

**On the Ramanujan's mathematics (Rogers-Ramanujan continued fractions, Hardy-Ramanujan number and Manuscript Book 1 formulae) applied to various sectors of String Theory: Further new possible mathematical connections XIII.**

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**Abstract**

*In this research thesis, we have analyzed and deepened further Ramanujan expressions (Rogers-Ramanujan continued fractions, Hardy-Ramanujan number and Manuscript Book 1 formulae) applied to some sectors of String Theory. We have therefore described other new possible mathematical connections.*

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<https://www.britannica.com/biography/Srinivasa-Ramanujan>

ff

(i)  $\frac{1+53x+9x^2}{1-82x-82x^2+x^3} = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$   
 or  $\frac{\alpha_0}{x} + \frac{\alpha_1}{x^2} + \frac{\alpha_2}{x^3} + \dots$

(ii)  $\frac{2-26x-12x^2}{1-82x-82x^2+x^3} = b_0 + b_1x + b_2x^2 + b_3x^3 + \dots$   
 or  $\frac{\beta_0}{x} + \frac{\beta_1}{x^2} + \frac{\beta_2}{x^3} + \dots$

(iii)  $\frac{2+8x-10x^2}{1-82x-82x^2+x^3} = c_0 + c_1x + c_2x^2 + c_3x^3 + \dots$   
 or  $\frac{\gamma_0}{x} + \frac{\gamma_1}{x^2} + \frac{\gamma_2}{x^3} + \dots$

then

$$\left. \begin{aligned} a_n^3 + b_n^3 &= c_n^3 + (-1)^n \\ \text{and } d_n^3 + \beta_n^3 &= \gamma_n^3 + (-1)^n \end{aligned} \right\}$$

Examples

$$135^3 + 138^3 = 172^3 - 1$$

$$11161^3 + 11468^3 = 14258^3 + 1$$

$$791^3 + 812^3 = 1010^3 - 1$$

$$9^3 + 10^3 = 12^3 + 1$$

$$6^3 + 8^3 = 9^3 - 1$$

<https://plus.maths.org/content/ramanujan>

## Ramanujan's manuscript

The representations of 1729 as the sum of two cubes appear in the bottom right corner. The equation expressing the near counter examples to Fermat's last theorem appears further up:  $\alpha^3 + \beta^3 = \gamma^3 + (-1)^n$ .

From Wikipedia

The **taxicab number**, typically denoted  $Ta(n)$  or  $Taxicab(n)$ , also called the  $n$ th **Hardy–Ramanujan number**, is defined as the smallest integer that can be expressed as a sum of two positive integer cubes in  $n$  distinct ways. The most famous taxicab number is  $1729 = Ta(2) = 1^3 + 12^3 = 9^3 + 10^3$ .

## A superfield constraint for $N = 2 \rightarrow N = 0$ breaking

*E. Dudas, S. Ferrara and A. Sagnotti* – arXiv:1707.03414v1 [hep-th] 11 Jul 2017

$$F = -\frac{1}{2}(m + ie) + \mathcal{O}(4 - \text{Fermi}), \quad D = -\frac{\xi}{2} + \mathcal{O}(4 - \text{Fermi}).$$

$$m = \pm \frac{\xi}{\sqrt{2}}.$$

$$m = \mp e_2$$

$$D = -\frac{\xi m}{2e_2 \sqrt{1 + \frac{\xi^2}{2e_2^2}}} \sqrt{1 + \frac{1}{m^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4m^4} (F \cdot \tilde{F})^2}.$$

$$F_{\mu\nu} \longrightarrow F_{\mu\nu} \left[ \frac{m}{e_2 \sqrt{1 + \frac{\xi^2}{2e_2^2}}} \right]^{\frac{1}{2}},$$

$$(1/(\sqrt{1+1/(2*1/2)}))^{1/2}$$

**Input:**

$$\sqrt{\frac{1}{\sqrt{1+\frac{1}{2 \times \frac{1}{2}}}}}$$

**Result:**

$$\frac{1}{\sqrt[4]{2}}$$

**Decimal approximation:**

0.840896415253714543031125476233214895040034262356784510813...

0.84089641525....

**Alternate form:**

$$\frac{2^{3/4}}{2}$$

**All 2nd roots of 1/sqrt(2):**

$$\frac{e^0}{\sqrt[4]{2}} \approx 0.84090 \text{ (real, principal root)}$$

$$\frac{e^{i\pi}}{\sqrt[4]{2}} \approx -0.8409 \text{ (real root)}$$

$$F_{\mu\nu} = 0.84089641525$$

$$F = -\frac{1}{2} (m + ie) + \mathcal{O}(4 - \text{Fermi}), \quad D = -\frac{\xi}{2} + \mathcal{O}(4 - \text{Fermi}).$$

$$D = -\frac{\xi m}{2 e_2 \sqrt{1 + \frac{\xi^2}{2 e_2^2}}} \sqrt{1 + \frac{1}{m^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4 m^4} (F \cdot \bar{F})^2}.$$

$$-1/(2*1/\sqrt{2}*\sqrt{1+1/(2*1/2)})*(((1+1/(0.5)*0.84089641525^2-1/(4*1/4) (-1/2(1/\sqrt{2}+i/\sqrt{2}))^4)))^{1/2}$$

**Input interpretation:**

$$\frac{\sqrt{1 + \frac{1}{0.5} \times 0.84089641525^2 - \frac{1}{4 \times \frac{1}{4}} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^4}}{2 \times \frac{1}{\sqrt{2}} \sqrt{1 + \frac{1}{2 \times \frac{1}{2}}}}$$

*i* is the imaginary unit

**Result:**

-0.78687889194599076898248593696226432570705302519174269029...

-0.7868788919

$$\mathcal{L} = \frac{e_1}{4m} F \cdot \tilde{F} + \frac{\xi}{2} D + \frac{m e_2}{2} \left[ 1 - \sqrt{1 - \frac{2D^2}{m^2} + \frac{1}{m^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4m^4} (F \cdot \tilde{F})^2} \right],$$

1/4\*(-1/2(1/sqrt2+i/sqrt2))^2+1/2\*(-0.78687889194599)+1/4\*(((1-(((1-2(-0.78687889194599)^2/(0.5)+1/(0.5)\*0.84089641525^2-1/(4\*1/4) (-1/2(1/sqrt2+i/sqrt2))^4))))^1/2))))

**Input interpretation:**

$$\frac{1}{4} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^2 + \frac{1}{2} \times (-0.78687889) + \frac{1}{4} \left( 1 - \sqrt{1 - 2 \times \frac{(-0.78687889)^2}{0.5} + \frac{1}{0.5} \times 0.8408964^2 - \frac{1}{4 \times \frac{1}{4}} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^4} \right)$$

*i* is the imaginary unit

**Result:**

-0.143439... +  
0.0624506... *i*

**Polar coordinates:**

*r* = 0.156445 (radius), *θ* = 156.473° (angle)

0.156445

The algebraic sum between the two results is:

$$0.156445 - \frac{1}{(2 \times \frac{1}{\sqrt{2}} \sqrt{1 + \frac{1}{2} \times \frac{1}{2}})} * \left( \left( \left( \left( 1 + \frac{1}{0.5} \times 0.84089641525^2 - \frac{1}{4 \times \frac{1}{4}} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right) \right)^4 - \frac{1}{(4 \times \frac{1}{4})} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^4 \right) \right)^{1/2}$$

**Input interpretation:**

0.156445 -

$$\frac{1}{2 \times \frac{1}{\sqrt{2}} \sqrt{1 + \frac{1}{2} \times \frac{1}{2}}} \sqrt{1 + \frac{1}{0.5} \times 0.84089641525^2 - \frac{1}{4 \times \frac{1}{4}} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^4}$$

*i* is the imaginary unit

**Result:**

-0.63043389194599076898248593696226432570705302519174269029...  
 -0.630433891945...

From which:

$$\left( \left( \left( \left( 0.012518 + 0.156445 - \frac{1}{(2 \times \frac{1}{\sqrt{2}} \sqrt{1 + \frac{1}{2} \times \frac{1}{2}})} * \left( \left( \left( \left( 1 + \frac{1}{0.5} \times 0.84089641525^2 - \frac{1}{4 \times \frac{1}{4}} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right) \right)^4 - \frac{1}{(4 \times \frac{1}{4})} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^4 \right) \right) \right)^{1/2} \right) \right)$$

**Input interpretation:**

0.012518 + 0.156445 -

$$\frac{1}{2 \times \frac{1}{\sqrt{2}} \sqrt{1 + \frac{1}{2} \times \frac{1}{2}}} \sqrt{1 + \frac{1}{0.5} \times 0.84089641525^2 - \frac{1}{4 \times \frac{1}{4}} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^4}$$

*i* is the imaginary unit

**Result:**

-0.61791589194599076898248593696226432570705302519174269029...  
 -0.6179158919...

$$\left( \left( \left( \left( \left( -1 / \left( \left( \left( \left( 0.012518 + 0.156445 - \frac{1}{(2 \times \frac{1}{\sqrt{2}} \sqrt{1 + \frac{1}{2} \times \frac{1}{2}})} * \left( \left( \left( \left( 1 + \frac{1}{0.5} \times 0.84089641525^2 - \frac{1}{4 \times \frac{1}{4}} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right) \right)^4 - \frac{1}{(4 \times \frac{1}{4})} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^4 \right) \right) \right)^{1/2} \right) \right) \right) \right)$$

**Input interpretation:**

$$- \left[ 1 / \left( 0.012518 + 0.156445 - \frac{1}{2 \times \frac{1}{\sqrt{2}} \sqrt{1 + \frac{1}{2 \times \frac{1}{2}}}} \right. \right. \\ \left. \left. \sqrt{1 + \frac{1}{0.5} \times 0.84089641525^2 - \frac{1}{4 \times \frac{1}{4}} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^4} \right) \right]$$

$i$  is the imaginary unit

**Result:**

1.618343229287596107556048273544210986232224332468988744858...

1.6183432292... result that is a very good approximation to the value of the golden ratio 1,618033988749...

$$\mathcal{L} = \frac{e_1}{4m} F \cdot \tilde{F} + \frac{\xi}{2} D \\ + \frac{m e_2}{2} \left[ 1 - \sqrt{1 - \frac{2D^2}{m^2} + \frac{1}{m^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4m^4} (F \cdot \tilde{F})^2} \right],$$

1/4\*(-1/2(1/sqrt2+i/sqrt2))^2+1/2\*(-0.78687889194599)+1/4\*(((1-(((1-2(-0.78687889194599)^2/(0.5)+1/(0.5)\*0.84089641525^2-1/(4\*1/4) (-1/2(1/sqrt2+i/sqrt2))^4))))^1/2))))

Now, we have that

$$\mathcal{L} = \frac{e_1}{4m} F \cdot \tilde{F} - \frac{m e_2}{2} \left[ \sqrt{1 + \frac{\xi^2}{2 e_2^2}} - 1 \right] \\ + \frac{m e_2}{2} \sqrt{1 + \frac{\xi^2}{2 e_2^2}} \left[ 1 - \sqrt{1 + \frac{1}{m^2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4m^4} (F \cdot \tilde{F})^2} \right]. \quad (6.6)$$

$$\frac{1}{4} * (-\frac{1}{2}(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}))^2 - \frac{1}{4}(((\sqrt{1 + \frac{1}{2 * \frac{1}{2}}}) - 1)) + \frac{1}{4} * ((\sqrt{1 + \frac{1}{2 * \frac{1}{2}}})) * (((1 - (((1 + \frac{1}{0.5}) * 0.84089641525^2 - \frac{1}{4 * \frac{1}{4}}) (-\frac{1}{2}(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}))^4)))^{\frac{1}{2}}))))$$

**Input interpretation:**

$$\frac{1}{4} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^2 - \frac{1}{4} \left( \sqrt{1 + \frac{1}{2 * \frac{1}{2}}} - 1 \right) + \frac{1}{4} \left( \sqrt{1 + \frac{1}{2 * \frac{1}{2}}} \left( 1 - \sqrt{1 + \frac{1}{0.5} * 0.84089641525^2 - \frac{1}{4 * \frac{1}{4}} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^4} \right) \right)$$

*i* is the imaginary unit

**Result:**

$$-0.306407... + 0.0625 i$$

**Polar coordinates:**

$r = 0.312717$  (radius),  $\theta = 168.471^\circ$  (angle)  
0.312717

$$2 * (((\frac{1}{4} * (-\frac{1}{2}(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}))^2 - \frac{1}{4}(((\sqrt{1 + \frac{1}{2 * \frac{1}{2}}}) - 1)) + \frac{1}{4} * ((\sqrt{1 + \frac{1}{2 * \frac{1}{2}}})) * (((1 - (((1 + \frac{1}{0.5}) * 0.84089641525^2 - \frac{1}{4 * \frac{1}{4}}) (-\frac{1}{2}(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}))^4)))^{\frac{1}{2}}))))))$$

**Input interpretation:**

$$2 \left( \frac{1}{4} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^2 - \frac{1}{4} \left( \sqrt{1 + \frac{1}{2 * \frac{1}{2}}} - 1 \right) + \frac{1}{4} \left( \sqrt{1 + \frac{1}{2 * \frac{1}{2}}} \left( 1 - \sqrt{1 + \frac{1}{0.5} * 0.84089641525^2 - \frac{1}{4 * \frac{1}{4}} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^4} \right) \right) \right)$$

*i* is the imaginary unit

**Result:**

$$-0.612815... + 0.125 i$$

**Polar coordinates:**

$r = 0.625433$  (radius),  $\theta = 168.471^\circ$  (angle)  
0.625433



$$\left[ \frac{1}{\left( \left( \left( \frac{1}{4} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^2 - \frac{1}{4} \left( \left( \sqrt{1 + \frac{1}{2 \times \frac{1}{2}} \right) - 1 \right) - 1 \right) + \frac{1}{4} \left( \left( \sqrt{1 + \frac{1}{2 \times \frac{1}{2}} \right) \right) \left( \left( 1 - \left( \left( \left( 1 + \frac{1}{0.5} \right) \times 0.84089641525^2 - \frac{1}{4 \times \frac{1}{4}} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^4 \right) \right)^{\frac{1}{2}} \right) \right) \right) \right)^4 + (34 + 8 + 1.618)i \right]$$

**Input interpretation:**

$$\left( \frac{1}{\left( \frac{1}{4} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^2 - \frac{1}{4} \left( \sqrt{1 + \frac{1}{2 \times \frac{1}{2}} - 1} \right) + \frac{1}{4} \left( \sqrt{1 + \frac{1}{2 \times \frac{1}{2}} \left( 1 - \sqrt{1 + \frac{1}{0.5} \times 0.84089641525^2 - \frac{1}{4 \times \frac{1}{4}} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^4} \right) \right)^{\frac{1}{2}} \right) \right) \right)^4 + (34 + 8 + 1.618)i \right]$$

*i* is the imaginary unit

**Result:**

$$72.4868... + 118.984... i$$

**Polar coordinates:**

$$r = 139.325 \text{ (radius)}, \quad \theta = 58.6495^\circ \text{ (angle)}$$

139.325 result practically equal to the rest mass of Pion meson 139.57 MeV

$$\left[ \frac{1}{\left( \left( \left( \frac{1}{4} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^2 - \frac{1}{4} \left( \left( \sqrt{1 + \frac{1}{2 \times \frac{1}{2}} \right) - 1 \right) - 1 \right) + \frac{1}{4} \left( \left( \sqrt{1 + \frac{1}{2 \times \frac{1}{2}} \right) \right) \left( \left( 1 - \left( \left( \left( 1 + \frac{1}{0.5} \right) \times 0.84089641525^2 - \frac{1}{4 \times \frac{1}{4}} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^4 \right) \right)^{\frac{1}{2}} \right) \right) \right) \right)^4 + (21 + 5 + 0.618)i \right]$$

**Input interpretation:**

$$\left( \frac{1}{\left( \frac{1}{4} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^2 - \frac{1}{4} \left( \sqrt{1 + \frac{1}{2 \times \frac{1}{2}} - 1} \right) + \frac{1}{4} \left( \sqrt{1 + \frac{1}{2 \times \frac{1}{2}} \left( 1 - \sqrt{1 + \frac{1}{0.5} \times 0.84089641525^2 - \frac{1}{4 \times \frac{1}{4}} \left( -\frac{1}{2} \left( \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right) \right)^4} \right) \right)^{\frac{1}{2}} \right) \right) \right)^4 + (21 + 5 + 0.618)i \right]$$

*i* is the imaginary unit

**Result:**

$$72.4868... + 101.984... i$$

**Polar coordinates:**

$$r = 125.12 \text{ (radius)}, \quad \theta = 54.5959^\circ \text{ (angle)}$$

125.12 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

From

## Two-Field Born-Infeld with Diverse Dualities

*S. Ferrara, A. Sagnotti and A. Yeranyan*

arXiv:1602.04566v3 [hep-th] 8 Jul 2016

### 2 One-Field Models: BI Theory and a Family of Extensions

$$\mathcal{L} = f^2 \left[ 1 - \sqrt{\left(1 + \frac{F^2 + \bar{F}^2}{2f^2}\right)^2 - \frac{1}{f^2} \sqrt{F^2 \bar{F}^2} \left(\frac{1}{f^2} \sqrt{F^2 \bar{F}^2} - \gamma\right)} \right. \quad (2.38)$$

$$\left. + \gamma \operatorname{ArcTanh} \left( \frac{1 + \frac{F^2 + \bar{F}^2}{2f^2} - \sqrt{\left(1 + \frac{F^2 + \bar{F}^2}{2f^2}\right)^2 - \frac{1}{f^2} \sqrt{F^2 \bar{F}^2} \left(\frac{1}{f^2} \sqrt{F^2 \bar{F}^2} - \gamma\right)}}{\frac{1}{f^2} \sqrt{F^2 \bar{F}^2} - \gamma} \right) \right],$$

$f=5$ ,  $F=8$ ,  $\bar{F}=13$  and  $\gamma=21$ , we obtain:

$$[1 - \sqrt{\left(\left(\left(1 + \frac{8^2 + 13^2}{2 \times 5^2}\right)^2 - \frac{1}{25} \sqrt{8^2 \times 13^2} \left(\frac{1}{25} \sqrt{8^2 \times 13^2} - 21\right)\right)\right)}]$$

**Input:**

$$1 - \sqrt{\left(1 + \frac{8^2 + 13^2}{2 \times 5^2}\right)^2 - \frac{1}{25} \sqrt{8^2 \times 13^2} \left(\frac{1}{25} \sqrt{8^2 \times 13^2} - 21\right)}$$

**Result:**

$$1 - \frac{\sqrt{10209}}{10}$$

**Decimal approximation:**

-9.10395961987180548530258111653070389318678159980168325011...

-9.103959619871805

$$25[(-9.103959619871805+21 \operatorname{atanh}(\frac{(((1+(8^2+13^2)/(2*5^2)-10.103959619871805))}{((1/25*(8^2*13^2)^{1/2}-21))))})]$$

**Input interpretation:**

$$25 \left( -9.103959619871805 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.103959619871805}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right)$$

$\tanh^{-1}(x)$  is the inverse hyperbolic tangent function

**Result:**

-85.69762719723360...

-85.69762719...

**Alternative representations:**

$$25 \left( -9.1039596198718050000 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.1039596198718050000}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) = 25 \left( -9.1039596198718050000 + 21 \operatorname{sn}^{-1} \left( \frac{-9.1039596198718050000 + \frac{8^2+13^2}{2 \times 5^2}}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \middle| 1 \right) \right)$$

$$25 \left( -9.1039596198718050000 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.1039596198718050000}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) = 25 \left( -9.1039596198718050000 - 21 i \operatorname{sc}^{-1} \left( \frac{i \left( -9.1039596198718050000 + \frac{8^2+13^2}{2 \times 5^2} \right)}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \middle| 0 \right) \right)$$

$$25 \left( -9.1039596198718050000 + \right. \\ \left. 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.1039596198718050000}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) = \\ 25 \left( -9.1039596198718050000 + \right. \\ \left. \frac{21}{2} \left( -\log \left( 1 - \frac{-9.1039596198718050000 + \frac{8^2+13^2}{2 \times 5^2}}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \right) + \right. \right. \\ \left. \left. \log \left( 1 + \frac{-9.1039596198718050000 + \frac{8^2+13^2}{2 \times 5^2}}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \right) \right) \right)$$

**Series representations:**

$$25 \left( -9.1039596198718050000 + \right. \\ \left. 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.1039596198718050000}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) = \\ -227.59899049679512500 + 525.000000000000000000 \\ \sum_{k=0}^{\infty} \frac{0.263893089066021674584^{1+2k}}{1+2k}$$

$$25 \left( -9.1039596198718050000 + \right. \\ \left. 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.1039596198718050000}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) = \\ -227.59899049679512500 + 262.500000000000000000 \\ \log(1.263893089066021674584) - 262.500000000000000000 \log(2) + \\ 262.500000000000000000 \sum_{k=1}^{\infty} \frac{0.631946544533010837292^k}{k}$$

$$\begin{aligned}
& 25 \left( -9.1039596198718050000 + \right. \\
& \quad \left. 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.1039596198718050000}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) = \\
& -227.59899049679512500 - 262.500000000000000000 \\
& \quad \log(0.736106910933978325416) + 262.500000000000000000 \log(2) - \\
& 262.500000000000000000 \sum_{k=1}^{\infty} \frac{(-0.736106910933978325416)^k \left(-\frac{1}{2}\right)^k}{k}
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& 25 \left( -9.1039596198718050000 + \right. \\
& \quad \left. 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.1039596198718050000}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) = \\
& -227.59899049679512500 + 138.543871759661379157 \\
& \quad \int_0^1 \frac{1}{1 - 0.069639562456807248298 t^2} dt
\end{aligned}$$

$$\begin{aligned}
& 25 \left( -9.1039596198718050000 + \right. \\
& \quad \left. 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.1039596198718050000}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) = \\
& -227.59899049679512500 - \frac{34.63596793991534479 i}{\pi^{3/2}} \\
& \quad \int_{-i\infty+\gamma}^{i\infty+\gamma} e^{0.0721832006668877818113 s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s)^2 ds \text{ for } 0 < \gamma < \frac{1}{2}
\end{aligned}$$

1-golden ratio\*(((25[(-9.1039596+21 atanh((((1+(8^2+13^2)/(2\*5^2)-10.1039596)))/(1/25\*(8^2\*13^2)^1/2-21)))))))))

**Input interpretation:**

$$1 - \phi \left( 25 \left( -9.1039596 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.1039596}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right)$$

$\tanh^{-1}(x)$  is the inverse hyperbolic tangent function

$\phi$  is the golden ratio

**Result:**

139.66167...

139.66167... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$1 - \phi 25 \left( -9.10396 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) =$$

$$1 - 25 \phi \left( -9.10396 + 21 \operatorname{sn}^{-1} \left( \frac{-9.10396 + \frac{8^2+13^2}{2 \times 5^2}}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \middle| 1 \right) \right)$$

$$1 - \phi 25 \left( -9.10396 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) =$$

$$1 - 25 \phi \left( -9.10396 - 21 i \operatorname{sc}^{-1} \left( \frac{i(-9.10396 + \frac{8^2+13^2}{2 \times 5^2})}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \middle| 0 \right) \right)$$

$$1 - \phi 25 \left( -9.10396 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) = 1 - 25 \phi \left( -9.10396 + \right.$$

$$\left. \frac{21}{2} \left( -\log \left( 1 - \frac{-9.10396 + \frac{8^2+13^2}{2 \times 5^2}}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \right) + \log \left( 1 + \frac{-9.10396 + \frac{8^2+13^2}{2 \times 5^2}}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \right) \right) \right)$$

**Series representations:**

$$1 - \phi 25 \left( -9.10396 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2 - 21}} \right) \right) =$$

$$1 + 227.599 \phi - 525 \phi \sum_{k=0}^{\infty} \frac{0.263893^{1+2k}}{1+2k}$$

$$1 - \phi 25 \left( -9.10396 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2 - 21}} \right) \right) =$$

$$1 + 227.599 \phi - 262.5 \phi \log(1.26389) + 262.5 \phi \log(2) - 262.5 \phi \sum_{k=1}^{\infty} \frac{0.631947^k}{k}$$

$$1 - \phi 25 \left( -9.10396 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2 - 21}} \right) \right) = 1 + 227.599 \phi +$$

$$262.5 \phi \log(0.736107) - 262.5 \phi \log(2) + 262.5 \phi \sum_{k=1}^{\infty} \frac{(-0.736107)^k \left(-\frac{1}{2}\right)^k}{k}$$

**Integral representations:**

$$1 - \phi 25 \left( -9.10396 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2 - 21}} \right) \right) =$$

$$1 + 227.599 \phi - 138.544 \phi \int_0^1 \frac{1}{1 - 0.0696396 t^2} dt$$

$$1 - \phi 25 \left( -9.10396 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2 - 21}} \right) \right) = 1 + 227.599 \phi +$$

$$\frac{34.636 \phi i}{\pi^{3/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} e^{0.0721832 s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s)^2 ds \text{ for } 0 < \gamma < \frac{1}{2}$$

$$-11-3+\left(\left(1-\text{golden ratio}\cdot\left(\left(\left(25\left[(-9.1039596+21 \operatorname{atanh}\left(\frac{1+(8^2+13^2)}{2\cdot 5^2}\right)-10.1039596\right)}{\frac{1}{25}\sqrt{8^2\times 13^2}-21}\right)\right)\right)\right)\right)\right)$$

**Input interpretation:**

$$-11-3+\left(1-\phi\left(25\left(-9.1039596+21 \operatorname{tanh}^{-1}\left(\frac{1+\frac{8^2+13^2}{2\cdot 5^2}-10.1039596}{\frac{1}{25}\sqrt{8^2\times 13^2}-21}\right)\right)\right)\right)$$

$\operatorname{tanh}^{-1}(x)$  is the inverse hyperbolic tangent function

$\phi$  is the golden ratio

**Result:**

125.6617...

125.6617... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$-11-3+\left(1-\phi\left(25\left(-9.10396+21 \operatorname{tanh}^{-1}\left(\frac{1+\frac{8^2+13^2}{2\cdot 5^2}-10.104}{\frac{1}{25}\sqrt{8^2\times 13^2}-21}\right)\right)\right)\right) =$$

$$-13-25\phi\left(-9.10396+21 \operatorname{sn}^{-1}\left(\frac{-9.10396+\frac{8^2+13^2}{2\cdot 5^2}}{-21+\frac{1}{25}\sqrt{8^2\times 13^2}}\middle|1\right)\right)$$

$$-11-3+\left(1-\phi\left(25\left(-9.10396+21 \operatorname{tanh}^{-1}\left(\frac{1+\frac{8^2+13^2}{2\cdot 5^2}-10.104}{\frac{1}{25}\sqrt{8^2\times 13^2}-21}\right)\right)\right)\right) =$$

$$-13-25\phi\left(-9.10396-21 i \operatorname{sc}^{-1}\left(\frac{i\left(-9.10396+\frac{8^2+13^2}{2\cdot 5^2}\right)}{-21+\frac{1}{25}\sqrt{8^2\times 13^2}}\middle|0\right)\right)$$



$$\begin{aligned}
& -11 - 3 + \left( 1 - \phi 25 \left( -9.10396 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) = \\
& -13 - 25 \phi \left( -9.10396 + \right. \\
& \quad \left. \frac{21}{2} \left( -\log \left( 1 - \frac{-9.10396 + \frac{8^2+13^2}{2 \times 5^2}}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \right) + \log \left( 1 + \frac{-9.10396 + \frac{8^2+13^2}{2 \times 5^2}}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \right) \right) \right)
\end{aligned}$$

### Series representations:

$$\begin{aligned}
& -11 - 3 + \left( 1 - \phi 25 \left( -9.10396 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) = \\
& -13 + 227.599 \phi - 525 \phi \sum_{k=0}^{\infty} \frac{0.263893^{1+2k}}{1+2k}
\end{aligned}$$

$$\begin{aligned}
& -11 - 3 + \left( 1 - \phi 25 \left( -9.10396 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) = \\
& -13 + 227.599 \phi - 262.5 \phi \log(1.26389) + 262.5 \phi \log(2) - 262.5 \phi \sum_{k=1}^{\infty} \frac{0.631947^k}{k}
\end{aligned}$$

$$\begin{aligned}
& -11 - 3 + \left( 1 - \phi 25 \left( -9.10396 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) = -13 + 227.599 \phi + \\
& 262.5 \phi \log(0.736107) - 262.5 \phi \log(2) + 262.5 \phi \sum_{k=1}^{\infty} \frac{(-0.736107)^k \left(-\frac{1}{2}\right)^k}{k}
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& -11 - 3 + \left( 1 - \phi 25 \left( -9.10396 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) = \\
& -13 + 227.599 \phi - 138.544 \phi \int_0^1 \frac{1}{1 - 0.0696396 t^2} dt
\end{aligned}$$



$$\begin{aligned}
& -8 + \frac{1}{2} \left( -11 + 1 - 1.61803 \left( 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) \right) 27 = \\
& -8 + \frac{27}{2} \left( -10 - 40.4509 \left( -9.10395 - 21 i \operatorname{sc}^{-1} \left( \frac{i(-9.10395 + \frac{8^2+13^2}{2 \times 5^2})}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \middle| 0 \right) \right) \right) \\
& -8 + \frac{1}{2} \left( -11 + 1 - 1.61803 \left( 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) \right) 27 = \\
& -8 + \frac{27}{2} \left( -10 - 40.4509 \left( -9.10395 + \right. \right. \\
& \quad \left. \left. \frac{21}{2} \left( -\log \left( 1 - \frac{-9.10395 + \frac{8^2+13^2}{2 \times 5^2}}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \right) + \log \left( 1 + \frac{-9.10395 + \frac{8^2+13^2}{2 \times 5^2}}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \right) \right) \right) \right)
\end{aligned}$$

**Series representations:**

$$\begin{aligned}
& -8 + \frac{1}{2} \left( -11 + 1 - 1.61803 \left( 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) \right) 27 = \\
& 4828.54 - 11467.8 \sum_{k=0}^{\infty} \frac{0.263893^{1+2k}}{1+2k}
\end{aligned}$$

$$\begin{aligned}
& -8 + \frac{1}{2} \left( -11 + 1 - 1.61803 \left( 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) \right) 27 = \\
& 4828.54 - 5733.91 \log(1.26389) + 5733.91 \log(2) - 5733.91 \sum_{k=1}^{\infty} \frac{0.631946^k}{k}
\end{aligned}$$

$$\begin{aligned}
& -8 + \frac{1}{2} \left( -11 + 1 - 1.61803 \left( 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) \right) 27 = \\
& 4828.54 + 5733.91 \log(0.736107) - 5733.91 \log(2) + 5733.91 \sum_{k=1}^{\infty} \frac{(-0.736107)^k \left(-\frac{1}{2}\right)^k}{k}
\end{aligned}$$



$$55 - 8 + \frac{1}{2} \left( -11 + 1 - 1.61803 \left( 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) \right) 27 =$$

$$47 + \frac{27}{2} \left( -10 - 40.4509 \left( -9.10395 - 21 i \operatorname{sc}^{-1} \left( \frac{i(-9.10395 + \frac{8^2+13^2}{2 \times 5^2})}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \middle| 0 \right) \right) \right)$$

$$55 - 8 + \frac{1}{2} \left( -11 + 1 - 1.61803 \left( 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) \right) 27 =$$

$$47 + \frac{27}{2} \left( -10 - 40.4509 \left( -9.10395 + \right. \right.$$

$$\left. \left. \frac{21}{2} \left( -\log \left( 1 - \frac{-9.10395 + \frac{8^2+13^2}{2 \times 5^2}}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \right) + \log \left( 1 + \frac{-9.10395 + \frac{8^2+13^2}{2 \times 5^2}}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \right) \right) \right) \right)$$

### Series representations:

$$55 - 8 + \frac{1}{2} \left( -11 + 1 - 1.61803 \left( 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) \right) 27 =$$

$$4883.54 - 11467.8 \sum_{k=0}^{\infty} \frac{0.263893^{1+2k}}{1+2k}$$

$$55 - 8 + \frac{1}{2} \left( -11 + 1 - 1.61803 \left( 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) \right) 27 =$$

$$4883.54 - 5733.91 \log(1.26389) + 5733.91 \log(2) - 5733.91 \sum_{k=1}^{\infty} \frac{0.631946^k}{k}$$

$$55 - 8 + \frac{1}{2} \left( -11 + 1 - 1.61803 \left( 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) \right) 27 =$$

$$4883.54 + 5733.91 \log(0.736107) - 5733.91 \log(2) + 5733.91 \sum_{k=1}^{\infty} \frac{(-0.736107)^k \left(-\frac{1}{2}\right)^k}{k}$$

**Integral representations:**

$$55 - 8 + \frac{1}{2} \left( -11 + 1 - 1.61803 \left( 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) \right) 27 =$$

$$4883.54 - 3026.27 \int_0^1 \frac{1}{1 - 0.0696393 t^2} dt$$

$$55 - 8 + \frac{1}{2} \left( -11 + 1 - 1.61803 \left( 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right) \right) 27 =$$

$$4883.54 + \frac{756.568 i}{\pi^{3/2}} \int_{-i \infty + \gamma}^{i \infty + \gamma} e^{0.0721829 s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1 - s) \Gamma(s)^2 ds \text{ for } 0 < \gamma < \frac{1}{2}$$

$$4-9*(((25[(-9.10395+21 \operatorname{atanh}((((1+(8^2+13^2))/(2*5^2)-10.10395)))/((1/25*(8^2*13^2)^{1/2}-21)))))))))$$

**Input interpretation:**

$$4 - 9 \left( 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.10395}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) \right)$$

$\tanh^{-1}(x)$  is the inverse hyperbolic tangent function

**Result:**

775.279...

775.279... result practically equal to the rest mass of Neutral rho meson 775.26

**Alternative representations:**

$$4 - 9 \times 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) =$$

$$4 - 225 \left( -9.10395 + 21 \operatorname{sn}^{-1} \left( \frac{-9.10395 + \frac{8^2+13^2}{2 \times 5^2}}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \middle| 1 \right) \right)$$

$$4 - 9 \times 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) =$$

$$4 - 225 \left( -9.10395 - 21 i \operatorname{sc}^{-1} \left( \frac{i \left( -9.10395 + \frac{8^2+13^2}{2 \times 5^2} \right)}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \middle| 0 \right) \right)$$

$$4 - 9 \times 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) = 4 - 225 \left( -9.10395 + \right.$$

$$\left. \frac{21}{2} \left( -\log \left( 1 - \frac{-9.10395 + \frac{8^2+13^2}{2 \times 5^2}}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \right) + \log \left( 1 + \frac{-9.10395 + \frac{8^2+13^2}{2 \times 5^2}}{-21 + \frac{1}{25} \sqrt{8^2 \times 13^2}} \right) \right) \right)$$

### Series representations:

$$4 - 9 \times 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) =$$

$$2052.39 - 4725 \sum_{k=0}^{\infty} \frac{0.263893^{1+2k}}{1+2k}$$

$$4 - 9 \times 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) =$$

$$2052.39 - 2362.5 \log(1.26389) + 2362.5 \log(2) - 2362.5 \sum_{k=1}^{\infty} \frac{0.631946^k}{k}$$

$$4 - 9 \times 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) =$$

$$2052.39 + 2362.5 \log(0.736107) - 2362.5 \log(2) + 2362.5 \sum_{k=1}^{\infty} \frac{(-0.736107)^k \left(-\frac{1}{2}\right)^k}{k}$$

### Integral representations:

$$4 - 9 \times 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) =$$

$$2052.39 - 1246.89 \int_0^1 \frac{1}{1 - 0.0696393 t^2} dt$$

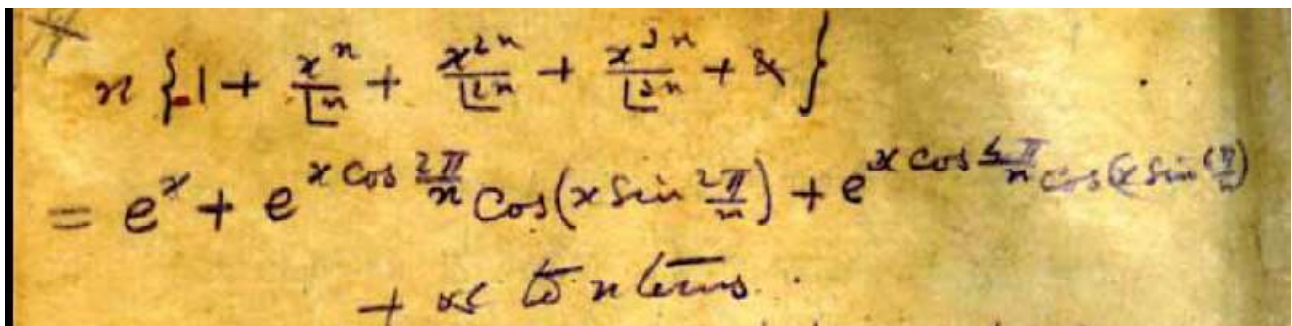
$$4 - 9 \times 25 \left( -9.10395 + 21 \tanh^{-1} \left( \frac{1 + \frac{8^2+13^2}{2 \times 5^2} - 10.104}{\frac{1}{25} \sqrt{8^2 \times 13^2} - 21} \right) \right) =$$

$$2052.39 + \frac{311.723 i}{\pi^{3/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} e^{0.0721829s} \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s)^2 ds \text{ for } 0 < \gamma < \frac{1}{2}$$

From:

**Manuscript Book 1 of Srinivasa Ramanujan**

Page 189



For  $x = 2$  and  $n = 24$

$$e^2 + e^{(2 \cos(2\pi)/24)} \cos((2 \sin(2\pi)/24)) + e^{(2 \cos(4\pi)/24)} \cos((2 \sin(4\pi)/24))$$

**Input:**

$$e^2 + e^{2(1/24 \cos(2\pi))} \cos\left(2\left(\frac{1}{24} \sin(2\pi)\right)\right) + e^{2(1/24 \cos(4\pi))} \cos\left(2\left(\frac{1}{24} \sin(4\pi)\right)\right)$$

**Exact result:**

$$2 \sqrt[12]{e + e^2}$$

**Decimal approximation:**

9.562864197973108004506986854601230775579590156093827120440...

9.56286419...

**Property:**

$2 \sqrt[12]{e + e^2}$  is a transcendental number

**Alternate form:**

$$\sqrt[12]{e} (2 + e^{23/12})$$



### Alternative representations:

$$e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) =$$

$$e^2 + \cosh\left(-\frac{2}{24} i \sin(2\pi)\right) e^{2/24 \cosh(-2i\pi)} + \cosh\left(-\frac{2}{24} i \sin(4\pi)\right) e^{2/24 \cosh(-4i\pi)}$$

$$e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) =$$

$$e^2 + \cosh\left(\frac{2}{24} i \sin(2\pi)\right) e^{2/24 \cosh(2i\pi)} + \cosh\left(\frac{2}{24} i \sin(4\pi)\right) e^{2/24 \cosh(4i\pi)}$$

$$e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) =$$

$$e^2 + \frac{1}{2} e^{1/24 (e^{-2i\pi} + e^{2i\pi})} (e^{-2/24 i \sin(2\pi)} + e^{2/24 i \sin(2\pi)}) +$$

$$\frac{1}{2} e^{1/24 (e^{-4i\pi} + e^{4i\pi})} (e^{-2/24 i \sin(4\pi)} + e^{2/24 i \sin(4\pi)})$$

### Integral representations:

$$e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) = \frac{1}{2i\pi}$$

$$\left( 2e^{2i\pi} + \exp\left(\frac{\sqrt{\pi}}{24i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\mathcal{A}^{-\pi^2/s+s}}{\sqrt{s}} ds\right) \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\mathcal{A}^{s-\sin^2(2\pi)/(576s)}}{\sqrt{s}} ds \right) \sqrt{\pi} + \right.$$

$$\left. \exp\left(\frac{\sqrt{\pi}}{24i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\mathcal{A}^{-(4\pi^2)/s+s}}{\sqrt{s}} ds\right) \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\mathcal{A}^{s-\sin^2(4\pi)/(576s)}}{\sqrt{s}} ds \right) \sqrt{\pi} \right) \text{ for } \gamma > 0$$

$$e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) =$$

$$\exp\left(-\frac{1}{12} \int_{\frac{\pi}{2}}^{2\pi} \sin(t) dt - \frac{1}{12} \int_{\frac{\pi}{2}}^{4\pi} \sin(t) dt\right)$$

$$\left( \exp\left(2 + \frac{1}{12} \int_{\frac{\pi}{2}}^{2\pi} \sin(t) dt + \frac{1}{12} \int_{\frac{\pi}{2}}^{4\pi} \sin(t) dt\right) - \right.$$

$$\left. e^{1/12 \int_{\pi/2}^{4\pi} \sin(t) dt} \int_{\frac{\pi}{2}}^{\frac{1}{12} \sin(2\pi)} \sin(t) dt - e^{1/12 \int_{\pi/2}^{2\pi} \sin(t) dt} \int_{\frac{\pi}{2}}^{\frac{1}{12} \sin(4\pi)} \sin(t) dt \right)$$

$$\begin{aligned}
& e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) = \\
& -\frac{1}{12} \exp\left(-\frac{\pi}{6} \int_0^1 \sin(2\pi t) dt - \frac{\pi}{3} \int_0^1 \sin(4\pi t) dt\right) \left(-12 e^{1/12+\pi/6} \int_0^1 \sin(2\pi t) dt - \right. \\
& \quad \left. 12 e^{1/12+\pi/3} \int_0^1 \sin(4\pi t) dt - 12 \exp\left(2 + \frac{\pi}{6} \int_0^1 \sin(2\pi t) dt + \frac{\pi}{3} \int_0^1 \sin(4\pi t) dt\right) + \right. \\
& \quad e^{1/12+\pi/3} \int_0^1 \sin(4\pi t) dt \sin(2\pi) \int_0^1 \sin\left(\frac{1}{12} t \sin(2\pi)\right) dt + \\
& \quad \left. e^{1/12+\pi/6} \int_0^1 \sin(2\pi t) dt \sin(4\pi) \int_0^1 \sin\left(\frac{1}{12} t \sin(4\pi)\right) dt\right)
\end{aligned}$$

**Multiple-argument formulas:**

$$\begin{aligned}
& e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) = \\
& e^2 + e^{1/12(1-2\sin^2(\pi))} \left(1 - 2 \sin^2\left(\frac{1}{24} \sin(2\pi)\right)\right) + e^{1/12(1-2\sin^2(2\pi))} \left(1 - 2 \sin^2\left(\frac{1}{24} \sin(4\pi)\right)\right)
\end{aligned}$$

$$\begin{aligned}
& e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) = \\
& e^2 + e^{1/12(-1+2\cos^2(\pi))} \left(-1 + 2 \cos^2\left(\frac{1}{24} \sin(2\pi)\right)\right) + \\
& e^{1/12(-1+2\cos^2(2\pi))} \left(-1 + 2 \cos^2\left(\frac{1}{24} \sin(4\pi)\right)\right)
\end{aligned}$$

$$\begin{aligned}
& e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) = \\
& e^2 + e^{-1/12+\cos^2(\pi)/6} \left(-1 + 2 \cos^2\left(\frac{1}{24} \sin(2\pi)\right)\right) + \\
& e^{-1/12+1/6\cos^2(2\pi)} \left(-1 + 2 \cos^2\left(\frac{1}{24} \sin(4\pi)\right)\right)
\end{aligned}$$

$$2\left(\left(e^2 + e^{(2 \cos(2\pi)/24)} \cos\left(\frac{2 \sin(2\pi)}{24}\right) + e^{(2 \cos(4\pi)/24)} \cos\left(\frac{2 \sin(4\pi)}{24}\right)\right)\right)^3 - 21 + 1$$

**Input:**

$$2\left(e^2 + e^{2(1/24 \cos(2\pi))} \cos\left(2\left(\frac{1}{24} \sin(2\pi)\right)\right) + e^{2(1/24 \cos(4\pi))} \cos\left(2\left(\frac{1}{24} \sin(4\pi)\right)\right)\right)^3 - 21 + 1$$

**Exact result:**

$$2\left(2\sqrt[12]{e + e^2}\right)^3 - 20$$

**Decimal approximation:**

1729.016718790462375715766763875040223182988802615077215159...

[1729.01671879...](#)

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Property:**

$-20 + 2 \left( 2 \sqrt[12]{e} + e^2 \right)^3$  is a transcendental number

**Alternate forms:**

$$2 \sqrt[4]{e} \left( 2 + e^{23/12} \right)^3 - 20$$

$$2 \left( \sqrt[4]{e} \left( 2 + e^{23/12} \right)^3 - 10 \right)$$

$$2 \left( -10 + 8 \sqrt[4]{e} + 12 e^{13/6} + 6 e^{49/12} + e^6 \right)$$

**Alternative representations:**

$$2 \left( e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) \right)^3 - 21 + 1 = -20 + 2 \left( e^2 + \cosh\left(-\frac{2}{24} i \sin(2\pi)\right) e^{2/24 \cosh(-2i\pi)} + \cosh\left(-\frac{2}{24} i \sin(4\pi)\right) e^{2/24 \cosh(-4i\pi)} \right)^3$$

$$2 \left( e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) \right)^3 - 21 + 1 = -20 + 2 \left( e^2 + \cosh\left(\frac{2}{24} i \sin(2\pi)\right) e^{2/24 \cosh(2i\pi)} + \cosh\left(\frac{2}{24} i \sin(4\pi)\right) e^{2/24 \cosh(4i\pi)} \right)^3$$

$$2 \left( e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) \right)^3 - 21 + 1 = -20 + 2 \left( e^2 + \frac{1}{2} e^{1/24 (e^{-2i\pi} + e^{2i\pi})} \left( e^{-2/24 i \sin(2\pi)} + e^{2/24 i \sin(2\pi)} \right) + \frac{1}{2} e^{1/24 (e^{-4i\pi} + e^{4i\pi})} \left( e^{-2/24 i \sin(4\pi)} + e^{2/24 i \sin(4\pi)} \right) \right)^3$$

**Series representations:**

$$2 \left( e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) \right)^3 - 21 + 1 = -20 + 2 \left( 2 \sqrt[12]{3 - \sum_{k=0}^{\infty} \frac{1+k}{(3+k)!}} + \left( -3 + \sum_{k=0}^{\infty} \frac{1+k}{(3+k)!} \right)^2 \right)^3$$

$$2 \left( e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) \right)^3 - 21 + 1 =$$

$$-20 + 16 \sqrt[4]{\sum_{k=0}^{\infty} \frac{1}{k!}} + 24 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{13/6} + 12 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{49/12} + 2 \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^6$$

$$2 \left( e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) \right)^3 - 21 + 1 =$$

$$-20 + 16 \sqrt[4]{\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}} + 24 \left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{13/6} +$$

$$12 \left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^{49/12} + 2 \left( \sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!} \right)^6$$

### Multiple-argument formulas:

$$2 \left( e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) \right)^3 - 21 + 1 =$$

$$-20 + 2 \left( e^2 + e^{1/12(1-2\sin^2(\pi))} \left( 1 - 2 \sin^2\left(\frac{1}{24} \sin(2\pi)\right) \right) + \right.$$

$$\left. e^{1/12(1-2\sin^2(2\pi))} \left( 1 - 2 \sin^2\left(\frac{1}{24} \sin(4\pi)\right) \right) \right)^3$$

$$2 \left( e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) \right)^3 - 21 + 1 =$$

$$-20 + 2 \left( e^2 + e^{1/12(-1+2\cos^2(\pi))} \left( -1 + 2 \cos^2\left(\frac{1}{24} \sin(2\pi)\right) \right) + \right.$$

$$\left. e^{1/12(-1+2\cos^2(2\pi))} \left( -1 + 2 \cos^2\left(\frac{1}{24} \sin(4\pi)\right) \right) \right)^3$$

$$2 \left( e^2 + e^{2/24 \cos(2\pi)} \cos\left(\frac{2}{24} \sin(2\pi)\right) + e^{2/24 \cos(4\pi)} \cos\left(\frac{2}{24} \sin(4\pi)\right) \right)^3 - 21 + 1 =$$

$$-20 + 2 \left( e^2 + e^{T_2(\cos(\pi))/12} T_{\frac{1}{12}}(\cos(\sin(2\pi))) + e^{T_4(\cos(\pi))/12} T_{\frac{1}{12}}(\cos(\sin(4\pi))) \right)^3$$

Now, we have that: (pag.149)

Can a

$$(i) \frac{24\sqrt{y}(1-y)(1-y^2)(1-y^3) \dots}{\sqrt[8]{3}} = \frac{24\sqrt{x} \sqrt[8]{1-x}}{\sqrt[8]{3}} \sqrt{1 + \frac{1.2}{3^2}x + \dots}$$

$$(ii) = \frac{24\sqrt{2} \sqrt[8]{1-x}}{\sqrt{2}} \sqrt{1 + \frac{1.3}{4^2}x + \dots} \quad (iii) = \frac{24\sqrt{\frac{2(1-x)}{432}}}{\sqrt[8]{3}} \sqrt{1 + \frac{1.5}{6^2}x + \dots}$$

$$\frac{12\sqrt{y}(1-y^2)(1-y^4)(1-y^6) \dots}{\sqrt{2}} = \frac{12\sqrt{x} \sqrt[8]{1-x}}{\sqrt{2}} \sqrt{1 + \frac{1.3}{2^2}x + \dots}$$

$$\frac{8\sqrt{y}(1-y^3)(1-y^6)(1-y^9) \dots}{\sqrt[8]{27}} = \frac{8\sqrt{\frac{x}{27}} \sqrt[8]{1-x}}{\sqrt[8]{27}} \sqrt{1 + \frac{1.2}{3^2}x + \dots}$$

$$1 + 240 \left( \frac{13y^3}{1-y^3} + \frac{2^8 y^6}{1-y^6} + \dots \right) = \left( 1 + \frac{1.2}{3^2}x + \dots \right) \left( 1 - \frac{2}{9}x \right)$$

$$= 1 - 504 \left( \frac{15y^3}{1-y^3} + \frac{2^8 y^6}{1-y^6} + \dots \right) = \left( 1 + \frac{1.2}{2^2}x + \dots \right) \left( 1 - \frac{2}{3}x + \frac{8}{27}x^2 \right)$$

For x = 2

$$\frac{24\sqrt{x} \sqrt[8]{1-x}}{\sqrt[8]{3}} \sqrt{1 + \frac{1.2}{3^2}x + \dots}$$

$$[\left(\frac{2^{1/24} \cdot (1-2)^{1/8}}{(3^{1/8})}\right) \cdot \left(1 + \frac{2}{9} \cdot 2\right)^{1/2}]$$

**Input:**

$$\frac{24\sqrt{2} \sqrt[8]{1-2}}{\sqrt[8]{3}} \sqrt{1 + \frac{2}{9} \times 2}$$

**Result:**

$$\frac{\sqrt[8]{-1} \cdot 24\sqrt{2} \sqrt{13}}{3 \sqrt[8]{3}}$$

**Decimal approximation:**

0.99625044180708626835197615333847194855815067069106445890... +  
 0.41266044451668302726007871592997912932817494536489701699... i

**Polar coordinates:** $r \approx 1.07833$  (radius),  $\theta \approx 22.5^\circ$  (angle)

1.0783337077498417.....

**Alternate forms:**

$$\frac{1}{9} \sqrt[24]{2} \sqrt[8]{-1} \sqrt{13} 3^{7/8}$$

$$\frac{\sqrt[24]{-2} \sqrt[12]{-1} \sqrt{13}}{3 \sqrt[8]{3}}$$

$$\frac{\sqrt[24]{2} \sqrt{13} \cos\left(\frac{\pi}{8}\right)}{3 \sqrt[8]{3}} + \frac{i \sqrt[24]{2} \sqrt{13} \sin\left(\frac{\pi}{8}\right)}{3 \sqrt[8]{3}}$$

$$\left( \left( \left( \left( 2^{1/24} \right) \left( (1-2)^{1/8} \right) \right) / \left( (3)^{1/8} \right) \right) * \left( (1+2/9*2)^{1/2} \right) \right)^{(Pi^4)/15}$$

**Input:**

$$\left( \frac{\sqrt[24]{2} \sqrt[8]{1-2}}{\sqrt[8]{3}} \sqrt{1 + \frac{2}{9} \times 2} \right)^{\pi^4/15}$$

**Exact result:**

$$(-1)^{\pi^4/120} 2^{\pi^4/360} \times 3^{-(3\pi^4)/40} \times 13^{\pi^4/30}$$

**Decimal approximation:**

-1.354724961582513803956447289686129352857847187607123292... +  
0.9098694501573817835460717550838290322254963981917706049... *i*

**Polar coordinates:** $r \approx 1.63191$  (radius),  $\theta \approx 146.114^\circ$  (angle)

1.63191

**Alternate forms:**

$$2^{\pi^4/360} \times 3^{-(3\pi^4)/40} \times 13^{\pi^4/30} \cos\left(\frac{\pi^5}{120}\right) + i 2^{\pi^4/360} \times 3^{-(3\pi^4)/40} \times 13^{\pi^4/30} \sin\left(\frac{\pi^5}{120}\right)$$

$$2^{\pi^4/360} \times 3^{-(3\pi^4)/40} \times 13^{\pi^4/30} e^{(i\pi^5)/120}$$

**Series representations:**

$$\left( \frac{\sqrt{1 + \frac{2 \times 2}{9}} \left( \sqrt[24]{2} \sqrt[8]{1-2} \right)}{\sqrt[8]{3}} \right)^{\pi^4/15} = (-1)^{3/4 \times \sum_{k=1}^{\infty} 1/k^4} 2^{1/4 \times \sum_{k=1}^{\infty} 1/k^4} \times 3^{-27/4 \times \sum_{k=1}^{\infty} 1/k^4} \times 2197^{\sum_{k=1}^{\infty} 1/k^4}$$

$$\left( \frac{\sqrt{1 + \frac{2 \times 2}{9}} \left( \sqrt[24]{2} \sqrt[8]{1-2} \right)}{\sqrt[8]{3}} \right)^{\pi^4/15} = (-1)^{4/5 \times \sum_{k=0}^{\infty} 1/(1+2k)^4} 2^{4/15 \times \sum_{k=0}^{\infty} 1/(1+2k)^4} \times 3^{-36/5 \times \sum_{k=0}^{\infty} 1/(1+2k)^4} \times 13^{16/5 \times \sum_{k=0}^{\infty} 1/(1+2k)^4}$$

$$\left( \frac{\sqrt{1 + \frac{2 \times 2}{9}} \left( \sqrt[24]{2} \sqrt[8]{1-2} \right)}{\sqrt[8]{3}} \right)^{\pi^4/15} = (-1)^{32/15 \left( \sum_{k=0}^{\infty} (-1)^k / (1+2k)^4 \right)} 2^{32/45 \left( \sum_{k=0}^{\infty} (-1)^k / (1+2k)^4 \right)} \times 3^{-96/5 \left( \sum_{k=0}^{\infty} (-1)^k / (1+2k)^4 \right)} \times 13^{128/15 \left( \sum_{k=0}^{\infty} (-1)^k / (1+2k)^4 \right)}$$

or, without the imaginary:

**Input:**

$$\left( \frac{\sqrt[24]{2} \sqrt[8]{1}}{\sqrt[8]{3}} \sqrt{1 + \frac{2}{9} \times 2} \right)^{\pi^4/15}$$

**Exact result:**

$$2^{\pi^4/360} \times 3^{-(3\pi^4)/40} \times 13^{\pi^4/30}$$

**Decimal approximation:**

1.631913642894267783481236013950185822073787320257943951835...

1.63191364...

**Series representations:**

$$\left( \frac{\sqrt{1 + \frac{2 \times 2}{9}} \left( \sqrt[24]{2} \sqrt[8]{1} \right)}{\sqrt[8]{3}} \right)^{\pi^4/15} = 2^{1/4 \times \sum_{k=1}^{\infty} 1/k^4} \times 3^{-27/4 \times \sum_{k=1}^{\infty} 1/k^4} \times 2197^{\sum_{k=1}^{\infty} 1/k^4}$$

$$\left( \frac{\sqrt{1 + \frac{2 \times 2}{9}} \left( {}^{24}\sqrt{2} \sqrt[8]{1} \right)}{\sqrt[8]{3}} \right)^{\pi^4/15} = 2^{4/15 \times \sum_{k=0}^{\infty} 1/(1+2k)^4} \times 3^{-36/5 \times \sum_{k=0}^{\infty} 1/(1+2k)^4} \times 13^{16/5 \times \sum_{k=0}^{\infty} 1/(1+2k)^4}$$

$$\left( \frac{\sqrt{1 + \frac{2 \times 2}{9}} \left( {}^{24}\sqrt{2} \sqrt[8]{1} \right)}{\sqrt[8]{3}} \right)^{\pi^4/15} = 2^{32/45 \left( \sum_{k=0}^{\infty} (-1)^k / (1+2k)^4 \right)} \times 3^{-96/5 \left( \sum_{k=0}^{\infty} (-1)^k / (1+2k)^4 \right)} \times 13^{128/15 \left( \sum_{k=0}^{\infty} (-1)^k / (1+2k)^4 \right)}$$

### Integral representations:

$$\left( \frac{\sqrt{1 + \frac{2 \times 2}{9}} \left( {}^{24}\sqrt{2} \sqrt[8]{1} \right)}{\sqrt[8]{3}} \right)^{\pi^4/15} = 2^{2/45 \left( \int_0^{\infty} 1/(1+t^2) dt \right)^4} \times 3^{-6/5 \left( \int_0^{\infty} 1/(1+t^2) dt \right)^4} \times 13^{8/15 \left( \int_0^{\infty} 1/(1+t^2) dt \right)^4}$$

$$\left( \frac{\sqrt{1 + \frac{2 \times 2}{9}} \left( {}^{24}\sqrt{2} \sqrt[8]{1} \right)}{\sqrt[8]{3}} \right)^{\pi^4/15} = 2^{2/45 \left( \int_0^{\infty} \sin(t)/t dt \right)^4} \times 3^{-6/5 \left( \int_0^{\infty} \sin(t)/t dt \right)^4} \times 13^{8/15 \left( \int_0^{\infty} \sin(t)/t dt \right)^4}$$

$$\left( \frac{\sqrt{1 + \frac{2 \times 2}{9}} \left( {}^{24}\sqrt{2} \sqrt[8]{1} \right)}{\sqrt[8]{3}} \right)^{\pi^4/15} = 2^{2/45 \left( \int_0^1 1/\sqrt{1-t^2} dt \right)^4} \times 3^{-6/5 \left( \int_0^1 1/\sqrt{1-t^2} dt \right)^4} \times 13^{8/15 \left( \int_0^1 1/\sqrt{1-t^2} dt \right)^4}$$



From the principal expression, we have that:

$$\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(\left(2^{1/24}\right)^{\left(\left(1\right)^{1/8}\right)}\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)^{1/16}$$

**Input:**

$$\sqrt[16]{\frac{1}{\frac{24\sqrt[2]{8}\sqrt[8]{1}}{8\sqrt[3]{3}}\sqrt{1+\frac{2}{9}\times 2}}}}$$

**Result:**

$$\frac{3^{9/128}}{\sqrt[384]{2}\sqrt[32]{13}}$$

**Decimal approximation:**

0.995297529741833477013435512450218049222021430270980144238...

0.995297529... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1}{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3} - 1}}} - \varphi + 1$$

and to the dilaton value **0.989117352243 =  $\phi$**

**Alternate form:**

$$\frac{1}{26} \times 3^{9/128} \times 2^{23/384} \sqrt[32]{13} 26^{15/16}$$

And:

$$8 \log_{\text{base } 0.995297529} [((((1/((((((2^{1/24}) * ((1)^{1/8})) / (((3)^{1/8})))) * ((1+2/9*2)^{1/2}))))))] - \pi + 1/\text{golden ratio}$$

**Input interpretation:**

$$8 \log_{0.995297529} \left( \frac{1}{\frac{\frac{24\sqrt{2} \sqrt[8]{1}}{\sqrt[8]{3}}}{\sqrt{1 + \frac{2}{9} \times 2}}}} \right) - \pi + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

125.4764...

125.4764... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternative representation:**

$$8 \log_{0.995298} \left( \frac{1}{\frac{\sqrt{1 + \frac{2 \times 2}{9}} \left( \frac{24\sqrt{2} \sqrt[8]{1}}{\sqrt[8]{3}} \right)}{\sqrt[8]{3}}} \right) - \pi + \frac{1}{\phi} = -\pi + \frac{1}{\phi} + \frac{8 \log \left( \frac{1}{\frac{\sqrt[8]{1} \cdot 24\sqrt{2} \sqrt{1 + \frac{4}{9}}}{\sqrt[8]{3}}} \right)}{\log(0.995298)}$$

**Series representations:**

$$8 \log_{0.995298} \left( \frac{1}{\frac{\sqrt{1 + \frac{2 \times 2}{9}} \left( \frac{24\sqrt{2} \sqrt[8]{1}}{\sqrt[8]{3}} \right)}{\sqrt[8]{3}}} \right) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi - \frac{8 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{3 \sqrt[8]{3}}{24\sqrt{2} \sqrt{13}} \right)^k}{k}}{\log(0.995298)}$$

$$8 \log_{0.995298} \left( \frac{1}{\frac{\sqrt{1 + \frac{2 \times 2}{9}} \left( \sqrt[24]{2} \sqrt[8]{1} \right)}{\sqrt[8]{3}}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - 1697.23 \log \left( \frac{3 \sqrt[8]{3}}{\sqrt[24]{2} \sqrt{13}} \right) - 8 \log \left( \frac{3 \sqrt[8]{3}}{\sqrt[24]{2} \sqrt{13}} \right) \sum_{k=0}^{\infty} (-0.00470247)^k G(k)$$

for  $G(0) = 0$  and  $\frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j}$

$$8 \log_{0.995298} \left( \frac{1}{\frac{\sqrt{1 + \frac{2 \times 2}{9}} \left( \sqrt[24]{2} \sqrt[8]{1} \right)}{\sqrt[8]{3}}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - 1697.23 \log \left( \frac{3 \sqrt[8]{3}}{\sqrt[24]{2} \sqrt{13}} \right) - 8 \log \left( \frac{3 \sqrt[8]{3}}{\sqrt[24]{2} \sqrt{13}} \right) \sum_{k=0}^{\infty} (-0.00470247)^k G(k)$$

for  $G(0) = 0$  and  $G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j}$

$8 \log_{\text{base } 0.995297529} [((((1/((((((2^{1/24}) * ((1)^{1/8})) / (((3)^{1/8})))) * ((1 + 2/9 * 2)^{1/2})))))))] + 11 + 1/\text{golden ratio}$

**Input interpretation:**

$$8 \log_{0.995297529} \left( \frac{1}{\frac{\frac{\sqrt[24]{2} \sqrt[8]{1}}{\sqrt[8]{3}}}{\sqrt{1 + \frac{2}{9} \times 2}}} \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

139.6180...

139.618... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representation:**

$$8 \log_{0.995298} \left( \frac{1}{\frac{\sqrt{1+\frac{2\sqrt{2}}{9}} \left( \sqrt[24]{2} \sqrt[8]{1} \right)}{\sqrt[8]{3}}}} \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} + \frac{8 \log \left( \frac{1}{\frac{\sqrt[8]{1} \sqrt[24]{2} \sqrt{1+\frac{4}{9}}}{\sqrt[8]{3}}}} \right)}{\log(0.995298)}$$

**Series representations:**

$$8 \log_{0.995298} \left( \frac{1}{\frac{\sqrt{1+\frac{2\sqrt{2}}{9}} \left( \sqrt[24]{2} \sqrt[8]{1} \right)}{\sqrt[8]{3}}}} \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} - \frac{8 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{3\sqrt[8]{3}}{2\sqrt[24]{2} \sqrt{13}} \right)^k}{k}}{\log(0.995298)}$$

$$8 \log_{0.995298} \left( \frac{1}{\frac{\sqrt{1+\frac{2\sqrt{2}}{9}} \left( \sqrt[24]{2} \sqrt[8]{1} \right)}{\sqrt[8]{3}}}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 1697.23 \log \left( \frac{3\sqrt[8]{3}}{2\sqrt[24]{2} \sqrt{13}} \right) - 8 \log \left( \frac{3\sqrt[8]{3}}{2\sqrt[24]{2} \sqrt{13}} \right) \sum_{k=0}^{\infty} (-0.00470247)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$$8 \log_{0.995298} \left( \frac{1}{\frac{\sqrt{1+\frac{2\sqrt{2}}{9}} \left( \sqrt[24]{2} \sqrt[8]{1} \right)}{\sqrt[8]{3}}}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 1697.23 \log \left( \frac{3\sqrt[8]{3}}{2\sqrt[24]{2} \sqrt{13}} \right) - 8 \log \left( \frac{3\sqrt[8]{3}}{2\sqrt[24]{2} \sqrt{13}} \right) \sum_{k=0}^{\infty} (-0.00470247)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$$27 \times 4 \log_{\text{base } 0.995297529} \left[ \left( \frac{1}{\left( \frac{(2^{1/24}) \cdot (1^{1/8})}{(3^{1/8})} \right)^2 \cdot \left( 1 + \frac{2}{9} \right)^{1/2}} \right) \right]$$

**Input interpretation:**

$$27 \times 4 \log_{0.995297529} \left( \frac{1}{\frac{\frac{2^{\frac{24}{\sqrt{2}}}}{\sqrt[8]{1}}}{\sqrt[8]{3}} \sqrt{1 + \frac{2}{9} \times 2}}}{\sqrt[8]{3}} \right)$$

$\log_b(x)$  is the base- $b$  logarithm

**Result:**

1728.000...

1728

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternative representation:**

$$27 \times 4 \log_{0.995298} \left( \frac{1}{\frac{\sqrt{1 + \frac{2 \times 2}{9}} \left( \frac{2^{\frac{24}{\sqrt{2}}}}{\sqrt[8]{1}} \right)}{\sqrt[8]{3}}}{\sqrt[8]{3}} \right) = \frac{108 \log \left( \frac{1}{\frac{\frac{\sqrt[8]{1} \cdot 2^{\frac{24}{\sqrt{2}}}}{\sqrt{1 + \frac{4}{9}}}}{\sqrt[8]{3}}}}{\log(0.995298)} \right)$$

**Series representations:**

$$27 \times 4 \log_{0.995298} \left( \frac{1}{\frac{\sqrt{1 + \frac{2 \times 2}{9}} \left( \frac{2^{\frac{24}{\sqrt{2}}}}{\sqrt[8]{1}} \right)}{\sqrt[8]{3}}}{\sqrt[8]{3}} \right) = - \frac{108 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{3 \sqrt[8]{3}}{2^{\frac{24}{\sqrt{2}}}} \sqrt{13} \right)^k}{k}}{\log(0.995298)}$$

$$27 \times 4 \log_{0.995298} \left( \frac{1}{\frac{\sqrt{1 + \frac{2 \times 2}{9} \left( \sqrt[24]{2} \sqrt[8]{1} \right)}}{\sqrt[8]{3}}}} \right) =$$

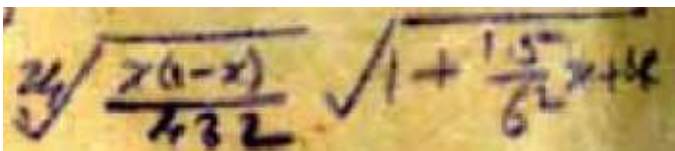
$$-22912.6 \log \left( \frac{3 \sqrt[8]{3}}{\sqrt[24]{2} \sqrt{13}} \right) - 108 \log \left( \frac{3 \sqrt[8]{3}}{\sqrt[24]{2} \sqrt{13}} \right) \sum_{k=0}^{\infty} (-0.00470247)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$$27 \times 4 \log_{0.995298} \left( \frac{1}{\frac{\sqrt{1 + \frac{2 \times 2}{9} \left( \sqrt[24]{2} \sqrt[8]{1} \right)}}{\sqrt[8]{3}}}} \right) =$$

$$-22912.6 \log \left( \frac{3 \sqrt[8]{3}}{\sqrt[24]{2} \sqrt{13}} \right) - 108 \log \left( \frac{3 \sqrt[8]{3}}{\sqrt[24]{2} \sqrt{13}} \right) \sum_{k=0}^{\infty} (-0.00470247)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$



$$((((((2(1-2)/432))^{1/24}) * ((1+5/36*2))^{1/2}))))$$

**Input:**

$$\sqrt[24]{2 \times \frac{1-2}{432}} \sqrt{1 + \frac{5}{36} \times 2}$$

**Result:**

$$\frac{\sqrt[24]{-1} \sqrt{23}}{3 \times 2^{5/8} \sqrt[8]{3}}$$

**Decimal approximation:**

0.89583355432310881682953988503540944783681626811484851799... +  
 0.11793872484923133574837811259814378639661932760750332985...  $i$

**Polar coordinates:**

$r \approx 0.903564$  (radius),  $\theta \approx 7.5^\circ$  (angle)

0.903564

**Alternate forms:**

$$\frac{1}{36} \sqrt[24]{-1} 6^{7/8} \sqrt{46}$$

$$\sqrt[8]{\frac{279841}{1259712} + \frac{279841 i}{419904 \sqrt{3}}}$$

$$\frac{\sqrt{23} \cos\left(\frac{\pi}{24}\right)}{3 \times 2^{5/8} \sqrt[8]{3}} + \frac{i \sqrt{23} \sin\left(\frac{\pi}{24}\right)}{3 \times 2^{5/8} \sqrt[8]{3}}$$

**Minimal polynomial:**

$$396718580736 x^{16} - 176259532896 x^8 + 78310985281$$

Or, without the imaginary:

**Input:**

$$\sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{5}{36} \times 2}$$

**Result:**

$$\frac{\sqrt{23}}{3 \times 2^{5/8} \sqrt[8]{3}}$$

**Decimal approximation:**

0.903563666749741023738621435771674160753810478778989574299...

0.9035636667...

**Alternate forms:**

$$\frac{1}{36} \times 6^{7/8} \sqrt{46}$$

$$\frac{1}{18} \left( 2^{3/8} \times 3^{7/8} \sqrt{23} \right)$$

$$\left[ \left( \left( \left( \left( \left( \frac{2(1-2)}{432} \right)^{1/24} \right) \times \left( \left( 1 + \frac{5}{36} \times 2 \right)^{1/2} \right) \right) \right) \right)^{1/16}$$

**Input:**

$$\sqrt[16]{\sqrt[24]{2 \times \frac{1-2}{432}} \sqrt{1 + \frac{5}{36} \times 2}}$$

**Result:**

$$\frac{\sqrt[384]{-1} \sqrt[32]{23}}{2^{5/128} \times 3^{9/128}}$$

**Decimal approximation:**

0.993648744350556310982770169660378561554790108932115583... +  
 0.00812945115569259619177463794215670052932371779368379856... i

**Polar coordinates:**

$r \approx 0.993682$  (radius),  $\theta \approx 0.46875^\circ$  (angle)

0.993682 result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\phi^5 \sqrt[4]{5^3}} - 1}}{\sqrt{5}} - \phi + 1$$

and to the dilaton value **0.989117352243 =  $\phi$**

**Alternate forms:**

$$\frac{1}{6} \times 6^{119/128} \sqrt[32]{46}$$



$$\frac{\sqrt[32]{23} \cos\left(\frac{\pi}{384}\right)}{2^{5/128} \times 3^{9/128}} + \frac{i \sqrt[32]{23} \sin\left(\frac{\pi}{384}\right)}{2^{5/128} \times 3^{9/128}}$$

$$\frac{\sqrt[32]{23} e^{(i\pi)/384}}{2^{5/128} \times 3^{9/128}}$$

And:

8 log base 0.99368199899441((((((((((2/432))^1/24))\*((1+5/36\*2))^1/2)))))))-  
Pi+1/golden ratio

**Input interpretation:**

$$8 \log_{0.99368199899441} \left( \sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{5}{36} \times 2} \right) - \pi + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

125.476441335...

125.476441335... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternative representation:**

$$8 \log_{0.993681998994410000} \left( \sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{2 \times 5}{36}} \right) - \pi + \frac{1}{\phi} =$$

$$-\pi + \frac{1}{\phi} + \frac{8 \log \left( \sqrt{1 + \frac{10}{36}} \sqrt[24]{\frac{2}{432}} \right)}{\log(0.993681998994410000)}$$

**Series representations:**

$$8 \log_{0.993681998994410000} \left( \sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{2 \times 5}{36}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - \frac{8 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{\sqrt{23}}{3 \times 2^{5/8} \sqrt[8]{3}} \right)^k}{k}}{\log(0.993681998994410000)}$$

$$8 \log_{0.993681998994410000} \left( \sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{2 \times 5}{36}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - 1262.223286910181 \log \left( \frac{\sqrt{23}}{3 \times 2^{5/8} \sqrt[8]{3}} \right) -$$

$$8 \log \left( \frac{\sqrt{23}}{3 \times 2^{5/8} \sqrt[8]{3}} \right) \sum_{k=0}^{\infty} (-0.006318001005590000)^k G(k)$$

$$\text{for } \left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

$$8 \log_{0.993681998994410000} \left( \sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{2 \times 5}{36}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - 1262.223286910181 \log \left( \frac{\sqrt{23}}{3 \times 2^{5/8} \sqrt[8]{3}} \right) -$$

$$8 \log \left( \frac{\sqrt{23}}{3 \times 2^{5/8} \sqrt[8]{3}} \right) \sum_{k=0}^{\infty} (-0.006318001005590000)^k G(k)$$

$$\text{for } \left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

8log base

0.99368199899441((((((((((2/432))^1/24))\*((1+5/36\*2))^1/2)))))))+11+1/golden ratio

**Input interpretation:**

$$8 \log_{0.99368199899441} \left( \sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{5}{36} \times 2} \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

139.618033989...

139.618033989... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representation:**

$$8 \log_{0.993681998994410000} \left( \sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{2 \times 5}{36}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{8 \log \left( \sqrt{1 + \frac{10}{36}} \sqrt[24]{\frac{2}{432}} \right)}{\log(0.993681998994410000)}$$

**Series representations:**

$$8 \log_{0.993681998994410000} \left( \sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{2 \times 5}{36}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - \frac{8 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{\sqrt{23}}{3 \times 2^{5/8} \sqrt[8]{3}} \right)^k}{k}}{\log(0.993681998994410000)}$$

$$8 \log_{0.993681998994410000} \left( \sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{2 \times 5}{36}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 1262.223286910181 \log \left( \frac{\sqrt{23}}{3 \times 2^{5/8} \sqrt[8]{3}} \right) -$$

$$8 \log \left( \frac{\sqrt{23}}{3 \times 2^{5/8} \sqrt[8]{3}} \right) \sum_{k=0}^{\infty} (-0.006318001005590000)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$$8 \log_{0.993681998994410000} \left( \sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{2 \times 5}{36}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 1262.223286910181 \log \left( \frac{\sqrt{23}}{3 \times 2^{5/8} \sqrt[8]{3}} \right) -$$

$$8 \log \left( \frac{\sqrt{23}}{3 \times 2^{5/8} \sqrt[8]{3}} \right) \sum_{k=0}^{\infty} (-0.006318001005590000)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$$27 \times 4 \log_{\text{base } 0.99368199899441}(((((((2/432))^{1/24}) \times ((1 + 5/36 \times 2))^{1/2}))))))$$

**Input interpretation:**

$$27 \times 4 \log_{0.99368199899441} \left( \sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{5}{36} \times 2} \right)$$

$\log_b(x)$  is the base- $b$  logarithm

**Result:**

1728.000000000...

1728

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

From Wikipedia:

*“The fundamental group of the complex form, compact real form, or any algebraic version of  $E_6$  is the cyclic group  $\mathbf{Z}/3\mathbf{Z}$ , and its outer automorphism group is the cyclic group  $\mathbf{Z}/2\mathbf{Z}$ . Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics,  $E_6$  plays a role in some grand unified theories”.*

**Alternative representation:**

$$27 \times 4 \log_{0.993681998994410000} \left( \sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{2 \times 5}{36}} \right) = \frac{108 \log \left( \sqrt{1 + \frac{10}{36}} \sqrt[24]{\frac{2}{432}} \right)}{\log(0.993681998994410000)}$$

**Series representations:**

$$27 \times 4 \log_{0.993681998994410000} \left( \sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{2 \times 5}{36}} \right) = - \frac{108 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{\sqrt{23}}{3 \times 2^{5/8} \sqrt[8]{3}} \right)^k}{k}}{\log(0.993681998994410000)}$$

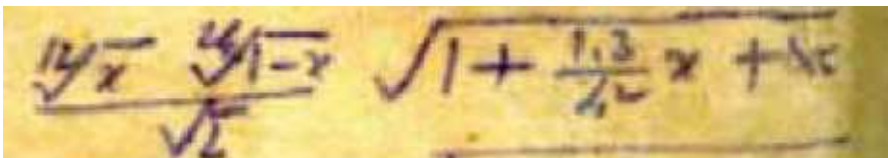
$$27 \times 4 \log_{0.993681998994410000} \left( \sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{2 \times 5}{36}} \right) = \log \left( \frac{\sqrt{23}}{3 \times 2^{5/8} \sqrt[8]{3}} \right) \left( -17040.01437328745 - 108.00000000000000 \sum_{k=0}^{\infty} (-0.006318001005590000)^k G(k) \right)$$

for  $\left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$$27 \times 4 \log_{0.993681998994410000} \left( \sqrt[24]{\frac{2}{432}} \sqrt{1 + \frac{2 \times 5}{36}} \right) = \log \left( \frac{\sqrt{23}}{3 \times 2^{5/8} \sqrt[8]{3}} \right) \left( -17040.01437328745 - 108.00000000000000 \sum_{k=0}^{\infty} (-0.006318001005590000)^k G(k) \right)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

Now, we have that:



$$1/\sqrt{2} * (2)^{(1/12)} * (1-2)^{(1/24)} * (1+3/16*2)^{1/2}$$

**Input:**

$$\frac{1}{\sqrt{2}} \sqrt[12]{2} \sqrt[24]{1-2} \sqrt{1 + \frac{3}{16} \times 2}$$

**Result:**

$$\frac{\sqrt[24]{-1} \sqrt{11}}{2 \times 2^{11/12}}$$

**Decimal approximation:**

0.87094504038371958062455436890270448603941132910620131206... +  
0.11466208982787202626000344204276087557463698338225347703... *i*

**Polar coordinates:**

$r \approx 0.87846$  (radius),  $\theta \approx 7.5^\circ$  (angle)

0.87846

**Alternate forms:**

$$\frac{1}{8} \times 2^{7/12} \sqrt[24]{-1} \sqrt{22}$$

$$\frac{1}{4} \sqrt[6]{1+i} \sqrt{11}$$

$$\frac{\sqrt{11} \cos\left(\frac{\pi}{24}\right)}{2 \times 2^{11/12}} + \frac{i \sqrt{11} \sin\left(\frac{\pi}{24}\right)}{2 \times 2^{11/12}}$$

**Minimal polynomial:**

$$8388608x^{12} - 5451776x^6 + 1771561$$

Or, without the imaginary:

$$1/\sqrt{2} * (2)^{(1/12)} * (1)^{(1/24)} * (1+3/16*2)^{1/2}$$

**Input:**

$$\frac{1}{\sqrt{2}} \sqrt[12]{2} \sqrt[24]{1} \sqrt{1 + \frac{3}{16} \times 2}$$

**Result:**

$$\frac{\sqrt{11}}{2 \times 2^{11/12}}$$

**Decimal approximation:**

0.878460390804670216313372885619023531722570983049875588333...

0.8784603908...

**Alternate forms:**

$$\frac{1}{8} \times 2^{7/12} \sqrt{22}$$

$$\frac{1}{4} \left( \sqrt[12]{2} \sqrt{11} \right)$$

$$\left( \left( \frac{1}{\sqrt{2}} * (2)^{(1/12)} * (1)^{(1/24)} * (1+3/16*2)^{1/2} \right) \right)^{1/16}$$

**Input:**

$$\sqrt[16]{\frac{1}{\sqrt{2}} \sqrt[12]{2} \sqrt[24]{1} \sqrt{1 + \frac{3}{16} \times 2}}$$

**Result:**

$$\frac{\sqrt[32]{11}}{2^{23/192}}$$

**Decimal approximation:**

0.991933680040897993668758581607226014402645354145273715639...

0.99193368... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\phi^5 \sqrt[4]{5^3} - 1}}}{\sqrt{5}} - \phi + 1$$

and to the dilaton value **0.989117352243 =  $\phi$**

**Alternate form:**

$$\frac{1}{2} \times 2^{163/192} \sqrt[32]{22}$$

8log base 0.99193368(((((((1/sqrt2 \* (2)^(1/12) \* (1)^(1/24) \* (1+3/16\*2)^1/2)))))))-  
Pi+1/golden ratio

**Input interpretation:**

$$8 \log_{0.99193368} \left( \frac{1}{\sqrt{2}} \sqrt[12]{2} \sqrt[24]{1} \sqrt{1 + \frac{3}{16} \times 2} \right) - \pi + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

125.476...

125.476... result very near to the dilaton mass calculated as a type of Higgs boson:  
125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternative representation:**

$$8 \log_{0.991934} \left( \frac{\sqrt[12]{2} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 3}{16}}}{\sqrt{2}} \right) - \pi + \frac{1}{\phi} = -\pi + \frac{1}{\phi} + \frac{8 \log \left( \frac{\sqrt[24]{1} \sqrt[12]{2} \sqrt{1 + \frac{6}{16}}}{\sqrt{2}} \right)}{\log(0.991934)}$$

**Series representations:**

$$8 \log_{0.991934} \left( \frac{\sqrt[12]{2} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 3}{16}}}{\sqrt{2}} \right) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi - \frac{8 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{\sqrt{11}}{2 \times 2^{5/12} \sqrt{2}} \right)^k}{k}}{\log(0.991934)}$$

$$8 \log_{0.991934} \left( \frac{\sqrt[12]{2} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 3}{16}}}{\sqrt{2}} \right) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi - 987.778 \log \left( \frac{\sqrt{11}}{2 \times 2^{5/12} \sqrt{2}} \right) - 8 \log \left( \frac{\sqrt{11}}{2 \times 2^{5/12} \sqrt{2}} \right) \sum_{k=0}^{\infty} (-0.00806632)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$



$$8 \log_{0.991934} \left( \frac{12\sqrt[2]{2} \ 24\sqrt[4]{1} \ \sqrt{1 + \frac{2 \times 3}{16}}}{\sqrt{2}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - 987.778 \log \left( \frac{\sqrt{11}}{2 \times 2^{5/12} \sqrt{2}} \right) - 8 \log \left( \frac{\sqrt{11}}{2 \times 2^{5/12} \sqrt{2}} \right) \sum_{k=0}^{\infty} (-0.00806632)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

8log base 0.99193368(((((((1/sqrt2 \* (2)^(1/12) \* (1)^(1/24) \* (1+3/16\*2)^1/2))))))))+11+1/golden ratio

**Input interpretation:**

$$8 \log_{0.99193368} \left( \frac{1}{\sqrt{2}} \ 12\sqrt[2]{2} \ 24\sqrt[4]{1} \ \sqrt{1 + \frac{3}{16} \times 2} \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

139.618...

139.618... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representation:**

$$8 \log_{0.991934} \left( \frac{12\sqrt[2]{2} \ 24\sqrt[4]{1} \ \sqrt{1 + \frac{2 \times 3}{16}}}{\sqrt{2}} \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} + \frac{8 \log \left( \frac{12\sqrt[2]{2} \ 24\sqrt[4]{1} \ \sqrt{1 + \frac{2 \times 3}{16}}}{\sqrt{2}} \right)}{\log(0.991934)}$$

**Series representations:**

$$8 \log_{0.991934} \left( \frac{12\sqrt[2]{2} \ 24\sqrt[4]{1} \ \sqrt{1 + \frac{2 \times 3}{16}}}{\sqrt{2}} \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} - \frac{8 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{\sqrt{11}}{2 \times 2^{5/12} \sqrt{2}} \right)^k}{k}}{\log(0.991934)}$$

$$8 \log_{0.991934} \left( \frac{12\sqrt[2]{2} \ 24\sqrt[4]{1} \ \sqrt{1 + \frac{2 \times 3}{16}}}{\sqrt{2}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 987.778 \log \left( \frac{\sqrt{11}}{2 \times 2^{5/12} \sqrt{2}} \right) - 8 \log \left( \frac{\sqrt{11}}{2 \times 2^{5/12} \sqrt{2}} \right) \sum_{k=0}^{\infty} (-0.00806632)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$$8 \log_{0.991934} \left( \frac{12\sqrt[2]{2} \ 24\sqrt[4]{1} \ \sqrt{1 + \frac{2 \times 3}{16}}}{\sqrt{2}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 987.778 \log \left( \frac{\sqrt{11}}{2 \times 2^{5/12} \sqrt{2}} \right) - 8 \log \left( \frac{\sqrt{11}}{2 \times 2^{5/12} \sqrt{2}} \right) \sum_{k=0}^{\infty} (-0.00806632)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$$27 \times 4 \log_{\text{base } 0.99193368} \left( \left( \left( \left( \left( \left( \frac{1}{\sqrt{2}} \times (2)^{1/12} \times (1)^{1/24} \times (1 + \frac{3}{16} \times 2)^{1/2} \right) \right) \right) \right) \right) \right) \right)$$

**Input interpretation:**

$$27 \times 4 \log_{0.99193368} \left( \frac{1}{\sqrt{2}} \ 12\sqrt[2]{2} \ 24\sqrt[4]{1} \ \sqrt{1 + \frac{3}{16} \times 2} \right)$$

$\log_b(x)$  is the base- $b$  logarithm

**Result:**

1728.00...

1728

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic

curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternative representation:**

$$27 \times 4 \log_{0.991934} \left( \frac{\sqrt[12]{2} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 3}{16}}}{\sqrt{2}} \right) = \frac{108 \log \left( \frac{\sqrt[24]{1} \sqrt[12]{2} \sqrt{1 + \frac{6}{16}}}{\sqrt{2}} \right)}{\log(0.991934)}$$

**Series representations:**

$$27 \times 4 \log_{0.991934} \left( \frac{\sqrt[12]{2} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 3}{16}}}{\sqrt{2}} \right) = - \frac{108 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{\sqrt{11}}{2 \times 2^{5/12} \sqrt{2}} \right)^k}{k}}{\log(0.991934)}$$

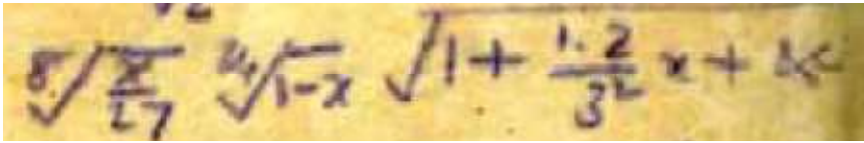
$$27 \times 4 \log_{0.991934} \left( \frac{\sqrt[12]{2} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 3}{16}}}{\sqrt{2}} \right) = -13335 \cdot \log \left( \frac{\sqrt{11}}{2 \times 2^{5/12} \sqrt{2}} \right) - 108 \log \left( \frac{\sqrt{11}}{2 \times 2^{5/12} \sqrt{2}} \right) \sum_{k=0}^{\infty} (-0.00806632)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$$27 \times 4 \log_{0.991934} \left( \frac{\sqrt[12]{2} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 3}{16}}}{\sqrt{2}} \right) = -13335 \cdot \log \left( \frac{\sqrt{11}}{2 \times 2^{5/12} \sqrt{2}} \right) - 108 \log \left( \frac{\sqrt{11}}{2 \times 2^{5/12} \sqrt{2}} \right) \sum_{k=0}^{\infty} (-0.00806632)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

We have that:



$$\left(\frac{2}{27}\right)^{1/8} (1-2)^{1/24} * \left(1+\frac{2}{9}x\right)^{1/2}$$

**Input:**

$$\sqrt[8]{\frac{2}{27}} \sqrt[24]{1-2} \sqrt{1 + \frac{2}{9} \times 2}$$

**Result:**

$$\frac{\sqrt[24]{-1} \sqrt[8]{2} \sqrt{13}}{3 \times 3^{3/8}}$$

**Decimal approximation:**

$$0.86065137424884676430197866130730198353406671589477785638... + 0.11330690297188522440906942574963763517935215682781351757... i$$

**Polar coordinates:**

$$r \approx 0.868078 \text{ (radius), } \theta \approx 7.5^\circ \text{ (angle)}$$

$$0.868078$$

**Alternate forms:**

$$\frac{1}{9} \sqrt[8]{2} 3^{5/8} \sqrt[24]{-1} \sqrt{13}$$

$$\sqrt[8]{\frac{28561}{177147} + \frac{28561 i}{59049 \sqrt{3}}}$$

$$\frac{\sqrt[8]{2} \sqrt{13} \cos\left(\frac{\pi}{24}\right)}{3 \times 3^{3/8}} + \frac{i \sqrt[8]{2} \sqrt{13} \sin\left(\frac{\pi}{24}\right)}{3 \times 3^{3/8}}$$

**Minimal polynomial:**

$$31381059609 x^{16} - 10118990934 x^8 + 3262922884$$

Or, without imaginary:

**Input:**

$$\sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2}{9} \times 2}$$

**Result:**

$$\frac{\sqrt[8]{2} \sqrt{13}}{3 \times 3^{3/8}}$$

**Decimal approximation:**

0.868077901030494329439330320894275629939857003729296684654...

0.868077901030...

**Alternate form:**

$$\frac{1}{9} \sqrt[8]{2} 3^{5/8} \sqrt{13}$$

$$\left( \left( \left( \frac{2}{27} \right)^{1/8} \left( 1 \right)^{1/24} * \left( 1 + \frac{2}{9} * 2 \right)^{1/2} \right) \right)^{1/16}$$

**Input:**

$$\sqrt[16]{\sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2}{9} \times 2}}$$

**Exact result:**

$$\frac{\sqrt[128]{2} \sqrt[32]{13}}{3^{11/128}}$$

**Decimal approximation:**

0.991196862736619632940254781123317938633777327991424025152...

0.9911968627... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} - \varphi + 1 \approx 0.9991104684$$

and to the dilaton value **0.989117352243 =  $\phi$**

8log base 0.991196862736(((((((2/27)^1/8 (1)^1/24 \* (1+2/9\*2)^1/2)))))))-  
 Pi+1/golden ratio

**Input interpretation:**

$$8 \log_{0.991196862736} \left( \sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2}{9} \times 2} \right) - \pi + \frac{1}{\phi}$$

$\log_b(x)$  is the base-  $b$  logarithm

$\phi$  is the golden ratio

**Result:**

125.4764413...

125.4764413... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternative representation:**

$$8 \log_{0.9911968627360000} \left( \sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 2}{9}} \right) - \pi + \frac{1}{\phi} =$$

$$-\pi + \frac{1}{\phi} + \frac{8 \log \left( \sqrt[24]{1} \sqrt{1 + \frac{4}{9}} \sqrt[8]{\frac{2}{27}} \right)}{\log(0.9911968627360000)}$$

**Series representations:**

$$8 \log_{0.9911968627360000} \left( \sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 2}{9}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - \frac{8 \sum_{k=1}^{\infty} \frac{\left( \frac{-1}{9} \right)^k \left( -9 + \sqrt[8]{2} \cdot 3^{5/8} \sqrt{13} \right)^k}{k}}{\log(0.9911968627360000)}$$

$$8 \log_{0.9911968627360000} \left( \sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 2}{9}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - 904.7669270722 \log \left( \frac{\sqrt[8]{2} \sqrt{13}}{3 \times 3^{3/8}} \right) -$$

$$8 \log \left( \frac{\sqrt[8]{2} \sqrt{13}}{3 \times 3^{3/8}} \right) \sum_{k=0}^{\infty} (-0.0088031372640000)^k G(k)$$

$$\text{for } \left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

$$8 \log_{0.9911968627360000} \left( \sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 2}{9}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - 904.7669270722 \log \left( \frac{\sqrt[8]{2} \sqrt{13}}{3 \times 3^{3/8}} \right) -$$

$$8 \log \left( \frac{\sqrt[8]{2} \sqrt{13}}{3 \times 3^{3/8}} \right) \sum_{k=0}^{\infty} (-0.0088031372640000)^k G(k)$$

$$\text{for } \left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

8log base 0.991196862736(((((((2/27)^1/8 (1)^1/24 \* (1+2/9\*2)^1/2)))))))+11+1/golden ratio

**Input interpretation:**

$$8 \log_{0.991196862736} \left( \sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2}{9} \times 2} \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$  is the base-  $b$  logarithm

$\phi$  is the golden ratio

**Result:**

139.6180340...

139.618034... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representation:**

$$8 \log_{0.9911968627360000} \left( \sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 2}{9}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{8 \log \left( \sqrt[24]{1} \sqrt{1 + \frac{4}{9}} \sqrt[8]{\frac{2}{27}} \right)}{\log(0.9911968627360000)}$$

**Series representations:**

$$8 \log_{0.9911968627360000} \left( \sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 2}{9}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - \frac{8 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{9}\right)^k \left(-9 + \sqrt[8]{2} \cdot 3^{5/8} \sqrt{13}\right)^k}{k}}{\log(0.9911968627360000)}$$

$$8 \log_{0.9911968627360000} \left( \sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 2}{9}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 904.7669270722 \log \left( \frac{\sqrt[8]{2} \sqrt{13}}{3 \times 3^{3/8}} \right) -$$

$$8 \log \left( \frac{\sqrt[8]{2} \sqrt{13}}{3 \times 3^{3/8}} \right) \sum_{k=0}^{\infty} (-0.0088031372640000)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$$8 \log_{0.9911968627360000} \left( \sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 2}{9}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 904.7669270722 \log \left( \frac{\sqrt[8]{2} \sqrt{13}}{3 \times 3^{3/8}} \right) -$$

$$8 \log \left( \frac{\sqrt[8]{2} \sqrt{13}}{3 \times 3^{3/8}} \right) \sum_{k=0}^{\infty} (-0.0088031372640000)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$



$$27 \times 4 \log_{\text{base } 0.991196862736}(((((((2/27)^{1/8} (1)^{1/24} * (1+2/9*2)^{1/2}))))))))$$

**Input interpretation:**

$$27 \times 4 \log_{0.991196862736} \left( \sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2}{9} \times 2} \right)$$

$\log_b(x)$  is the base- $b$  logarithm

**Result:**

1728.000000...

1728

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternative representation:**

$$27 \times 4 \log_{0.9911968627360000} \left( \sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 2}{9}} \right) = \frac{108 \log \left( \sqrt[24]{1} \sqrt{1 + \frac{4}{9}} \sqrt[8]{\frac{2}{27}} \right)}{\log(0.9911968627360000)}$$

**Series representations:**

$$27 \times 4 \log_{0.9911968627360000} \left( \sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 2}{9}} \right) = \frac{108 \sum_{k=1}^{\infty} \frac{\left( \frac{-1}{9} \right)^k \left( -9 + \sqrt[8]{2} \cdot 3^{5/8} \sqrt{13} \right)^k}{k}}{\log(0.9911968627360000)}$$

$$27 \times 4 \log_{0.9911968627360000} \left( \sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 2}{9}} \right) = \log \left( \frac{\sqrt[8]{2} \sqrt{13}}{3 \times 3^{3/8}} \right)$$

$$\left( -12\,214.353515475 - 108.000000000000 \sum_{k=0}^{\infty} (-0.0088031372640000)^k G(k) \right)$$

for  $\left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$$27 \times 4 \log_{0.9911968627360000} \left( \sqrt[8]{\frac{2}{27}} \sqrt[24]{1} \sqrt{1 + \frac{2 \times 2}{9}} \right) = \log \left( \frac{\sqrt[8]{2} \sqrt{13}}{3 \times 3^{3/8}} \right)$$

$$\left( -12\,214.353515475 - 108.000000000000 \sum_{k=0}^{\infty} (-0.0088031372640000)^k G(k) \right)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

From the sum of this four results

0.868077901030... 0.8784603908... 0.9035636667... 1.0783337077498417.....

we obtain:

$$1 + 1 / (((((0.8680779 + 0.8784603 + 0.9035636 + 1.07833370774))))^{1/3})$$

**Input interpretation:**

$$1 + \frac{1}{\sqrt[3]{0.8680779 + 0.8784603 + 0.9035636 + 1.07833370774}}$$

**Result:**

1.6448981...

$$1.6448981... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

and:

$(((((0.8680779 + 0.8784603 + 0.9035636 + 1.07833370774))))^4 - 64 - \pi - \frac{1}{\text{golden ratio}})$

**Input interpretation:**

$$(0.8680779 + 0.8784603 + 0.9035636 + 1.07833370774)^4 - 64 - \pi - \frac{1}{\phi}$$

$\phi$  is the golden ratio

**Result:**

125.485...

125.485... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$(0.868078 + 0.87846 + 0.903564 + 1.078333707740000)^4 - 64 - \pi - \frac{1}{\phi} = -64 - \pi + 3.72844^4 - \frac{1}{2 \cos(216^\circ)}$$

$$(0.868078 + 0.87846 + 0.903564 + 1.078333707740000)^4 - 64 - \pi - \frac{1}{\phi} = -64 - 180^\circ + 3.72844^4 - \frac{1}{2 \cos(216^\circ)}$$

$$(0.868078 + 0.87846 + 0.903564 + 1.078333707740000)^4 - 64 - \pi - \frac{1}{\phi} = -64 - \pi + 3.72844^4 - \frac{1}{2 \cos\left(\frac{\pi}{5}\right)}$$

**Series representations:**

$$(0.868078 + 0.87846 + 0.903564 + 1.078333707740000)^4 - 64 - \pi - \frac{1}{\phi} = 129.244 - \frac{1}{\phi} - 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}$$

$$(0.868078 + 0.87846 + 0.903564 + 1.078333707740000)^4 - 64 - \pi - \frac{1}{\phi} =$$

$$131.244 - \frac{1}{\phi} - 2 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$(0.868078 + 0.87846 + 0.903564 + 1.078333707740000)^4 - 64 - \pi - \frac{1}{\phi} =$$

$$129.244 - \frac{1}{\phi} - \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}}$$

### Integral representations:

$$(0.868078 + 0.87846 + 0.903564 + 1.078333707740000)^4 - 64 - \pi - \frac{1}{\phi} =$$

$$129.244 - \frac{1}{\phi} - 2 \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$(0.868078 + 0.87846 + 0.903564 + 1.078333707740000)^4 - 64 - \pi - \frac{1}{\phi} =$$

$$129.244 - \frac{1}{\phi} - 4 \int_0^1 \sqrt{1-t^2} dt$$

$$(0.868078 + 0.87846 + 0.903564 + 1.078333707740000)^4 - 64 - \pi - \frac{1}{\phi} =$$

$$129.244 - \frac{1}{\phi} - 2 \int_0^{\infty} \frac{\sin(t)}{t} dt$$

### Appendix

From:

*Modular equations and approximations to  $\pi$  – S. Ramanujan*  
 Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

1. If we suppose that

$$(1 + e^{-\pi\sqrt{n}})(1 + e^{-3\pi\sqrt{n}})(1 + e^{-5\pi\sqrt{n}}) \dots = 2^{\frac{1}{4}} e^{-\pi\sqrt{n}/24} G_n \quad (1)$$

and

$$(1 - e^{-\pi\sqrt{n}})(1 - e^{-3\pi\sqrt{n}})(1 - e^{-5\pi\sqrt{n}}) \dots = 2^{\frac{1}{4}} e^{-\pi\sqrt{n}/24} g_n, \quad (2)$$

then  $G_n$  and  $g_n$  can always be expressed as roots of algebraical equations when  $n$  is any rational number. For we know that

3. In order to obtain approximations for  $\pi$  we take logarithms of (1) and (2). Thus

$$\left. \begin{aligned} \pi &= \frac{24}{\sqrt{n}} \log(2^{\frac{1}{4}} G_n) \\ \pi &= \frac{24}{\sqrt{n}} \log(2^{\frac{1}{4}} g_n) \end{aligned} \right\}, \quad (10)$$

approximately, the error being nearly  $\frac{24}{\sqrt{n}} e^{-\pi\sqrt{n}}$  in both cases. These equations may also be written as

$$e^{\pi\sqrt{n}/24} = 2^{\frac{1}{4}} G_n, \quad e^{\pi\sqrt{n}/24} = 2^{\frac{1}{4}} g_n \quad (11)$$

For  $n = (\sqrt{5}+1)/2$ , we obtain:

$$24/\text{sqrt}(((\text{sqrt}5+1)/2)) \ln(e^{((\text{Pi}*\text{sqrt}((\text{sqrt}5+1)/2))/24))$$

**Input:**

$$\frac{24}{\sqrt{\frac{1}{2}(\sqrt{5} + 1)}} \log \left( e^{\frac{1}{24} \left( \pi \sqrt{\frac{1}{2}(\sqrt{5} + 1)} \right)} \right)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$\pi$

**Decimal approximation:**

3.141592653589793238462643383279502884197169399375105820974...

3.1415926535...

**Property:**

$\pi$  is a transcendental number

**Alternative representations:**

$$\frac{\log \left( e^{\frac{1}{24} \pi \sqrt{\frac{1}{2}(\sqrt{5}+1)}} \right)^{24}}{\sqrt{\frac{1}{2}(\sqrt{5}+1)}} = \frac{24 \pi \sqrt{\frac{1}{2}(1+\sqrt{5})}}{24 \sqrt{\frac{1}{2}(1+\sqrt{5})}}$$

$$\frac{\log \left( e^{\frac{1}{24} \pi \sqrt{\frac{1}{2}(\sqrt{5}+1)}} \right)^{24}}{\sqrt{\frac{1}{2}(\sqrt{5}+1)}} = \frac{24 \log_e \left( e^{\frac{1}{24} \pi \sqrt{\frac{1}{2}(1+\sqrt{5})}} \right)}{\sqrt{\frac{1}{2}(1+\sqrt{5})}}$$

$$\frac{\log \left( e^{\frac{1}{24} \pi \sqrt{\frac{1}{2}(\sqrt{5}+1)}} \right)^{24}}{\sqrt{\frac{1}{2}(\sqrt{5}+1)}} = \frac{24 \log(a) \log_a \left( e^{\frac{1}{24} \pi \sqrt{\frac{1}{2}(1+\sqrt{5})}} \right)}{\sqrt{\frac{1}{2}(1+\sqrt{5})}}$$

**Series representations:**

$$\frac{\log \left( e^{\frac{1}{24} \pi \sqrt{\frac{1}{2}(\sqrt{5}+1)}} \right)^{24}}{\sqrt{\frac{1}{2}(\sqrt{5}+1)}} = 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{\log \left( e^{\frac{1}{24} \pi \sqrt{\frac{1}{2}(\sqrt{5}+1)}} \right)^{24}}{\sqrt{\frac{1}{2}(\sqrt{5}+1)}} = \sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}$$

$$\frac{\log \left( e^{\frac{1}{24} \pi \sqrt{\frac{1}{2}(\sqrt{5}+1)}} \right)^{24}}{\sqrt{\frac{1}{2}(\sqrt{5}+1)}} = \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right)$$

**Integral representations:**

$$\frac{\log\left(e^{\frac{1}{24}\pi\sqrt{\frac{1}{2}(\sqrt{5}+1)}}\right)_{24}}{\sqrt{\frac{1}{2}(\sqrt{5}+1)}} = 4 \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{\log\left(e^{\frac{1}{24}\pi\sqrt{\frac{1}{2}(\sqrt{5}+1)}}\right)_{24}}{\sqrt{\frac{1}{2}(\sqrt{5}+1)}} = 2 \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{\log\left(e^{\frac{1}{24}\pi\sqrt{\frac{1}{2}(\sqrt{5}+1)}}\right)_{24}}{\sqrt{\frac{1}{2}(\sqrt{5}+1)}} = 2 \int_0^\infty \frac{1}{1+t^2} dt$$

$$24/\sqrt{((\sqrt{5}+1)/2)) \ln(e^{((\pi*\sqrt{((\sqrt{5}+1)/2))/x)}) = \pi$$

**Input:**

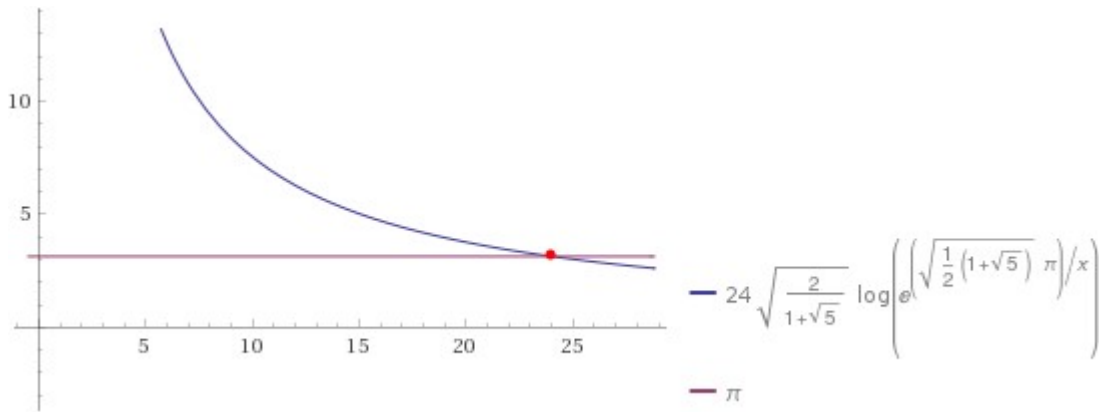
$$\frac{24}{\sqrt{\frac{1}{2}(\sqrt{5}+1)}} \log\left(e^{\left(\pi\sqrt{\frac{1}{2}(\sqrt{5}+1)}\right)/x}\right) = \pi$$

log(x) is the natural logarithm

**Exact result:**

$$24 \sqrt{\frac{2}{1+\sqrt{5}}} \log\left(e^{\left(\sqrt{\frac{1}{2}(1+\sqrt{5})}\pi\right)/x}\right) = \pi$$

**Plot:**



**Alternate forms:**

$$12 \sqrt{2(\sqrt{5}-1)} \log \left( e^{\left( \sqrt{\frac{1}{2}(1+\sqrt{5})} \pi \right) / x} \right) = \pi$$

$$e^{\frac{\left( \frac{1}{2} \sqrt{1-2i} + \frac{1}{2} \sqrt{1+2i} \right) \pi}{x}} = e^{\pi / \left( 12 \sqrt{2(\sqrt{5}-1)} \right)}$$

**Alternate form assuming x>0:**

$$\frac{24 \pi}{x} = \pi$$

**Solution:**

$$x = \frac{\sqrt{\frac{1}{2}(1+\sqrt{5})} \pi}{\frac{1}{24} \sqrt{\frac{1}{2}(1+\sqrt{5})} \pi + 2i\pi c_1} \text{ for } c_1 \in \mathbb{Z}$$

$\mathbb{Z}$  is the set of integers

**Solution:**

$$x \approx \frac{61.057}{2.5440 + 96in}, \quad n \in \mathbb{Z}$$

$\mathbb{Z}$  is the set of integers

**Integer solution:**

$$x = 24$$

24

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.



5. Since  $G_n$  and  $g_n$  can be expressed as roots of algebraical equations with rational coefficients, the same is true of  $G_n^{24}$  or  $g_n^{24}$ . So let us suppose that

$$1 = ag_n^{-24} - bg_n^{-48} + \dots,$$

or

$$g_n^{24} = a - bg_n^{-24} + \dots.$$

But we know that

$$\begin{aligned} 64e^{-\pi\sqrt{n}}g_n^{24} &= 1 - 24e^{-\pi\sqrt{n}} + 276e^{-2\pi\sqrt{n}} - \dots, \\ 64g_n^{24} &= e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \dots, \\ 64a - 64bg_n^{-24} + \dots &= e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \dots, \\ 64a - 4096be^{-\pi\sqrt{n}} + \dots &= e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \dots, \end{aligned}$$

that is

$$e^{\pi\sqrt{n}} = (64a + 24) - (4096b + 276)e^{-\pi\sqrt{n}} + \dots \tag{13}$$

$$e^{\pi\sqrt{n}} = (64a - 24) - (4096b + 276)e^{-\pi\sqrt{n}} + \dots$$

$$e^{(\pi\sqrt{1/5})} = (64a - 24) - (4096b + 276)e^{-(\pi\sqrt{1/5})}$$

**Input:**

$$e^{\pi\sqrt{1/5}} = (64a - 24) - (4096b + 276)e^{-\pi\sqrt{1/5}}$$

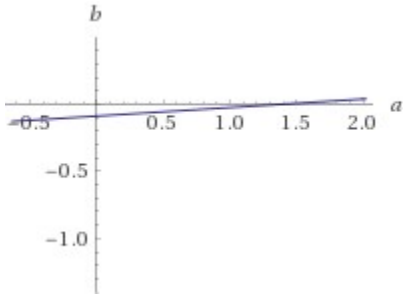
**Exact result:**

$$e^{\pi/\sqrt{5}} = 64a - e^{-\pi/\sqrt{5}}(4096b + 276) - 24$$

**Geometric figure:**

line

**Implicit plot:**



**Alternate forms:**

$$e^{\pi/\sqrt{5}} = 8(8a - 3) - 4e^{-\pi/\sqrt{5}}(1024b + 69)$$

$$b = \frac{1}{64}e^{\pi/\sqrt{5}}a + \frac{-276 - 24e^{\pi/\sqrt{5}} - e^{(2\pi)/\sqrt{5}}}{4096}$$

$$e^{\pi/\sqrt{5}} = 4e^{-\pi/\sqrt{5}}(16e^{\pi/\sqrt{5}}a - 1024b - 6e^{\pi/\sqrt{5}} - 69)$$

**Expanded form:**

$$e^{\pi/\sqrt{5}} = 64a - 4096e^{-\pi/\sqrt{5}}b - 276e^{-\pi/\sqrt{5}} - 24$$

**Real solution:**

$$b \approx 0.0636777a - 0.0953168$$

**Solution:**

$$b \approx 0.063678a - 0.095317$$

**Solution for the variable b:**

$$b = \frac{64e^{\pi/\sqrt{5}}a - e^{(2\pi)/\sqrt{5}} - 24e^{\pi/\sqrt{5}} - 276}{4096}$$

Thence:

$$e^{\pi\sqrt{n}} = (64a - 24) - (4096b + 276)e^{-\pi\sqrt{n}} + \dots$$

$$4.0753757303457 \approx 4.07537573033485$$

Indeed:

$$e^{(\pi\sqrt{1/5})} = (64(0.490939) - 24) - (4096(0.0636777(0.490939) - 0.0953168) + 276) * e^{(-\pi\sqrt{1/5})}$$

$$e^{(\pi \cdot \sqrt{1/5})}$$

**Input:**

$$e^{\pi \sqrt{1/5}}$$

**Exact result:**

$$e^{\pi/\sqrt{5}}$$

**Decimal approximation:**

4.075375730345737651273044453067495042599043332439503696709...

4.0753757303457....

**Property:**

$e^{\pi/\sqrt{5}}$  is a transcendental number

**Series representations:**

$$e^{\pi \sqrt{1/5}} = \sum_{k=0}^{\infty} \frac{5^{-k/2} \pi^k}{k!}$$

$$e^{\pi \sqrt{1/5}} = \sum_{k=-\infty}^{\infty} I_k\left(\frac{\pi}{\sqrt{5}}\right)$$

$$e^{\pi \sqrt{1/5}} = \sum_{k=0}^{\infty} \frac{5^{-k} \pi^{-1+2k} (2\sqrt{5} k + \pi)}{(2k)!}$$

**Integral representation:**

$$(1+z)^\alpha = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-\alpha-s)}{z^s} ds}{(2\pi i)\Gamma(-\alpha)} \text{ for } (0 < \gamma < -\text{Re}(\alpha) \text{ and } |\arg(z)| < \pi)$$

$$(64(0.490939)-24)-(4096(0.0636777(0.490939)-0.0953168)+276)*e^{(-\pi \cdot \sqrt{1/5})}$$

**Input interpretation:**

$$(64 \times 0.490939 - 24) - (4096 (0.0636777 \times 0.490939 - 0.0953168) + 276) e^{-\pi \sqrt{1/5}}$$

**Result:**

4.075375730334850801646054008644231570457697237459017213138...

4.07537573033485.....

**Series representations:**

$$(64 \times 0.490939 - 24) - (4096 (0.0636777 \times 0.490939 - 0.0953168) + 276) e^{-\pi \sqrt{1/5}} =$$

$$7.4201 - 13.631 \exp\left(-\pi \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{4}{5}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$(64 \times 0.490939 - 24) - (4096 (0.0636777 \times 0.490939 - 0.0953168) + 276) e^{-\pi \sqrt{1/5}} =$$

$$7.4201 - 13.631 \exp\left(-\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{1}{5} - z_0\right)^k z_0^{-k}}{k!}\right)$$

for not  $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$(64 \times 0.490939 - 24) - (4096 (0.0636777 \times 0.490939 - 0.0953168) + 276) e^{-\pi \sqrt{1/5}} =$$

$$7.4201 - 13.631 \exp\left(\frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-j} \left(-\frac{4}{5}\right)^{-s} \Gamma\left(-\frac{1}{2} - s\right) \Gamma(s)}{2 \sqrt{\pi}}\right)$$

$$(x(0.490939)-24)-(4096(0.0636777(0.490939)-0.0953168)+276)*e^{(-\text{Pi}*\text{sqrt}(1/5))} =$$

$$4.07537573$$

**Input interpretation:**

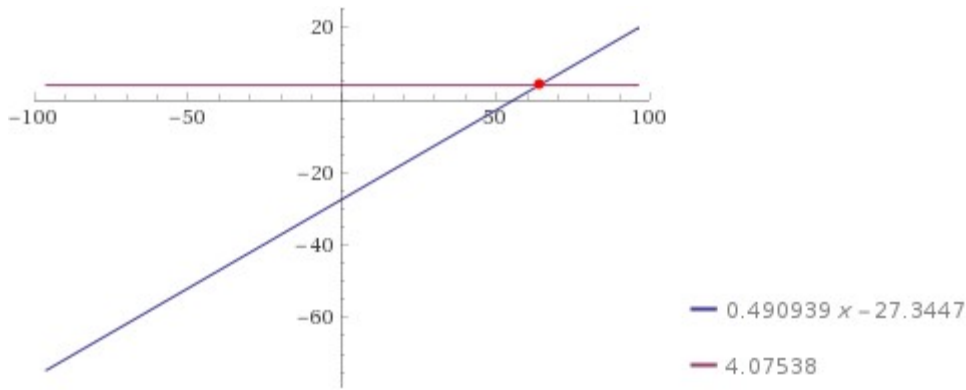
$$(x \times 0.490939 - 24) - (4096 (0.0636777 \times 0.490939 - 0.0953168) + 276) e^{-\pi \sqrt{1/5}} =$$

$$4.07537573$$

**Result:**

$$0.490939 x - 27.3447 = 4.07538$$

**Plot:**



**Alternate forms:**

$$0.490939 x - 31.4201 = 0$$

$$0.490939 (x - 55.6988) = 4.07538$$

**Solution:**

$$x \approx 64.$$

$$64 = 8^2$$

$$(64(0.490939) - 24) - (x(0.0636777(0.490939) - 0.0953168) + 276) * e^{(-\pi * \text{sqrt}(1/5))} = 4.07537573$$

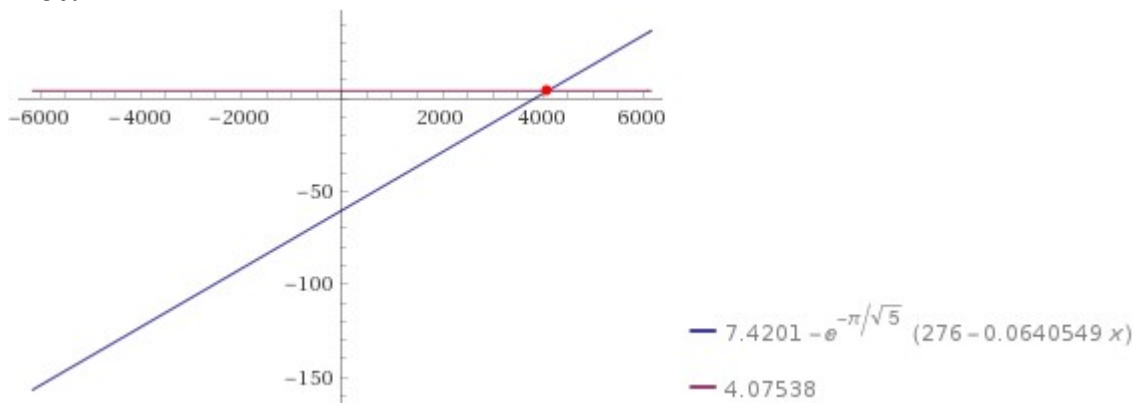
**Input interpretation:**

$$(64 \times 0.490939 - 24) - (x (0.0636777 \times 0.490939 - 0.0953168) + 276) e^{-\pi \sqrt{1/5}} = 4.07537573$$

**Result:**

$$7.4201 - e^{-\pi/\sqrt{5}} (276 - 0.0640549 x) = 4.07538$$

**Plot:**



**Alternate forms:**

$$0.0157176 x - 64.3791 = 0$$

$$0.0157176 x - 60.3037 = 4.07538$$

$$0.0157176 (x - 4308.8) + 7.4201 = 4.07538$$

**Solution:**

$$x \approx 4096.$$

$$4096 = 8^4$$

$$(((x/2+3/2))(0.490939)-24)-(4096(0.0636777(0.490939)-0.0953168)+276)*e^{(-\pi*\sqrt{1/5})} = 4.07537573$$

**Input interpretation:**

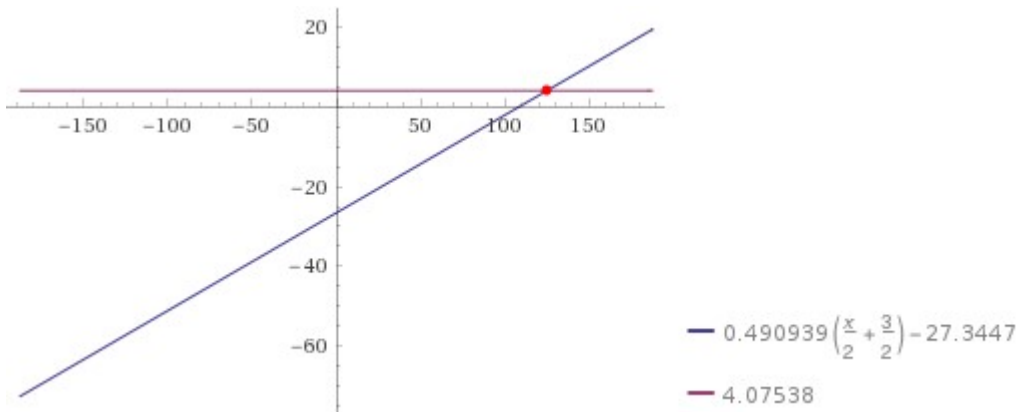
$$\left(\left(\frac{x}{2} + \frac{3}{2}\right) \times 0.490939 - 24\right) -$$

$$(4096 (0.0636777 \times 0.490939 - 0.0953168) + 276) e^{-\pi \sqrt{1/5}} = 4.07537573$$

**Result:**

$$0.490939 \left(\frac{x}{2} + \frac{3}{2}\right) - 27.3447 = 4.07538$$

**Plot:**



**Alternate forms:**

$$0.24547 x - 30.6837 = 0$$

$$0.24547 x - 26.6083 = 4.07538$$

$$0.24547 (x - 108.398) = 4.07538$$

**Solution:**

$$x \approx 125.$$

125 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

$$(((x/2-11/2))(0.490939)-24)-(4096(0.0636777(0.490939)-0.0953168)+276)*e^{(-\pi*\sqrt{1/5})} = 4.07537573$$

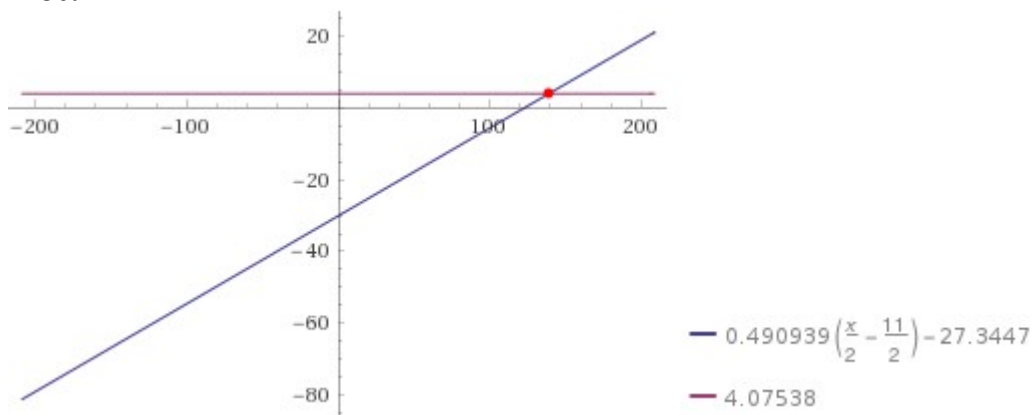
**Input interpretation:**

$$\left(\left(\frac{x}{2} - \frac{11}{2}\right) \times 0.490939 - 24\right) - (4096 (0.0636777 \times 0.490939 - 0.0953168) + 276) e^{-\pi \sqrt{1/5}} = 4.07537573$$

**Result:**

$$0.490939 \left(\frac{x}{2} - \frac{11}{2}\right) - 27.3447 = 4.07538$$

**Plot:**



**Alternate forms:**

$$0.24547 x - 34.1203 = 0$$

$$0.24547 x - 30.0449 = 4.07538$$

$$0.24547 (x - 122.398) = 4.07538$$

**Solution:**

$$x \approx 139.$$

139 result practically equal to the rest mass of Pion meson 139.57 MeV

$$\left(\left(\frac{x-1}{27}\right)(0.490939)-24\right)-\left(4096(0.0636777(0.490939)-0.0953168)+276\right)e^{-\pi\sqrt{1/5}} = 4.07537573$$

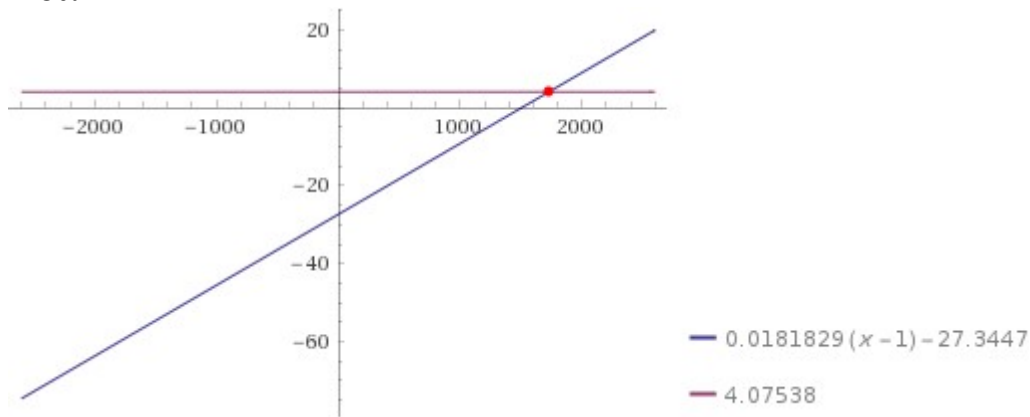
**Input interpretation:**

$$\left(\frac{x-1}{27} \times 0.490939 - 24\right) - (4096 (0.0636777 \times 0.490939 - 0.0953168) + 276) e^{-\pi \sqrt{1/5}} = 4.07537573$$

**Result:**

$$0.0181829(x - 1) - 27.3447 = 4.07538$$

**Plot:**



**Alternate forms:**

$$0.0181829x - 31.4383 = 0$$

$$0.0181829x - 27.3629 = 4.07538$$

$$0.0181829(x - 1504.87) = 4.07538$$

**Solution:**

$$x \approx 1729.$$

1729

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729



$$(64(0.490939)-24)-((4x+20)(0.0636777(0.490939)-0.0953168)+276)*e^{(-\pi*\sqrt{1/5})} = 4.07537573$$

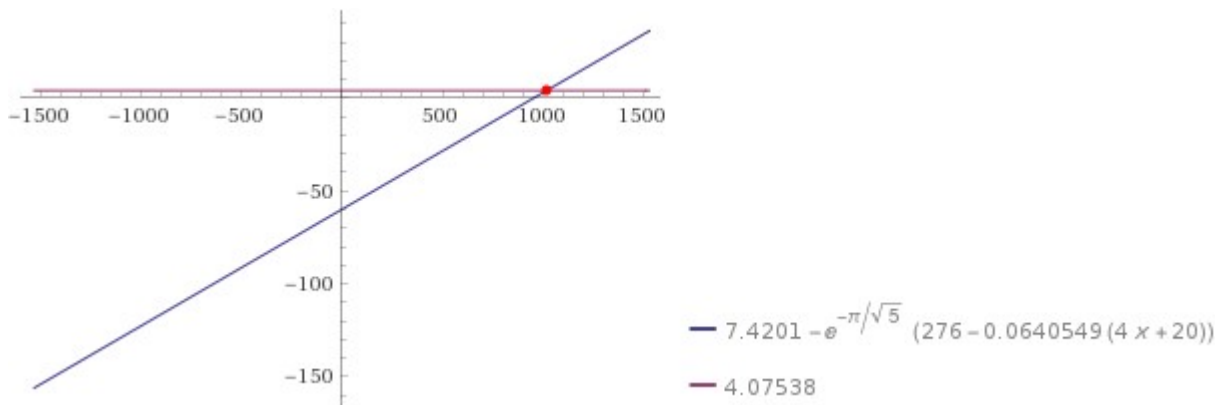
**Input interpretation:**

$$(64 \times 0.490939 - 24) - ((4x + 20)(0.0636777 \times 0.490939 - 0.0953168) + 276) e^{-\pi \sqrt{1/5}} = 4.07537573$$

**Result:**

$$7.4201 - e^{-\pi/\sqrt{5}} (276 - 0.0640549 (4x + 20)) = 4.07538$$

**Plot:**



**Alternate forms:**

$$0.0628702 x - 64.0647 = 0$$

$$0.0628702 (x - 1072.2) + 7.4201 = 4.07538$$

$$1.82071 (0.0345305 x - 32.9483) = 4.07538$$

**Expanded form:**

$$0.0628702 x - 59.9894 = 4.07538$$

**Solution:**

$$x \approx 1019.$$

1019 result practically equal to the rest mass of Phi meson 1019.445

## Appendix

From:

<https://www.altrogiornale.org/david-wilcock-la-scienza-delluno-capitolo-14-yoga-vedico-seth-e-cosmologia-multidimensionale-parte-3/?fbclid=IwAR0qHbTcRV-sPcM8OID6WnflmYx2-wJXwX-gNOp0q07Ee8XLEnSPuLe2i6A>

### 14.22.2 THE RAMANUJAN SYSTEM

In the case of Ramanujan, modular functions are defined as mathematical operations in which there is an incredible and almost never seen level of symmetry, a symmetry that works with this higher density geometry. In this symmetry, moreover, in many different and synchronic ways, Ramanujan's modular functions bring us back to number eight as the key force of organization behind the structure of dimensions or density in this universe.

This can be seen in excerpts from Dr. Mikio Kaku's Hyperspace book. Now we should keep in mind that the "Superstring" theory is very similar to the etheric concepts, in which it is considered a quantum realm seen as a product of vibrating energy "Strings".

Srinivasa Ramanujan was the strangest man in the whole history of mathematics, probably in the whole history of science. He was confronted with an exploding supernova, which illuminated the darkness in the darkest corners of mathematics, before tragically dying from tuberculosis at the age of 33 like Riemann before him.

Working in total isolation from the main currents of his field, he managed to laugh 100 years of Occidental mathematics alone. The tragedy of his life is that much of his work was wasted rediscovering known mathematics. Among the dark equations in his notebooks are these modular functions, among the strangest ever discovered ...

In Ramanujan's work, the number 24 ( $8 \times 3$ ) appears repeatedly. This is an example of what mathematicians call magic numbers, which appear continuously where we least expect them, for reasons that nobody understands. Miraculously, the Ramanujan function also appears in this theory ... .. In string theory, each of the 24 modes in the Ramanujan function corresponds to a physical vibration of the string ...

When the Ramanujan function is generalized, the number 24 is replaced by the number 8. That is, the critical number for the sperstring is  $8 + 2$ , therefore 10. This is

the origin of the tenth dimension. The string vibrates in ten dimensions because it requires these generalized Ramanujan functions (based on the number 8) in order to remain self-consistent. In other words, physicists do not have the slightest understanding of why 10 and 26 dimensions are rendered as the single dimension of the string.

(Now read the next sentence carefully, remember that this belongs to an authoritative figure in official science :)

There is thought to be some kind of profound numerology manifested in these functions that nobody understands ...

In the final analysis, the origin of the ten-dimensional theory is as mysterious as Ramanujan himself. When asked why nature should exist in ten dimensions, physicists are forced to respond, "We don't know."

As we can see from the aforementioned passage, modern physicists who support the Superstrings Theory, feel that the energies that create the dimensions "are not symmetrical" in the Ramanujan octave-based system, therefore arbitrarily add two extra dimensions for everything to level up mathematical.

The ten dimensions of the conventional "Superstring theory" come from this abstraction and always in the same inelegant way, the String theorists took the group of three octaves or 24 dimensions of Ramanujan and added two to get to 26.

One might think that having three different octave systems, each of them with tremendous musical symmetry, we would not want to break this symmetry in this way, adding two to the whole group, but many of them are probably not musicians!

In note n.13 at the bottom of page 346 of the book *Hyperspace*, Kaku shows us how the Octave can be reintroduced by removing the two "extra" dimensions that they added:

However, two of these vibrator modes can be removed when we break the symmetry of the string, remaining with 24 vibrator modes, which are the ones that appear in the Ramanujan function.

Now that we understand the vibration and the forms it takes, it should be easy to see how this apparent error was produced. As we will see later, our entire understanding of energy and quantum physics has several distortions.

When these distortions are clarified and we see geometry at work, we find the exact "symmetry" that String theorists think they must preserve with two extra "dimensions". With someone brilliant like Ramanujan it is more than easy for him or

his source of information to be well aware of what he was doing; the simple fact that we still don't understand many of his theorems should be a great clue to understand that we haven't "solved the puzzle" yet.

The addition of the two extra dimensions is simply a convenient shortcut for everything to look good on paper.

### **14.23 TANIYAMA-SHIMURA: MODULAR FUNCTIONS AS GEOMETRIC OBJECTS**

While our research in this area continued after writing *The Shift of the Ages*, we were interested in finding out that the official scientific model already existed and associated Platonic geometry with modular functions based on Ramanujan's Octave!

This comes from the Taniyama-Shimura conjecture, proven mathematically only in the 1900s. This conjecture essentially stated that all Ramanujan's modular functions, "based on the Octave", could essentially be modeled as elliptic curves.

While the full definition of "elliptic curves" is quite complex, the main point is that these curves are truly toroidal or donut shaped and can be seen wrapped around Platonic geometries, specifically the cube. We were very impressed to discover this fact.

(The math that described this configuration led to the discovery of Andrew Wiles' math test in the mid-1900s relating to Fermat's Last Theorem, considered to be the "largest mathematical puzzle of the past 300 years.")

So to put it in simple terms, modern mathematical theories actually support the results of our models of a fluid in vibration, e.g. Platonic geometries surrounded and created by spiraling or curved lines. As the Taniyama-Shimura conjecture shows us, Ramanujan's octave-based modular functions are geometric in nature and the geometry surprisingly corresponds to what we would have expected in the harmonic model.

## Conclusions

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and  $4096 = 64^2$

**(Modular equations and approximations to  $\pi$  - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)**

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants  $\pi$ ,  $\phi$ ,  $1/\phi$ , the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

## References

**A superfield constraint for  $N = 2 \rightarrow N = 0$  breaking** - *E. Dudas, S. Ferrara and A. Sagnotti* – arXiv:1707.03414v1 [hep-th] 11 Jul 2017

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**Modular equations and approximations to  $\pi$**  – *S. Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372