On contra $\pi gp-$ continuous functions

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Abstract

In this paper, we introduce and investigate the notion of contra π gp-continuous functions by utilizing Park's $\pi gp-$ closed sets [18]. We obtain fundamental properties of contra π gp-continuous functions and discuss the relationships between contra π gp-continuity and other related functions.

1 Introduction

In 1996, Dontchev [5] introduced a new class of functions called contra-continuous functions. He defined a function $f : X \to Y$ to be contra-continuous if the pre image of every open set of Y is closed in X. In 2007, Caldas et.al. [4] introduced and investigate the notion of contra $g-$ continuity. In 1968, V. Zaitsev [25] introduced the notion of π-open sets as a finite union of regular open sets. Zolotarev [26] proved that in a metric space every closed set is open [Theorem 1] (i.e. every closed set is the intersection of finitely many regular closed sets). This notion received a proper attention and some research articles came to existence. J. Dontchev and T. Noiri [6] introduced and investigated, among others, continuity and almost continuity. Ekici and Baker [8] and Ekici [9] used this notion to introduce and present some fundamental properties of a new type of generalized closed set and new forms of continuities. In [14], Kalantan introduced and investigated π-normality. The digital n-space is not a metric space, since it is not T_1. But recently S. Takigawa and H. Maki [24] showed that in the digital n-space every closed set is π-open.

Recently, Ekici [11] introduced and studied contra πg-continuous functions. In this paper, we present a new generalization of a contra-continuity called contra πgp-continuity. It turns out that the notion of contra πgp-continuity is a weaker form of contra-continuity and contra πg-continuity.

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2 Preliminaries

Throughout this paper, spaces (X,τ) and (Y,σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The closure of A and the interior of A are denoted by $Cl(A)$ and $Int(A)$, respectively. A subset A is said to be regular open[23] (resp.regular closed) if $A = Int(Cl(A))$ (resp. $A = Cl(Int(A))$. The finite union of regular open sets is said to be π-open[25]. The complement of a π-open set is said to be π-closed.

Definition 2.1. A subset A of a space X is said to be

1. g-closed [15] if $cl(A) \subset U$ whenever $A \subset U$ and U is open in X ;
2. gs-closed [1] if $scl(A) \subset U$ whenever $A \subset U$ and U is open in X ;
3. gp-closed [17] if $pcl(A) \subset U$ whenever $A \subset U$ and U is open in X ;
4. πg-closed [6] if $cl(A) \subset U$ whenever $A \subset U$ and U is π-open in X ;
5. πgs-closed [2] if $scl(A) \subset U$ whenever $A \subset U$ and U is π-open in X ;
6. πgp-closed [18] if $pcl(A) \subset U$ whenever $A \subset U$ and U is π-open in X .

The family of all $\pi gp-$ open (resp. $\pi gp-$ closed, closed) sets of X containing a point $x \in X$ is denoted by $\pi GPO(X,x)$ (resp. $\pi GPC(X,x), C(X,x)$). The family of all $\pi gp-$ open (resp. $\pi gp-$ closed, closed, semiopen) sets of X is denoted by $\pi GPO(X)$ (resp. $\pi GPC(X), C(X), SO(X)$).

Definition 2.2. A function $f : X \to Y$ is said to be π-continuous [6] (resp. πgp-continuous [19]) if $f^{-1}(V)$ is π-open (resp. πgp-open) in X for every open set V of Y.

Definition 2.3. Let A be a subset of a space (X,τ)

1. The set $\bigcap\{U \in \tau : A \subset U\}$ is called the kernel of A[16] and is denoted by $ker(A)$;
2. The set $\bigcap\{F : F$ is $\pi gp-$ closed in $X; A \subset F\}$ is called the $\pi gp-$ closure of A [19] and is denoted by $\pi gp - Cl(A)$.

Lemma 2.4. *[13] The following properties hold for subsets U and V of a space (X,τ)*

1. *$x \in ker(U)$ if and only if $U \cap F \neq \emptyset$ for any closed set $F \in C(X,x)$;*
2. *$U \subset ker(U)$ and $U = ker(U)$ if U is open in X;*
3. *If $U \subset V$, then $ker(U) \subset ker(V)$.*

Lemma 2.5. *[19] Let A be a subset of a space (X,τ) , then*

1. *$\pi gp - cl(X \backslash A) = X \backslash \pi gp - int(A)$;*
2. *$x \in \pi gp - cl(A)$ if and only if $A \cap U \neq \emptyset$ for each $U \in \pi GPO(X,x)$;*
3. *If A is πgp-closed in X , then $A = \pi gp - cl(A)$.*

3 contra $\pi gp-$ continuous functions

Definition 3.1. A function $f : X \to Y$ is called contra πgp-continuous if $f^{-1}(V)$ is πgp-closed in X for every open set V of Y.

Theorem 3.2. *The following are equivalent for a function $f : X \to Y$:*

1. *f is contra πgp-continuous;*
2. *The inverse image of every closed set of Y is πgp-open in X;*
3. *For each $x \in X$ and each closed set V in Y with $f(x) \in V$, there exists a π gp-open set U in X such that $x \in U$ and $f(U) \subset V$;*
4. *$f(\pi gp - Cl(A)) \subset Ker(f(A))$ for every subset A of X;*
5. *$\pi gp - Cl(f^{-1}(B)) \subset f^{-1}(Ker(B))$ for every subset B of Y.*

Proof. (1) $\Rightarrow$ *(2) Let U be any closed set of Y. Since $Y \backslash U$ is open, then by (1), it follows that $f^{-1}(Y \backslash U) = X \backslash f^{-1}(U)$ is $\pi gp-$ closed. This shows that $f^{-1}(U)$ is $\pi gp-$ open in X.*

(1) $\Rightarrow$ (3) *Let $x \in X$ and V be a closed set in Y with $f(x) \in V$. By (1), it follows that $f^{-1}(Y \backslash V) = X \backslash f^{-1}(V)$ is $\pi gp-$ closed. Take $U = f^{-1}(V)$ We obtain that $x \in U$ and $f(U) \subset V$.*

(3) $\Rightarrow$ (2) *Let V be a closed set in Y with $x \in f^{-1}(V)$. Since $f(x) \in V$, by (3) there exists a $\pi gp-$ open set U in X containing x such that $f(U) \subset V$. It follows that $x \in U \subset f^{-1}(V)$. Hence $f^{-1}(V)$ is $\pi gp-$ open.*

(2) $\Rightarrow$ (4) *Let A be any subset of X. Let $y \notin Ker(f(A))$. Then by Lemma 2.4, there exist a closed set F containing y such that $f(A) \cap F = \emptyset$. We have $A \cap f^{-1}(F) = \emptyset$ and since $f^{-1}(F)$ is $\pi gp-$ open then we have $\pi gp - Cl(A) \cap f^{-1}(F) = \emptyset$ .Hence we obtain $f(\pi gp - Cl(A)) \cap F = \emptyset$ and $y \notin f(\pi gp - Cl(A))$. Thus $f(\pi gp - Cl(A)) \subset Ker(f(A))$.*

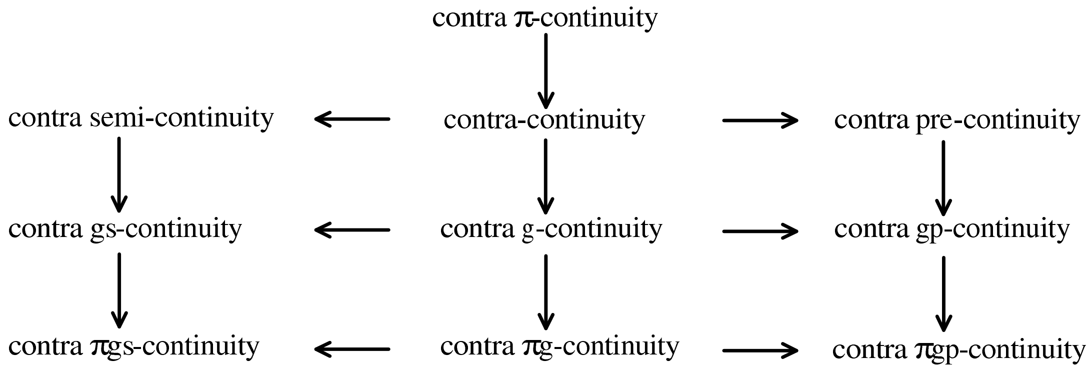
(4) $\Rightarrow$ (5) *Let B be any subset of Y. By (4), $f(\pi gp - Cl(f^{-1}(B))) \subset Ker(B)$ and $\pi gp - Cl(f^{-1}(B)) \subset f^{-1}(Ker(B))$.*

(5) $\Rightarrow$ (1) *Let B be any open set of Y. By (5), $\pi gp - Cl(f^{-1}(B)) \subset f^{-1}(Ker(B)) = f^{-1}(B)$ and $\pi gp - Cl(f^{-1}(B)) = f^{-1}(B)$. So we obtain that $f^{-1}(B)$ is $\pi gp-$ closed in X.* □

Definition 3.3. A function $f : X \to Y$ is said to be

1. perfectly continuous [5] if $f^{-1}(V)$ is clopen in X for every open set V of Y;
2. contra-continuous [5] (resp. contra-precontinuous [12], contra-semicontinuous[7]) if $f^{-1}(V)$ is closed (resp. pre-closed , semi-closed) in X for every open set V of Y;
3. contra $g-$ continuous [4] (resp. contra $gp-$ continuous, contra $gs-$ continuous [7]) if $f^{-1}(V)$ is $g-$ closed (resp. $gp-$ closed, $gs-$ closed) in X for every open set V of Y;
4. contra $\pi-$ continuous (resp. contra π g-continuous [11], contra $\pi gs-$ continuous) if $f^{-1}(V)$ is $\pi-$ closed (resp. $\pi g-$ closed, $\pi gs-$ closed) in X for every open set V of Y.

For the functions defined above, we have the following implications :



Remark 3.4. *None of these implications is reversible as shown by the following examples.*

Example 3.5. *Let* $X=\{a,b,c\}$, $\tau=\{\emptyset, \{a\}, \{b\}, \{a,b\}, \{b,c\}, X\}$ *and* $\sigma=\{\emptyset, \{a\}, \{c\}, \{a,c\}, X\}$*. Then the identity function* $f: (X,\tau) \to (X,\sigma)$ *is contra continuous but not contra* π *-continuous.*

Example 3.6. *Let* $X=\{a,b,c\}$, $\tau=\{\emptyset, \{a\}, \{b\}, \{a,b\}, \{b,c\}, X\}$ *and* $\sigma=\{\emptyset, \{a\}, \{b\}, \{a,b\}, X\}$*. Then the identity function* $f: (X,\tau) \to (X,\sigma)$ *is contra* π *gp-continuous but not contra gp-continuous.*

Example 3.7. *Let* $X=\{a,b,c,d,e\}$, $\tau = \{\emptyset, \{b\}, \{b,c\}, \{a,d\}, \{a,b,d\}, \{a,b,c,d\}, X\}$ *and* $\sigma = \{\emptyset, \{a\}, X\}$*. Then the identity function* $f: (X,\tau) \to (X,\sigma)$ *is contra* π *gp-continuous but not contra* π *g-continuous.*

Remark 3.8. *The following examples shows that the concept of contra* π *gp-continuity and contra* π *gs-continuity are independent.*

Example 3.9. *Let* $X=\{a,b,c,d\}$, $\tau=\{\emptyset, \{a\}, \{b\}, \{a,b\}, \{b,c\},\{a,b,c\}, X\}$ *and* $\sigma=\{\emptyset, \{a\}, X\}$*. Then the identity function* $f: (X,\tau) \to (X,\sigma)$ *is contra* π *gs-continuous but not contra* π *gp-continuous.*

Example 3.10. *Let* $X=\{a,b,c,d,e\}$, $\tau=\{\emptyset, \{b\}, \{b,c\}, \{a,d\}, \{a,b,d\}, \{a,b,c,d\}, X\}$ *and* $\sigma=\{\emptyset, \{d\}, X\}$*. Then the identity function* $f: (X,\tau) \to (X,\sigma)$ *is contra* π *gp-continuous but not contra* π *gs-continuous.*

Definition 3.11. A function $f : X \to Y$ is said to be

1. πgp-semiopen if $f(U) \in SO(Y)$ for every πgp-open set of X.
2. contra-$I(\pi$ gp)-continuous if for each $x \in X$ and each $F \in C(Y, f(x))$, there exists $U \in \pi GPO(X,x)$ such that $Int(f(U)) \subset F$.

Theorem 3.12. *If a function* $f : X \to Y$ *is contra-*$I(\pi$ *gp)-continuous and* πgp *-semiopen, then* f *is contra* π *gp-continuous.*

Proof. *Suppose that* $x \in X$ *and* $F \in C(Y, f(x))$*. Since* f *is contra-*$I(\pi$ *gp)-continuous, there exists* $U \in \pi GPO(X,x)$ *such that* $Int(f(U)) \subset F$*. By hypothesis* f *is* πgp *-semiopen, therefore* $f(U) \in SO(Y)$ *and* $f(U) \subset Cl(Int(f(U))) \subset F$*. This shows that* f *is contra* πgp *-continuous.*

Lemma 3.13. *[19] If* A *is* π *-open and* π *gp-closed in a space* (X,τ)*, then* A *is clopen.*

Theorem 3.14. *If a function $f : X \to Y$ is contra π gp-continuous and π -continuous, then f is perfectly continuous.*

Proof. *Let U be an open set in Y. Since f is contra π gp-continuous and π -continuous, $f^{-1}(U)$ is $\pi gp-$ closed and π -open, by Lemma 3.13, $f^{-1}(U)$ is clopen. Then f is perfectly continuous.* □

Theorem 3.15. *If a function $f : X \to Y$ is contra $\pi gp-$ continuous and Y is regular, then f is π gp-continuous.*

Proof. *Let x be an arbitrary point of X and U be an open set of Y containing $f(x)$. Since Y is regular, there exists an open set W in Y containing $f(x)$ such that $Cl(W) \subset U$. Since f is contra $\pi gp-$ continuous, there exists $V \in \pi GPO(X, x)$ such that $f(V) \subset Cl(W)$. Then $f(V) \subset Cl(W) \subset U$. Hence f is $\pi gp-$ continuous.* □

Theorem 3.16. *Let $\{X_i, i \in \Omega\}$ be any family of topological spaces. If a function $f : X \to \prod X_i$ is contra π gp-continuous, then $Pr_i \circ f : X \to X_i$ is contra π gp-continuous for each $i \in \Omega$, where Pr_i is the projection of $\prod X_i$ onto X_i*

Proof. *For a fixed $i \in \Omega$, let V_i be any open set of X_i. Since Pr_i is continuous, $Pr_i^{-1}(V_i)$ is open in $\prod X_i$. Since f is contra π gp-continuous, $f^{-1}(Pr_i^{-1}(V_i)) = (Pr_i \circ f)^{-1}(V_i)$ is $\pi gp-$ closed in X. Therefore, $Pr_i \circ f$ is contra π gp-continuous for each $i \in \Omega$.* □

Theorem 3.17. *Let $f : X \to Y$ and $g : Y \to Z$ be a function. Then the following hold:*

1. *If f is contra π gp-continuous and g is continuous, then $g \circ f : X \to Z$ is contra π gp-continuous;*
2. *If f is π gp-continuous and g is contra-continuous, then $g \circ f : X \to Z$ is contra π gp-continuous;*
3. *If f is contra π gp-continuous and g is contra-continuous, then $g \circ f : X \to Z$ is π gp-continuous.*

Definition 3.18. A space (X, τ) is called $\pi gp - T_{1/2}$ [19] if every $\pi gp-$ closed set is preclosed.

Remark 3.19. *Every contra $\pi gp-$ continuous function defined on a $\pi gp - T_{1/2}$ space is contra-precontinuous.*

Theorem 3.20. *Let $f : X \to Y$ be a function. Suppose that X is a $\pi gp - T_{1/2}$ space. Then the following are equivalent*

1. *f is contra π gp-continuous;*
2. *f is contra gp -continuous;*
3. *f is contra-pre continuous.*

Proof. *Obvious* □

Definition 3.21. [19] For a space (X, τ), ${}_{\pi}\tau^* = \{U \subset X : \pi gp - Cl(X \backslash U) = X \backslash U\}$.

Theorem 3.22. *[19] Let (X, τ) be a space. Then every πgp -closed set is closed if and only if ${}_{\pi}\tau^* = \tau$.*

Theorem 3.23. *If* $_{\pi}\tau^{*} = \tau$ *in* X *, then for a function* $f : X \rightarrow Y$ *the following are equivalent*

1. f *is contra* π *gp-continuous;*
2. f *is contra* πg *-continuous;*
3. f *is contra* g *-continuous;*
4. f *is contra-continuous.*

Proof. *Follows from Theorem 2.11 in [19].* □

4 Properties of contra π gp -continuous functions

Definition 4.1. A space X is said to be $\pi gp{-}T_1$ if for each pair of distinct points x and y in X , there exist π gp-open sets U and V containing x and y respectively, such that $y \notin U$ and $x \notin V$.

Definition 4.2. [10] A space X is said to be $\pi gp - T_2$ if for each pair of distinct points x and y in X, there exist $U \in \pi GP0(X, x)$ and $V \in \pi GP0(X, y)$ such that $U \cap V = \emptyset$.

Theorem 4.3. *Let* X *be a topological space. Suppose that for each pair of distinct points* x_1 *and* x_2 *in* X *there exists a function* f *of* X *into a Urysohn space* Y *such that* $f(x_1) \neq f(x_2)$ *. Moreover, let* f *be contra* $\pi gp-$ *continuous at* x_1 *and* x_2 *. Then* X *is* $\pi gp - T_2$ *.*

Proof. *Let* x_1 *and* x_2 *be any distinct points in* X *. Then there exist an Urysohn space* Y *and a function* $f : X \rightarrow Y$ *such that* $f(x_1) \neq f(x_2)$ *and* f *is contra* πgp *-continuous at* x_1 *and* x_2 *. Let* $w = f\ (x_1\ )$ *and* $z = f\ (x_2\ )$*. Then* $w \neq z$ *. Since* Y *is Urysohn, there exist open sets* U *and* V *containing* w *and* z *respectively such that* $Cl(U) \cap Cl(V) = \emptyset$ *. Since* f *is contra* πgp *-continuous at* x_1 *and* x_2 *, then there exist* πgp *-open sets* A *and* B *containing* x_1 *and* x_2 *respectively such that* $f(A) \subset Cl(U)$ *and* $f(B) \subset Cl(V)$ *. So we have* $A \cap B = \emptyset$ *since* $Cl(U) \cap Cl(V) = \emptyset$ *. Hence,* X *is* $\pi gp - T_2$ *.* □

Corollary 4.4. *If* f *is a contra* $\pi gp-$ *continuous injection of a topological space* X *into a Urysohn space* Y *, then* X *is* $\pi gp - T_2$ *.*

Proof. *For each pair of disdinct points* x_1 *and* x_2 *in* X *and* f *is a contra* $\pi gp-$ *continuous function of* X *into a Urysohn space* Y *such that* $f(x_1) \neq f(x_2)$ *because* f *is injective. Hence by Theorem 4.3,* X *is* $\pi gp - T_2$ *.*

Definition 4.5. [19] A space (X, τ) is said to be πgp- connected if X cannot be expressed as the disjoint union of two non-empty $\pi gp-$ open sets.

Remark 4.6. *[19] Every* π *gp-connected space is connected.*

Theorem 4.7. *For a space* X*, the following are equivalent:*

1. X *is* $\pi gp-$ *connected;*
2. *The only subsets of* X *which are both* πgp *-open and* πgp *-closed are the empty set* $\emptyset$ *and* X;

3. *Each contra πgp -continuous function of X into a discrete space Y with at least two points is a constant function.*

Proof.

(1) $\Leftrightarrow$ (2) *Follows from Proposition 6.2 [19]*

(2) $\Rightarrow$ (3) *Let $f : X \rightarrow Y$ be contra π gp-continuous function where Y is a discrete space with at least two points. Then $f^{-1}(\{y\})$ is π gp-closed and π gp-open for each $y \in Y$ and $X = \cup\{f^{-1}(\{y\}) : y \in Y\}$ .By hypothesis $f^{-1}(\{y\}) = \emptyset$ or X . If $f^{-1}(\{y\}) = \emptyset$ for all $y \in Y$, then f is not a function. Also there cannot exist more than one $y \in Y$ such that $f^{-1}(\{y\}) = X$. Hence there exists only one $y \in Y$ such that $f^{-1}(\{y\}) = X$ and $f^{-1}(\{y_1\}) = \emptyset$ where $y \neq y_1 \in Y$. This shows that f is a constant function.*

(3) $\Rightarrow$ (2) *Let P be a non-empty set which is both π gp-open and π gp- closed in X . Suppose $f : X \rightarrow Y$ is a contra πgp - continuous function defined by $f(P) = \{a\}$ and $f(X \backslash P) = \{b\}$ where $a \neq b$ and $a, b \in Y$ .By hypothesis, f is constant. Therefore $P = X$.* □

Theorem 4.8. *If f is a contra π gp- continuous function from a π gp- connected space X onto any space Y , then Y is not a discrete space.*

Proof. *Suppose that Y is discrete. Let A be a proper non-empty open and closed subset of Y . Then $f^{-1}(A)$ is a proper non-empty π gp-clopen subset of X which is a contradiction to the fact that X is π gp-connected.* □

Theorem 4.9. *If $f : X \rightarrow Y$ is a contra π gp-continuous surjection and X is π gp-connected, then Y is connected.*

Proof. *Suppose that Y is not a connected space. There exist non-empty disjoint open sets U_1 and U_2 such that $Y = U_1 \cup U_2$ .Therefore U_1 and U_2 are clopen in Y . Since f is contra π gp-continuous. $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are π gp-open in X . Moreover, $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are non-empty disjoint and $X = f^{-1}(U_1) \cup f^{-1}(U_2)$ .This shows that X is not $\pi gp-$ connected. This contradicts that Y is not connected assumed. Hence Y is connected.* □

Definition 4.10. The graph $G(f)$ of a function $f : X \rightarrow Y$ is said to be contra π gp-graph if for each $(x, y) \in (X \times Y) \backslash G(f)$,there exist a π gp-open set U in X containing x and a closed set V in Y containing y such that $(U \times V) \cap G(f) = \emptyset$.

Lemma 4.11. *A graph $G(f)$ of a function $f : X \rightarrow Y$ is contra π gp-graph in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) \backslash G(f)$, there exists $U \in \pi GPO(X)$ containing x and $V \in C(Y)$ containing y such that $f(U) \cap V = \emptyset$.*

Theorem 4.12. *If $f : X \rightarrow Y$ is contra $\pi gp-$ continuous and Y is Urysohn, $G(f)$ is contra π gp-graph in $X \times Y$.*

Proof. *Let $(x, y) \in (X \times Y) \backslash G(f)$. It follows that $f(x) \neq y$. since Y is Urysohn, there exist open sets V and W such that $f(x) \in V$, $y \in W$ and $Cl(V) \cap Cl(W) = \emptyset$. Since f is contra π gp-continuous, there exist a $U \in \pi GPO(X, x)$ such that $f(U) \subset Cl(V)$ and $f(U) \cap Cl(W) = \emptyset$. Hence $G(f)$ is contra π gp-graph in $X \times Y$.* □

Theorem 4.13. *Let $f : X \rightarrow Y$ be a function and $g : X \rightarrow X \times Y$ the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is contra π gp-continuous, then f is contra π gp -continuous.*

Proof. *Let* U *be an open set in* Y, *then* $X \times U$ *is an open set in* $X \times Y$. *It follows that* $f^{-1}(U) = g^{-1}(X \times U) \in \pi GPC(X)$. *Thus* f *is contra* π *gp-continuous.* □

Definition 4.14. A space (X, τ) is said to be submaximal [3] if every dense subset of X is open in X.

Note that (X, τ) is submaximal if and only if every preopen set is open [20].

Lemma 4.15. [10] *Let* (X, τ) *be a topological space. If* $U, V \in \pi GPO(X)$ *and* X *is a submaximal space, then* $U \cap V \in \pi GPO(X)$.

Theorem 4.16. *If* $f : X \to Y$ *and* $g : X \to Y$ *are contra* π *gp-continuous,* X *is submaximal and* Y *is Urysohn, then* $K = \{x \in X : f(x) = g(x)\}$ *is* $\pi gp-$ *closed in* X.

Proof. *Let* $x \in X \backslash K$. *Then* $f(x) \neq g(x)$. *Since* Y *is Urysohn, there exist open sets* U *and* V *such that* $f(x) \in U, g(x) \in V$ *and* $Cl(U) \cap Cl(V) = \emptyset$. *Since* f *and* g *are contra* $\pi gp-$ *continuous,* $f^{-1}(Cl(U)) \in \pi GPO(X)$ *and* $g^{-1}(Cl(V)) \in \pi GPO(X)$. *Let* $A = f^{-1}(Cl(U))$ *and* $B = g^{-1}(Cl(V))$. *Then* A *and* B *contains* x. *Set* $C = A \cap B$. C *is* π *gp-open in* X. *Hence* $f(C) \cap g(C) = \emptyset$ *and* $x \notin \pi gp - Cl(K)$. *Thus,* K *is* π *gp-closed in* X. □

Definition 4.17. A subset A of a topological space X is said to be $\pi gp-$ dense in X if $\pi gp - Cl(A) = X$

Theorem 4.18. *Let* $f : X \to Y$ *and* $g : X \to Y$ *be contra* π *gp-continuous. If* Y *is Urysohn and* $f = g$ *on a* π *gp-dense set* $A \subset X$, *then* $f = g$ *on* X.

Proof. *Since* f *and* g *are contra* πgp *-continuous and* Y *is Urysohn, by Theorem 4.16,* $K = \{x \in X : f(x) = g(x)\}$ *is* $\pi gp-$ *closed in* X. *We have* $f = g$ *on* π *gp-dense set* $A \subset X$. *Since* $A \subset K$ *and* A *is* $\pi gp-$ *dense set in* X, *then* $X = \pi gp - Cl(A) \subset \pi gp - Cl(K) \subset K$. *Hence,* $f = g$ *on* X. □

Definition 4.19. A space X is said to be $weakly Hausdorff$ [21] if each element of X is an intersection of regular closed sets.

Theorem 4.20. *If* $f : X \to Y$ *is a contra* $\pi gp-$ *continuous injection and* Y *is weakly Hausdroff, then* X *is* $\pi gp - T_1$.

Proof. *Suppose that* Y *is weakly Hausdorff. For any distinct points* x_1 *and* x_2 *in* X, *there exist regular closed sets* U *and* V *in* Y *such that* $f(x_1) \in U, f(x_2) \notin U, f(x_1) \notin V$ *and* $f(x_2) \in V$. *Since* f *is contra* π *gp-continuous,* $f^{-1}(U)$ *and* $f^{-1}(V)$ *are* π *gp-open subsets of* X *such that* $x_1 \in f^{-1}(U), x_2 \notin f^{-1}(U), x_1 \notin f^{-1}(V)$ *and* $x_2 \in f^{-1}(V)$. *This shows that* X *is* $\pi gp - T_1$. □

Theorem 4.21. *Let* $f : X \to Y$ *have a contra* πgp *-graph. If* f *is injective, then* X *is* $\pi gp - T_1$.

Proof. *Let* x_1 *and* x_2 *be any two distinct points of* X. *Then, we have* $(x_1, f(x_2)) \in (X \times Y) \backslash G(f)$. *Then, there exist a* π *gp-open set* U *in* X *containing* x_1 *and* $F \in C(Y, f(x_2))$ *such that* $f(U) \cap F = \emptyset$. *Hence* $U \cap f^{-1}(F) = \emptyset$. *Therefore, we have* $x_2 \notin U$. *This implies that* X *is* $\pi gp - T_1$. □

Definition 4.22. A topological space X is said to be $Ultra Hausdorff$ [22] if for each pair of distinct points x and y in X there exist clopen sets A and B containing x and y, respectively such that $A \cap B = \emptyset$.

Theorem 4.23. *Let $f : X \to Y$ be a contra π gp-continuous injection .If Y is an Ultra Hausdorff space, then X is $\pi gp - T_2$.*

Proof. *Let x_1 and x_2 be any distinct point in X , then $f(x_1) \neq f(x_2)$ and there exist clopen sets U and V containing $f(x_1)$ and $f(x_2)$ respectively such that $U \cap V = \emptyset$. since f is contra π gp-continuous, then $f^{-1}(U) \in \pi GP0(X)$ and $f^{-1}(V) \in \pi GP0(X)$ such that $f^{-1}(U) \cap f^{-1}(V) = \emptyset$. Hence, X is $\pi gp - T_2$.* □

Definition 4.24. A topological space X is said to be

1. πgp-normal if each pair of non-empty disjoint closed sets can be separated by disjoint πgp-open sets.
2. Ultra normal [22] if for each pair of non-empty distinct closed sets can be separated by disjoint clopen sets.

Theorem 4.25. *If $f : X \to Y$ is a contra π gp-continuous, closed injection and Y is Ultra normal, then X is πgp-normal.*

Proof. *Let F_1 and F_2 be disjoint closed subsets of X . Since f is closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint closed subsets of Y . Since Y is Ultra normal, $f(F_1)$ and $f(F_2)$ are separated by disjoint clopen sets V_1 and V_2 respectively. Hence $F_i \subset f^{-1}(V_i), f^{-1}(V_i) \in \pi GP0(X, x)$ for $i = 1, 2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$ and thus X is πgp-normal.*

References

[1] Arya .S.P and Noiri .T, Characterization of s-normal spaces, Indian J.Pure Appl. Math. 21(1990), 717-719.

[2] Aslim .G, Guler .A.C and Noiri .T, On πgs-closed sets in topological spaces, Acta Math. Hungar. 112, 6(2006), 275-283.

[3] Bourbaki .N, General topology , Part I, Reading, MA: Addison Wesley, 1966.

[4] Caldas .M, Jafari .S, Noiri .T and Simoes .M, A new generalization of contra-continuity via Levine's g-closed set, Chaos,Solitons and Fractals, 32(2007),1597-1603.

[5] Dontchev .J, Contra-continuous functions and strongly s-closed spaces, Internat. J. Math. & Math. Sci., 19(1996), 303-310.

[6] Dontchev .J, and Noiri .T, Quasi-normal spaces and πg-closed sets Acta Math. Hungar., 89,3(2000),211-219.

[7] Dontchev .J and Noiri .T, Contra-Semi continuous functions, Math. Pannon. 10, 2(1999), 159-168.

[8] Ekici .E and Baker .C.W, On πg-closed sets and πg-continuity, Kochi J. Math. 2(2007), 35-42.

[9] Ekici .E, On (g, s)-continuous and $(\pi g, s)$-continuous functions, saracvo J. Math. 3(15)(2007), 99-113.

[10] Ekici .E, On almost $\pi gp-$ continuous functions, Chaos,Solitons and Fractals,32(2007),1935-1944.

[11] Ekici .E, On contra $\pi g-$ continuous functions, Chaos,Solitons and Fractals,35(2008),71-81.

[12] Jafari .S and Noiri .T, On contra precontinuous functions, Bull. Malaysian Math. Sci. Soc., 25(2002),115-128.

[13] Jafari.S and Noiri.T, Contra-super-continuous functions, Ann. Univ. Sci. Budapest. Eotvos Scct. Math. 42(1999),27-34.

[14] Kalantan .L.N, π-normal topological spaces, Filomat 22(1)(2008), 173-181.

[15] Levine .N, Generalized closed sets in topology, Rend. Circ. Mat. Palermc (2), 19 (1970), 89-96.

[16] Mrsevic .M, On pairwise R_0 and pairwise R_1 bitopological spaces, Bull. Math. Soc. Sci. Math RS Roumane, 30(1996),141-148.

[17] Noiri .T, Maki .H and Umehara .J, Generalized preclosed functions, Mem. Fac. Sci. Kochi Univ. Ser. A (Math), 19(1998),13-20.

[18] Park .J.H, On $\pi gp-$ closed sets in topological spaces, Indian J. Pure Appl. Math., (In press)

[19] Park .J.H and Park .J.K, On $\pi gp-$ continuous functions in topological spaces, Chaos, Solitons and Fractals, 20(2004),467-477.

[20] Reilly .I.L and Vamanamurthy .M.K, On some questions concerning preopen sets, Kyungpook Math. J. 30(1)(1990),87-93.

[21] Soundararajan .T, Weakly Hausdorff spaces and the cardinality of topological spaces. In: General topology and its relation to modern analysis and algebra. III, Proc. Conf. Kanpur, 1968, Academia, Prague 1971. p. $301 - 306$.

[22] Staum .R, The algebra of bounded continuous functions into a nonarchimedean field, Pacific J. Math., 50 (1974),169-85.

[23] Stone .M.H, Applications of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc., 41 (1937),375-481.

[24] Takigawa .S and Maki .H, Every nonempty open set of the digital n-space is expressible as the union of finitely many nonempty regular open sets, Sci. Math. Japan. Online. E 2007, 601 612.

[25] Zaitsev .V, On certain classes of topological spaces and their bicompactifications, Dokl Akad Nauk SSSR, 178(1968),778-9.

[26] Zolotarev .V.P, π-sets, Soviet Math. Dokl, 17(1976), 1277 1279.