Integration in finite terms

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ABSTRACT. Elementary Integral

ELEMENTARY INTEGRAL:

$$\pi\left(\frac{1}{2}-\frac{1}{2\sqrt{2}}\right)=\int_{1/\sqrt{2}}^{\infty}\sin^{-1}(f(x))\,dx \tag{1}$$

$$f(x)=-\frac{2}{3}+\sqrt[3]{\frac{1}{27}+\frac{1}{2x^4}+\frac{1}{x^2}\sqrt{\frac{1}{27}+\frac{1}{4x^4}}}+\sqrt[3]{\frac{1}{27}+\frac{1}{2x^4}-\frac{1}{x^2}\sqrt{\frac{1}{27}+\frac{1}{4x^4}}} \tag{2}$$

$$\pi=4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} \tag{3}$$

References

[1] Boros, G. and Moll, V.H.: Irresistible Integrals , Cambridge University Press, 2004.

[2] Marchisotto, E. A. and Zakeri, G.: An invitation to integration in finite terms. College Math. Jour., 25 , 1994 , 295-308.