NEUTROSOPHIC CONTRA-CONTINUOUS MULTIFUNCTIONS

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Abstract. This paper is devoted to the concepts of neutrosophic upper and neutrosophic lower contra-continuous multifunctions and also some of their characterizations are considered.

1. Introduction

In the last three decades, the theory of multifunctions has advanced in a variety of ways and applications of this theory can be found, specially in functional analysis and fixed point theory. In recent years, several authors have studied some new forms of contra-continuity for functions and multifunctions. In the present paper, we study the notions of neutrosophic upper and neutrosophic lower contra-continuous multifunctions. Also, some characterizations and properties of such notions are discussed. Since the advent of the concepts of neutrosophy and neutrosophic sets by F. Smarandache [[1],[2]] , the theory of neutrosophy has found wide applications in economics, engineering, medicine, information sciences, programming, optimization, graphs etc. Moreover, we believe that neutrosophic multifunctions will be important in many applications mentioned above.

2. Preliminaries

Definition 2.1. [1] *Let X be a non-empty fixed set. A neutrosophic set A is an object having the form $A =< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >$, where $\mu_A(x)$, $\sigma_A(x)$ and $\gamma_A(x)$ which represent the degree of member ship function, the degree of indeterminacy, and the degree of non-membership, respectively of each element $x \in X$ to the set A.*

Definition 2.2. [3] *A neutrosophic topology on a nonempty set X is a family τ of neutrosophic subsets of X which satisfies the following three conditions:*

(1) $0, 1 \in \tau$,
(2) *If $g, h \in \tau$, their $g \wedge h \in \tau$,*
(3) *If $f_i \in \tau$ for each $i \in I$, then $\vee_{i \in I} f_i \in \tau$.*

The pair (X, τ) is called a neutrosophic topological space.

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Definition 2.3. *Members of τ are called neutrosophic open sets and complement of neutrosophic open sets are called neutrosophic closed sets, where the complement of a neutrosophic set A, denoted by A^c, is $1 - A$.*

3. neutrosophic Upper and Lower Contra-Continuous Multifunctions

Definition 3.1. *Let (X, τ) be a topological space in the classical sense and (Y, σ) be a neutrosophic topological space. Then $F : (X, \tau) \to (Y, \sigma)$ is called a neutrosophic multifunction if and only if for each $x \in X, F(x)$ is a neutrosophic set in Y.*

Definition 3.2. *For a neutrosophic multifunction $F : (X, \tau) \to (Y, \sigma)$, the upper inverse $F^+(\lambda)$ and lower inverse $F^-(\lambda)$ of a neutrosophic set λ in Y are defined as follows:*
$F^+(\lambda) = \{x \in X : F(x) \subset \lambda\}$ and $F^-(\lambda) = \{x \in X : F(x)q\lambda\}$.

Lemma 3.3. *For a fuzzy multifunction $F : (X, \tau) \to (Y, \sigma)$, we have $F^-(1 - \lambda) = X - F^+(\lambda)$ for any neutrosophic set λ in Y.*

Definition 3.4. *A neutrosophic multifunction $F : (X, \tau) \to (Y, \sigma)$ is called neutrosophic lower contra-continuous if for any neutrosophic closed set A in Y with $x \in F^-(A)$, there exists an open set B in X containing x such that $B \subset F^-(A)$.*

Definition 3.5. *A neutrosophic multifunction $F : (X, \tau) \to (Y, \sigma)$ is called neutrosophic upper contra-continuous if for each neutrosophic closed set A in Y with $x \in F^+(A)$, there exists an open set B in X containing x such that $B \subset F^+(A)$.*

Theorem 3.6. *The following are equivalent for a neutrosophic multifunction $F : (X, \tau) \to (Y, \sigma)$:*
1. *F is neutrosophic upper contra-continuous,*
2. *For each neutrosophic closed set A and $x \in X$ such that $F(x) \subset A$, there exists an open set B containing x such that if $y \in B$, then $F(y) \subset A$,*
3. *$F^+(A)$ is open for any neutrosophic closed set A in Y,*
4. *$F^-(B)$ is closed for any neutrosophic open set B in Y.*

Proof. (1) $\Rightarrow$ (2): Obvious.
(1) $\Rightarrow$ (3): Let A be any neutrosophic closed set in Y and $x \in F^+(A)$. By (1), there exists an open set A_x containing x such that $A_x \subset F^+(A)$. Thus, $x \in \mathrm{Int}(F^+(A))$ and hence $F^+(A)$ is an open set in X.
(3) $\Rightarrow$ (4): Let A be a neutrosophic open set in Y. Then $Y \backslash A$ is a neutrosophic closed set in Y. By (3), $F^+(Y \backslash A)$ is open. Since $F^+(1 \backslash A) = X \backslash F^-(A)$, then $F^-(A)$ is closed in X.
(4) $\Rightarrow$ (3): It is similar to that of (3) $\Rightarrow$ (4).
(3) $\Rightarrow$ (1): Let A be any neutrosophic closed set in Y and $x \in F^+(A)$.

By (3), $F^+(A)$ is an open set in X. Take $B = F^+(A)$. Then, $B \subset F^+(A)$. Thus, F is neutrosophic upper contra-continuous. $\square$

Definition 3.7. *The set $\wedge\{A \in \tau : B \subset A\}$ is called the neutrosophic kernel of a neutrosophic set A in a neutrosophic topological space (X, τ) and is denoted by $Ker(A)$.*

Lemma 3.8. *If $A \in \tau$ rfor a neutrosophic set A in a neutrosophic topological space (X, τ), then $A = Ker(A)$.*

Theorem 3.9. *Let $F : (X, \tau) \to (Y, \sigma)$ be a neutrosophic multifunction. If $\mathrm{Cl}(F^-(A)) \subset F^-(Ker(A))$ for any neutrosophic set A in Y, then F is neutrosophic upper contra-continuous.*

Proof. Suppose that $\mathrm{Cl}(F^-(A)) \subset F^-(Ker(A))$ for every neutrosophic set A in Y. Let $B \in \sigma$. By Lemma 3.8, $\mathrm{Cl}(F^-(B)) \subset F^-(Ker(B)) = F^-(B)$. This implies that $\mathrm{Cl}((F^-(B)) = F^-(B)$ and hence $F^-(B)$ is closed in X. Thus, by Theorem 3.6, F is neutrosophic upper contra-continuous. $\square$

Definition 3.10. *A neutrosophic multifunction $F : (X, \tau) \to (Y, \sigma)$ is called*

(1) *neutrosophic lower semi-continuous if for any neutrosophic open subset $A \subset Y$ with $x \in F^-(A)$, there exists an open set B in X containing x such that $B \subset F^-(A)$.*
(2) *neutrosophic upper semi-continuous if for any neutrosophic open subset $A \subset Y$ with $x \in F^+(A)$, there exists an open set B in X containing x such that $B \subset F^+(A)$.*

Remark 3.11. *The notions of neutrosophic upper contra-continuous multifunctions and neutrosophic upper semi-continuous multifunctions are independent as shown in the following examples.*

Example 3.12. *Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{a\}\}$ and $Y = [0, 1]$, $\sigma = \{Y, 0, A, B, C\}$, where $A(y) =< 0.5, 0, 0.5 >$, $B(y) =< 0.6, 0, 0.4 >$ and $C(y) =< 0.7, 0, 0.3 >$ for $y \in Y$. Define a neutrosophic multifunction as follows: $F(a) = A$, $F(b) = B$, $F(c) = C$. Then the neutrosophic multifunction $F : (X, \tau) \to (Y, \sigma)$ is neutrosophic upper contra-continuous but it is not neutrosophic upper semi-continuous.*

Example 3.13. *Let $X = \{a, b, c\}$, $\tau = \{X, \emptyset, \{b, c\}\}$ and $Y = [0, 1]$, $\sigma = \{Y, 0, A, B, C\}$, where $A(y) =< 0.3, 0, 0.7 >$, $B(y) =< 0.2, 0, 0.8 >$, $C(y) =< 0.6, 0, 0.4 >$, $D(y) =< 0.4, 0, 0.6 >$, and $E(y) =< 0.5, 0, 0.5 >$ for $y \in Y$. Define a neutrosophic multifunction as follows: $F(a) = D$, $F(b) = E$, $F(c) = C$. Then the neutrosophic multifunction $F : (X, \tau) \to (Y, \sigma)$ is neutrosophic upper semi-continuous, but it is not neutrosophic upper contra-continuous.*

Theorem 3.14. *The following are equivalent for a neutrosophic multifunction $F : (X, \tau) \to (Y, \sigma)$:*

(1) *F is neutrosophic lower contra-continuous,*
(2) *For each neutrosophic closed set A and $x \in X$ such that $F(x)qA$, there exists an open set B containing x such that if $y \in B$, then $F(y)qA$,*
(3) *$F^-(A)$ is open for any neutrosophic closed set A in Y,*
(4) *$F^+(B)$ is closed for any neutrosophic open set B in Y.*

Proof. It is similar to that of Theorem 3.6. □

Theorem 3.15. *For a neutrosophic multifunction $F : (X, \tau) \to (Y, \sigma)$, if $\mathrm{Cl}(F^+(A)) \subset F^+(Ker(A))$ for every neutrosophic set A in Y, then F is neutrosophic lower contra-continuous.*

Proof. Suppose that $\mathrm{Cl}(F^+(A)) \subset F^+(Ker(A))$ for every neutrosophic set A in Y. Let $A \in \sigma$. We have $\mathrm{Cl}(F^+(A)) \subset F^+(Ker(A)) = F^+(A)$. Thus, $\mathrm{Cl}(F^+(A)) = F^+(A)$ and hence $F^+(A)$ is closed in X. Then F is neutrosophic lower contra-continuous. □

Definition 3.16. *Given a family $\{F_i : (X, \tau) \to (Y, \sigma) : i \in I\}$ of neutrosophic multifunctions, we define the union $\underset{i \in I}{\vee} F_i$ and the intersection $\underset{i \in I}{\wedge} F_i$ as follows: $\underset{i \in I}{\vee} F_i : (X, \tau) \to (Y, \sigma)$, $(\underset{i \in I}{\vee} F_i)(x) = \underset{i \in I}{\vee} F_i(x)$ and $\underset{i \in I}{\wedge} F_i : (X, \tau) \to (Y, \sigma)$, $(\underset{i \in I}{\wedge} F_i)(x) = \underset{i \in I}{\wedge} F_i(x)$.*

Theorem 3.17. *If $F_i : X \to Y$ are neutrosophic upper contra-continuous multifunctions for $i = 1, 2, ..., n$, then $\overset{n}{\underset{i \in I}{\vee}} F_i$ is a neutrosophic upper contra-continuous multifunction.*

Proof. Let A be a neutrosophic closed set of Y. We will show that $(\overset{n}{\underset{i \in I}{\vee}} F_i)^+(A) = \{x \in X : \overset{n}{\underset{i \in I}{\vee}} F_i(x) \subset A\}$ is open in X. Let $x \in (\overset{n}{\underset{i \in I}{\vee}} F_i)^+(A)$. Then $F_i(x) \subset A$ for $i = 1, 2, ..., n$. Since $F_i : X \to Y$ is neutrosophic upper contra-continuous multifunction for $i = 1, 2, ..., n$, then there exists an open set U_x containing x such that for all $z \in U_x$, $F_i(z) \subset A$. Let $U = \overset{n}{\underset{i \in I}{\cup}} U_x$. Then $U \subset (\overset{n}{\underset{i \in I}{\vee}} F_i)^+(A)$. Thus, $(\overset{n}{\underset{i \in I}{\vee}} F_i)^+(A)$ is open and hence $\overset{n}{\underset{i \in I}{\vee}} F_i$ is a neutrosophic upper contra-continuous multifunction. □

Lemma 3.18. *Let $\{A_i\}_{i \in I}$ be a family of neutrosophic sets in a neutrosophic topological space X. Then a neutrosophic point x is quasi-coincident with $\vee A_i$ if and only if there exists an $i_0 \in I$ such that xqA_{i_0}.*

Theorem 3.19. *If $F_i : X \to Y$ are neutrosophic lower contra-continuous multifunctions for $i = 1, 2, ..., n$, then $\overset{n}{\underset{i \in I}{\vee}} F_i$ is a neutrosophic lower contra-continuous multifunction.*

Proof. Let A be a neutrosophic closed set of Y. We will show that $(\overset{n}{\underset{i\in I}{\vee}} F_i)^-(A) = \{x \in X : (\overset{n}{\underset{i\in I}{\vee}} F_i)(x)qA\}$ is open in X. Let $x \in (\overset{n}{\underset{i\in I}{\vee}} F_i)^-(A)$. Then $(\overset{n}{\underset{i\in I}{\vee}} F_i)(x)qA$ and hence $F_{i0}(x)qA$ for an i_0. Since $F_i : X \to Y$ is neutrosophic lower contra-continuous multifunction, there exists an open set U_x containing x such that for all $z \in U$, $F_{i0}(z)qA$. Then $(\overset{n}{\underset{i\in I}{\vee}} F_i)(z)qA$ and hence $U \subset (\overset{n}{\underset{i\in I}{\vee}} F_i)^-(A)$. Thus, $(\overset{n}{\underset{i\in I}{\vee}} F_i)^-(A)$ is open and hence $\overset{n}{\underset{i\in I}{\vee}} F_i$ is a neutrosophic lower contra-continuous multifunction. □

Theorem 3.20. *Let $F : (X, \tau) \to (Y, \sigma)$ be a neutrosophic multifunction and $\{U_i : i \in I\}$ be an open cover for X. Then the following are equivalent:*

(1) *$F_i = F_{|U_i}$ is a neutrosophic lower contra-continuous multifunction for all $i \in I$,*
(2) *F is neutrosophic lower contra-continuous.*

Proof. (1) ⇒ (2): Let $x \in X$ and A be a neutrosophic closed set in Y with $x \in F^-(A)$. Since $\{U_i : i \in I\}$ is an open cover for X, then $x \in U_{i0}$ for an $i_0 \in I$. We have $F(x) = F_{i0}(x)$ and hence $x \in F^-_{i0}(A)$. Since $F_{|U_i0}$ is neutrosophic lower contra-continuous, there exists an open set $B = G \cap U_{i0}$ in U_{i0} such that $x \in B$ and $F^-(A) \cap U_{i_0} = F_{|U_i}(A) \supset B = G \cap U_{i0}$, where G is open in X. We have $x \in B = G \cap U_{i0} \subset F^-_{|U_i0}(A) = F^-(A) \cap U_{i0} \subset F^-(A)$. Hence, F is neutrosophic lower contra-continuous.
(2) ⇒ (1): Let $x \in X$ and $x \in U_i$. Let A be a neutrosophic closed set in Y with $F_i(x)qA$. Since F is lower contra-continuous and $F(x) = F_i(x)$, there exists an open set U containing x such that $U \subset F^-(A)$. Take $B = U_i \cap U$. Then B is open in U_i containing x. We have $B \subset F^-i(A)$. Thus F_i is a neutrosophic lower contra-continuous. □

Theorem 3.21. *Let $F : (X, \tau) \to (Y, \sigma)$ be a neutrosophic multifunction and $\{U_i : i \in I\}$ be an open cover for X. Then the following are equivalent:*

(1) *$F_i = F_{|U_i}$ is a neutrosophic upper contra-continuous multifunction for all $i \in I$,*
(2) *F is neutrosophic upper contra-continuous.*

Proof. It is similar to that of Theorem 3.20. □

Recall that for a multifunction $F_1 : (X, \tau) \to (Y, \sigma)$ and a neutrosophic multifunction $F_2 : (Y, \sigma) \to (Z, \eta)$, the neutrosophic multifunction $F_2 \circ F_1 : (X, \tau) \to (Z, \eta)$ is defined by $(F_2 \circ F_1)(x) = F_2(F_1(x))$ for $x \in X$.

Definition 3.22. *Let X and Y be topological spaces. A multifunction $F : (X, \tau) \to (Y, \sigma)$ is called*

(1) *lower semi-continuous if for any open subset $A \subset Y$ with $x \in F^-(A)$, there exists an open set B in X containing x such that $B \subset F^-(A)$.*
(2) *upper semi-continuous if for any open subset $A \subset Y$ with $x \in F^+(A)$, there exists an open set B in X containing x such that $B \subset F^+(A)$.*

Theorem 3.23. *If $F_1 : X \to Y$ is an upper semi-continuous multifunction, where X and Y are topological spaces and $F_2 : Y \to Z$ is a neutrosophic upper contra-continuous multifunction, where Z is a neutrosophic topological space, then $F_2 \circ F_1$ is neutrosophic upper contra-continuous.*

Proof. Let $x \in X$ and A be a neutrosophic closed set in Z. We have $(F_2 \circ F_1)^+(A) = F_1^+(F_2^+(A))$. Since F_2 is neutrosophic upper contra-continuous, $F_2^+(A)$ is open in Y. Since F_1 is upper semi-continuous, $F_1^+(F_2^+(A)) = (F_2 \circ F_1)^+(A)$ is open in X. Thus, $F_2 \circ F_1$ is neutrosophic upper contra-continuous. □

Definition 3.24. *A neutrosophic set A in a neutrosophic topological space X is called:*

(1) *a neutrosophic cl-neighbourhood of a neutrosophic point x in X if there exists a neutrosophic closed set B in X such that $x \in B \subset A$.*
(2) *a neutrosophic cl-neighbourhood of a neutrosophic set B in X if there exists a neutrosophic closed set C in X such that $B \subset C \subset A$.*

Theorem 3.25. *If $F : (X, \tau) \to (Y, \sigma)$ is a neutrosophic upper contra-continuous multifunction, then for each point x of X and each neutrosophic cl-neighbourhood A of $F(x)$, $F^+(A)$ is a neighbourhood of x.*

Proof. Let $x \in X$ and A be a neutrosophic cl-neighbourhood of $F(x)$. There exists a neutrosophic closed set B in Y such that $F(x) \subset B \subset A$. We have $x \in F^+(B) \subset F^+(A)$. Since $F^+(B)$ is an open set, $F^+(A)$ is a neighbourhood of x. □

Remark 3.26. *A subset A of a topological space (X, τ) can be considered as a neutrosophic set with characteristic function defined by*

$$A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

Let (Y, σ) be a neutrosophic topological space. The neutrosophic sets of the form $A \times B$ with $A \in \tau$ and $B \in \sigma$ form a basis for the product neutrosophic topology $\tau \times \sigma$ on $X \times Y$, where for any $(x, y) \in X \times Y$, $(A \times B)(x, y) = min\{A(x), B(y)\}$.

Definition 3.27. *For a neutrosophic multifunction $F : (X, \tau) \to (Y, \sigma)$, the neutrosophic graph multifunction $G_F : X \to X \times Y$ of F is defined by $G_F(x) = x_1 \times F(x)$ for every $x \in X$.*

Theorem 3.28. *If the neutrosophic graph multifunction G_F of a neutrosophic multifunction $F : (X, \tau) \to (Y, \sigma)$ is neutrosophic lower contra-continuous, then F is neutrosophic lower contra-continuous.*

Proof. Suppose that G_F is neutrosophic lower contra-continuous and $x \in X$. Let A be a neutrosophic closed set in Y such that $F(x)qA$. Then there exists $y \in Y$ such that $(F(x))(y) + A(y) > 1$. Then $(G_F(x))(x, y)+(X\times A)(x, y) = (F(x))(y)+A(y) > 1$. Hence, $G_F(x)q(X\times A)$. Since G_F is neutrosophic lower contra-continuous, there exists an open set B in X such that $x \in B$ and $G_F(b)q(X \times A)$ for all $b \in B$. Let there exists $b_0 \in B$ such that $F(b_0)qA$. Then for all $y \in Y$, $(F(b_0))(y) + A(y) < 1$. For any $(a, c) \in X \times Y$, we have $(G_F(b_0))(a, c) \subset (F(b_0))(c)$ and $(X \times A)(a, c) \subset A(c)$. Since for all $y \in Y$, $(F(b_0))(y) + A(y) < 1$, $(G_F(b_0))(a, c) + (X \times A)(a, c) < 1$. Thus, $G_F(b_0)q(X \times A)$, where $b_0 \in B$. This is a contradiction since $G_F(b)q(X \times A)$ for all $b \in B$. Hence, F is neutrosophic lower contra-continuous. □

Theorem 3.29. *If the neutrosophic graph multifunction G_F of a neutrosophic multifunction $F : X \to Y$ is neutrosophic upper contra-continuous, then F is neutrosophic upper contra-continuous.*

Proof. Suppose that G_F is neutrosophic upper contra-continuous and let $x \in X$. Let A be neutrosophic closed in Y with $F(x) \subset A$. Then $G_F(x) \subset X \times A$. Since G_F is neutrosophic upper contra-continuous, there exists an open set B containing x such that $G_F(B) \subset X \times A$. For any $b \in B$ and $y \in Y$, we have $(F(b))(y) = (G_F(b))(b, y) \subset (X \times A)(b, y) = A(y)$. Then $(F(b))(y) \subset A(y)$ for all $y \in Y$. Thus, $F(b) \subset A$ for any $b \in B$. Hence, F is neutrosophic upper contra-continuous. □

Theorem 3.30. *Let $F : (X, \tau) \to (Y, \sigma)$ be a neutrosophic multifunction. Then the following are equivalent:*

1. *F is neutrosophic lower contra-continuous,*
2. *For any $x \in X$ and any net $(x_i)_{i\in I}$ converging to x in X and each neutrosophic closed set B in Y with $x \in F^-(B)$, the net $(x_i)_{i\in I}$ is eventually in $F^-(B)$.*

Proof. (1) $\Rightarrow$ (2): Let (x_i) be a net converging to x in X and B be any neutrosophic closed set in Y with $x \in F^-(B)$. Since F is neutrosophic lower contra-continuous, there exists an open set $A \subset X$ containing x such that $A \subset F^-(B)$. Since $x_i \to x$, there exists an index $i_0 \in I$ such that $x_i \in A$ for every $i \geq i_0$. We have $x_i \in A \subset F^-(B)$ for all $i \geq i_0$. Hence, $(x_i)_{i\in I}$ is eventually in $F^-(B)$.
(2) $\Rightarrow$ (1): Suppose that F is not neutrosophic lower contra-continuous. There exists a point x and a neutrosophic closed set A with $x \in F^-(A)$ such that $B \nsubseteq F^-(A)$ for any open set $B \subset X$ containing x. Let $x_i \in B$ and $x_i \notin F^-(A)$ for each open set $B \subset X$ containing x. Then

the neighborhood net (x_i) converges to x but $(x_i)_{i\in I}$ is not eventually in $F^-(A)$. This is a contradiction. □

Theorem 3.31. *Let $F : (X,\tau) \to (Y,\sigma)$ be a neutrosophic multifunction. Then the following are equivalent:*

(1) *F is neutrosophic upper contra-continuous,*
(2) *For any $x \in X$ and any net (x_i) converging to x in X and any neutrosophic closed set B in Y with $x \in F^+(B)$, the net (x_i) is eventually in $F^+(B)$.*

Proof. The proof is similar to that of Theorem 3.30. □

Theorem 3.32. *The set of all points of X at which a neutrosophic multifunction $F : (X,\tau) \to (Y,\sigma)$ is not neutrosophic upper contra-continuous is identical with the union of the frontier of the upper inverse image of neutrosophic closed sets containing $F(x)$.*

Proof. Suppose F is not neutrosophic upper contra-continuous at $x \in X$. Then there exists a neutrosophic closed set A in Y containing $F(x)$ such that $A \cap (X \backslash F^+(B)) \neq \emptyset$ for every open set A containing x. We have $x \in \mathrm{Cl}(X \backslash F^+(B)) = X \backslash \mathrm{Int}(F^+(B))$ and $x \in F^+(B)$. Thus, $x \in Fr(F^+(B))$. Conversely, let B be a neutrosophic closed set in Y containing $F(x)$ with $x \in Fr(F^+(B))$. Suppose that F is neutrosophic upper contra-continuous at x. There exists an open set A containing x such that $A \subset F^+(B)$. We have $x \in \mathrm{Int}(F^+(B))$. This is a contradiction. Thus, F is not neutrosophic upper contra-continuous at x. □

Theorem 3.33. *The set of all points of X at which a neutrosophic multifunction $F : (X,\tau) \to (Y,\sigma)$ is not neutrosophic lower contra-continuous is identical with the union of the frontier of the lower inverse image of neutrosophic closed sets which are quasi-coincident with $F(x)$.*

Proof. It is similar to that of Theorem 3.32. □

Definition 3.34. *A neutrosophic topological space X is called neutrosophic strongly S-closed if every neutrosophic closed cover of X has a finite subcover.*

Theorem 3.35. *Let $F : (X,\tau) \to (Y,\sigma)$ be a neutrosophic upper contra-continuous surjective multifunction. Suppose that $F(x)$ is neutrosophic strongly S-closed for each $x \in X$. If X is compact, then Y is neutrosophic strongly S-closed.*

Proof. Let $\{A_k\}_{k\in I}$ be a neutrosophic closed cover of Y. Since $F(x)$ is neutrosophic strongly S-closed for any $x \in X$, there exists a finite subset I_x of I such that $F(x) \subset \underset{k\in I_x}{\vee} A_k$. Take $A_x = \underset{k\in I_x}{\vee} A_k$. Since F is neutrosophic upper contra-continuous, there exists a neutrosophic open set U_x of X containing x such that $F(U_x) \subset A_x$. Then $\{U_x\}_{x\in X}$

is an open cover of X. Since X is compact, there exist x_1, x_2, x_3, ...,x_n in X such that $X = \overset{n}{\underset{i=1}{\cup}} U_{x_i}$. We have $Y = F(X) = F(\overset{n}{\underset{i=1}{\cup}} U_{x_i}) \leq \overset{n}{\underset{i=1}{\vee}} F(U_{x_i}) \leq \overset{n}{\underset{i=1}{\vee}} U_{x_i} A_{x_i} = \overset{n}{\underset{i=1}{\vee}} \underset{k \in I_{x_i}}{\vee} U_k$. Thus, Y is neutrosophic strongly S-closed. □

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