Neutrosophic Nano Aψ Closed Sets In Neutrosophic Nano Topological Spaces

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Abstract- In this article we introduce the notation of neutrosophic nano semi closed, neutrosophic nano α closed, neutrosophic nano pre closed, neutrosophic nano semi pre closed and neutrosophic nano regular closed and investigate some of their properties. Further we study the concept of neutrosophic nano sg closed, neutrosophic nano ψ closed and neutrosophic nano αψ closed and derive some of their properties.

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Keywords and Phrases: neutrosophic nano semi closed, neutrosophic nano α closed, neutrosophic nano pre closed, neutrosophic nano semi pre closed, neutrosophic nano regular closed, neutrosophic nano sg closed, neutrosophic nano ψ closed and neutrosophic nano αψ closed.

1. Introduction

The nation of α closed sets in topological spaces was introduced by O.Njastad[1]. M.K.R.S.Veerakumar [2] was explored the notion of ψ closed. The new concept of αψ closed set in topology was introduced by R.Devi et.al [3]. Nanotopology was introduced by LellisThivagar et.al [4]. It contains approximations and boundary region. The open set contains only five set that is empty, universe, Lower and upper approximation, boundary region. Fuzzy and intuitionistic fuzzy were introduced by Zadeh [5] and K.Atanassav [6]. The new theory neutrosophic set described by membership, indeterminacy and non-membership were introduced by Smarandache [7]. The neutrosophic set in (X, τ_N)is having the form $S=\{<x,M_S(x),I_S(x),N_S(x)>: x\in(X,\tau_N)\}$, where the functions $M_S:S\to[0,1]$, $I_S:S\to[0,1]$, $N_S:S\to[0,1]$ denoted the degree of membership, indeterminacy, degree of non-membership. The neutrosophic set $S=\{<x,M_S(x),I_S(x),N_S(x)>: x\in(X,\tau_N)\}$ is called a subset of $T=\{<x,M_T(x),I_T(x),N_T(x)>: x\in(X,\tau_N)\}$ [in short S ⊂ T] if degree of membership and indeterminacy is minimum in S and degree of non-membership is maximum in S or degree of membership is minimum and degree of non-membership and indeterminacy is maximum in S. The complement on NTS $S=\{<x,M_S(x),I_S(x),N_S(x)>: x\in(X,\tau_N)\}$ is $S^C=\{<x,N_S(x),I_S(x),M_S(x)>: x\in(X,\tau_N)\}$. Parimala et.al [8] introduced and studied the concept of neutrosophic αψ-closed sets.

Now LellisThivagar et.al [9] explored a new concept of neutrosophic nano topology. In that paper he discussed about neutrosophic nano interior and neutrosophic nano closure.

In this paper, basic properties of neutrosophic nano semi closed, neutrosophic nano α closed, neutrosophic nano pre closed, neutrosophic nano semi pre closed and neutrosophic nano regular closed were introduced. It also established the notion of neutrosophic nano sg closed, neutrosophic nano ψ closed and neutrosophic nano αψ closed. Further, studied some of their related attributes were discussed.

2. Preliminaries

This section shows that some related definition and properties.

Definition 2.1.[4] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Let X is

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a subset of U, then the lower approximation of X with respect to R is is denoted by $\underline{R} = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where R(X) denotes the equivalence class determined by $x \in U$.

Definition 2.2. [4] The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and its is denoted by $\overline{R} = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$.

Definition 2.3. [4] The boundary region of X with respect to R is the set of all objects, which can be possibly classified neither as X nor as not X with respect to R and its is denoted by $B_R = \overline{R} - \underline{R}$.

Definition 2.4. [4]If (U, R) is an approximation space and X, Y ⊆ U. Then

1. $\underline{R} \subseteq X \subseteq \overline{R}$
2. $\underline{R}(\phi) = \overline{R}(\phi) = \phi$ and $\underline{R}(U) = \overline{R}(U) = U$
3. $\overline{R}(X \cup Y) = \overline{R}(X) \cup \overline{R}(Y)$
4. $\overline{R}(X \cap Y) = \overline{R}(X) \cap \overline{R}(Y)$
5. $\underline{R}(X \cup Y) = \underline{R}(X) \cup \underline{R}(Y)$
6. $\underline{R}(X \cap Y) = \underline{R}(X) \cap \underline{R}(Y)$
7. $\overline{R}(X) \subseteq \overline{R}(Y)$ and $\underline{R}(X) \subseteq \underline{R}(Y)$ whenever $X \subseteq Y$
8. $\overline{R}(X^c) = (\underline{R})^c$ and $\underline{R}(X^c) = (\overline{R})^c$
9. $\underline{R}(\underline{R}) = \overline{R}(\underline{R}) = \underline{R}$
10. $\overline{R}(\overline{R}) = \underline{R}(\overline{R}) = \overline{R}$

Definition 2.5. [9] Let U be an universe and R be an equivalence relation on U

and Let S be a neutrosophic subset of U. Then the neutrosophic nano topology

is defined by $\tau_N(S) = \{0_N, 1_N, \underline{N}(S), \overline{N}(S), B_N(S)\}$, where

1. $\underline{N}(S) = \{\langle y, M_{\underline{R}(y)}, I_{\underline{R}(y)}, N_{\underline{R}(y)} \rangle / z \in [y]_R, y \in U\}$

2. $\overline{N}(S) = \{\langle y, M_{\overline{R}(y)}, I_{\overline{R}(y)}, N_{\overline{R}(y)} \rangle / z \in [y]_R, y \in U\}$

3. $B_N(S) = \overline{N} - \underline{N}$

Wher $M_{\underline{R}(y)} = \wedge_{z \in [y]_R} M_S(z), I_{\underline{R}(y)} = \wedge_{z \in [y]_R} I_S(z), N_{\underline{R}(y)} = \vee_{z \in [y]_R} N_S(z),$ $M_{\overline{R}(y)} = \vee_{z \in [y]_R} M_S(z), I_{\overline{R}(y)} = \vee_{z \in [y]_R} I_S(z), N_{\overline{R}(y)} = \wedge_{z \in [y]_R} N_S(z)$.

Definition 2.6. [9] Let (U, $\tau_N(S)$) be a neutrosophic nano topological spaces,

where S ⊆ U. Assume S and T be neutrosophic subset of U. Then the following

hold:

1. S ⊆ $N_N$cl(S).

2. S is neutrosophic nano closed iff$N_N$cl(S) = S.

3. $N_N$cl(0_N) = 0_N and $N_N$cl(1_N) = 1.

4. S $\subseteq$ T implies $N_N$cl(S) $\subseteq$ $N_N$cl(T).

5. $N_N$cl(S $\bigcup$ T) = $N_N$cl(S) $\bigcup$ $N_N$cl(T).

6. $N_N$cl(S $\bigcap$ T) $\subseteq$ $N_N$cl(S) $\bigcap$ $N_N$cl(T).

7. $N_N$cl($N_N$cl(A)) = $N_N$cl(S).

Definition 2.7. [9] Let (U, $\tau_N(S)$) be a neutrosophic nano topological spaces, where S $\subseteq$ U. Assume S and T be neutrosophic subset of U. Then the following hold:

1. $N_N$int(S) $\subseteq$ S.

2. S is neutrosophic nano open iff $N_N$int(S) = S.

3. $N_N$int(0_N) = 0_N and $N_N$int(1_N) = 1_N.

4. S $\subseteq$ T implies $N_N$int(S) $\subseteq$ $N_N$int(T).

5. $N_N$int(S) $\bigcup$ $N_N$int(T) $\subseteq$ $N_N$int(S [T).

6. $N_N$int(S $\bigcap$ T) = $N_N$int(S) $\bigcap$ $N_N$int(T).

7. $N_N$int($N_N$int(A)) = $N_N$int(S).

Definition 2.8. [10] Let (X, τ_N) be a non-empty fixed set. A neutrosophic set A is an object having the form $S=\{<x, M_S(x), I_S(x), N_S(x)>: x\in(X,\tau_N)\}$. Where M_S(x), I_S(x), N_S(x) which represent the degree of membership, the degree of indeterminacy, and the degree of non-membership of each element $x\in(X,\tau_N)$ to the set S.

Definition 2.9. [11] Let S and T be NS of the form $S=\{<x, M_S(x), I_S(x), N_S(x)>: x\in(X,\tau_N)\}$ and $T=\{<x, M_T(x), I_T(x), N_T(x)>: x\in(X,\tau_N)\}$. Then

(1) S $\subseteq$ T if and only if $M_S(x)\le M_T(x)$, $I_S(x)\ge I_T(x)$ and $N_S(x)\ge N_T(x)$ for all $x\in(X,\tau_N)$ or $M_S(x)\ge M_T(x)$, $I_S(x)\le I_T(x)$ and $N_S(x)\le N_T(x)$ for all $x\in(X,\tau_N)$.

(2) S = T if and only if S $\subseteq$ T and T $\subseteq$ S.

(3) $S^C=\{<x, N_S(x), I_S(x), M_S(x)>: x\in(X,\tau_N)\}$.

(4) $S\bigcup T=\{<x, M_S(x)\vee M_T(x), I_S(x)\wedge I_T(x), N_S(x)\wedge N_T(x)>: x\in(X,\tau_N)\}$.

(5) $S\bigcap T=\{<x, M_S(x)\wedge M_T(x), I_S(x)\vee I_T(x), N_S(x)\vee N_T(x)>: x\in(X,\tau_N)\}$.

Definition 2.10. [11] Let (X, τ_N) be NTS and $S=\{<x, M_S(x), I_S(x), N_S(x)>: x\in(X,\tau_N)\}$ be a NS in X. Then the neutrosophic closure and neutrosophic interior of S are defined by

(1) Ncl(S) = $\bigcap$ {K:K is an NCS in X and S $\subseteq$ K}

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(2) Nint(S) = $\bigcup$ {K:K is an NOS in X and K $\subseteq$ S}

3. Neutrosophic Nano Aψ Closed Sets

Definition 3.1. Let (U, $\tau_N(S)$) be a neutrosophic nano topological space. Then

A neutrosophic nano subset S in (U, $\tau_N(S)$) is said to be:

(a) Neutrosophic nano semi closed (NNSC) if $N_N$int($N_N$cl(S)) $\subseteq$ S.

(b) Neutrosophic nano α closed (NNαC) if $N_N$cl($N_N$int($N_N$cl(S))) $\subseteq$ S.

(c) Neutrosophic nano pre closed (NNPC) if $N_N$cl($N_N$int(S)) $\subseteq$ S.

(d) Neutrosophic nano semi pre closed (NNSPC) if $N_N$int($N_N$cl($N_N$int(S))) $\subseteq$ S.

(e) Neutrosophic nano regular closed (NNRC) if $N_N$cl($N_N$int(S)) = S.

(f) Neutrosophic nano sg closed set (NNSGC) if $N_N$scl(S) $\subseteq$ V whenever S $\subseteq$ V

and V is neutrosophic nano semi open.

(g) neutrosophic nano ψ closed set (NNψC) if $N_N$scl(S) $\subseteq$ V whenever S $\subseteq$ V

and V is neutrosophic nanosg open.

(h) neutrosophic nano αψ closed set (NNαψC) if N_N cl(S) $\subseteq$ V whenever

S $\subseteq$ V and V is neutrosophic nano α open.

This shows that the following example.

Example 3.2. Assume $U=\{n_1,n_2,n_3\}$ be the universe set and the equivalence relation is $U/R=\{\{n_1\},\{n_2,n_3\}\}$ Let $S=\left\{\left\langle\frac{n_1}{(0.3,0.4,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.6,0.3,0.1)}\right\rangle,\left\langle\frac{n_3}{(0.2,0.6,0.2)}\right\rangle\right\}$ be neutrosophic nano subset of U. Then $N_*(S)=\left\{\left\langle\frac{n_1}{(0.2,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.5,0.3,0.2)}\right\rangle,\left\langle\frac{n_3}{(0.2,0.5,0.3)}\right\rangle\right\}$, $N^*(S)=\left\{\left\langle\frac{n_1}{(0.3,0.5,0.2)}\right\rangle,\left\langle\frac{n_2}{(0.6,0.4,0.1)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.5,0.2)}\right\rangle\right\}$ and $B_N(S)=\left\{\left\langle\frac{n_1}{(0.2,0.3,0.4)}\right\rangle,\left\langle\frac{n_2}{(0.5,0.4,0.2)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.4,0.2)}\right\rangle\right\}$. Here $\tau_N(S)=\{0_N,1_N,N_*(S),N^*(S),B_N(S)\}$ be a neutrosophic nano open set and a neutrosophic nano closed set is $[\tau_N(S)]^C=\{0_N,1_N,N_*^C(S),N^{*C}(S),B_N(S)\}$ where $N_*^C(S)=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.2)}\right\rangle,\left\langle\frac{n_2}{(0.2,0.3,0.5)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.5,0.2)}\right\rangle\right\}$, $N^{*C}(S)=\left\{\left\langle\frac{n_1}{(0.2,0.5,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.1,0.4,0.6)}\right\rangle,\left\langle\frac{n_3}{(0.2,0.5,0.3)}\right\rangle\right\}$ and $B^C{}_N(S)=\left\{\left\langle\frac{n_1}{(0.4,0.3,0.2)}\right\rangle,\left\langle\frac{n_2}{(0.2,0.4,0.5)}\right\rangle,\left\langle\frac{n_3}{(0.2,0.4,0.3)}\right\rangle\right\}$. Assume $R=\left\{\left\langle\frac{n_1}{(0.2,0.3,0.2)}\right\rangle,\left\langle\frac{n_2}{(0.2,0.2,0.6)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.2,0.3)}\right\rangle\right\}$ be a neutrosophic nano semi closed because $N_N$int($N_N$cl(R)) = N_N($N_N$cl(R)) = N_N(N_*^C)=$0_N \subseteq$ R. Similarlly, R is also neutrosophic nano α closed, neutrosophic nano pre closed and neutrosophic nano semi pre closed.

Example 3.3. Assume $U=\{n_1,n_2,n_3\}$ be the universe set and the equivalence relation is $U/R=\{\{n_1\},\{n_2,n_3\}\}$ Let $S=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.2)}\right\rangle,\left\langle\frac{n_2}{(0.2,0.3,0.1)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.2,0.3)}\right\rangle\right\}$ be neutrosophic nano subset of U. Then $N_*(S)=\left\{\left\langle\frac{n_1}{(0.2,0.3,0.2)}\right\rangle,\left\langle\frac{n_2}{(0.2,0.3,0.2)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.5,0.3)}\right\rangle\right\}$, $N^*(S)=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.2)}\right\rangle,\left\langle\frac{n_2}{(0.3,0.4,0.3)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.5,0.3)}\right\rangle\right\}$ and $B_N(S)=N_*(S)$. Here $\tau_N(S)=\{0_N,1_N,N_*(S),N^*(S),B_N(S)\}$ be a neutrosophic nano open set and a neutrosophic nano closed set is $[\tau_N(S)]^C = \{0_N, 1_N, N_*^C(S), N^{*C}(S), B_N(S)\}$ where $N_*^C(S) = \left\{\left\langle \frac{n_1}{(0.2,0.3,0.2)} \right\rangle, \left\langle \frac{n_2}{(0.2,0.3,0.2)} \right\rangle, \left\langle \frac{n_3}{(0.3,0.5,0.3)} \right\rangle\right\}$, $N^{*C}(S) = \left\{\left\langle \frac{n_1}{(0.3,0.3,0.2)} \right\rangle, \left\langle \frac{n_2}{(0.3,0.4,0.3)} \right\rangle, \left\langle \frac{n_3}{(0.3,0.5,0.3)} \right\rangle\right\}$

. Assume $R = \left\{\left\langle \frac{n_1}{(0.2,0.3,0.2)} \right\rangle, \left\langle \frac{n_2}{(0.2,0.2,0.6)} \right\rangle, \left\langle \frac{n_3}{(0.3,0.2,0.3)} \right\rangle\right\}$ be a neutrosophic nano regular

closed because $N_N cl(N_N int(R)) = R$.

Example 3.4. Assume $U = \{n_1, n_2, n_3\}$ be the universe set and the equivalence relation is $U/R = \{\{n_1\}, \{n_2, n_3\}\}$ Let $S = \left\{\left\langle \frac{n_1}{(0.3,0.3,0.2)} \right\rangle, \left\langle \frac{n_2}{(0.2,0.3,0.1)} \right\rangle, \left\langle \frac{n_3}{(0.3,0.2,0.3)} \right\rangle\right\}$ be neutrosophic nano subset of U. Then $N_*(S) = \left\{\left\langle \frac{n_1}{(0.3,0.3,0.4)} \right\rangle, \left\langle \frac{n_2}{(0.1,0.3,0.5)} \right\rangle, \left\langle \frac{n_3}{(0.1,0.2,0.5)} \right\rangle\right\}$, $N^*(S) = \left\{\left\langle \frac{n_1}{(0.4,0.3,0.2)} \right\rangle, \left\langle \frac{n_2}{(0.3,0.4,0.1)} \right\rangle, \left\langle \frac{n_3}{(0.4,0.3,0.2)} \right\rangle\right\}$ and $B_N(S) = \left\{\left\langle \frac{n_1}{(0.3,0.3,0.3)} \right\rangle, \left\langle \frac{n_2}{(0.3,0.3,0.2)} \right\rangle, \left\langle \frac{n_3}{(0.3,0.3,0.4)} \right\rangle\right\}$. Here $\tau_N(S) = \{0_N, 1_N, N_*(S), N^*(S), B_N(S)\}$ be a neutrosophic nano open set and a neutrosophic nano closed set is $[\tau_N(S)]^C = \{0_N, 1_N, N_*^C(S), N^{*C}(S), B_N(S)\}$ where $N_*^C(S) = \left\{\left\langle \frac{n_1}{(0.4,0.3,0.3)} \right\rangle, \left\langle \frac{n_2}{(0.5,0.3,0.1)} \right\rangle, \left\langle \frac{n_3}{(0.5,0.2,0.1)} \right\rangle\right\}$, $N^{*C}(S) = \left\{\left\langle \frac{n_1}{(0.2,0.3,0.4)} \right\rangle, \left\langle \frac{n_2}{(0.1,0.4,0.3)} \right\rangle, \left\langle \frac{n_3}{(0.2,0.3,0.4)} \right\rangle\right\}$ and $B^C{}_N(S) = \left\{\left\langle \frac{n_1}{(0.3,0.3,0.3)} \right\rangle, \left\langle \frac{n_2}{(0.2,0.3,0.3)} \right\rangle, \left\langle \frac{n_3}{(0.4,0.3,0.3)} \right\rangle\right\}$

. Assume $R = \left\{\left\langle \frac{n_1}{(0.3,0.3,0.3)} \right\rangle, \left\langle \frac{n_2}{(0.1,0.3,0.5)} \right\rangle, \left\langle \frac{n_3}{(0.1,0.2,0.5)} \right\rangle\right\}$ be a neutrosophic nano sg closed and $C = \left\{\left\langle \frac{n_1}{(0.3,0.3,0.3)} \right\rangle, \left\langle \frac{n_2}{(0.1,0.3,0.2)} \right\rangle, \left\langle \frac{n_3}{(0.3,0.2,0.4)} \right\rangle\right\}$ be a neutrosophic nano ψ closed and also neutrosophic nano αψ closed.

Theorem 3.5. Let (U, $\tau_N(S)$) be a neutrosophic nano topological space. Then

the following are hold:

(a) Every neutrosophic nano closed set is neutrosophic nano semi closed set.

(b) Every neutrosophic nano closed set is neutrosophic nano α closed set.

(c) Every neutrosophic nano closed set is neutrosophic nano pre closed.

(d) Every neutrosophic nano closed set is neutrosophic nano semi pre closed set.

(e) Every neutrosophic nano regular closed set is neutrosophic nano closed set.

(f) Every neutrosophic nano closed set is neutrosophic nano semi closed set.

(g) Every neutrosophic nano α αclosed set is neutrosophic nano pre closed set.

Proof.

(a) Let S be a neutrosophic nano closed set then $N_N cl(S) = S$. This implies

$N_N int(N_N cl(S)) \subseteq N_N int(S) = S$. Therefore $N_N int(N_N cl(S)) \subseteq S$. Then neutrosophic nano closed set is neutrosophic nano semi closed set.

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(b) Let S be a neutrosophic nano closed set then $N_Ncl(S)$ = S. This implies $N_Nint(N_Ncl(S)) \subseteq N_Nint(S) \subseteq S$ implies that $N_Ncl(N_Nint(N_Ncl(S)) \subseteq N_Ncl(S)$ =S. Therefore $N_Ncl(N_Nint(N_Ncl(S)) \subseteq S$. Then S is neutrosophic nano α closed set.

(c) Let S be a neutrosophic nano closed set then $N_Ncl(S)$ = S. We know that $N_Nint(S) \subseteq S$. This implies $N_Ncl(N_Nint(S)) \subseteq N_Ncl(S)$ = S . Therefore $N_Ncl(N_Nint(S)) \subseteq S$. Then S is neutrosophic nano pre closed.

(d) Let S be a neutrosophic nano closed set then $N_Ncl(S)$ = S. We know that $N_Nint(S) \subseteq S$. This implies $N_Ncl(N_Nint(S)) \subseteq N_Ncl(S)$ = S implies that $N_Nint(N_Ncl(N_Nint(S)) \subseteq N_Nint(S) \subseteq S$. Then S is neutrosophic nano semi pre closed set.

(e) Let S be a neutrosophic nano regular closed set then $N_Ncl(N_Nint(S))$ =S. This implies $N_Ncl(N_Ncl(N_Nint(S)))$ = $N_Ncl(S)$ implies that $N_Ncl(N_Nint(S))$ =$N_Ncl(S)$ = S . Therefore S is neutrosophic nano closed set.

(f) Let S be a neutrosophic nano α closed set then $N_Ncl(N_Nint(N_Ncl(S)) \subseteq S$ implies that $N_Nint(N_Ncl(S)) \subseteq S$. Hence S is neutrosophic nano semi closed set

(g) Let S be a neutrosophic nano α closed set then $N_Ncl(N_Nint(N_Ncl(S)) \subseteq S$. We know that S is neutrosophic nano closed set so $N_Ncl(S)$ = S implies that $N_Ncl(N_Nint(S)) \subseteq S$. Hence S is neutrosophic nano pre closed set

The inverse part not true by the following examples.

Example 3.6. By using Example [3.2],

(a) Let us take $R=\left\{\left\langle\frac{n_1}{(0.2,0.3,0.2)}\right\rangle,\left\langle\frac{n_2}{(0.2,0.2,0.6)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.2,0.3)}\right\rangle\right\}$ be a neutrosophic nano semi closed but it is not nutrosophicnano closed.

(b) Let us take $R=\left\{\left\langle\frac{n_1}{(0.2,0.3,0.2)}\right\rangle,\left\langle\frac{n_2}{(0.2,0.2,0.6)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.2,0.3)}\right\rangle\right\}$ be a neutrosophic nanoαclosed but it is not nutrosophicnano closed.

(c) Let us take $R=\left\{\left\langle\frac{n_1}{(0.2,0.3,0.2)}\right\rangle,\left\langle\frac{n_2}{(0.2,0.2,0.6)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.2,0.3)}\right\rangle\right\}$ be a neutrosophic nano pre closed but it is not nutrosophicnano closed.

(d) Let us take $R=\left\{\left\langle\frac{n_1}{(0.2,0.3,0.2)}\right\rangle,\left\langle\frac{n_2}{(0.2,0.2,0.6)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.2,0.3)}\right\rangle\right\}$ be a neutrosophic nano semi pre closed but it is not nutrosophicnano closed.

(e) Let us take $R=\left\{\left\langle\frac{n_1}{(0.2,0.3,0.2)}\right\rangle,\left\langle\frac{n_2}{(0.2,0.2,0.6)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.2,0.3)}\right\rangle\right\}$ be a neutrosophic nano closed but it is not nutrosophicnano regular closed.

(f) Assume $U=\{n_1,n_2,n_3\}$ be the universe set and the equivalence relation is $U/R=\{\{n_1\},\{n_2,n_3\}\}$ Let $S=\left\{\left\langle\frac{n_1}{(0.3,0.4,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.6,0.3,0.1)}\right\rangle,\left\langle\frac{n_3}{(0.2,0.6,0.2)}\right\rangle\right\}$ be neutrosophic nano subset of U. Then $N_*(S)=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.4)}\right\rangle,\left\langle\frac{n_2}{(0.1,0.3,0.5)}\right\rangle,\left\langle\frac{n_3}{(0.1,0.2,0.5)}\right\rangle\right\}$, $N^*(S)=\left\{\left\langle\frac{n_1}{(0.4,0.3,0.2)}\right\rangle,\left\langle\frac{n_2}{(0.3,0.4,0.1)}\right\rangle,\left\langle\frac{n_3}{(0.4,0.3,0.2)}\right\rangle\right\}$ and $B_N(S)=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.2,0.3,0.3)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.3,0.4)}\right\rangle\right\}$. Here $\tau_N(S)=\{0_N,1_N,N_*(S),N^*(S),B_N(S)\}$ be a neutrosophic nano open set and a neutrosophic nano closed set is $[\tau_N(S)]^C=\{0_N,1_N,N_*{}^C(S),N^{*C}(S),B_N(S)\}$ where $N_*{}^C(S)=\left\{\left\langle\frac{n_1}{(0.4,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.5,0.3,0.1)}\right\rangle,\left\langle\frac{n_3}{(0.5,0.2,0.1)}\right\rangle\right\}$, $N^{*C}(S)=\left\{\left\langle\frac{n_1}{(0.2,0.3,0.4)}\right\rangle,\left\langle\frac{n_2}{(0.1,0.4,0.3)}\right\rangle,\left\langle\frac{n_3}{(0.2,0.3,0.4)}\right\rangle\right\}$ and

$B^C{}_N(S)=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.2,0.3,0.3)}\right\rangle,\left\langle\frac{n_3}{(0.4,0.3,0.3)}\right\rangle\right\}$. Assume $R=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.1,0.3,0.4)}\right\rangle,\left\langle\frac{n_3}{(0.2,0.2,0.4)}\right\rangle\right\}$ be a neutrosophic nano semi closed but it is not neutrosophic nano α closed.

(g) Assume $U=\{n_1,n_2,n_3\}$ be the universe set and the equivalence relation is $U/R=\{\{n_1\},\{n_2,n_3\}\}$ Let $S=\left\{\left\langle\frac{n_1}{(0.3,0.4,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.6,0.3,0.1)}\right\rangle,\left\langle\frac{n_3}{(0.2,0.6,0.2)}\right\rangle\right\}$ be neutrosophic nano subset of U. Then $N_*(S)=\left\{\left\langle\frac{n_1}{(0.4,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.5,0.3,0.1)}\right\rangle,\left\langle\frac{n_3}{(0.5,0.2,0.1)}\right\rangle\right\}$, $N^*(S)=\left\{\left\langle\frac{n_1}{(0.2,0.3,0.4)}\right\rangle,\left\langle\frac{n_2}{(0.1,0.4,0.3)}\right\rangle,\left\langle\frac{n_3}{(0.2,0.3,0.4)}\right\rangle\right\}$ and $B_N(S)=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.2,0.3,0.3)}\right\rangle,\left\langle\frac{n_3}{(0.4,0.3,0.3)}\right\rangle\right\}$. Here $\tau_N(S)=\{0_N,1_N,N_*(S),N^*(S),B_N(S)\}$ be a neutrosophic nano open set and a neutrosophic nano closed set is $[\tau_N(S)]^C=\{0_N,1_N,N_*{}^C(S),N^{*C}(S),B_N(S)\}$ where $N_*{}^C(S)=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.4)}\right\rangle,\left\langle\frac{n_2}{(0.1,0.3,0.5)}\right\rangle,\left\langle\frac{n_3}{(0.1,0.2,0.5)}\right\rangle\right\}$, $N^{*C}(S)=\left\{\left\langle\frac{n_1}{(0.4,0.3,0.2)}\right\rangle,\left\langle\frac{n_2}{(0.3,0.4,0.1)}\right\rangle,\left\langle\frac{n_3}{(0.4,0.3,0.2)}\right\rangle\right\}$ and $B^C{}_N(S)=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.2,0.3,0.3)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.3,0.4)}\right\rangle\right\}$. Assume $R=\left\{\left\langle\frac{n_1}{(0.4,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.5,0.3,0.2)}\right\rangle,\left\langle\frac{n_3}{(0.5,0.2,0.4)}\right\rangle\right\}$ be a neutrosophic nano semi closed but it is not neutrosophic nano α closed.

Theorem 3.7. Let (U, $\tau_N(S)$) be a neutrosophic topological space. Then the following are hold:

(a) Every neutrosophic nano closed set is neutrosophic nano αψ closed set.

(b) Every neutrosophic nano α closed set is neutrosophic nano αψ closed set.

(c) Every neutrosophic nano semi closed set is neutrosophic nano sg closed.

(d) Every neutrosophic nano ψ closed set is neutrosophic nano αψ closed set.

Proof.

(a) Let S be a neutrosophic nano closed set. This implies $N_N$cl(S) = S. Now assume that T be a neutrosophic nano α open set and S $\subseteq$ T, then N_N cl(S) $\subseteq$ $N_N$cl(S) = S $\subseteq$ T. Therefore N_N cl(S) $\subseteq$ T. Hence S is neutrosophic nano αψ closed.

(b) Let S be a neutrosophic nano α closed set then $N_N$cl(S) = S. Now assume that S $\subseteq$ T and T be a neutrosophic nano αψ open set, then N_N cl(S) $\subseteq$ $N_N$cl(S) $\subseteq$ T. Therefore it is neutrosophic nano αψ closed set.

(c) Let S be a neutrosophic nano semi closed then $N_N$scl(S) = S. Assume that S $\subseteq$ T, T is neutrosophic nano semi open, then $N_N$scl(S) $\subseteq$ T. Then S is neutrosophic nano sg closed set.

(d) Let S be a neutrosophic nano α closed. Every neutrosophic nano α open set is neutrosophic nano semi open and neutrosophic nano semi open is neutrosophic nano sg open. Assume that S $\subseteq$ T, T is neutrosophic nano sg open, then $N_N$cl($N_N$scl(S)) $\subseteq$ T. Then S is neutrosophic nano αψ closed set.

The inverse part not true by the following examples.

Example 3.8. By using Example [3.5 (f)]

(a) Let $C=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.1,0.3,0.2)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.2,0.4)}\right\rangle\right\}$ be a neutrosophic nano semi open and also neutrosophic nano α open. Here $R=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.1,0.3,0.5)}\right\rangle,\left\langle\frac{n_3}{(0.1,0.2,0.5)}\right\rangle\right\}$ be a neutrosophic nanosg closed and $V=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.5,0.3,0.1)}\right\rangle,\left\langle\frac{n_3}{(0.5,0.2,0.1)}\right\rangle\right\}$ be a neutrosophic nanosg open and C $\subseteq$ V . Hence C is neutrosophic nano αψ closed but it is not neutrosophic nano closed set.

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(b) Let $C=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.1,0.3,0.2)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.2,0.4)}\right\rangle\right\}$ be a neutrosophic nano semi open and also neutrosophic nano α open. Here $R=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.1,0.3,0.5)}\right\rangle,\left\langle\frac{n_3}{(0.1,0.2,0.5)}\right\rangle\right\}$ be a neutrosophic nano sg closed and $V=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.5,0.3,0.1)}\right\rangle,\left\langle\frac{n_3}{(0.5,0.2,0.1)}\right\rangle\right\}$ be a neutrosophic nanosg open and $C\subseteq V$. Hence C is neutrosophic nano αψ closed but it is not neutrosophic nano α closed set.

(c) Let $U=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.1,0.3,0.2)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.2,0.4)}\right\rangle\right\}$ be a neutrosophic nano semi open .Here $C=\left\{\left\langle\frac{n_1}{(0.2,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.3,0.3,0.4)}\right\rangle,\left\langle\frac{n_3}{(0.2,0.3,0.4)}\right\rangle\right\}$. Hence C is neutrosophic nano sg closed but it is not neutrosophic nano semi closed set.

(d) Let $C=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.1,0.3,0.2)}\right\rangle,\left\langle\frac{n_3}{(0.3,0.2,0.4)}\right\rangle\right\}$ be a neutrosophic nano semi open and also neutrosophic nano α open. Here $R=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.1,0.3,0.4)}\right\rangle,\left\langle\frac{n_3}{(0.2,0.2,0.4)}\right\rangle\right\}$ is not neutrosophic nano sg closed and $V=\left\{\left\langle\frac{n_1}{(0.3,0.3,0.3)}\right\rangle,\left\langle\frac{n_2}{(0.4,0.3,0.1)}\right\rangle,\left\langle\frac{n_3}{(0.4,0.2,0.2)}\right\rangle\right\}$ is not neutrosophic nano sg open and $C\subseteq U$. Hence C is neutrosophic nano αψ closed but it is not neutrosophic nano ψ closed set.

Theorem 3.9. The union of two neutrosophic nano αψ closed set is also neutrosophic nano αψ closed set.

Proof. Let us assume that S and T be two neutrosophic nano αψ closed sets. Let $S \cup T \subseteq V$, V is neutrosophic nano α open. By definition of neutrosophic nano αψ closed set, $N_N\,\psi cl(S) \subseteq V$ and $N_N\psi cl(T) \subseteq V$. This implies that $N_N\psi cl(S \cup T) \subseteq V$. Hence $S \cup T$ is neutrosophic nano αψ closed set.

Theorem3.10. Let S is neutrosophic nano αψ closed set if S is both neutrosophic nano α open and neutrosophic nano ψ closedset.

Proof. Let us assume that S is both neutrosophic nano α open and neutrosophic nano ψ closed set, then by definition of neutrosophic nano ψ closed set, $N_Nscl(S) \subseteq T$ this implies that $N_N\psi cl(S) \subseteq N_Nscl(S) \subseteq T$. Therefore $N_N\psi cl(S) \subseteq T$. Hence S is neutrosophic nano αψ closed set.

Theorem 3.11. Assume S be a neutrosophic nano αψ closed set in neutrosophic nano topological spaces U. Then $N_N\psi cl(S) - S$ does not contain any non-empty neutrosophic nano α closed set.

Proof. Let us assume T be non-empty neutrosophic nano α closed subset of $N_N\psi cl(S)-S$. Then $S \subset U - T$, where $U - T$ is neutrosophic nano α open. Thus $N_N\psi cl(S) \subset U - T$ or equivalently $T \subset U - N_N\psi cl(S)$. This is contradiction by assumption. Hence $N_N\psi cl(S) - S$ does not contain any non-empty neutrosophic nano α closed s topological space.

Theorem 3.12. Let S be neutrosophic nano αψ closed subset of neutrosophic topological spaces such that $S \subset T \subset N_N\psi cl(S)$, then T is also neutrosophic nano αψ closed subset of neutrosophic nano topological space.

Proof. Let S be a neutrosophic nano αψ closed set, by definition $S \subseteq W$ and W is neutrosophic nano α open set in neutrosophic nano topological spaces then $N_N\psi cl(S) \subset W$. Now assume that $T \subset W$ and W is neutrosophic nano α open set. Here $N_N\psi cl(S)$ is neutrosophic nano ψ closed set. Therefore $N_N\psi cl(T) \subseteq N_N\psi cl(N_N\psi cl(S)) = N_N\psi cl(S) \subset W$. Hence $N_N\psi cl(S) \subset W$. Therefore S is also neutrosophic nano αψ closed subset of neutrosophic nano topological space.

Theorem 3.13. Let S be a subset of neutrosophic nano topological space then the following conditions are equivalent

(i) S is neutrosophic nano semi open and neutrosophic nano ψ closed.

(ii) S is neutrosophic nano regular open.

Proof. (i) ⇒ (ii) Let us assume that S is neutrosophic nano semi open and neutrosophic nano ψ closed sets. Every neutrosophic nano semi open is neutrosophic nanosg open. Then $N_N$scl(S) ⊆ S ⇒ $N_N$cl($N_N$int(S)) ⊆ S ⇒ $N_N$int($N_N$cl($N_N$int(S)) ⊆ $N_N$int(S). As S is neutrosophic nano open sets then it is neutrosophic nano α open and so S ⊆ $N_N$int(N_N cl($N_N$int(S)). Then $N_N$int($N_N$cl(S)) ⊆ S ⊆ $N_N$int($N_N$cl(S)). Therefore S = $N_N$int($N_N$cl(S)). Hence S is neutrosophic nano regular open set.

(ii) ⇒ (i) Each neutrosophic nano regular open set is neutrosophic nano open and every neutrosophic nano open set is neutrosophic nano semi open. By assumption S is neutrosophic nano semi open and by definition of neutrosophic nano regular open set, S = $N_N$int($N_N$cl(S)) then $N_N$int($N_N$cl(S)) ⊆ S, therefore it is also neutrosophic nano semi closed. Henec it is also neutrosophic nano ψ closed set.

4. Conclusion

In this paper it is discussed about the new concept of neutrosophic nano semi closed, neutrosophic nano α closed, neutrosophic nano pre closed, neutrosophic nano semi pre closed and neutrosophic nano regular closed and investigate some of their properties. Also the work is extended as neutrosophic nano sg closed, neutrosophic nano ψ closed and neutrosophic nano αψ closed and derive some of their properties and theorems.

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