

On the various Ramanujan equations (Rogers-Ramanujan continued fractions) linked to some sectors of String Theory and Particle Physics: Further new possible mathematical connections VI.

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Abstract

In this research thesis, we have analyzed and deepened further Ramanujan expressions applied to some sectors of String Theory and Particle Physics. We have therefore described other new possible mathematical connections.

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<https://www.sciencephoto.com/media/228058/view/indian-mathematician-srinivasa-ramanujan>

We have that:

From:

On Classical Stability with Broken Supersymmetry

I. Basile, J. Mourad and A. Sagnotti - arXiv:1811.11448v2 [hep-th] 10 Jan 2019

Scalar perturbations

$$(\ell \geq 2)$$

In order to refer to the BF bound in Appendix C one should add 6 to these expressions and compare the result with -4 . All in all, *there are no modes below the BF bound in this sector*. The vector modes are massive for $\ell > 1$ in the region $\sigma_7 > 3$, while they become massless for $\ell = 1$ and all allowed values of $\sigma_7 > 3$, and for all values of ℓ in the singular limit $\sigma_7 = 3$, which would correspond to a three-sphere of infinite radius. All in all, for $\ell = 1$ there are 6 massless vectors arising from one of the two eigenvalues above in the heterotic vacuum. According to Appendix

$$L_7 = \ell(\ell + 2) . \quad (4.27)$$

$$L_7 = 2(2+2) = 8$$

In most of the parameter space, two eigenvalues are not problematic, but there is one bad eigenvalue in the tree-level heterotic potential, which corresponds to $\sigma_7 = 15$ and $\tau_7 = 75$. It obtains for $\ell = 1$ and $k = 0$ from

$$4 \left[16 + 3 L_7 - 4 \sqrt{34 + 15 L_7} \cos \left(\frac{\delta - 2\pi k}{3} \right) \right] \quad (k = 0, 1, 2) , \quad (4.34)$$

where

$$\delta = \text{Arg} \left(152 - 45L_7 + 3i \sqrt{3(5L_7 + 3)[(5L_7 + 14)^2 + 4]} \right) . \quad (4.35)$$

Still, there is again a stability region for values of σ_7 that are close to 12, for negative V_0 , and typically for positive τ_7 , i.e. for potentials that are convex close to the vacuum configuration.. These results are displayed in figs. 5 and 6.

$$4 \left[16 + 3 L_7 - 4 \sqrt{34 + 15 L_7} \cos \left(\frac{\delta - 2\pi k}{3} \right) \right] \quad (k = 0, 1, 2)$$

$$\delta = \text{Arg} \left(152 - 45L_7 + 3i \sqrt{3(5L_7 + 3)[(5L_7 + 14)^2 + 4]} \right)$$

We obtain:

$$\text{arg}(152-45*8+3i*\text{sqrt}((((3(5*8+3))((5*8+14)^2+4))))))$$

Input:

$$\arg\left(152 - 45 \times 8 + 3i \sqrt{3(5 \times 8 + 3)((5 \times 8 + 14)^2 + 4)}\right)$$

$\arg(z)$ is the complex argument

i is the imaginary unit

Exact result:

$$\pi - \tan^{-1}\left(\frac{3\sqrt{\frac{47085}{2}}}{52}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Decimal approximation:

1.683287508359747777680923835832472202981834257024700684395...

$$1.68328750835\dots = \delta$$

Alternate form:

$$\pi - \frac{1}{2}i \log\left(1 - \frac{3}{52}i \sqrt{\frac{47085}{2}}\right) + \frac{1}{2}i \log\left(1 + \frac{3}{52}i \sqrt{\frac{47085}{2}}\right)$$

$\log(x)$ is the natural logarithm

Alternative representations:

$$\arg\left(152 - 45 \times 8 + 3i \sqrt{3(5 \times 8 + 3)((5 \times 8 + 14)^2 + 4)}\right) = -i \log\left(\operatorname{sgn}\left(-208 + 3i \sqrt{129(4 + 54^2)}\right)\right)$$

$$\arg\left(152 - 45 \times 8 + 3i \sqrt{3(5 \times 8 + 3)((5 \times 8 + 14)^2 + 4)}\right) = i \left(\log\left(\left|-208 + 3i \sqrt{129(4 + 54^2)}\right|\right) - \log\left(-208 + 3i \sqrt{129(4 + 54^2)}\right)\right)$$

$$\arg\left(152 - 45 \times 8 + 3i \sqrt{3(5 \times 8 + 3)((5 \times 8 + 14)^2 + 4)}\right) = -i \log\left(\frac{-208 + 3i \sqrt{129(4 + 54^2)}}{\left|-208 + 3i \sqrt{129(4 + 54^2)}\right|}\right)$$

Series representations:

$$\arg\left(152 - 45 \times 8 + 3i \sqrt{3(5 \times 8 + 3)((5 \times 8 + 14)^2 + 4)}\right) = \frac{\pi}{2} + \sum_{k=0}^{\infty} \frac{(-1)^k 2^{2+4k+1/2(1+2k)} \times 3^{-1+1/2(-1-2k)-2k} \times 13^{1+2k} \times 15695^{1/2(-1-2k)}}{1+2k}$$

$$\arg\left(152 - 45 \times 8 + 3i \sqrt{3(5 \times 8 + 3)((5 \times 8 + 14)^2 + 4)}\right) = \frac{\pi}{2} + \sum_{k=0}^{\infty} \frac{(-1)^k 2^{5/2+5k} \times 3^{-3/2-3k} \times 13^{1+2k} \times 15695^{-1/2-k}}{1+2k}$$

$$\arg\left(152 - 45 \times 8 + 3i \sqrt{3(5 \times 8 + 3)((5 \times 8 + 14)^2 + 4)}\right) = \pi - \sum_{k=0}^{\infty} \frac{1}{1+2k} (-1)^k 2^{-1+1/2(-1-2k)-2k} \times 3^{1+2k+1/2(1+2k)} \times 5^{-k+1/2(1+2k)} \times 3139^{1/2(1+2k)} \left(13 \left(1 + \frac{\sqrt{\frac{86105}{2}}}{26}\right)\right)^{-1-2k} F_{1+2k}$$

F_n is the n^{th} Fibonacci number

Integral representations:

$$\arg\left(152 - 45 \times 8 + 3i \sqrt{3(5 \times 8 + 3)((5 \times 8 + 14)^2 + 4)}\right) = \pi - \frac{3 \sqrt{\frac{47085}{2}}}{52} \int_0^1 \frac{1}{1 + \frac{423765 t^2}{5408}} dt$$

$$\arg\left(152 - 45 \times 8 + 3i \sqrt{3(5 \times 8 + 3)((5 \times 8 + 14)^2 + 4)}\right) = \pi + \frac{3i \sqrt{\frac{47085}{2}}}{208 \pi^{3/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \left(\frac{5408}{429173}\right)^s \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s)^2 ds \text{ for } 0 < \gamma < \frac{1}{2}$$

$$\arg\left(152 - 45 \times 8 + 3i \sqrt{3(5 \times 8 + 3)((5 \times 8 + 14)^2 + 4)}\right) = \pi + \frac{3i \sqrt{\frac{47085}{2}}}{208 \pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(\frac{5408}{423765}\right)^s \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s)}{\Gamma\left(\frac{3}{2} - s\right)} ds \text{ for } 0 < \gamma < \frac{1}{2}$$

$\Gamma(x)$ is the gamma function

We note that from the following equation (see eq. (6.19) **Integrable Scalar Cosmologies I. Foundations and links with String Theory**-P. Fré , A. Sagnotti and A.S. Sorin - arXiv:1307.1910v3 [hep-th] 16 Oct 2013)

$$\gamma_9 = \sqrt{\frac{d^2 - 14d + 184}{24(d - 4)}}$$

concerning the orientifold Vacua and exponential potentials, we obtain, for $d = 6$:

$$\text{sqrt}(((6^2-14*6+184)/(24(6-4))))$$

Input:

$$\sqrt{\frac{6^2 - 14 \times 6 + 184}{24(6 - 4)}}$$

Result:

$$\sqrt{\frac{17}{6}}$$

Decimal approximation:

1.683250823060346325560564319511600118433983160746602156975...

1.68325082306...

Alternate form:

$$\frac{\sqrt{102}}{6}$$

The two results are very near: 1.68328750835... and 1.68325082306...

Now, from:

$$4 \left[16 + 3 L_7 - 4 \sqrt{34 + 15 L_7} \cos \left(\frac{\delta - 2\pi k}{3} \right) \right]$$

For $k = 2$ and $L_7 = 8$, we obtain:

$$4((((16+3*8-4*\sqrt{34+15*8}) \cos ((1.68328750835 - 4\pi)/3))))$$

Input interpretation:

$$4\left(16 + 3 \times 8 - \left(4 \sqrt{34 + 15 \times 8}\right) \cos\left(\frac{1}{3} (1.68328750835 - 4 \pi)\right)\right)$$

Result:

335.5543483878...

335.5543483878... result practically equal to the value of $f_0(500)$ scalar meson BREIT-WIGNER width 335 ± 67 MeV

Alternative representations:

$$4\left(16 + 3 \times 8 - \cos\left(\frac{1}{3} (1.683287508350000 - 4 \pi)\right) 4 \sqrt{34 + 15 \times 8}\right) = 4\left(40 - 4 \cosh\left(\frac{1}{3} i (1.683287508350000 - 4 \pi)\right) \sqrt{154}\right)$$

$$4\left(16 + 3 \times 8 - \cos\left(\frac{1}{3} (1.683287508350000 - 4 \pi)\right) 4 \sqrt{34 + 15 \times 8}\right) = 4\left(40 - 4 \cosh\left(-\frac{1}{3} i (1.683287508350000 - 4 \pi)\right) \sqrt{154}\right)$$

$$4\left(16 + 3 \times 8 - \cos\left(\frac{1}{3} (1.683287508350000 - 4 \pi)\right) 4 \sqrt{34 + 15 \times 8}\right) = 4\left(40 - \frac{4 \sqrt{154}}{\sec\left(\frac{1}{3} (1.683287508350000 - 4 \pi)\right)}\right)$$

Series representations:

$$4\left(16 + 3 \times 8 - \cos\left(\frac{1}{3} (1.683287508350000 - 4 \pi)\right) 4 \sqrt{34 + 15 \times 8}\right) = -16 \left(-10 + J_0(0.561095836116667 - 1.333333333333333 \pi) \sqrt{153} \sum_{k=0}^{\infty} 153^{-k} \binom{\frac{1}{2}}{k} + 2 \sqrt{153} \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} (-1)^{k_1} 153^{-k_2} J_{2k_1}(0.561095836116667 - 1.333333333333333 \pi) \binom{\frac{1}{2}}{k_2} \right)$$

$$4 \left(16 + 3 \times 8 - \cos \left(\frac{1}{3} (1.683287508350000 - 4\pi) \right) 4 \sqrt{34 + 15 \times 8} \right) =$$

$$-16 \left(-10 + \exp \left(i\pi \left\lfloor \frac{\arg(154 - x)}{2\pi} \right\rfloor \right) \sqrt{x} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 9^{-k_1} (1.683287508350000 - 4\pi)^{2k_1} (154 - x)^{k_2} x^{-k_2} \left(-\frac{1}{2}\right)_{k_2}}{(2k_1)! k_2!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$4 \left(16 + 3 \times 8 - \cos \left(\frac{1}{3} (1.683287508350000 - 4\pi) \right) 4 \sqrt{34 + 15 \times 8} \right) =$$

$$29.333333333333333$$

$$\left(5.45454545454545 + 1.000000000000000 \exp \left(i\pi \left\lfloor \frac{\arg(154 - x)}{2\pi} \right\rfloor \right) \sqrt{x} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{(1+2k_1)! k_2!} (-1)^{k_1+k_2} e^{1.212271607140631k_1} (0.3060522742454545 - \pi)^{1+2k_1} (154 - x)^{k_2} x^{-k_2} \left(-\frac{1}{2}\right)_{k_2} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$J_n(z)$ is the Bessel function of the first kind

$\binom{n}{m}$ is the binomial coefficient

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

\mathbb{R} is the set of real numbers

Integral representations:

$$4 \left(16 + 3 \times 8 - \cos \left(\frac{1}{3} (1.683287508350000 - 4\pi) \right) 4 \sqrt{34 + 15 \times 8} \right) =$$

$$160 + 16 \sqrt{154} \int_{\frac{\pi}{2}}^{0.561095836116667 - 1.33333333333333\pi} \sin(t) dt$$

$$4 \left(16 + 3 \times 8 - \cos \left(\frac{1}{3} (1.683287508350000 - 4\pi) \right) 4 \sqrt{34 + 15 \times 8} \right) =$$

$$160 + \int_0^1 (8.97753337786667 - 21.3333333333333\pi) \sin(-1.33333333333333$$

$$(-0.4208218770875000 + \pi)t) \sqrt{154} dt - 16 \sqrt{154}$$

$$4 \left(16 + 3 \times 8 - \cos \left(\frac{1}{3} (1.683287508350000 - 4\pi) \right) 4 \sqrt{34 + 15 \times 8} \right) = 160 -$$

$$\frac{8 \sqrt{154} \sqrt{\pi}}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-(0.444444444444444 (-0.4208218770875000+\pi)^2)/s+s}}{\sqrt{s}} ds \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$4 \left(16 + 3 \times 8 - \cos \left(\frac{1}{3} (1.683287508350000 - 4\pi) \right) 4 \sqrt{34 + 15 \times 8} \right) =$$

$$4 \left(40 - 4 (-1 + 2 \cos^2(0.2805479180583333 - 0.6666666666666667 \pi)) \sqrt{154} \right)$$

$$4 \left(16 + 3 \times 8 - \cos \left(\frac{1}{3} (1.683287508350000 - 4\pi) \right) 4 \sqrt{34 + 15 \times 8} \right) =$$

$$4 \left(40 + 4 (-1 + 2 \sin^2(0.2805479180583333 - 0.6666666666666667 \pi)) \sqrt{154} \right)$$

$$4 \left(16 + 3 \times 8 - \cos \left(\frac{1}{3} (1.683287508350000 - 4\pi) \right) 4 \sqrt{34 + 15 \times 8} \right) =$$

$$4 \left(40 - 4 \cos(0.1870319453722222 - 0.4444444444444444 \pi) \right.$$

$$\left. (-3 + 4 \cos^2(0.1870319453722222 - 0.4444444444444444 \pi)) \sqrt{154} \right)$$

From Wikipedia

It is most often used to model resonances (unstable particles) in high-energy physics. In this case, E is the center-of-mass energy that produces the resonance, M is the mass of the resonance, and Γ is the resonance width (or decay width), related to its mean lifetime according to τ = 1/Γ. (With units included, the formula is τ = ħ/Γ.)

The relativistic Breit–Wigner distribution (after the 1936 nuclear resonance formula of Gregory Breit and Eugene Wigner) is a continuous probability distribution with the following probability density function,

$$f(E) = \frac{k}{(E^2 - M^2)^2 + M^2\Gamma^2},$$

where k is a constant of proportionality, equal to

$$k = \frac{2\sqrt{2}M\Gamma\gamma}{\pi\sqrt{M^2 + \gamma^2}} \quad \text{with} \quad \gamma = \sqrt{M^2 (M^2 + \Gamma^2)}.$$

$$\gamma = \text{sqrt}(\text{((((512-188i)^2((512-188i)^2+(335.5543483878)^2))))))$$

Input interpretation:

$$\sqrt{(512 - 188 i)^2 ((512 - 188 i)^2 + 335.5543483878^2)}$$

Result:

1.001836045330590... +
0.0006692729843439119... *i*

Polar coordinates:

$r = 1.001836268883276$ (radius), $\theta = 0.03827623473846^\circ$ (angle)

1.001836268883276....result practically equal to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5} - \varphi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}} \approx 1.0018674362$$

$$k = (((2\sqrt{2}*(512-188i)* 335.5543483878*340702.0662294)))/((((Pi*\sqrt{((512-188i)^2)+ 340702.0662294)}))))$$

Input interpretation:

$$\frac{2\sqrt{2} (512 - 188 i) \times 335.5543483878 \times 340\,702.0662294}{\pi \sqrt{(512 - 188 i)^2 + 340\,702.0662294}}$$

i is the imaginary unit

Result:

7.123701838923... $\times 10^7$ -
1.358073943236... $\times 10^7$ *i*

Polar coordinates:

$r = 7.25199922264 \times 10^7$ (radius), $\theta = -10.7934432565^\circ$ (angle)

$7.25199922264 \times 10^7 = k$

Series representations:

$$\frac{2(\sqrt{2} (512 - i 188) 335.55434838780000 \times 340 702.06622940000)}{\pi \sqrt{(512 - i 188)^2 + 340 702.06622940000}} =$$

$$\left(1.0000000000000000 \left(1.17067837263855 \times 10^{11} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2 - z_0)^k z_0^{-k}}{k!} - 4.2985846495322 \times 10^{10} i \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2 - z_0)^k z_0^{-k}}{k!} \right) \right) /$$

$$\left(\pi \sum_{k=0}^{\infty} \frac{1}{k!} (-35 344.0000000000000)^k \left(-\frac{1}{2}\right)_k (17.056531977970801 - 5.4468085106382979 i + i^2 - 0.000028293345405160706 z_0)^k z_0^{-k} \right) \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{2(\sqrt{2} (512 - i 188) 335.55434838780000 \times 340 702.06622940000)}{\pi \sqrt{(512 - i 188)^2 + 340 702.06622940000}} =$$

$$\left(1.0000000000000000 \left(1.1706783726385 \times 10^{11} \exp\left(\pi \mathcal{A} \left[\frac{\arg(2 - x)}{2\pi} \right]\right) \sum_{k=0}^{\infty} \frac{(-1)^k (2 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} - 4.2985846495322 \times 10^{10} i \exp\left(\pi \mathcal{A} \left[\frac{\arg(2 - x)}{2\pi} \right]\right) \sum_{k=0}^{\infty} \frac{(-1)^k (2 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) /$$

$$\left(\pi \exp\left(\pi \mathcal{A} \left[\frac{1}{2\pi} \arg(602 846.0662294000 - 192 512.0000000000000 i + 35 344.0000000000000 i^2 - 1.0000000000000000 x) \right]\right) \sum_{k=0}^{\infty} \frac{1}{k!} (-35 344.0000000000000)^k (17.056531977970801 - 5.4468085106382979 i + i^2 - 0.000028293345405160706 x)^k x^{-k} \left(-\frac{1}{2}\right)_k \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\begin{aligned}
& \frac{2(\sqrt{2}(512 - i188)335.55434838780000 \times 340702.06622940000)}{\pi \sqrt{(512 - i188)^2 + 340702.06622940000}} = \\
& \left(1.0000000000000000 \right. \\
& \left. \left(\frac{1}{z_0} \right)^{1/2 [\arg(2-z_0)/(2\pi)] - 1/2 [\arg(340702.06622940000 + (512 - 188i)^2 - z_0)/(2\pi)]} \right. \\
& \left. z_0^{-1/2 [\arg(340702.06622940000 + (512 - 188i)^2 - z_0)/(2\pi)]} \right. \\
& \left. \left(1.1706783726385500 \times 10^{11} z_0^{1/2 [\arg(2-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2 - z_0)^k z_0^{-k}}{k!} - \right. \right. \\
& \left. \left. 4.2985846495321757 \times 10^{10} i z_0^{1/2 [\arg(2-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2 - z_0)^k z_0^{-k}}{k!} \right) \right) / \left(\pi \sum_{k=0}^{\infty} \frac{1}{k!} (-35344.000000000000)^k \right. \\
& \left. \left(-\frac{1}{2}\right)_k (17.056531977970801 - 5.4468085106382979 i + \right. \\
& \left. i^2 - 0.000028293345405160706 z_0)^k z_0^{-k} \right)
\end{aligned}$$

We note that:

$$\left(\left(\left(\left(\left(\left(2\sqrt{2}(512-188i) \cdot 335.5543483878 \cdot 340702.0662294 \right) \right) \right) \right) \right) \right) / \left(\left(\left(\left(\left(\left(\pi \cdot \sqrt{((512-188i)^2 + 340702.0662294)} \right) \right) \right) \right) \right) \right) \right)^{1/3 + 76 + 4 + 1/\text{golden ratio}}$$

Where 76 and 4 are Lucas numbers

Input interpretation:

$$\sqrt[3]{\frac{2\sqrt{2}(512 - 188i) \times 335.5543483878 \times 340702.0662294}{\pi \sqrt{(512 - 188i)^2 + 340702.0662294}}} + 76 + 4 + \frac{1}{\phi}$$

i is the imaginary unit
 ϕ is the golden ratio

Result:

496.8120170605... -
26.16876719114... *i*

Polar coordinates:

r = 497.5007383633 (radius), $\theta = -3.01517579495^\circ$ (angle)

497.5007383633 result practically equal to the rest mass of Kaon meson 497.614

Series representations:

$$\sqrt[3]{\frac{2(\sqrt{2}(512 - i188)335.55434838780000 \times 340702.06622940000)}{\pi \sqrt{(512 - i188)^2 + 340702.06622940000}} + 76 + 4 + \frac{1}{\phi}} =$$

$$\frac{1}{\phi} 970.679538934413 \left(0.001030206118383595 + 0.0824164894706876 \phi + \right.$$

$$1.0000000000000000 \phi \left(- \left((-128 + 47i) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2 - z_0)^k z_0^{-k}}{k!} \right) / \right.$$

$$\left. \left(\pi \sum_{k=0}^{\infty} \frac{1}{k!} (-35344.00000000000000)^k \left(-\frac{1}{2}\right)_k \right. \right.$$

$$\left. \left. (17.056531977970801 - 5.4468085106382979i + i^2 - \right. \right.$$

$$\left. \left. 0.000028293345405160706 z_0)^k z_0^{-k} \right) \right) \wedge$$

$$(1/3) \left. \right) \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\sqrt[3]{\frac{2(\sqrt{2}(512 - i188)335.55434838780000 \times 340702.06622940000)}{\pi \sqrt{(512 - i188)^2 + 340702.06622940000}} + 76 + 4 + \frac{1}{\phi}} =$$

$$\frac{1}{\phi} 970.679538934413$$

$$\left(0.001030206118383595 + 0.0824164894706876 \phi + 1.0000000000000000 \phi \right.$$

$$\left(- \left((-128 + 47i) \exp\left(\pi \mathcal{A} \left[\frac{\arg(2 - x)}{2\pi} \right]\right) \sum_{k=0}^{\infty} \frac{(-1)^k (2 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \right.$$

$$\left(\pi \exp\left(\pi \mathcal{A} \left[\frac{\arg(340702.06622940000 + (512 - 188i)^2 - x)}{2\pi} \right]\right) \right.$$

$$\left. \sum_{k=0}^{\infty} \frac{1}{k!} (-35344.00000000000000)^k \right.$$

$$\left. \left. (17.056531977970801 - 5.4468085106382979i + \right. \right.$$

$$\left. \left. i^2 - 0.000028293345405160706 x)^k \right. \right.$$

$$\left. \left. x^{-k} \left(-\frac{1}{2}\right)_k \right) \right) \wedge (1/3) \left. \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\sqrt[3]{\frac{2(\sqrt{2}(512-i188)335.55434838780000 \times 340702.06622940000)}{\pi \sqrt{(512-i188)^2 + 340702.06622940000}} + 76 + 4 + \frac{1}{\phi}} + \frac{1}{\phi} 970.679538934413 \left(0.001030206118383595 + 0.0824164894706876 \phi + 1.0000000000000000 \phi \left(- \left(\left((-128 + 47i) \left(\frac{1}{z_0} \right)^{1/2 [\arg(2-z_0)/(2\pi)] - 1/2 [\arg(340702.06622940000 + (512-188i)^2 - z_0)/(2\pi)]} \right)^{1/2 [\arg(2-z_0)/(2\pi)] - 1/2 [\arg(340702.06622940000 + (512-188i)^2 - z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) / \left(\pi \sum_{k=0}^{\infty} \frac{1}{k!} (-35344.000000000000)^k \left(-\frac{1}{2}\right)_k (17.056531977970801 - 5.4468085106382979i + i^2 - 0.000028293345405160706 z_0)^k z_0^{-k} \right) \right) \right)^{(1/3)}$$

And:

$$1 + (76/10^2) * 1 / \left(\left(\left(\left(\left(\left(\left(\left(2\sqrt{2}(512-188i) * 335.5543483878 * 340702.0662294 \right) \right) \right) \right) \right) \right) \right) / \left(\left(\left(\left(\left(\left(\left(\pi * \sqrt{((512-188i)^2 + 340702.0662294)} \right) \right) \right) \right) \right) \right) \right) \right) \right)^{1/3}$$

where 76 is a Lucas number

Input interpretation:

$$1 + \frac{76}{10^2} \times \frac{1}{\sqrt[3]{\frac{2\sqrt{2}(512-188i) \times 335.5543483878 \times 340702.0662294}{\pi \sqrt{(512-188i)^2 + 340702.0662294}}}}$$

i is the imaginary unit

Result:

$$1.00181888073755... + 0.000114364619637493... i$$

Polar coordinates:

$$r = 1.001818887265311 \text{ (radius), } \theta = 0.00654071322509^\circ \text{ (angle)}$$

1.001818887265311 result practically equal to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}}} \approx 1.0018674362$$

Series representations:

$$1 + \frac{76}{\sqrt[3]{\frac{2\sqrt{2}(512-i188)335.55434838780000 \times 340702.06622940000}{\pi \sqrt{(512-i188)^2 + 340702.06622940000}} 10^2}}} =$$

$$\left(1.0000000000000000 \left(-2.723404255319149 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} + \right. \right.$$

$$1.0000000000000000 i \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} -$$

$$0.00001665865212705388 \pi \left(-\left((-128 + 47i) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) / \right.$$

$$\left. \left(\pi \sum_{k=0}^{\infty} \frac{1}{k!} (-35344.000000000000)^k \left(-\frac{1}{2}\right)_k \right. \right.$$

$$\left. \left. (17.056531977970801 - 5.4468085106382979 \right. \right.$$

$$\left. \left. \left. \left. \left. \left. i + i^2 - 0.000028293345405160706 z_0 \right)^k \right. \right. \right. \right. \right. \left. \left. \left. \left. \left. \left. z_0^{-k} \right) \right) \right) \right) \right) \sum_{k=0}^{\infty} \frac{1}{k!} (-35344.000000000000)^k \right.$$

$$\left. \left. \left. \left. \left. \left. \left(-\frac{1}{2}\right)_k (17.056531977970801 - 5.4468085106382979 i + \right. \right. \right. \right. \right. \right. \left. \left. \left. \left. \left. \left. i^2 - 0.000028293345405160706 z_0 \right)^k z_0^{-k} \right) \right) \right) \right) \right) /$$

$$\left((-2.72340425531915 + 1.0000000000000000 i) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right)$$

for not ((z₀ ∈ ℝ and -∞ < z₀ ≤ 0))

$$\begin{aligned}
& 1 + \frac{76}{\sqrt[3]{\frac{2\sqrt{2}(512-i188)335.55434838780000 \times 340702.06622940000}{\pi \sqrt{(512-i188)^2 + 340702.06622940000}}}} 10^2 = \left(1.0000000000000000 \right. \\
& \left. \left(-2.723404255319149 \exp\left(\pi \mathcal{A} \left[\frac{\arg(2-x)}{2\pi} \right] \right) \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \right. \\
& \quad 1.0000000000000000 i \exp\left(\pi \mathcal{A} \left[\frac{\arg(2-x)}{2\pi} \right] \right) \\
& \quad \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} - 0.00001665865212705388 \pi \\
& \quad \left. \exp\left(\pi \mathcal{A} \left[\frac{\arg(340702.06622940000 + (512-188i)^2 - x)}{2\pi} \right] \right) \right) \\
& \left(\left((-128 + 47i) \exp\left(\pi \mathcal{A} \left[\frac{\arg(2-x)}{2\pi} \right] \right) \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) / \\
& \quad \left(\pi \exp\left(\pi \mathcal{A} \left[\frac{1}{2\pi} \arg(340702.06622940000 + (512-188 \right. \right. \right. \\
& \quad \quad \quad \left. \left. \left. i)^2 - x) \right] \right) \sum_{k=0}^{\infty} \frac{1}{k!} (-35344.000000000000)^k \right. \\
& \quad \quad \quad (17.056531977970801 - 5.4468085106382979 \\
& \quad \quad \quad \left. i + i^2 - 0.000028293345405160706 x)^k x^{-k} \right. \\
& \quad \quad \quad \left. \left. \left. \left(-\frac{1}{2}\right)_k \right) \right) \right) \sum_{k=0}^{\infty} \frac{1}{k!} (-35344.000000000000)^k \\
& \quad \quad \quad (17.056531977970801 - 5.4468085106382979 i + i^2 - \\
& \quad \quad \quad 0.000028293345405160706 x)^k x^{-k} \left(-\frac{1}{2}\right)_k \Big) / \\
& \left((-2.72340425531915 + 1.0000000000000000 i) \right. \\
& \quad \exp\left(\right. \\
& \quad \quad \pi \\
& \quad \quad \quad \mathcal{A} \\
& \quad \quad \quad \left. \left[\frac{\arg(2-x)}{2\pi} \right] \right) \\
& \quad \quad \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \\
& \text{for} \\
& (x \in \\
& \quad \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& 1 + \frac{76}{\sqrt[3]{\frac{2\sqrt{2}(512-i188)335.55434838780000 \times 340702.06622940000}{\pi \sqrt{(512-i188)^2 + 340702.06622940000}}}} = \\
& \left(1.0000000000000000 \left(\frac{1}{z_0}\right)^{-1/2 [\arg(2-z_0)/(2\pi)]} \right. \\
& \quad z_0^{-1/2 [\arg(2-z_0)/(2\pi)]} \left[-2.723404255319149 \left(\frac{1}{z_0}\right)^{1/2 [\arg(2-z_0)/(2\pi)]} \right. \\
& \quad \quad z_0^{1/2 [\arg(2-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} + \\
& \quad 1.0000000000000000 i \left(\frac{1}{z_0}\right)^{1/2 [\arg(2-z_0)/(2\pi)]} z_0^{1/2 [\arg(2-z_0)/(2\pi)]} \\
& \quad \quad \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} - 0.00001665865212705388 \\
& \quad \quad \pi \left(\frac{1}{z_0}\right)^{1/2 [\arg(340702.06622940000 + (512-188i)^2 - z_0)/(2\pi)]} \\
& \quad \quad z_0^{1/2 [\arg(340702.06622940000 + (512-188i)^2 - z_0)/(2\pi)]} \left[- \left((-128 + 47i) \right. \right. \\
& \quad \quad \left. \left. \left(\frac{1}{z_0}\right)^{1/2 [\arg(2-z_0)/(2\pi)] - 1/2 [\arg(340702.06622940000 + (512-188i)^2 - z_0)/(2\pi)]} \right. \right. \\
& \quad \quad z_0^{1/2 [\arg(2-z_0)/(2\pi)] - 1/2 [\arg(340702.06622940000 + (512-188i)^2 - z_0)/(2\pi)]} \\
& \quad \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) \right] / \\
& \quad \left(\pi \sum_{k=0}^{\infty} \frac{1}{k!} (-35344.00000000000000)^k \left(-\frac{1}{2}\right)_k \right. \\
& \quad \quad (17.056531977970801 - 5.4468085106382979 i + \\
& \quad \quad \quad i^2 - 0.000028293345405160706 z_0)^k \\
& \quad \quad \left. \left. z_0^k \right) \right]^{2/3} \sum_{k=0}^{\infty} \frac{1}{k!} (-35344.00000000000000)^k \\
& \quad \left(-\frac{1}{2}\right)_k (17.056531977970801 - 5.4468085106382979 i + \\
& \quad \quad i^2 - 0.000028293345405160706 z_0)^k z_0^{-k} \Big) \Big) / \\
& \left(-2.72340425531915 + 1.0000000000000000 \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

From:

$$f(E) = \frac{k}{(E^2 - M^2)^2 + M^2 \Gamma^2}$$

$$(7.25199922264e+7)/((((((x^2-(512-188i)^2))^2+(512-188i)^2*335.5543483878^2)))) = y$$

Where $x = E$ and $y = f(E)$

Input interpretation:

$$\frac{7.25199922264 \times 10^7}{(x^2 - (512 - 188i)^2)^2 + (512 - 188i)^2 \times 335.5543483878^2} = y$$

i is the imaginary unit

Result:

$$\frac{7.25199922264 \times 10^7}{(x^2 - (226800 - 192512i))^2 + (2.553693625974 \times 10^{10} - 2.167621989963 \times 10^{10}i)} = y$$

Alternate forms:

$y =$

$$\frac{7.25199922264 \times 10^7}{x^4 - (453600 - 385024i)x^2 + (3.991430611574 \times 10^{10} - 1.0899966309963 \times 10^{11}i)}$$

$$7.25199922264 \times 10^7 / (x^4 - (453600.000000 - 385024.000000i)x^2 + (3.99143061157 \times 10^{10} - 1.089996630996 \times 10^{11}i)) = y$$

$$7.2519992226 \times 10^7 / ((1.000000000000x - (538.75186704 - 19.21865602i))(1.000000000000x - (530.62531359 - 343.28915060i))(1.000000000000x + (530.62531359 - 343.28915060i))(1.000000000000x + (538.75186704 - 19.21865602i))) = y$$

Alternate form assuming x and y are real:

$$\begin{aligned}
 & -((3.28950684739 \times 10^{13} x^2) / ((385\,024 (x^2 - 226\,800) - 2.167621989963 \times 10^{10})^2 + \\
 & \quad ((x^2 - 226\,800)^2 - 1.152393388426 \times 10^{10})^2)) + \\
 & \quad i(7.90465472067 \times 10^{18} / ((385\,024 (x^2 - 226\,800) - 2.167621989963 \times 10^{10})^2 + \\
 & \quad ((x^2 - 226\,800)^2 - 1.152393388426 \times 10^{10})^2) - \\
 & \quad (2.79219374870 \times 10^{13} x^2) / \\
 & \quad ((385\,024 (x^2 - 226\,800) - 2.167621989963 \times 10^{10})^2 + \\
 & \quad ((x^2 - 226\,800)^2 - 1.152393388426 \times 10^{10})^2)) + \\
 & \quad 2.89458516924 \times 10^{18} / ((385\,024 (x^2 - 226\,800) - 2.167621989963 \times 10^{10})^2 + \\
 & \quad ((x^2 - 226\,800)^2 - 1.152393388426 \times 10^{10})^2) + \\
 & \quad (7.25199922264 \times 10^7 x^4) / ((385\,024 (x^2 - 226\,800) - 2.167621989963 \times 10^{10})^2 + \\
 & \quad ((x^2 - 226\,800)^2 - 1.152393388426 \times 10^{10})^2) = y
 \end{aligned}$$

Solution:

$$\begin{aligned}
 & x^4 - (453\,600.0000000000 - 385\,024.0000000000 i) x^2 + \\
 & \quad (3.991430611600000 \times 10^{10} - 1.0899966310000000 \times 10^{11} i) \neq 0, \\
 & y \approx 7.251999222641509 \times 10^7 / \\
 & \quad (x^4 - (453\,600.0000000000 - 385\,024.0000000000 i) x^2 + \\
 & \quad (3.991430611600000 \times 10^{10} - 1.0899966310000000 \times 10^{11} i))
 \end{aligned}$$

Solutions:

$$\begin{aligned}
 x & \approx -532.0698829675344, \quad y \approx -0.008680425259864120 \\
 x & \approx 532.0698829675344, \quad y \approx -0.008680425259864120
 \end{aligned}$$

$$x = 532.0698829675344$$

Partial derivatives:

$$\begin{aligned}
 \frac{\partial}{\partial x} & (7.25199922264 \times 10^7 / ((x^2 - (226\,800 - 192\,512 i))^2 + \\
 & \quad (2.553693625974 \times 10^{10} - 2.167621989963 \times 10^{10} i))) = \\
 & -((2.90079968906 \times 10^8 x (x^2 - (226\,800 - 192\,512 i))) / \\
 & \quad (x^4 - (453\,600 - 385\,024 i) x^2 + \\
 & \quad (3.99143061157 \times 10^{10} - 1.089996630996 \times 10^{11} i)^2)) \\
 \frac{\partial}{\partial y} & (7.25199922264 \times 10^7 / ((x^2 - (226\,800 - 192\,512 i))^2 + \\
 & \quad (2.553693625974 \times 10^{10} - 2.167621989963 \times 10^{10} i))) = 0
 \end{aligned}$$

Implicit derivatives:

$$\begin{aligned}
 \frac{\partial x(y)}{\partial y} & = \\
 & -((53 ((8\,621\,490\,121\,000 - 23\,543\,927\,229\,519 i) - (97\,977\,600 - 83\,165\,184 i) x^2 + \\
 & \quad 216 x^4)^2) / (717\,300\,464\,550\,912 x ((-226\,800 + 192\,512 i) + x^2)))
 \end{aligned}$$

$$\frac{\partial y(x)}{\partial x} = -\left(\frac{717300464550912x(-226800 + 192512i + x^2)}{53((8621490121000 - 23543927229519i) - (97977600 - 83165184i)x^2 + 216x^4)^2}\right)$$

$$(7.25199922264e+7)/((((((532.0698829675344^2 - (512 - 188i)^2))^2 + (512 - 188i)^2 * 335.5543483878^2))))$$

Input interpretation:

$$\frac{7.25199922264 \times 10^7}{(532.0698829675344^2 - (512 - 188i)^2)^2 + (512 - 188i)^2 \times 335.5543483878^2}$$

i is the imaginary unit

Result:

$$-0.00868042525994... - 4.20949600289... \times 10^{-14} i$$

Polar coordinates:

$r = 0.0086804252599$ (radius), $\theta = -179.9999999997^\circ$ (angle)
0.0086804252599

$$1/(1+0.0086804252599)$$

Input interpretation:

$$\frac{1}{1 + 0.0086804252599}$$

Result:

$$0.991394276083365678129357234482051195509335768350142845099... \\ 0.991394276...$$

$$(((1/(1+0.0086804252599))))^{1/16}$$

Input interpretation:

$$\sqrt[16]{\frac{1}{1 + 0.0086804252599}}$$

Result:

0.99945996043753...

0.99945996043753... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\phi^5 \sqrt[4]{5^3} - 1}}}{\sqrt{5}} - \phi + 1$$

and to the dilaton value **0.989117352243 = ϕ**

8*log base 0.99945996043753((1/(1+0.0086804252599)))-Pi+1/golden ratio

where 8 is a Fibonacci number

Input interpretation:

$$8 \log_{0.99945996043753} \left(\frac{1}{1 + 0.0086804252599} \right) - \pi + \frac{1}{\phi}$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

125.47644133...

125.47644133... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

Alternative representation:

$$8 \log_{0.999459960437530000} \left(\frac{1}{1 + 0.00868042525990000} \right) - \pi + \frac{1}{\phi} =$$

$$-\pi + \frac{1}{\phi} + \frac{8 \log \left(\frac{1}{1.00868042525990000} \right)}{\log(0.999459960437530000)}$$

Series representations:

$$8 \log_{0.999459960437530000} \left(\frac{1}{1 + 0.00868042525990000} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - \frac{8 \sum_{k=1}^{\infty} \frac{(-1)^k (-0.00860572391663432)^k}{k}}{\log(0.999459960437530000)}$$

$$8 \log_{0.999459960437530000} \left(\frac{1}{1 + 0.00868042525990000} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - 14809.7295042054 \log(0.99139427608336568) -$$

$$8 \log(0.99139427608336568) \sum_{k=0}^{\infty} (-0.000540039562470000)^k G(k)$$

for $\left(G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

8*log base 0.99945996043753((1/(1+0.0086804252599)))+11+1/golden ratio

where 11 is a Lucas number

Input interpretation:

$$8 \log_{0.99945996043753} \left(\frac{1}{1 + 0.0086804252599} \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

139.61803399...

139.61803399... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternative representation:

$$8 \log_{0.999459960437530000} \left(\frac{1}{1 + 0.00868042525990000} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{8 \log \left(\frac{1}{1.00868042525990000} \right)}{\log(0.999459960437530000)}$$

Series representations:

$$8 \log_{0.999459960437530000} \left(\frac{1}{1 + 0.00868042525990000} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - \frac{8 \sum_{k=1}^{\infty} \frac{(-1)^k (-0.00860572391663432)^k}{k}}{\log(0.999459960437530000)}$$

$$\begin{aligned}
& 8 \log_{0.999459960437530000} \left(\frac{1}{1 + 0.00868042525990000} \right) + 11 + \frac{1}{\phi} = \\
& 11 + \frac{1}{\phi} - 14809.7295042054 \log(0.99139427608336568) - \\
& 8 \log(0.99139427608336568) \sum_{k=0}^{\infty} (-0.000540039562470000)^k G(k) \\
& \text{for } \left(G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)
\end{aligned}$$

From:

Integrable Scalar Cosmologies I. Foundations and links with String Theory
P. Fré, A. Sagnotti and A.S. Sorin - <https://arxiv.org/abs/1307.1910v3>

Now, we have that:

$$t \geq 0, \quad a = -2.5, -1.9, 2.5$$

From:

$$x(t) = \sinh(\Omega t), \quad (4.229)$$

$$y(t) = \left[a - \frac{1}{1+\gamma} \int_0^{\sinh^2(\Omega t)} du u^{\frac{1-\gamma}{2(1+\gamma)}} (1+u)^{-\frac{1}{2}} \right] \cosh(\Omega t) + \left[\sinh(\Omega t) \right]^{\frac{3+\gamma}{1+\gamma}},$$

$$x(t) = \cosh(\Omega t), \quad (4.230)$$

$$y(t) = \left[a + \frac{1}{1+\gamma} \int_1^{\cosh^2(\Omega t)} du u^{\frac{1-\gamma}{2(1+\gamma)}} (u-1)^{-\frac{1}{2}} \right] \sinh(\Omega t) - \left[\cosh(\Omega t) \right]^{\frac{3+\gamma}{1+\gamma}},$$

the integrals of eqs. (4.229) and (4.230) become particularly simple and the solutions read

$$\mathcal{A} = \frac{3}{4} \log \left\{ \sinh^2 \left(\frac{2t}{3} \right) \left[\cosh^2 \left(\frac{2t}{3} \right) + (a-2) \cosh \left(\frac{2t}{3} \right) + 1 \right] \right\}, \quad (4.232)$$

$$\varphi = \frac{3}{4} \log \left\{ \frac{\sinh^2 \left(\frac{2t}{3} \right)}{\cosh^2 \left(\frac{2t}{3} \right) + (a-2) \cosh \left(\frac{2t}{3} \right) + 1} \right\} \quad (4.233)$$

and

$$\mathcal{A} = \frac{3}{4} \log \left\{ \cosh^2 \left(\frac{2t}{3} \right) \left[\sinh^2 \left(\frac{2t}{3} \right) + a \sinh \left(\frac{2t}{3} \right) - 1 \right] \right\}, \quad (4.234)$$

$$\varphi = \frac{3}{4} \log \left\{ \frac{\cosh^2 \left(\frac{2t}{3} \right)}{\sinh^2 \left(\frac{2t}{3} \right) + a \sinh \left(\frac{2t}{3} \right) - 1} \right\}. \quad (4.235)$$

For $t = 5$ and $a = 2.5$, we obtain:

From (4.232), we obtain:

$$\frac{3}{4} \ln(((((((\sinh^2(10/3))(((\cosh^2(10/3)+(2.5-2) \cosh(10/3)+1))))))))))$$

Input:

$$\frac{3}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right)$$

$\sinh(x)$ is the hyperbolic sine function
 $\cosh(x)$ is the hyperbolic cosine function
 $\log(x)$ is the natural logarithm

Result:

7.9504807...

7.9504807...

Alternative representations:

$$\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 =$$

$$\frac{3}{4} \log_e \left(\left(1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right) \right) \sinh^2 \left(\frac{10}{3} \right) \right)$$

$$\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 =$$

$$\frac{3}{4} \log(a) \log_a \left(\left(1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right) \right) \sinh^2 \left(\frac{10}{3} \right) \right)$$

$$\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 =$$

$$\frac{3}{4} \log \left(\left(1 + 0.5 \cos \left(\frac{10i}{3} \right) + \cos^2 \left(\frac{10i}{3} \right) \right) \left(\frac{1}{2} \left(-\frac{1}{e^{10/3}} + e^{10/3} \right) \right)^2 \right)$$

Series representation:

$$\frac{1}{4} \log\left(\sinh^2\left(\frac{10}{3}\right)\left(\cosh^2\left(\frac{10}{3}\right) + (2.5 - 2) \cosh\left(\frac{10}{3}\right) + 1\right)\right) 3 =$$

$$\frac{3}{4} \log\left(-1 + \left(1 + 0.5 \cosh\left(\frac{10}{3}\right) + \cosh^2\left(\frac{10}{3}\right)\right) \sinh^2\left(\frac{10}{3}\right)\right) -$$

$$\frac{3}{4} \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \left(1 + 0.5 \cosh\left(\frac{10}{3}\right) + \cosh^2\left(\frac{10}{3}\right)\right) \sinh^2\left(\frac{10}{3}\right)\right)^{-k}}{k}$$

Integral representations:

$$\frac{1}{4} \log\left(\sinh^2\left(\frac{10}{3}\right)\left(\cosh^2\left(\frac{10}{3}\right) + (2.5 - 2) \cosh\left(\frac{10}{3}\right) + 1\right)\right) 3 =$$

$$\frac{3}{4} \int_1^{1+0.5 \cosh\left(\frac{10}{3}\right) + \cosh^2\left(\frac{10}{3}\right) \sinh^2\left(\frac{10}{3}\right)} \frac{1}{t} dt$$

$$\frac{1}{4} \log\left(\sinh^2\left(\frac{10}{3}\right)\left(\cosh^2\left(\frac{10}{3}\right) + (2.5 - 2) \cosh\left(\frac{10}{3}\right) + 1\right)\right) 3 =$$

$$\frac{3}{4} \log\left(123.457 \left(\int_0^1 \cosh\left(\frac{10t}{3}\right) dt\right)^2\right.$$

$$\left. \left(0.225 + 0.75 \int_0^1 \sinh\left(\frac{10t}{3}\right) dt + \left(\int_0^1 \sinh\left(\frac{10t}{3}\right) dt\right)^2\right)\right)$$

$$\frac{1}{4} \log\left(\sinh^2\left(\frac{10}{3}\right)\left(\cosh^2\left(\frac{10}{3}\right) + (2.5 - 2) \cosh\left(\frac{10}{3}\right) + 1\right)\right) 3 =$$

$$\frac{3}{4} \log\left(11.1111 \left(\int_0^1 \cosh\left(\frac{10t}{3}\right) dt\right)^2 \left(1 + 0.5 \int_{\frac{i\pi}{2}}^{\frac{10}{3}} \sinh(t) dt + \left(\int_{\frac{i\pi}{2}}^{\frac{10}{3}} \sinh(t) dt\right)^2\right)\right)$$

From (4.234), we obtain:

$$\frac{3}{4} \ln\left(\left(\left(\left(\left(\left(\cosh^2\left(\frac{10}{3}\right)\left(\left(\sinh^2\left(\frac{10}{3}\right) + (2.5) \sinh\left(\frac{10}{3}\right) - 1\right)\right)\right)\right)\right)\right)\right)\right)$$

Input:

$$\frac{3}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1\right)\right)$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

log(x) is the natural logarithm

Result:

8.0405451...

8.0405451...

Alternative representations:

$$\begin{aligned} & \frac{1}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1\right)\right) 3 = \\ & \frac{3}{4} \log_e\left(\cosh^2\left(\frac{10}{3}\right)\left(-1 + \sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right)\right)\right) \\ & \frac{1}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1\right)\right) 3 = \\ & \frac{3}{4} \log(a) \log_a\left(\cosh^2\left(\frac{10}{3}\right)\left(-1 + \sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right)\right)\right) \\ & \frac{1}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1\right)\right) 3 = \\ & \frac{3}{4} \log\left(\cos^2\left(\frac{10}{3}\right)\left(-1 + 1.25\left(-\frac{1}{e^{10/3}} + e^{10/3}\right) + \left(\frac{1}{2}\left(-\frac{1}{e^{10/3}} + e^{10/3}\right)\right)^2\right)\right) \end{aligned}$$

Series representation:

$$\begin{aligned} & \frac{1}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1\right)\right) 3 = \\ & \frac{3}{4} \log\left(-1 + \cosh^2\left(\frac{10}{3}\right)\left(-1 + 2.5 \sinh\left(\frac{10}{3}\right) + \sinh^2\left(\frac{10}{3}\right)\right)\right) - \\ & \frac{3}{4} \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \cosh^2\left(\frac{10}{3}\right)\left(-1 + 2.5 \sinh\left(\frac{10}{3}\right) + \sinh^2\left(\frac{10}{3}\right)\right)\right)^k}{k} \end{aligned}$$

Integral representations:

$$\begin{aligned} & \frac{1}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1\right)\right) 3 = \\ & \frac{3}{4} \int_1^{\cosh^2\left(\frac{10}{3}\right)\left(-1 + 2.5 \sinh\left(\frac{10}{3}\right) + \sinh^2\left(\frac{10}{3}\right)\right)} \frac{1}{t} dt \\ & \frac{1}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1\right)\right) 3 = \\ & \frac{3}{4} \log\left(123.457\left(-0.09 + 0.75 \int_0^1 \cosh\left(\frac{10t}{3}\right) dt + \left(\int_0^1 \cosh\left(\frac{10t}{3}\right) dt\right)^2\right)\right. \\ & \quad \left.(0.3 + \int_0^1 \sinh\left(\frac{10t}{3}\right) dt\right)^2\right) \\ & \frac{1}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1\right)\right) 3 = \frac{3}{4} \log\left(11.1111\right. \\ & \quad \left.(-0.09 + 0.75 \int_0^1 \cosh\left(\frac{10t}{3}\right) dt + \left(\int_0^1 \cosh\left(\frac{10t}{3}\right) dt\right)^2\right) \left(\int_{\frac{i\pi}{2}}^{\frac{10}{3}} \sinh(t) dt\right)^2\right) \end{aligned}$$

From (4.233), we obtain:

$$\varphi = \frac{3}{4} \log \left\{ \frac{\sinh^2 \left(\frac{2t}{3} \right)}{\cosh^2 \left(\frac{2t}{3} \right) + (a-2) \cosh \left(\frac{2t}{3} \right) + 1} \right\}$$

$$3/4 \ln \left(\frac{\sinh^2(10/3)}{\cosh^2(10/3) + (2.5-2) \cosh(10/3) + 1} \right)$$

Input:

$$\frac{3}{4} \log \left(\frac{\sinh^2 \left(\frac{10}{3} \right)}{\cosh^2 \left(\frac{10}{3} \right) + (2.5-2) \cosh \left(\frac{10}{3} \right) + 1} \right)$$

$\sinh(x)$ is the hyperbolic sine function
 $\cosh(x)$ is the hyperbolic cosine function
 $\log(x)$ is the natural logarithm

Result:

-0.0337426...

-0.0337426...

Alternative representations:

$$\frac{1}{4} \log \left(\frac{\sinh^2 \left(\frac{10}{3} \right)}{\cosh^2 \left(\frac{10}{3} \right) + (2.5-2) \cosh \left(\frac{10}{3} \right) + 1} \right) 3 = \frac{3}{4} \log_e \left(\frac{\sinh^2 \left(\frac{10}{3} \right)}{1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right)} \right)$$

$$\frac{1}{4} \log \left(\frac{\sinh^2 \left(\frac{10}{3} \right)}{\cosh^2 \left(\frac{10}{3} \right) + (2.5-2) \cosh \left(\frac{10}{3} \right) + 1} \right) 3 =$$

$$\frac{3}{4} \log(a) \log_a \left(\frac{\sinh^2 \left(\frac{10}{3} \right)}{1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right)} \right)$$

$$\frac{1}{4} \log \left(\frac{\sinh^2 \left(\frac{10}{3} \right)}{\cosh^2 \left(\frac{10}{3} \right) + (2.5-2) \cosh \left(\frac{10}{3} \right) + 1} \right) 3 = \frac{3}{4} \log \left(\frac{\left(\frac{1}{2} \left(-\frac{1}{e^{10/3}} + e^{10/3} \right) \right)^2}{1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right)} \right)$$

Series representation:

$$\frac{1}{4} \log \left(\frac{\sinh^2 \left(\frac{10}{3} \right)}{\cosh^2 \left(\frac{10}{3} \right) + (2.5-2) \cosh \left(\frac{10}{3} \right) + 1} \right) 3 =$$

$$-\frac{3}{4} \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{\sinh^2 \left(\frac{10}{3} \right)}{1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right)} \right)^k}{k}$$

Integral representations:

$$\frac{1}{4} \log \left(\frac{\sinh^2\left(\frac{10}{3}\right)}{\cosh^2\left(\frac{10}{3}\right) + (2.5 - 2) \cosh\left(\frac{10}{3}\right) + 1} \right) 3 = \frac{3}{4} \int_1^{\frac{\sinh^2\left(\frac{10}{3}\right)}{1+0.5 \cosh\left(\frac{10}{3}\right)+\cosh^2\left(\frac{10}{3}\right)}} \frac{1}{t} dt$$

$$\frac{1}{4} \log \left(\frac{\sinh^2\left(\frac{10}{3}\right)}{\cosh^2\left(\frac{10}{3}\right) + (2.5 - 2) \cosh\left(\frac{10}{3}\right) + 1} \right) 3 = \frac{3}{4} \log \left(\frac{\left(\int_0^1 \cosh\left(\frac{10t}{3}\right) dt\right)^2}{0.225 + 0.75 \int_0^1 \sinh\left(\frac{10t}{3}\right) dt + \left(\int_0^1 \sinh\left(\frac{10t}{3}\right) dt\right)^2} \right)$$

$$\frac{1}{4} \log \left(\frac{\sinh^2\left(\frac{10}{3}\right)}{\cosh^2\left(\frac{10}{3}\right) + (2.5 - 2) \cosh\left(\frac{10}{3}\right) + 1} \right) 3 = \frac{3}{4} \log \left(\frac{11.1111 \left(\int_0^1 \cosh\left(\frac{10t}{3}\right) dt\right)^2}{1 + 0.5 \int_{\frac{i\pi}{2}}^{\frac{10}{3}} \sinh(t) dt + \left(\int_{\frac{i\pi}{2}}^{\frac{10}{3}} \sinh(t) dt\right)^2} \right)$$

From (4.235), we obtain:

$$\varphi = \frac{3}{4} \log \left\{ \frac{\cosh^2\left(\frac{2t}{3}\right)}{\sinh^2\left(\frac{2t}{3}\right) + a \sinh\left(\frac{2t}{3}\right) - 1} \right\}$$

$$3/4 \ln \left(\frac{\cosh^2(10/3)}{\sinh^2(10/3) + (2.5) \sinh(10/3) - 1} \right)$$

Input:

$$\frac{3}{4} \log \left(\frac{\cosh^2\left(\frac{10}{3}\right)}{\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1} \right)$$

cosh(x) is the hyperbolic cosine function
sinh(x) is the hyperbolic sine function
log(x) is the natural logarithm

Result:

-0.116171...

-0.116171...

Alternative representations:

$$\frac{1}{4} \log \left(\frac{\cosh^2\left(\frac{10}{3}\right)}{\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1} \right) 3 = \frac{3}{4} \log_e \left(\frac{\cosh^2\left(\frac{10}{3}\right)}{-1 + \sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right)} \right)$$

$$\frac{1}{4} \log \left(\frac{\cosh^2\left(\frac{10}{3}\right)}{\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1} \right) 3 = \frac{3}{4} \log(a) \log_a \left(\frac{\cosh^2\left(\frac{10}{3}\right)}{-1 + \sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right)} \right)$$

$$\frac{1}{4} \log \left(\frac{\cosh^2\left(\frac{10}{3}\right)}{\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1} \right) 3 =$$

$$\frac{3}{4} \log \left(\frac{\cos^2\left(\frac{10i}{3}\right)}{-1 + 1.25 \left(-\frac{1}{e^{10/3}} + e^{10/3} \right) + \left(\frac{1}{2} \left(-\frac{1}{e^{10/3}} + e^{10/3} \right) \right)^2} \right)$$

Series representation:

$$\frac{1}{4} \log \left(\frac{\cosh^2\left(\frac{10}{3}\right)}{\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1} \right) 3 = -\frac{3}{4} \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{\cosh^2\left(\frac{10}{3}\right)}{-1 + 2.5 \sinh\left(\frac{10}{3}\right) + \sinh^2\left(\frac{10}{3}\right)} \right)^k}{k}$$

Integral representations:

$$\frac{1}{4} \log \left(\frac{\cosh^2\left(\frac{10}{3}\right)}{\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1} \right) 3 = \frac{3}{4} \int_1^{\frac{\cosh^2\left(\frac{10}{3}\right)}{-1 + 2.5 \sinh\left(\frac{10}{3}\right) + \sinh^2\left(\frac{10}{3}\right)}} \frac{1}{t} dt$$

$$\frac{1}{4} \log \left(\frac{\cosh^2\left(\frac{10}{3}\right)}{\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1} \right) 3 =$$

$$\frac{3}{4} \log \left(\frac{\left(0.3 + \int_0^1 \sinh\left(\frac{10t}{3}\right) dt \right)^2}{-0.09 + 0.75 \int_0^1 \cosh\left(\frac{10t}{3}\right) dt + \left(\int_0^1 \cosh\left(\frac{10t}{3}\right) dt \right)^2} \right)$$

$$\frac{1}{4} \log \left(\frac{\cosh^2\left(\frac{10}{3}\right)}{\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1} \right) 3 =$$

$$\frac{3}{4} \log \left(\frac{0.09 \left(\int_{\frac{2}{3}}^{\frac{10}{3}} \sinh(t) dt \right)^2}{-0.09 + 0.75 \int_0^1 \cosh\left(\frac{10t}{3}\right) dt + \left(\int_0^1 \cosh\left(\frac{10t}{3}\right) dt \right)^2} \right)$$

From the sum of the four results, we obtain:

$$\frac{3}{4} \ln\left[\left(\sinh^2\left(\frac{10}{3}\right)\left(\cosh^2\left(\frac{10}{3}\right)+\left(2.5-2\right)\cosh\left(\frac{10}{3}\right)+1\right)\right)\right]+\frac{3}{4} \ln\left[\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right)+\left(2.5\right)\sinh\left(\frac{10}{3}\right)-1\right)\right)\right]-0.149913730627$$

Input interpretation:

$$\frac{3}{4} \log\left(\sinh^2\left(\frac{10}{3}\right)\left(\cosh^2\left(\frac{10}{3}\right)+\left(2.5-2\right)\cosh\left(\frac{10}{3}\right)+1\right)\right)+\frac{3}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right)+2.5\sinh\left(\frac{10}{3}\right)-1\right)\right)-0.149913730627$$

sinh(x) is the hyperbolic sine function
cosh(x) is the hyperbolic cosine function
log(x) is the natural logarithm

Result:

15.841112...

15.841112... result very near to the black hole entropy 15.8174

Alternative representations:

$$\frac{1}{4} \log\left(\sinh^2\left(\frac{10}{3}\right)\left(\cosh^2\left(\frac{10}{3}\right)+\left(2.5-2\right)\cosh\left(\frac{10}{3}\right)+1\right)\right)3+\frac{1}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right)+2.5\sinh\left(\frac{10}{3}\right)-1\right)\right)3-0.1499137306270000=-0.1499137306270000+\frac{3}{4} \log_a\left(\left(1+0.5\cosh\left(\frac{10}{3}\right)+\cosh^2\left(\frac{10}{3}\right)\right)\sinh^2\left(\frac{10}{3}\right)\right)+\frac{3}{4} \log_a\left(\cosh^2\left(\frac{10}{3}\right)\left(-1+\sinh^2\left(\frac{10}{3}\right)+2.5\sinh\left(\frac{10}{3}\right)\right)\right)$$

$$\frac{1}{4} \log\left(\sinh^2\left(\frac{10}{3}\right)\left(\cosh^2\left(\frac{10}{3}\right)+\left(2.5-2\right)\cosh\left(\frac{10}{3}\right)+1\right)\right)3+\frac{1}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right)+2.5\sinh\left(\frac{10}{3}\right)-1\right)\right)3-0.1499137306270000=-0.1499137306270000+\frac{3}{4} \log_e\left(\left(1+0.5\cosh\left(\frac{10}{3}\right)+\cosh^2\left(\frac{10}{3}\right)\right)\sinh^2\left(\frac{10}{3}\right)\right)+\frac{3}{4} \log_e\left(\cosh^2\left(\frac{10}{3}\right)\left(-1+\sinh^2\left(\frac{10}{3}\right)+2.5\sinh\left(\frac{10}{3}\right)\right)\right)$$

$$\frac{1}{4} \log\left(\sinh^2\left(\frac{10}{3}\right)\left(\cosh^2\left(\frac{10}{3}\right)+\left(2.5-2\right)\cosh\left(\frac{10}{3}\right)+1\right)\right)3+\frac{1}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right)+2.5\sinh\left(\frac{10}{3}\right)-1\right)\right)3-0.1499137306270000=-0.1499137306270000+\frac{3}{4} \log\left(\left(1+0.5\cos\left(\frac{10i}{3}\right)+\cos^2\left(\frac{10i}{3}\right)\right)\left(\frac{1}{2}\left(-\frac{1}{e^{10/3}}+e^{10/3}\right)\right)^2\right)+\frac{3}{4} \log\left(\cos^2\left(\frac{10i}{3}\right)\left(-1+1.25\left(-\frac{1}{e^{10/3}}+e^{10/3}\right)+\left(\frac{1}{2}\left(-\frac{1}{e^{10/3}}+e^{10/3}\right)\right)^2\right)\right)$$

Series representation:

$$\begin{aligned}
& \frac{1}{4} \log\left(\sinh^2\left(\frac{10}{3}\right)\left(\cosh^2\left(\frac{10}{3}\right) + (2.5 - 2) \cosh\left(\frac{10}{3}\right) + 1\right)\right) 3 + \\
& \frac{1}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1\right)\right) 3 - \\
& 0.1499137306270000 = -0.1499137306270000 + \\
& 0.7500000000000000 \log\left(-1 + \left(1 + 0.5 \cosh\left(\frac{10}{3}\right) + \cosh^2\left(\frac{10}{3}\right)\right) \sinh^2\left(\frac{10}{3}\right)\right) + \\
& 0.7500000000000000 \log\left(-1 + \cosh^2\left(\frac{10}{3}\right)\left(-1 + 2.5 \sinh\left(\frac{10}{3}\right) + \sinh^2\left(\frac{10}{3}\right)\right)\right) + \\
& 0.7500000000000000 \\
& \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \left(-\left(-1 + \left(1 + 0.5 \cosh\left(\frac{10}{3}\right) + \cosh^2\left(\frac{10}{3}\right)\right) \sinh^2\left(\frac{10}{3}\right)\right)^{-k} - \right. \\
& \quad \left. \left(-1 + \cosh^2\left(\frac{10}{3}\right)\left(-1 + 2.5 \sinh\left(\frac{10}{3}\right) + \sinh^2\left(\frac{10}{3}\right)\right)\right)^{-k} \right)
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& \frac{1}{4} \log\left(\sinh^2\left(\frac{10}{3}\right)\left(\cosh^2\left(\frac{10}{3}\right) + (2.5 - 2) \cosh\left(\frac{10}{3}\right) + 1\right)\right) 3 + \\
& \frac{1}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1\right)\right) 3 - \\
& 0.1499137306270000 = -0.1499137306270000 + \\
& \int_1^{1+0.5 \cosh\left(\frac{10}{3}\right) + \cosh^2\left(\frac{10}{3}\right) \sinh^2\left(\frac{10}{3}\right)} \left(\left(\cosh^2\left(\frac{10}{3}\right)\left(0.75 - 1.875 \sinh\left(\frac{10}{3}\right)\right) + \right. \right. \\
& \quad \left. \left. 0.75 \sinh^2\left(\frac{10}{3}\right) + 0.375 \cosh\left(\frac{10}{3}\right) \sinh^2\left(\frac{10}{3}\right) + \right. \right. \\
& \quad \left. \left. t \left(-1.5 + \cosh^2\left(\frac{10}{3}\right)\left(-1.5 + 3.75 \sinh\left(\frac{10}{3}\right) + 1.5 \sinh^2\left(\frac{10}{3}\right)\right)\right)\right) \right) / \\
& \left(t \left(\cosh^2\left(\frac{10}{3}\right)\left(1 - 2.5 \sinh\left(\frac{10}{3}\right)\right) + \sinh^2\left(\frac{10}{3}\right) + 0.5 \cosh\left(\frac{10}{3}\right) \sinh^2\left(\frac{10}{3}\right) + \right. \right. \\
& \quad \left. \left. t \left(-1 + \cosh^2\left(\frac{10}{3}\right)\left(-1 + 2.5 \sinh\left(\frac{10}{3}\right) + \sinh^2\left(\frac{10}{3}\right)\right)\right)\right) \right) dt
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} \log\left(\sinh^2\left(\frac{10}{3}\right)\left(\cosh^2\left(\frac{10}{3}\right) + (2.5 - 2) \cosh\left(\frac{10}{3}\right) + 1\right)\right) 3 + \\
& \frac{1}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1\right)\right) 3 - 0.1499137306270000 = \\
& -0.1499137306270000 + \frac{0.3750000000000000}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{\Gamma(1-s)} \Gamma(-s)^2 \Gamma(1+s) \\
& \left(\left(-1 + \left(1 + 0.5 \cosh\left(\frac{10}{3}\right) + \cosh^2\left(\frac{10}{3}\right)\right) \sinh^2\left(\frac{10}{3}\right)\right)^{-s} + \left(-1 + \cosh^2\left(\frac{10}{3}\right)\right) \right. \\
& \quad \left. \left(-1 + 2.5 \sinh\left(\frac{10}{3}\right) + \sinh^2\left(\frac{10}{3}\right)\right)^{-s} \right) ds \text{ for } -1 < \gamma < 0
\end{aligned}$$

$$\begin{aligned} & \frac{1}{4} \log\left(\sinh^2\left(\frac{10}{3}\right)\left(\cosh^2\left(\frac{10}{3}\right) + (2.5 - 2) \cosh\left(\frac{10}{3}\right) + 1\right)\right) 3 + \\ & \frac{1}{4} \log\left(\cosh^2\left(\frac{10}{3}\right)\left(\sinh^2\left(\frac{10}{3}\right) + 2.5 \sinh\left(\frac{10}{3}\right) - 1\right)\right) 3 - 0.1499137306270000 = \\ & 0.7500000000000000 \left(-0.1998849741693333 + 1.0000000000000000 \right. \\ & \quad \log\left(123.457 \left(-0.09 + 0.75 \int_0^1 \cosh\left(\frac{10t}{3}\right) dt + \left(\int_0^1 \cosh\left(\frac{10t}{3}\right) dt \right)^2 \right) \right. \\ & \quad \left. \left(0.3 + \int_0^1 \sinh\left(\frac{10t}{3}\right) dt \right)^2 \right) + \\ & \quad \left. 1.0000000000000000 \log\left(123.457 \left(\int_0^1 \cosh\left(\frac{10t}{3}\right) dt \right)^2 \right. \right. \\ & \quad \left. \left. \left(0.225 + 0.75 \int_0^1 \sinh\left(\frac{10t}{3}\right) dt + \left(\int_0^1 \sinh\left(\frac{10t}{3}\right) dt \right)^2 \right) \right) \right) \end{aligned}$$

$8(7.9504807+8.0405451-0.0337426 -0.116171)$ -golden ratio

where 8 is a Fibonacci number

Input interpretation:

$$8(7.9504807 + 8.0405451 - 0.0337426 - 0.116171) - \phi$$

ϕ is the golden ratio

Result:

125.11086...

125.11086... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV

$8(7.9504807+8.0405451-0.0337426 -0.116171)+11$ +golden ratio

where 11 is a Lucas number

Input interpretation:

$$8(7.9504807 + 8.0405451 - 0.0337426 - 0.116171) + 11 + \phi$$

ϕ is the golden ratio

Result:

139.34693...

139.34693... result practically equal to the rest mass of Pion meson 139.57 MeV

$$27*4(7.9504807+8.0405451-0.0337426 -0.116171)+18$$

where 18 and 4 are Lucas numbers

From Wikipedia:

“The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group $\mathbf{Z}/3\mathbf{Z}$, and its outer automorphism group is the cyclic group $\mathbf{Z}/2\mathbf{Z}$. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories”.

Input interpretation:

$$27 \times 4 (7.9504807 + 8.0405451 - 0.0337426 - 0.116171) + 18$$

Result:

$$1728.8401176$$

$$1728.8401176$$

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

$$27*4(((3/4 \ln[((\sinh^2(10/3))(((\cosh^2(10/3)+(2.5-2) \cosh(10/3)+1)))))]+3/4 \ln[((\cosh^2(10/3))(((\sinh^2(10/3)+(2.5) \sinh(10/3)-1)))))]-0.149913730627)))+18+4/25$$

Input interpretation:

$$27 \times 4 \left(\frac{3}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) + \frac{3}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) - 0.149913730627 \right) + 18 + \frac{4}{25}$$

sinh(x) is the hyperbolic sine function
cosh(x) is the hyperbolic cosine function
log(x) is the natural logarithm

Result:

1729.000102247452326132762174521823226506405503470291949862...

1729.0001022...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Alternative representations:

$$\begin{aligned}
& 27 \times 4 \left(\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 + \right. \\
& \quad \left. \frac{1}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) 3 - \right. \\
& \quad \left. 0.1499137306270000 \right) + \\
& 18 + \frac{4}{25} = 18 + 108 \left(-0.1499137306270000 + \right. \\
& \quad \frac{3}{4} \log_e \left(\left(1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right) \right) \sinh^2 \left(\frac{10}{3} \right) \right) + \\
& \quad \left. \frac{3}{4} \log_e \left(\cosh^2 \left(\frac{10}{3} \right) \left(-1 + \sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) \right) \right) \right) + \frac{4}{25} \\
& 27 \times 4 \left(\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 + \right. \\
& \quad \left. \frac{1}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) 3 - \right. \\
& \quad \left. 0.1499137306270000 \right) + 18 + \frac{4}{25} = 18 + 108 \\
& \quad \left(-0.1499137306270000 + \frac{3}{4} \log_e \left(\left(1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right) \right) \sinh^2 \left(\frac{10}{3} \right) \right) + \right. \\
& \quad \left. \frac{3}{4} \log_e \left(\cosh^2 \left(\frac{10}{3} \right) \left(-1 + \sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) \right) \right) \right) + \frac{4}{25}
\end{aligned}$$

$$\begin{aligned}
& 27 \times 4 \left(\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 + \right. \\
& \quad \left. \frac{1}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) 3 - \right. \\
& \quad \left. 0.1499137306270000 \right) + \\
& 18 + \frac{4}{25} = 18 + 108 \left(-0.1499137306270000 + \right. \\
& \quad \left. \frac{3}{4} \log \left(\left(1 + 0.5 \cos \left(\frac{10i}{3} \right) + \cos^2 \left(\frac{10i}{3} \right) \right) \left(\frac{1}{2} \left(-\frac{1}{e^{10/3}} + e^{10/3} \right) \right)^2 \right) + \right. \\
& \quad \left. \frac{3}{4} \log \left(\cos^2 \left(\frac{10i}{3} \right) \left(-1 + 1.25 \left(-\frac{1}{e^{10/3}} + e^{10/3} \right) + \left(\frac{1}{2} \left(-\frac{1}{e^{10/3}} + e^{10/3} \right) \right)^2 \right) \right) \right) + \frac{4}{25}
\end{aligned}$$

Series representation:

$$\begin{aligned}
& 27 \times 4 \left(\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 + \right. \\
& \quad \left. \frac{1}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) 3 - \right. \\
& \quad \left. 0.1499137306270000 \right) + 18 + \frac{4}{25} = 1.96931709228400 + \\
& 81.0000000000000000 \log \left(-1 + \left(1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right) \right) \sinh^2 \left(\frac{10}{3} \right) \right) + \\
& 81.0000000000000000 \log \left(-1 + \cosh^2 \left(\frac{10}{3} \right) \left(-1 + 2.5 \sinh \left(\frac{10}{3} \right) + \sinh^2 \left(\frac{10}{3} \right) \right) \right) + \\
& 81.0000000000000000 \\
& \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \left(-\left(-1 + \left(1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right) \right) \sinh^2 \left(\frac{10}{3} \right) \right)^{-k} - \right. \\
& \quad \left. \left(-1 + \cosh^2 \left(\frac{10}{3} \right) \left(-1 + 2.5 \sinh \left(\frac{10}{3} \right) + \sinh^2 \left(\frac{10}{3} \right) \right) \right)^{-k} \right)
\end{aligned}$$

Integral representations:

$$\begin{aligned}
 & 27 \times 4 \left(\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 + \right. \\
 & \quad \left. \frac{1}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) 3 - \right. \\
 & \quad \left. 0.1499137306270000 \right) + 18 + \frac{4}{25} = 1.96931709228400 + \\
 & \int_1^{\left(1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right) \right) \sinh^2 \left(\frac{10}{3} \right)} \left(\left(\cosh^2 \left(\frac{10}{3} \right) \left(81 - 202.5 \sinh \left(\frac{10}{3} \right) \right) + \right. \right. \\
 & \quad \left. \left. 81 \sinh^2 \left(\frac{10}{3} \right) + 40.5 \cosh \left(\frac{10}{3} \right) \sinh^2 \left(\frac{10}{3} \right) + \right. \right. \\
 & \quad \left. \left. t \left(-162 + \cosh^2 \left(\frac{10}{3} \right) \left(-162 + 405 \sinh \left(\frac{10}{3} \right) + 162 \sinh^2 \left(\frac{10}{3} \right) \right) \right) \right) / \right. \\
 & \quad \left. \left(t \left(\cosh^2 \left(\frac{10}{3} \right) \left(1 - 2.5 \sinh \left(\frac{10}{3} \right) \right) + \sinh^2 \left(\frac{10}{3} \right) + 0.5 \cosh \left(\frac{10}{3} \right) \sinh^2 \left(\frac{10}{3} \right) + \right. \right. \right. \\
 & \quad \left. \left. \left. t \left(-1 + \cosh^2 \left(\frac{10}{3} \right) \left(-1 + 2.5 \sinh \left(\frac{10}{3} \right) + \sinh^2 \left(\frac{10}{3} \right) \right) \right) \right) \right) \right) dt
 \end{aligned}$$

$$\begin{aligned}
 & 27 \times 4 \left(\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 + \right. \\
 & \quad \left. \frac{1}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) 3 - \right. \\
 & \quad \left. 0.1499137306270000 \right) + 18 + \frac{4}{25} = \\
 & 1.969317092284 + \frac{40.500000000000}{i \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{1}{\Gamma(1-s)^2 \Gamma(1+s)} \\
 & \quad \left(\left(-1 + \left(1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right) \right) \sinh^2 \left(\frac{10}{3} \right) \right)^{-s} + \right. \\
 & \quad \left. \left(-1 + \cosh^2 \left(\frac{10}{3} \right) \left(-1 + 2.5 \sinh \left(\frac{10}{3} \right) + \sinh^2 \left(\frac{10}{3} \right) \right) \right)^{-s} \right) \\
 & \quad ds \text{ for } -1 < \gamma < 0
 \end{aligned}$$

$$\begin{aligned}
 & 27 \times 4 \left(\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 + \right. \\
 & \quad \left. \frac{1}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) 3 - \right. \\
 & \quad \left. 0.1499137306270000 \right) + 18 + \frac{4}{25} = \\
 & 81.00000000000000 \left(0.0243125566948642 + 1.0000000000000000 \right. \\
 & \quad \left. \log \left(123.457 \left(-0.09 + 0.75 \int_0^1 \cosh \left(\frac{10t}{3} \right) dt + \left(\int_0^1 \cosh \left(\frac{10t}{3} \right) dt \right)^2 \right) \right. \right. \\
 & \quad \left. \left. \left(0.3 + \int_0^1 \sinh \left(\frac{10t}{3} \right) dt \right)^2 \right) + \right. \\
 & \quad \left. 1.0000000000000000 \log \left(123.457 \left(\int_0^1 \cosh \left(\frac{10t}{3} \right) dt \right)^2 \right. \right. \\
 & \quad \left. \left. \left(0.225 + 0.75 \int_0^1 \sinh \left(\frac{10t}{3} \right) dt + \left(\int_0^1 \sinh \left(\frac{10t}{3} \right) dt \right)^2 \right) \right) \right)
 \end{aligned}$$

$$48 * \left(\left(\frac{3}{4} \ln \left[\left(\sinh^2 \left(\frac{10}{3} \right) \left(\left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) \right] \right) + \frac{3}{4} \ln \left[\left(\cosh^2 \left(\frac{10}{3} \right) \left(\left(\sinh^2 \left(\frac{10}{3} \right) + (2.5) \sinh \left(\frac{10}{3} \right) - 1 \right) \right) \right) \right] - 0.149913730627 \right) - 29 - 3 - \frac{1}{2 * \text{golden ratio}} \right)$$

where 29 and 3 are Lucas numbers

Input interpretation:

$$48 \left(\frac{3}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) + \frac{3}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) - 0.149913730627 \right) - 29 - 3 - \frac{1}{2\phi}$$

sinh(x) is the hyperbolic sine function
cosh(x) is the hyperbolic cosine function
log(x) is the natural logarithm
φ is the golden ratio

Result:

728.06436...

728.06436... result practically equal to the Ramanujan cube $728 = 9^3 - 1$

Alternative representations:

$$48 \left(\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 + \frac{1}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) 3 - 0.1499137306270000 \right) - 29 - 3 - \frac{1}{2\phi} = -32 + 48 \left(-0.1499137306270000 + \frac{3}{4} \log(a) \log_a \left(\left(1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right) \right) \sinh^2 \left(\frac{10}{3} \right) \right) + \frac{3}{4} \log(a) \log_a \left(\cosh^2 \left(\frac{10}{3} \right) \left(-1 + \sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) \right) \right) \right) - \frac{1}{2\phi}$$

$$48 \left(\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 + \frac{1}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) 3 - 0.1499137306270000 \right) - 29 - 3 - \frac{1}{2\phi} = -32 + 48 \left(-0.1499137306270000 + \frac{3}{4} \log_e \left(\left(1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right) \right) \sinh^2 \left(\frac{10}{3} \right) \right) + \frac{3}{4} \log_e \left(\cosh^2 \left(\frac{10}{3} \right) \left(-1 + \sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) \right) \right) \right) - \frac{1}{2\phi}$$

$$\begin{aligned}
& 48 \left(\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 + \right. \\
& \quad \left. \frac{1}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) 3 - \right. \\
& \quad \left. 0.1499137306270000 \right) - \\
& 29 - 3 - \frac{1}{2\phi} = -32 + 48 \left(-0.1499137306270000 + \right. \\
& \quad \left. \frac{3}{4} \log \left(\left(1 + 0.5 \cos \left(\frac{10i}{3} \right) + \cos^2 \left(\frac{10i}{3} \right) \right) \left(\frac{1}{2} \left(-\frac{1}{e^{10/3}} + e^{10/3} \right) \right)^2 \right) + \right. \\
& \quad \left. \frac{3}{4} \log \left(\cos^2 \left(\frac{10i}{3} \right) \left(-1 + 1.25 \left(-\frac{1}{e^{10/3}} + e^{10/3} \right) + \left(\frac{1}{2} \left(-\frac{1}{e^{10/3}} + e^{10/3} \right) \right)^2 \right) \right) \right) - \frac{1}{2\phi}
\end{aligned}$$

Series representation:

$$\begin{aligned}
& 48 \left(\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 + \right. \\
& \quad \left. \frac{1}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) 3 - \right. \\
& \quad \left. 0.1499137306270000 \right) - 29 - 3 - \frac{1}{2\phi} = \\
& -39.19585907009600 - \frac{0.5000000000000000}{\phi} + \\
& 36.0000000000000000 \log \left(-1 + \left(1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right) \right) \sinh^2 \left(\frac{10}{3} \right) \right) + \\
& 36.0000000000000000 \log \left(-1 + \cosh^2 \left(\frac{10}{3} \right) \left(-1 + 2.5 \sinh \left(\frac{10}{3} \right) + \sinh^2 \left(\frac{10}{3} \right) \right) \right) + \\
& 36.0000000000000000 \\
& \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k \left(-\left(-1 + \left(1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right) \right) \sinh^2 \left(\frac{10}{3} \right) \right)^{-k} - \right. \\
& \quad \left. \left(-1 + \cosh^2 \left(\frac{10}{3} \right) \left(-1 + 2.5 \sinh \left(\frac{10}{3} \right) + \sinh^2 \left(\frac{10}{3} \right) \right) \right)^{-k} \right)
\end{aligned}$$

Integral representations:

$$\begin{aligned}
 & 48 \left(\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 + \right. \\
 & \quad \left. \frac{1}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) 3 - \right. \\
 & \quad \left. 0.1499137306270000 \right) - 29 - 3 - \frac{1}{2\phi} = \\
 & -39.19585907009600 - \frac{0.5000000000000000}{\phi} + \\
 & \int_1^{(1+0.5 \cosh(\frac{10}{3}) + \cosh^2(\frac{10}{3})) \sinh^2(\frac{10}{3})} \left(\left(\cosh^2 \left(\frac{10}{3} \right) \left(36 - 90 \sinh \left(\frac{10}{3} \right) \right) + \right. \right. \\
 & \quad \left. \left. 36 \sinh^2 \left(\frac{10}{3} \right) + 18 \cosh \left(\frac{10}{3} \right) \sinh^2 \left(\frac{10}{3} \right) + \right. \right. \\
 & \quad \left. \left. t \left(-72 + \cosh^2 \left(\frac{10}{3} \right) \left(-72 + 180 \sinh \left(\frac{10}{3} \right) + 72 \sinh^2 \left(\frac{10}{3} \right) \right) \right) \right) / \right. \\
 & \quad \left. \left(t \left(\cosh^2 \left(\frac{10}{3} \right) \left(1 - 2.5 \sinh \left(\frac{10}{3} \right) \right) + \sinh^2 \left(\frac{10}{3} \right) + 0.5 \cosh \left(\frac{10}{3} \right) \sinh^2 \left(\frac{10}{3} \right) + \right. \right. \right. \\
 & \quad \left. \left. \left. t \left(-1 + \cosh^2 \left(\frac{10}{3} \right) \left(-1 + 2.5 \sinh \left(\frac{10}{3} \right) + \sinh^2 \left(\frac{10}{3} \right) \right) \right) \right) \right) dt
 \end{aligned}$$

$$\begin{aligned}
 & 48 \left(\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 + \right. \\
 & \quad \left. \frac{1}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) 3 - \right. \\
 & \quad \left. 0.1499137306270000 \right) - 29 - 3 - \frac{1}{2\phi} = \\
 & -39.1958590700960 - \frac{0.5000000000000000}{\phi} + \\
 & \frac{18.0000000000000000}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{\Gamma(1-s)} \Gamma(-s)^2 \Gamma(1+s) \\
 & \quad \left(\left(-1 + \left(1 + 0.5 \cosh \left(\frac{10}{3} \right) + \cosh^2 \left(\frac{10}{3} \right) \right) \sinh^2 \left(\frac{10}{3} \right) \right)^{-s} + \right. \\
 & \quad \left. \left(-1 + \cosh^2 \left(\frac{10}{3} \right) \left(-1 + 2.5 \sinh \left(\frac{10}{3} \right) + \sinh^2 \left(\frac{10}{3} \right) \right) \right)^{-s} \right) \\
 & \quad ds \text{ for } -1 < \gamma < 0
 \end{aligned}$$

$$\begin{aligned}
 & 48 \left(\frac{1}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) 3 + \right. \\
 & \quad \left. \frac{1}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) 3 - \right. \\
 & \quad \left. 0.1499137306270000 \right) - 29 - 3 - \frac{1}{2\phi} = \frac{1}{\phi} 36.00000000000000 \\
 & \left(-0.01388888888888889 - 1.08877386305822\phi + 1.000000000000000\phi \right. \\
 & \quad \left. \log \left(123.457 \left(-0.09 + 0.75 \int_0^1 \cosh \left(\frac{10t}{3} \right) dt + \left(\int_0^1 \cosh \left(\frac{10t}{3} \right) dt \right)^2 \right) \right. \right. \\
 & \quad \left. \left. \left(0.3 + \int_0^1 \sinh \left(\frac{10t}{3} \right) dt \right)^2 \right) + \right. \\
 & \quad \left. 1.000000000000000\phi \log \left(123.457 \left(\int_0^1 \cosh \left(\frac{10t}{3} \right) dt \right)^2 \right. \right. \\
 & \quad \left. \left. \left(0.225 + 0.75 \int_0^1 \sinh \left(\frac{10t}{3} \right) dt + \left(\int_0^1 \sinh \left(\frac{10t}{3} \right) dt \right)^2 \right) \right) \right)
 \end{aligned}$$

And:

$$\left(\left(\frac{1}{\left(\frac{3}{4} \ln \left[\left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right] \right) \right) \right) \right) + \frac{3}{4} \ln \left[\left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + (2.5) \sinh \left(\frac{10}{3} \right) - 1 \right) \right) \right] - 0.149913730627 \right) \right)^{1/4096}$$

Input interpretation:

$$\left(1 / \left(\frac{3}{4} \log \left(\sinh^2 \left(\frac{10}{3} \right) \left(\cosh^2 \left(\frac{10}{3} \right) + (2.5 - 2) \cosh \left(\frac{10}{3} \right) + 1 \right) \right) + \frac{3}{4} \log \left(\cosh^2 \left(\frac{10}{3} \right) \left(\sinh^2 \left(\frac{10}{3} \right) + 2.5 \sinh \left(\frac{10}{3} \right) - 1 \right) \right) - 0.149913730627 \right) \right)^{(1/4096)}$$

sinh(x) is the hyperbolic sine function
 cosh(x) is the hyperbolic cosine function
 log(x) is the natural logarithm

Result:

0.99932576241...

0.99932576241... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1}{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3} - 1}}} - \varphi + 1$$

and to the dilaton value **0.989117352243 = ϕ**

Now, we have that:

2. The potential with parameters $C > 0$, $\theta = 0$, $\gamma = \frac{5}{8}$ that is bounded from below possesses one admissible fixed point (4.152) with $k = -1$, the node of eq. (4.160):

$$v_{-1c} = 0, \quad \varphi_{-1c} = \frac{3}{15} \log \left(\frac{1 + \cos \frac{5\pi}{8}}{1 - \cos \frac{5\pi}{8}} \right). \quad (4.163)$$

3. The potential with parameters $C > 0$, $\theta = 0$, $\gamma = \frac{3}{4}$ that is bounded from below possesses one admissible fixed point (4.152) with $k = -1$, the improper node of eq. (4.161):

$$v_{-1c} = 0, \quad \varphi_{-1c} = \frac{1}{3} \log \left(\frac{\sqrt{2} - 1}{\sqrt{2} + 1} \right). \quad (4.164)$$

We obtain:

$$\varphi_{-1c} = \frac{3}{15} \log \left(\frac{1 + \cos \frac{5\pi}{8}}{1 - \cos \frac{5\pi}{8}} \right)$$

$$3/15 \ln(((1+\cos((5\text{Pi})/8))/(1-\cos((5\text{Pi})/8))))$$

Input:

$$\frac{3}{15} \log \left(\frac{1 + \cos \left(\frac{5\pi}{8} \right)}{1 - \cos \left(\frac{5\pi}{8} \right)} \right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{1}{5} \log \left(\frac{1 - \sin \left(\frac{\pi}{8} \right)}{1 + \sin \left(\frac{\pi}{8} \right)} \right)$$

Decimal approximation:

-0.16127988766460459832141241439431398067031323531550863187...

-0.1612798876646....

Property:

$\frac{1}{5} \log \left(\frac{1 - \sin \left(\frac{\pi}{8} \right)}{1 + \sin \left(\frac{\pi}{8} \right)} \right)$ is a transcendental number

Alternate forms:

$$\frac{1}{5} \log \left(\frac{2}{1 + \sin\left(\frac{\pi}{8}\right)} - 1 \right)$$

$$\frac{1}{5} \log \left(7 - 4\sqrt{2} - 2\sqrt{2(10 - 7\sqrt{2})} \right)$$

$$\frac{1}{5} \log \left(\frac{1 - \sqrt[4]{-1} + 2(-1)^{5/8}}{(\sqrt[8]{-1} + i)^2} \right)$$

Alternative representations:

$$\frac{1}{15} \log \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right) 3 = \frac{3}{15} \log \left(\frac{1 + \cosh\left(-\frac{5i\pi}{8}\right)}{1 - \cosh\left(-\frac{5i\pi}{8}\right)} \right)$$

$$\frac{1}{15} \log \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right) 3 = \frac{3}{15} \log \left(\frac{1 + \cosh\left(\frac{5i\pi}{8}\right)}{1 - \cosh\left(\frac{5i\pi}{8}\right)} \right)$$

$$\frac{1}{15} \log \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right) 3 = \frac{3}{15} \log_e \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right)$$

Series representations:

$$\frac{1}{15} \log \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right) 3 = -\frac{1}{5} \sum_{k=1}^{\infty} \frac{(-1)^{2k} \left(\frac{2 \sin\left(\frac{\pi}{8}\right)}{1 + \sin\left(\frac{\pi}{8}\right)} \right)^k}{k}$$

$$\frac{1}{15} \log \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right) 3 = \frac{1}{5} \log \left(\frac{1 - \sum_{k=0}^{\infty} \frac{(-1)^{3k} \left(\frac{3\pi}{8}\right)^{2k}}{(2k)!}}{1 + \sum_{k=0}^{\infty} \frac{(-1)^{3k} \left(\frac{3\pi}{8}\right)^{2k}}{(2k)!}} \right)$$

$$\frac{1}{15} \log \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right) 3 = \frac{1}{5} \log \left(\frac{1 - \sum_{k=0}^{\infty} \frac{(-1)^k 8^{-1-2k} \pi^{1+2k}}{(1+2k)!}}{1 + \sum_{k=0}^{\infty} \frac{(-1)^k 8^{-1-2k} \pi^{1+2k}}{(1+2k)!}} \right)$$

Integral representations:

$$\frac{1}{15} \log \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right) 3 = \frac{1}{5} \int_1^{\frac{1 - \sin\left(\frac{\pi}{8}\right)}{1 + \sin\left(\frac{\pi}{8}\right)}} \frac{1}{t} dt$$

$$\frac{1}{15} \log \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right) 3 = \frac{1}{5} \log \left(\frac{8 - \pi \int_0^1 \cos\left(\frac{\pi t}{8}\right) dt}{8 + \pi \int_0^1 \cos\left(\frac{\pi t}{8}\right) dt} \right)$$

$$\frac{1}{15} \log \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right) 3 = \frac{1}{5} \log \left(\frac{32i - \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-\pi^2/(256s)+s}}{s^{3/2}} ds}{32i + \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-\pi^2/(256s)+s}}{s^{3/2}} ds} \right) \text{ for } \gamma > 0$$

$$\frac{1}{15} \log \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right) 3 = \frac{1}{5} \log \left(\frac{2\sqrt{\pi} + i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(\frac{\pi}{16}\right)^{1-2s} \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} ds}{2\sqrt{\pi} - i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(\frac{\pi}{16}\right)^{1-2s} \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} ds} \right) \text{ for } 0 < \gamma < 1$$

And:

$$\varphi_{-1c} = \frac{1}{3} \log \left(\frac{\sqrt{2} - 1}{\sqrt{2} + 1} \right)$$

$$\frac{1}{3} \ln \left(\frac{\sqrt{2}-1}{\sqrt{2}+1} \right)$$

Input:

$$\frac{1}{3} \log \left(\frac{\sqrt{2} - 1}{\sqrt{2} + 1} \right)$$

log(x) is the natural logarithm

Decimal approximation:

-0.58758239134636201682173954998652820601877355217442360716...

-0.587582391346....

Property:

$$\frac{1}{3} \log \left(\frac{-1 + \sqrt{2}}{1 + \sqrt{2}} \right) \text{ is a transcendental number}$$

Alternate forms:

$$\frac{1}{3} \log(3 - 2\sqrt{2})$$

$$\frac{1}{3} \left(\log(\sqrt{2} - 1) - \log(1 + \sqrt{2}) \right)$$

$$\frac{1}{3} \log(\sqrt{2} - 1) - \frac{1}{3} \log(1 + \sqrt{2})$$

Alternative representations:

$$\frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = \frac{1}{3} \log_e\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}\right)$$

$$\frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = \frac{1}{3} \log(a) \log_a\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}\right)$$

$$\frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = -\frac{1}{3} \operatorname{Li}_1\left(1 - \frac{-1+\sqrt{2}}{1+\sqrt{2}}\right)$$

Series representations:

$$\frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = -\frac{1}{3} \sum_{k=1}^{\infty} \frac{(-1)^k (2-2\sqrt{2})^k}{k}$$

$$\frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = \frac{2}{3} i\pi \left[\frac{\arg(3-2\sqrt{2}-x)}{2\pi} \right] + \frac{\log(x)}{3} - \frac{1}{3} \sum_{k=1}^{\infty} \frac{(-1)^k (3-2\sqrt{2}-x)^k x^{-k}}{k}$$

for $x < 0$

$$\frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = \frac{2}{3} i\pi \left[\frac{\arg\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}-x\right)}{2\pi} \right] + \frac{\log(x)}{3} - \frac{1}{3} \sum_{k=1}^{\infty} \frac{(-1)^k (3-2\sqrt{2}-x)^k x^{-k}}{k}$$

for $x < 0$

Integral representation:

$$\frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = \frac{1}{3} \int_1^{3-2\sqrt{2}} \frac{1}{t} dt$$

From the algebraic sum of the two expressions, we obtain:

$$\left[\left(\left(\left(-\frac{3}{15} \ln\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) \right) - \frac{1}{3} \ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) \right) \right) \right]$$

Input:

$$-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$-\frac{1}{3} \log\left(\frac{\sqrt{2}-1}{1+\sqrt{2}}\right) - \frac{1}{5} \log\left(\frac{1-\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)$$

Decimal approximation:

0.748862279010966615143151964380842186689086787489932239044...

0.74886227901....

Alternate forms:

$$\begin{aligned} & \frac{2}{5} \tanh^{-1}\left(\sin\left(\frac{\pi}{8}\right)\right) - \frac{1}{3} \log(3-2\sqrt{2}) \\ & \frac{1}{15} \left(-5 \log(3-2\sqrt{2}) - 3 \log\left(1-\sin\left(\frac{\pi}{8}\right)\right) + 3 \log\left(1+\sin\left(\frac{\pi}{8}\right)\right)\right) \\ & -\frac{1}{3} \log(3-2\sqrt{2}) - \frac{1}{5} \log\left(7-4\sqrt{2}-2\sqrt{2(10-7\sqrt{2})}\right) \end{aligned}$$

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

Alternative representations:

$$\begin{aligned} & \frac{1}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) (-3) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = \\ & -\frac{1}{3} \log\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}\right) - \frac{3}{15} \log\left(\frac{1+\cosh\left(-\frac{5i\pi}{8}\right)}{1-\cosh\left(-\frac{5i\pi}{8}\right)}\right) \end{aligned}$$

$$\begin{aligned} & \frac{1}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) (-3) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = \\ & -\frac{1}{3} \log\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}\right) - \frac{3}{15} \log\left(\frac{1+\cosh\left(\frac{5i\pi}{8}\right)}{1-\cosh\left(\frac{5i\pi}{8}\right)}\right) \end{aligned}$$

$$\begin{aligned} & \frac{1}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) (-3) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = \\ & -\frac{1}{3} \log(a) \log_a\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}\right) - \frac{3}{15} \log(a) \log_a\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) \end{aligned}$$

Series representations:

$$\frac{1}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) (-3) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = \sum_{k=1}^{\infty} \frac{(-1)^k \left(5(2-2\sqrt{2})^k + 3\left(-\frac{2\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)^k\right)}{15k}$$

$$\frac{1}{15} \log \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right) (-3) - \frac{1}{3} \log \left(\frac{\sqrt{2} - 1}{\sqrt{2} + 1} \right) =$$

$$\sum_{k=1}^{\infty} \left(-\frac{(-1)^{-1+k} \left(-1 + \frac{-1+\sqrt{2}}{1+\sqrt{2}}\right)^k}{3k} - \frac{(-1)^{-1+k} \left(-1 + \frac{1-\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)^k}{5k} \right)$$

Integral representations:

$$\frac{1}{15} \log \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right) (-3) - \frac{1}{3} \log \left(\frac{\sqrt{2} - 1}{\sqrt{2} + 1} \right) =$$

$$\int_1^{3-2\sqrt{2}} \frac{8t \sin\left(\frac{\pi}{8}\right) + 5(-1 + \sqrt{2} + (-2 + \sqrt{2}) \sin\left(\frac{\pi}{8}\right))}{15t(-1 + \sqrt{2} + (-2 + \sqrt{2}) \sin\left(\frac{\pi}{8}\right) + t \sin\left(\frac{\pi}{8}\right))} dt$$

$$\frac{1}{15} \log \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right) (-3) - \frac{1}{3} \log \left(\frac{\sqrt{2} - 1}{\sqrt{2} + 1} \right) =$$

$$\frac{1}{15} \left(-5 \log(3 - 2\sqrt{2}) - 3 \log \left(\frac{8 - \pi \int_0^1 \cos\left(\frac{\pi t}{8}\right) dt}{8 + \pi \int_0^1 \cos\left(\frac{\pi t}{8}\right) dt} \right) \right)$$

$$\frac{1}{15} \log \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right) (-3) - \frac{1}{3} \log \left(\frac{\sqrt{2} - 1}{\sqrt{2} + 1} \right) =$$

$$\int_1^{3-2\sqrt{2}} \left(-\frac{1}{3t} - \frac{(-2 + 2\sqrt{2}) \left(-1 + \frac{1-\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)}{5(2 - 2\sqrt{2}) \left(-3 + 2\sqrt{2} + t + \frac{1-\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)} - \frac{t(1-\sin\left(\frac{\pi}{8}\right))}{1+\sin\left(\frac{\pi}{8}\right)}\right)} \right) dt$$

Multiple-argument formula:

$$\frac{1}{15} \log \left(\frac{1 + \cos\left(\frac{5\pi}{8}\right)}{1 - \cos\left(\frac{5\pi}{8}\right)} \right) (-3) - \frac{1}{3} \log \left(\frac{\sqrt{2} - 1}{\sqrt{2} + 1} \right) =$$

$$\frac{1}{3} \left(-\log(-1 + \sqrt{2}) + \log(1 + \sqrt{2}) \right) + \frac{1}{5} \left(-\log\left(1 - \sin\left(\frac{\pi}{8}\right)\right) + \log\left(1 + \sin\left(\frac{\pi}{8}\right)\right) \right)$$

From which:

$$\sqrt[2]{2 \times \frac{1}{\left(-\frac{3}{15} \ln\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right)}}$$

Input:

$$\sqrt[2]{2 \times \frac{1}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)}}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\sqrt[2]{\frac{2}{-\frac{1}{3} \log\left(\frac{\sqrt{2}-1}{1+\sqrt{2}}\right) - \frac{1}{5} \log\left(\frac{1-\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)}}$$

Decimal approximation:

1.634233166543416228388539660999984146741084989691121141309...

1.63423316654...

Alternate forms:

$$\sqrt[2]{\frac{30}{6 \tanh^{-1}\left(\sin\left(\frac{\pi}{8}\right)\right) - 5 \log(3 - 2\sqrt{2})}}$$

$$\sqrt[2]{\frac{30}{5 \log(3 - 2\sqrt{2}) + 3 \log\left(1 - \sin\left(\frac{\pi}{8}\right)\right) - 3 \log\left(1 + \sin\left(\frac{\pi}{8}\right)\right)}}$$

$$\sqrt[2]{\frac{2}{-\frac{1}{3} \log(3 - 2\sqrt{2}) - \frac{1}{5} \log\left(7 - 4\sqrt{2} - 2\sqrt{2(10 - 7\sqrt{2})}\right)}}$$

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

All 2nd roots of $2/(-1/3 \log((\sqrt{2}-1)/(1+\sqrt{2}))) - 1/5 \log((1-\sin(\pi/8))/(1+\sin(\pi/8)))$):

$$e^0 \sqrt[2]{\frac{2}{-\frac{1}{3} \log\left(\frac{\sqrt{2}-1}{1+\sqrt{2}}\right) - \frac{1}{5} \log\left(\frac{1-\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)}} \approx 1.6342 \text{ (real, principal root)}$$

$$e^{i\pi} \sqrt[2]{\frac{2}{-\frac{1}{3} \log\left(\frac{\sqrt{2}-1}{1+\sqrt{2}}\right) - \frac{1}{5} \log\left(\frac{1-\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)}} \approx -1.6342 \text{ (real root)}$$

Alternative representations:

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)}} = \sqrt{\frac{2}{-\frac{1}{3} \log\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}\right) - \frac{3}{15} \log\left(\frac{1+\cosh\left(-\frac{5i\pi}{8}\right)}{1-\cosh\left(-\frac{5i\pi}{8}\right)}\right)}}$$

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)}} = \sqrt{\frac{2}{-\frac{1}{3} \log\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}\right) - \frac{3}{15} \log\left(\frac{1+\cosh\left(\frac{5i\pi}{8}\right)}{1-\cosh\left(\frac{5i\pi}{8}\right)}\right)}}$$

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)}} = \sqrt{\frac{2}{-\frac{1}{3} \log_e\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}\right) - \frac{3}{15} \log_e\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right)}}$$

Series representations:

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)}} = \sqrt{30} \sqrt{\frac{1}{\sum_{k=1}^{\infty} \frac{(-1)^k \left(5(2-2\sqrt{2})^k + 3\left(-\frac{2\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)^k\right)}{k}}}}$$

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)}} = \sqrt{2} \sqrt{\frac{1}{\sum_{k=1}^{\infty} \frac{(-1)^k \left(5(2-2\sqrt{2})^k + 3\left(-\frac{2\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)^k\right)}{15k}}}}$$

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)}} = \sqrt{2} \sqrt{\frac{1}{\sum_{k=1}^{\infty} \left(-\frac{(-1)^{-1+k} \left(-1+\frac{-1+\sqrt{2}}{1+\sqrt{2}}\right)^k}{3k} - \frac{(-1)^{-1+k} \left(-1+\frac{1-\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)^k}{5k} \right)}}$$

Integral representations:

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)}} = \sqrt{30} \sqrt{\frac{1}{5 \log\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}\right) + 3 \log\left(\frac{1-\frac{\pi}{8} \int_0^1 \cos\left(\frac{\pi t}{8}\right) dt}{1+\frac{\pi}{8} \int_0^1 \cos\left(\frac{\pi t}{8}\right) dt}\right)}}$$

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)}} =$$

$$\sqrt{30} \sqrt{\frac{1}{\int_1^{3-2\sqrt{2}} \left(-\frac{5}{t} - \frac{3(-2+2\sqrt{2})\left(-1+\frac{1-\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)}{(2-2\sqrt{2})\left(-3+2\sqrt{2}+t+\frac{1-\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)} - \frac{t(1-\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)} \right) dt}}$$

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)}} =$$

$$\sqrt{30} \sqrt{\frac{1}{5 \log\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}\right) + 3 \log\left(\frac{1+\frac{i\sqrt{\pi}}{32} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-\pi^2/(256s)+s}}{s^{3/2}} ds}{1-\frac{i\sqrt{\pi}}{32} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-\pi^2/(256s)+s}}{s^{3/2}} ds}\right)}}} \quad \text{for } \gamma > 0$$

Multiple-argument formula:

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)}} =$$

$$\sqrt{\frac{2}{\frac{1}{3} (-\log(-1+\sqrt{2}) + \log(1+\sqrt{2})) + \frac{1}{5} (-\log(1-\sin\left(\frac{\pi}{8}\right)) + \log(1+\sin\left(\frac{\pi}{8}\right)))}}$$

And:

$$\text{sqrt}[2*1/(((-3/15 \ln(((1+\cos((5\text{Pi})/8))/(1-\cos((5\text{Pi})/8)))) - 1/3 \ln ((\text{sqrt}2-1)/(\text{sqrt}2+1)))))] - 16/10^3$$

Input:

$$\sqrt{2 \times \frac{1}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)} - \frac{16}{10^3}}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\sqrt{-\frac{1}{3} \log\left(\frac{\sqrt{2}-1}{1+\sqrt{2}}\right) - \frac{1}{5} \log\left(\frac{1-\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)} - \frac{2}{125}$$

Decimal approximation:

1.618233166543416228388539660999984146741084989691121141309...

1.61823316654341.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

Alternate forms:

$$\sqrt{\frac{30}{6 \tanh^{-1}\left(\sin\left(\frac{\pi}{8}\right)\right) - 5 \log(3 - 2\sqrt{2})} - \frac{2}{125}}$$

$$\sqrt{-\frac{30}{5 \log(3 - 2\sqrt{2}) + 3 \log\left(1 - \sin\left(\frac{\pi}{8}\right)\right) - 3 \log\left(1 + \sin\left(\frac{\pi}{8}\right)\right)} - \frac{2}{125}}$$

$$\sqrt{\frac{2}{-\frac{1}{3} \log(3 - 2\sqrt{2}) - \frac{1}{5} \log\left(7 - 4\sqrt{2} - 2\sqrt{2(10 - 7\sqrt{2})}\right)} - \frac{2}{125}}$$

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

Alternative representations:

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)} - \frac{16}{10^3} =$$

$$-\frac{16}{10^3} + \sqrt{\frac{2}{-\frac{1}{3} \log\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}\right) - \frac{3}{15} \log\left(\frac{1+\cosh\left(-\frac{5i\pi}{8}\right)}{1-\cosh\left(-\frac{5i\pi}{8}\right)}\right)}}$$

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)} - \frac{16}{10^3} =$$

$$-\frac{16}{10^3} + \sqrt{\frac{2}{-\frac{1}{3} \log\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}\right) - \frac{3}{15} \log\left(\frac{1+\cosh\left(\frac{5i\pi}{8}\right)}{1-\cosh\left(\frac{5i\pi}{8}\right)}\right)}}$$

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)} - \frac{16}{10^3}} =$$

$$-\frac{16}{10^3} + \sqrt{\frac{2}{-\frac{1}{3} \log_e\left(\frac{-1+\sqrt{2}}{1+\sqrt{2}}\right) - \frac{3}{15} \log_e\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right)}}$$

Series representations:

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)} - \frac{16}{10^3}} =$$

$$-\frac{2}{125} + \sqrt{30} \sqrt{\frac{1}{\sum_{k=1}^{\infty} \frac{(-1)^k \left(5(2-2\sqrt{2})^k + 3 \left(-\frac{2 \sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)^k\right)}{k}}}$$

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)} - \frac{16}{10^3}} =$$

$$-\frac{2}{125} + \sqrt{2} \sqrt{\frac{1}{\sum_{k=1}^{\infty} \frac{(-1)^k \left(5(2-2\sqrt{2})^k + 3 \left(-\frac{2 \sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)^k\right)}{15k}}}$$

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)} - \frac{16}{10^3}} =$$

$$-\frac{2}{125} + \sqrt{2} \sqrt{\frac{1}{\sum_{k=1}^{\infty} \left(\frac{(-1)^{-1+k} \left(-1 + \frac{-1+\sqrt{2}}{1+\sqrt{2}}\right)^k}{3k} - \frac{(-1)^{-1+k} \left(-1 + \frac{1-\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)}\right)^k}{5k} \right)}}$$

Integral representations:

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)} - \frac{16}{10^3} = \frac{1}{125} \left(-2 + 125 \sqrt{30} \sqrt{-\frac{1}{5 \log(3-2\sqrt{2}) + 3 \log\left(\frac{8-\pi \int_0^1 \cos\left(\frac{\pi t}{8}\right) dt}{8+\pi \int_0^1 \cos\left(\frac{\pi t}{8}\right) dt}\right)}} \right)$$

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)} - \frac{16}{10^3} = -\frac{2}{125} + \sqrt{30} \sqrt{\frac{1}{\int_1^{3-2\sqrt{2}} \frac{-8t \sin\left(\frac{\pi}{8}\right) - 5(-1+\sqrt{2} + (-2+\sqrt{2}) \sin\left(\frac{\pi}{8}\right))}{t(-1+\sqrt{2} + (-2+\sqrt{2}) \sin\left(\frac{\pi}{8}\right) + t \sin\left(\frac{\pi}{8}\right))} dt}}$$

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)} - \frac{16}{10^3} = -\frac{2}{125} + \sqrt{30} \sqrt{\frac{1}{\int_1^{3-2\sqrt{2}} \left(-\frac{5}{t} - \frac{3(-2+2\sqrt{2}) \left(-1 + \frac{1-\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)} \right)}{(2-2\sqrt{2}) \left(-3+2\sqrt{2} + t + \frac{1-\sin\left(\frac{\pi}{8}\right)}{1+\sin\left(\frac{\pi}{8}\right)} - \frac{t(1-\sin\left(\frac{\pi}{8}\right))}{1+\sin\left(\frac{\pi}{8}\right)} \right)} \right) dt}}$$

Multiple-argument formula:

$$\sqrt{\frac{2}{-\frac{3}{15} \log\left(\frac{1+\cos\left(\frac{5\pi}{8}\right)}{1-\cos\left(\frac{5\pi}{8}\right)}\right) - \frac{1}{3} \log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)} - \frac{16}{10^3} = -\frac{2}{125} + \sqrt{\frac{2}{\frac{1}{3} (-\log(-1 + \sqrt{2}) + \log(1 + \sqrt{2})) + \frac{1}{5} (-\log(1 - \sin\left(\frac{\pi}{8}\right)) + \log(1 + \sin\left(\frac{\pi}{8}\right)))}}$$

Now, we have that:

Let us now turn to the detailed fixed–point analysis of an interesting class of potentials, not all integrable but whose choice is inspired the families of potentials in Table 1. We shall occasionally distinguish various ranges of the relevant parameters, and for brevity we shall mostly leave out fixed points at infinity, unless they are the only ones present, as will be the case for the last examples. In the corresponding lists we shall reserve boldface characters to the physically more relevant cases of potentials bounded from below and we shall treat the two cases of systems evolving from a Big Bang (corresponding to $\sigma = 1$ in eqs. (4.4) or (4.7)) or evolving toward a Big Crunch (corresponding to $\sigma = -1$ in eqs. (4.4) or (4.7)).

1. The potentials $\mathcal{V}(\varphi) = C \cosh(w\varphi) + \Lambda$

The class of potentials

$$\mathcal{V}(\varphi) = C \cosh(w\varphi) + \Lambda, \quad (C \neq 0, \quad w \neq 0). \quad (4.35)$$

possesses an isolated fixed point,

$$v_c = 0, \quad \varphi_c = 0, \quad (4.36)$$

which is admissible provided

$$\mathcal{V}(\varphi_c) = C + \Lambda \geq 0. \quad (4.37)$$

As we shall see in the following subsections, the condition (4.37) has an important physical consequence: the exact solutions for potential wells of this type will show indeed that when it is not fulfilled a scalar trying to settle at the extremum will readily run away. This behavior reflects, all in all, a familiar fact, the absence of spatially flat AdS slices.

The eigenvalues of eq. (4.13) for the potentials (4.35) read

$$\lambda_{\pm} = -\sigma \sqrt{\frac{C + \Lambda}{2}} \pm \sqrt{\frac{C + \Lambda}{2} - Cw^2}, \quad (4.38)$$

so that in this case the admissible fixed point is simple (not degenerate) since $\lambda_{\pm} \neq 0$. Depending on the values of the parameters, these eigenvalues can correspond to a hyperbolic fixed point or alternatively to an elliptic one.

For $C + \Lambda = 34$; $C = 13$; $w = 8$

$-\sqrt{17} + \sqrt{17 - 13 \cdot 64}$

Input:

$-\sqrt{17} + \sqrt{17 - 13 \times 64}$

Result:

$-\sqrt{17} + i\sqrt{815}$

Decimal approximation:

-4.1231056256176605498214098559740770251471992253736204343... +
28.548204847240395265910819287672173583484520269628371294... i

Polar coordinates:

$r \approx 28.8444$ (radius), $\theta \approx 98.2182^\circ$ (angle)

28.8444

Alternate form:

$$-\sqrt{2(-399 - i\sqrt{13855})}$$

Minimal polynomial:

$$x^4 + 1596x^2 + 692224$$

From which:

$$((- \sqrt{17} + \sqrt{-(17 - 13 \cdot 64)})^2$$

Input:

$$\left(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)}\right)^2$$

Result:

$$\left(\sqrt{815} - \sqrt{17}\right)^2$$

Decimal approximation:

596.5854719861155212990779794796614192785758277489773582149...

596.5854719...

Alternate forms:

$$2(416 - \sqrt{13855})$$

$$832 - 2\sqrt{13855}$$

$$\left(\sqrt{17} - \sqrt{815}\right)^2$$

Minimal polynomial:

$$x^2 - 1664x + 636804$$

$$(1+1/((-sqr(17)+sqr-(17-13*64)))^2)$$

Input:

$$1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2}$$

Result:

$$1 + \frac{1}{(\sqrt{815} - \sqrt{17})^2}$$

Decimal approximation:

1.001676205752498232546750526096758717879789423703762888803...

1.0016762057... result very near to the following to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{\frac{2\pi}{5}}}{\sqrt{\phi\sqrt{5} - \phi}} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}} \approx 1.0018674362$$

Alternate forms:

$$\frac{318818 + \sqrt{13855}}{318402}$$

$$\frac{\sqrt{13855}}{318402} + \frac{159409}{159201}$$

$$1 + \frac{1}{832 - 2\sqrt{13855}}$$

Minimal polynomial:

$$636804x^2 - 1275272x + 638469$$

$$[1*1/(1+1/((-sqr(17)+sqr-(17-13*64)))^2)]^1/2$$

Input:

$$\sqrt{1 \times \frac{1}{1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2}}}$$

Result:

$$\frac{1}{\sqrt{1 + \frac{1}{(\sqrt{815} - \sqrt{17})^2}}}$$

Decimal approximation:

0.999162949278809895840493185366552794874156055136422553192...

0.99916294927... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = ϕ**

Alternate forms:

$$\frac{\sqrt{141882(318818 - \sqrt{13855})}}{212823}$$

$$\frac{1}{\sqrt{1 + \frac{1}{832 - 2\sqrt{13855}}}}$$

$$\frac{1}{3} \sqrt{\frac{2(318818 - \sqrt{13855})}{70941}}$$

Minimal polynomial:

$$638469x^4 - 1275272x^2 + 636804$$

$$[(1+1/((-sqr(17)+sqr-(17-13*64)))^2)]^1/4096$$

Input:

$$\sqrt[4096]{1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2}}$$

Exact result:

$$\sqrt[4096]{1 + \frac{1}{(\sqrt{815} - \sqrt{17})^2}}$$

Decimal approximation:

1.000000408887409650159029727801964351565272441982968263068...

1.000000408887409... result very near to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{\frac{2\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \frac{e^{-6\pi\sqrt{5}}}{1 + \frac{e^{-8\pi\sqrt{5}}}{1 + \dots}}}} \approx 1.0000007913$$

Alternate forms:

$$\sqrt[4096]{1 + \frac{1}{832 - 2\sqrt{13855}}}$$

$$\frac{\sqrt[4096]{833 - 2\sqrt{13855}}}{\sqrt[2048]{\sqrt{815} - \sqrt{17}}}$$

2sqrt((((log base 1.0000004088874096 [(1+1/((-sqr(17)+sqr-(17-13*64)))^2)]))))-
Pi+1/golden ratio

Input interpretation:

$$2 \sqrt{\log_{1.0000004088874096} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right) - \pi + \frac{1}{\phi}}$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

125.47644134...

125.47644134... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV

Alternative representation:

$$2 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)} - \pi + \frac{1}{\phi} =$$

$$-\pi + \frac{1}{\phi} + 2 \sqrt{\frac{\log \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{815})^2} \right)}{\log(1.00000040888740960000)}}$$

Series representations:

$$2 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)} - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + 2 \sqrt{-\frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{1}{(\sqrt{17} - \sqrt{815})^2} \right)^k}{k}}{\log(1.00000040888740960000)}}$$

$$2 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)} - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + 2 \sqrt{-1 + \log_{1.00000040888740960000} \left(1 + \frac{1}{(\sqrt{17} - \sqrt{815})^2} \right)}$$

$$\sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-1 + \log_{1.00000040888740960000} \left(1 + \frac{1}{(\sqrt{17} - \sqrt{815})^2} \right) \right)^{-k}$$

$$2 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + 2 \sqrt{\left(-1.0000000000000000 \log \left(1 + \frac{1}{(\sqrt{17} - \sqrt{815})^2} \right) \right.$$

$$\left. \left(-2.44566152188929 \times 10^6 + \sum_{k=0}^{\infty} 4.0888740960000 \times 10^{-7k} G(k) \right) \right)}$$

for $G(0) = 0$ and $G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j}$

2sqrt((((log base 1.0000004088874096 [(1+1/((-sqr(17)+sqr-(17-13*64)))^2])))))+11+1/golden ratio

where 11 is a Lucas number

Input interpretation:

$$2 \sqrt{\log_{1.0000004088874096} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right) + 11 + \frac{1}{\phi}}$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

139.61803400...

139.618034... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternative representation:

$$2 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + 2 \sqrt{\frac{\log \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{815})^2} \right)}{\log(1.00000040888740960000)}}$$

Series representations:

$$2 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)} + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + 2 \sqrt{-\frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{1}{(\sqrt{17} - \sqrt{815})^2} \right)^k}{k}}{\log(1.00000040888740960000)}}$$

$$2 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)} + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + 2 \sqrt{-1 + \log_{1.00000040888740960000} \left(1 + \frac{1}{(\sqrt{17} - \sqrt{815})^2} \right)}$$

$$\sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-1 + \log_{1.00000040888740960000} \left(1 + \frac{1}{(\sqrt{17} - \sqrt{815})^2} \right) \right)^{-k}$$

$$2 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)} + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + 2 \sqrt{\left(-1.0000000000000000 \log \left(1 + \frac{1}{(\sqrt{17} - \sqrt{815})^2} \right) \right.$$

$$\left. \left. - 2.44566152188929 \times 10^6 + \sum_{k=0}^{\infty} 4.0888740960000 \times 10^{-7k} G(k) \right) \right)}$$

$$\text{for } \left(G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

$$27 * \text{sqrt}(\text{((((log base 1.0000004088874096 [(1+1/((-sqrt(17)+sqr-(17-13*64)))^2]))))))$$

Input interpretation:

$$27 \sqrt{\log_{1.0000004088874096} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)}$$

$\log_b(x)$ is the base- b logarithm

Result:

1728.0000001...

1728.0000001...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Alternative representation:

$$27 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)} =$$

$$27 \sqrt{\frac{\log \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{815})^2} \right)}{\log(1.00000040888740960000)}}$$

Series representations:

$$27 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)} =$$

$$27 \sqrt{\frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{1}{(\sqrt{17} - \sqrt{815})^2} \right)^k}{k}}{\log(1.00000040888740960000)}}$$

$$27 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)} =$$

$$27 \sqrt{-1 + \log_{1.00000040888740960000} \left(1 + \frac{1}{(\sqrt{17} - \sqrt{815})^2} \right)}$$

$$\sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-1 + \log_{1.00000040888740960000} \left(1 + \frac{1}{(\sqrt{17} - \sqrt{815})^2} \right) \right)^{-k}$$

$$27 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)} =$$

$$27 \sqrt{\left(-1.0000000000000000 \log \left(1 + \frac{1}{(\sqrt{17} - \sqrt{815})^2} \right) \right.$$

$$\left. \left(-2.44566152188929 \times 10^6 + \sum_{k=0}^{\infty} 4.0888740960000 \times 10^{-7k} G(k) \right) \right)}$$

for $G(0) = 0$ and $\frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j}$

$$27 * \text{sqrt}(\text{((log base 1.0000004088874096 [(1+1/((-sqrt(17)+sqrt-(17-13*64)))^2])))) + 55$$

where 55 is a Fibonacci number

Input interpretation:

$$27 \sqrt{\log_{1.0000004088874096} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)} + 55$$

$\log_b(x)$ is the base- b logarithm

Result:

1783.0000001...

1783.0000001... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

Alternative representation:

$$27 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)} + 55 =$$

$$55 + 27 \sqrt{\frac{\log \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{815})^2} \right)}{\log(1.00000040888740960000)}}$$

Series representations:

$$27 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)} + 55 =$$

$$55 + 27 \sqrt{-\frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{1}{(\sqrt{17} - \sqrt{815})^2} \right)^k}{k}}{\log(1.00000040888740960000)}}$$

$$27 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)} + 55 =$$

$$55 + 27 \sqrt{-1 + \log_{1.00000040888740960000} \left(1 + \frac{1}{(\sqrt{17} - \sqrt{815})^2} \right)}$$

$$\sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(-1 + \log_{1.00000040888740960000} \left(1 + \frac{1}{(\sqrt{17} - \sqrt{815})^2} \right) \right)^{-k}$$

$$27 \sqrt{\log_{1.00000040888740960000} \left(1 + \frac{1}{(-\sqrt{17} + \sqrt{-(17 - 13 \times 64)})^2} \right)} + 55 =$$

$$55 + 27 \sqrt{\left(-1.0000000000000000 \log \left(1 + \frac{1}{(\sqrt{17} - \sqrt{815})^2} \right) \right.}$$

$$\left. \left(-2.44566152188929 \times 10^6 + \sum_{k=0}^{\infty} 4.0888740960000 \times 10^{-7k} G(k) \right) \right)}$$

for $\left(G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

Now, we have that:

$$v_{kc} = 0, \quad \varphi_{kc} = \frac{1}{4\gamma} \log \left(\frac{1 + \cos \frac{\frac{\pi}{2} + \pi k - \theta}{\frac{1}{\gamma} - 2}}{1 - \cos \frac{\frac{\pi}{2} + \pi k - \theta}{\frac{1}{\gamma} - 2}} \right), \quad (4.152)$$

For:

$$\gamma = 0.8, \quad 1/\gamma = 1.25, \quad k = 3, \quad \theta = \pi$$

$$1/(4 \times 0.8) \ln \left(\frac{1 + \cos\left(\frac{\pi/2 + 3\pi - \pi}{1.25 - 2}\right)}{1 - \cos\left(\frac{\pi/2 + 3\pi - \pi}{1.25 - 2}\right)} \right)$$

Input:

$$\frac{1}{4 \times 0.8} \log \left(\frac{1 + \cos\left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2}\right)}{1 - \cos\left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2}\right)} \right)$$

log(x) is the natural logarithm

Result:

-0.343316...

-0.343316...

Alternative representations:

$$\frac{\log \left(\frac{1 + \cos\left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2}\right)}{1 - \cos\left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2}\right)} \right)}{4 \times 0.8} = \frac{\log \left(\frac{1 + \cosh\left(\frac{5i\pi}{2(-0.75)}\right)}{1 - \cosh\left(\frac{5i\pi}{2(-0.75)}\right)} \right)}{3.2}$$

$$\frac{\log \left(\frac{1 + \cos\left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2}\right)}{1 - \cos\left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2}\right)} \right)}{4 \times 0.8} = \frac{\log_e \left(\frac{1 + \cos\left(\frac{5\pi}{2(-0.75)}\right)}{1 - \cos\left(\frac{5\pi}{2(-0.75)}\right)} \right)}{3.2}$$

$$\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8} = \frac{\log(a) \log_a\left(\frac{1+\cos\left(\frac{5\pi}{2(-0.75)}\right)}{1-\cos\left(\frac{5\pi}{2(-0.75)}\right)}\right)}{3.2}$$

Series representations:

$$\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8} = -0.3125 \sum_{k=1}^{\infty} \frac{(-2)^k \left(\frac{\cos(-3.33333\pi)}{1-\cos(-3.33333\pi)}\right)^k}{k}$$

$$\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8} = 0.3125 \log\left(\frac{1 + \sum_{k=0}^{\infty} \frac{(-1)^k e^{2.40795k} (-\pi)^{2k}}{(2k)!}}{1 - \sum_{k=0}^{\infty} \frac{(-1)^k e^{2.40795k} (-\pi)^{2k}}{(2k)!}}\right)$$

$$\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8} = 0.3125 \log\left(\frac{1 + \pi \sum_{k=0}^{\infty} \frac{(-1)^k 3.83333^{1+2k} (-\pi)^{2k}}{(1+2k)!}}{-1 + \pi \sum_{k=0}^{\infty} \frac{(-1)^k 3.83333^{1+2k} (-\pi)^{2k}}{(1+2k)!}}\right)$$

Integral representations:

$$\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8} = 0.3125 \int_1^{\frac{1+\cos(-3.33333\pi)}{1-\cos(-3.33333\pi)}} \frac{1}{t} dt$$

$$\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8} = 0.3125 \log\left(-1 - \frac{0.6}{\pi \int_0^1 \sin(-3.33333\pi t) dt}\right)$$

$$\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8} = 0.3125 \log\left(\frac{1 - \int_{\frac{\pi}{2}}^{-3.33333\pi} \sin(t) dt}{1 + \int_{\frac{\pi}{2}}^{-3.33333\pi} \sin(t) dt}\right)$$

$$\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8} = 0.3125 \log\left(\frac{2i\pi + \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-(2.77778\pi^2)/s+s}}{\sqrt{s}} ds}{2i\pi - \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-(2.77778\pi^2)/s+s}}{\sqrt{s}} ds}\right) \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8} = 0.3125 \log\left(-1 + \frac{1}{\sin^2(-1.66667\pi)}\right)$$

$$\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8} = 0.3125 \log\left(-\frac{\cos^2(-1.66667\pi)}{-1 + \cos^2(-1.66667\pi)}\right)$$

$$\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8} = 0.3125 \left(\log\left(\frac{1}{1 - \cos(-3.33333\pi)}\right) + \log(1 + \cos(-3.33333\pi))\right)$$

From which:

$$\sqrt{\left(\left(-\frac{1}{4 \times 0.8} \ln \left(\frac{1 + \cos\left(\frac{\pi}{2} + 3\pi - \pi\right)}{1.25 - 2}\right)\right) / \left(1 - \cos\left(\frac{\pi}{2} + 3\pi - \pi\right) / (1.25 - 2)\right)\right)\right)}$$

Input:

$$\sqrt{-\left(\frac{1}{4 \times 0.8} \log \left(\frac{1 + \cos\left(\frac{\pi}{2} + 3\pi - \pi\right)}{1.25 - 2}\right)\right)}$$

log(x) is the natural logarithm

Result:

0.585932026952601626228196884945220556485133085171369120457...

0.58593202695.... result near to the following Ramanujan continued fraction:

$$4 \int_0^{\infty} \frac{t dt}{e^{\sqrt{5}t} \cosh t} = \frac{1}{1 + \frac{1^2}{1 + \frac{1^2}{1 + \frac{2^2}{1 + \frac{2^2}{1 + \frac{3^2}{1 + \frac{3^2}{1 + \dots}}}}}}}} \approx 0.5683000031$$

All 2nd roots of 0.343316:

$0.585932 e^0 \approx 0.58593$ (real, principal root)

$0.585932 e^{i\pi} \approx -0.58593$ (real root)

Alternative representations:

$$\sqrt{-\frac{\log\left(\frac{1 + \cos\left(\frac{\pi}{2} + 3\pi - \pi\right)}{1.25 - 2}\right)}{4 \times 0.8}} = \sqrt{-\frac{\log\left(\frac{1 + \cosh\left(\frac{5i\pi}{2(-0.75)}\right)}{1 - \cosh\left(\frac{5i\pi}{2(-0.75)}\right)}\right)}{3.2}}$$

$$\sqrt{-\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8}} = \sqrt{-\frac{\log_e\left(\frac{1+\cos\left(\frac{5\pi}{2(-0.75)}\right)}{1-\cos\left(\frac{5\pi}{2(-0.75)}\right)}\right)}{3.2}}$$

$$\sqrt{-\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8}} = \sqrt{-\frac{\log(a) \log_a\left(\frac{1+\cos\left(\frac{5\pi}{2(-0.75)}\right)}{1-\cos\left(\frac{5\pi}{2(-0.75)}\right)}\right)}{3.2}}$$

Series representations:

$$\sqrt{-\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1 - 0.3125 \log\left(\frac{1+\cos(-3.33333\pi)}{1-\cos(-3.33333\pi)}\right)\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{-\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8}} = \exp\left(i\pi \left[\frac{\arg\left(-x - 0.3125 \log\left(\frac{1+\cos(-3.33333\pi)}{1-\cos(-3.33333\pi)}\right)\right)}{2\pi}\right]\right) \sqrt{x}$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} \left(-x - 0.3125 \log\left(\frac{1+\cos(-3.33333\pi)}{1-\cos(-3.33333\pi)}\right)\right)^k \left(-\frac{1}{2}\right)_k}{k!} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\sqrt{-\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8}} = \left(\frac{1}{z_0}\right)^{1/2} \left[\arg\left(-0.3125 \log\left(\frac{1+\cos(-3.33333\pi)}{1-\cos(-3.33333\pi)}\right)\right) - z_0\right] / (2\pi)$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-0.3125 \log\left(\frac{1+\cos(-3.33333\pi)}{1-\cos(-3.33333\pi)}\right) - z_0\right)^k z_0^{-k}}{k!}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\text{arg}(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

\mathbb{R} is the set of real numbers

Integral representations:

$$\sqrt{-\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8}} = \sqrt{-0.3125 \int_1^{\frac{1+\cos(-3.33333\pi)}{1-\cos(-3.33333\pi)}} \frac{1}{t} dt}$$

$$\sqrt{-\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8}} = \sqrt{-0.3125 \log\left(-1 - \frac{0.6}{\pi \int_0^1 \sin(-3.33333\pi t) dt}\right)}$$

$$\sqrt{-\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8}} = \sqrt{-0.3125 \log\left(\frac{1 - \int_{\frac{\pi}{2}}^{-3.33333\pi} \sin(t) dt}{1 + \int_{\frac{\pi}{2}}^{-3.33333\pi} \sin(t) dt}\right)}$$

$$\sqrt{-\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8}} = \sqrt{-0.3125 \log\left(\frac{2i\pi + \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-(2.77778\pi^2)/s+s}}{\sqrt{s}} ds}{2i\pi - \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-(2.77778\pi^2)/s+s}}{\sqrt{s}} ds}\right)} \text{ for } \gamma > 0$$

Multiple-argument formula:

$$\sqrt[2048]{-\frac{\log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)}{4 \times 0.8}} = \exp\left(i\pi \left[-\frac{-\pi + \arg(-0.3125) + \arg\left(\log\left(\frac{1+\cos(-3.33333\pi)}{1-\cos(-3.33333\pi)}\right)\right)}{2\pi}\right]\right)$$

$$\sqrt{-0.3125} \sqrt{\log\left(\frac{1+\cos(-3.33333\pi)}{1-\cos(-3.33333\pi)}\right)}$$

$(((-1/(4*0.8) \ln (((1+\cos((\pi/2+3\pi-\pi)/(1.25-2)))/(1-\cos((\pi/2+3\pi-\pi)/(1.25-2))))))))^1/2048$

Input:

$$\sqrt[2048]{-\left(\frac{1}{4 \times 0.8} \log\left(\frac{1+\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}{1-\cos\left(\frac{\frac{\pi}{2}+3\pi-\pi}{1.25-2}\right)}\right)\right)}$$

log(x) is the natural logarithm

Result:

0.99947811...

0.99947811... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{1 + \sqrt[5]{\sqrt{\varphi^5 4 \sqrt{5^3} - 1}}}{- \varphi + 1}$$

and to the dilaton value **0.989117352243 = ϕ**

$\frac{1}{16} \cdot \log_{\text{base } 0.99947811} \left(\left(\left(\left(\left(\left(\left(\frac{1}{4 \cdot 0.8} \right) \ln \left(\frac{1 + \cos\left(\frac{\pi}{2} + 3\pi - \pi\right)}{1.25 - 2}\right)}{1 - \cos\left(\frac{\pi}{2} + 3\pi - \pi\right)}{1.25 - 2}\right)} \right) \right) \right) \right) \right) - \pi + \frac{1}{\text{golden ratio}}$

Input interpretation:

$$\frac{1}{16} \log_{0.99947811} \left(\left(\left(\frac{1}{4 \times 0.8} \log \left(\frac{1 + \cos\left(\frac{\pi}{2} + 3\pi - \pi\right)}{1.25 - 2}\right)}{1 - \cos\left(\frac{\pi}{2} + 3\pi - \pi\right)}{1.25 - 2}\right) \right) \right) - \pi + \frac{1}{\phi}$$

$\log(x)$ is the natural logarithm

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

125.476...

125.476... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV

Alternative representations:

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos\left(\frac{\pi}{2} + 3\pi - \pi\right)}{1.25 - 2}\right)}{1 - \cos\left(\frac{\pi}{2} + 3\pi - \pi\right)}{1.25 - 2} \right) - \frac{\pi}{4 \times 0.8} + \frac{1}{\phi} =$$

$$-\pi + \frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cosh\left(\frac{5i\pi}{2(-0.75)}\right)}{1 - \cosh\left(\frac{5i\pi}{2(-0.75)}\right)} \right)}{3.2} \right) + \frac{1}{\phi}$$

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) - \pi + \frac{1}{\phi} =$$

$$-\pi + \frac{1}{16} \log_{0.999478} \left(\frac{\log_e \left(\frac{1 + \cos \left(\frac{5\pi}{2(-0.75)} \right)}{1 - \cos \left(\frac{5\pi}{2(-0.75)} \right)} \right)}{3.2} \right) + \frac{1}{\phi}$$

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) - \pi + \frac{1}{\phi} = -\pi + \frac{1}{\phi} + \frac{\log \left(\frac{\log \left(\frac{1 + \cos \left(\frac{5\pi}{2(-0.75)} \right)}{1 - \cos \left(\frac{5\pi}{2(-0.75)} \right)} \right)}{3.2} \right)}{16 \log(0.999478)}$$

Series representations:

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + \frac{1}{16} \log_{0.999478} \left(0.3125 \sum_{k=1}^{\infty} \frac{(-2)^k \left(\frac{\cos(-3.33333\pi)}{1 - \cos(-3.33333\pi)} \right)^k}{k} \right)$$

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\pi + 3\pi - \pi}{2} \right)}{1.25 - 2} \right)}{1 - \cos \left(\frac{\pi + 3\pi - \pi}{2} \right)} \right)}{4 \times 0.8} - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 - 0.3125 \log \left(\frac{1 + \cos(-3.33333\pi)}{1 - \cos(-3.33333\pi)} \right) \right)^k}{k}}{16 \log(0.999478)}$$

Integral representations:

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\pi + 3\pi - \pi}{2} \right)}{1.25 - 2} \right)}{1 - \cos \left(\frac{\pi + 3\pi - \pi}{2} \right)} \right)}{4 \times 0.8} - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + \frac{1}{16} \log_{0.999478} \left(-0.3125 \int_1^{\frac{1 + \cos(-3.33333\pi)}{1 - \cos(-3.33333\pi)}} \frac{1}{t} dt \right)$$

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\pi + 3\pi - \pi}{2} \right)}{1.25 - 2} \right)}{1 - \cos \left(\frac{\pi + 3\pi - \pi}{2} \right)} \right)}{4 \times 0.8} - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + \frac{1}{16} \log_{0.999478} \left(-0.3125 \log \left(-1 - \frac{0.6}{\pi \int_0^1 \sin(-3.33333\pi t) dt} \right) \right)$$

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{-16 + 16 \phi \pi - \phi \log_{0.999478} \left(-0.3125 \log \left(\frac{1 - \int_{\frac{\pi}{2}}^{-3.33333 \pi} \sin(t) dt}{2} \right) \right)}{16 \phi}$$

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{-16 + 16 \phi \pi - \phi \log_{0.999478} \left(-0.3125 \log \left(\frac{2i\pi + \sqrt{\pi} \int_{-i\infty + \gamma}^{i\infty + \gamma} \frac{e^{-(2.77778 \pi^2) / (s+s)} \sqrt{s}}{\sqrt{s}} ds}{2i\pi - \sqrt{\pi} \int_{-i\infty + \gamma}^{i\infty + \gamma} \frac{e^{-(2.77778 \pi^2) / (s+s)} \sqrt{s}}{\sqrt{s}} ds} \right) \right)}{16 \phi} \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi +$$

$$\frac{1}{16} \log_{0.999478} \left(-0.3125 \left(\log \left(\frac{1}{1 - \cos(-3.33333 \pi)} \right) + \log(1 + \cos(-3.33333 \pi)) \right) \right)$$

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\pi + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\pi + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + \frac{1}{16} \log_{0.999478} \left(\left[-0.3125 \left(2i\pi \left[-\frac{-\pi + \arg \left(\frac{1}{1 - \cos(-3.33333\pi)} \right) + \arg(1 + \cos(-3.33333\pi))}{2\pi} \right] + \log \left(\frac{1}{1 - \cos(-3.33333\pi)} \right) + \log(1 + \cos(-3.33333\pi)) \right] \right) \right)$$

1/16*log base 0.99947811(((((-1/(4*0.8) ln (((1+cos((Pi/2+3Pi-Pi)/(1.25-2)))/(1-cos((Pi/2+3Pi-Pi)/(1.25-2)))))))))))+11+1/golden ratio

where 11 is a Lucas number

Input interpretation:

$$\frac{1}{16} \log_{0.99947811} \left(\left(\frac{1}{4 \times 0.8} \log \left(\frac{1 + \cos \left(\frac{\pi + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\pi + 3\pi - \pi}{1.25 - 2} \right)} \right) \right) + 11 + \frac{1}{\phi} \right)$$

log(x) is the natural logarithm

log_b(x) is the base- b logarithm

φ is the golden ratio

Result:

139.617...

139.617... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternative representations:

$$\frac{1}{16} \log_{0.999478} \left(- \frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{16} \log_{0.999478} \left(- \frac{\log \left(\frac{1 + \cosh \left(\frac{5i\pi}{2(-0.75)} \right)}{1 - \cosh \left(\frac{5i\pi}{2(-0.75)} \right)} \right)}{3.2} \right) + \frac{1}{\phi}$$

$$\frac{1}{16} \log_{0.999478} \left(- \frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{16} \log_{0.999478} \left(- \frac{\log_e \left(\frac{1 + \cos \left(\frac{5\pi}{2(-0.75)} \right)}{1 - \cos \left(\frac{5\pi}{2(-0.75)} \right)} \right)}{3.2} \right) + \frac{1}{\phi}$$

$$\frac{1}{16} \log_{0.999478} \left(- \frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} + \frac{\log \left(- \frac{\log \left(\frac{1 + \cos \left(\frac{5\pi}{2(-0.75)} \right)}{1 - \cos \left(\frac{5\pi}{2(-0.75)} \right)} \right)}{3.2} \right)}{16 \log(0.999478)}$$

Series representations:

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{1}{16} \log_{0.999478} \left(0.3125 \sum_{k=1}^{\infty} \frac{(-2)^k \left(\frac{\cos(-3.33333\pi)}{1 - \cos(-3.33333\pi)} \right)^k}{k} \right)$$

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 - 0.3125 \log \left(\frac{1 + \cos(-3.33333\pi)}{1 - \cos(-3.33333\pi)} \right) \right)^k}{k}}{16 \log(0.999478)}$$

Integral representations:

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{1}{16} \log_{0.999478} \left(-0.3125 \int_1^{\frac{1 + \cos(-3.33333\pi)}{1 - \cos(-3.33333\pi)}} \frac{1}{t} dt \right)$$

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{1}{16} \log_{0.999478} \left(-0.3125 \log \left(-1 - \frac{0.6}{\pi \int_0^1 \sin(-3.33333 \pi t) dt} \right) \right)$$

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) + 11 + \frac{1}{\phi} =$$

$$\frac{16 + 176 \phi + \phi \log_{0.999478} \left(-0.3125 \log \left(\frac{1 - \int_{\frac{\pi}{2}}^{-3.33333 \pi} \sin(t) dt}{1 + \int_{\frac{\pi}{2}}^{-3.33333 \pi} \sin(t) dt} \right) \right)}{16 \phi}$$

$$\frac{1}{16} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) + 11 + \frac{1}{\phi} =$$

$$\frac{16 + 176 \phi + \phi \log_{0.999478} \left(-0.3125 \log \left(\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-(2.77778 \pi^2)/s+s}}{\sqrt{s}} ds}{\int_{-i\infty-\gamma}^{i\infty-\gamma} \frac{e^{-(2.77778 \pi^2)/s+s}}{\sqrt{s}} ds} \right) \right)}{16 \phi} \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{1}{16} \log_{0.999478} \left[\frac{\log \left(\frac{1 + \cos \left(\frac{\pi + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\pi + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right] + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} +$$

$$\frac{1}{16} \log_{0.999478} \left(-0.3125 \left(\log \left(\frac{1}{1 - \cos(-3.33333 \pi)} \right) + \log(1 + \cos(-3.33333 \pi)) \right) \right)$$

$$\frac{1}{16} \log_{0.999478} \left[\frac{\log \left(\frac{1 + \cos \left(\frac{\pi + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\pi + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right] + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} + \frac{1}{16} \log_{0.999478} \left(\right.$$

$$\left. -0.3125 \left[2 i \pi \left[-\frac{-\pi + \arg \left(\frac{1}{1 - \cos(-3.33333 \pi)} \right) + \arg(1 + \cos(-3.33333 \pi))}{2 \pi} \right] + \right.$$

$$\left. \left. \log \left(\frac{1}{1 - \cos(-3.33333 \pi)} \right) + \log(1 + \cos(-3.33333 \pi)) \right] \right)$$

27*1/32*log base 0.99947811(((1/(4*0.8) ln (((1+cos((Pi/2+3Pi-Pi)/(1.25-2)))/(1-cos((Pi/2+3Pi-Pi)/(1.25-2))))))))))

Input interpretation:

$$27 \times \frac{1}{32} \log_{0.99947811} \left(\left(\frac{1}{4 \times 0.8} \log \left(\frac{1 + \cos \left(\frac{\pi + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\pi + 3\pi - \pi}{1.25 - 2} \right)} \right) \right) \right)$$

Result:

1727.99...

1727.99... This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Alternative representations:

$$\frac{27}{32} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) = \frac{27}{32} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cosh \left(\frac{5i\pi}{2(-0.75)} \right)}{1 - \cosh \left(\frac{5i\pi}{2(-0.75)} \right)} \right)}{3.2} \right)$$

$$\frac{27}{32} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) = \frac{27}{32} \log_{0.999478} \left(\frac{\log_e \left(\frac{1 + \cos \left(\frac{5\pi}{2(-0.75)} \right)}{1 - \cos \left(\frac{5\pi}{2(-0.75)} \right)} \right)}{3.2} \right)$$

$$\frac{27}{32} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) = \frac{27 \log \left(\frac{\log \left(\frac{1 + \cos \left(\frac{5\pi}{2(-0.75)} \right)}{1 - \cos \left(\frac{5\pi}{2(-0.75)} \right)} \right)}{3.2} \right)}{32 \log(0.999478)}$$

Series representations:

$$\frac{27}{32} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) =$$

$$\frac{27}{32} \log_{0.999478} \left(0.3125 \sum_{k=1}^{\infty} \frac{(-2)^k \left(\frac{\cos(-3.33333\pi)}{1 - \cos(-3.33333\pi)} \right)^k}{k} \right)$$

$$\frac{27}{32} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) = - \frac{27 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 - 0.3125 \log \left(\frac{1 + \cos(-3.33333\pi)}{1 - \cos(-3.33333\pi)} \right) \right)^k}{k}}{32 \log(0.999478)}$$

Integral representations:

$$\frac{27}{32} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) = \frac{27}{32} \log_{0.999478} \left(-0.3125 \int_1^{\frac{1 + \cos(-3.33333\pi)}{1 - \cos(-3.33333\pi)}} \frac{1}{t} dt \right)$$

$$\frac{27}{32} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) =$$

$$\frac{27}{32} \log_{0.999478} \left(-0.3125 \log \left(-1 - \frac{0.6}{\pi \int_0^1 \sin(-3.33333\pi t) dt} \right) \right)$$

$$\frac{27}{32} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) =$$

$$\frac{27}{32} \log_{0.999478} \left(-0.3125 \log \left(\frac{1 - \int_{\frac{\pi}{2}}^{-3.33333 \pi} \sin(t) dt}{1 + \int_{\frac{\pi}{2}}^{-3.33333 \pi} \sin(t) dt} \right) \right)$$

$$\frac{27}{32} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) =$$

$$\frac{27}{32} \log_{0.999478} \left(-0.3125 \log \left(\frac{2i\pi + \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-(2.77778 \pi^2)/s+s}}{\sqrt{s}} ds}{2i\pi - \sqrt{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-(2.77778 \pi^2)/s+s}}{\sqrt{s}} ds} \right) \right) \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{27}{32} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) =$$

$$\frac{27}{32} \log_{0.999478} \left(-0.3125 \left(\log \left(\frac{1}{1 - \cos(-3.33333 \pi)} \right) + \log(1 + \cos(-3.33333 \pi)) \right) \right)$$

$$\frac{27}{32} \log_{0.999478} \left(\frac{\log \left(\frac{1 + \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}{1 - \cos \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)} \right)}{4 \times 0.8} \right) = \frac{27}{32}$$

$$\log_{0.999478} \left(-0.3125 \left(2i\pi \left[\frac{-\pi + \arg \left(\frac{1}{1 - \cos(-3.33333\pi)} \right) + \arg(1 + \cos(-3.33333\pi))}{2\pi} \right] + \log \left(\frac{1}{1 - \cos(-3.33333\pi)} \right) + \log(1 + \cos(-3.33333\pi)) \right) \right)$$

Now, from

$$\mathcal{V}(\varphi_{kc}) = C(-1)^{k+1} \left(\sin \frac{\frac{\pi}{2} + \pi k - \theta}{\frac{1}{\gamma} - 2} \right)^{2 - \frac{1}{\gamma}}, \quad (4.155)$$

For

$\gamma = 0.8$, $1/\gamma = 1.25$, $k = 3$, $\theta = \pi$ and $C(-1)^{k+1} = 1$

we obtain:

$$\left(\left(\sin \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) \right)^{2 - 1.25}$$

Input:

$$\sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)$$

Result:

0.897735...

0.897735...

Or, for $C(-1)^{k+1} = 0.7$:

$$0.7 \left(\sin \left(\frac{\pi/2 + 3\pi - \pi}{1.25 - 2} \right) \right)^{2-1.25}$$

Input:

$$0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)$$

Result:

0.628414...

$$0.628414... \approx \pi / 5$$

Alternative representations:

$$0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) = 0.7 \left(\frac{1}{\csc \left(\frac{5\pi}{2(-0.75)} \right)} \right)^{0.75}$$

$$0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) = 0.7 \cos^{0.75} \left(\frac{\pi}{2} - \frac{5\pi}{2(-0.75)} \right)$$

$$0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) = 0.7 \left(-\cos \left(\frac{\pi}{2} + \frac{5\pi}{2(-0.75)} \right) \right)^{0.75}$$

Series representations:

$$0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) = 1.17725 \left(\sum_{k=0}^{\infty} (-1)^k J_{1+2k}(-3.33333\pi) \right)^{0.75}$$

$$0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) = 0.7 \left(\sum_{k=0}^{\infty} \frac{(-1)^k e^{2.68747k} (-\pi)^{2k}}{(2k)!} \right)^{0.75}$$

$$0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) = 1.17725 \left(\pi \sum_{k=0}^{\infty} \frac{(-1)^k (-1.66667 + k) ((-3.33333)_k)^3}{(k!)^3} \right)^{0.75}$$

$J_n(z)$ is the Bessel function of the first kind

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

Multiple-argument formulas:

$$0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) = 1.17725 (\cos(-1.66667\pi) \sin(-1.66667\pi))^{0.75}$$

$$0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) = 0.7 (3 \sin(-1.11111\pi) - 4 \sin^3(-1.11111\pi))^{0.75}$$

$$0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) = 0.7 (U_{-4.33333}(\cos(\pi)) \sin(\pi))^{0.75}$$

$U_n(x)$ is the Chebyshev polynomial of the second kind

From which:

$$(((0.7 ((\sin(((\pi/2+3\pi-\pi)/(1.25-2))))))^{(2-1.25)}))^{1/512}$$

Input:

$$\sqrt[512]{0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right)}$$

Result:

0.99909308...

0.99909308... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = ϕ**

$\frac{1}{4} \log_{0.99909308}(((0.7 ((\sin(((\pi/2+3\pi-\pi)/(1.25-2))))))^{(2-1.25)})))-$
 $\pi+1/\text{golden ratio}$

Input interpretation:

$$\frac{1}{4} \log_{0.99909308} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) - \pi + \frac{1}{\phi}$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

125.477...

125.477... result very near to the dilaton mass calculated as a type of Higgs boson:
 125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV

Alternative representations:

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) - \pi + \frac{1}{\phi} = -\pi + \frac{1}{\phi} + \frac{\log \left(0.7 \sin^{0.75} \left(\frac{5\pi}{2(-0.75)} \right) \right)}{4 \log(0.999093)}$$

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) - \pi + \frac{1}{\phi} =$$

$$-\pi + \frac{1}{4} \log_{0.999093} \left(0.7 \left(\frac{1}{\csc \left(\frac{5\pi}{2(-0.75)} \right)} \right)^{0.75} \right) + \frac{1}{\phi}$$

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) - \pi + \frac{1}{\phi} =$$

$$-\pi + \frac{1}{4} \log_{0.999093} \left(0.7 \cos^{0.75} \left(\frac{\frac{\pi}{2} - \frac{5\pi}{2(-0.75)}}{2} \right) \right) + \frac{1}{\phi}$$

Series representations:

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (-1+0.7 \sin^{0.75}(-3.33333 \pi))^k}{k}}{4 \log(0.999093)}$$

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + \frac{1}{4} \log_{0.999093} \left(1.17725 \left(\sum_{k=0}^{\infty} (-1)^k J_{1+2k}(-3.33333 \pi) \right)^{0.75} \right)$$

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + \frac{1}{4} \log_{0.999093} \left(0.7 \left(\sum_{k=0}^{\infty} \frac{(-1)^k e^{2.68747k} (-\pi)^{2k}}{(2k)!} \right)^{0.75} \right)$$

$J_n(z)$ is the Bessel function of the first kind

$n!$ is the factorial function

Multiple-argument formulas:

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + \frac{1}{4} (\log_{0.999093}(0.7) + \log_{0.999093}(\sin^{0.75}(-3.33333 \pi)))$$

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + \frac{1}{4} \log_{0.999093}(1.17725 (\cos(-1.66667 \pi) \sin(-1.66667 \pi))^{0.75})$$

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + \frac{1}{4} \log_{0.999093}(0.7 (3 \sin(-1.11111 \pi) - 4 \sin^3(-1.11111 \pi))^{0.75})$$

$\frac{1}{4} \log_{0.99909308}(((0.7 ((\sin(((\pi/2+3\pi-\pi)/(1.25-2))))))^{(2-1.25)})))+11+1/\text{golden ratio}$

where 11 is a Lucas number

Input interpretation:

$$\frac{1}{4} \log_{0.99909308} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

139.619...

139.619... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternative representations:

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} + \frac{\log \left(0.7 \sin^{0.75} \left(\frac{5\pi}{2(-0.75)} \right) \right)}{4 \log(0.999093)}$$

$$\begin{aligned} &\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) + 11 + \frac{1}{\phi} = \\ &11 + \frac{1}{4} \log_{0.999093} \left(0.7 \left(\frac{1}{\csc \left(\frac{5\pi}{2(-0.75)} \right)} \right)^{0.75} \right) + \frac{1}{\phi} \end{aligned}$$

$$\begin{aligned} &\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) + 11 + \frac{1}{\phi} = \\ &11 + \frac{1}{4} \log_{0.999093} \left(0.7 \cos^{0.75} \left(\frac{\pi}{2} - \frac{5\pi}{2(-0.75)} \right) \right) + \frac{1}{\phi} \end{aligned}$$

$\log(x)$ is the natural logarithm

$\csc(x)$ is the cosecant function

Series representations:

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (-1+0.7 \sin^{0.75}(-3.33333 \pi))^k}{k}}{4 \log(0.999093)}$$

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{1}{4} \log_{0.999093} \left(1.17725 \left(\sum_{k=0}^{\infty} (-1)^k J_{1+2k}(-3.33333 \pi) \right)^{0.75} \right)$$

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{1}{4} \log_{0.999093} \left(0.7 \left(\sum_{k=0}^{\infty} \frac{(-1)^k e^{2.68747k} (-\pi)^{2k}}{(2k)!} \right)^{0.75} \right)$$

$J_n(z)$ is the Bessel function of the first kind

$n!$ is the factorial function

Multiple-argument formulas:

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{1}{4} (\log_{0.999093}(0.7) + \log_{0.999093}(\sin^{0.75}(-3.33333 \pi)))$$

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{1}{4} \log_{0.999093}(1.17725 (\cos(-1.66667 \pi) \sin(-1.66667 \pi))^{0.75})$$

$$\frac{1}{4} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{1}{4} \log_{0.999093}(0.7 (3 \sin(-1.11111 \pi) - 4 \sin^3(-1.11111 \pi))^{0.75})$$

$$27 \times \frac{1}{8} \log_{\text{base } 0.99909308} \left(\left(0.7 \left(\sin \left(\frac{\pi}{2} + 3\pi - \pi \right) / (1.25 - 2) \right) \right)^{2 - 1.25} \right)$$

Input interpretation:

$$27 \times \frac{1}{8} \log_{0.99909308} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right)$$

$\log_b(x)$ is the base- b logarithm

Result:

1728.01...

1728.01...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Alternative representations:

$$\frac{27}{8} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) = \frac{27 \log \left(0.7 \sin^{0.75} \left(\frac{5\pi}{2(-0.75)} \right) \right)}{8 \log(0.999093)}$$

$$\frac{27}{8} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) = \frac{27}{8} \log_{0.999093} \left(0.7 \left(\frac{1}{\csc \left(\frac{5\pi}{2(-0.75)} \right)} \right)^{0.75} \right)$$

$$\frac{27}{8} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) = \frac{27}{8} \log_{0.999093} \left(0.7 \cos^{0.75} \left(\frac{\pi}{2} - \frac{5\pi}{2(-0.75)} \right) \right)$$

Series representations:

$$\frac{27}{8} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) = - \frac{27 \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + 0.7 \sin^{0.75}(-3.33333\pi))^k}{k}}{8 \log(0.999093)}$$

$$\frac{27}{8} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) =$$

$$\frac{27}{8} \log_{0.999093} \left(1.17725 \left(\sum_{k=0}^{\infty} (-1)^k J_{1+2k}(-3.33333 \pi) \right)^{0.75} \right)$$

$$\frac{27}{8} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) =$$

$$\frac{27}{8} \log_{0.999093} \left(0.7 \left(\sum_{k=0}^{\infty} \frac{(-1)^k e^{2.68747k} (-\pi)^{2k}}{(2k)!} \right)^{0.75} \right)$$

Multiple-argument formulas:

$$\frac{27}{8} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) =$$

$$\frac{27}{8} (\log_{0.999093}(0.7) + \log_{0.999093}(\sin^{0.75}(-3.33333 \pi)))$$

$$\frac{27}{8} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) =$$

$$\frac{27}{8} \log_{0.999093}(1.17725 (\cos(-1.66667 \pi) \sin(-1.66667 \pi))^{0.75})$$

$$\frac{27}{8} \log_{0.999093} \left(0.7 \sin^{2-1.25} \left(\frac{\frac{\pi}{2} + 3\pi - \pi}{1.25 - 2} \right) \right) =$$

$$\frac{27}{8} \log_{0.999093}(0.7 (3 \sin(-1.11111 \pi) - 4 \sin^3(-1.11111 \pi))^{0.75})$$

And now:

$$\mathcal{V}''(\varphi_{kc}) = -4(1 - \gamma)(1 - 2\gamma)\mathcal{V}(\varphi_{kc}), \quad (4.156)$$

$$C(-1)^{k+1} > 0 \quad (4.157)$$

From (4.156), we obtain:

$$-4(1-0.8)(1-2*0.8)*0.628414$$

Input interpretation:

$$-4(1 - 0.8)((1 - 2 \times 0.8) \times 0.628414)$$

Result:

$$0.30163872$$

$$0.30163872$$

From which:

$$(((-4(1-0.8)(1-2*0.8)*0.628414)))^{1/2048}$$

Input interpretation:

$$\sqrt[2048]{-4(1 - 0.8)((1 - 2 \times 0.8) \times 0.628414)}$$

Result:

$$0.99941495\dots$$

0.99941495... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$\frac{\sqrt{5}}{1 + \sqrt[5]{\sqrt{\varphi^5 4\sqrt{5^3} - 1}}} - \varphi + 1$$

and to the dilaton value **0.989117352243 = ϕ**

1/16 log base 0.99941495(((−4(1−0.8)(1−2*0.8)*0.628414))−Pi+1/golden ratio

Input interpretation:

$$\frac{1}{16} \log_{0.99941495}(-4(1-0.8)((1-2 \times 0.8) \times 0.628414)) - \pi + \frac{1}{\phi}$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

125.476...

125.476... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV

Alternative representation:

$$\frac{1}{16} \log_{0.999415}(-4(1-0.8)((1-2 \times 0.8) 0.628414)) - \pi + \frac{1}{\phi} = -\pi + \frac{1}{\phi} + \frac{\log(0.301639)}{16 \log(0.999415)}$$

Series representations:

$$\frac{1}{16} \log_{0.999415}(-4(1-0.8)((1-2 \times 0.8) 0.628414)) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.698361)^k}{k}}{16 \log(0.999415)}$$

$$\frac{1}{16} \log_{0.999415}(-4(1-0.8)((1-2 \times 0.8) 0.628414)) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - 106.797 \log(0.301639) - \frac{1}{16} \log(0.301639) \sum_{k=0}^{\infty} (-0.00058505)^k G(k)$$

$$\text{for } \left(G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

1/16 log base 0.99941495(((−4(1−0.8)(1−2*0.8)*0.628414)))+11+1/golden ratio

where 11 is a Lucas number

Input interpretation:

$$\frac{1}{16} \log_{0.99941495}(-4(1-0.8)((1-2 \times 0.8) \times 0.628414)) + 11 + \frac{1}{\phi}$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

139.617...

139.617... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternative representation:

$$\frac{1}{16} \log_{0.999415}(-4(1-0.8)((1-2 \times 0.8)0.628414)) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{\log(0.301639)}{16 \log(0.999415)}$$

$\log(x)$ is the natural logarithm

Series representations:

$$\frac{1}{16} \log_{0.999415}(-4(1-0.8)((1-2 \times 0.8)0.628414)) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.698361)^k}{k}}{16 \log(0.999415)}$$

$$\frac{1}{16} \log_{0.999415}(-4(1-0.8)((1-2 \times 0.8)0.628414)) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 106.797 \log(0.301639) - \frac{1}{16} \log(0.301639) \sum_{k=0}^{\infty} (-0.00058505)^k G(k)$$

for $\left(G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$27 * \frac{1}{32} \log \text{ base } 0.99941495(((-4(1-0.8)(1-2*0.8)*0.628414)))$

Input interpretation:

$$27 \times \frac{1}{32} \log_{0.99941495}(-4(1-0.8)((1-2 \times 0.8) \times 0.628414))$$

$\log_b(x)$ is the base- b logarithm

Result:

1727.99...

1727.99...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Alternative representation:

$$\frac{27}{32} \log_{0.999415}(-4(1-0.8)((1-2 \times 0.8)0.628414)) = \frac{27 \log(0.301639)}{32 \log(0.999415)}$$

Series representations:

$$\frac{27}{32} \log_{0.999415}(-4(1-0.8)((1-2 \times 0.8)0.628414)) = -\frac{27 \sum_{k=1}^{\infty} \frac{(-1)^k (-0.698361)^k}{k}}{32 \log(0.999415)}$$

$$\begin{aligned} \frac{27}{32} \log_{0.999415}(-4(1-0.8)((1-2 \times 0.8)0.628414)) = \\ -1441.76 \log(0.301639) - 0.84375 \log(0.301639) \sum_{k=0}^{\infty} (-0.00058505)^k G(k) \\ \text{for } \left(G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right) \end{aligned}$$

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References

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