LOW SEPARATION AXIOMS ASSOCIATED WITH $\hat{g}^*s$-CLOSED SETS

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Abstract. In this paper, we introduce ${}_\kappa T_{1/2}$-spaces, ${}_\kappa^* T_{1/2}$-spaces, ${}_\kappa T_b$-spaces, ${}_\kappa T_c$-spaces, ${}_\kappa T_d$-spaces, ${}_\kappa T_f$-spaces, ${}_\kappa T_{\hat{g}^*}$-spaces and T_b^κ-spaces and investigate their characterizations.

1 Introduction

Levine [7], Mashhour et al. [13] and Njastad [14] have introduced the concept of semi-open sets, preopen sets and α-open sets, respectively. Levine [8] introduced generalised closed sets and studied their properties. Bhattacharya and Lahiri[3], Arya and Nour [2], Maki et al. [10, 9] introduced semi-generalised closed sets, generalised semi-closed sets and α-generalised closed sets and generalised α-closed sets, respectively. Veerakumar [22] defined $\hat{g}$-closed sets. Chandrasekararao and Narasimhan [4] intoduced and studied g^*s-closed sets. Pious Missier and Anto [16] intoduced $\hat{g}^*s$-closed sets in topological spaces.

The rest of this paper is organized as follows. In Section 2, we present the fundamental concepts and sets in topological spaces. In Section 3, we introduce ${}_\kappa T_{1/2}$-spaces, ${}_\kappa^* T_{1/2}$-spaces, ${}_\kappa T_b$-spaces, ${}_\kappa T_c$-spaces, ${}_\kappa T_d$-spaces, ${}_\kappa T_f$-spaces, ${}_\kappa T_{\hat{g}^*}$-spaces and T_b^κ-spaces and investigate their characterizations.

2 Preliminaries

[0]Keywords and Phrases: ${}_\kappa T_{1/2}$-spaces, ${}_\kappa^* T_{1/2}$-spaces, ${}_\kappa T_b$-spaces, ${}_\kappa T_c$-spaces, ${}_\kappa T_d$-spaces, ${}_\kappa T_f$-spaces, ${}_\kappa T_{\hat{g}^*}$-spaces and T_b^κ-spaces.

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Throughout this paper, (X,τ) represents a non-empty topological spaces. For a subset A of a topological space (X,τ), $cl(A), int(A)$ and A^c or $X-A$ denote the closure, the interior and the complement of A, respectively. The power set of A is denoted by $P(A)$.

Definition 2.1. A subset A of a space (X,τ) is called

(i) semi-open [7] if $A \subseteq cl(int(A))$ and semi-closed if $int(cl(A)) \subseteq A$,

(ii) pre-open [13] if $A \subseteq int(cl(A))$ and pre-closed if $cl(int(A)) \subseteq A$,

(iii) α-open [14] if $A \subseteq int(cl(int(A)))$ and α-closed if $cl(int(cl(A))) \subseteq A$,

Definition 2.2. [21] A subset A of a topological space (X,τ) is called $\hat{g}$-closed if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in (X,τ)

Definition 2.3. [8] Let A be a subset of a topological space. Then A is called g-closed if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Definition 2.4. [15] A subset A of a topological space (X,τ) is called sg-closed if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in (X,τ)

Definition 2.5. [17] Let A be a subset of a topological space.Then A is called g^*-closed if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open.

Definition 2.6. [6] Let A be a subset of a topological space.Then A is called g^*s-closed if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open.

The semi-closure (resp. pre closure, α-closure and g^*s closure) of a subset A of a space (X,τ) is the intersection of all semi-closed (resp. pre closed, α-closed and g^*s closed) sets containing A and is denoted by scl(A) (resp. pcl(A), α-$cl(A)$ and $g^*scl(A)$).

Definition 2.7. [1] A subset A of a topological space (X,τ) is called gs-closed if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ)

Definition 2.8. [19] A subset A of a topological space (X,τ) is called $\hat{g}^*$-closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open.

Remark 2.9. *Note that M. K. R. S. Veerakumar called this $\hat{g}^{*}$-closed set as $^{*}g$-closed in his paper [19]*

Definition 2.10. [23] A subset A of a topological space (X,τ) is called $\hat{g}^{*}\alpha$-closed if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open in (X,τ)

Definition 2.11. [11] A subset A of a topological space (X,τ) is called $g\alpha$-closed if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X,τ)

Definition 2.12. [12] A subset A of a topological space (X,τ) is called αg-closed if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ)

Definition 2.13. [18] A subset A of a topological space (X,τ) is called a ψ-closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open.

Definition 2.14. [13] A topological space (X,τ) is called T_b if every gs-closed set in it is closed.

Definition 2.15. [17] A topological space (X,τ) is called T_c if every gs-closed set is g^{*}-closed

Definition 2.16. [5] A topological space (X,τ) is called T_d if every gs-closed set in it is g-closed.

Definition 2.17. [20] A topological space (X,τ) is called a T_f space if every g-closed set in it is $\hat{g}$-closed.

Definition 2.18. [8] A topological space (X,τ) is called $T_{1/2}$ if every g-closed set in it is closed.

Definition 2.19. [17] A topological space (X,τ) is called $T^{*}_{1/2}$ if every g^{*}-closed set in it is closed.

Definition 2.20. [17] A topological space (X,τ) is called $^{*}T_{1/2}$ if every g-closed set in it is g^{*}-closed.

Definition 2.21. [6] A topological space (X,τ) is called a strongly semi-$T_{1/2}$ space if every gs-closed set is $g^{*}s$-closed

Definition 2.22. [6] A topological space (X, τ) is called a semi-T_b space if every gs-closed set is semi closed

Definition 2.23. [16] A subset A of a topological space (X, τ) is called a $\hat{g}^*s$-closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$-open.

Definition 2.24. [12] A subset A of a topological space (X, τ) is called gp-closed if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

If A is gp-closed, then A^c is gp-open.

Definition 2.25. [6]A topological space (X, τ) is called a semi-T_p space if every g^*s-closed set is closed.

Lemma 2.26. *[19] Let a subset A of a topological space (X, τ) be $\hat{g}^*$-closed. Then $cl(A) - A$ contains no non-empty $\hat{g}$-closed set.*

Proposition 2.27. *[16] Let A be a $\hat{g}^*s$-closed subset of X. Then $scl(A) - A$ contains no non-empty $\hat{g}$ closed set in X*

Proposition 2.28. *[16] Every semi closed set in X is $\hat{g}^*s$-closed in X.*

3 Low separation axioms associated with $\hat{g}^*s$-closed sets

Definition 3.1. A topological space (X, τ) is called ${}_{\kappa}T_{1/2}$ if every $\hat{g}^*s$-closed set in it is semi closed.

Definition 3.2. A topological space (X, τ) is called ${}_{\kappa}^{*}T_{1/2}$ if every $\hat{g}^*s$-closed set in it is g^*s-closed.

Definition 3.3. A topological space (X, τ) is called ${}_{\kappa}T_b$ if every $\hat{g}^*s$-closed set in it is closed.

Definition 3.4. A topological space (X,τ) is called ${}_\kappa T_c$ if every $\hat{g}^*s$-closed set in it is g^*-closed.

Definition 3.5. A topological space (X,τ) is called ${}_\kappa T_d$ if every $\hat{g}^*s$-closed set in it is g-closed.

Definition 3.6. A topological space (X,τ) is called ${}_\kappa T_f$ if every $\hat{g}^*s$-closed set in it is $\hat{g}$-closed.

Definition 3.7. A topological space (X,τ) is called ${}_\kappa T_{\hat{g}^*}$ if every $\hat{g}^*s$-closed set in it is $\hat{g}^*$-closed.

Definition 3.8. A topological space (X,τ) is called T_b^κ if every gs-closed set in it is $\hat{g}^*s$-closed.

Proposition 3.9. *Every strongly semi-$T_{1/2}$ space is a T_b^κ space.*

Proof. Suppose (X,τ) is strongly semi-$T_{1/2}$. Let $A \subseteq X$ be gs-closed. Since (X,τ) is strongly semi-$T_{1/2}$, A is g^*s-closed. But every g^*s-closed set is $\hat{g}^*s$-closed. Therefore A is $\hat{g}^*s$-closed. Hence (X,τ) is T_b^κ. □

Remark 3.10. *The converse of the above Proposition is not true as seen from the following Example.*

Example 3.11. *Let (X,τ) be a topological space, where $X = \{a,b,c,d\}$ with $\tau = \{\phi, X, \{a\}\}$ and $\tau^c = \{\phi, X, \{b,c,d\}\}$. We have $g^*sC(X,\tau) = \{\phi, X, \{b,c,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{d\}, \{c\}, \{b\}\}$; $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{a,b,c\}, \{c,d\}, \{b,d\}, \{b,c\}, \{a,d\}, \{a,c\}, \{a,b\}, \{d\}, \{c\}, \{b\}\}$ and $gsC(X,\tau) = \{\phi, X, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{a,b,c\}, \{c,d\}, \{b,d\}, \{b,c\}, \{a,d\}, \{a,c\}, \{a,b\}, \{d\}, \{c\}, \{b\}\}$. Since gs-closed sets are all $\hat{g}^*s$-closed, (X,τ) is T_b^κ. As, gs-closed sets are all not g^*s-closed, (X,τ) is not strongly semi-$T_{1/2}$.*

Proposition 3.12. *Every strongly semi-$T_{1/2}$ space is a ${}^*_\kappa T_{1/2}$ space.*

Proof. Suppose (X,τ) be strongly semi-$T_{1/2}$. Let $A \subseteq X$ be $\hat{g}^*s$-closed. But every $\hat{g}^*s$-closed set is gs-closed. Thus A is gs-closed. Since (X,τ) is strongly semi-$T_{1/2}$, A is g^*s-closed. Hence (X,τ) is ${}^*_\kappa T_{1/2}$. □

Remark 3.13. *The converse of the above Proposition is not true as seen from the following Example.*

Example 3.14. *Let* (X,τ) *be a topological space, where* $X=\{a,b,c\}$ *with* $\tau=\{\phi,X,\{a\},\{b,c\}\}$ *and* $\tau^c=\{\phi,X,\{b,c\},\{a\}\}$. *We have* $g^*sC(X,\tau)=\{\phi,X,\{b,c\},\{a\}\}$; $\hat{g}^*sC(X,\tau)=\{\phi,X,\{b,c\},\{a\ \}$ *and* $gsC(X,\tau)=\{\phi,X,\{b,c\},$
$\{a,c\},\{a,b\},\{c\},\{b\},\{a\}\}$ *.Since* $\hat{g}^*s$ *-closed sets are all* g^*s *-closed,* (X,τ) *is* ${}^*_\kappa T_{1/2}$ *. As,* gs *-closed sets are all not* g^*s *-closed,* (X,τ) *is not strongly semi-*$T_{1/2}$ *.*

Proposition 3.15. *A topological space* (X,τ) *is strongly semi-*$T_{1/2}$ *iff it is both* ${}^*_\kappa T_{1/2}$ *and* T_b^κ *.*

Proof. Suppose (X,τ) is strongly semi-$T_{1/2}$. Then by Proposition 3.12, (X,τ) is ${}^*_\kappa T_{1/2}$. And by Proposition 3.9, (X,τ) is T_b^κ.

Conversely, let (X,τ) be both ${}^*_\kappa T_{1/2}$ and T_b^κ. Let A be gs-closed in (X,τ). Since (X,τ) is T_b^κ, by Definition 3.8, A is $\hat{g}^*s$-closed. Since (X,τ) is ${}^*_\kappa T_{1/2}$, by Definition 3.2 , A is g^*s-closed. Therefore (X,τ) is strongly semi-$T_{1/2}$. □

Proposition 3.16. *Every semi-*T_b *space is a* T_b^κ *space.*

Proof. Let (X,τ) be semi-T_b and $A\subseteq X$, gs-closed. Since (X,τ) is semi-T_b, A is semi closed. But every semi closed set is $\hat{g}^*s$-closed. Therefore A is $\hat{g}^*s$-closed. Hence (X,τ) is T_b^κ. □

Remark 3.17. *The converse of the above Proposition is not true as seen from the following Example.*

Example 3.18. *Let* (X,τ) *be a topological space where* $X=\{a,b,c,d\}$ *with* $\tau=\{\phi,X,\{a,b\},\{a,b,c\},\{a,b,d\}\}$ *and* $\tau^c=\{\phi,X,\{c,d\},\{d\},\{c\}\}$ *.We have* $SC(X,\tau)=\{\phi,X,\{c,d\},\{d\},\{c\}\}$; $\hat{g}^*sC(X,\tau)=\{\phi,X,\{b,c,d\},\{a,c,d\},$
$\{c,d\},\{d\},\{c\}\}$ *and* $gsC(X,\tau)=\{\phi,X,\{b,c,d\},\{a,c,d\},\{c,d\},\{d\},\{c\}\}$ *. Since* gs *-closed sets are all* $\hat{g}^*s$ *-closed,* (X,τ) *is* T_b^κ *. As,* gs *-closed sets are all not semi-closed,* (X,τ) *is not semi-*T_b *.*

Proposition 3.19. *Every semi-T_b space is a ${}_\kappa T_{1/2}$ space.*

Proof. Suppose (X,τ) be semi-T_b and let $A \subseteq X$ be $\hat{g}^*s$-closed. But every $\hat{g}^*s$-closed set is gs-closed. Therefore A is gs-closed. Since (X,τ) is semi-T_b, A is semi closed. Therefore (X,τ) is ${}_\kappa T_{1/2}$. □

Remark 3.20. *The converse of the above Proposition is not true as seen from the following Example.*

Example 3.21. *Let (X,τ) be a topological space where $X = \{a,b,c\}$ with $\tau = \{\phi, X, \{a\}, \{b,c\}\}$ and $\tau^c = \{\phi, X, \{b,c\}, \{a\}\}$. We have $SC(X,\tau) = \{\phi, X, \{b,c\}, \{a\}\}$; $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a\ \}$ and $gsC(X,\tau) = \{\phi, X,$ $\{b,c\}, \{a,c\}, \{a,b\}, \{c\}, \{b\}, \{a\}\}$. Since $\hat{g}^*s$-closed sets are all semi closed, (X,τ) is $\kappa T_{1/2}$. As, gs-closed sets are all not semi closed, (X,τ) is not semi-T_b.*

Proposition 3.22. *A topological space (X,τ) is semi-T_b iff it is both ${}_\kappa T_{1/2}$ and T_b^κ.*

Proof. Suppose (X,τ) is semi-T_b. Then by Proposition 3.19, (X,τ) is ${}_\kappa T_{1/2}$. Thus by Proposition 3.16, (X,τ) is T_b^κ.

Conversely, let (X,τ) be both ${}_\kappa T_{1/2}$ and T_b^κ. Let also A be gs-closed in (X,τ). Since (X,τ) is T_b^κ, by Definition 3.8, A is $\hat{g}^*s$-closed. Since (X,τ) is ${}_\kappa T_{1/2}$, by Definition 3.1, A is semi closed. Therefore (X,τ) is semi-T_b. □

Proposition 3.23. *Every ${}_\kappa T_b$ space is a semi-T_p space.*

Proof. Let (X,τ) be a ${}_\kappa T_b$ space. Let $A \subseteq X$ be g^*s-closed. But every g^*s-closed set is a $\hat{g}^*s$-closed set. Therefore A is $\hat{g}^*s$-closed in X. Since X is ${}_\kappa T_b$ space, A is closed. Hence (X,τ) is a semi-T_p space. □

Remark 3.24. *The converse of the above Proposition is not true as seen from the following Example.*

Example 3.25. *Let* (X,τ) *be a topological space where* $X=\{a,b,c\}$ *with* $\tau=\{\phi,X,\{a\},\{b,c\}\}$ *and* $\tau^c = \{\phi,X,\{b,c\},\{a\}\}$ *. We have* $SC(X,\tau) = \{\phi,X,\{b,c\},\{a\ \}$ *;* $\hat{g}^*sC(X,\tau) = \{\phi,X,\{b,c\},\{a\ \}$ *and* $gsC(X,\tau)=\{\phi,X,$
$\{b,c\},\{a,c\},\{a,b\},\{c\},\{b\},\{a\}\}$ *. Since* $\hat{g}^*s$ *-closed sets are all semi closed,* (X,τ) *is* $\kappa T_{1/2}$ *. As,* gs *-closed sets are all not semi closed,* (X,τ) *is not semi-*T_p *.*

Proposition 3.26. *Every* T_b *space is a* ${}_\kappa T_b$ *space.*

Proof. Let (X,τ) be a T_b space. Let $A\subseteq X$ be $\hat{g}^*s$-closed. But every $\hat{g}^*s$-closed set is gs-closed. Therefore A is gs-closed in X. Since (X,τ) is T_b, A is closed. Therefore X is ${}_\kappa T_b$. □

Remark 3.27. *The converse is not true as seen from the following example.*

Example 3.28. *Let* (X,τ) *be a topological space where* $X=\{a,b,c\}$ *with* $\tau=\{\phi,X,\{a\},\{b,c\}\}$ *and* $\tau^c = \{\phi,X,\{b,c\},\{a\}\}$ *. We have* $\hat{g}^*sC(X,\tau) = \{\phi,X,\{b,c\},\{a\ \}$ *and* $gsC(X,\tau) = \{\phi,X,\{b,c\},\{a,c\},\{a,b\},\{c\},\{b\},\{a\}\}$ *. Since* $\hat{g}^*s$ *-closed sets are all closed,* (X,τ) *is* κT_b *. Since* gs *-closed sets are all not closed,* (X,τ) *is not* T_b *.*

Proposition 3.29. *Every* ${}_\kappa T_b$ *space is a* ${}_\kappa T_{1/2}$ *space.*

Proof. Let (X,τ) be a ${}_\kappa T_b$ space. Let $A\subseteq X$ be $\hat{g}^*s$-closed. Since X is κT_b, A is closed. Since X is closed, A is semi closed. Therefore X is ${}_\kappa T_{1/2}$. □

Remark 3.30. *The converse is not true as seen by the following Example.*

Example 3.31. *Let* (X,τ) *be a topological space where* $X = \{a,b,c\}$ *with* $\tau = \{\phi,X,\{a\},\{b\},\{a,b\}\}$ *and* $\tau^c = \{\phi,X,\{b,c\},\{a,c\},\{c\}\}$ *. We have* $SC(X,\tau) = \{\phi,X,\{b,c\},\{a,c\},\{c\},\{b\},\{a\}\}$ *and* $\hat{g}^*sC(X,\tau)=\{\phi,X,\{b,c\},\{a,c\},\{c\},\{b\},\{a\},\}$ *. Since all* $\hat{g}^*s$ *-closed sets are semi closed,* (X,τ) *is* ${}_\kappa T_{1/2}$ *. As* $\hat{g}^*s$ *-closed sets are all not closed,* (X,τ) *is not* ${}_\kappa T_b$ *.*

Proposition 3.32. *Every* ${}_\kappa T_{1/2}$ *space is a* ${}^*_\kappa T_{1/2}$ *space.*

Proof. Let (X,τ) be a ${}_{\kappa}T_{1/2}$ space. Let $A \subseteq X$ be $\hat{g}^*s$-closed. Since X is ${}_{\kappa}T_{1/2}$, A is semi closed. But every semi closed set is g^*s-closed. Therefore X is ${}^{*}_{\kappa}T_{1/2}$ space. □

Remark 3.33. *The converse is not true as seen from the following example.*

Example 3.34. *Let* (X,τ) *be a topological space where* $X = \{a,b,c\}$ *with* $\tau = \{\phi, X, \{a,b\}\}$ *and* $\tau^c = \{\phi, X, \{c\}\}$ *.We have* $SC(X,\tau) = \{\phi, X, \{c\}\}$ *;* $g^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$ *and* $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\},$
$\{c\}\}$ *. Since Since every* $\hat{g}^*s$ *-closed set is* g^*s *-closed,* (X,τ) *is a* ${}^{*}_{\kappa}T_{1/2}$ *. By the fact that every* $\hat{g}^*s$ *-closed set is not semi closed,* (X,τ) *is not* ${}_{\kappa}T_{1/2}$

Proposition 3.35. *Every* ${}_{\kappa}T_b$ *space is a* ${}^{*}_{\kappa}T_{1/2}$ *space.*

Proof. By Proposition 3.29, every ${}_{\kappa}T_b$ space is a ${}_{\kappa}T_{1/2}$ space. By Proposition 3.32, every ${}_{\kappa}T_{1/2}$ space is a ${}^{*}_{\kappa}T_{1/2}$ space. Therefore, every ${}_{\kappa}T_b$ space is a ${}^{*}_{\kappa}T_{1/2}$ space. □

Note 3.36. *The converse of the above Proposition is not true as seen by the following Example.*

Example

3.37. *Let* (X,τ) *be a topological space where* $X = \{a,b,c\}$ *with* $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$ *and* $\tau^c = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$ *.We have* $g^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}, \{b\}, \{a\},\}$ *and* $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\},$
$\{a,c\}, \{c\}, \{b\}, \{a\},\}$ *.Since all* $\hat{g}^*s$ *-closed sets are* g^*s *-closed,* (X,τ) *is* ${}^{*}_{\kappa}T_{1/2}$ *. As* $\hat{g}^*s$ *-closed sets are all not closed,* (X,τ) *is not* ${}_{\kappa}T_b$ *.*

Proposition 3.38. *A topological space* (X,τ) *is* ${}_{\kappa}T_b$ *iff it is both* ${}^{*}_{\kappa}T_{1/2}$ *and semi-*T_p *.*

Proof. Suppose (X,τ) is ${}_{\kappa}T_b$. Then by Proposition 3.35, (X,τ) is ${}^{*}_{\kappa}T_{1/2}$. Due to Proposition 3.23, (X,τ) is semi-T_p.

Conversely, let (X,τ) be both ${}^{*}_{\kappa}T_{1/2}$ and semi-T_p. Let A be $\hat{g}^*s$-closed in (X,τ). Since (X,τ) is ${}^{*}_{\kappa}T_{1/2}$, by Definition 3.2, A is g^*s-closed. By the fact that (X,τ) is semi-T_p, A is closed. □

Proposition 3.39. *Every* T_c *space is a* ${}_{\kappa}T_c$ *space.*

Proof. Let (X,τ) be a T_c space. Let $A \subseteq X$ be $\hat{g}^*s$-closed. But every $\hat{g}^*s$-closed set is gs-closed. Therefore A is gs-closed in X. Since X is T_c, A is g^*-closed. Therefore X is ${}_\kappa T_c$. □

Remark 3.40. *The converse is not true as seen from the following example.*

Example 3.41. *Let (X,τ) be a topological space where $X=\{a,b,c\}$ with $\tau=\{\phi,X,\{a\},\{b,c\}\}$ and $\tau^c=\{\phi,X,\{b,c\},\{a\}\}$. We have $g^*C(X,\tau)=\{\phi,X,\{b,c\},\{a\}\}$; $\hat{g}^*sC(X,\tau)=\{\phi,X,\{b,c\},\{a\ \}$ and $gsC(X,\tau)=\{\phi,X,$*

$\{b,c\},\{a,c\},\{a,b\},\{c\},\{b\},\{a\}\}$. Since $\hat{g}^*s$-closed sets are all g^*-closed, (X,τ) is κT_c. As, gs-closed sets are all not g^*-closed, (X,τ) is not T_c.

Proposition 3.42. *Every κT_c space is a ${}^*_\kappa T_{1/2}$ space.*

Proof. Let (X,τ) be a ${}_\kappa T_c$ space. Let $A \subseteq X$ be $\hat{g}^*s$-closed. Since (X,τ) is ${}_\kappa T_c$, A is g^*-closed. But every g^*-closed set is g^*s-closed. Therefore (X,τ) is a ${}^*_\kappa T_{1/2}$ space. □

Remark 3.43. *The converse is not true as seen from the following Example.*

Example 3.44. *Let (X,τ) be a topological space where $X=\{a,b,c\}$ with $\tau=\{\phi,X,\{a\},\{b\},\{a,b\}\}$ and $\tau^c=\{\phi,X,\{b,c\},\{a,c\},\{c\}\}$. We have $g^*C(X,\tau)=\{\phi,X,\{b,c\},\{a,c\},\{c\}\}$; $g^*sC(X,\tau)=\{\phi,X,\{b,c\},\{a,c\},\{c\},$*

$\{b\},\{a\}\}$ and $\hat{g}^*sC(X,\tau)=\{\phi,X,\{b,c\},\{a,c\},\{c\},\{b\},\{a\}\}$. Since all $\hat{g}^*s$-closed sets are g^*s-closed, (X,τ) is ${}^*_\kappa T_{1/2}$. Since all $\hat{g}^*s$-closed sets are not g^*-closed, (X,τ) is not ${}_\kappa T_c$.

Proposition 3.45. *Every ${}_\kappa T_b$ space is a ${}_\kappa T_c$ space.*

Proof. Let (X,τ) be a ${}_\kappa T_b$ space. Let $A \subseteq X$ be $\hat{g}^*s$-closed. Since (X,τ) is ${}_\kappa T_b$, A is closed. But every closed set is g^*-closed. Therefore A is g^*-closed. *i.e*, (X,τ) is a ${}_\kappa T_c$ space. □

Remark 3.46. *The converse is not true as seen from the following Example.*

Example 3.47. *Let (X,τ) be a topological space where $X=\{a,b,c\}$ with $\tau=\{\phi,X,\{a,b\}\}$ and $\tau^c=\{\phi,X,\{c\}\}$. We have $g^*C(X,\tau)=\{\phi,X,\{b,c\},$*

$\{a,c\},\{c\}\}$ and $\hat{g}^*sC(X,\tau)=\{\phi,X,\{b,c\},\{a,c\},\{c\}\}$ .Since all $\hat{g}^*s$ -closed sets are g^* -closed, (X,τ) is ${}_{\kappa}T_c$. Since all $\hat{g}^*s$ -closed sets are not closed, (X,τ) is ${}_{\kappa}T_b$.

Proposition 3.48. *Every T_c space is a T_b^{κ} space.*

Proof. Let (X,τ) be a T_c space. Let $A\subseteq X$ be gs-closed. Since (X,τ) is T_c, A is g^*-closed. But every g^*-closed set is $\hat{g}^*s$-closed. Therefore A is $\hat{g}^*s$-closed. *i.e.* (X,τ) is a T_b^{κ} space. □

Remark 3.49. *The converse is not true as seen from the following example.*

Example 3.50. *Let (X,τ) be a topological space where $X=\{a,b,c\}$ with $\tau=\{\phi,X,\{a\},\{b\},\{a,b\}\}$ and $\tau^c=\{\phi,X,\{b,c\},\{a,c\},\{c\}\}$ .We have $g^*C(X,\tau)=\{\phi,X,\{b,c\},\{a,c\},\{c\}\}$; $\hat{g}^*sC(X,\tau)=\{\phi,X,\{b,c\},\{a,c\},\{c\},$ $\{b\},\{a\}\}$ and $gsC(X,\tau)=\{\phi,X,\{b,c\},\{a,c\},\{c\},\{b\},\{a\}\}$ .Since all gs -closed sets are $\hat{g}^*s$ -closed, (X,τ) is T_b^{κ} . Since all gs -closed sets are not g^* -closed, (X,τ) is not T_c .*

Proposition 3.51. *A topological space (X,τ) is T_c iff it is both ${}_{\kappa}T_c$ and T_b^{κ} .*

Proof. Suppose (X,τ) is T_c. Then by Proposition 3.39, (X,τ) is ${}_{\kappa}T_c$. And by Proposition 3.48, (X,τ) is T_b^{κ} .

Conversely, let (X,τ) be both ${}_{\kappa}T_c$ and T_b^{κ}. Let A be gs-closed in (X,τ). Since (X,τ) is T_b^{κ}, by Definition 3.8, A is $\hat{g}^*s$-closed. Since (X,τ) is ${}_{\kappa}T_c$, by Definition 3.4, A is g^*-closed. Therefore (X,τ) is a T_c space. □

Proposition 3.52. *Every T_b space is T_b^{κ} .*

Proof. Let (X,τ) be a T_b space. Let $A\subseteq X$ be gs-closed. Since (X,τ) is T_b, A is closed. But every closed set is $\hat{g}^*s$-closed. Therefore A is $\hat{g}^*s$-closed. *i.e.* (X,τ) is a T_b^{κ} space. □

Remark 3.53. *The converse is not true as seen from the following example.*

Example 3.54. *Let (X,τ) be a topological space where $X=\{a,b,c\}$ with $\tau=\{\phi,X,\{a\},\{b\},\{a,b\}\}$ and $\tau^c=\{\phi,X,\{b,c\},\{a,c\},\{c\}\}$. We have $\hat{g}^*sC(X,\tau)=$*

$\{\phi, X, \{b,c\}, \{a,c\}, \{c\}, \{b\}, \{a\}\}$ *and* $gsC(X,\tau) = \{\phi, X, \{b,c\},$ $\{a,c\}, \{c\}, \{b\}, \{a\}\}$ *.Since all* gs *-closed sets are* $\hat{g}^*s$ *-closed,* (X,τ) *is* T_b^κ *. Since all* gs *-closed sets are not closed,* (X,τ) *is not* T_b *.*

Proposition 3.55. *A topological space* (X,τ) *is* T_b *iff it is both* ${}_\kappa T_b$ *and* T_b^κ *.*

Proof. Suppose (X,τ) is T_b. Then by Proposition 3.26, (X,τ) is ${}_\kappa T_b$. And also by Proposition 3.52 , (X,τ) is T_b^κ.

Conversely, let (X,τ) be both ${}_\kappa T_b$ and T_b^κ. Suppose A is gs-closed in (X,τ). Since (X,τ) is T_b^κ, by Definition 3.8, A is $\hat{g}^*s$-closed. Further, since (X,τ) is ${}_\kappa T_b$, by Definition 3.3 A is closed. Therefore (X,τ) is a T_b space. □

Proposition 3.56. *If a topological space* (X,τ) *is both* ${}_\kappa T_d$ *and* $T_{1/2}$ *, then it is* ${}_\kappa T_b$ *.*

Proof. Let (X,τ) be both ${}_\kappa T_d$ and $T_{1/2}$. Let A be $\hat{g}^*s$-closed in (X,τ). Since (X,τ) is ${}_\kappa T_d$, by Definition 3.5, A is g-closed. Since (X,τ) is $T_{1/2}$, A is closed. Therefore (X,τ) is a ${}_\kappa T_b$ space. □

Proposition 3.57. *If a topological space* (X,τ) *is both* ${}_\kappa T_d$ *and* ${}^*T_{1/2}$ *, then it is* ${}_\kappa T_c$ *.*

Proof. Let (X,τ) be both ${}_\kappa T_d$ and ${}^*T_{1/2}$. Suppose that A is $\hat{g}^*s$-closed in (X,τ). Since (X,τ) is ${}_\kappa T_d$, by Definition 3.5, A is g-closed. But the space (X,τ) is ${}^*T_{1/2}$ and hence A is g^*-closed. Therefore (X,τ) is a ${}_\kappa T_c$ space. □

Proposition 3.58. *Every* ${}_\kappa T_b$ *space is a* $T_{1/2}^*$ *space.*

Proof. Let (X,τ) be a ${}_\kappa T_b$ space. Let $A \subseteq X$ be g^*-closed. But every g^*-closed set is $\hat{g}^*s$-closed. Therefore A is $\hat{g}^*s$-closed. Since (X,τ) is ${}_\kappa T_b$, A is closed. Therefore A is closed. *i.e.* (X,τ) is a $T_{1/2}^*$. □

Remark 3.59. *The converse is not true as seen from the following example.*

Example 3.60. *Let* (X,τ) *be a topological space, where* $X = \{a,b,c\}$ *with* $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$ *and* $\tau^c = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$. *We have* $g^*C(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$ *and* $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\},$ $\{c\}, \{b\}, \{a\}\}$. *Since all* g^* *-closed sets are closed,* (X,τ) *is* $T^*_{1/2}$. *By the fact that all* $\hat{g}^*s$ *-closed sets are not closed,* (X,τ) *is not* ${}_\kappa T_b$.

Proposition 3.61. *A topological space* (X,τ) *is* ${}_\kappa T_b$ *iff it is both* ${}_\kappa T_c$ *and* $T^*_{1/2}$.

Proof. Suppose (X,τ) is ${}_\kappa T_b$. Then by Proposition 3.45, (X,τ) is ${}_\kappa T_c$. And by Proposition 3.58, (X,τ) is $T^*_{1/2}$.

Conversely, let (X,τ) be both ${}_\kappa T_c$ and $T^*_{1/2}$. Let A be $\hat{g}^*s$-closed in (X,τ). Since (X,τ) is ${}_\kappa T_c$, by Definition 3.4, A is g^*-closed. Since (X,τ) is $T^*_{1/2}$, A is closed. Therefore (X,τ) is a ${}_\kappa T_b$. □

Remark 3.62. T_c *space and* ${}_\kappa T_b$ *space are independent of each other as seen from the following two examples.*

Example 3.63. *Let* (X,τ) *be a topological space where* $X = \{a,b,c\}$ *with* $\tau = \{\phi, X, \{a\}, \{b,c\}\}$ *and* $\tau^c = \{\phi, X, \{b,c\}, \{a\}\}$. *We have* $g^*C(X,\tau) = \{\phi, X, \{b,c\}, \{a\}\}$; $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a\ \}$ *and* $gsC(X,\tau) = \{\phi, X,$ $\{b,c\}, \{a,c\}, \{a,b\}, \{c\}, \{b\}, \{a\}\}$. *Since all* $\hat{g}^*s$ *-closed sets are closed,* (X,τ) *is* ${}_\kappa T_b$. *Since all* gs *-closed sets are not* g^* *-closed,* (X,τ) *is not* T_c.

Example 3.64. *Let* (X,τ) *be a topological space where* $X = \{a,b,c\}$ *with* $\tau = \{\phi, X, \{a,b\}\}$ *and* $\tau^c = \{\phi, X, \{c\}\}$. *We have* $g^*C(X,\tau) = \{\phi, X,$ $\{b,c\}, \{a,c\}, \{c\}\}$; $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\},\}$ *and* $gsC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$. *Since all* gs *-closed sets are* g^* *-closed,* (X,τ) *is* T_c. *But since all* $\hat{g}^*s$ *-closed sets are not closed, the space* (X,τ) *is not* ${}_\kappa T_b$.

It follows from Examples 3.63 and 3.64, ${}_\kappa T_b$ *space and* T_c *space are independent of each other.*

Remark 3.65. ${}^*T_{1/2}$ *space and* ${}_\kappa T_c$ *are independent of each other as seen from the following two examples.*

Example 3.66. *Let* (X,τ) *be a topological space where* $X = \{a,b,c,d\}$ *with* $\tau = \{\phi, X, \{a\}, \{a,b,c\}\}$ *and* $\tau^c = \{\phi, X, \{b,c,d\}, \{d\}\}$ *. We have*

$gC(X,\tau) = \{\phi, X, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{c,d\}, \{b,d\}, \{a,d\}, \{d\}\}$ *;*

$g^*C(X,\tau) = \{\phi, X, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{c,d\}, \{b,d\}, \{a,d\}, \{d\}\}$ *and* $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{a,d\}, \{d\},$
$\{c\}, \{b\}\}$ *. Since all* g *-closed sets are* g^* *-closed,* (X,τ) *is* ${}^*T_{1/2}$ *. Since all* $\hat{g}^*s$ *-closed sets are not* g^* *-closed,* (X,τ) *is not* ${}_\kappa T_c$ *.*

Example 3.67. *Let* (X,τ) *be a topological space where* $X = \{a,b,c\}$ *with* $\tau = \{\phi, X, \{a\}, \{b,c\}\}$ *and* $\tau^c = \{\phi, X, \{b,c\}, \{a\}\}$ *. We have* $gC(X,\tau) = P(X)$ *;* $g^*C(X,\tau) = \{\phi, X, \{b,c\}, \{a\}\}$ *and* $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a\}\}$ *. Since all* $\hat{g}^*s$ *-closed sets are* g^* *-closed,* (X,τ) *is* ${}_\kappa T_c$ *. And Since all* g *-closed sets are not* g^* *-closed,* (X,τ) *is not* ${}^*T_{1/2}$

It follows from Examples 3.66 and 3.67, ${}^*T_{1/2}$ *space and* ${}_\kappa T_c$ *space are independent of each other.*

Remark 3.68. ${}^*T_{1/2}$ *space and* ${}_\kappa T_b$ *space are independent of each other as seen from the following two examples.*

Example 3.69. *Let* (X,τ) *be a topological space where* $X = \{a,b,c,d\}$ *with* $\tau = \{\phi, X, \{a\}, \{a,b,c\}\}$ *and* $\tau^c = \{\phi, X, \{b,c,d\}, \{d\}\}$ *. We have* $gC(X,\tau) = \{\phi, X, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{c,d\}, \{b,d\}, \{a,d\}, \{d\}\}$ *;* $g^*C(X,\tau) = \{\phi, X,$
$\{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{c,d\}, \{b,d\}, \{a,d\}, \{d\}\}$ *and* $\hat{g}^*sC(X,\tau) = \{\phi, X,$
$\{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{c,d\}, \{b,d\}, \{b,c\}, \{a,d\}, \{d\}, \{c\}, \{b\}\}$ *. Since all* g *-closed sets are* g^* *-closed,* (X,τ) *is* ${}^*T_{1/2}$ *. Since all* $\hat{g}^*s$ *-closed sets are not closed,* (X,τ) *is not* ${}_\kappa T_b$ *.*

Example 3.70. *Let* (X,τ) *be a topological space where* $X = \{a,b,c\}$ *with* $\tau = \{\phi, X, \{a\}, \{b,c\}\}$ *and* $\tau^c = \{\phi, X, \{a\}, \{b,c\}\}$ *. We have* $gC(X,\tau) = P(X)$ *;* $g^*C(X,\tau) = \{\phi, X, \{b,c\}, \{a\ \}$ *and* $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a\}\}$ *.*

Since all $\hat{g}^*s$ *-closed sets are closed,* (X,τ) *is* ${}_\kappa T_b$ *. It is obvious that all* g *-closed sets are not* g^* *-closed, thus* (X,τ) *is not* ${}^*T_{1/2}$

Therefore from Examples 3.69 and 3.70, $^*T_{1/2}$ *space and* $_\kappa T_b$ *space are independent of each other.*

Remark 3.71. *semi-*$T_{1/2}$ *space and* $_\kappa T_{1/2}$ *space are independent of each other as seen from the following two examples.*

Example 3.72. *Let* (X,τ) *be a topological space where* $X=\{a,b,c,d\}$ *with* $\tau=\{\phi,X,\{a\},\{a,b,c\}\}$ *and* $\tau^c=\{\phi,X,\{b,c,d\},\{d\}\}$ *. We have* $SC(X,\tau)=\{\phi,X,\{b,c,d\},\{c,d\},\{b,d\},\{b,c\},\{d\},\{c\},\{b\}\}$ *;* $sgC(X,\tau)=\{\phi,X,\{b,c,d\},\{c,d\},\{b,d\},\{b,c\},\{d\},\{c\},\{b\}\}$ *and* $\hat{g}^*sC(X,\tau)=\{\phi,X,\{b,c,d\},\{a,c,d\},\{a,b,d\},\{c,d\},\{b,d\},\{b,c\},\{a,d\},\{d\},\{c\},\{b\}\}$ *. Since all* sg *-closed sets are semi closed,* (X,τ) *is semi-*$T_{1/2}$ *. Moreover, since all* $\hat{g}^*s$ *-closed sets are not semi closed,* (X,τ) *is not* $_\kappa T_{1/2}$ *.*

Example 3.73. *Let* (X,τ) *be a topological space where* $X=\{a,b,c\}$ *with* $\tau=\{\phi,X,\{a\},\{b,c\}\}$ *and* $\tau^c=\{\phi,X,\{b,c\},\{a\}\}$ *. We have* $SC(X,\tau)=\{\phi,X,\{b,c\},\{a\ \}$ *;* $sgC(X,\tau)=P(X)$ *and* $\hat{g}^*sC(X,\tau)=\{\phi,X,\{b,c\},\{a\}\}$ *.*

Since all $\hat{g}^*s$ *-closed sets are semi closed,* (X,τ) *is* $_\kappa T_{1/2}$ *. Since all* sg *-closed sets are not semi closed,* (X,τ) *is not semi-*$T_{1/2}$ *.*

Therefore from Examples 3.72 and 3.73,semi-$T_{1/2}$ *space and* $_\kappa T_{1/2}$ *space are independent of each other.*

Remark 3.74. $_\kappa T_{1/2}$ *and* $_\kappa T_c$ *are independent of each other as seen from the following two examples.*

Example 3.75. *Let* (X,τ) *be a topological space where* $X=\{a,b,c\}$ *with* $\tau=\{\phi,X,\{a\},\{b\},\{a,b\}\}$ *and* $\tau^c=\{\phi,X,\{b,c\},\{a,c\},\{c\}\}$ *. We have* $SC(X,\tau)=\{\phi,X,\{b,c\},\{a,c\},\{c\},\{b\},\{a\}\}$ *;* $g^*C(X,\tau)=\{\phi,X,\{b,c\},\{a,c\},\{c\}\}$ *and* $\hat{g}^*sC(X,\tau)=\{\phi,X,\{b,c\},\{a,c\},\{c\},\{b\},\{a\}\}$ *. Since all* $\hat{g}^*s$ *-closed sets are semi closed,* (X,τ) *is* $_\kappa T_{1/2}$ *. Since all* $\hat{g}^*s$ *-closed sets are not* g^* *-closed,* (X,τ) *is not* $_\kappa T_c$ *.*

Example 3.76. *Let* (X,τ) *be a topological space where* $X=\{a,b,c\}$ *with* $\tau=\{\phi,X,\{a,b\}\}$ *and* $\tau^c=\{\phi,X,\{c\}\}$ *. We have* $SC(X,\tau)=\{\phi,X,\{c\}\}$ *;*

$g^*C(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$ *and* $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\},$ $\{c\}\}$ *.Since all* $\hat{g}^*s$ *-closed sets are not semi closed,* (X,τ) *is not* ${}_\kappa T_{1/2}$ *. Since all* $\hat{g}^*s$ *-closed sets are* g^* *-closed,* (X,τ) *is* ${}_\kappa T_c$ *.*

It follows from Examples 3.75 and 3.76, ${}_\kappa T_{1/2}$ *and* ${}_\kappa T_c$ *are independent of each other.*

Remark 3.77. ${}_\kappa T_b$ *and* T_b^κ *are independent of each other as seen from the following two examples.*

Example 3.78. *Let* (X,τ) *be a topological space where* $X = \{a,b,c\}$ *with* $\tau = \{\phi, X, \{a\}, \{b,c\}\}$ *and* $\tau^c = \{\phi, X, \{b,c\}, \{a\}\}$ *.We have* $gsC(X,\tau) = P(X)$ *and* $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a\}\}$ *.Since all* $\hat{g}^*s$ *-closed sets are closed,* (X,τ) *is* ${}_\kappa T_b$ *. Since all* gs *-closed sets are not* $\hat{g}^*s$ *-closed,* (X,τ) *is not* T_b^κ *.*

Example 3.79. *Let* (X,τ) *be a topological space where* $X = \{a,b,c\}$ *with* $\tau = \{\phi, X, \{a,b\}\}$ *and* $\tau^c = \{\phi, X, \{c\}\}$ *.* $gsC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\},$ $\{c\}\}$ *and* $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$ *.Since all* gs *-closed sets are* $\hat{g}^*s$ *-closed,* (X,τ) *is* T_b^κ *. Since all* $\hat{g}^*s$ *-closed sets are not closed,* (X,τ) *is* ${}_\kappa T_b$ *.*

It follows from Examples 3.78 and 3.79, T_b^κ *and* ${}_\kappa T_b$ *are independent of each other.*

Remark 3.80. T_b^κ *and* ${}_\kappa T_c$ *are independent of each other as seen from the following two examples.*

Example 3.81. *Let* (X,τ) *be a topological space where* $X = \{a,b,c\}$ *with* $\tau = \{\phi, X, \{a\}, \{b,c\}\}$ *and* $\tau^c = \{\phi, X, \{b,c\}, \{a\}\}$ *. We have* $g^*C(X,\tau) = \{\phi, X, \{b,c\}, \{a\}\}$ *;* $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a\}\}$ *and* $gsC(X,\tau) = P(X)$ *.*

Since all $\hat{g}^*s$ *-closed sets are* g^* *-closed,* (X,τ) *is* ${}_\kappa T_c$ *. And since all* gs *-closed sets are not* $\hat{g}^*s$ *-closed,* (X,τ) *is not* T_b^κ *.*

Example 3.82. *Let* (X,τ) *be a topological space where* $X = \{a,b,c\}$ *with* $\tau = \{\phi, X, \{a\}, \{a,b\}\}$ *and* $\tau^c = \{\phi, X, \{b,c\}, \{c\}\}$ *.We have* $g^*C(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$ *;* $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}, \{b\}\}$ *and*

$gsC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}, \{b\}\}$. *Since all* $\hat{g}^*s$ *-closed sets are not* g^* *-closed,* (X,τ) *is not* ${}_\kappa T_c$. *Since all* gs *-closed sets are* $\hat{g}^*s$ *-closed,* (X,τ) *is* T_b^κ .

Therefore from Examples 3.81 and 3.82, T_b^κ *and* ${}_\kappa T_c$ *are independent of each other.*

Remark 3.83. *From the above Definitions and Remarks, we obtain the following comparative diagram.*

Proposition 3.84. *In a topological space* (X,τ) *which is both* T_d *and* T_f *, every* $\hat{g}^*s$ *-closed set in it is* $\hat{g}$ *-closed.*

Proof. Let (X,τ) be both T_d and T_f . Let $A \subseteq X$ be $\hat{g}^*s$-closed. But every $\hat{g}^*s$-closed set is gs-closed a set. Therefore A is gs-closed. Since X is T_d , A is g-closed. By the fact that X is T_d , then A is g-closed. □

Proposition 3.85. *Every* $\hat{g}^*s$ *-closed set in a* ${}_\kappa T_b$ *space is*

(i) g *-closed.*

(ii) $\hat{g}$ *-closed.*

(iii) g^* *-closed.*

(iv) $\hat{g}^*$ *-closed.*

(v) semi closed.

(vi) g^*s *-closed.*

(vii) sg *-closed.*

(viii) gs *-closed.*

(ix) ψ *-closed.*

(x) α *-closed.*

(xi) αg *-closed.*

(xii) $g\alpha$ *-closed.*

(xiii) $\hat{g}^*\alpha$ *-closed.*

(xiv) pre closed.

(xv) gp closed.

Proof. (i) Let X be a ${}_{\kappa}T_b$ space and A $\hat{g}^*s$-closed in X.

Consequently, by Definition 3.3, A is closed. But every closed set is g-closed. Therefore A is g-closed.

(ii) Since every closed set is $\hat{g}$-closed.

(iii) Since every closed set is g^*-closed.

(iv) Since every closed set is $\hat{g}^*$-closed.

(v) Since every closed set is semi closed.

(vi) Since every semi closed set is g^*s-closed.

(vii) Since every semi closed set is sg-closed.

$(viii)$ Since every semi closed set is gs-closed.

(ix) Since every semi closed set is ψ-closed.

(x) Since every closed set is α-closed.

(xi) Since every α-closed set is $g\alpha$-closed.

(xii) Since every $g\alpha$-closed set is αg-closed.

$(xiii)$ Since every α-closed set is $\hat{g}^*\alpha$-closed.

(xiv) Since every closed set is pre-closed.

(xv) Since every pre closed set is gp-closed. □

Proposition 3.86. *Every gs-closed set in a T_b space is $\hat{g}^*s$-closed.*

Proof. Let X be a T_b space and A a gs-closed set in X.

Consequently, A is closed. But every closed set is semi closed and every semi closed set is $\hat{g}^*s$-closed. Therefore A is $\hat{g}^*s$-closed. □

Proposition 3.87. *Every gs-closed set in a T_c space is $\hat{g}^*s$-closed.*

Proof. Let X be a T_c space and A a gs-closed set in X.

Consequently, A is g^* closed. But every g^*-closed set is g^*s-closed and every g^*s-closed set is $\hat{g}^*s$-closed. Therefore A is $\hat{g}^*s$-closed. □

Proposition 3.88. *Every $\hat{g}^*s$-closed set in a ${}_\kappa T_{1/2}$ space is*

(i) sg-closed.

(ii) g^*s-closed.

(iii) ψ-closed.

Proof. (i) Let X be a ${}_\kappa T_{1/2}$ space and A be $\hat{g}^*s$-closed in X.

Consequently, A is semi closed. But every semi closed set is sg-closed. Hence A is sg-closed.

(ii) Since every semi closed set is g^*s-closed.

(iii) Since every semi closed set is ψ-closed. □

Proposition 3.89. *Every $\hat{g}^*s$-closed set in a ${}_{\kappa}T_c$ space is*

(i) $\hat{g}^*$-closed and hence g-closed.

(ii) g^*s-closed.

Proof. (i) Let X be a ${}_{\kappa}T_c$ space and A be $\hat{g}^*s$-closed in X.

Consequently, A is g^*-closed.But every g^*-closed set is $\hat{g}^*$-closed and every $\hat{g}^*$-closed set is g-closed.Hence (i) follows.

(ii) Since every g^*-closed set is g^*s-closed. Therefore A is g^*s-closed. □

Proposition 3.90. *Every $\hat{g}^*s$-closed set in a ${}_{\kappa}T_{\hat{g}^*}$ space is g-closed.*

Proof. Let X be a ${}_{\kappa}T_{\hat{g}^*}$ space and $A \subseteq X$ be a $\hat{g}^*s$-closed set. Since X is ${}_{\kappa}T_{\hat{g}^*}$, A is $\hat{g}^*$-closed. But every $\hat{g}^*$-closed set is g-closed. Therefore, A is g-closed. □

Proposition 3.91. *Let the topological space (X,τ) be a ${}_{\kappa}T_c$ space. Then for each $x \in X, \{x\}$ is $\hat{g}$-closed or g^*-open.*

Proof. Suppose $\{x\}$ is not $\hat{g}$-closed. Then $X - \{x\}$ is not $\hat{g}$-open. Therefore X is the only $\hat{g}$-open set containing $X - \{x\}$. Hence $scl\,(X - \{x\}) \subseteq X$.

Therefore $X - \{x\}$ is $\hat{g}^*s$-closed. But in a ${}_{\kappa}T_c$ space, every $\hat{g}^*s$-closed set is g^*-closed. Thus, $X - \{x\}$ is g^*-closed. Hence $\{x\}$ is g^*-open. □

Remark 3.92. *The converse need not be true as seen from the following example.*

Example 3.93. *Let (X,τ) be a topological space where $X = \{a,b,c\}$ with $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$ and $\tau^c = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$. We have $g^*C(X,\tau) = \{\phi, X, \{c\}, \{a,c\}, \{b,c\}\}$; $g^*O(X,\tau) = \{\phi, X, \{a,b\}, \{b\}, \{a\}\}$; $\hat{g}C(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$ and $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\},$*

$\{c\}, \{b\}, \{a\}, \}$. *In this example, for every* $x \in X, \{x\}$ *is either* $\hat{g}$*-closed or* g^{*}*-open. But, not every* $\hat{g}^{*}s$*-closed set is* g^{*}*-closed. i.e.,* (X, τ) *is not* ${}_{\kappa}T_{c}$.

Proposition 3.94. *Let the topological space* (X, τ) *be a* ${}_{\kappa}T_{b}$ *space. Then for each* $x \in X, \{x\}$ *is* $\hat{g}$*-closed or open.*

Proof. Suppose $\{x\}$ is not $\hat{g}$-closed. Then $X - \{x\}$ is not $\hat{g}$-open. Therefore X is the only $\hat{g}$-open set containing $X - \{x\}$. Thus $scl\,(X - \{x\}) \subseteq X$. Therefore $X - \{x\}$ is $\hat{g}^{*}s$-closed. But in a ${}_{\kappa}T_{b}$ space, every $\hat{g}^{*}s$-closed set is closed. Hence $X - \{x\}$ is closed. It follows that $\{x\}$ is open. □

Remark 3.95. *The converse of the above Proposition need not be true as seen from the following Example.*

Example 3.96. *Let* (X, τ) *be a topological space where* $X = \{a, b, c\}$ *with* $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ *and* $\tau^{c} = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$*. We have* $\hat{g}C(X, \tau) = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$ *and* $\hat{g}^{*}sC(X, \tau) = \{\phi, X, \{b, c\}, \{a, c\},$
$\{c\}, \{b\}, \{a\}, \}$. *In this example, for every* $x \in X, \{x\}$ *is either* $\hat{g}$*-closed or open. But, not every* $\hat{g}^{*}s$*-closed set is closed. i.e.,* (X, τ) *is not* ${}_{\kappa}T_{b}$.

Proposition 3.97. *Let the topological space* (X, τ) *be a* ${}_{\kappa}T_{d}$ *space. Then for each* $x \in X, \{x\}$ *is* $\hat{g}$*-closed or* g*-open.*

Proof. Suppose $\{x\}$ is not $\hat{g}$-closed. Then $X - \{x\}$ is not $\hat{g}$-open. Then X is the only $\hat{g}$-open set containing $X - \{x\}$. Therefore $scl\,(X - \{x\}) \subseteq X$.

Therefore $X - \{x\}$ is $\hat{g}^{*}s$-closed. But in a ${}_{\kappa}T_{d}$ space, every $\hat{g}^{*}s$-closed set is g-closed. Hence $X - \{x\}$ is g-closed. Therefore $\{x\}$ is g-open. □

Remark 3.98. *The converse of the above Proposition need not be true as seen from the following Example.*

Example 3.99. *Let* (X, τ) *be a topological space where* $X = \{a, b, c\}$ *with* $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ *and* $\tau^{c} = \{\phi, X, \{b, c\}, \{a, c\}, \{c\}\}$*. We* *have* $\hat{g}C(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$ *;* $gO(X,\tau) = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$ *;* $gC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$ *and* $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}, \{b\}, \{a\}\}$ *. From this Example, we find out* $\{x\}$ *is either* $\hat{g}$ *-closed or* g *-open. But not every* $\hat{g}^*s$ *-closed set is* g *-closed. Therefore* (X,τ) *is not a* ${}_\kappa T_d$ *space.*

Proposition 3.100. *Let the topological space* (X,τ) *be a* ${}_\kappa T_f$ *space. Then for each* $x \in X, \{x\}$ *is* $\hat{g}$ *-closed or* $\hat{g}$ *-open.*

Proof. Suppose $\{x\}$ is not $\hat{g}$-closed. Then $X - \{x\}$ is not $\hat{g}$-open. Then X is the only $\hat{g}$-open set containing $X - \{x\}$. Therefore $scl\,(X - \{x\}) \subseteq X$ .Therefore $X - \{x\}$ is $\hat{g}^*s$-closed.But in a ${}_\kappa T_f$ space, every $\hat{g}^*s$-closed set is $\hat{g}$-closed.Thus $X-\{x\}$ is $\hat{g}$-closed. Hence $\{x\}$ is $\hat{g}$-open. □

Remark 3.101. *The converse of the above Proposition need not be true as seen from the following Example.*

Example 3.102. *Let* (X,τ) *be a topological space, where* $X = \{a,b,c\}$ *with* $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$ *and* $\tau^c = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$ *.We have* $\hat{g}C(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$ *;* $\hat{g}O(X,\tau) = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$ *and* $\hat{g}^*sC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}, \{b\}, \{a\}, \}$ *. In this example, for every* $x \in X, \{x\}$ *is either* $\hat{g}$ *-closed or* $\hat{g}$ *-open. But, not every* $\hat{g}^*s$ *-closed set is* $\hat{g}$ *-closed. This means that* (X,τ) *is not* ${}_\kappa T_f$ *.*

Proposition 3.103. *Let the topological space* (X,τ) *be a* ${}_\kappa T_{\hat{g}^*}$ *space.Then for each* $x \in X, \{x\}$ *is either* $\hat{g}$ *-closed or* $\hat{g}^*$ *-open.*

Proof. Suppose that for some $x \in X, \{x\}$ is not $\hat{g}$-closed. Then $X - \{x\}$ is not $\hat{g}$-open. Thus X is the only $\hat{g}$-open set containing $X - \{x\}$. Therefore $scl\,(X - \{x\}) \subseteq X$. Hence $X - \{x\}$ is $\hat{g}^*s$-closed. Since X is ${}_\kappa T_{\hat{g}^*}, X - \{x\}$ is $\hat{g}^*$-closed.It follows that $\{x\}$ is $\hat{g}^*$-open. □

Remark 3.104. *The converse need not be true as seen from the following example.*

Example 3.105. *Let* (X,τ) *be a topological space where* $X = \{a,b,c\}$ *with* $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$ *and* $\tau^{c} = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$. *We have* $\hat{g}C(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$ *;* $\hat{g}^{*}O(X,\tau) = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$ *;*

$\hat{g}^{*}C(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}\}$ *and* $\hat{g}^{*}sC(X,\tau) = \{\phi, X, \{b,c\}, \{a,c\}, \{c\}, \{b\}, \{a\}\}$. *In this example, for every* $x \in X, \{x\}$ *is either* $\hat{g}$ *-closed or* $\hat{g}^{*}$ *-open. But, not every* $\hat{g}^{*}s$ *-closed set is* $\hat{g}^{*}$ *-closed. i.e.,* (X,τ) *is not* ${}_{\kappa}T_{\hat{g}^{*}}$

Proposition 3.106. *For a topological space* (X,τ), *the following are equivalent.*

(i) Suppose (X,τ) *is* ${}_{\kappa}T_{1/2}$.

(ii) Every $\hat{g}^{*}$ *-closed set in it is semi closed.*

(iii) Every singleton $\{x\}$ *of* X *is either semi open or* $\hat{g}$ *-closed.*

Proof. $(i) \Rightarrow (iii)$

Suppose that for some $x \in$ X , $\{x\}$ is not $\hat{g}$-closed.Then $X - \{x\}$ is not $\hat{g}$-open. Then X is the only $\hat{g}$-open set containing $X - \{x\}$. Thus $scl\,(X - \{x\}) \subseteq X$. Therefore $X - \{x\}$ is $\hat{g}^{*}s$-closed. Since X is ${}_{\kappa}T_{1/2}$, $X - \{x\}$ is semi closed. Hence $\{x\}$ is semi open.

$(iii) \Rightarrow (i)$

Let A be $\hat{g}^{*}s$-closed in X. To show that A is semi closed in X, it means to show that $scl\,(A) = A$.

Let $x \in scl\,(A)$. Then $\{x\}$ is $\hat{g}$-closed or semi open.

$Case\,(i)$: Let $\{x\}$ be $\hat{g}$-closed.

Suppose $x \notin A$. Therefore $x \in scl\,(A) - A$. Thus $\{x\} \subseteq scl\,(A) - A$. Then by Proposition 2.27, A is not $\hat{g}^{*}s$-closed in X which is a contradiction. Therefore $x \in A$.

$Case\,(ii)$: Suppose $\{x\}$ is semi open. Then $\{x\} \cap A \neq \phi$. Therefore $x \in A$.Thus in both cases $x \in A$. Thus $scl\,(A) \subseteq A$. Hence A is semi closed in X. It follows that X is ${}_{\kappa}T_{1/2}$.

$(ii) \Rightarrow (iii)$

Let $x \in X$. Suppose that for some $x \in X, \{x\}$ is not $\hat{g}$-closed. Then $X - \{x\}$ is not $\hat{g}$-open. It follows that X is the only $\hat{g}$-open set containing $X - \{x\}$. Thus $cl\,(X - \{x\}) \subseteq X$ .Therefore $X - \{x\}$ is $\hat{g}^*$-closed. Therefore, by assumption, $X - \{x\}$ is semi closed. Hence $\{x\}$ is semi open.

$(iii) \Rightarrow (ii)$

Let A be $\hat{g}^*$-closed in X . Showing that A is semi closed in X , it means that to show that $scl\,(A) = A$ .Let $x \in scl\,(A)$. Then $\{x\}$ is $\hat{g}$-closed or semi open.

$Case\,(i)$: Let $\{x\}$ be $\hat{g}$-closed.Suppose $x \notin A$ .Therefore $x \in scl\,(A) - A$. Thus $\{x\} \subseteq scl\,(A) - A$. Therefore $\{x\} \subseteq cl\,(A) - A$. It follows, by Lemma 2.26, A is not $\hat{g}^*$-closed in X which is a contradiction. Therefore $x \in A$.

$Case\,(ii)$: Suppose $\{x\}$ is semi open. Then $\{x\} \cap A \neq \phi$. Therefore $x \in A$. Thus in both cases $x \in A$. This means that $scl\,(A) \subseteq A$. Therefore A is semi closed. □

Proposition 3.107. *For a topological space* (X, τ) *, the following are equivalent.*

(i) *Suppose* (X, τ) *is* ${}^{*}_{\kappa}T_{1/2}$.

(ii) *Every* $\hat{g}^*$ *-closed set in it is* g^*s *-closed.*

(iii) *Every singleton* $\{x\}$ *of* X *is* $\hat{g}$ *-closed or* g^*s *-open.*

Proof. $(i) \Rightarrow (iii)$

Suppose that for some $x \in X, \{x\}$ is not $\hat{g}$-closed. Then $X - \{x\}$ is not $\hat{g}$-open. Thus X is the only $\hat{g}$-open set containing $X - \{x\}$. Therefore $scl\,(X - \{x\}) \subseteq X$. It follows that $X - \{x\}$ is $\hat{g}^*s$-closed. Since X is ${}^{*}_{\kappa}T_{1/2}, X - \{x\}$ is g^*s-closed. Therefore $\{x\}$ is g^*s-open.

$(iii) \Rightarrow (i)$

Let A be $\hat{g}^*s$-closed in X .To show that A is g^*s-closed in X. *i.e.* to show that $g^*scl\,(A) = A$ .Let $x \in g^*scl\,(A)$ .Then $\{x\}$ is $\hat{g}$-closed or g^*s-open.

$Case\,(i)$: Let $\{x\}$ be $\hat{g}$-closed. Since $x \in g^*scl\,(A)$,by Proposition 2.28, we have $x \in$

$scl(A)$.Suppose $x \notin A$. Therefore $x \in scl(A) - A$. It follows that $\{x\} \subseteq scl(A) - A$.Then, by Proposition 2.27, A is not $g^{*}s$-closed which is a contradiction. Hence $x \in A$.

Case (ii) : Suppose $\{x\}$ is $g^{*}s$-open.Then $\{x\} \cap A \neq \phi$.Therefore $x \in A$.Thus in both cases $x \in A$. Thus $g^{*}scl(A) \subseteq A$. Therefore A is $\hat{g}^{*}s$-closed in X. Hence X is ${}^{*}_{\kappa}T_{1/2}$.

$(ii) \Rightarrow (iii)$

Suppose that for some $x \in$ X, $\{x\}$ is not $\hat{g}$-closed. Then $X - \{x\}$ is not $\hat{g}$-open. This implies that X is the only $\hat{g}$-open set containing $X - \{x\}$. Therefore $cl(X - \{x\}) \subseteq X$. Thus $X - \{x\}$ is $\hat{g}^{*}$-closed. By our assumption, every $\hat{g}^{*}$-closed set is $g^{*}s$-closed.Therefore $X - \{x\}$ is $g^{*}s$-closed. Hence $\{x\}$ is $g^{*}s$-open.

$(iii) \Rightarrow (ii)$

Let A be $\hat{g}^{*}$-closed in X.To show that A is $g^{*}s$-closed in X. *i.e.* to show that $g^{*}scl(A) = A$. Let $x \in g^{*}scl(A)$. Then $\{x\}$ is $\hat{g}$-closed or $g^{*}s$-open.

Case (i) : Let $\{x\}$ be $\hat{g}$-closed.Since, every closed set is $g^{*}s$-closed, we have $g^{*}scl(A) \subseteq cl(A)$, we have $x \in cl(A)$.Suppose $x \notin A$. Therefore $x \in cl(A) - A$. It follows that $\{x\} \subseteq cl(A) - A$. Thus, by Lemma 2.26 , A is not $\hat{g}^{*}$-closed in X which is a contradiction. Hence $x \in A$.

Case (ii) : Suppose $\{x\}$ is $g^{*}s$-open. Then $\{x\} \cap A \neq \phi$. Therefore $x \in A$. Thus in both cases $x \in A$. It follows that $g^{*}scl(A) \subseteq A$. Therefore A is $g^{*}s$-closed in X. Hence X is ${}^{*}_{\kappa}T_{1/2}$. □

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References

[1] S. P. Arya and T. Nour, *Characterizations of S-normal spaces*, Indian J. Pure Appl. Math.,21(1990),717-719.

[2] K. Balachandran, H. Maki and P. Sundaram, *On generalized continuous maps in topological spaces*,Mem. Fac. Sci. Kochi Univ. Ser.A. Math.,12(1991), 5-13.

[3] P. Bhattacharya and B. K. Lahiri, *Semi-generalized closed sets in Topology*, Indian J. Math.,29(1990),No.8,717-719.

[4] K. Chandrasekararao and D. Narasimhan, *Pairwise T_S -spaces*,The journal of Mathematics, Vol.6, (2) (2008),Number 1 : 1-8.

[5] R. Devi,H. Maki and K. Balachandran, *Semi generalised closed maps and generalised semi closed maps*,Mem. Fac. Sci. Kochi Univ. Ser.A. Math.,14(1993),41-54.

[6] A. I. El-Maghrabi and A. A. Nasaf,*Between semi closed and gs -closed sets*, Journal of Taibah University for Scince 2: 78-87(2009).

[7] N. Levine,*Semi open sets and semi continuity in topological spaces*, Amer. Math. Monthly 70 (1963) 36-41.

[8] N. Levine,*Generalised closed sets in topology*, Rend. Circ. Mat. Palermo 19 (2) (1970) 89-96.

[9] H. Maki,R. Devi and K. Balachandran, *Generalized α -closed maps and α -generalized closed maps*, Indian J. Pure Appl. Math.,29(1998),No.1,37-49

[10] H.Maki,R.Devi and K.Balachandran, *Semi-generalized closed maps and generalized semi closed maps*, Mem. Fac. Kochi UNiv. sec. A Math., 14(1993),41-54.

[11] H. Maki,R. Devi and K. Balachandran, *Generalized α -closed sets in topology*,Bull. Fukuoka Univ. Ed. PartIII, 42(1993), 13-21.

[12] H.Maki,R.Devi and K.Balachandran, *Associated topologies of generalized α -closed sets and α -generalised closed sets*,Mem. Fac. Sci. Kochi Univ. Ser. A. Math.,15(1994), 51-63.

[13] A. S. Mashhour, M. E. Abd. El - Monsef and S. N. El-Deeb, *On pre-continuous and weak pre-continuous mappings*, Proc. Math. and Phys. Soc. Egypt. 53 (1982),47-53.

[14] O. Njastad, *On some classes of nearly open sets*, Pacific J. Math. 15(1965), 961-970.

[15] P. Bhattacharya and B. K. Lahiri, *Semi generalized closed sets*, Indian J. of Mathematics, Vol. 29, No.3. 1987, 375-382.

[16] S. Poius Missier and M. Anto, *$\hat{g}^*s$-closed sets in topological spaces*,Int. J. modern Eng. Research, 4(11)(2014),32-38

[17] M. K. R. S. Veerakumar, *Between closed sets and g-closed sets*, Mem. Fac. Sci. Kochi Univ. Ser.A. Math.,21(2000), 1-19.

[18] M. K. R. S. Veerakumar, *Between semi closed sets and semi pre closed sets*, Rend.Istit. Mat. Univ. Trieste, Vol.XXXI: 25-41(2000).

[19] M. K. R. S. Veerakumar, *Between g^*-closed sets and g-closed sets*,Antartica J. Math.,3(1)(2006),43-65.

[20] M. K. R. S. Veerakumar, *On $\hat{g}$-locally closed sets and $\hat{g}LC$-functions*, Indian Journal of Mathematics, Vol.43, No.2,2001,231-247.

[21] M. K. R. S. Veerakumar,*On $\hat{g}$-closed sets in topological spaces*,Bull. Allahabad Math. Soc., 18,pp.99-112,(2003).

[22] M. K. R. S. Veerakumar,*On $\hat{g}$-closed sets in topological spaces*,Bull. Allahabad Math. Soc., 18,pp.99-112,(2003).

[23] M. K .R. S. Veerakumar, *μ-closed sets in topological spaces*, Antartica J. Math.,2(1)(2005),1-18.

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