ON $\mathcal{D}$-SETS, $\mathcal{DS}$-SETS AND DECOMPOSITIONS OF CONTINUOUS, $\mathcal{A}$-CONTINUOUS AND $\mathcal{AB}$-CONTINUOUS FUNCTIONS

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Abstract. The main purpose of this paper is to introduce the notions of $\mathcal{D}$-sets, $\mathcal{DS}$-sets, $\mathcal{D}$-continuity and $\mathcal{DS}$-continuity and to obtain decompositions of continuous functions, $\mathcal{A}$-continuous functions and $\mathcal{AB}$-continuous functions. Also, properties of the classes of $\mathcal{D}$-sets and $\mathcal{DS}$-sets are discussed.

Key words and phrases: $\mathcal{D}$-set, $\mathcal{DS}$-set, $\mathcal{D}$-continuity, $\mathcal{DS}$-continuity.

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1. Introduction and preliminaries

In [24] Tong, introduced a new class of sets namely $\mathcal{A}$-sets and established a new decomposition of continuity. Also, in [25] Tong introduced a new class of sets namely $\mathcal{B}$-sets and established an other decomposition of continuity. In 1998, Dontchev [12] introduced a class of sets called $\mathcal{AB}$-sets which are weaker than $\mathcal{A}$-sets and stronger than $\mathcal{B}$-sets. On the other hand, the class of LC-sets which introduced by Bourbaki [5] play important role when continuous functions are decomposition. In this paper, we introduce two new classes of sets called $\mathcal{D}$-sets and $\mathcal{DS}$-sets. The class of $\mathcal{D}$-sets is properly placed between $\mathcal{A}$-sets and LC-sets and the class of $\mathcal{DS}$-sets is properly placed between $\mathcal{AB}$-sets and $\mathcal{B}$-sets. Also, some new decompositions of continuity, $\mathcal{A}$-continuity and $\mathcal{AB}$-continuity via the notions of $\mathcal{D}$-sets and $\mathcal{DS}$-sets are established.

In this paper (X,τ) and (Y,σ) represent topological spaces. For a subset P of a space X, $cl(P)$ and $int(P)$ denote the closure of P and the interior of P, respectively.

Definition 1. A subset P of a space (X,τ) is called

(1) semiopen [18] if $P \subset cl(int(P))$,

(2) semi-regular [8] if it is both semiopen and semiclosed,

(3) an $\mathcal{AB}$-set [12] if $P \in \mathcal{AB}(X) = \{A \cap B : A \in \tau$ and B is semi-regular$\}$,

(4) an LC-set [5] if $P \in LC(X) = \{A \cap B : A \in \tau,\ cl(B) = B\}$,

(5) an $\mathcal{A}$-set [24] if $P \in \mathcal{A}(X) = \{A \cap B : A \in \tau,\ B = cl(int(B))\}$,

(6) a $\mathcal{B}$-set [25] if $P \in \mathcal{B}(X) = \{A \cap B : A \in \tau,\ int(cl(B)) \subset B\}$,

(7) α-open [21] if $P \subset int(cl(int(P)))$,

(8) β-open [1] or semi-preopen [3] if $P \subset cl(int(cl(P)))$,

(9) b-open [4] or γ-open [14] or sp-open [11] if $P \subset int(cl(P)) \cup cl(int(P))$,

(10) preopen [19] or locally dense [7] if $P \subset int(cl(P))$.

A subset P of a space X is called regular open (resp regular closed) [23] if $P = int(cl(P))$ (resp. $P = cl(int(P))$). If for each $x \in P$, there exists a regular open set A such that $x \in A \subset P$, P is called δ-open [26]. A point $x \in X$ is called a δ-cluster point of P [26] if $P \cap int(cl(U)) \neq \varnothing$ for each open set U containing x. The set of all δ-cluster points of P is called the δ-closure of P and is denoted by δ-$cl(P)$. If δ-$cl(P) = P$, then P is said to be δ-closed. The set $\{x \in X : x \in U \subset P$ for some regular open set U of $X\}$ is called the δ-interior of P and is denoted by δ-$int(P)$. A subset P of a topological space X is said to be δ-semiopen [22] if $P \subset cl(\delta\text{-}int(P))$. The complement of a δ-semiopen set is called a δ-semiclosed set. The union (resp. intersection) of all δ-semiopen (resp. δ-semiclosed) sets, each contained in (resp. containing) a set P in a topological space X is called the δ-semiinterior (resp. δ-semiclosure) of P and it is denoted by δ-$sint(P)$ (resp. δ-$scl(P)$) [22].

Lemma 2. ([6]) *Let X be a topological space and $P \subset X$. The following hold:*

(1) δ-$cl(\delta$-$cl(P)) = \delta$-$cl(P)$.

(2) *δ-$cl(P)$ is δ-closed.*

2. $\mathcal{D}$-sets and $\mathcal{DS}$-sets in topological spaces

Definition 3. A subset P of a topological space (X,τ) is called

(1) a $\mathcal{D}$-set if $P = A \cap B$, where A is open and B is δ-closed.

(2) a $\mathcal{DS}$-set if $P = A \cap B$, where A is open and B is δ-semiclosed.

Remark 4.

(1) Every open and every δ-closed set is a $\mathcal{D}$-set.

(2) Every open and every δ-semiclosed set is a $\mathcal{DS}$-set.

The following example shows that these implications are not reversible.

Example 5. Let $X = \{a, b, c, d\}$ and let $\tau = \{\varnothing,\ X,\ \{a\},\ \{c\},\ \{a, b\},\ \{a, c\},\ \{a, b, c\},\ \{a, c, d\}\}$. The set $\{a, d\}$ is a $\mathcal{D}$-set and so it is a $\mathcal{DS}$-set but it is neither δ-semiclosed nor an open set.

Remark 6. The following diagram holds for a subset P of a space X:

$$\begin{array}{ccccc} \mathcal{AB}\text{-set} & \Longrightarrow & \mathcal{DS}\text{-set} & \Longrightarrow & \mathcal{B}\text{-set} \\ \Uparrow & & \Uparrow & & \Uparrow \\ \mathcal{A}\text{-set} & \Longrightarrow & \mathcal{D}\text{-set} & \Longrightarrow & LC\text{-set} \end{array}$$

The following examples show that these implications are not reversible.

Example 7. Let $X = \{a, b, c, d\}$ and let $\tau = \{\varnothing,\ \{a\},\ \{c\},\ \{a, b\},\ \{a, c\},\ \{a, b, c\},\ \{a, c, d\},\ X\}$. The set $\{b, c, d\}$ is an LC-set and so a $\mathcal{B}$-set but it is neither a $\mathcal{D}$-set nor a $\mathcal{DS}$-set. The set $\{d\}$ is a $\mathcal{D}$-set and so a $\mathcal{DS}$-set but it is neither an $\mathcal{A}$-set nor an $\mathcal{AB}$-set.

Example 8. Let $X = \{a, b, c, d\}$ and let $\tau = \{\varnothing, \{a\}, \{b\}, \{a, b\}, X\}$. The set $\{b, d\}$ is an $\mathcal{AB}$-set and so it is a $\mathcal{DS}$-set but it is neither a $\mathcal{D}$-set nor an LC-set.

The family of all $\mathcal{D}$-sets (resp. $\mathcal{DS}$-sets) of a topological space (X, τ) will be denoted by $\mathcal{D}(X)$ (resp. $\mathcal{DS}(X)$).

Theorem 9. *The following are equivalent for a subset P of a space X:*

(1) $P \in \mathcal{D}(X)$,

(2) $P = A \cap \delta\text{-}cl(P)$ *for some open set A.*

Proof. $(\Longrightarrow)$: Let $P \in \mathcal{D}(X)$. This implies that $P = A \cap B$, where A is open and B is δ-closed. Since $P \subset B$, $\delta\text{-}cl(P) \subset \delta\text{-}cl(B) = B$. Moreover, $A \cap \delta\text{-}cl(P) \subset A \cap B = P \subset A \cap \delta\text{-}cl(P)$ and hence $P = A \cap \delta\text{-}cl(P)$.

$(\Longleftarrow)$: Let $P = A \cap \delta\text{-}cl(P)$ for some open set A. Since $\delta\text{-}cl(P)$ is δ-closed, $P \in \mathcal{D}(X)$. ∎

Theorem 10. *The following are equivalent for a subset P of a space X:*

(1) $P \in \mathcal{DS}(X)$,

(2) $P = A \cap \delta\text{-}scl(P)$ *for some open set A.*

Proof. Similar to that of Theorem 9. ∎

Theorem 11. *The following are equivalent for a subset P of a space X:*

(1) *P is an $\mathcal{AB}$-set,*

(2) *P is semiopen and a $\mathcal{DS}$-set,*

(3) *P is b-open and a $\mathcal{DS}$-set,*

(4) *P is β-open and a $\mathcal{DS}$-set.*

Proof. $(1) \Longrightarrow (2)$: Since every $\mathcal{AB}$-set is both semiopen and $\mathcal{DS}$-set, the proof is completed.

$(2) \Longrightarrow (3) \Longrightarrow (4)$: Obvious.

$(4) \Longrightarrow (1)$: Let P be β-open and a $\mathcal{DS}$-set. Since P is a $\mathcal{B}$-set, by Theorem 2.4 [12] P is an $\mathcal{AB}$-set. ∎

Definition 12. Let (X, τ) be a topological space and $A \subset X$. Then A is called interior-closed [16] if $int(A)$ is closed in A.

Theorem 13. *The following are equivalent for a subset P of a space X:*

(1) *P is open,*

(2) *P is α-open and an $\mathcal{AB}$-set,*

(3) *P is α-open and a $\mathcal{D}$-set,*

(4) *P is α-open and a $\mathcal{DS}$-set,*

(5) *P is preopen and an $\mathcal{AB}$-set,*

(6) *P is preopen and a $\mathcal{D}$-set,*

(7) *P is preopen and a $\mathcal{DS}$-set,*

(8) *P is a semiopen $\mathcal{D}$-set and P is either preopen or interior-closed,*

(9) *P is a semiopen $\mathcal{DS}$-set and P is either preopen or interior-closed.*

Proof. $(1) \Longrightarrow (2)$: It follows from the fact that every open set is both α-open and an $\mathcal{AB}$-set.

$(2) \Longrightarrow (5) \Longrightarrow (7)$: Obvious.

$(1) \Longrightarrow (3)$: Since every open set is both α-open and a $\mathcal{D}$-set, the proof is completed.

$(3) \Longrightarrow (4) \Longrightarrow (7)$: Obvious.

$(3) \Longrightarrow (6) \Longrightarrow (7)$: Obvious.

(7) $\Longrightarrow$ (1) : Let P be preopen and a $\mathcal{DS}$-set. Then P is a $\mathcal{B}$-set. By Proposition 9 [25], P is open.

(1) $\Longrightarrow$ (8) : It follows from the fact that every open set is a semiopen $\mathcal{D}$-set and preopen.

(8) $\Longrightarrow$ (9) : It follows from Remark 6.

(9) $\Longrightarrow$ (1) : Let P be a semiopen $\mathcal{DS}$-set and P be either preopen or interior-closed. By Remark 6, P is a $\mathcal{B}$-set. So, by Proposition 9 [25], P is open since P is preopen and a $\mathcal{B}$-set. On the other hand, by Theorem 1 [16], P is open since P is interior-closed and semiopen. ∎

Theorem 14. *The following are equivalent for a subset P of a space X:*

(1) *P is an $\mathcal{A}$-set,*

(2) *P is semiopen and a $\mathcal{D}$-set,*

(3) *P is b-open and a $\mathcal{D}$-set,*

(4) *P is β-open and a $\mathcal{D}$-set.*

Proof. (1) $\Longrightarrow$ (2) : By Remark 6, every $\mathcal{A}$-set is a $\mathcal{D}$-set. Also by Theorem 3.1 [24], every $\mathcal{A}$-set is semiopen.

(2) $\Longrightarrow$ (3) $\Longrightarrow$ (4) : Obvious.

(4) $\Longrightarrow$ (1) : Let P be β-open and a $\mathcal{D}$-set. Then by Remark 6, it is also an LC-set. Hence by Theorem 2.4 [9], P is an $\mathcal{A}$-set. ∎

A topological space X is called a partition space or locally indiscrete [13] if every open subset of X is closed.

Theorem 15. *The following are equivalent for a space X:*

(1) *X is a partition space,*

(2) *every $\mathcal{DS}$-set is clopen,*

(3) *every $\mathcal{D}$-set is clopen,*

(4) *every $\mathcal{DS}$-set is closed,*

(5) *every $\mathcal{D}$-set is closed.*

Proof. (1) $\Longrightarrow$ (2) : Let P be a $\mathcal{DS}$-set. Then there exist an open set A and a δ-semiclosed set B such that $P = A \cap B$. By (1), A is clopen and then P is semiclosed. Since X is a partition space, by [2] P is clopen.

(2) $\Longrightarrow$ (3) : It follows from the fact that every $\mathcal{D}$-set set is a $\mathcal{DS}$-set.

(2) $\Longrightarrow$ (4) : Obvious.

(3) $\Longrightarrow$ (5) : Obvious.

(4) $\Longrightarrow$ (1) : Let $P \subset X$ be an open set. By Remark 4, P is a $\mathcal{DS}$-set. By (4), P is closed. Thus, X is a partition space.

(5) $\Longrightarrow$ (1) : It is similar to that of (4) $\Longrightarrow$ (1). ∎

Theorem 16. *For a space X, the following are equivalent:*

(1) *X is indiscrete,*

(2) *the $\mathcal{DS}$-sets in X are only the trivial ones,*

(3) *the $\mathcal{D}$-sets in X are only the trivial ones.*

Proof. (1) $\Longrightarrow$ (2) : Let A be a $\mathcal{DS}$-set in X. Then there exist an open set V and a δ-semiclosed set B such that $A = V \cap B$. Suppose $A \neq \varnothing$. Then $V \neq \varnothing$. By (1), we have $V = X$ and $A = B$. Thus, $X = \delta\text{-}scl(A) \subset A$ and hence, $A = X$.

(2) $\Longrightarrow$ (3) : Obvious.

(3) $\Longrightarrow$ (1) : Since every open set is a $\mathcal{D}$-set, then open sets in X are only the trivial ones. Thus, X is indiscrete. ∎

A topological space X is called submaximal [5] if every dense subset of X is open. ∎

Theorem 17. *For a space X the following are equivalent:*

(1) *X is submaximal,*

(2) *every dense subset of X is a $\mathcal{D}$-set,*

(3) *every dense subset of X is a $\mathcal{DS}$-set.*

Proof. (1) $\Longrightarrow$ (2) : Let $M \subset X$ be a dense subset. Since X submaximal, then M is open and hence M is a $\mathcal{D}$-set.

(2) $\Longrightarrow$ (3) : By Remark 6, the proof is obvious.

(3) $\Longrightarrow$ (1) : Let $M \subset X$ be a dense subset. Then M is a $\mathcal{DS}$-set. Also M is preopen since M is dense. By Theorem 13, M is open. Thus, X is submaximal. ∎

Definition 18. A subset A of a space X is called δ-generalized closed [10] in X if $\delta\text{-}cl(A) \subset W$ whenever $A \subset W$ and W is open in X.

Theorem 19. *Let M be a subset of a space X. Then M is δ-closed if and only if M is a $\mathcal{D}$-set and δ-generalized closed.*

Proof. Let M be δ-closed. Then it is a $\mathcal{D}$-set and δ-generalized closed. Conversely, let M be a $\mathcal{D}$-set and δ-generalized closed. Then there exists an open set P such that $M = P \cap \delta\text{-}cl(M)$. Since M is δ-generalized closed and $M \subset P$, then $\delta\text{-}cl(M) \subset P$. We have $\delta\text{-}cl(M) \subset P \cap \delta\text{-}cl(M) = M$. Hence, M is δ-closed. ∎

3. Decompositions of continuity, $\mathcal{A}$-continuity and $\mathcal{AB}$-continuity

Definition 20. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called

(1) $\mathcal{D}$-continuous if $f^{-1}(N) \in \mathcal{D}(X)$ for each $N \in \sigma$.

(2) $\mathcal{DS}$-continuous if $f^{-1}(N) \in \mathcal{DS}(X)$ for each $N \in \sigma$.

Definition 21. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called

(1) β-continuous [1] if $f^{-1}(N)$ is β-open for each $N \in \sigma$.

(2) α-continuous [20] if $f^{-1}(N)$ is α-open for each $N \in \sigma$.

(3) γ-continuous [14] if $f^{-1}(N)$ is γ-open for each $N \in \sigma$.

(4) quasi-continuous [17] if $f^{-1}(N)$ is semiopen for each $N \in \sigma$.

(5) precontinuous [19] if $f^{-1}(N)$ is preopen for each $N \in \sigma$.

Definition 22. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called $\mathcal{A}$ -continuous [24] (resp. $\mathcal{AB}$-continuous [12], $\mathcal{B}$-continuous [25], LC-continuous [15]) if $f^{-1}(N) \in \mathcal{A}(X)$ (resp. $f^{-1}(N) \in \mathcal{AB}(X)$, $f^{-1}(N) \in \mathcal{B}(X)$, $f^{-1}(N) \in LC(X)$) for each $N \in \sigma$.

Remark 23. The following diagram holds for a function $f : X \longrightarrow Y$:

$$\begin{array}{ccc}
LC\text{-continuous} & \Longrightarrow & \mathcal{B}\text{-continuous} \\
\Uparrow & & \Uparrow \\
\mathcal{D}\text{-continuous} & \Longrightarrow & \mathcal{DS}\text{-continuous} \\
\Uparrow & & \Uparrow \\
\mathcal{A}\text{-continuous} & \Longrightarrow & \mathcal{AB}\text{-continuous}
\end{array}$$

None of these implications is reversible as shown in the following examples:

Example 24. Let $X = Y = \{a, b, c, d\}$ and $\tau = \sigma = \{\varnothing, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$. Then

(1) the function $f : (X, \tau) \longrightarrow (Y, \sigma)$, defined as: $f(a) = b$, $f(b) = a$, $f(c) = c$, $f(d)=d$, is LC-continuous and so $\mathcal{B}$-continuous but it is neither $\mathcal{D}$-continuous nor $\mathcal{DS}$ -continuous.

(2) the function $g : (X, \tau) \longrightarrow (Y, \sigma)$, defined as: $g(a) = c$, $g(b) = d$, $g(c) = b$, $g(d)=a$, is $\mathcal{D}$-continuous and so $\mathcal{DS}$-continuous but it is neither $\mathcal{A}$-continuous nor $\mathcal{AB}$-continuous.

Example 25. Let $X = \{a, b, c, d\}$ and $\tau = \{\varnothing, X, \{a\}, \{b\}, \{a, b\}\}$. Then the function $f : (X, \tau) \longrightarrow (X, \tau)$, defined as: $f(a) = c$, $f(b) = a$, $f(c) = c$, $f(d) = a$, is $\mathcal{DS}$-continuous and $\mathcal{AB}$-continuous but it is neither $\mathcal{D}$-continuous nor LC-continuous.

Theorem 26. *For a function $f : (X, \tau) \longrightarrow (Y, \sigma)$, the following are equivalent:*

(1) *f is $\mathcal{AB}$-continuous,*

(2) *f is quasi-continuous and $\mathcal{DS}$-continuous,*

(3) *f is γ-continuous and $\mathcal{DS}$-continuous,*

(4) *f is β-continuous and $\mathcal{DS}$-continuous.*

Proof. It follows from Theorem 11. ∎

Definition 27. A function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is called ic-continuous [16] if $f^{-1}(N)$ is interior-closed for each $N \in \sigma$.

Theorem 28. *For a function $f : (X, \tau) \longrightarrow (Y, \sigma)$, the following are equivalent:*

(1) *f is continuous,*

(2) *f is α-continuous and $\mathcal{AB}$-continuous,*

(3) *f is α-continuous and $\mathcal{D}$-continuous,*

(4) *f is α-continuous and $\mathcal{DS}$-continuous,*

(5) *f is precontinuous and $\mathcal{AB}$-continuous,*

(6) *f is precontinuous and $\mathcal{D}$-continuous,*

(7) *f is precontinuous and $\mathcal{DS}$-continuous,*

(8) *f is quasi-continuous, $\mathcal{D}$-continuous and f is either precontinuous or ic-continuous,*

(9) *f is quasi-continuous, $\mathcal{DS}$-continuous and f is either precontinuous or ic-continuous.*

Proof. It is immediate consequence of Theorem 13. ∎

Theorem 29. *For a function $f : (X, \tau) \longrightarrow (Y, \sigma)$, the following are equivalent:*

(1) *f is $\mathcal{A}$-continuous,*

(2) *f is quasi-continuous and $\mathcal{D}$-continuous,*

(3) *f is γ-continuous and $\mathcal{D}$-continuous,*

(4) *f is β-continuous and $\mathcal{D}$-continuous.*

Proof. It is immediate consequence of Theorem 14. ∎

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