5 - 0.618$$

Input interpretation:

$$\frac{1}{\pi} \left(-535.4916^{-1/8} \sum_{n=0}^{\pi} (-1)^{n+1} \times \frac{535.4916^{(n+1)^2} (1 - 535.4916) (1 - 535.4916^3) (1 - 535.4916^{2n-1})}{(1 + 535.4916)^2 (1 + 535.4916^3)^2 (1 + 535.4916^{2n+1})^2} \right)^{(1/3) - 5 - 0.618}$$

Result:

125.329

125.329 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV

$27/2 * (((((1/\pi)[-535.49^{(-1/8)} \sum_{n=0}^{\pi} (-1)^{n+1} \times \frac{535.49^{(n+1)^2} (1 - 535.49)(1 - 535.49^3)(1 - 535.49^{2n-1})}{(1 + 535.49)^2 (1 + 535.49^3)^2 (1 + 535.49^{2n+1})^2})^{(1/3 - 2 - 0.618)} - \pi$

Input interpretation:

$$\frac{27}{2} \left(\frac{1}{\pi} \left(-535.49^{-1/8} \sum_{n=0}^{\pi} (-1)^{n+1} \times \frac{535.49^{(n+1)^2} (1 - 535.49)(1 - 535.49^3)(1 - 535.49^{2n-1})}{(1 + 535.49)^2 (1 + 535.49^3)^2 (1 + 535.49^{2n+1})^2} \right)^{(1/3 - 2 - 0.618)} - \pi \right)$$

Result:

1729.29

1729.29

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From

Replica Wormholes and the Entropy of Hawking Radiation

Ahmed Almheiri, Thomas Hartman, Juan Maldacena, Edgar Shaghoulian and Amirhossein Tajdini - arXiv:1911.12333v1 [hep-th] 27 Nov 2019

We have that:

This metric should be joined to the flat space outside. We consider a finite temperature configuration where $\tau \sim \tau + 2\pi$. For general temperatures, all we need to do is to rescale $\phi_r \rightarrow 2\pi\phi_r/\beta$. In other words, the only dimensionful scale is ϕ_r , so the only dependence on the temperature for dimensionless quantities is through ϕ_r/β . We define the coordinate $v = e^y$. So the physical half cylinder $\sigma \geq 0$ corresponds to $|v| \geq 1$. At the boundary we have that $w = e^{i\theta(\tau)}$, $v = e^{i\tau}$. Unfortunately, we cannot extend this to a holomorphic map in the interior of the disk. However, we can find another coordinate z such that there are holomorphic maps from $|w| \leq 1$ and $|v| \geq 1$ to the coordinate z , see figure 10.

We now review the computation of the entropy of the region $B = [0, b]$ which includes the AdS_2 boundary, see figure 11. In gravity this will involve an interval $[-a, b]$, with $a, b > 0$, see figure 12.

$$S_{\text{gen}}([-a, b]) = S_0 + \frac{2\pi\phi_r}{\beta} \frac{1}{\tanh\left(\frac{2\pi a}{\beta}\right)} + \frac{c}{6} \log\left(\frac{2\beta \sinh^2\left(\frac{\pi}{\beta}(a+b)\right)}{\pi\epsilon \sinh\left(\frac{2\pi a}{\beta}\right)}\right). \quad (3.10)$$

$$\partial_a S_{\text{gen}} = 0 \quad \rightarrow \quad \sinh\left(\frac{2\pi a}{\beta}\right) = \frac{12\pi\phi_r}{\beta c} \frac{\sinh\left(\frac{\pi}{\beta}(b+a)\right)}{\sinh\left(\frac{\pi}{\beta}(a-b)\right)}$$

For $\beta = 2\pi$, $a = 3$ and $b = 2$, we obtain:

$$12\pi \times \frac{1}{(2\pi)x} \times \frac{\sinh\left(\frac{\pi}{2\pi}(2+3)\right)}{\sinh\left(\frac{\pi}{2\pi}\right)}$$

Input:

$$12\pi \times \frac{1}{(2\pi)x} \times \frac{\sinh\left(\frac{\pi}{2\pi}(2+3)\right)}{\sinh\left(\frac{\pi}{2\pi}\right)}$$

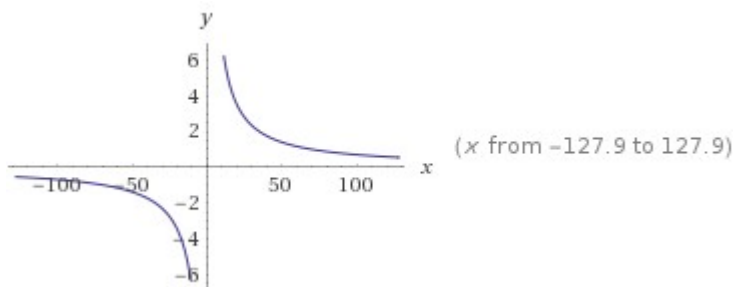
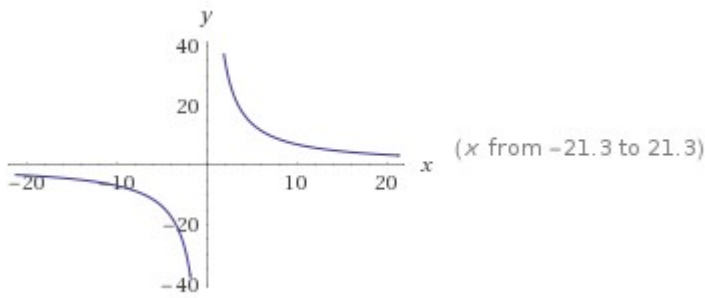
$\sinh(x)$ is the hyperbolic sine function

Exact result:

$$\frac{6 \sinh\left(\frac{5}{2}\right) \operatorname{csch}\left(\frac{1}{2}\right)}{x}$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

Plots:



Alternate forms:

$$\frac{6(1 + e + e^2 + e^3 + e^4)}{e^2 x}$$

$$\frac{6\left(e^{5/2} - \frac{1}{e^{5/2}}\right)}{\left(\sqrt{e} - \frac{1}{\sqrt{e}}\right)x}$$

$$\frac{6(1 - \sqrt{e} + e - e^{3/2} + e^2)(1 + \sqrt{e} + e + e^{3/2} + e^2)}{e^2 x}$$

Alternate form assuming x is real:

$$-\frac{12 \sinh\left(\frac{1}{2}\right) \sinh\left(\frac{5}{2}\right)}{x(1 - \cosh(1))}$$

cosh(x) is the hyperbolic cosine function

Roots:

(no roots exist)

Properties as a real function:

Domain

{x ∈ ℝ : x ≠ 0}

Range

{y ∈ ℝ : y ≠ 0}

Injectivity

injective (one-to-one)

Parity

odd

\mathbb{R} is the set of real numbers

Derivative:

$$\frac{d}{dx} \left(\frac{12 \pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right)}{(2\pi)x \sinh\left(\frac{\pi}{2\pi}\right)} \right) = -\frac{6 \sinh\left(\frac{5}{2}\right) \operatorname{csch}\left(\frac{1}{2}\right)}{x^2}$$

Indefinite integral:

$$\int \frac{6 \operatorname{csch}\left(\frac{1}{2}\right) \sinh\left(\frac{5}{2}\right)}{x} dx = 6 \sinh\left(\frac{5}{2}\right) \operatorname{csch}\left(\frac{1}{2}\right) \log(x) + \text{constant}$$

(assuming a complex-valued logarithm)

$\log(x)$ is the natural logarithm

Limit:

$$\lim_{x \rightarrow \pm\infty} \frac{6 \operatorname{csch}\left(\frac{1}{2}\right) \sinh\left(\frac{5}{2}\right)}{x} = 0$$

Alternative representations:

$$\frac{(12\pi) \sinh\left(\frac{\pi(2+3)}{2\pi}\right)}{(2\pi)x \sinh\left(\frac{\pi}{2\pi}\right)} = \frac{12\pi}{(2\pi)x \operatorname{csch}\left(\frac{5\pi}{2\pi}\right) \operatorname{csch}\left(\frac{\pi}{2\pi}\right)}$$

$$\frac{(12\pi) \sinh\left(\frac{\pi(2+3)}{2\pi}\right)}{(2\pi)x \sinh\left(\frac{\pi}{2\pi}\right)} = \frac{12i\pi \cos\left(\frac{\pi}{2} + \frac{5i\pi}{2\pi}\right)}{(2\pi)x \left(i \cos\left(\frac{\pi}{2} + \frac{i\pi}{2\pi}\right)\right)}$$

$$\frac{(12\pi) \sinh\left(\frac{\pi(2+3)}{2\pi}\right)}{(2\pi)x \sinh\left(\frac{\pi}{2\pi}\right)} = \frac{12i\pi \cosh\left(\frac{i\pi}{2} - \frac{5\pi}{2\pi}\right)}{(2\pi)x \left(i \cosh\left(\frac{i\pi}{2} - \frac{\pi}{2\pi}\right)\right)}$$

Series representations:

$$\frac{(12\pi) \sinh\left(\frac{\pi(2+3)}{2\pi}\right)}{(2\pi)x \sinh\left(\frac{\pi}{2\pi}\right)} = -\frac{12 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{2}{5}\right)^{-1-2k_2} q^{-1+2k_1}}{(1+2k_2)!}}{x} \quad \text{for } q = \sqrt{e}$$

$$\frac{(12 \pi) \sinh\left(\frac{\pi(2+3)}{2 \pi}\right)}{(2 \pi) x \sinh\left(\frac{\pi}{2 \pi}\right)} = \frac{12 \sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} \left(\frac{2}{5}\right)^{-1-2 k_2}}{(1+2 k_2)! (1+4 \pi^2 k_1^2)}}{x}$$

$$\frac{(12 \pi) \sinh\left(\frac{\pi(2+3)}{2 \pi}\right)}{(2 \pi) x \sinh\left(\frac{\pi}{2 \pi}\right)} = \frac{12 \left(1 + 2 \sum_{k=1}^{\infty} \frac{(-1)^k}{1+4 k^2 \pi^2}\right) \sum_{k=0}^{\infty} \frac{\left(\frac{5}{2}\right)^{1+2 k}}{(1+2 k)!}}{x}$$

Integral representations:

$$\frac{(12 \pi) \sinh\left(\frac{\pi(2+3)}{2 \pi}\right)}{(2 \pi) x \sinh\left(\frac{\pi}{2 \pi}\right)} = \frac{30 \int_0^1 \cosh\left(\frac{5 t}{2}\right) d t}{x \int_0^1 \cosh\left(\frac{t}{2}\right) d t}$$

$$\frac{(12 \pi) \sinh\left(\frac{\pi(2+3)}{2 \pi}\right)}{(2 \pi) x \sinh\left(\frac{\pi}{2 \pi}\right)} = \frac{30 \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{25/(16 s)+s}}{s^{3/2}} d s}{x \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{1/(16 s)+s}}{s^{3/2}} d s} \quad \text{for } \gamma > 0$$

For $c = 1$, we obtain:

$$12 \pi * 1 / ((2 \pi)) * \sinh \left(\left(\left(\left(\pi / (2 \pi) \right) * (2+3) \right) \right) / \sinh \left(\pi / (2 \pi) \right) \right)$$

Input:

$$12 \pi \times \frac{1}{2 \pi} \times \frac{\sinh\left(\frac{\pi}{2 \pi} (2+3)\right)}{\sinh\left(\frac{\pi}{2 \pi}\right)}$$

$\sinh(x)$ is the hyperbolic sine function

Exact result:

$$6 \sinh\left(\frac{5}{2}\right) \operatorname{csch}\left(\frac{1}{2}\right)$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

Decimal approximation:

69.66331591078650285648142918236969349074603204715890369018...

69.66331591.....

Property:

$6 \operatorname{csch}\left(\frac{1}{2}\right) \sinh\left(\frac{5}{2}\right)$ is a transcendental number

Alternate forms:

$$\frac{6(1 + e + e^2 + e^3 + e^4)}{e^2}$$

$$-\frac{12 \sinh\left(\frac{1}{2}\right) \sinh\left(\frac{5}{2}\right)}{1 - \cosh(1)}$$

$$\frac{6\left(e^{5/2} - \frac{1}{e^{5/2}}\right)}{\sqrt{e} - \frac{1}{\sqrt{e}}}$$

$\cosh(x)$ is the hyperbolic cosine function

Alternative representations:

$$\frac{(12 \pi) \sinh\left(\frac{\pi(2+3)}{2 \pi}\right)}{(2 \pi) \sinh\left(\frac{\pi}{2 \pi}\right)} = \frac{12 \pi}{\frac{(2 \pi) \operatorname{csch}\left(\frac{5 \pi}{2 \pi}\right)}{\operatorname{csch}\left(\frac{\pi}{2 \pi}\right)}}$$

$$\frac{(12 \pi) \sinh\left(\frac{\pi(2+3)}{2 \pi}\right)}{(2 \pi) \sinh\left(\frac{\pi}{2 \pi}\right)} = \frac{12 i \pi \cos\left(\frac{\pi}{2} + \frac{5 i \pi}{2 \pi}\right)}{(2 \pi) \left(i \cos\left(\frac{\pi}{2} + \frac{i \pi}{2 \pi}\right)\right)}$$

$$\frac{(12 \pi) \sinh\left(\frac{\pi(2+3)}{2 \pi}\right)}{(2 \pi) \sinh\left(\frac{\pi}{2 \pi}\right)} = \frac{12 i \pi \cosh\left(\frac{i \pi}{2} - \frac{5 \pi}{2 \pi}\right)}{(2 \pi) \left(i \cosh\left(\frac{i \pi}{2} - \frac{\pi}{2 \pi}\right)\right)}$$

Series representations:

$$\frac{(12 \pi) \sinh\left(\frac{\pi(2+3)}{2 \pi}\right)}{(2 \pi) \sinh\left(\frac{\pi}{2 \pi}\right)} = -12 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{2}{5}\right)^{-1-2k_2} q^{-1+2k_1}}{(1+2k_2)!} \text{ for } q = \sqrt{e}$$

$$\frac{(12 \pi) \sinh\left(\frac{\pi(2+3)}{2 \pi}\right)}{(2 \pi) \sinh\left(\frac{\pi}{2 \pi}\right)} = 12 \sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} \left(\frac{2}{5}\right)^{-1-2k_2}}{(1+2k_2)! (1+4\pi^2 k_1^2)}$$

$$\frac{(12 \pi) \sinh\left(\frac{\pi(2+3)}{2 \pi}\right)}{(2 \pi) \sinh\left(\frac{\pi}{2 \pi}\right)} = 12 \left(1 + 2 \sum_{k=1}^{\infty} \frac{(-1)^k}{1+4k^2 \pi^2}\right) \sum_{k=0}^{\infty} \frac{\left(\frac{5}{2}\right)^{1+2k}}{(1+2k)!}$$

Integral representations:

$$\frac{(12\pi) \sinh\left(\frac{\pi(2+3)}{2\pi}\right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} = \frac{30 \int_0^1 \cosh\left(\frac{5t}{2}\right) dt}{\int_0^1 \cosh\left(\frac{t}{2}\right) dt}$$

$$\frac{(12\pi) \sinh\left(\frac{\pi(2+3)}{2\pi}\right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} = \frac{30 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{25/(16s)+s}}{s^{3/2}} ds}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{1/(16s)+s}}{s^{3/2}} ds} \quad \text{for } \gamma > 0$$

$$2 * ((12\pi * 1/(2\pi)) * \sinh(((\pi/(2\pi)) * (2+3)))) / \sinh(\pi/(2\pi))$$

Input:

$$2 \left(12\pi \times \frac{1}{2\pi} \times \frac{\sinh\left(\frac{\pi}{2\pi} (2+3)\right)}{\sinh\left(\frac{\pi}{2\pi}\right)} \right)$$

$\sinh(x)$ is the hyperbolic sine function

Exact result:

$$12 \sinh\left(\frac{5}{2}\right) \operatorname{csch}\left(\frac{1}{2}\right)$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

Decimal approximation:

139.3266318215730057129628583647393869814920640943178073803...

139.3266318... result practically equal to the rest mass of Pion meson 139.57 MeV

Property:

$12 \operatorname{csch}\left(\frac{1}{2}\right) \sinh\left(\frac{5}{2}\right)$ is a transcendental number

Alternate forms:

$$\frac{12(1 + e + e^2 + e^3 + e^4)}{e^2}$$

$$\frac{24 \sinh\left(\frac{1}{2}\right) \sinh\left(\frac{5}{2}\right)}{1 - \cosh(1)}$$

$$\frac{12 \left(e^{5/2} - \frac{1}{e^{5/2}} \right)}{\sqrt{e} - \frac{1}{\sqrt{e}}}$$

$\cosh(x)$ is the hyperbolic cosine function

Alternative representations:

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} = \frac{24\pi}{\frac{(2\pi) \operatorname{csch}\left(\frac{5\pi}{2\pi}\right)}{\operatorname{csch}\left(\frac{\pi}{2\pi}\right)}}$$

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} = \frac{24 i \pi \cos\left(\frac{\pi}{2} + \frac{5i\pi}{2\pi}\right)}{(2\pi) \left(i \cos\left(\frac{\pi}{2} + \frac{i\pi}{2\pi}\right) \right)}$$

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} = \frac{24 i \pi \cosh\left(\frac{i\pi}{2} - \frac{5\pi}{2\pi}\right)}{(2\pi) \left(i \cosh\left(\frac{i\pi}{2} - \frac{\pi}{2\pi}\right) \right)}$$

Series representations:

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} = -24 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{2}{5}\right)^{-1-2k_2} q^{-1+2k_1}}{(1+2k_2)!} \quad \text{for } q = \sqrt{e}$$

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} = 24 \sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} \left(\frac{2}{5}\right)^{-1-2k_2}}{(1+2k_2)! (1+4\pi^2 k_1^2)}$$

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} = 24 \left(1 + 2 \sum_{k=1}^{\infty} \frac{(-1)^k}{1+4k^2\pi^2} \right) \sum_{k=0}^{\infty} \frac{\left(\frac{5}{2}\right)^{1+2k}}{(1+2k)!}$$

Integral representations:

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} = \frac{60 \int_0^1 \cosh\left(\frac{5t}{2}\right) dt}{\int_0^1 \cosh\left(\frac{t}{2}\right) dt}$$

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} = \frac{60 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{25/(16s)+s}}{s^{3/2}} ds}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{1/(16s)+s}}{s^{3/2}} ds} \quad \text{for } \gamma > 0$$

$$2 * ((12\pi * 1 / ((2\pi)) * \sinh(((\pi / ((2\pi))) * (2+3)))) / \sinh(\pi / ((2\pi)))) - 2$$

Input:

$$2 \left(12\pi \times \frac{1}{2\pi} \times \frac{\sinh\left(\frac{\pi}{2\pi} (2+3)\right)}{\sinh\left(\frac{\pi}{2\pi}\right)} \right) - 2$$

$\sinh(x)$ is the hyperbolic sine function

Exact result:

$$12 \sinh\left(\frac{5}{2}\right) \operatorname{csch}\left(\frac{1}{2}\right) - 2$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

Decimal approximation:

137.3266318215730057129628583647393869814920640943178073803...

137.3266318...

This result is very near to the inverse of fine-structure constant 137,035

Property:

$-2 + 12 \operatorname{csch}\left(\frac{1}{2}\right) \sinh\left(\frac{5}{2}\right)$ is a transcendental number

Alternate forms:

$$2 \left(6 \sinh\left(\frac{5}{2}\right) \operatorname{csch}\left(\frac{1}{2}\right) - 1 \right)$$

$$10 + 12e + 12e^2 + \frac{12(1+e)}{e^2}$$

$$-2 - \frac{24 \sinh\left(\frac{1}{2}\right) \sinh\left(\frac{5}{2}\right)}{1 - \cosh(1)}$$

$\cosh(x)$ is the hyperbolic cosine function

Alternative representations:

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 2 = -2 + \frac{24\pi}{\frac{(2\pi) \operatorname{csch}\left(\frac{5\pi}{2\pi}\right)}{\operatorname{csch}\left(\frac{\pi}{2\pi}\right)}}$$

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 2 = -2 + \frac{24 i \pi \cos\left(\frac{\pi}{2} + \frac{5 i \pi}{2\pi}\right)}{(2\pi) \left(i \cos\left(\frac{\pi}{2} + \frac{i \pi}{2\pi}\right) \right)}$$

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 2 = -2 + \frac{24 i \pi \cosh\left(\frac{i \pi}{2} - \frac{5 \pi}{2\pi}\right)}{(2\pi) \left(i \cosh\left(\frac{i \pi}{2} - \frac{\pi}{2\pi}\right) \right)}$$

Series representations:

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 2 = -2 \left(1 + 12 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{2}{5}\right)^{-1-2k_2} q^{-1+2k_1}}{(1+2k_2)!} \right) \text{ for } q = \sqrt{e}$$

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 2 = 2 \left(-1 + 12 \sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} \left(\frac{2}{5}\right)^{-1-2k_2}}{(1+2k_2)! (1+4\pi^2 k_1^2)} \right)$$

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 2 = -2 i \left(-i + 12 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(-\frac{1}{2} i (5i + \pi)\right)^{2k_2} q^{-1+2k_1}}{(2k_2)!} \right)$$

for $q = \sqrt{e}$

Integral representations:

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 2 = - \frac{2 \left(\int_0^1 \cosh\left(\frac{t}{2}\right) dt - 30 \int_0^1 \cosh\left(\frac{5t}{2}\right) dt \right)}{\int_0^1 \cosh\left(\frac{t}{2}\right) dt}$$

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 2 = - \frac{2 \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{1/(16s)+s}}{s^{3/2}} ds - 30 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{25/(16s)+s}}{s^{3/2}} ds \right)}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{1/(16s)+s}}{s^{3/2}} ds} \text{ for } \gamma > 0$$

$2 * ((12\pi * 1/((2\pi)) * \sinh (((\pi/((2\pi))) * (2+3)))) / \sinh (\pi/((2\pi)))) - 13 - 1/\text{golden ratio}$

Input:

$$2 \left(12 \pi \times \frac{1}{2 \pi} \times \frac{\sinh\left(\frac{\pi}{2\pi} (2+3)\right)}{\sinh\left(\frac{\pi}{2\pi}\right)} \right) - 13 - \frac{1}{\phi}$$

$\sinh(x)$ is the hyperbolic sine function

ϕ is the golden ratio

Exact result:

$$-\frac{1}{\phi} - 13 + 12 \sinh\left(\frac{5}{2}\right) \operatorname{csch}\left(\frac{1}{2}\right)$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

Decimal approximation:

125.7085978328231108647582715303737488637717549145120445182...

125.708597832... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV

Property:

$-13 - \frac{1}{\phi} + 12 \operatorname{csch}\left(\frac{1}{2}\right) \sinh\left(\frac{5}{2}\right)$ is a transcendental number

Alternate forms:

$$-\frac{1}{\phi} - 1 + 12 e + 12 e^2 + \frac{12 (1 + e)}{e^2}$$

$$-13 - \frac{2}{1 + \sqrt{5}} + 12 \sinh\left(\frac{5}{2}\right) \operatorname{csch}\left(\frac{1}{2}\right)$$

$$\frac{1}{2} (-25 - \sqrt{5}) + 12 \sinh\left(\frac{5}{2}\right) \operatorname{csch}\left(\frac{1}{2}\right)$$

Alternative representations:

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 13 - \frac{1}{\phi} = -13 - \frac{1}{\phi} + \frac{24\pi}{\frac{(2\pi) \operatorname{csch}\left(\frac{5\pi}{2\pi}\right)}{\operatorname{csch}\left(\frac{\pi}{2\pi}\right)}}$$

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 13 - \frac{1}{\phi} = -13 - \frac{1}{\phi} + \frac{24 i \pi \cos\left(\frac{\pi}{2} + \frac{5i\pi}{2\pi}\right)}{(2\pi) \left(i \cos\left(\frac{\pi}{2} + \frac{i\pi}{2\pi}\right) \right)}$$

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 13 - \frac{1}{\phi} = -13 - \frac{1}{\phi} + \frac{24 i \pi \cosh\left(\frac{i\pi}{2} - \frac{5\pi}{2\pi}\right)}{(2\pi) \left(i \cosh\left(\frac{i\pi}{2} - \frac{\pi}{2\pi}\right) \right)}$$

Series representations:

$$\begin{aligned} & \frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 13 - \frac{1}{\phi} = \\ & -\frac{1}{1+\sqrt{5}} \left(15 + 13\sqrt{5} + 24 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{2}{5}\right)^{-1-2k_2} q^{-1+2k_1}}{(1+2k_2)!} + \right. \\ & \left. 24\sqrt{5} \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{2}{5}\right)^{-1-2k_2} q^{-1+2k_1}}{(1+2k_2)!} \right) \text{ for } q = \sqrt{e} \end{aligned}$$

$$\begin{aligned} & \frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 13 - \frac{1}{\phi} = \frac{1}{1+\sqrt{5}} \\ & \left(-15 - 13\sqrt{5} + 24 \sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} \left(\frac{2}{5}\right)^{-1-2k_2}}{(1+2k_2)! (1+4\pi^2 k_1^2)} + \right. \\ & \left. 24\sqrt{5} \sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} \left(\frac{2}{5}\right)^{-1-2k_2}}{(1+2k_2)! (1+4\pi^2 k_1^2)} \right) \end{aligned}$$

$$\begin{aligned} & \frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 13 - \frac{1}{\phi} = \\ & -\frac{1}{1+\sqrt{5}} i \left(-15i - 13i\sqrt{5} + 24 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(-\frac{1}{2}i(5i+\pi)\right)^{2k_2} q^{-1+2k_1}}{(2k_2)!} + \right. \\ & \left. 24\sqrt{5} \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(-\frac{1}{2}i(5i+\pi)\right)^{2k_2} q^{-1+2k_1}}{(2k_2)!} \right) \text{ for } q = \sqrt{e} \end{aligned}$$

Integral representations:

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 13 - \frac{1}{\phi} =$$

$$\frac{\int_0^1 \cosh\left(\frac{t}{2}\right) dt + 13 \phi \int_0^1 \cosh\left(\frac{t}{2}\right) dt - 60 \phi \int_0^1 \cosh\left(\frac{5t}{2}\right) dt}{\phi \int_0^1 \cosh\left(\frac{t}{2}\right) dt}$$

$$\frac{2 \times 12 \left(\pi \sinh\left(\frac{\pi(2+3)}{2\pi}\right) \right)}{(2\pi) \sinh\left(\frac{\pi}{2\pi}\right)} - 13 - \frac{1}{\phi} =$$

$$\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{1/(16s)+s}}{s^{3/2}} ds + 13 \phi \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{1/(16s)+s}}{s^{3/2}} ds - 60 \phi \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{25/(16s)+s}}{s^{3/2}} ds}{\phi \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{1/(16s)+s}}{s^{3/2}} ds} \quad \text{for } \gamma > 0$$

Now, we have that:

$$S_{\text{gen}}([-a, b]) = S_0 + \frac{2\pi\phi_r}{\beta} \frac{1}{\tanh\left(\frac{2\pi a}{\beta}\right)} + \frac{c}{6} \log\left(\frac{2\beta \sinh^2\left(\frac{\pi}{\beta}(a+b)\right)}{\pi \epsilon \sinh\left(\frac{2\pi a}{\beta}\right)}\right). \quad (3.10)$$

For $\beta = 2\pi$, $a = 3$, $b = 2$ and $c = 1$, we obtain:

$$2\pi/(2\pi \cdot \tanh 3) + 1/6 \ln\left[\frac{4\pi \sinh^2\left(\frac{5\pi}{2\pi}\right)}{0.0864055\pi \sinh 3}\right]$$

Input interpretation:

$$2 \times \frac{\pi}{2\pi \tanh(3)} + \frac{1}{6} \log\left(4\pi \times \frac{\sinh^2\left(\frac{5\pi}{2\pi}\right)}{0.0864055 \pi \sinh(3)}\right)$$

$\tanh(x)$ is the hyperbolic tangent function

$\sinh(x)$ is the hyperbolic sine function

$\log(x)$ is the natural logarithm

Result:

1.860105...

1.860105...

Alternative representations:

$$\frac{2\pi}{2\pi \tanh(3)} + \frac{1}{6} \log\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2\pi}\right)}{0.0864055 \pi \sinh(3)}\right) =$$

$$\frac{1}{6} \log_e\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2\pi}\right)}{0.0864055 \pi \sinh(3)}\right) + \frac{2\pi}{2\pi \left(-1 + \frac{2}{1+\frac{1}{e^6}}\right)}$$

$$\frac{2\pi}{2\pi \tanh(3)} + \frac{1}{6} \log\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2\pi}\right)}{0.0864055 \pi \sinh(3)}\right) =$$

$$\frac{1}{6} \log(a) \log_a\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2\pi}\right)}{0.0864055 \pi \sinh(3)}\right) + \frac{2\pi}{2\pi \left(-1 + \frac{2}{1+\frac{1}{e^6}}\right)}$$

$$\frac{2\pi}{2\pi \tanh(3)} + \frac{1}{6} \log\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2\pi}\right)}{0.0864055 \pi \sinh(3)}\right) =$$

$$\frac{1}{6} \log(a) \log_a\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2\pi}\right)}{0.0864055 \pi \sinh(3)}\right) + \frac{2\pi}{2i\pi \cot\left(3i + \frac{\pi}{2}\right)}$$

Series representations:

$$\frac{2\pi}{2\pi \tanh(3)} + \frac{1}{6} \log\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2\pi}\right)}{0.0864055 \pi \sinh(3)}\right) =$$

$$\left(1 + 4 \log\left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)}\right) \sum_{k=1}^{\infty} \frac{1}{36 + (1 - 2k)^2 \pi^2} - \right.$$

$$\left. 4 \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_2} \left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)}\right)^{-k_2}}{(36 + \pi^2 (1 - 2k_1)^2) k_2} \right) / \left(24 \sum_{k=1}^{\infty} \frac{1}{36 + (1 - 2k)^2 \pi^2}\right)$$

$$\frac{2\pi}{2\pi \tanh(3)} + \frac{1}{6} \log\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2}\right)}{0.0864055\pi \sinh(3)}\right) =$$

$$\left(-6 + \log\left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)}\right) + 2 \log\left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)}\right) \sum_{k=1}^{\infty} (-1)^k q^{2k} - \right.$$

$$\left. \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)}\right)^{-k}}{k} - \right.$$

$$\left. 2 \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_1+k_2} q^{2k_1} \left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)}\right)^{-k_2}}{k_2} \right) /$$

$$\left(6 \left(1 + 2 \sum_{k=1}^{\infty} (-1)^k q^{2k} \right) \right) \text{ for } q = e^3$$

$$\frac{2\pi}{2\pi \tanh(3)} + \frac{1}{6} \log\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2}\right)}{0.0864055\pi \sinh(3)}\right) =$$

$$\left(-6 + \log\left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)}\right) \sum_{k=0}^{\infty} \left(\delta_k + \frac{2^{1+k} \text{Li}_{-k}(-e^{2z_0})}{k!} \right) (3 - z_0)^k - \right.$$

$$\left. \sum_{k_1=0}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_2} \left(\delta_{k_1} + \frac{2^{1+k_1} \text{Li}_{-k_1}(-e^{2z_0})}{k_1!} \right) \left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)} \right)^{-k_2} (3 - z_0)^{k_1}}{k_2} \right) /$$

$$\left(6 \sum_{k=0}^{\infty} \left(\delta_k + \frac{2^{1+k} \text{Li}_{-k}(-e^{2z_0})}{k!} \right) (3 - z_0)^k \right) \text{ for } \frac{1}{2} + \frac{iz_0}{\pi} \notin \mathbb{Z}$$

Now, we have that:

$$x + 2\pi / (2\pi \cdot \tanh 3) + 1/6 \ln\left[\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2}\right)}{0.0864055\pi \sinh 3}\right)\right] = 69.66331591$$

Input interpretation:

$$x + 2 \times \frac{\pi}{2\pi \tanh(3)} + \frac{1}{6} \log\left(4\pi \times \frac{\sinh^2\left(\frac{5\pi}{2}\right)}{0.0864055\pi \sinh(3)}\right) = 69.66331591$$

$\tanh(x)$ is the hyperbolic tangent function

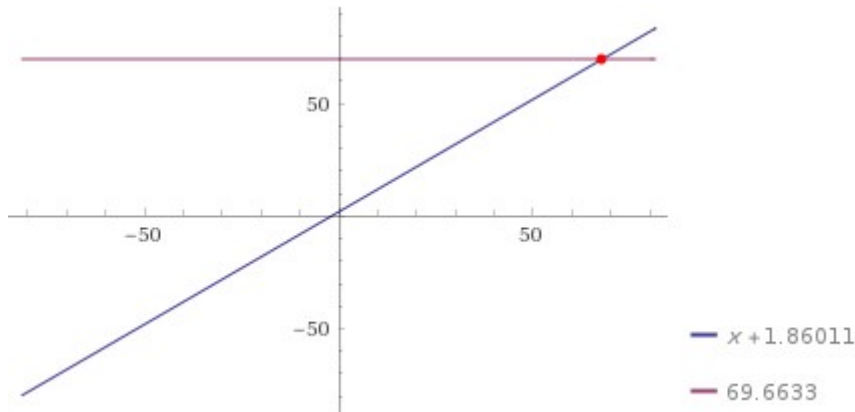
$\sinh(x)$ is the hyperbolic sine function

$\log(x)$ is the natural logarithm

Result:

$$x + 1.86011 = 69.6633$$

Plot:



Alternate forms:

$$x - 67.8032 = 0$$

$$x + 1.86011 = 69.6633$$

Solution:

$$x \approx 67.8032$$

67.8032

$$67.8032 + 2\pi / (2\pi \tanh 3) + 1/6 \ln[(((4\pi \sinh^2(((5\pi)/(2\pi)))) / (0.0864055\pi \sinh 3)))]$$

Input interpretation:

$$67.8032 + 2 \times \frac{\pi}{2\pi \tanh(3)} + \frac{1}{6} \log \left(4\pi \times \frac{\sinh^2\left(\frac{5\pi}{2\pi}\right)}{0.0864055\pi \sinh(3)} \right)$$

$\tanh(x)$ is the hyperbolic tangent function

$\sinh(x)$ is the hyperbolic sine function

$\log(x)$ is the natural logarithm

Result:

69.6633...

69.6633...

Alternative representations:

$$67.8032 + \frac{2\pi}{2\pi \tanh(3)} + \frac{1}{6} \log\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2}\right)}{0.0864055\pi \sinh(3)}\right) =$$

$$67.8032 + \frac{1}{6} \log_e\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2}\right)}{0.0864055\pi \sinh(3)}\right) + \frac{2\pi}{2\pi \left(-1 + \frac{2}{1 + \frac{1}{e^6}}\right)}$$

$$67.8032 + \frac{2\pi}{2\pi \tanh(3)} + \frac{1}{6} \log\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2}\right)}{0.0864055\pi \sinh(3)}\right) =$$

$$67.8032 + \frac{1}{6} \log(a) \log_a\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2}\right)}{0.0864055\pi \sinh(3)}\right) + \frac{2\pi}{2\pi \left(-1 + \frac{2}{1 + \frac{1}{e^6}}\right)}$$

$$67.8032 + \frac{2\pi}{2\pi \tanh(3)} + \frac{1}{6} \log\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2}\right)}{0.0864055\pi \sinh(3)}\right) =$$

$$67.8032 + \frac{1}{6} \log(a) \log_a\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2}\right)}{0.0864055\pi \sinh(3)}\right) + \frac{2\pi}{2i\pi \cot\left(3i + \frac{\pi}{2}\right)}$$

Series representations:

$$67.8032 + \frac{2\pi}{2\pi \tanh(3)} + \frac{1}{6} \log\left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2}\right)}{0.0864055\pi \sinh(3)}\right) =$$

$$\frac{1}{\sum_{k=1}^{\infty} \frac{1}{36 + (1-2k)^2 \pi^2}} 0.166667 \left(0.25 + 406.819 \sum_{k=1}^{\infty} \frac{1}{36 + (1-2k)^2 \pi^2} + \log\left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)}\right) \sum_{k=1}^{\infty} \frac{1}{36 + (1-2k)^2 \pi^2} - \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_2} \left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)}\right)^{-k_2}}{(36 + \pi^2 (1-2k_1)^2) k_2} \right)$$

$$\begin{aligned}
& 67.8032 + \frac{2\pi}{2\pi \tanh(3)} + \frac{1}{6} \log \left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2}\right)}{0.0864055 \pi \sinh(3)} \right) = \\
& \left(0.166667 \left[200.41 + 0.5 \log \left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)} \right) + \right. \right. \\
& \quad 406.819 \sum_{k=1}^{\infty} (-1)^k q^{2k} + \log \left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)} \right) \sum_{k=1}^{\infty} (-1)^k q^{2k} - \\
& \quad 0.5 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)} \right)^{-k}}{k} - \\
& \quad \left. \left. \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_1+k_2} q^{2k_1} \left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)} \right)^{-k_2}}{k_2} \right] \right) / \\
& \left(0.5 + \sum_{k=1}^{\infty} (-1)^k q^{2k} \right) \text{ for } q = e^3
\end{aligned}$$

$$\begin{aligned}
& 67.8032 + \frac{2\pi}{2\pi \tanh(3)} + \frac{1}{6} \log \left(\frac{4\pi \sinh^2\left(\frac{5\pi}{2}\right)}{0.0864055 \pi \sinh(3)} \right) = \\
& \left(0.166667 \left[-6 + 406.819 \sum_{k=0}^{\infty} \left(\delta_k + \frac{2^{1+k} \text{Li}_{-k}(-e^{2z_0})}{k!} \right) (3-z_0)^k + \right. \right. \\
& \quad \log \left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)} \right) \sum_{k=0}^{\infty} \left(\delta_k + \frac{2^{1+k} \text{Li}_{-k}(-e^{2z_0})}{k!} \right) (3-z_0)^k - \\
& \quad \sum_{k_1=0}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_2} \left(\delta_{k_1} + \frac{2^{1+k_1} \text{Li}_{-k_1}(-e^{2z_0})}{k_1!} \right) \left(-1 + \frac{46.2933 \sinh^2\left(\frac{5}{2}\right)}{\sinh(3)} \right)^{-k_2} (3-z_0)^{k_1}}{k_2} \left. \right] \right) \\
& / \left(\sum_{k=0}^{\infty} \left(\delta_k + \frac{2^{1+k} \text{Li}_{-k}(-e^{2z_0})}{k!} \right) (3-z_0)^k \right) \text{ for } \frac{1}{2} + \frac{iz_0}{\pi} \notin \mathbb{Z}
\end{aligned}$$

We insert the value 69.6633 in the Hawking radiation calculator as entropy (S) and obtain the surface area 1.675666e-67

Thence, adding this result to the previous expression

$$2 \times \frac{\pi}{2\pi \tanh(3)} + \frac{1}{6} \log \left(4\pi \times \frac{\sinh^2\left(\frac{5\pi}{2\pi}\right)}{0.0864055 \pi \sinh(3)} \right)$$

we obtain the generalized entropy (S_{gen}):

$$1.675666e-67 + 2\pi / (2\pi * \tanh 3) + 1/6 \ln[(((4\pi \sinh^2(((5\pi)/(2\pi)))) / (0.0864055\pi \sinh 3)))]$$

Input interpretation:

$$1.675666 \times 10^{-67} + 2 \times \frac{\pi}{2\pi \tanh(3)} + \frac{1}{6} \log \left(4\pi \times \frac{\sinh^2\left(\frac{5\pi}{2\pi}\right)}{0.0864055 \pi \sinh(3)} \right)$$

$\tanh(x)$ is the hyperbolic tangent function

$\sinh(x)$ is the hyperbolic sine function

$\log(x)$ is the natural logarithm

Result:

1.860105...

1.860105...

For this value correspond a mass and a radius of 1.270786e-8, 1.886929e-35 respectively.

Inserting the above values and the temperature 1.227203e+11

Mass = 1.270786e-8

Radius = 1.886929e-35

Temperature = 1.227203e+11

from the Ramanujan-Nardelli mock formula, we obtain:

$$\sqrt{\left[\frac{1}{\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.270786 \times 10^{-8}} \right) \sqrt{\left[-\frac{1.227203 \times 10^{11} \times 4 \pi (1.886929 \times 10^{-35})^3 - (1.886929 \times 10^{-35})^2}{6.67 \times 10^{-11}} \right]}} \right]}$$

Input interpretation:

$$\sqrt{\left(\frac{1}{\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.270786 \times 10^{-8}} \right) \sqrt{\left[-\frac{1.227203 \times 10^{11} \times 4 \pi (1.886929 \times 10^{-35})^3 - (1.886929 \times 10^{-35})^2}{6.67 \times 10^{-11}} \right]}} \right)}$$

Result:

1.61732...

[1.61732...](#)

$$1 / \sqrt{\left[\frac{1}{\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{1.270786 \times 10^{-8}} \right) \sqrt{\left[-\frac{1.227203 \times 10^{11} \times 4 \pi (1.886929 \times 10^{-35})^3 - (1.886929 \times 10^{-35})^2}{6.67 \times 10^{-11}} \right]}} \right]}$$

Input interpretation:

$$1 / \left(\frac{1}{\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{1.270786 \times 10^{-8}} \right) \sqrt{\left[-\frac{1.227203 \times 10^{11} \times 4 \pi (1.886929 \times 10^{-35})^3 - (1.886929 \times 10^{-35})^2}{6.67 \times 10^{-11}} \right]}} \right)}$$

Result:

0.618306...

[0.618306...](#)

Now, we have that:

The generalized entropy, including the island, is

$$S_{\text{gen}}(I \cup R) = \frac{\phi_r}{a} + \frac{c}{6} \log \frac{(a+b)^2}{a}.$$

For $\phi_r \cong 1$, $a = 3$, $b = 2$ and $c = 1$, we obtain:

$$1/3 + 1/6 \ln((2+3)^2 * 1/3)$$

Input:

$$\frac{1}{3} + \frac{1}{6} \log\left((2+3)^2 \times \frac{1}{3}\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{1}{3} + \frac{1}{6} \log\left(\frac{25}{3}\right)$$

Decimal approximation:

0.686710589366681842967712238254974929067285358452380998681...

0.686710589366681842967....

Property:

$\frac{1}{3} + \frac{1}{6} \log\left(\frac{25}{3}\right)$ is a transcendental number

Alternate forms:

$$\frac{1}{6} \left(2 + \log\left(\frac{25}{3}\right) \right)$$

$$\frac{1}{3} - \frac{\log(3)}{6} + \frac{\log(5)}{3}$$

$$\frac{1}{6} (2(1 + \log(5)) - \log(3))$$

Alternative representations:

$$\frac{1}{3} + \frac{1}{6} \log\left(\frac{1}{3} (2+3)^2\right) = \frac{1}{3} + \frac{\log_e\left(\frac{5^2}{3}\right)}{6}$$

$$\frac{1}{3} + \frac{1}{6} \log\left(\frac{1}{3} (2+3)^2\right) = \frac{1}{3} + \frac{1}{6} \log(a) \log_a\left(\frac{5^2}{3}\right)$$

$$\frac{1}{3} + \frac{1}{6} \log\left(\frac{1}{3} (2+3)^2\right) = \frac{1}{3} - \frac{1}{6} \text{Li}_1\left(1 - \frac{5^2}{3}\right)$$

Series representations:

$$\frac{1}{3} + \frac{1}{6} \log\left(\frac{1}{3} (2+3)^2\right) = \frac{1}{3} + \frac{1}{6} \log\left(\frac{22}{3}\right) - \frac{1}{6} \sum_{k=1}^{\infty} \frac{\left(-\frac{3}{22}\right)^k}{k}$$

$$\frac{1}{3} + \frac{1}{6} \log\left(\frac{1}{3} (2+3)^2\right) = \frac{1}{3} + \frac{1}{3} i\pi \left[\frac{\arg\left(\frac{25}{3} - x\right)}{2\pi} \right] + \frac{\log(x)}{6} - \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{25}{3} - x\right)^k x^{-k}}{k}$$

for $x < 0$

$$\frac{1}{3} + \frac{1}{6} \log\left(\frac{1}{3} (2+3)^2\right) = \frac{1}{3} + \frac{1}{6} \left[\frac{\arg\left(\frac{25}{3} - z_0\right)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) +$$

$$\frac{\log(z_0)}{6} + \frac{1}{6} \left[\frac{\arg\left(\frac{25}{3} - z_0\right)}{2\pi} \right] \log(z_0) - \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{25}{3} - z_0\right)^k z_0^{-k}}{k}$$

Integral representations:

$$\frac{1}{3} + \frac{1}{6} \log\left(\frac{1}{3} (2+3)^2\right) = \frac{1}{3} + \frac{1}{6} \int_1^{\frac{25}{3}} \frac{1}{t} dt$$

$$\frac{1}{3} + \frac{1}{6} \log\left(\frac{1}{3} (2+3)^2\right) = \frac{1}{3} - \frac{i}{12\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(\frac{3}{22}\right)^s \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

Inserting this entropy value in the Hawking radiation calculator, we obtain:

Surface area: 1.651799e-69

Mass: 7.721303e-9

Radius: 1.146499e-35

Temperature: 1.227203e+11

Entropy: 0.686710589366681842967

Practically, we have a very low entropy value!

This result can also be expressed as follows:

0.68671058936668184.....

Input:

$$\frac{-797 + 827 e - 338 e^2}{117 - 1324 e + 265 e^2}$$

Decimal approximation:

0.686710589366681843005715267802990503833913039596173167478...

0.686710589...

Property:

$$\frac{-797 + 827 e - 338 e^2}{117 - 1324 e + 265 e^2} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{797 - 827 e + 338 e^2}{-117 + 1324 e - 265 e^2}$$

$$\frac{(827 - 338 e) e - 797}{117 + e(265 e - 1324)}$$

$$-\frac{797 - 827 e + 338 e^2}{117 - 1324 e + 265 e^2}$$

Continued fraction:

$$\begin{array}{c}
 1 \\
 \hline
 1 + \frac{1}{\hline} \\
 2 + \frac{1}{\hline} \\
 5 + \frac{1}{\hline} \\
 4 + \frac{1}{\hline} \\
 1 + \frac{1}{\hline} \\
 3 + \frac{1}{\hline} \\
 5 + \frac{1}{\hline} \\
 1 + \frac{1}{\hline} \\
 1 + \frac{1}{\hline} \\
 7 + \frac{1}{\hline} \\
 1 + \frac{1}{\hline} \\
 1 + \frac{1}{\hline} \\
 1 + \frac{1}{\hline} \\
 4 + \frac{1}{\hline} \\
 1 + \frac{1}{\hline} \\
 4 + \frac{1}{\hline} \\
 3 + \frac{1}{\hline} \\
 1 + \frac{1}{\hline} \\
 3 + \frac{1}{\hline} \\
 3 + \frac{1}{\hline} \\
 4 + \frac{1}{\hline} \\
 4 + \frac{1}{\hline} \\
 1 + \frac{1}{\hline} \\
 \dots
 \end{array}$$

Alternative representation:

$$\frac{-797 + 827 e - 338 e^2}{117 - 1324 e + 265 e^2} = \frac{-797 + 827 \exp(z) - 338 \exp^2(z)}{117 - 1324 \exp(z) + 265 \exp^2(z)} \text{ for } z = 1$$

Series representations:

$$\frac{-797 + 827 e - 338 e^2}{117 - 1324 e + 265 e^2} = - \frac{-797 + 827 e - 338 \sum_{k=0}^{\infty} \frac{2^k}{k!}}{-117 + 1324 e - 265 \sum_{k=0}^{\infty} \frac{2^k}{k!}}$$

$$\frac{-797 + 827 e - 338 e^2}{117 - 1324 e + 265 e^2} = - \frac{797 - 827 \sum_{k=0}^{\infty} \frac{1}{k!} + 338 \left(\sum_{k=0}^{\infty} \frac{1}{k!} \right)^2}{117 - 1324 \sum_{k=0}^{\infty} \frac{1}{k!} + 265 \left(\sum_{k=0}^{\infty} \frac{1}{k!} \right)^2}$$

$$\frac{-797 + 827 e - 338 e^2}{117 - 1324 e + 265 e^2} = - \frac{338 - 827 \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} + 797 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \right)^2}{265 - 1324 \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} + 117 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \right)^2}$$

n! is the factorial function

Furthermore, the result is very near to the following Rogers-Ramanujan expression

$$\sqrt{\frac{e\pi}{2}} \operatorname{erfc}\left(\frac{\sqrt{2}}{2}\right) \approx 0.6556795424$$

If we insert instead of 2, $2 \cdot 0.937$, we obtain:

$$\operatorname{sqrt}((e \cdot \pi) / 2) \operatorname{erfc}((\operatorname{sqrt}(2 \cdot 0.937) / 2))$$

Input:

$$\sqrt{\frac{e\pi}{2}} \operatorname{erfc}\left(\frac{\sqrt{2 \times 0.937}}{2}\right)$$

$\operatorname{erfc}(x)$ is the complementary error function

Result:

0.688204...

0.688204...

Alternative representations:

$$\sqrt{\frac{e\pi}{2}} \operatorname{erfc}\left(\frac{\sqrt{2 \times 0.937}}{2}\right) = \left(1 - \operatorname{erf}\left(\frac{\sqrt{1.874}}{2}\right)\right) \sqrt{\frac{e\pi}{2}}$$

$$\sqrt{\frac{e\pi}{2}} \operatorname{erfc}\left(\frac{\sqrt{2 \times 0.937}}{2}\right) = \left(1 + i \operatorname{erfi}\left(\frac{i\sqrt{1.874}}{2}\right)\right) \sqrt{\frac{e\pi}{2}}$$

$$\sqrt{\frac{e\pi}{2}} \operatorname{erfc}\left(\frac{\sqrt{2 \times 0.937}}{2}\right) = \operatorname{erf}\left(\frac{\sqrt{1.874}}{2}, \infty\right) \sqrt{\frac{e\pi}{2}}$$

Series representations:

$$\sqrt{\frac{e\pi}{2}} \operatorname{erfc}\left(\frac{\sqrt{2 \times 0.937}}{2}\right) = \frac{\sqrt{-1 + \frac{e\pi}{2}} \left(\sum_{k=0}^{\infty} \left(-1 + \frac{e\pi}{2}\right)^{-k} \binom{\frac{1}{2}}{k} \right) \left(\sqrt{\pi} - 2 \sum_{k=0}^{\infty} \frac{(-1)^k 2^{-1-2k} \sqrt{1.874}^{1+2k}}{(1+2k)k!} \right)}{\sqrt{\pi}}$$

$$\sqrt{\frac{e\pi}{2}} \operatorname{erfc}\left(\frac{\sqrt{2 \times 0.937}}{2}\right) = \frac{1}{\sqrt{\pi}} \exp\left(i\pi \left\lfloor \frac{\arg\left(\frac{e\pi}{2} - x\right)}{2\pi} \right\rfloor\right) \sqrt{x} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{e\pi}{2} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \left(\sqrt{\pi} - 2 \sum_{k=0}^{\infty} \frac{(-1)^k 2^{-1-2k} \sqrt{1.874}^{1+2k}}{(1+2k)k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\sqrt{\frac{e\pi}{2}} \operatorname{erfc}\left(\frac{\sqrt{2 \times 0.937}}{2}\right) = \frac{1}{\sqrt{\pi}} \exp\left(i\pi \left\lfloor \frac{\arg\left(\frac{e\pi}{2} - x\right)}{2\pi} \right\rfloor\right) \sqrt{x} \left(\sqrt{\pi} - \sum_{k=0}^{\infty} \frac{(-1)^k 2^{-1/2-3k} H_{1+2k}\left(\frac{\sqrt{1.874}}{2}\right)}{(1+2k)k!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{e\pi}{2} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

Integral representations:

$$\sqrt{\frac{e\pi}{2}} \operatorname{erfc}\left(\frac{\sqrt{2 \times 0.937}}{2}\right) = \frac{2\sqrt{\frac{e\pi}{2}}}{\sqrt{\pi}} \int_{\frac{\sqrt{1.874}}{2}}^{\infty} \mathcal{A}^{-t^2} dt$$

$$\sqrt{\frac{e\pi}{2}} \operatorname{erfc}\left(\frac{\sqrt{2 \times 0.937}}{2}\right) = \sqrt{\frac{e\pi}{2}} - \frac{2\sqrt{\frac{e\pi}{2}}}{\pi} \int_0^{\infty} \frac{\mathcal{A}^{-t^2} \sin(t\sqrt{1.874})}{t} dt$$

$$\sqrt{\frac{e\pi}{2}} \operatorname{erfc}\left(\frac{\sqrt{2 \times 0.937}}{2}\right) = \frac{\sqrt{\frac{e\pi}{2}}}{2i\pi\sqrt{\pi}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{4^s \Gamma(s) \Gamma\left(\frac{1}{2} + s\right) \sqrt{1.874}^{-2s}}{\Gamma(1+s)} ds \text{ for } 0 < \gamma$$

$$\sqrt{\frac{e\pi}{2}} \operatorname{erfc}\left(\frac{\sqrt{2 \times 0.937}}{2}\right) = \sqrt{\frac{e\pi}{2}} - \frac{\sqrt{\frac{e\pi}{2}}}{2i\pi\sqrt{\pi}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{4^s \Gamma(-s) \Gamma\left(\frac{1}{2} + s\right) \sqrt{1.874}^{-2s}}{\Gamma(1-s)} ds$$

for $\gamma > -\frac{1}{2}$

Note that the result 0.688204 is very near to the value of generalized entropy 0.686710589366681842967....

From this above value of generalized entropy 0.686710589, we obtain

$$\text{Mass} = 7.721303e-9$$

$$\text{Radius} = 1.146499e-35$$

$$\text{Temperature} = 1.227203e+11$$

And from the Ramanujan-Nardelli mock formula, we obtain:

$$\sqrt{\left[\left[\left[\left[\left[\left[\frac{1}{\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{7.721303 \times 10^{-9}} \right] \sqrt{\left[- \left(\left(1.227203 \times 10^{11} \times 4 \pi (1.146499 \times 10^{-35})^3 - (1.146499 \times 10^{-35})^2 \right) \right) \right] / \left((6.67 \times 10^{-11}) \right) \right] \right] \right] \right] \right]$$

Input interpretation:

$$\sqrt{\left(1 / \left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{7.721303 \times 10^{-9}} \right) \sqrt{- \frac{1.227203 \times 10^{11} \times 4 \pi (1.146499 \times 10^{-35})^3 - (1.146499 \times 10^{-35})^2}{6.67 \times 10^{-11}}} \right)}$$

Result:

1.61732...

1.61732...

and:

$$1/\sqrt{\left[\left[\left[\left[\left[\frac{1}{\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \right) \times \frac{1}{7.721303 \times 10^{-9}} \right] \sqrt{\left[- \left(\left(1.227203 \times 10^{11} \times 4 \pi (1.146499 \times 10^{-35})^3 - (1.146499 \times 10^{-35})^2 \right) \right) \right] / \left((6.67 \times 10^{-11}) \right) \right] \right] \right] \right] \right]$$

Input interpretation:

$$1 / \left(\sqrt[3]{ \left(\sqrt[3]{ \left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{7.721303 \times 10^{-9}} \right. \right. \right. \\ \left. \left. \left. \sqrt{-\frac{1.227203 \times 10^{11} \times 4\pi (1.146499 \times 10^{-35})^3 - (1.146499 \times 10^{-35})^2}{6.67 \times 10^{-11}}} \right) \right) \right)}$$

Result:

0.618306...

0.618306...

Now, we have that:

$$S_{\text{fermions}}(I \cup R) = \frac{c}{3} \log \left[\frac{2 \cosh t_a \cosh t_b |\cosh(t_a - t_b) - \cosh(a + b)|}{\sinh a \cosh\left(\frac{a+b-t_a-t_b}{2}\right) \cosh\left(\frac{a+b+t_a+t_b}{2}\right)} \right] \quad (5.7)$$

$$S_{\text{matter}}(I \cup R) \approx 2S_{\text{matter}}([P_1, P_2]) - \frac{c}{3} \log \left(\frac{2|\cosh(a + b) - \cosh(t_a - t_b)|}{\sinh a} \right) . \quad (5.13)$$

For $\beta = 2\pi$, $a = 3$, $b = 2$ and $t_a = 8$ $t_b = 5$ and $c = 1$, we obtain:

$$\frac{1}{3} \ln \left(\frac{2(\cosh 8 \cosh 5) (\cosh(8-5) - \cosh(3+2))}{\sinh 3 \cosh\left(\frac{3+2-8-5}{2}\right) \cosh\left(\frac{3+2+8+5}{2}\right)} \right)$$

Input:

$$\frac{1}{3} \log \left(\frac{2 (\cosh(8) \cosh(5)) (\cosh(8 - 5) - \cosh(3 + 2))}{\sinh(3) \cosh\left(\frac{1}{2} (3 + 2 - 8 - 5)\right) \cosh\left(\frac{1}{2} (3 + 2 + 8 + 5)\right)} \right)$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{1}{3} (\log(-2 (\cosh(3) - \cosh(5)) \cosh(5) \cosh(8) \operatorname{csch}(3) \operatorname{sech}(4) \operatorname{sech}(9)) + i \pi)$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

$\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

$$0.84986337432782627143532208812886523171519615427360533536... + 1.0471975511965977461542144610931676280657231331250352736... i$$

Polar coordinates:

$$r \approx 1.34866 \text{ (radius), } \theta \approx 50.9386^\circ \text{ (angle)}$$

$$1.34866$$

Alternate forms:

$$\frac{1}{3} (\log(2 \cosh(5) (\cosh(5) - \cosh(3)) \cosh(8) \operatorname{csch}(3) \operatorname{sech}(4) \operatorname{sech}(9)) + i \pi)$$

$$\frac{1}{3} i (\pi - i \log(-2 (\cosh(3) - \cosh(5)) \cosh(5) \cosh(8) \operatorname{csch}(3) \operatorname{sech}(4) \operatorname{sech}(9)))$$

$$\frac{1}{3} \log(-2 (\cosh(3) - \cosh(5)) \cosh(5) \cosh(8) \operatorname{csch}(3) \operatorname{sech}(4) \operatorname{sech}(9)) + \frac{i \pi}{3}$$

Alternative representations:

$$\frac{1}{3} \log \left(\frac{2 (\cosh(8) \cosh(5)) (\cosh(8 - 5) - \cosh(3 + 2))}{\sinh(3) \cosh\left(\frac{1}{2} (3 + 2 - 8 - 5)\right) \cosh\left(\frac{1}{2} (3 + 2 + 8 + 5)\right)} \right) = \frac{1}{3} \log_e \left(\frac{2 (\cosh(3) - \cosh(5)) \cosh(5) \cosh(8)}{\cosh(-4) \cosh(9) \sinh(3)} \right)$$

$$\frac{1}{3} \log \left(\frac{2 (\cosh(8) \cosh(5)) (\cosh(8 - 5) - \cosh(3 + 2))}{\sinh(3) \cosh\left(\frac{1}{2} (3 + 2 - 8 - 5)\right) \cosh\left(\frac{1}{2} (3 + 2 + 8 + 5)\right)} \right) = \frac{1}{3} \log(a) \log_a \left(\frac{2 (\cosh(3) - \cosh(5)) \cosh(5) \cosh(8)}{\cosh(-4) \cosh(9) \sinh(3)} \right)$$

$$\frac{1}{3} \log \left(\frac{2 (\cosh(8) \cosh(5)) (\cosh(8 - 5) - \cosh(3 + 2))}{\sinh(3) \cosh\left(\frac{1}{2} (3 + 2 - 8 - 5)\right) \cosh\left(\frac{1}{2} (3 + 2 + 8 + 5)\right)} \right) = \frac{1}{3} \log \left(\frac{2 (\cos(-3 i) - \cos(-5 i)) \cos(-5 i) \cos(-8 i)}{\frac{1}{2} \cos(4 i) \cos(-9 i) \left(-\frac{1}{e^3} + e^3\right)} \right)$$

Series representation:

$$\frac{1}{3} \log \left(\frac{2 (\cosh(8) \cosh(5)) (\cosh(8 - 5) - \cosh(3 + 2))}{\sinh(3) \cosh\left(\frac{1}{2} (3 + 2 - 8 - 5)\right) \cosh\left(\frac{1}{2} (3 + 2 + 8 + 5)\right)} \right) =$$

$$\frac{i\pi}{3} + \frac{1}{3} \log(-1 - 2 (\cosh(3) - \cosh(5)) \cosh(5) \cosh(8) \operatorname{csch}(3) \operatorname{sech}(4) \operatorname{sech}(9)) -$$

$$\frac{1}{3} \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{-1 - 2 (\cosh(3) - \cosh(5)) \cosh(5) \cosh(8) \operatorname{csch}(3) \operatorname{sech}(4) \operatorname{sech}(9)} \right)^k}{k}$$

Integral representations:

$$\frac{1}{3} \log \left(\frac{2 (\cosh(8) \cosh(5)) (\cosh(8 - 5) - \cosh(3 + 2))}{\sinh(3) \cosh\left(\frac{1}{2} (3 + 2 - 8 - 5)\right) \cosh\left(\frac{1}{2} (3 + 2 + 8 + 5)\right)} \right) =$$

$$\frac{i\pi}{3} + \frac{1}{3} \int_1^{-2 (\cosh(3) - \cosh(5)) \cosh(5) \cosh(8) \operatorname{csch}(3) \operatorname{sech}(4) \operatorname{sech}(9)} \frac{1}{t} dt$$

$$\frac{1}{3} \log \left(\frac{2 (\cosh(8) \cosh(5)) (\cosh(8 - 5) - \cosh(3 + 2))}{\sinh(3) \cosh\left(\frac{1}{2} (3 + 2 - 8 - 5)\right) \cosh\left(\frac{1}{2} (3 + 2 + 8 + 5)\right)} \right) =$$

$$\frac{i\pi}{3} - \frac{i}{6\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{\Gamma(1-s)} \Gamma(-s)^2 \Gamma(1+s)$$

$$(-1 - 2 (\cosh(3) - \cosh(5)) \cosh(5) \cosh(8) \operatorname{csch}(3) \operatorname{sech}(4) \operatorname{sech}(9))^{-s}$$

$$ds \text{ for } -1 < \gamma < 0$$

We have that:

$$8 * \left(\left(\frac{1}{3} \ln \left(\frac{2 (\cosh(8) \cosh(5)) (\cosh(8 - 5) - \cosh(3 + 2))}{\sinh(3) \cosh\left(\frac{1}{2} (3 + 2 - 8 - 5)\right) \cosh\left(\frac{1}{2} (3 + 2 + 8 + 5)\right)} \right) \right) \right)^{16}$$

Input:

$$8 \left(\frac{1}{3} \log \left(\frac{2 (\cosh(8) \cosh(5)) (\cosh(8 - 5) - \cosh(3 + 2))}{\sinh(3) \cosh\left(\frac{1}{2} (3 + 2 - 8 - 5)\right) \cosh\left(\frac{1}{2} (3 + 2 + 8 + 5)\right)} \right) \right)^{16}$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{8 (\log(-2 (\cosh(3) - \cosh(5)) \cosh(5) \cosh(8) \operatorname{csch}(3) \operatorname{sech}(4) \operatorname{sech}(9)) + i \pi)^{16}}{43\,046\,721}$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

$\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

$$-83.824214837761897634075953385479942107959074931812014999\dots + 954.71737063413632987588750442525000930592145279690083453\dots i$$

Polar coordinates:

$$r \approx 958.39 \text{ (radius)}, \quad \theta \approx 95.0177^\circ \text{ (angle)}$$

958.39

Alternate forms:

$$\frac{8 (\pi - i \log(-2 (\cosh(3) - \cosh(5)) \cosh(5) \cosh(8) \operatorname{csch}(3) \operatorname{sech}(4) \operatorname{sech}(9)))^{16}}{43\,046\,721}$$

$$\frac{1}{43\,046\,721}$$

$$8 (i \pi + \log(\operatorname{sech}(4)) + \log(\operatorname{sech}(9)) + \log(-2 (\cosh(3) - \cosh(5))) + \log(\cosh(5)) + \log(\cosh(8)) + \log(\operatorname{csch}(3)))^{16}$$

$$\frac{8 \left(\log \left(- \frac{4 \left(\frac{1}{e^5} + e^5 \right) \left(\frac{1}{e^8} + e^8 \right) \left(\frac{1}{2} \left(\frac{1}{e^3} + e^3 \right) + \frac{1}{2} \left(-\frac{1}{e^5} - e^5 \right) \right)}{\left(e^3 - \frac{1}{e^3} \right) \left(\frac{1}{e^4} + e^4 \right) \left(\frac{1}{e^9} + e^9 \right)} \right) + i \pi \right)^{16}}{43\,046\,721}$$

Alternative representations:

$$8 \left(\frac{1}{3} \log \left(\frac{2 (\cosh(8) \cosh(5)) (\cosh(8 - 5) - \cosh(3 + 2))}{\sinh(3) \cosh\left(\frac{1}{2} (3 + 2 - 8 - 5)\right) \cosh\left(\frac{1}{2} (3 + 2 + 8 + 5)\right)} \right) \right)^{16} = 8 \left(\frac{1}{3} \log_e \left(\frac{2 (\cosh(3) - \cosh(5)) \cosh(5) \cosh(8)}{\cosh(-4) \cosh(9) \sinh(3)} \right) \right)^{16}$$

$$8 \left(\frac{1}{3} \log \left(\frac{2 (\cosh(8) \cosh(5)) (\cosh(8 - 5) - \cosh(3 + 2))}{\sinh(3) \cosh\left(\frac{1}{2} (3 + 2 - 8 - 5)\right) \cosh\left(\frac{1}{2} (3 + 2 + 8 + 5)\right)} \right) \right)^{16} = 8 \left(\frac{1}{3} \log(a) \log_a \left(\frac{2 (\cosh(3) - \cosh(5)) \cosh(5) \cosh(8)}{\cosh(-4) \cosh(9) \sinh(3)} \right) \right)^{16}$$

$$8 \left(\frac{1}{3} \log \left(\frac{2 (\cosh(8) \cosh(5)) (\cosh(8-5) - \cosh(3+2))}{\sinh(3) \cosh\left(\frac{1}{2} (3+2-8-5)\right) \cosh\left(\frac{1}{2} (3+2+8+5)\right)} \right) \right)^{16} =$$

$$8 \left(\frac{1}{3} \log \left(\frac{2 (\cos(-3i) - \cos(-5i)) \cos(-5i) \cos(-8i)}{\frac{1}{2} \cos(4i) \cos(-9i) \left(-\frac{1}{e^3} + e^3\right)} \right) \right)^{16}$$

Series representation:

$$8 \left(\frac{1}{3} \log \left(\frac{2 (\cosh(8) \cosh(5)) (\cosh(8-5) - \cosh(3+2))}{\sinh(3) \cosh\left(\frac{1}{2} (3+2-8-5)\right) \cosh\left(\frac{1}{2} (3+2+8+5)\right)} \right) \right)^{16} = \frac{1}{43046721}$$

$$8 \left(i\pi + \log(-1 - 2 (\cosh(3) - \cosh(5)) \cosh(5) \cosh(8) \operatorname{csch}(3) \operatorname{sech}(4) \operatorname{sech}(9)) - \sum_{k=1}^{\infty} \left(\frac{-1 - 2 (\cosh(3) - \cosh(5)) \cosh(5) \cosh(8) \operatorname{csch}(3) \operatorname{sech}(4) \operatorname{sech}(9)}{k} \right)^k \right)^{16}$$

From the result, we obtain:

$$(-83.824214837 + 954.717370634i) - 21i + (1/\text{golden ratio})i$$

Input interpretation:

$$(-83.824214837 + 954.717370634i) - 21i + \frac{1}{\phi} i$$

i is the imaginary unit
 ϕ is the golden ratio

Result:

$$-83.824214837 + 934.335404623... i$$

Polar coordinates:

$$r = 938.08802749 \text{ (radius)}, \quad \theta = 95.1265851558^\circ \text{ (angle)}$$

938.08802749 result practically equal to the proton mass in MeV

Alternative representations:

$$(-83.8242148370000 + 954.7173706340000 i) - i 21 + \frac{i}{\phi} =$$

$$-83.8242148370000 + 933.7173706340000 i + \frac{i}{2 \sin(54^\circ)}$$

$$(-83.8242148370000 + 954.7173706340000 i) - i 21 + \frac{i}{\phi} =$$

$$-83.8242148370000 + 933.7173706340000 i + -\frac{i}{2 \cos(216^\circ)}$$

$$(-83.8242148370000 + 954.7173706340000 i) - i 21 + \frac{i}{\phi} =$$

$$-83.8242148370000 + 933.7173706340000 i + -\frac{i}{2 \sin(666^\circ)}$$

$$\frac{1}{3} \ln((2 * \cosh(3+2) - \cosh(8-5)) / ((\sinh(3))))$$

Input:

$$\frac{1}{3} \log\left(\frac{2 \cosh(3 + 2) - \cosh(8 - 5)}{\sinh(3)}\right)$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{1}{3} \log((2 \cosh(5) - \cosh(3)) \operatorname{csch}(3))$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

Decimal approximation:

0.875143957033453614479519096149462979703497629170650583036...

0.875143957...

Alternate forms:

$$\frac{1}{3} \log(2 \cosh(5) \operatorname{csch}(3) - \operatorname{coth}(3))$$

$$\frac{1}{3} (\log(2 \cosh(5) - \cosh(3)) + \log(\operatorname{csch}(3)))$$

$$\frac{1}{3} (-2 - \log(e^6 - 1) + \log(2 - e^2 - e^8 + 2 e^{10}))$$

$\operatorname{coth}(x)$ is the hyperbolic cotangent function

Alternative representations:

$$\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right) = \frac{1}{3} \log_e\left(\frac{-\cosh(3) + 2 \cosh(5)}{\sinh(3)}\right)$$

$$\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right) = \frac{1}{3} \log(a) \log_a\left(\frac{-\cosh(3) + 2 \cosh(5)}{\sinh(3)}\right)$$

$$\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right) = \frac{1}{3} \log\left(\frac{\frac{1}{e^5} + \frac{1}{2}\left(-\frac{1}{e^3} - e^3\right) + e^5}{\frac{1}{2}\left(-\frac{1}{e^3} + e^3\right)}\right)$$

Series representations:

$$\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right) =$$

$$\frac{1}{3} \log(-1 - \coth(3) + 2 \cosh(5) \operatorname{csch}(3)) - \frac{1}{3} \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1+\coth(3)-2 \cosh(5) \operatorname{csch}(3)}\right)^k}{k}$$

$$\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right) =$$

$$\frac{1}{3} \log(-1 + (-\cosh(3) + 2 \cosh(5)) \operatorname{csch}(3)) - \frac{1}{3} \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1+\coth(3)-2 \cosh(5) \operatorname{csch}(3)}\right)^k}{k}$$

Integral representations:

$$\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right) = \frac{1}{3} \int_1^{-\coth(3)+2 \cosh(5) \operatorname{csch}(3)} \frac{1}{t} dt$$

$$\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right) =$$

$$-\frac{i}{6\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1 - \coth(3) + 2 \cosh(5) \operatorname{csch}(3))^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

Note that the result [0.875143957...](#) is very near to the following second 7th order Ramanujan mock theta function value:

$$\left(\frac{0.449329}{(1-0.449329)} + \frac{(0.449329)^4}{((1-0.449329^2)(1-0.449329^3))}\right) + \left(\frac{(0.449329)^9}{((1-0.449329^3)(1-0.449329^4)(1-0.449329^5))}\right)$$

Input interpretation:

$$\frac{\left(\frac{0.449329}{1 - 0.449329} + \frac{0.449329^4}{(1 - 0.449329^2)(1 - 0.449329^3)} \right) + \frac{0.449329^9}{(1 - 0.449329^3)(1 - 0.449329^4)(1 - 0.449329^5)}}{0.449329^9}$$

Result:

0.873007700790297068938379062120625965241700531051591249067...
[0.8730077...](#)

We have also:

$$1/6 * 11 * (((1/3 \ln((2 * \cosh(3+2) - \cosh(8-5)) / (\sinh(3)))))) + (11+3)/10^3$$

Input:

$$\frac{1}{6} \times 11 \left(\frac{1}{3} \log \left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)} \right) \right) + \frac{11+3}{10^3}$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{7}{500} + \frac{11}{18} \log((2 \cosh(5) - \cosh(3)) \operatorname{csch}(3))$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

Decimal approximation:

1.618430587894664959879118342940682129456412320146192735566...

[1.6184305878...](#) result that is a very good approximation to the value of the golden ratio [1,618033988749...](#)

Alternate forms:

$$\frac{7}{500} + \frac{11}{18} \log(2 \cosh(5) \operatorname{csch}(3) - \operatorname{coth}(3))$$

$$\frac{63 + 2750 \log((2 \cosh(5) - \cosh(3)) \operatorname{csch}(3))}{4500}$$

$$\frac{7}{500} + \frac{11}{18} (\log(2 \cosh(5) - \cosh(3)) + \log(\operatorname{csch}(3)))$$

$\coth(x)$ is the hyperbolic cotangent function

Alternative representations:

$$\frac{11 \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)}{3 \times 6} + \frac{11+3}{10^3} = \frac{11 \log_e\left(\frac{-\cosh(3)+2 \cosh(5)}{\sinh(3)}\right)}{3 \times 6} + \frac{14}{10^3}$$

$$\frac{11 \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)}{3 \times 6} + \frac{11+3}{10^3} = \frac{11 \log(a) \log_a\left(\frac{-\cosh(3)+2 \cosh(5)}{\sinh(3)}\right)}{3 \times 6} + \frac{14}{10^3}$$

$$\frac{11 \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)}{3 \times 6} + \frac{11+3}{10^3} = \frac{11 \log\left(\frac{\frac{1}{e^5} + \frac{1}{2} \left(-\frac{1}{e^3} - e^3\right) + e^5}{\frac{1}{2} \left(-\frac{1}{e^3} + e^3\right)}\right)}{3 \times 6} + \frac{14}{10^3}$$

Series representations:

$$\frac{11 \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)}{3 \times 6} + \frac{11+3}{10^3} = \frac{7}{500} + \frac{11}{18} \log(-1 - \coth(3) + 2 \cosh(5) \operatorname{csch}(3)) - \frac{11}{18} \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1 + \coth(3) - 2 \cosh(5) \operatorname{csch}(3)}\right)^k}{k}$$

$$\frac{11 \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)}{3 \times 6} + \frac{11+3}{10^3} = \frac{7}{500} + \frac{11}{18} \log(-1 + (-\cosh(3) + 2 \cosh(5)) \operatorname{csch}(3)) - \frac{11}{18} \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1 + \coth(3) - 2 \cosh(5) \operatorname{csch}(3)}\right)^k}{k}$$

Integral representations:

$$\frac{11 \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)}{3 \times 6} + \frac{11+3}{10^3} = \frac{7}{500} + \frac{11}{18} \int_1^{-\coth(3)+2 \cosh(5) \operatorname{csch}(3)} \frac{1}{t} dt$$

$$\frac{11 \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)}{3 \times 6} + \frac{11+3}{10^3} = \frac{7}{500} - \frac{11 i}{36 \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{(-1 - \coth(3) + 2 \cosh(5) \operatorname{csch}(3))^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$$\frac{11 \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)}{3 \times 6} + \frac{11+3}{10^3} =$$

$$\frac{7}{500} - \frac{11i}{36\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1 + (-\cosh(3) + 2 \cosh(5)) \operatorname{csch}(3))^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

From the inversion of previous expression, we obtain:

$$1/\left(\left(\left(\frac{1}{3} \ln\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)\right)\right)\right)$$

Input:

$$\frac{1}{3 \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)}$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{1}{3 \log(2 \cosh(5) - \cosh(3)) \operatorname{csch}(3)}$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

Decimal approximation:

1.142669148273366397519468167771273089986708880034033515463...

[1.14266914...](#)

Alternate forms:

$$\frac{1}{3 \log(2 \cosh(5) \operatorname{csch}(3) - \operatorname{coth}(3))}$$

$$\frac{1}{3 \log(2 \cosh(5) - \cosh(3)) + \log(\operatorname{csch}(3))}$$

$$-\frac{1}{2 + \log\left(\frac{e^6 - 1}{2 - e^{-2} - e^{-8} + 2e^{10}}\right)}$$

$\operatorname{coth}(x)$ is the hyperbolic cotangent function

Alternative representations:

$$\frac{1}{\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)} = \frac{1}{\frac{1}{3} \log_e\left(\frac{-\cosh(3)+2 \cosh(5)}{\sinh(3)}\right)}$$

$$\frac{1}{\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)} = \frac{1}{\frac{1}{3} \log(a) \log_a\left(\frac{-\cosh(3)+2 \cosh(5)}{\sinh(3)}\right)}$$

$$\frac{1}{\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)} = \frac{1}{\frac{1}{3} \log\left(\frac{\frac{1}{e^5} + \frac{1}{2} \left(-\frac{1}{e^3} - e^3\right) + e^5}{\frac{1}{2} \left(-\frac{1}{e^3} + e^3\right)}\right)}$$

Series representations:

$$\frac{1}{\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)} = \frac{3}{\log(-1 - \coth(3) + 2 \cosh(5) \operatorname{csch}(3)) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1 + \coth(3) - 2 \cosh(5) \operatorname{csch}(3)}\right)^k}{k}}$$

$$\frac{1}{\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)} = \frac{3}{\log(-1 + (-\cosh(3) + 2 \cosh(5)) \operatorname{csch}(3)) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1 + \coth(3) - 2 \cosh(5) \operatorname{csch}(3)}\right)^k}{k}}$$

Integral representations:

$$\frac{1}{\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)} = \frac{3}{\int_1^{-\coth(3)+2 \cosh(5) \operatorname{csch}(3)} \frac{1}{t} dt}$$

$$\frac{1}{\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)} = \frac{6 i \pi}{\int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{(-1 - \coth(3) + 2 \cosh(5) \operatorname{csch}(3))^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}$$

for $-1 < \gamma < 0$

Note that the result [1.1426691482733...](#) is almost equal to the following first 5th order Ramanujan mock theta function value:

$$\frac{(((1+(0.449329)^2/(1+0.449329) + (0.449329)^6 / ((1+0.449329)+(1+0.449329^2)))) + (((0.449329)^{12} / ((1+0.449329)(1+0.449329^2)(1+0.449329^3)))))$$

Input interpretation:

$$\frac{\left(1 + \frac{0.449329^2}{1 + 0.449329} + \frac{0.449329^6}{(1 + 0.449329) + (1 + 0.449329^2)}\right) + \frac{0.449329^{12}}{(1 + 0.449329)(1 + 0.449329^2)(1 + 0.449329^3)}}$$

Result:

1.142443242201380904097917635488946328383797361320962332093...
[f\(q\) = 1.1424432422...](#)

We have also:

$$1/10^{27}(((1+1/\sqrt{((((1/((1/3 \ln((2*\cosh(3+2)-\cosh(8-5)))/((\sinh 3))))))))))^{(2\pi)))+(13+2)/10^3))$$

Input:

$$\frac{1}{10^{27}} \left(1 + \frac{1}{\sqrt{\left(\frac{1}{\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)\right)^{2\pi}}} + \frac{13+2}{10^3} \right)$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

log(x) is the natural logarithm

Exact result:

$$\frac{\frac{203}{200} + \left(\frac{1}{3} \log((2 \cosh(5) - \cosh(3)) \operatorname{csch}(3))\right)^\pi}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

csch(x) is the hyperbolic cosecant function

Decimal approximation:

1.6727144407468413501328914758068737665473280418445729... × 10⁻²⁷

1.672714440746... * 10⁻²⁷ result practically equal to the proton mass in kg

Alternate forms:

$$\frac{\frac{203}{200} + \left(\frac{1}{3} \log(2 \cosh(5) \operatorname{csch}(3) - \operatorname{coth}(3))\right)^\pi}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{203 + 200 \left(\frac{1}{3} \log((2 \cosh(5) - \cosh(3)) \operatorname{csch}(3))\right)^\pi}{200\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{203}{200\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{\left(\frac{1}{3} \log((2 \cosh(5) - \cosh(3)) \operatorname{csch}(3))\right)^\pi}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

coth(x) is the hyperbolic cotangent function

Alternative representations:

$$1 + \frac{1}{\sqrt{\left(\frac{1}{\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)}\right)^{2\pi}} + \frac{13+2}{10^3} = 1 + \frac{15}{10^3} + \frac{1}{\sqrt{\left(\frac{1}{\frac{1}{3} \log_e\left(\frac{-\cosh(3)+2 \cosh(5)}{\sinh(3)}\right)}\right)^{2\pi}}}$$

$$1 + \frac{1}{\sqrt{\left(\frac{1}{\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)}\right)^{2\pi}} + \frac{13+2}{10^3} = 1 + \frac{15}{10^3} + \frac{1}{\sqrt{\left(\frac{1}{\frac{1}{3} \log(a) \log_a\left(\frac{-\cosh(3)+2 \cosh(5)}{\sinh(3)}\right)}\right)^{2\pi}}}$$

$$1 + \frac{1}{\sqrt{\left(\frac{1}{\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)}\right)^{2\pi}} + \frac{13+2}{10^3} = 1 + \frac{15}{10^3} + \frac{1}{\sqrt{\left(\frac{1}{\frac{1}{3} \log\left(\frac{-\cos(-3i)+2 \cos(-5i)}{\frac{1}{2}\left(-\frac{1}{e^3}+e^3\right)}\right)}\right)^{2\pi}}}$$

Series representations:

$$1 + \frac{1}{\sqrt{\left(\frac{1}{\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)\right)^{2\pi}}} + \frac{13+2}{10^3}$$

$$= \frac{203}{200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} +$$

$$\frac{3^{-\pi} \left(\log(-1 - \coth(3) + 2 \cosh(5) \operatorname{csch}(3)) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1 + \coth(3) - 2 \cosh(5) \operatorname{csch}(3)}\right)^k}{k} \right)^{\pi}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$1 + \frac{1}{\sqrt{\left(\frac{1}{\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)\right)^{2\pi}}} + \frac{13+2}{10^3}$$

$$= \frac{203}{200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} +$$

$$\frac{3^{-\pi} \left(\log(-1 + (-\cosh(3) + 2 \cosh(5)) \operatorname{csch}(3)) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1 + \coth(3) - 2 \cosh(5) \operatorname{csch}(3)}\right)^k}{k} \right)^{\pi}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

Integral representations:

$$1 + \frac{1}{\sqrt{\left(\frac{1}{\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)\right)^{2\pi}}} + \frac{13+2}{10^3}$$

$$= \frac{203}{200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{3^{-\pi} \left(\int_1^{-\coth(3)+2 \cosh(5) \operatorname{csch}(3)} \frac{1}{t} dt \right)^{\pi}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$1 + \frac{1}{\sqrt{\left(\frac{1}{\frac{1}{3} \log\left(\frac{2 \cosh(3+2) - \cosh(8-5)}{\sinh(3)}\right)\right)^{2\pi}}} + \frac{13+2}{10^3}$$

$$= \frac{203}{200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{2^{-27-\pi} (3\pi)^{-\pi} \left(-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1 - \coth(3) + 2 \cosh(5) \operatorname{csch}(3))^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^{\pi}}{7\,450\,580\,596\,923\,828\,125} \text{ for } -1 < \gamma < 0$$

Conclusions

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π , ϕ , $1/\phi$, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

References

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Replica Wormholes and the Entropy of Hawking Radiation

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