Selected papers of the 2014 International Conference on Topology and its Applications

2014 International Conference on Topology and its Applications,
July 3-7, 2014, Nafpaktos,
Greece

Selected papers of the 2014 International Conference on Topology and its Applications

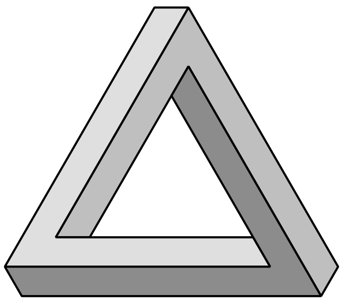
Editors

D.N. Georgiou
S.D. Iliadis
I.E. Kougias
A.C. Megaritis

Department of Mathematics, University of Patras, Greece

**2014 International Conference on Topology and its Applications,
July 3-7, 2014, Nafpaktos,
Greece**

Selected papers of the 2014 International Conference on Topology and its Applications



Editors

D.N. Georgiou
S.D. Iliadis
I.E. Kougias
A.C. Megaritis

Department of Mathematics, University of Patras, Greece

ISBN: 978-618-80609-1-3

Preface

The **2014 International Conference on Topology and its Applications** took place from July 3 to 7 in the **3rd High School of Nafpaktos, Greece**. It covered all areas of Topology and its Applications (especially General Topology, Set-Theoretic Topology, Geometric Topology, Topological Groups, Dimension Theory, Dynamical Systems and Continua Theory, Computational Topology, History of Topology). This conference was attended by 235 participants from 44 countries and the program consisted by 147 talks.

The Organizing Committee consisted of S.D. Iliadis (University of Patras), D.N. Georgiou (University of Patras), I.E. Kougias (Technological Educational Institute of Western Greece), A.C. Megaritis (Technological Educational Institute of Western Greece), and I. Boules (Mayor of the city of Nafpaktos).

The Organizing Committee is very much indebted to the City of Nafpaktos for its hospitality and for its excellent support during the conference.

The conference was sponsored by University of Patras, Technological Educational Institute of Western Greece, Municipality of Nafpaktos, New Media Soft – Internet Solutions, Loux Marlafekas A.B.E.E., TAXYTYPO – TAXYEKTYPOSEIS GRAVANIS EPE, Alpha Bank, and Wizard Solutions LTD.

This volume is a special volume under the title: "Selected papers of the 2014 International Conference on Topology and its Applications" which will be edited by the organizers (D.N. Georgiou, S.D. Iliadis, I.E. Kougias, and A.C. Megaritis) and published by the University of Patras. We thank the authors for their submissions.

Editors

D.N. Georgiou
S.D. Iliadis
I.E. Kougias
A.C. Megaritis

Selected papers of the 2014 International Conference on Topology and its Applications
(2015) Pages 77–86

Department of Mathematics
University of Patras
Greece

New types of continuous functions via $\widetilde{G}\alpha$-open sets

S. Jafari [1], A. Selvakumar [2]

[1] *College of Vestsjaelland South, Herrestraede 11, 4200 Slagelse, Denmark*
[2] *Info Institute of Engineering Coimbatore-107, Tamilnadu, India*

Abstract

In this paper , we will continue the study of related irresolute functions with $\widetilde{g}\alpha$-open sets [6]. We introduce and study the notion of completely $\widetilde{g}\alpha$-irresolute functions. Further, we discuss the notion of $\widetilde{g}\alpha$-quotient functions and study some of their properties.

Key words: $\widetilde{g}\alpha$-open set, $\widetilde{g}\alpha$-irresolute, completely $\widetilde{g}\alpha$-irresolute, $\widetilde{g}\alpha$-quotient.
1991 MSC: 54A05, 54D05 54D10, 54D45.

1. Introduction

The first step of generalizing closed set was done by Levine in 1970 [10]. Recently, as generalization of closed sets, the notion of $\widetilde{g}\alpha$-closed sets were introduced and studied by R.Devi et al. [6]. Functions and of course irresolute functions stand amoung the most important researched points in the whole of Mathematical Science. Crossley and Hildebrand [2] introduced the notion of irresoluteness in 1972. Its importance is significant in various areas of Mathematics and related sciences. In this paper, we will continue the study of related irresolute functions with $\widetilde{g}\alpha$-open sets. We introduce and study the notion of completely $\widetilde{g}\alpha$-irresolute functions. Further, we discuss the notion of $\widetilde{g}\alpha$-quotient functions and study some of their properties.

All through this paper, (X, τ), (Y, σ) and (Z, η) stand for topological spaces with no separation axioms assumed, unless otherwise stated. Let $A \subseteq X$, the closure of A and the interior of A will be denoted by $cl(A)$ and $int(A)$

[1] jafaripersia@gmail.com
[2] selvam_mphil@yahoo.com

respectively. A is regular open [17] if $A = int(cl(A))$ and A is regular closed [17] if its complement is regular open; equivalently A is regular closed if $A = cl(int(A))$.

2. Preliminaries

We recall the following definitions, which are useful in the sequel.

Definition 2.1. A subset A of a space (X, τ) is called

1. a semi-open set [11] if $A \subseteq cl(int(A))$ and a semi-closed set [11] if $int(cl(A)) \subseteq A$ and
2. an α-open set [15] if $A \subseteq int(cl(int(A)))$ and an α-closed set [15] if $cl(int(cl(A))) \subseteq A$.

The semi-closure (resp. α-closure) of a subset A of a space (X, τ) is the intersection of all semi-closed (resp. α-closed) sets that contain A and is denoted by $scl(A)$ (resp. $\alpha cl(A)$).

Definition 2.2. A subset A of a space (X, τ) is called a

1. $\widehat{g}$-closed set [21] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ); the complement of $\widehat{g}$-closed set is $\widehat{g}$-open set,
2. ${}^*g$-closed set [19] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\widehat{g}$-open in (X, τ); the complement of ${}^*g$-closed set is ${}^*g$-open set,
3. ${}^\sharp gs$-closed set [20] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is ${}^*g$-open in (X, τ); the complement of ${}^\sharp gs$-closed set is ${}^\sharp gs$-open set and
4. $\widetilde{g}\alpha$-closed set [6] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ${}^\sharp gs$-open in (X, τ); the complement of $\widetilde{g}\alpha$-closed set is $\widetilde{g}\alpha$-open set.

For a topological space (X, τ), $RO(X)$ (resp. $RC(X)$, $\widetilde{G}\alpha O(X)$) denotes the class of all regular open (resp. regular closed, $\widetilde{g}s$-open) subsets of (X, τ).

Definition 2.3. A function $f : (X, \tau) \to (Y, \sigma)$ is called

1. strongly continuous [12] if $f^{-1}(V)$ is both open and closed in (X, τ) for every subset V of (Y, σ),
2. completely continuous [1] if $f^{-1}(V)$ is regular open in (X, τ) for every open set V of (Y, σ),
3. $\widetilde{g}\alpha$-continuous [5] if $f^{-1}(V)$ is $\widetilde{g}\alpha$-closed in (X, τ) for every closed set V of (Y, σ),
4. $\widetilde{g}\alpha$-irresolute [5] if $f^{-1}(V)$ is $\widetilde{g}\alpha$-closed in (X, τ) for every $\widetilde{g}\alpha$-closed set V of (Y, σ),
5. $\widetilde{g}\alpha$-open [5] if $f(V)$ is $\widetilde{g}\alpha$-open in (Y, σ) for every open set V of (X, τ),
6. $\widetilde{g}\alpha$-closed [5] if $f(V)$ is $\widetilde{g}\alpha$-closed in (Y, σ) for every closed set V of (X, τ) and

7. quasi $\widetilde{g}\alpha$-open [3] if $f(V)$ is open in (Y,σ) for every $\widetilde{g}\alpha$-open set V of (X,τ).

3. Completely $\widetilde{g}\alpha$-irresolute functions

Definition 3.1. A function $f : (X,\tau) \to (Y,\sigma)$ is called completely $\widetilde{g}\alpha$-irresolute if the inverse image of each $\widetilde{g}\alpha$-open subset of Y is regular open in X.

Theorem 3.2.

(a) Every strongly continuous function is completely $\widetilde{g}\alpha$-irresolute.
(b) Every completely $\widetilde{g}\alpha$-irresolute function is $\widetilde{g}\alpha$-irresolute and hence $\widetilde{g}\alpha$-continuous.

Proof. It follows from the definitions. ■

The converse of the above Theorem need not be true in general as seen from the following examples.

Example 3.3.

(a) Let $X = \{a,b,c\} = Y$, $\tau = \{X, \phi, \{b\}, \{b,c\}\}$ and

$$\sigma = \{Y, \phi, \{a\}, \{a,b\}, \{a,c\}\}.$$

Define a function $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = c$, $f(b) = a$ and $f(c) = b$. Clearly, f is completely $\widetilde{g}\alpha$-irresolute but not strongly continuous.
(b) Let $X = \{a,b,c\} = Y$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}\}$ and $\sigma = \{Y, \phi, \{a,b\}\}$. Define an identity function $f : (X,\tau) \to (Y,\sigma)$. Clearly, f is $\widetilde{g}\alpha$-irresolute but not completely $\widetilde{g}\alpha$-irresolute.

Theorem 3.4. If a function $f : (X,\tau) \to (Y,\sigma)$ is completely $\widetilde{g}\alpha$-irresolute then $f^{-1}(F)$ is regular closed in X for every $\widetilde{g}\alpha$-closed set F of Y.

Proof. Let F be any $\widetilde{g}\alpha$-closed set of Y. Then $Y/f \in \widetilde{G}\alpha O(Y)$. By hypothesis, $f^{-1}(Y/F) = X/f^{-1}(F) \in RO(X)$. We have $f^{-1}(F) \in RC(X)$.
Converse is similar. ■

Lemma 3.5. [13] Let S be an open subset of a space (X,τ). Then the following hold:

(i) If U is regular open in X, then so is $U \cap S$ in the subspace (S,τ_S).
(ii) If $B \subset S$ is regular open in (S,τ_S), then there exists a regular open set U in (X,τ) such that $B = U \cap S$.

Theorem 3.6. If $f : (X, \tau) \to (Y, \sigma)$ is a completely $\widetilde{g}\alpha$-irresolute function and A is any open subset of X, then the restriction $f/A : A \to Y$ is completely $\widetilde{g}\alpha$-irresolute.

Proof. Let F be a $\widetilde{g}\alpha$-open subset of Y. By hypothesis, $f^{-1}(F)$ is regular open in X. Since A is open in X, it follows from Lemma 3.5 that $(f/A)^{-1}(F) = A \cap f^{-1}(F)$, which is regular open in A. Therefore, f/A is completely $\widetilde{g}\alpha$-irresolute. ■

Definition 3.7. [4] A topological space X is said to be $\widetilde{g}\alpha$-normal if each pair of non-empty disjoint closed sets can be separated by disjoint $\widetilde{g}\alpha$-open sets.

Theorem 3.8. If $f : (X, \tau) \to (Y, \sigma)$ is completely $\widetilde{g}\alpha$-irresolute, $\widetilde{g}\alpha$-closed surjection and X is $\widetilde{g}\alpha$-normal, then Y is $\widetilde{g}\alpha$-normal.

Proof. Let F_1 and F_2 be any distinct $\widetilde{g}\alpha$-closed sets of Y. Since f is completely $\widetilde{g}\alpha$-irresolute, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are disjoint $\widetilde{g}\alpha$-closed sets of X. By $\widetilde{g}\alpha$-normal of X, there exist $U_1, U_2 \in \widetilde{G}\alpha O(X)$ such that $f^{-1}(F_1) \subset U_1$ and $f^{-1}(F_2) \subset U_2$ and $U_1 \cap U_2 = \phi$. Let $V_i = Y - f(X - U_i)$ for $i = 1, 2$. Since f is $\widetilde{g}\alpha$-closed, the sets V_1, V_2 are $\widetilde{g}\alpha$-open in Y and $F_i \subset V_i$ for $i = 1, 2$. Since U_1 and U_2 are disjoint and $f^{-1}(F_i) \subset U_i$ for $i = 1, 2$, we obtain $V_1 \cap V_2 = \phi$. This shows that Y is $\widetilde{g}\alpha$-normal. ■

Definition 3.9. A space X is said to be almost connected [8] (resp. $\widetilde{g}\alpha$-connected [4]) if there does not exist disjoint regular open (resp. $\widetilde{g}\alpha$-open) sets A and B such that $A \cup B = X$.

Theorem 3.10. If $f : (X, \tau) \to (Y, \sigma)$ is completely $\widetilde{g}\alpha$-irresolute surjective function and X is almost connected, then Y is $\widetilde{g}\alpha$-connected.

Proof. Suppose that Y is not $\widetilde{g}\alpha$-connected. Then there exists disjoint $\widetilde{g}\alpha$-open sets A and B of Y such that $A \cup B = Y$. Since f is completely $\widetilde{g}\alpha$-irresolute surjective, $f^{-1}(A)$ and $f^{-1}(B)$ are regular open sets in X. Moreover, $f^{-1}(A) \cup f^{-1}(B) = X$, $f^{-1}(A) \neq \phi$ and $f^{-1}(B) \neq \phi$. This shows that X is not almost connected, which is contradiction to the assumption that X is almost connected. By contradiction, Y is $\widetilde{g}\alpha$-connected. ■

Definition 3.11. A space (X, τ) is said to be $\widetilde{g}\alpha$-T_1 [4] (resp. r-T_1 [8]) if for each pair of distinct points x and y of X, there exist $\widetilde{g}\alpha$-open (resp. regular open) sets U_1 and U_2 such that $x \in U_1$ and $y \in U_2$, $x \notin U_2$ and $y \notin U_1$.

Theorem 3.12. If $f : (X, \tau) \to (Y, \sigma)$ is completely $\widetilde{g}\alpha$-irresolute injective function and Y is $\widetilde{g}\alpha$-T_1, then X is r-T_1.

Proof. Suppose that Y is $\widetilde{g}\alpha$-T_1. For any two distinct points x and y of X, there exist $\widetilde{g}\alpha$-open sets F_1 and F_2 in Y such that $f(x) \in F_1$, $f(y) \in F_2$, $f(x) \notin F_2$ and $f(y) \notin F_1$. Since f is injective completely $\widetilde{g}\alpha$-irresolute function, we have X is r-T_1. ■

Definition 3.13. A space (X, τ) is said to be $\widetilde{g}\alpha$-T_2 [4] for each pair of distinct points x and y in X, there exist distinct $\widetilde{g}\alpha$-open set A and B in X such that $x \in A$ and $y \in B$.

Theorem 3.14. If $f : (X, \tau) \to (Y, \sigma)$ is completely $\widetilde{g}\alpha$-irresolute injective function and Y is $\widetilde{g}\alpha$-T_2, then X is T_2.

Proof. Similar to the proof of Theorem 3.12. ■

Definition 3.15. A space X is said to be

(i) Nearly compact [16] if every regular open cover of X has a finite subcover.
(ii) Nearly countably compact [9] if every countable cover by regular open sets has a finite subcover.
(iii) Nearly Lindelof [8] if every cover of X by regular open sets has a countable subcover.
(iv) $\widetilde{g}\alpha$-compact if every $\widetilde{g}\alpha$-open cover of X has a finite subcover.
(v) countably $\widetilde{g}\alpha$-compact if every $\widetilde{g}\alpha$-open countable cover of X has a finite subcover.
(vi) $\widetilde{g}\alpha$-Lindelof if every cover of X by $\widetilde{g}\alpha$-open sets has a countable subcover.

Theorem 3.16. If $f : (X, \tau) \to (Y, \sigma)$ is completely $\widetilde{g}\alpha$-irresolute surjective function. Then the following statements hold:

(i) If X is nearly compact, then Y is $\widetilde{g}\alpha$-compact
(ii) If X is nearly Lindelof, then Y is $\widetilde{g}\alpha$-Lindelof
(i) If X is nearly countably compact, then Y is countably $\widetilde{g}\alpha$-compact

Proof. (i) Let $f : (X, \tau) \to (Y, \sigma)$ be a completely $\widetilde{g}\alpha$-irresolute function of nearly compact space X onto a space Y. Let $\{U_\alpha : \alpha \in \Delta\}$ be any $\widetilde{g}\alpha$-open cover of Y. Then, $\{f^{-1}(U_\alpha) : \alpha \in \Delta\}$ is a regular open cover of X. Since X is nearly compact, there exists a finite subfamily, $\{f^{-1}(U_{\alpha_i})/i = 1, 2, ...n\}$ of $\{f^{-1}(U_\alpha) : \alpha \in \Delta\}$ which cover X. It follows that $\{U_{\alpha_i} : i = 1, 2, ..n\}$ is a finite subfamily of $\{U_\alpha : \alpha \in \Delta\}$ which cover Y. Hence, space Y is a $\widetilde{g}\alpha$-compact. The proof of other cases are similar. ■

Definition 3.17. A space (X, τ) is said to be:

(i) S-closed [18] (resp. $\widetilde{g}\alpha$-closed compact) if every regular closed (resp. $\widetilde{g}\alpha$-closed) cover of X has a finite subcover.
(ii) countably S-closed compact [7] (resp. countably $\widetilde{g}\alpha$-closed compact) if every countable cover of X by regular closed (resp. $\widetilde{g}\alpha$-closed) sets has a finite subcover.
(iii) S-Lindelof [14] (resp. $\widetilde{g}\alpha$-closed Lindelof) if every cover of X by regular closed (resp. $\widetilde{g}\alpha$-closed) sets has a countable subcover.

Theorem 3.18. Let $f : (X, \tau) \to (Y, \sigma)$ be a completely $\widetilde{g}\alpha$-irresolute surjective function. Then the following statements hold:

(i) If X is S-closed, then Y is $\widetilde{g}\alpha$-closed compact

(ii) If X is S-Lindelof, then Y is $\widetilde{g}\alpha$-closed Lindelof

(iii) If X is countably S-closed-compact, then Y is countably $\widetilde{g}\alpha$-closed compact

Proof. It can be obtained similarly as the Theorem 3.16. ■

Theorem 3.19. The following hold for function $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$.

(i) If f is completely $\widetilde{g}\alpha$-irresolute and g is strongly $\widetilde{g}\alpha$-continuous, then $g \circ f$ is completely continuous.

(ii) If f is completely $\widetilde{g}\alpha$-irresolute and g is $\widetilde{g}\alpha$-irresolute, then $g \circ f$ is completely $\widetilde{g}\alpha$-irresolute.

(iii) If f is completely continuous and g is completely $\widetilde{g}\alpha$-irresolute functions, then $g \circ f$ is completely $\widetilde{g}\alpha$-irresolute.

Proof. It is obvious. ■

4. $\widetilde{G}\alpha$-Quotient function

Definition 4.1. A surjective function $f : (X, \tau) \to (Y, \sigma)$ is said to be a $\widetilde{g}\alpha$-quotient function if f is $\widetilde{g}\alpha$-continuous and $\widetilde{g}\alpha$-open.

Theorem 4.2. Every quotient function is $\widetilde{g}\alpha$-quotient function.

Proof. Follows from the definitions. ■

The following example shows that $\widetilde{g}\alpha$-quotient function need not be a quotient function in general.

Example 4.3. Let $X = \{a, b, c\} = Y$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a, b\}\}$. Clearly, the identity function $f : (X, \tau) \to (Y, \sigma)$ is $\widetilde{g}\alpha$-quotient but not quotient function.

Theorem 4.4. Let $f : (X, \tau) \to (Y, \sigma)$ be an open surjective $\widetilde{g}\alpha$-irresolute function and $g : (Y, \sigma) \to (Z, \eta)$ be a $\widetilde{g}\alpha$-quotient function. Then the composition $g \circ f : (X, \tau) \to (Z, \eta)$ is a $\widetilde{g}\alpha$-quotient function.

Proof. Let V be any open set in (Z, η). Then $g^{-1}(V)$ is a $\widetilde{g}\alpha$-open set, since g is a $\widetilde{g}\alpha$-quotient function. Since f is $\widetilde{g}\alpha$-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is a $\widetilde{g}\alpha$-open in X. This shows that $g \circ f$ is $\widetilde{g}\alpha$-continuous. Also, assume that V is open in (X, τ). Since f is open in (Y, σ). Then $g(f(V))$ is also open in (Z, η), because g is $\widetilde{g}\alpha$-quotient function. It follows that $(g \circ f)(V)$ is open in (Z, η). Therefore, $(g \circ f)(V)$ is $\widetilde{g}\alpha$-open in (Z, η). Thus, $(g \circ f)$ is $\widetilde{g}\alpha$-quotient function. ■

Theorem 4.5. If $h : (X, \tau) \to (Y, \sigma)$ is a $\widetilde{g}\alpha$-quotient function and g :

$(X,\tau) \to (Z,\eta)$is a continuous function where (Z,η) is a space that is constant on each set $h^{-1}(\{y\})$, for $y \in Y$, then g induces a $\widetilde{g}\alpha$-continuous function $f : (Y,\sigma) \to (Z,\eta)$ such that $f \circ h = g$.

Proof. Since g is constant on $h^{-1}(\{y\})$, for each $y \in Y$, the set $g(h^{-1}(\{y\}))$ is a point set in (Z,η). Let $f(y)$ denote this point $x \in X$, $f(h(x)) = g(x)$. We claim that f is $\widetilde{g}\alpha$-continuous. Let V be any open set on (Z,η), then $g^{-1}(V)$ is open, as g is continuous. But $g^{-1}(V) = h^{-1}(f^{-1}(V))$ is open in (X,τ). Since h is a $\widetilde{g}\alpha$-quotient function, $f(V)$ is $\widetilde{g}\alpha$-open in Y. ■

Definition 4.6. A surjective function $f : (X,\tau) \to (Y,\sigma)$ is said to be a strongly $\widetilde{g}\alpha$-quotient function if f is $\widetilde{g}\alpha$-continuous and quasi $\widetilde{g}\alpha$-open.

Theorem 4.7. Every strongly $\widetilde{g}\alpha$-quotient function is $\widetilde{g}\alpha$-quotient function.

Proof. It follows from the definitions. ■

The converse of the above Theorem need not be true by the following example.

Example 4.8. Let $X = \{a,b,c\} = Y$, $\tau = \{X,\phi,\{a\},\{b\},\{a,b\}\}$ and $\sigma = \{Y,\phi,\{a,b\}\}$. Clearly, the identity function $f : (X,\tau) \to (Y,\sigma)$ is $\widetilde{g}\alpha$-quotient but not strongly $\widetilde{g}\alpha$-quotient function.

Definition 4.9. A function $f : (X,\tau) \to (Y,\sigma)$ is called a completely $\widetilde{g}\alpha$-quotient function if f is $\widetilde{g}\alpha$-irresolute and quasi $\widetilde{g}\alpha$-open.

Theorem 4.10. Every completely $\widetilde{g}\alpha$-quotient function is strongly $\widetilde{g}\alpha$-quotient function.

Proof. Suppose V is an open set in Y, then it is a $\widetilde{g}\alpha$-open in Y. Since f is $\widetilde{g}\alpha$-irresolute, $f^{-1}(V)$ is a $\widetilde{g}\alpha$-open in X. Thus V is open in Y gives $f^{-1}(V)$ is a $\widetilde{g}\alpha$-open set in X. Suppose $f^{-1}(V)$ is a $\widetilde{g}\alpha$-open set in X. Since f is a completely $\widetilde{g}\alpha$-quotient function, V is open set in Y. Hence, f is strongly $\widetilde{g}\alpha$-quotient function. ■

Definition 4.11. A function $f : (X,\tau) \to (Y,\sigma)$ is called a pre $\widetilde{g}\alpha$-open [4] if the image of every $\widetilde{g}\alpha$-open set in X is an $\widetilde{g}\alpha$-open in Y.

Theorem 4.12. Let $f : (X,\tau) \to (Y,\sigma)$ be a surjective pre $\widetilde{g}\alpha$-open and $\widetilde{g}\alpha$-irresolute function and $g : (Y,\sigma) \to (Z,\eta)$ be a completely $\widetilde{g}\alpha$-quotient function. Then $g \circ f$ is completely $\widetilde{g}\alpha$-quotient function.

Proof. Let V be a $\widetilde{g}\alpha$-open set in Z. Then $g^{-1}(V)$ is a $\widetilde{g}\alpha$-open in Y because g is a completely $\widetilde{g}\alpha$-quotient function. We claim that $g \circ f$ is $\widetilde{g}\alpha$-irresolute. Since f is $\widetilde{g}\alpha$-irresolute, $f^{-1}(g^{-1}(V))$ is a $\widetilde{g}\alpha$-open set in X, that is $g \circ f$ is $\widetilde{g}\alpha$-irresolute. Suppose V be a $\widetilde{g}\alpha$-open set in X. Since f is pre $\widetilde{g}\alpha$-open , $f(V)$ is a $\widetilde{g}\alpha$-open in Y. Since g is completely $\widetilde{g}\alpha$-quotient function, $g(f(V))$ is open in Z. Therefore, $(g \circ f)(V)$ is open in Z. Hence, $g \circ f$ is completely $\widetilde{g}\alpha$-quotient function. ■

Theorem 4.13. Let $f : (X, \tau) \to (Y, \sigma)$ be a completely $\widetilde{g}\alpha$-quotient function and $g : (Y, \sigma) \to (Z, \eta)$ be a $\widetilde{g}\alpha$-irresolute, quasi $\widetilde{g}\alpha$-open, then $g \circ f$ is completely $\widetilde{g}\alpha$-quotient function.

Proof. Let V be a $\widetilde{g}\alpha$-open set in Z. Then $g^{-1}(V)$ is a $\widetilde{g}\alpha$-open in Y, $f^{-1}(g^{-1}(V))$ is a $\widetilde{g}\alpha$-open set in X. Hence $g \circ f$ is $\widetilde{g}\alpha$-irresolute. Assume that V be a $\widetilde{g}\alpha$-open in X. Since f is completely $\widetilde{g}\alpha$-quotient, $f(V)$ is open in Y. Implies that $f(V)$ is $\widetilde{g}\alpha$-open in Y. Then $g(f(V))$ is open in Z. Therefore, $g \circ f$ is completely $\widetilde{g}\alpha$-quotient function. ■

Corollary 4.14. Let $f : (X, \tau) \to (Y, \sigma)$ be a strongly $\widetilde{g}\alpha$-quotient function and $g : (Y, \sigma) \to (Z, \eta)$ be a strongly $\widetilde{g}\alpha$-quotient, then $g \circ f$ is strongly $\widetilde{g}\alpha$-quotient function.

Proof. It follows from the Theorem 4.13. ■

Theorem 4.15. If $f : (X, \tau) \to (Y, \sigma)$ be $\widetilde{g}\alpha$-quotient surjective function and X is $\widetilde{g}\alpha$-connected (resp. Y is $\widetilde{g}\alpha$-connected), then Y is connected (resp. X is connected).

Proof. Suppose that Y is not connected. Then there exist disjoint open sets A and B of Y such that $A \cup B = Y$. Since f is $\widetilde{g}\alpha$-quotient surjective, $f^{-1}(A)$ and $f^{-1}(B)$ are $\widetilde{g}\alpha$-open sets in X. Moreover, $f^{-1}(A) \cap f^{-1}(B) = X$, $f^{-1}(A) \neq \phi$ and $f^{-1}(B) \neq \phi$. This shows that X is not $\widetilde{g}\alpha$-connected, which is a contradiction to the assumption that X is $\widetilde{g}\alpha$-connected. By contradiction, Y is connected. ■

Theorem 4.16. If $f : (X, \tau) \to (Y, \sigma)$ is completely $\widetilde{g}\alpha$-quotient surjective function and X is $\widetilde{g}\alpha$-connected (resp. Y is connected), then Y is $\widetilde{g}\alpha$-connected (resp. X is $\widetilde{g}\alpha$-connected).

Proof. Suppose that Y is not $\widetilde{g}\alpha$-connected. Then there exist disjoint $\widetilde{g}\alpha$-open sets A and B of Y such that $A \cup B = Y$. Since f is completely $\widetilde{g}\alpha$-quotient surjective, $f^{-1}(A)$ and $f^{-1}(B)$ are $\widetilde{g}\alpha$-open sets in X. Moreover, $f^{-1}(A) \cap f^{-1}(B) = X$, $f^{-1}(A) \neq \phi$ and $f^{-1}(B) \neq \phi$. This shows that X is not $\widetilde{g}\alpha$-connected, which is a contradiction to the assumption that X is $\widetilde{g}\alpha$-connected. By contradiction, Y is $\widetilde{g}\alpha$-connected. ■

Theorem 4.17. If $f : (X, \tau) \to (Y, \sigma)$ is strongly $\widetilde{g}\alpha$-quotient surjective function and X is $\widetilde{g}\alpha$-connected (resp. Y is connected), then Y is connected (resp. X is $\widetilde{g}\alpha$-connected).

Proof. Suppose that Y is not connected. Then there exist disjoint open sets A and B of Y such that $A \cup B = Y$. Since f is strongly $\widetilde{g}\alpha$-quotient surjective function, $f^{-1}(A)$ and $f^{-1}(B)$ are $\widetilde{g}\alpha$-open sets in X. Moreover, $f^{-1}(A) \cap f^{-1}(B) = X$, $f^{-1}(A) \neq \phi$ and $f^{-1}(B) \neq \phi$. This shows that X is not $\widetilde{g}\alpha$-connected, which is a contradiction to the assumption that X is $\widetilde{g}\alpha$-connected. By contradiction, Y is connected. ■

References

[1] S. P. Arya and R. Gupta, *On strongly continuous maps*, Kyungpook Math. J., 14 (1974), 131–143.

[2] S. G. Crossley and S. K. Hildebrand, *Semi Topological properties*, Fund. Math., 74 (1972), 233–254.

[3] R. Devi and A. Selvakumar, *Quasi* $\widetilde{g}\alpha$*-open function*, (submitted).

[4] R. Devi, A. Selvakumar and M. Caldas, *Contra* $\widetilde{g}\alpha$*-continuous functions*, (submitted).

[5] R. Devi, A. Selvakumar and S. Jafari, *Applications of* $\widetilde{g}\alpha$*-closed sets*, (submitted).

[6] R. Devi, A. Selvakumar and S. Jafari, $\widetilde{g}\alpha$*-closed sets in topological spaces*, (submitted).

[7] K. Dlaska, N. Ergun and M. Ganster, *Countable* S*-closed spaces*, Math. Slovaca, 44 (1994), 337–348.

[8] E. Ekici, *Generalization of perfectly continuous, Regular set-connected and clopen functions*, Acta Math. Hungar., 107 (3)(2005), 193–206.

[9] N. Ergun, *On nearly paracompact spaces*, Intanbul. Univ. Fen. Mec. Ser. A, 45 (1980), 65–87.

[10] N. Levine, *Generalized closed sets in topology*, Rend. Circ. Mat. Palermo, 19 (1970), 89–96.

[11] N. Levine, *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly, 70 (1963), 36–41.

[12] N. Levine, *Strong continuity in topological spaces*, Amer. Math. Monthly, 67 (1960), 269.

[13] P. E. Long and L. L. Herrington, *Basic properties of regular-closed functions*, Rend. Circ. Mat. Palermo, 27 (1978), 20–28.

[14] G. D. Maio, S*-closed spaces,* S*-sets and* S*-continuous functions*, Acad. Sci. Torino, 118 (1984), 125–134.

[15] O. Njastad, *On some classes of nearly open sets*, Pacific J. Math., 15(1965), 961–970.

[16] T. Noiri and G. D. Maio, *Properties of* α*-compact spaces*, Rend. Circ. Mat. Palermo. Ser(II), (1998), 359–369.

[17] M. H. Stone, *Applications of the theory Boolean rings to general topology*, Trans. Amer. Math. Soc., 41 (1937), 375–381.

[18] T. Thomson, S*-closed spaces*, Proc. Amer. Math. Soc., 60 (1976), 335–338.

[19] M.K.R.S. Veera kumar, *Between g^*-closed sets and g-closed sets*, Antartica J. Math., 3(1)(2006), 43–65.

[20] M.K.R.S. Veera kumar, *$^\sharp g$-semi-closed sets in topological spaces*, Antartica J. Math., 2:2(2005), 201–222.

[21] M.K.R.S. Veera Kumar, *$\widehat{g}$-locally closed sets and $\widehat{G}LC$-functions*, Indian J. Math., 43(2)(2001), 231–247.

2014 INTERNATIONAL CONFERENCE
on TOPOLOGY and its APPLICATIONS

July 3-7, 2014 Nafpaktos, Greece

Homepage: http://www.lepantotopology.gr

E-mail: lepanto@math.upatras.gr

Organizing committee:

- S. D. Iliadis (Moscow State University (M.V. Lomonosov)) Chairman
- D. N. Georgiou (University of Patras)
- I. E. Kougias (Technological Educational Institute of Western Greece)
- A. C. Megaritis (Technological Educational Institute of Western Greece)
- I. Boules (Mayor of the city of Nafpaktos)

Department of Mathematics
University of Patras,

Computer and Informatics Engineering Department,
Technological Educational Institute of Western Greece

Municipality
of Nafpaktos

ISBN: 978-618-80609-1-3