

On the Ramanujan mathematics applied to some sectors of String Theory and Particle Physics: Further new possible mathematical connections V.

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Abstract

In this research thesis, we have analyzed and deepened further Ramanujan expressions applied to some sectors of String Theory and Particle Physics. We have therefore described new possible mathematical connections.

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https://www.storyofmathematics.com/20th_hardy.html

From:

Integrable Scalar Cosmologies

I. Foundations and links with String Theory

P. Fré, A. Sagnotti and A.S. Sorin - arXiv:1307.1910v3 [hep-th] 16 Oct 2013

We have that:

$$\varphi = \frac{2(d-1)}{3(d-2)} \phi . \quad (6.13)$$

For $d = 6$, we obtain:

$$(2(6-1))/(3(6-2))x = y$$

Input:

$$\frac{2(6-1)}{3(6-2)} x = y$$

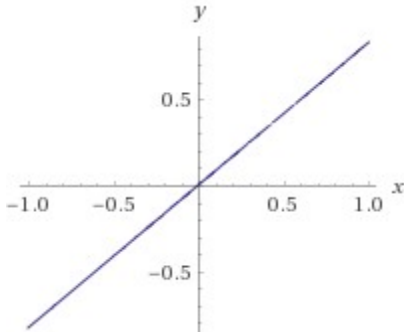
Result:

$$\frac{5x}{6} = y$$

Geometric figure:

line

Implicit plot:



Alternate forms:

$$x = \frac{6y}{5}$$

$$\frac{5x}{6} - y = 0$$

Real solution:

$$y = \frac{5x}{6}$$

Solution:

$$y = \frac{5x}{6}$$

Integer solution:

$$x = 6n, \quad y = 5n, \quad n \in \mathbb{Z}$$

\mathbb{Z} is the set of integers

Partial derivatives:

$$\frac{\partial}{\partial x} \left(\frac{5x}{6} \right) = \frac{5}{6}$$

$$\frac{\partial}{\partial y} \left(\frac{5x}{6} \right) = 0$$

$$(2(6-1))/(3(6-2))6$$

Input:

$$\frac{2(6-1)}{3(6-2)} \times 6$$

Result:

5

5

Thence, we have $\varphi = 5$ and $\phi = 6$

$$(\sigma = -1)$$

$$(\sigma = +1)$$

$$\Phi_t = \sqrt{\frac{d-2}{2(d-1)}} \left(\frac{3}{2} \phi - \frac{10-d}{d-2} \sigma \right), \quad (6.7)$$

$$\Phi_s = \sqrt{\frac{10-d}{2(d-1)}} \left(\frac{1}{2} \phi + 3\sigma \right), \quad (6.8)$$

$$\left(\left(\frac{6-2}{2(6-1)} \right) \right)^{1/2} * \left(\frac{3}{2} * 6 - \frac{10-6}{6-2} \right)$$

Input:

$$\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3}{2} * 6 - \frac{10-6}{6-2} \right)$$

Result:

$$8\sqrt{\frac{2}{5}}$$

Decimal approximation:

5.059644256269406931198229671092349653951288222920346922972...

5.059644256269..... = Φ_t

Alternate form:

$$\frac{\sqrt{10} 8}{5}$$

$$(((10-6)/(2(6-1)))^{1/2} * (1/2 * 6 + 3))$$

Input:

$$\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{1}{2} \times 6 + 3 \right)$$

Result:

$$6 \sqrt{\frac{2}{5}}$$

Decimal approximation:

3.794733192202055198398672253319262240463466167190260192229...

$$3.7947331922\dots = \Phi_s$$

Alternate form:

$$\frac{\sqrt{10} 6}{5}$$

From the sum of the two results, we obtain:

$$(((((((6-2)/(2(6-1))))))^{1/2} * (3/2 * 6 - (10-6)/(6-2)))) + (((((10-6)/(2(6-1)))^{1/2} * (1/2 * 6 + 3))))$$

Input:

$$\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3}{2} \times 6 - \frac{10-6}{6-2} \right) + \sqrt{\frac{10-6}{2(6-1)}} \left(\frac{1}{2} \times 6 + 3 \right)$$

Result:

$$14 \sqrt{\frac{2}{5}}$$

Decimal approximation:

8.854377448471462129596901924411611894414754390110607115201...

8.85437744847....

Alternate form:

$$\frac{14\sqrt{10}}{5}$$

From the product:

$$\left(\left(\left(\left(\left(\frac{6-2}{2(6-1)}\right)\right)\right)\right)^{1/2} * \left(\frac{3/2*6-(10-6)}{6-2}\right)\right) * \left(\left(\left(\frac{10-6}{2(6-1)}\right)\right)\right)^{1/2} * \left(\frac{1/2*6+3}{2}\right)$$

Input:

$$\left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3}{2} \times 6 - \frac{10-6}{6-2}\right)\right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{1}{2} \times 6 + 3\right)\right)$$

Exact result:

$$\frac{96}{5}$$

Decimal form:

19.2
19.2

And:

$$2 * \left[\left(\left(\left(\left(\frac{6-2}{2(6-1)}\right)\right)\right)\right)^{1/2} * \left(\frac{3/2*6-(10-6)}{6-2}\right)\right) * \left(\left(\left(\frac{10-6}{2(6-1)}\right)\right)\right)^{1/2} * \left(\frac{1/2*6+3}{2}\right)\right]^2 + 47 - \pi + \left(\frac{\sqrt{5}+1}{2}\right)$$

where 47 is a Lucas number

Input:

$$2 \left(\left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3}{2} \times 6 - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{1}{2} \times 6 + 3 \right) \right) \right)^2 + 47 - \pi + \frac{1}{2} (\sqrt{5} + 1)$$

Result:

$$\frac{19607}{25} + \frac{1}{2} (1 + \sqrt{5}) - \pi$$

Decimal approximation:

782.7564413351601016097419434510861352335231397804306570411...

782.75644.... result practically equal to the rest mass of Omega meson 782.65

Property:

$\frac{19607}{25} + \frac{1}{2}(1 + \sqrt{5}) - \pi$ is a transcendental number

Alternate forms:

$$\frac{1}{50} (39239 + 25\sqrt{5} - 50\pi)$$

$$\frac{39239}{50} + \frac{\sqrt{5}}{2} - \pi$$

$$\frac{1}{50} (39239 + 25\sqrt{5}) - \pi$$

Series representations:

$$2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 + 47 - \pi + \frac{1}{2}(\sqrt{5} + 1) = \frac{39239}{50} - \pi + \frac{1}{2} \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}$$

$$2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 + 47 - \pi + \frac{1}{2}(\sqrt{5} + 1) = \frac{39239}{50} - \pi + \frac{1}{2} \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 + 47 - \pi + \frac{1}{2}(\sqrt{5} + 1) = \frac{39239}{50} - \pi + \frac{\sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 4^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{4\sqrt{\pi}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function
 $\text{Res } f$ is a complex residue
 $\neq 0$

$$2 * [(((((((6-2)/(2(6-1))))))^{1/2} * (3/2*6-(10-6)/(6-2)))) * (((10-6)/(2(6-1)))^{1/2} * (1/2*6+3)))]^{2-2\pi-3}$$

where 3 is a Fibonacci/Lucas number

Input:

$$2 \left(\left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3}{2} \times 6 - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{1}{2} \times 6 + 3 \right) \right) \right)^2 - 2\pi - 3$$

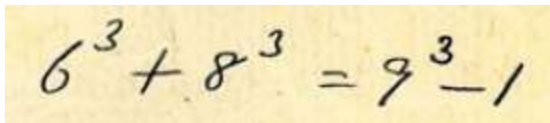
Result:

$$\frac{18\,357}{25} - 2\pi$$

Decimal approximation:

727.9968146928204135230747132334409942316056612012497883580...

727.996814692... $\approx 728 = 9^3 - 1$ that is the following Ramanujan cube:



A photograph of a piece of yellowed paper with the handwritten equation $6^3 + 8^3 = 9^3 - 1$ in black ink.

Property:

$\frac{18\,357}{25} - 2\pi$ is a transcendental number

Alternate form:

$$\frac{1}{25} (18\,357 - 50\pi)$$

Series representations:

$$2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 = \frac{18\,357}{25} - 8 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 = \frac{18357}{25} + \sum_{k=0}^{\infty} \frac{8(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}$$

$$2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 = \frac{18357}{25} - 2 \sum_{k=0}^{\infty} \left(-\frac{1}{4} \right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right)$$

Integral representations:

$$2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 = \frac{18357}{25} - 8 \int_0^1 \sqrt{1-t^2} dt$$

$$2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 = \frac{18357}{25} - 4 \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 = \frac{18357}{25} - 4 \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$987+13+2*[\((((((6-2)/(2(6-1))))))^{1/2} * (3/2*6-(10-6)/(6-2)))) * (((10-6)/(2(6-1))))^{1/2}*(1/2*6+3))]^2-2\pi-3$$

where 987 and 13 are Fibonacci numbers

Input:

$$987+13+2 \left(\left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3}{2} \times 6 - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{1}{2} \times 6 + 3 \right) \right) \right)^2 - 2\pi - 3$$

Result:

$$\frac{43357}{25} - 2\pi$$

Decimal approximation:

1727.996814692820413523074713233440994231605661201249788358...

1727.9968146...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Property:

$\frac{43\,357}{25} - 2\pi$ is a transcendental number

Alternate form:

$\frac{1}{25} (43\,357 - 50\pi)$

Series representations:

$$987 + 13 + 2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 =$$

$$\frac{43\,357}{25} - 8 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$987 + 13 + 2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 =$$

$$\frac{43\,357}{25} + \sum_{k=0}^{\infty} \frac{8(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}$$

$$987 + 13 + 2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 =$$

$$\frac{43\,357}{25} - 2 \sum_{k=0}^{\infty} \left(-\frac{1}{4} \right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right)$$

Integral representations:

$$987 + 13 + 2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 = \frac{43357}{25} - 8 \int_0^1 \sqrt{1-t^2} dt$$

$$987 + 13 + 2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 = \frac{43357}{25} - 4 \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$987 + 13 + 2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 = \frac{43357}{25} - 4 \int_0^\infty \frac{1}{1+t^2} dt$$

$$987+13+55+2*[\((((((6-2)/(2(6-1))))))^{1/2} * (3/2*6-(10-6)/(6-2)))] * (((((10-6)/(2(6-1))))^{1/2}*(1/2*6+3))))^2-2\pi-3$$

where 55 is a Fibonacci number

Input:

$$987 + 13 + 55 + 2 \left(\left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3}{2} \times 6 - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{1}{2} \times 6 + 3 \right) \right) \right)^2 - 2\pi - 3$$

Result:

$$\frac{44732}{25} - 2\pi$$

Decimal approximation:

1782.996814692820413523074713233440994231605661201249788358...

1782.9968146... result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

Property:

$$\frac{44732}{25} - 2\pi \text{ is a transcendental number}$$

Alternate forms:

$$\frac{2}{25} (22\,366 - 25\pi)$$

$$-\frac{2}{25} (25\pi - 22\,366)$$

Series representations:

$$987 + 13 + 55 + 2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 =$$

$$\frac{44\,732}{25} - 8 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$987 + 13 + 55 + 2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 =$$

$$\frac{44\,732}{25} + \sum_{k=0}^{\infty} \frac{8(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}$$

$$987 + 13 + 55 + 2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 =$$

$$\frac{44\,732}{25} - 2 \sum_{k=0}^{\infty} \left(-\frac{1}{4} \right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right)$$

Integral representations:

$$987 + 13 + 55 + 2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 =$$

$$\frac{44\,732}{25} - 8 \int_0^1 \sqrt{1-t^2} \, dt$$

$$987 + 13 + 55 + 2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 =$$

$$\frac{44\,732}{25} - 4 \int_0^1 \frac{1}{\sqrt{1-t^2}} \, dt$$

$$987 + 13 + 55 + 2 \left(\left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) \sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right)^2 - 2\pi - 3 =$$

$$\frac{44\,732}{25} - 4 \int_0^{\infty} \frac{1}{1+t^2} \, dt$$

$$2*144+5+2*[\((((((6-2)/(2(6-1))))))^{1/2} * (3/2*6-(10-6)/(6-2)))) * (((10-6)/(2(6-1)))^{1/2}*(1/2*6+3)))]^2-11-((\text{sqrt}5-1)/2)+2/5$$

where 144 and 5 are Fibonacci numbers, while 11 is a Lucas number

Input:

$$2 \times 144 + 5 + 2 \left(\left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3}{2} \times 6 - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{1}{2} \times 6 + 3 \right) \right) \right)^2 - 11 - \frac{1}{2} (\sqrt{5} - 1) + \frac{2}{5}$$

Result:

$$\frac{25492}{25} + \frac{1}{2} (1 - \sqrt{5})$$

Decimal approximation:

1019.061966011250105151795413165634361882279690820194237137...

1019.061966... result practically equal to the rest mass of Phi meson 1019.445

Alternate forms:

$$\frac{1}{50} (51009 - 25 \sqrt{5})$$

$$\frac{51009}{50} - \frac{\sqrt{5}}{2}$$

Minimal polynomial:

$$625x^2 - 1275225x + 650478739$$

$$8*[\((((((6-2)/(2(6-1))))))^{1/2} * (3/2*6-(10-6)/(6-2)))) * (((10-6)/(2(6-1)))^{1/2}*(1/2*6+3)))]-13-((\text{sqrt}5-1)/2)-2/5$$

where 8 and 13 are Fibonacci numbers

Input:

$$8 \left(\left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3}{2} \times 6 - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{1}{2} \times 6 + 3 \right) \right) \right)^2 - 13 - \frac{1}{2} (\sqrt{5} - 1) - \frac{2}{5}$$

Result:

$$\frac{701}{5} + \frac{1}{2} (1 - \sqrt{5})$$

Decimal approximation:

139.5819660112501051517954131656343618822796908201942371378...

139.581966011... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternate forms:

$$\frac{1}{10} (1407 - 5 \sqrt{5})$$

$$\frac{1407}{10} - \frac{\sqrt{5}}{2}$$

Minimal polynomial:

$$25x^2 - 7035x + 494881$$

$$8 * [((((((6-2)/(2(6-1))))))^{1/2} * (3/2*6-(10-6)/(6-2)))] * (((10-6)/(2(6-1)))^{1/2} * (1/2*6+3))] - 29 + ((\sqrt{5}+1)/2) - 4/5$$

where 29 is a Lucas number

Input:

$$8 \left(\left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3}{2} \times 6 - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{1}{2} \times 6 + 3 \right) \right) \right) - 29 + \frac{1}{2} (\sqrt{5} + 1) - \frac{4}{5}$$

Result:

$$\frac{619}{5} + \frac{1}{2} (1 + \sqrt{5})$$

Decimal approximation:

125.4180339887498948482045868343656381177203091798057628621...

125.4180339887... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

Alternate forms:

$$\frac{1}{10} (1243 + 5 \sqrt{5})$$

$$\frac{1243}{10} + \frac{\sqrt{5}}{2}$$

Minimal polynomial:

$$25x^2 - 6215x + 386231$$

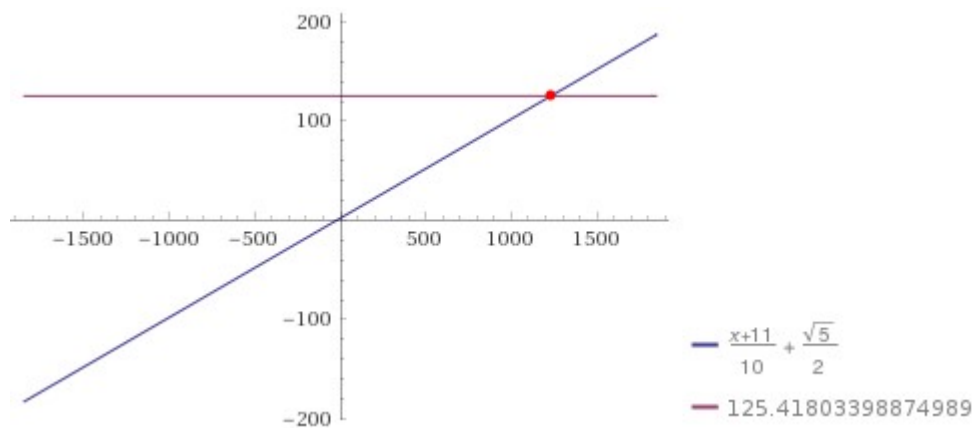
$$(x+11)/10 + \sqrt{5}/2 = 125.41803398874989$$

Input interpretation:

$$\frac{x+11}{10} + \frac{\sqrt{5}}{2} = 125.41803398874989$$

Result:

$$\frac{x+11}{10} + \frac{\sqrt{5}}{2} = 125.41803398874989$$

Plot:**Alternate forms:**

$$\frac{x}{10} - 123.20000000000000 = 0$$

$$\frac{1}{10} (x + 5\sqrt{5} + 11) = 125.41803398874989$$

Expanded form:

$$\frac{x}{10} + \frac{\sqrt{5}}{2} + \frac{11}{10} = 125.41803398874989$$

Solution:

$$x \approx 1232.0000000000000$$

Integer solution:

$$x = 1232$$

1232 result equal to the rest mass of Delta baryon

$$(377+89+21)*[\frac{(((((6-2)/(2(6-1))))))^{1/2} * (3/2*6-(10-6)/(6-2))) * (((10-6)/(2(6-1)))^{1/2}*(1/2*6+3)))]-47-e-1/\text{golden ratio}$$

where 377, 89 and 21 are Fibonacci numbers, while 47 is a Lucas number

Input:

$$(377 + 89 + 21) \left(\left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3}{2} \times 6 - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{1}{2} \times 6 + 3 \right) \right) \right) - 47 - e - \frac{1}{\phi}$$

ϕ is the golden ratio

Result:

$$-\frac{1}{\phi} + \frac{46517}{5} - e$$

Decimal approximation:

9300.063684182791059916435125694281699384522443726494277562...

9300.063684... result practically equal to the rest mass of Bottom eta meson 9300

Property:

$\frac{46517}{5} - e - \frac{1}{\phi}$ is a transcendental number

Alternate forms:

$$\frac{1}{10} (93039 - 5\sqrt{5} - 10e)$$

$$\frac{-46517\phi + 5e\phi + 5}{5\phi}$$

$$\frac{1}{10} (93039 - 5\sqrt{5}) - e$$

Series representations:

$$(377 + 89 + 21) \left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) - 47 - e - \frac{1}{\phi} =$$

$$\frac{46517}{5} - \frac{1}{\phi} - \sum_{k=0}^{\infty} \frac{1}{k!}$$

$$(377 + 89 + 21) \left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) - 47 - e - \frac{1}{\phi} =$$

$$\frac{46502}{5} - \frac{1}{\phi} + \sum_{k=0}^{\infty} \frac{1+k}{(3+k)!}$$

$$(377 + 89 + 21) \left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) - 47 - e - \frac{1}{\phi} =$$

$$\frac{46517}{5} - \frac{1}{\phi} - \frac{\sum_{k=0}^{\infty} \frac{-1+k+z}{k!}}{z}$$

$$(233+89)*[((((((6-2)/(2(6-1))))))^{1/2} * (3/2*6-(10-6)/(6-2)))] * (((((10-6)/(2(6-1))))^{1/2}*(1/2*6+3)))]-34-13-e-1/golden\ ratio+144$$

where 233, 89, 34, 13 and 144 are Fibonacci numbers

Input:

$$(233 + 89) \left(\left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3}{2} \times 6 - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{1}{2} \times 6 + 3 \right) \right) \right) -$$

$$34 - 13 - e - \frac{1}{\phi} + 144$$

ϕ is the golden ratio

Result:

$$-\frac{1}{\phi} + \frac{31397}{5} - e$$

Decimal approximation:

6276.063684182791059916435125694281699384522443726494277562...

6276.063684... result practically equal to the rest mass of Charmed B meson 6276

Property:

$\frac{31397}{5} - e - \frac{1}{\phi}$ is a transcendental number

Alternate forms:

$$\frac{1}{10} \left(62799 - 5\sqrt{5} - 10e \right) - \frac{-31397\phi + 5e\phi + 5}{5\phi}$$

$$\frac{1}{10} \left(62799 - 5\sqrt{5} \right) - e$$

Series representations:

$$(233 + 89) \left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) - 34 - 13 - e - \frac{1}{\phi} + 144 = \frac{31397}{5} - \frac{1}{\phi} - \sum_{k=0}^{\infty} \frac{1}{k!}$$

$$(233 + 89) \left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) - 34 - 13 - e - \frac{1}{\phi} + 144 = \frac{31382}{5} - \frac{1}{\phi} + \sum_{k=0}^{\infty} \frac{1+k}{(3+k)!}$$

$$(233 + 89) \left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3 \times 6}{2} - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{6}{2} + 3 \right) \right) - 34 - 13 - e - \frac{1}{\phi} + 144 = \frac{31397}{5} - \frac{1}{\phi} - \sum_{k=0}^{\infty} \frac{-1+k+z}{k!}$$

$(89+5)*[((((((6-2)/(2(6-1))))))^{1/2} * (3/2*6-(10-6)/(6-2)))) * (((10-6)/(2(6-1)))^{1/2}*(1/2*6+3))]+64+1/\text{golden ratio}$

where 89 and 5 are Fibonacci numbers

Input:

$$(89 + 5) \left(\left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3}{2} \times 6 - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{1}{2} \times 6 + 3 \right) \right) \right) + 64 + \frac{1}{\phi}$$

ϕ is the golden ratio

Result:

$$\frac{1}{\phi} + \frac{9344}{5}$$

Decimal approximation:

1869.418033988749894848204586834365638117720309179805762862...

1869.41803398... result practically equal to the rest mass of D meson 1869.62

Alternate forms:

$$\frac{1}{10} (18683 + 5\sqrt{5})$$

$$\frac{9344\phi + 5}{5\phi}$$

$$\frac{\sqrt{5}}{2} + \frac{18683}{10}$$

$$(21+5) * [((((((6-2)/(2(6-1))))))^{1/2} * (3/2*6-(10-6)/(6-2)))) * (((10-6)/(2(6-1))))^{1/2} * (1/2*6+3))] - 3 + \text{golden ratio}$$

where 21, 5 and 3 are Fibonacci numbers

Input:

$$(21 + 5) \left(\left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3}{2} \times 6 - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{1}{2} \times 6 + 3 \right) \right) \right) - 3 + \phi$$

ϕ is the golden ratio

Result:

$$\phi + \frac{2481}{5}$$

Decimal approximation:

497.8180339887498948482045868343656381177203091798057628621...

497.81803398... result practically equal to the rest mass of Kaon meson 497.614

Alternate forms:

$$\frac{1}{10} (4967 + 5 \sqrt{5})$$

$$\frac{1}{5} (5 \phi + 2481)$$

$$\frac{4967}{10} + \frac{\sqrt{5}}{2}$$

$$(144-13)*[((((((6-2)/(2(6-1))))))^{1/2} * (3/2*6-(10-6)/(6-2)))) * (((10-6)/(2(6-1)))^{1/2}*(1/2*6+3)))]+\text{golden ratio}^2$$

where 144 and 13 are Fibonacci numbers

Input:

$$(144 - 13) \left(\left(\sqrt{\frac{6-2}{2(6-1)}} \left(\frac{3}{2} \times 6 - \frac{10-6}{6-2} \right) \right) \left(\sqrt{\frac{10-6}{2(6-1)}} \left(\frac{1}{2} \times 6 + 3 \right) \right) \right) + \phi^2$$

ϕ is the golden ratio

Result:

$$\phi^2 + \frac{12576}{5}$$

Decimal approximation:

2517.818033988749894848204586834365638117720309179805762862...

2517.81803398... result about equal to the rest mass of charmed Sigma baryon
2517.9

Alternate forms:

$$\frac{1}{10} (25167 + 5 \sqrt{5})$$

$$\frac{1}{5} (5 \phi^2 + 12576)$$

$$\frac{25167}{10} + \frac{\sqrt{5}}{2}$$

The two results are:

$$5.059644256269..... = \Phi_t$$

$$3.7947331922..... = \Phi_s$$

Now, we have that:

The potential $\mathcal{V}(\varphi) = \mathcal{V}_0 e^{2\gamma\varphi} + \Lambda$

if $\mathcal{V}_0 \geq 0$, $\Lambda \geq 0$:

To begin with, one can notice that independent shifts of Φ_s and Φ_t can turn the general scalar potential resulting from $D9$ and $D3$ branes into the universal form

$$V = V_0 \left(e^{\sqrt{3} \Phi_t} + e^{\sqrt{3} \left(\frac{1}{2} \Phi_t - \frac{\sqrt{3}}{2} \Phi_s \right)} \right), \quad (6.17)$$

For $V_0 = 1$, we obtain:

$$(e^{(\sqrt{3} * 5.0596442562)} + e^{(\sqrt{3}(1/2 * 5.0596442562 - (\sqrt{3})/2 * 3.794733192))})$$

Input interpretation:

$$e^{\sqrt{3} \times 5.0596442562} + \exp\left(\sqrt{3} \left(\frac{1}{2} \times 5.0596442562 + \frac{\sqrt{3}}{2} \times (-3.794733192) \right)\right)$$

Result:

6397.119473...

6397.119473...

We have that from the following 7th order Ramanujan mock theta function

$$(i) \quad 1 + \frac{q}{1 - q^2} + \frac{q^4}{(1 - q^3)(1 - q^4)} + \frac{q^9}{(1 - q^4)(1 - q^5)(1 - q^6)} + \dots$$

From the (i), we have:

$$0.9239078 + 0.000433255 + (-1.8754140254243246404383299476354805043847163776 \times 10^{-7})$$

[Input interpretation:](#)

$$0.9239078 + 0.000433255 - 1.8754140254243246404383299476354805043847163776 \times 10^{-7}$$

[Open code](#)

[Enlarge Data](#) [Customize A](#) [Plaintext](#) [Interactive](#)

[Result](#)

$$0.92434086745859745756753595616700523645194956152836224$$

[Open code](#)

The result is

$$0.92434086745859745756753595616700523645194956152836224$$

0.9243408674589...

For $V_0 = 0.9243408674589$ we obtain:

$$0.9243408674589(e^{(\sqrt{3} * 5.0596442562)} + e^{(\sqrt{3}(1/2 * 5.0596442562 - (\sqrt{3})/2 * 3.794733192))})$$

Input interpretation:

$$0.9243408674589$$

$$\left(e^{\sqrt{3} \times 5.0596442562} + \exp\left(\sqrt{3} \left(\frac{1}{2} \times 5.0596442562 + \frac{\sqrt{3}}{2} \times (-3.794733192) \right)\right) \right)$$

Result:

$$5913.118963...$$

$$5913.118963...$$

Series representations:

$$0.92434086745890000 \left(e^{\sqrt{3} \cdot 5.05964425620000} + e^{\sqrt{3} \left(5.05964425620000/2 - (3.79473 \sqrt{3})/2 \right)} \right) =$$

$$0.9243408674589000$$

$$\left(1.0000000000000000 e^{5.05964425620000 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + 1.0000000000000000 \right.$$

$$\left. \exp \left(\sqrt{2} \left(\sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k} \right) \right) \left(2.52982212810000 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k} \right) \right)$$

$$0.92434086745890000 \left(e^{\sqrt{3} \cdot 5.05964425620000} + e^{\sqrt{3} \left(5.05964425620000/2 - (3.79473 \sqrt{3})/2 \right)} \right) =$$

$$0.9243408674589000 \left(1.0000000000000000 e^{5.05964425620000 \sqrt{2} \sum_{k=0}^{\infty} \frac{(-1)^k \binom{-1/2}{k}}{k!}} + \right.$$

$$1.0000000000000000 \exp \left(\sqrt{2} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \binom{-1/2}{k}}{k!} \right) \right)$$

$$\left(2.52982212810000 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} \frac{(-1)^k \binom{-1/2}{k}}{k!} \right) \right)$$

$$0.92434086745890000 \left(e^{\sqrt{3} \cdot 5.05964425620000} + e^{\sqrt{3} \left(5.05964425620000/2 - (3.79473 \sqrt{3})/2 \right)} \right) =$$

$$0.9243408674589000 \left(1.0000000000000000 \right.$$

$$\exp \left(5.05964425620000 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \binom{-1/2}{k} (3 - z_0)^k z_0^{-k}}{k!} \right) +$$

$$1.0000000000000000 \exp \left(\sqrt{z_0} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \binom{-1/2}{k} (3 - z_0)^k z_0^{-k}}{k!} \right) \right)$$

$$\left(2.52982212810000 - 1.89737 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \binom{-1/2}{k} (3 - z_0)^k z_0^{-k}}{k!} \right) \right)$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

\mathbb{R} is the set of real numbers

From which:

$$0.9243408674589(e^{(\sqrt{3} \cdot 5.0596442562)} + e^{(\sqrt{3} \cdot (1/2 \cdot 5.0596442562 - (\sqrt{3})/2 \cdot 3.794733192))}) + 29 + \text{Pi}$$

where 29 is a Lucas number

Input interpretation:

$$0.9243408674589 \left(e^{\sqrt{3} \times 5.0596442562} + \exp\left(\sqrt{3} \left(\frac{1}{2} \times 5.0596442562 + \frac{\sqrt{3}}{2} \times (-3.794733192) \right)\right) \right) + 29 + \pi$$

Result:

5945.260556...

5945.260556... result about equal to the rest mass of bottom Xi baryon 5945.5

Series representations:

$$0.92434086745890000 \left(e^{\sqrt{3} \cdot 5.059644256200000} + e^{\sqrt{3} \left(5.059644256200000/2 - (3.79473 \sqrt{3})/2 \right)} \right) + 29 + \pi = 0.92434086745890000 \left(31.37370749356104 + 1.0000000000000000000 e^{5.059644256200000 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + 1.0000000000000000000 \exp\left(\sqrt{2} \left(\sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k} \right)\right) \left(2.52982212810000 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k} \right) + 1.0818519825365875 \pi \right)$$

$$0.92434086745890000 \left(e^{\sqrt{3} \cdot 5.059644256200000} + e^{\sqrt{3} \left(5.059644256200000/2 - (3.79473 \sqrt{3})/2 \right)} \right) + 29 + \pi = 0.92434086745890000 \left(31.37370749356104 + 1.0000000000000000000 e^{5.059644256200000 \sqrt{2} \sum_{k=0}^{\infty} \frac{\binom{-1}{2}^k \binom{-1}{2}_k}{k!}} + 1.0000000000000000000 \exp\left(\sqrt{2} \left(\sum_{k=0}^{\infty} \frac{\binom{-1}{2}^k \binom{-1}{2}_k}{k!} \right)\right) \left(2.52982212810000 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} \frac{\binom{-1}{2}^k \binom{-1}{2}_k}{k!} \right) + 1.0818519825365875 \pi \right)$$

$$\begin{aligned}
& 0.92434086745890000 \left(e^{\sqrt{3} \cdot 5.05964425620000} + e^{\sqrt{3} \left(5.05964425620000/2 - (3.79473 \sqrt{3})/2 \right)} \right) + \\
& 29 + \pi = 0.9243408674589000 \left(31.37370749356104 + 1.0000000000000000 \right. \\
& \quad \exp \left(5.05964425620000 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!} \right) + \\
& \quad 1.0000000000000000 \exp \left(\sqrt{z_0} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!} \right) \right. \\
& \quad \left. \left(2.52982212810000 - 1.89737 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!} \right) \right) + \\
& \quad \left. 1.0818519825365875 \pi \right) \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

\mathbb{R} is the set of real numbers

Now, we have that:

$$V = V_0 \left(e^{\sqrt{3} \gamma_9 \Phi_t} + e^{\sqrt{3} \gamma_\Lambda \Phi_t} \right), \quad (6.18)$$

where

$$\gamma_9 = \sqrt{\frac{d^2 - 14d + 184}{24(d - 4)}}, \quad \gamma_\Lambda = -\frac{10}{3} \frac{(d - 4)(d - 10)}{\sqrt{2(d^2 - 14d + 184)}} \quad (6.19)$$

$\text{sqrt}(((6^2-14*6+184)/(24(6-4))))$

Input:

$$\sqrt{\frac{6^2 - 14 \times 6 + 184}{24(6 - 4)}}$$

Result:

$$\sqrt{\frac{17}{6}}$$

Decimal approximation:

1.683250823060346325560564319511600118433983160746602156975...

1.68325082...

Alternate form:

$$\frac{\sqrt{102}}{6}$$

$$\left(-\frac{10}{3}(6-4)(6-10)\right) \times \frac{1}{\sqrt{2(6^2-14 \times 6+184)}}$$

Input:

$$\left(-\frac{10}{3}(6-4)(6-10)\right) \times \frac{1}{\sqrt{2(6^2-14 \times 6+184)}}$$

Result:

$$\frac{20}{3\sqrt{17}}$$

Decimal approximation:

1.616904166908886490126043080774147852998901657009262915450...

1.6169041669..... result that is a good approximation to the value of the golden ratio
1,618033988749...

Alternate form:

$$\frac{20\sqrt{17}}{51}$$

$$\left(\left(e^{\sqrt{3} \times 1.68325082 \times 5.059644256269}\right) + e^{\sqrt{3} \times 1.6169041669 \times 5.059644256269}\right)$$

Input interpretation:

$$e^{\sqrt{3} \times 1.68325082 \times 5.059644256269} + e^{\sqrt{3} \times 1.6169041669 \times 5.059644256269}$$

Result:

3.97437349772313604868572629687922895277844703622987151... $\times 10^6$

3.974373497723136... $\times 10^6$

For $V_0 = 0.9243408674589$ we obtain:

$$0.9243408674589((e^{(\sqrt{3} \times 1.68325082 \times 5.059644256269)} + e^{(\sqrt{3} \times 1.6169041669 \times 5.059644256269)}))$$

Input interpretation:

$$0.9243408674589 \left(e^{\sqrt{3} \times 1.68325082 \times 5.059644256269} + e^{\sqrt{3} \times 1.6169041669 \times 5.059644256269} \right)$$

Result:

$$3.6736758... \times 10^6$$

$$3.6736758... * 10^6$$

From the ratio of (6.17) and (6.18), we obtain:

$$\left(\frac{0.9243408674589(e^{(\sqrt{3} \times 5.0596442562)} + e^{(\sqrt{3} \times (1/2 \times 5.0596442562 - (\sqrt{3})/2 \times 3.794733192)}))}{0.9243408674589((e^{(\sqrt{3} \times 1.68325082 \times 5.059644256269)} + e^{(\sqrt{3} \times 1.6169041669 \times 5.059644256269)}))} \right)$$

Input interpretation:

$$\left(\frac{0.9243408674589 \left(e^{\sqrt{3} \times 5.0596442562} + \exp \left(\sqrt{3} \left(\frac{1}{2} \times 5.0596442562 + \frac{\sqrt{3}}{2} \times (-3.794733192) \right) \right) \right)}{0.9243408674589 \left(e^{\sqrt{3} \times 1.68325082 \times 5.059644256269} + e^{\sqrt{3} \times 1.6169041669 \times 5.059644256269} \right)} \right)$$

Result:

$$0.0016095919...$$

$$0.0016095919...$$

Series representations:

$$\begin{aligned}
& \left(0.92434086745890000 \left(e^{\sqrt{3} \cdot 5.05964425620000} + e^{\sqrt{3} \left(5.05964425620000/2 - (\sqrt{3} \cdot 3.79473)/2 \right)} \right) \right) / \\
& \left(0.92434086745890000 \right. \\
& \quad \left. \left(e^{\sqrt{3} \cdot 1.68325 \times 5.0596442562690000} + e^{\sqrt{3} \cdot 1.616904166900000 \times 5.0596442562690000} \right) \right) = \\
& \left(1.0000000000000000 \left(1.0000000000000000 e^{5.05964425620000 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + \right. \right. \\
& \quad 1.0000000000000000 \exp \left(\sqrt{2} \left(\sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k} \right) \right) \\
& \quad \left. \left. \left(2.52982212810000 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k} \right) \right) \right) / \\
& \left(1.0000000000000000 e^{8.18095988099300 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + \right. \\
& \quad \left. 1.0000000000000000 e^{8.51665 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(0.92434086745890000 \left(e^{\sqrt{3} \cdot 5.05964425620000} + e^{\sqrt{3} \left(5.05964425620000/2 - (\sqrt{3} \cdot 3.79473)/2 \right)} \right) \right) / \\
& \left(0.92434086745890000 \right. \\
& \quad \left. \left(e^{\sqrt{3} \cdot 1.68325 \times 5.0596442562690000} + e^{\sqrt{3} \cdot 1.616904166900000 \times 5.0596442562690000} \right) \right) = \\
& \left(1.0000000000000000 \left(1.0000000000000000 e^{5.05964425620000 \sqrt{2} \sum_{k=0}^{\infty} \frac{\binom{-1}{2}^k \binom{-1}{2}_k}{k!}} + \right. \right. \\
& \quad 1.0000000000000000 \exp \left(\sqrt{2} \left(\sum_{k=0}^{\infty} \frac{\binom{-1}{2}^k \binom{-1}{2}_k}{k!} \right) \right) \\
& \quad \left. \left. \left(2.52982212810000 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} \frac{\binom{-1}{2}^k \binom{-1}{2}_k}{k!} \right) \right) \right) / \\
& \left(1.0000000000000000 e^{8.18095988099300 \sqrt{2} \sum_{k=0}^{\infty} \frac{\binom{-1}{2}^k \binom{-1}{2}_k}{k!}} + \right. \\
& \quad \left. 1.0000000000000000 e^{8.51665 \sqrt{2} \sum_{k=0}^{\infty} \frac{\binom{-1}{2}^k \binom{-1}{2}_k}{k!}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(0.92434086745890000 \left(e^{\sqrt{3} \cdot 5.05964425620000} + e^{\sqrt{3} \left(5.05964425620000/2 - (\sqrt{3} \cdot 3.79473)/2 \right)} \right) \right) / \\
& \left(0.92434086745890000 \right. \\
& \quad \left. \left(e^{\sqrt{3} \cdot 1.68325 \times 5.0596442562690000} + e^{\sqrt{3} \cdot 1.61690416690000 \times 5.0596442562690000} \right) \right) = \\
& \left(1.000000000000000 \left(1.000000000000000 \exp \left[5.05964425620000 \sqrt{z_0} \right. \right. \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right] + 1.000000000000000 \right. \\
& \quad \left. \exp \left[\sqrt{z_0} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right) \right] \left(2.52982212810000 - \right. \right. \\
& \quad \left. \left. 1.89737 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right) \right] \right) / \\
& \left(1.000000000000000 \exp \left[8.18095988099300 \sqrt{z_0} \right. \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right] + \right. \\
& \quad \left. 1.000000000000000 \exp \left[8.51665 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right] \right) \\
& \text{for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

And:

$$\begin{aligned}
& ((0.9243408674589((e^{(\sqrt{3} \cdot 1.68325082 \cdot 5.059644256269)} + e^{(\sqrt{3} \cdot 1.6169041669 \cdot 5.059644256269)}))) / \\
& ((0.9243408674589(e^{(\sqrt{3} \cdot 5.0596442562)} + e^{(\sqrt{3} \cdot (1/2 \cdot 5.0596442562 - (\sqrt{3})/2 \cdot 3.794733192)})))
\end{aligned}$$

Input interpretation:

$$\begin{aligned}
& \left(0.9243408674589 \left(e^{\sqrt{3} \times 1.68325082 \times 5.059644256269} + e^{\sqrt{3} \times 1.6169041669 \times 5.059644256269} \right) \right) / \\
& \left(0.9243408674589 \right. \\
& \quad \left. \left(e^{\sqrt{3} \times 5.0596442562} + \exp \left[\sqrt{3} \left(\frac{1}{2} \times 5.0596442562 + \frac{\sqrt{3}}{2} \times (-3.794733192) \right) \right] \right) \right)
\end{aligned}$$

Result:

621.27548...

621.27548...

Series representations:

$$\begin{aligned}
& \left(0.92434086745890000 \right. \\
& \quad \left. \left(e^{\sqrt{3} \cdot 1.68325 \times 5.0596442562690000} + e^{\sqrt{3} \cdot 1.61690416690000 \times 5.0596442562690000} \right) \right) / \\
& \left(0.92434086745890000 \right. \\
& \quad \left. \left(e^{\sqrt{3} \cdot 5.05964425620000} + e^{\sqrt{3} \cdot (5.05964425620000/2 - (\sqrt{3} \cdot 3.79473)/2)} \right) \right) = \\
& \left(1.0000000000000000 \left(1.0000000000000000 e^{8.18095988099300 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + \right. \right. \\
& \quad \left. \left. 1.0000000000000000 e^{8.51665 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \right) \right) / \\
& \left(1.0000000000000000 e^{5.05964425620000 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + 1.0000000000000000 \right. \\
& \quad \left. \exp \left(\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{k} \right) \left(2.52982212810000 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{k} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(0.92434086745890000 \right. \\
& \quad \left. \left(e^{\sqrt{3} \cdot 1.68325 \times 5.0596442562690000} + e^{\sqrt{3} \cdot 1.61690416690000 \times 5.0596442562690000} \right) \right) / \\
& \left(0.92434086745890000 \right. \\
& \quad \left. \left(e^{\sqrt{3} \cdot 5.05964425620000} + e^{\sqrt{3} \cdot (5.05964425620000/2 - (\sqrt{3} \cdot 3.79473)/2)} \right) \right) = \\
& \left(1.0000000000000000 \left(1.0000000000000000 e^{8.18095988099300 \sqrt{2} \sum_{k=0}^{\infty} \frac{(-1)^k (-1/2)_k}{k!}} + \right. \right. \\
& \quad \left. \left. 1.0000000000000000 e^{8.51665 \sqrt{2} \sum_{k=0}^{\infty} \frac{(-1)^k (-1/2)_k}{k!}} \right) \right) / \\
& \left(1.0000000000000000 e^{5.05964425620000 \sqrt{2} \sum_{k=0}^{\infty} \frac{(-1)^k (-1/2)_k}{k!}} + \right. \\
& \quad \left. 1.0000000000000000 \exp \left(\sqrt{2} \sum_{k=0}^{\infty} \frac{(-1)^k (-1/2)_k}{k!} \right) \right. \\
& \quad \left. \left(2.52982212810000 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} \frac{(-1)^k (-1/2)_k}{k!} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left(0.92434086745890000 \right. \\
& \quad \left. \left(e^{\sqrt{3} \cdot 1.68325 \times 5.0596442562690000} + e^{\sqrt{3} \cdot 1.61690416690000 \times 5.0596442562690000} \right) \right) / \\
& \left(0.92434086745890000 \right. \\
& \quad \left. \left(e^{\sqrt{3} \cdot 5.05964425620000} + e^{\sqrt{3} \cdot (5.05964425620000/2 - (\sqrt{3} \cdot 3.79473)/2)} \right) \right) = \\
& \left(1.0000000000000000 \left(1.0000000000000000 \right. \right. \\
& \quad \exp \left(\frac{4.09047994049650 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma(-\frac{1}{2}-s) \Gamma(s)}{\sqrt{\pi}} \right) + \\
& \quad \left. 1.0000000000000000 \exp \left(\frac{4.25833 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma(-\frac{1}{2}-s) \Gamma(s)}{\sqrt{\pi}} \right) \right) \right) / \\
& \left(1.0000000000000000 \exp \left(\frac{2.52982212810000 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma(-\frac{1}{2}-s) \Gamma(s)}{\sqrt{\pi}} \right) \right. \\
& \quad \left. 1.0000000000000000 \exp \left(\frac{1}{\sqrt{\pi}^2} \left(1.26491 \sqrt{\pi} - 0.474342 \right. \right. \right. \\
& \quad \quad \left. \left. \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma(-\frac{1}{2}-s) \Gamma(s) \right) \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma(-\frac{1}{2}-s) \Gamma(s) \right) \right)
\end{aligned}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

Res_f is a complex residue
 $z=z_0$

We have also:

$$\begin{aligned}
& 1/[1+((((0.9243408674589(e^{(\sqrt{3}*5.0596442562)}+e^{(\sqrt{3}(1/2*5.0596442562- \\
& (\sqrt{3})/2*3.794733192)))))))/ \\
& (((0.9243408674589((e^{(\sqrt{3}*1.68325082*5.059644256269)}+e^{(\sqrt{3}*1.6169041 \\
& 669*5.059644256269))))))]
\end{aligned}$$

Input interpretation:

1

$$1 + \frac{0.9243408674589 \left(e^{\sqrt{3} \times 5.0596442562} + \exp\left(\sqrt{3} \left(\frac{1}{2} \times 5.0596442562 + \frac{\sqrt{3}}{2} \times (-3.794733192) \right) \right) \right)}{0.9243408674589 \left(e^{\sqrt{3} \times 1.68325082 \times 5.059644256269} + e^{\sqrt{3} \times 1.6169041669 \times 5.059644256269} \right)}$$

Result:

0.99839299470...

0.99839299470...

Series representations:

$$\begin{aligned} & 1 / \left(1 + \left(0.92434086745890000 \right. \right. \\ & \quad \left. \left. \left(e^{\sqrt{3} \cdot 5.05964425620000} + e^{\sqrt{3} \left(5.05964425620000/2 - (\sqrt{3} \cdot 3.79473)/2 \right)} \right) \right) \right) / \\ & \quad \left(0.92434086745890000 \left(e^{\sqrt{3} \cdot 1.68325 \times 5.0596442562690000} + \right. \right. \\ & \quad \quad \left. \left. e^{\sqrt{3} \cdot 1.61690416690000 \cdot 5.0596442562690000} \right) \right) = \\ & \left(1.0000000000000000 \left(1.0000000000000000 e^{8.18095988099300 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + \right. \right. \\ & \quad \left. \left. 1.0000000000000000 e^{8.51665 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \right) \right) / \\ & \left(1.0000000000000000 e^{5.05964425620000 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + \right. \\ & \quad 1.0000000000000000 e^{8.18095988099300 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + \\ & \quad \left. 1.0000000000000000 e^{8.51665 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + 1.0000000000000000 \right. \\ & \quad \left. \exp\left(\sqrt{2} \left(\sum_{k=0}^{\infty} 2^{-k} \binom{1}{k} \right) \right) \left(2.52982212810000 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{k} \right) \right) \end{aligned}$$

$$\begin{aligned}
& 1 / \left(1 + \left(0.92434086745890000 \right. \right. \\
& \quad \left. \left. \left(e^{\sqrt{3} \cdot 5.05964425620000} + e^{\sqrt{3} \left(5.05964425620000/2 - (\sqrt{3} \cdot 3.79473)/2 \right)} \right) \right) \right) / \\
& \quad \left(0.92434086745890000 \left(e^{\sqrt{3} \cdot 1.68325 \times 5.0596442562690000} + \right. \right. \\
& \quad \quad \left. \left. e^{\sqrt{3} \cdot 1.61690416690000 \times 5.0596442562690000} \right) \right) = \\
& \left(1.0000000000000000 \left(1.0000000000000000 e^{8.18095988099300 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + \right. \right. \\
& \quad \left. \left. 1.0000000000000000 e^{8.51665 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} \right) \right) / \\
& \left(1.0000000000000000 e^{5.05964425620000 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + \right. \\
& \quad 1.0000000000000000 e^{8.18095988099300 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + \\
& \quad 1.0000000000000000 e^{8.51665 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + \\
& \quad 1.0000000000000000 \exp \left(\sqrt{2} \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right) \\
& \quad \left. \left. \left(2.52982212810000 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 1 / \left(1 + \left(0.92434086745890000 \right. \right. \\
& \quad \left. \left. \left(e^{\sqrt{3} \cdot 5.05964425620000} + e^{\sqrt{3} \left(5.05964425620000/2 - (\sqrt{3} \cdot 3.79473)/2 \right)} \right) \right) \right) / \\
& \quad \left(0.92434086745890000 \left(e^{\sqrt{3} \cdot 1.68325 \times 5.0596442562690000} + \right. \right. \\
& \quad \quad \left. \left. e^{\sqrt{3} \cdot 1.61690416690000 \times 5.0596442562690000} \right) \right) = \\
& \left(1.0000000000000000 \left(1.0000000000000000 \right. \right. \\
& \quad \left. \left. \exp \left(\frac{4.09047994049650 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) + \right. \right. \\
& \quad \left. \left. 1.0000000000000000 \exp \left(\frac{4.25833 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) \right) \right) / \\
& \left(1.0000000000000000 \exp \left(\frac{2.52982212810000 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) + \right. \\
& \quad 1.0000000000000000 \\
& \quad \left. \exp \left(\frac{4.09047994049650 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) + \right. \\
& \quad 1.0000000000000000 \exp \left(\frac{4.25833 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) + \\
& \quad 1.0000000000000000 \\
& \quad \left. \exp \left(\frac{1}{\sqrt{\pi}^2} \left(1.26491 \sqrt{\pi} - 0.474342 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right) \right. \right. \\
& \quad \left. \left. \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right) \right)
\end{aligned}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

Res_f is a complex residue
 $z=20$

$$1/\left[1+\left(\left(0.9243408674589\left(e^{\sqrt{3}\cdot 5.05964425}\right)+e^{\sqrt{3}\left(\frac{1}{2}\cdot 5.05964425-\left(\sqrt{3}\right)/2\cdot 3.794733192\right)}\right)\right)\right] / \left(\left(0.9243408674589\left(e^{\sqrt{3}\cdot 1.68325082\cdot 5.05964425}\right)+e^{\sqrt{3}\cdot 1.6169041669\cdot 5.05964425}\right)\right)\right]^{1/2}$$

Input interpretation:

$$\frac{1}{\sqrt{1 + \frac{0.9243408674589 \left(e^{\sqrt{3} \times 5.05964425} + \exp\left(\sqrt{3} \left(\frac{1}{2} \times 5.05964425 + \frac{\sqrt{3}}{2} \times (-3.794733192) \right) \right) \right)}{0.9243408674589 \left(e^{\sqrt{3} \times 1.68325082 \times 5.05964425} + e^{\sqrt{3} \times 1.6169041669 \times 5.05964425} \right)}}$$

Result:

0.99919617428...

0.99919617428... result practically equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt{5^3} - 1}}} - \varphi + 1 = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = φ**

Series representations:

$$\frac{1}{\sqrt{1 + \frac{0.92434086745890000 \left(e^{\sqrt{3} \cdot 5.05964} + e^{\sqrt{3} \left(5.05964/2 - (\sqrt{3} \cdot 3.79473)/2 \right)} \right)}{0.92434086745890000 \left(e^{\sqrt{3} \cdot 1.68325 \times 5.05964} + e^{\sqrt{3} \cdot 1.61690416690000 \times 5.05964} \right)}} = \frac{1}{\sqrt{1 + \left(1.0000000000000000 \left(e^{5.05964 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + \exp\left(\sqrt{2} \left(\sum_{k=0}^{\infty} 2^{-k} \binom{1}{k} \right) \left(2.52982 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{k} \right) \right) \right) \left(e^{8.18096 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + e^{8.51665 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \right) \right)}}$$

$$\begin{aligned}
& \frac{1}{\sqrt{1 + \frac{0.92434086745890000 \left(e^{\sqrt{3}} 5.05964 + e^{\sqrt{3}} \left(5.05964/2 - (\sqrt{3} 3.79473)/2 \right) \right)}{0.92434086745890000 \left(e^{\sqrt{3}} 1.68325 \times 5.05964 + e^{\sqrt{3}} 1.61690416690000 \times 5.05964 \right)}}} \\
& 1 / \left(\sqrt{1 + \left(1.000000000000000000 \right. \right. \\
& \left. \left. \left(e^{5.05964 \sqrt{2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2} \right)^k \left(-\frac{1}{2} \right)_k}{k!} + \exp \left(\sqrt{2} \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2} \right)^k \left(-\frac{1}{2} \right)_k}{k!} \right) \right. \right. \right. \\
& \left. \left. \left. \left(2.52982 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2} \right)^k \left(-\frac{1}{2} \right)_k}{k!} \right) \right) \right) \right) / \\
& \left. \left. \left(e^{8.18096 \sqrt{2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2} \right)^k \left(-\frac{1}{2} \right)_k}{k!} + e^{8.51665 \sqrt{2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2} \right)^k \left(-\frac{1}{2} \right)_k}{k!} \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\sqrt{1 + \frac{0.92434086745890000 \left(e^{\sqrt{3}} 5.05964 + e^{\sqrt{3}} \left(5.05964/2 - (\sqrt{3} 3.79473)/2 \right) \right)}{0.92434086745890000 \left(e^{\sqrt{3}} 1.68325 \times 5.05964 + e^{\sqrt{3}} 1.61690416690000 \times 5.05964 \right)}}} \\
& 1 / \left(\sqrt{1 + \left(1.000000000000000000 \right. \right. \\
& \left. \left. \left(\exp \left(\frac{2.52982 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) \right. \right. \\
& \left. \left. \exp \left(\frac{1}{2 \sqrt{\pi}} \left(\sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right) \left(2.52982 - \right. \right. \right. \\
& \left. \left. \left. \frac{0.948683 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) \right) \right) \right) / \\
& \left. \left. \left(\exp \left(\frac{4.09048 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) \right. \right. \\
& \left. \left. \exp \left(\frac{4.25833 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) \right) \right) \right)
\end{aligned}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

Res_f is a complex residue
 $\Rightarrow 0$

And:

$$3 * [((0.9243408674589((e^{(\sqrt{3} * 1.68325082 * 5.05964425)} + e^{(\sqrt{3} * 1.616904166 * 5.05964425)}))) / ((0.9243408674589(e^{(\sqrt{3} * 5.05964425)} + e^{(\sqrt{3}(1/2 * 5.05964425 - (\sqrt{3})/2 * 3.794733192)})))))] + 1/\text{golden ratio}$$

Input interpretation:

$$3 \times \left(0.9243408674589 \left(e^{\sqrt{3} \times 1.68325082 \times 5.05964425} + e^{\sqrt{3} \times 1.6169041669 \times 5.05964425} \right) \right) / \left(0.9243408674589 \left(e^{\sqrt{3} \times 5.05964425} + \exp \left(\sqrt{3} \left(\frac{1}{2} \times 5.05964425 + \frac{\sqrt{3}}{2} \times (-3.794733192) \right) \right) \right) \right) + \frac{1}{\phi}$$

ϕ is the golden ratio

Result:

1864.444...

1864.444... result almost equal to the rest mass of D meson 1864.84

Series representations:

$$\frac{3 \left(0.92434086745890000 \left(e^{\sqrt{3} \cdot 1.68325 \cdot 5.05964} + e^{\sqrt{3} \cdot 1.61690416690000 \cdot 5.05964} \right) \right)}{0.92434086745890000 \left(e^{\sqrt{3} \cdot 5.05964} + e^{\sqrt{3} \left(5.05964/2 - (\sqrt{3} \cdot 3.79473)/2 \right)} \right)} + \frac{1}{\phi} =$$

$$\left(1.0000000000000000 \right) \left(1.0000000000000000 e^{5.05964 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + 1.0000000000000000 \right.$$

$$\left. \exp \left(\sqrt{2} \left(\sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k} \right) \right) \left(2.52982 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k} \right) \right) +$$

$$\frac{3.0000000000000000 e^{8.18096 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \phi + 3.0000000000000000 e^{8.51665 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \phi}{\left(1.0000000000000000 e^{5.05964 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + 1.0000000000000000 \right) \exp \left(\sqrt{2} \left(\sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k} \right) \right) \left(2.52982 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k} \right) \phi$$

$$\begin{aligned}
& \frac{3 \left(0.92434086745890000 \left(e^{\sqrt{3} \cdot 1.68325 \times 5.05964} + e^{\sqrt{3} \cdot 1.61690416690000 \times 5.05964} \right) \right)}{0.92434086745890000 \left(e^{\sqrt{3} \cdot 5.05964} + e^{\sqrt{3} \cdot (5.05964/2 - (\sqrt{3} \cdot 3.79473)/2)} \right)} + \frac{1}{\phi} = \\
& \left(1.0000000000000000 \right. \\
& \quad \left(1.0000000000000000 e^{5.05964 \sqrt{2} \sum_{k=0}^{\infty} \frac{(-1)^k (-1/2)_k}{k!}} + 1.0000000000000000 \right. \\
& \quad \quad \left. \exp \left(\sqrt{2} \left(\sum_{k=0}^{\infty} \frac{(-1)^k (-1/2)_k}{k!} \right) \left(2.52982 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} \frac{(-1)^k (-1/2)_k}{k!} \right) \right) + \right. \\
& \quad \quad 3.0000000000000000 e^{8.18096 \sqrt{2} \sum_{k=0}^{\infty} \frac{(-1)^k (-1/2)_k}{k!}} \phi + \\
& \quad \quad \left. \left. 3.0000000000000000 e^{8.51665 \sqrt{2} \sum_{k=0}^{\infty} \frac{(-1)^k (-1/2)_k}{k!}} \phi \right) \right) / \\
& \left(\left(1.0000000000000000 e^{5.05964 \sqrt{2} \sum_{k=0}^{\infty} \frac{(-1)^k (-1/2)_k}{k!}} + 1.0000000000000000 \right. \right. \\
& \quad \left. \left. \exp \left(\sqrt{2} \left(\sum_{k=0}^{\infty} \frac{(-1)^k (-1/2)_k}{k!} \right) \left(2.52982 - 1.89737 \sqrt{2} \sum_{k=0}^{\infty} \frac{(-1)^k (-1/2)_k}{k!} \right) \right) \right) \phi \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{3 \left(0.92434086745890000 \left(e^{\sqrt{3} \cdot 1.68325 \times 5.05964} + e^{\sqrt{3} \cdot 1.61690416690000 \times 5.05964} \right) \right)}{0.92434086745890000 \left(e^{\sqrt{3} \cdot 5.05964} + e^{\sqrt{3} \cdot (5.05964/2 - (\sqrt{3} \cdot 3.79473)/2)} \right)} + \frac{1}{\phi} = \\
& \left(1.0000000000000000 \right. \\
& \quad \left(1.0000000000000000 \exp \left[5.05964 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right] + \right. \\
& \quad 1.0000000000000000 \exp \left[\sqrt{z_0} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right) \right. \\
& \quad \left. \left. \left(2.52982 - 1.89737 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right) \right] \right) + \\
& \quad 3.0000000000000000 \exp \left[8.18096 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right] \phi + \\
& \quad 3.0000000000000000 \\
& \quad \left. \left. \left. \exp \left[8.51665 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right] \phi \right] \right) \right) / \\
& \quad \left(\left(1.0000000000000000 \exp \left[5.05964 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right] + \right. \right. \\
& \quad 1.0000000000000000 \exp \left[\sqrt{z_0} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right) \right. \\
& \quad \left. \left. \left. \left(2.52982 - 1.89737 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!} \right) \right] \right) \right) \phi \right) \\
& \text{for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

Now, we have that:

$$\frac{dy}{dx} = \frac{\arctan\left(\frac{y}{x}\right)}{\log \sqrt{x^2 + y^2} + \sqrt{\left(\log \sqrt{x^2 + y^2}\right)^2 + \left(\arctan\left(\frac{y}{x}\right)\right)^2}}, \quad (4.259)$$

which clearly tends to zero for large values of x and y , so that indeed $\varphi \rightarrow +\infty$. Notice that a cosmological term Λ would displace $\frac{y}{x}$ in the argument of the inner arctan to $\frac{y}{x} + \alpha \Lambda$, with α positive. If $\Lambda > 0$ the contour still flattens so that x/y grows and $\varphi \rightarrow \infty$ at late times. However, if $\Lambda < 0$ the slope at some point changes sign and y is driven to zero, which signals the expected Big Crunch.

For $y = 144$, $x = 89$ (that are Fibonacci numbers) we obtain:

$$\text{atan}(144/89) * 1/((((\ln \text{sqrt}(89^2+144^2))+((\ln \text{sqrt}(89^2+144^2))^2+(\text{atan}(144/89))^2)))^1/2)))$$

Input:

$$\tan^{-1}\left(\frac{144}{89}\right) \times \frac{1}{\log\left(\sqrt{89^2 + 144^2}\right) + \sqrt{\log^2\left(\sqrt{89^2 + 144^2}\right) + \tan^{-1}\left(\frac{144}{89}\right)^2}}$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Exact Result:

$$\frac{\tan^{-1}\left(\frac{144}{89}\right)}{\frac{\log(28\,657)}{2} + \sqrt{\frac{\log^2(28\,657)}{4} + \tan^{-1}\left(\frac{144}{89}\right)^2}}$$

(result in radians)

Decimal approximation:

0.098157523250448659905738796019256424398657719500538685926...

(result in radians)

0.09815752325....

Alternate forms:

$$\frac{2 \tan^{-1}\left(\frac{144}{89}\right)}{\log(28\,657) + \sqrt{\log^2(28\,657) + 4 \tan^{-1}\left(\frac{144}{89}\right)^2}}$$

$$\frac{i \left(\log\left(1 - \frac{144i}{89}\right) - \log\left(1 + \frac{144i}{89}\right) \right)}{2 \left(\frac{\log(28\,657)}{2} + \sqrt{\frac{\log^2(28\,657)}{4} - \frac{1}{4} \left(\log\left(1 - \frac{144i}{89}\right) - \log\left(1 + \frac{144i}{89}\right) \right)^2} \right)}$$

Alternative representations:

$$\frac{\tan^{-1}\left(\frac{144}{89}\right)}{\log\left(\sqrt{89^2 + 144^2}\right) + \sqrt{\log^2\left(\sqrt{89^2 + 144^2}\right) + \tan^{-1}\left(\frac{144}{89}\right)^2}} =$$

$$\frac{\tan^{-1}\left(1, \frac{144}{89}\right)}{\log\left(\sqrt{89^2 + 144^2}\right) + \sqrt{\tan^{-1}\left(1, \frac{144}{89}\right)^2 + \log^2\left(\sqrt{89^2 + 144^2}\right)}}$$

$$\frac{\tan^{-1}\left(\frac{144}{89}\right)}{\log\left(\sqrt{89^2 + 144^2}\right) + \sqrt{\log^2\left(\sqrt{89^2 + 144^2}\right) + \tan^{-1}\left(\frac{144}{89}\right)^2}} =$$

$$\frac{\tan^{-1}\left(\frac{144}{89}\right)}{\log_e\left(\sqrt{89^2 + 144^2}\right) + \sqrt{\tan^{-1}\left(\frac{144}{89}\right)^2 + \log_e^2\left(\sqrt{89^2 + 144^2}\right)}} =$$

$$\frac{\tan^{-1}\left(\frac{144}{89}\right)}{\log\left(\sqrt{89^2 + 144^2}\right) + \sqrt{\log^2\left(\sqrt{89^2 + 144^2}\right) + \tan^{-1}\left(\frac{144}{89}\right)^2}} =$$

$$\frac{\tan^{-1}\left(1, \frac{144}{89}\right)}{\log_e\left(\sqrt{89^2 + 144^2}\right) + \sqrt{\tan^{-1}\left(1, \frac{144}{89}\right)^2 + \log_e^2\left(\sqrt{89^2 + 144^2}\right)}}$$

Series representation:

$$\frac{\tan^{-1}\left(\frac{144}{89}\right)}{\log\left(\sqrt{89^2 + 144^2}\right) + \sqrt{\log^2\left(\sqrt{89^2 + 144^2}\right) + \tan^{-1}\left(\frac{144}{89}\right)^2}} =$$

$$\left(2 \tan^{-1}(z_0) + i \sum_{k=1}^{\infty} \frac{(-(-i - z_0)^{-k} + (i - z_0)^{-k}) \left(\frac{144}{89} - z_0\right)^k}{k} \right) /$$

$$\left(\log(28\,656) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{28\,656}\right)^k}{k} + 2 \sqrt{\left(\frac{1}{4} \left(\log(28\,656) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{28\,656}\right)^k}{k} \right)^2 + \right.} \right.$$

$$\left. \left. \left(\tan^{-1}(z_0) + \frac{1}{2} i \sum_{k=1}^{\infty} \frac{(-(-i - z_0)^{-k} + (i - z_0)^{-k}) \left(\frac{144}{89} - z_0\right)^k}{k} \right)^2 \right) \right)$$

for ($i z_0 \notin \mathbb{R}$ or (not ($1 \leq i z_0 < \infty$) and not ($-\infty < i z_0 \leq -1$)))

\mathbb{R} is the set of real numbers

For $y = 8$, $x = 5$ (that are Fibonacci numbers) we obtain:

$$\operatorname{atan}(8/5) * 1/((((\ln \operatorname{sqrt}(5^2+8^2)))+((((\ln \operatorname{sqrt}(5^2+8^2))^2+(\operatorname{atan}(8/5))^2))))^1/2)))$$

Input:

$$\tan^{-1}\left(\frac{8}{5}\right) \times \frac{1}{\log\left(\sqrt{5^2 + 8^2}\right) + \sqrt{\log^2\left(\sqrt{5^2 + 8^2}\right) + \tan^{-1}\left(\frac{8}{5}\right)^2}}$$

$\tan^{-1}(x)$ is the inverse tangent function
 $\log(x)$ is the natural logarithm

Exact Result:

$$\frac{\tan^{-1}\left(\frac{8}{5}\right)}{\frac{\log(89)}{2} + \sqrt{\frac{\log^2(89)}{4} + \tan^{-1}\left(\frac{8}{5}\right)^2}}$$

(result in radians)

Decimal approximation:

0.215071362395606031430139199289691497266228918734609327741...

(result in radians)

0.215071362395606....

Alternate forms:

$$\frac{2 \tan^{-1}\left(\frac{8}{5}\right)}{\log(89) + \sqrt{\log^2(89) + 4 \tan^{-1}\left(\frac{8}{5}\right)^2}}$$

$$\frac{i \left(\log\left(1 - \frac{8i}{5}\right) - \log\left(1 + \frac{8i}{5}\right) \right)}{2 \left(\frac{\log(89)}{2} + \sqrt{\frac{\log^2(89)}{4} - \frac{1}{4} \left(\log\left(1 - \frac{8i}{5}\right) - \log\left(1 + \frac{8i}{5}\right) \right)^2} \right)}$$

Alternative representations:

$$\frac{\tan^{-1}\left(\frac{8}{5}\right)}{\log\left(\sqrt{5^2 + 8^2}\right) + \sqrt{\log^2\left(\sqrt{5^2 + 8^2}\right) + \tan^{-1}\left(\frac{8}{5}\right)^2}} =$$

$$\frac{\tan^{-1}\left(1, \frac{8}{5}\right)}{\log\left(\sqrt{5^2 + 8^2}\right) + \sqrt{\tan^{-1}\left(1, \frac{8}{5}\right)^2 + \log^2\left(\sqrt{5^2 + 8^2}\right)}}$$

$$\frac{\tan^{-1}\left(\frac{8}{5}\right)}{\log\left(\sqrt{5^2 + 8^2}\right) + \sqrt{\log^2\left(\sqrt{5^2 + 8^2}\right) + \tan^{-1}\left(\frac{8}{5}\right)^2}} =$$

$$\frac{\tan^{-1}\left(\frac{8}{5}\right)}{\log_e\left(\sqrt{5^2 + 8^2}\right) + \sqrt{\tan^{-1}\left(\frac{8}{5}\right)^2 + \log_e^2\left(\sqrt{5^2 + 8^2}\right)}}$$

$$\frac{\tan^{-1}\left(\frac{8}{5}\right)}{\log\left(\sqrt{5^2+8^2}\right)+\sqrt{\log^2\left(\sqrt{5^2+8^2}\right)+\tan^{-1}\left(\frac{8}{5}\right)^2}} = \frac{\tan^{-1}\left(1, \frac{8}{5}\right)}{\log_e\left(\sqrt{5^2+8^2}\right)+\sqrt{\tan^{-1}\left(1, \frac{8}{5}\right)^2+\log_e^2\left(\sqrt{5^2+8^2}\right)}}$$

Series representation:

$$\frac{\tan^{-1}\left(\frac{8}{5}\right)}{\log\left(\sqrt{5^2+8^2}\right)+\sqrt{\log^2\left(\sqrt{5^2+8^2}\right)+\tan^{-1}\left(\frac{8}{5}\right)^2}} = \left(\frac{2 \tan^{-1}(z_0) + i \sum_{k=1}^{\infty} \frac{(-(-i-z_0)^{-k} + (i-z_0)^{-k}) \left(\frac{8}{5} - z_0\right)^k}{k}}{\left(\log(88) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{88}\right)^k}{k} + 2 \sqrt{\frac{1}{4} \left(\log(88) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{88}\right)^k}{k} \right)^2 + \left(\tan^{-1}(z_0) + \frac{1}{2} i \sum_{k=1}^{\infty} \frac{(-(-i-z_0)^{-k} + (i-z_0)^{-k}) \left(\frac{8}{5} - z_0\right)^k}{k} \right)^2} \right)} \right)$$

for $(i z_0 \notin \mathbb{R}$ or $(\text{not } (1 \leq i z_0 < \infty)$ and $\text{not } (-\infty < i z_0 \leq -1))$)

$$1/\left(\left(7 \cdot \left(\left(\tan^{-1}\left(\frac{8}{5}\right) \cdot \frac{1}{\left(\left(\ln \sqrt{5^2+8^2}\right) + \left(\left(\ln \sqrt{5^2+8^2}\right)^2 + \left(\tan^{-1}\left(\frac{8}{5}\right)\right)^2\right)^{1/2}\right)}\right)\right)\right)\right)$$

where 7 is a Lucas number

Input:

$$\frac{1}{7 \left(\tan^{-1}\left(\frac{8}{5}\right) \times \frac{1}{\log\left(\sqrt{5^2+8^2}\right)+\sqrt{\log^2\left(\sqrt{5^2+8^2}\right)+\tan^{-1}\left(\frac{8}{5}\right)^2}} \right)}$$

$\tan^{-1}(x)$ is the inverse tangent function
 $\log(x)$ is the natural logarithm

Exact Result:

$$\frac{\frac{\log(89)}{2} + \sqrt{\frac{\log^2(89)}{4} + \tan^{-1}\left(\frac{8}{5}\right)^2}}{7 \tan^{-1}\left(\frac{8}{5}\right)}$$

(result in radians)

Decimal approximation:

0.664231356819923454615359246608224462981367973844637840413...

(result in radians)

0.6642313568...

Alternate forms:

$$\frac{\log(89) + \sqrt{\log^2(89) + 4 \tan^{-1}\left(\frac{8}{5}\right)^2}}{14 \tan^{-1}\left(\frac{8}{5}\right)}$$

$$\frac{\sqrt{\frac{\log^2(89)}{4} + \tan^{-1}\left(\frac{8}{5}\right)^2}}{7 \tan^{-1}\left(\frac{8}{5}\right)} + \frac{\log(89)}{14 \tan^{-1}\left(\frac{8}{5}\right)}$$

$$-\frac{2i \sqrt{\frac{\log^2(89)}{4} - \frac{1}{4} \left(\log\left(1 - \frac{8i}{5}\right) - \log\left(1 + \frac{8i}{5}\right) \right)^2}}{7 \left(\log\left(1 - \frac{8i}{5}\right) - \log\left(1 + \frac{8i}{5}\right) \right)} - \frac{i \log(89)}{7 \left(\log\left(1 - \frac{8i}{5}\right) - \log\left(1 + \frac{8i}{5}\right) \right)}$$

Alternative representations:

$$\frac{1}{7 \tan^{-1}\left(\frac{8}{5}\right)} = \frac{1}{7 \tan^{-1}\left(1, \frac{8}{5}\right)}$$

$$\frac{\log\left(\sqrt{5^2+8^2}\right) + \sqrt{\log^2\left(\sqrt{5^2+8^2}\right) + \tan^{-1}\left(\frac{8}{5}\right)^2}}{7 \tan^{-1}\left(\frac{8}{5}\right)} = \frac{\log\left(\sqrt{5^2+8^2}\right) + \sqrt{\tan^{-1}\left(1, \frac{8}{5}\right)^2 + \log_e^2\left(\sqrt{5^2+8^2}\right)}}{7 \tan^{-1}\left(1, \frac{8}{5}\right)}$$

$$\frac{1}{7 \tan^{-1}\left(\frac{8}{5}\right)} = \frac{1}{7 \tan^{-1}\left(\frac{8}{5}\right)}$$

$$\frac{\log\left(\sqrt{5^2+8^2}\right) + \sqrt{\log^2\left(\sqrt{5^2+8^2}\right) + \tan^{-1}\left(\frac{8}{5}\right)^2}}{7 \tan^{-1}\left(\frac{8}{5}\right)} = \frac{\log_e\left(\sqrt{5^2+8^2}\right) + \sqrt{\tan^{-1}\left(\frac{8}{5}\right)^2 + \log_e^2\left(\sqrt{5^2+8^2}\right)}}{7 \tan^{-1}\left(\frac{8}{5}\right)}$$

$$\frac{1}{7 \tan^{-1}\left(\frac{8}{5}\right)} = \frac{1}{7 \tan^{-1}\left(1, \frac{8}{5}\right)}$$

$$\frac{\log\left(\sqrt{5^2+8^2}\right) + \sqrt{\log^2\left(\sqrt{5^2+8^2}\right) + \tan^{-1}\left(\frac{8}{5}\right)^2}}{7 \tan^{-1}\left(\frac{8}{5}\right)} = \frac{\log_e\left(\sqrt{5^2+8^2}\right) + \sqrt{\tan^{-1}\left(1, \frac{8}{5}\right)^2 + \log_e^2\left(\sqrt{5^2+8^2}\right)}}{7 \tan^{-1}\left(1, \frac{8}{5}\right)}$$

Series representation:

$$\frac{1}{\frac{7 \tan^{-1}\left(\frac{8}{5}\right)}{\log\left(\sqrt{5^2+8^2}\right) + \sqrt{\log^2\left(\sqrt{5^2+8^2}\right) + \tan^{-1}\left(\frac{8}{5}\right)^2}} =$$

$$\left(\log(88) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{88}\right)^k}{k} + 2 \sqrt{\left(\frac{1}{4} \left(\log(88) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{88}\right)^k}{k} \right)^2 + \left(\tan^{-1}(z_0) + \frac{1}{2} i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{8}{5} - z_0\right)^k}{k} \right)^2} \right) \right) /$$

$$\left(7 \left(2 \tan^{-1}(z_0) + i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{8}{5} - z_0\right)^k}{k} \right) \right)$$

for ($i z_0 \notin \mathbb{R}$ or (not ($1 \leq i z_0 < \infty$) and not ($-\infty < i z_0 \leq -1$)))

For $y = 4096 = 64^2$ and $x = 2304 = 48^2$, we obtain:

$$\left[\operatorname{atan}\left(\frac{4096}{2304}\right) * \frac{1}{\left(\left(\ln \sqrt{2304^2+4096^2}\right) + \left(\left(\ln \sqrt{2304^2+4096^2}\right)^2 + \left(\operatorname{atan}\left(\frac{4096}{2304}\right)\right)^2\right)^{1/2}}\right)} \right]$$

Input:

$$\tan^{-1}\left(\frac{4096}{2304}\right) \times \frac{1}{\log\left(\sqrt{2304^2 + 4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Exact Result:

$$\frac{\tan^{-1}\left(\frac{16}{9}\right)}{\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}$$

(result in radians)

Decimal approximation:

0.062345686523848593389829477150855610035009036144693460916...

(result in radians)

0.0623456865...

Alternate forms:

$$\frac{2 \tan^{-1}\left(\frac{16}{9}\right)}{\log(22\,085\,632) + \sqrt{\log^2(22\,085\,632) + 4 \tan^{-1}\left(\frac{16}{9}\right)^2}}$$

$$\frac{\tan^{-1}\left(\frac{16}{9}\right)}{\log(256) + \frac{\log(337)}{2} + \sqrt{\left(\log(256) + \frac{\log(337)}{2}\right)^2 + \tan^{-1}\left(\frac{16}{9}\right)^2}}$$

$$\frac{2 \tan^{-1}\left(\frac{16}{9}\right)}{16 \log(2) + \log(337) + \sqrt{(16 \log(2) + \log(337))^2 + 4 \tan^{-1}\left(\frac{16}{9}\right)^2}}$$

Alternative representations:

$$\frac{\tan^{-1}\left(\frac{4096}{2304}\right)}{\log\left(\sqrt{2304^2 + 4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}} =$$

$$\frac{\tan^{-1}\left(1, \frac{4096}{2304}\right)}{\log\left(\sqrt{2304^2 + 4096^2}\right) + \sqrt{\tan^{-1}\left(1, \frac{4096}{2304}\right)^2 + \log^2\left(\sqrt{2304^2 + 4096^2}\right)}}$$

$$\frac{\tan^{-1}\left(\frac{4096}{2304}\right)}{\log\left(\sqrt{2304^2 + 4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}} =$$

$$\frac{\tan^{-1}\left(\frac{4096}{2304}\right)}{\log_e\left(\sqrt{2304^2 + 4096^2}\right) + \sqrt{\tan^{-1}\left(\frac{4096}{2304}\right)^2 + \log_e^2\left(\sqrt{2304^2 + 4096^2}\right)}}$$

$$\frac{\tan^{-1}\left(\frac{4096}{2304}\right)}{\log\left(\sqrt{2304^2 + 4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}} =$$

$$\frac{\tan^{-1}\left(1, \frac{4096}{2304}\right)}{\log_e\left(\sqrt{2304^2 + 4096^2}\right) + \sqrt{\tan^{-1}\left(1, \frac{4096}{2304}\right)^2 + \log_e^2\left(\sqrt{2304^2 + 4096^2}\right)}}$$

Series representation:

$$\frac{\tan^{-1}\left(\frac{4096}{2304}\right)}{\log\left(\sqrt{2304^2 + 4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}} =$$

$$\frac{\left(2 \tan^{-1}(z_0) + i \sum_{k=1}^{\infty} \frac{(-(-i - z_0)^{-k} + (i - z_0)^{-k})\left(\frac{16}{9} - z_0\right)^k}{k}\right)}{\left(2 \log(-1 + 256 \sqrt{337}) - 2 \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256 \sqrt{337}}\right)^k}{k} + 2 \sqrt{\left(\log(-1 + 256 \sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256 \sqrt{337}}\right)^k}{k}\right)^2 + \left(\tan^{-1}(z_0) + \frac{1}{2} i \sum_{k=1}^{\infty} \frac{(-(-i - z_0)^{-k} + (i - z_0)^{-k})\left(\frac{16}{9} - z_0\right)^k}{k}\right)^2}\right)}$$

for ($i z_0 \notin \mathbb{R}$ or (not ($1 \leq i z_0 < \infty$) and not ($-\infty < i z_0 \leq -1$)))

$$1/\left[\operatorname{atan}\left(\frac{4096}{2304}\right) * 1/\left(\left(\left(\ln \sqrt{2304^2+4096^2}\right)\right)+\left(\left(\left(\ln \sqrt{2304^2+4096^2}\right)\right)^2+\left(\operatorname{atan}\left(\frac{4096}{2304}\right)\right)^2\right)\right)^{1/2}\right]$$

Input:

$$\frac{1}{\tan^{-1}\left(\frac{4096}{2304}\right) \times \frac{1}{\log\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2+4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}}$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Exact Result:

$$\frac{\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\tan^{-1}\left(\frac{16}{9}\right)}$$

(result in radians)

Decimal approximation:

16.03960202792021631041702383835265760607863951423263695222...

(result in radians)

16.039602027...

Alternate forms:

$$\frac{\log(256) + \frac{\log(337)}{2} + \sqrt{\left(\log(256) + \frac{\log(337)}{2}\right)^2 + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\tan^{-1}\left(\frac{16}{9}\right)}$$

$$\frac{16 \log(2) + \log(337) + \sqrt{(16 \log(2) + \log(337))^2 + 4 \tan^{-1}\left(\frac{16}{9}\right)^2}}{2 \tan^{-1}\left(\frac{16}{9}\right)}$$

$$\frac{\sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\tan^{-1}\left(\frac{16}{9}\right)} + \frac{\log(256 \sqrt{337})}{\tan^{-1}\left(\frac{16}{9}\right)}$$

Alternative representations:

$$\frac{1}{\tan^{-1}\left(\frac{4096}{2304}\right)} =$$

$$\frac{\log\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2+4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{1}$$

$$\frac{\log\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\tan^{-1}\left(1, \frac{4096}{2304}\right)^2 + \log^2\left(\sqrt{2304^2+4096^2}\right)}}{\tan^{-1}\left(1, \frac{4096}{2304}\right)}$$

$$\frac{1}{\tan^{-1}\left(\frac{4096}{2304}\right)} =$$

$$\frac{\log\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2+4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{1}$$

$$\frac{\log_e\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\tan^{-1}\left(\frac{4096}{2304}\right)^2 + \log_e^2\left(\sqrt{2304^2+4096^2}\right)}}{\tan^{-1}\left(\frac{4096}{2304}\right)}$$

$$\frac{1}{\tan^{-1}\left(\frac{4096}{2304}\right)} =$$

$$\frac{\log\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2+4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{1}$$

$$\frac{\log_e\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\tan^{-1}\left(1, \frac{4096}{2304}\right)^2 + \log_e^2\left(\sqrt{2304^2+4096^2}\right)}}{\tan^{-1}\left(1, \frac{4096}{2304}\right)}$$

Series representation:

$$\frac{1}{\tan^{-1}\left(\frac{4096}{2304}\right)} = \frac{1}{\log\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2+4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}$$

$$\left(2 \log\left(-1 + 256 \sqrt{337}\right) - 2 \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256 \sqrt{337}}\right)^k}{k} + \right.$$

$$\left. 2 \sqrt{\left(\log\left(-1 + 256 \sqrt{337}\right) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256 \sqrt{337}}\right)^k}{k} \right)^2 + \left(\tan^{-1}(z_0) + \frac{1}{2} i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2} \right) /$$

$$\left(2 \tan^{-1}(z_0) + i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)$$

for ($i z_0 \notin \mathbb{R}$ or (not ($1 \leq i z_0 < \infty$) and not ($-\infty < i z_0 \leq -1$)))

64/[atan(4096/2304) * 1/((((ln sqrt(2304^2+4096^2))+(((ln sqrt(2304^2+4096^2))^2+(atan(4096/2304))^2)))^1/2)))]-5-golden ratio

where 5 is a Fibonacci number

Input:

$$\frac{64}{\tan^{-1}\left(\frac{4096}{2304}\right) \times \frac{1}{\log\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2+4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}} - 5 - \phi$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Exact Result:

$$-\phi - 5 + \frac{64 \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)}{\tan^{-1}\left(\frac{16}{9}\right)}$$

(result in radians)

Decimal approximation:

1019.916495798143949018484938820204448671312619731083002080...

(result in radians)

1019.9164957... result almost equal to the rest mass of Phi meson 1019.445

Alternate forms:

$$-\phi - 5 + \frac{32 \left(\log(22\,085\,632) + \sqrt{\log^2(22\,085\,632) + 4 \tan^{-1}\left(\frac{16}{9}\right)^2} \right)}{\tan^{-1}\left(\frac{16}{9}\right)}$$

$$-\phi - 5 + \frac{32 \left(16 \log(2) + \log(337) + \sqrt{(16 \log(2) + \log(337))^2 + 4 \tan^{-1}\left(\frac{16}{9}\right)^2} \right)}{\tan^{-1}\left(\frac{16}{9}\right)}$$

$$-5 + \frac{1}{2}(-1 - \sqrt{5}) + \frac{64 \left(\log(256) + \frac{\log(337)}{2} + \sqrt{\left(\log(256) + \frac{\log(337)}{2}\right)^2 + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)}{\tan^{-1}\left(\frac{16}{9}\right)}$$

Alternative representations:

$$\frac{64}{\tan^{-1}\left(\frac{4096}{2304}\right)} - 5 - \phi = \frac{\log\left(\sqrt{2304^2 + 4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{64} - 5 - \phi + \frac{\log\left(\sqrt{2304^2 + 4096^2}\right) + \sqrt{\tan^{-1}\left(1, \frac{4096}{2304}\right)^2 + \log^2\left(\sqrt{2304^2 + 4096^2}\right)}}{\tan^{-1}\left(1, \frac{4096}{2304}\right)}$$

$$\begin{aligned}
& \frac{64}{\tan^{-1}\left(\frac{4096}{2304}\right)} - 5 - \phi = \\
& \frac{\log\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2+4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{64} \\
& -5 - \phi + \frac{\log_e\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\tan^{-1}\left(\frac{4096}{2304}\right)^2 + \log_e^2\left(\sqrt{2304^2+4096^2}\right)}}{\tan^{-1}\left(\frac{4096}{2304}\right)}
\end{aligned}$$

$$\begin{aligned}
& \frac{64}{\tan^{-1}\left(\frac{4096}{2304}\right)} - 5 - \phi = \\
& \frac{\log\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2+4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{64} \\
& -5 - \phi + \frac{\log_e\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\tan^{-1}\left(1, \frac{4096}{2304}\right)^2 + \log_e^2\left(\sqrt{2304^2+4096^2}\right)}}{\tan^{-1}\left(1, \frac{4096}{2304}\right)}
\end{aligned}$$

Series representation:

$$\frac{64}{\frac{\tan^{-1}\left(\frac{4096}{2304}\right)}{\log\left(\sqrt{2304^2+4096^2}\right)+\sqrt{\log^2\left(\sqrt{2304^2+4096^2}\right)+\tan^{-1}\left(\frac{4096}{2304}\right)^2}}}-5-\phi =$$

$$-\left(\left(22 \tan^{-1}(z_0) + 2 \sqrt{5} \tan^{-1}(z_0) - 256 \log\left(-1 + 256 \sqrt{337}\right) + 256 \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256 \sqrt{337}}\right)^k}{k} - 256 \sqrt{\left(\log\left(-1 + 256 \sqrt{337}\right) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256 \sqrt{337}}\right)^k}{k}\right)^2} + \left(\tan^{-1}(z_0) + \frac{1}{2} i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k}\right)^2\right) + 11 i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} + i \sqrt{5} \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k}\right) / \left(2 \left(2 \tan^{-1}(z_0) + i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k}\right)\right)$$

for ($i z_0 \notin \mathbb{R}$ or (not ($1 \leq i z_0 < \infty$) and not ($-\infty < i z_0 \leq -1$)))

And:

$$48 / \left[\operatorname{atan}\left(\frac{4096}{2304}\right) * \frac{1}{\left(\left(\left(\ln \sqrt{2304^2+4096^2}\right) + \left(\left(\left(\ln \sqrt{2304^2+4096^2}\right)^2 + \left(\operatorname{atan}\left(\frac{4096}{2304}\right)\right)^2\right)\right)^{1/2}\right)\right)} \right] + 13$$

where 13 is a Fibonacci number

Input:

$$\frac{48}{\frac{\tan^{-1}\left(\frac{4096}{2304}\right) \times \frac{1}{\log\left(\sqrt{2304^2+4096^2}\right)+\sqrt{\log^2\left(\sqrt{2304^2+4096^2}\right)+\tan^{-1}\left(\frac{4096}{2304}\right)^2}}}} + 13$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Exact Result:

$$13 + \frac{48 \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)}{\tan^{-1}\left(\frac{16}{9}\right)}$$

(result in radians)

Decimal approximation:

782.9008973401703829000171442409275650917746966831665737068...

(result in radians)

782.90089734... result almost equal to the rest mass of Omega meson 782.65

Alternate forms:

$$13 + \frac{24 \left(\log(22\,085\,632) + \sqrt{\log^2(22\,085\,632) + 4 \tan^{-1}\left(\frac{16}{9}\right)^2} \right)}{\tan^{-1}\left(\frac{16}{9}\right)}$$

$$13 + \frac{24 \left(16 \log(2) + \log(337) + \sqrt{(16 \log(2) + \log(337))^2 + 4 \tan^{-1}\left(\frac{16}{9}\right)^2} \right)}{\tan^{-1}\left(\frac{16}{9}\right)}$$

$$13 + \frac{48 \left(\log(256) + \frac{\log(337)}{2} + \sqrt{\left(\log(256) + \frac{\log(337)}{2}\right)^2 + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)}{\tan^{-1}\left(\frac{16}{9}\right)}$$

Alternative representations:

$$\frac{48}{\tan^{-1}\left(\frac{4096}{2304}\right)} + 13 =$$

$$13 + \frac{48}{\tan^{-1}\left(1, \frac{4096}{2304}\right)}$$

$$\frac{\log\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2+4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{\log\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\tan^{-1}\left(1, \frac{4096}{2304}\right)^2 + \log^2\left(\sqrt{2304^2+4096^2}\right)}}$$

$$\frac{48}{\tan^{-1}\left(\frac{4096}{2304}\right)} + 13 =$$

$$\frac{\log\left(\sqrt{2304^2+4096^2}\right)+\sqrt{\log^2\left(\sqrt{2304^2+4096^2}\right)+\tan^{-1}\left(\frac{4096}{2304}\right)^2}}{13 + \frac{48}{\tan^{-1}\left(\frac{4096}{2304}\right)}} = \frac{\log_e\left(\sqrt{2304^2+4096^2}\right)+\sqrt{\tan^{-1}\left(\frac{4096}{2304}\right)^2+\log_e^2\left(\sqrt{2304^2+4096^2}\right)}}{\log_e\left(\sqrt{2304^2+4096^2}\right)+\sqrt{\tan^{-1}\left(\frac{4096}{2304}\right)^2+\log_e^2\left(\sqrt{2304^2+4096^2}\right)}}$$

$$\frac{48}{\tan^{-1}\left(\frac{4096}{2304}\right)} + 13 =$$

$$\frac{\log\left(\sqrt{2304^2+4096^2}\right)+\sqrt{\log^2\left(\sqrt{2304^2+4096^2}\right)+\tan^{-1}\left(\frac{4096}{2304}\right)^2}}{13 + \frac{48}{\tan^{-1}\left(1, \frac{4096}{2304}\right)}} = \frac{\log_e\left(\sqrt{2304^2+4096^2}\right)+\sqrt{\tan^{-1}\left(1, \frac{4096}{2304}\right)^2+\log_e^2\left(\sqrt{2304^2+4096^2}\right)}}{\log_e\left(\sqrt{2304^2+4096^2}\right)+\sqrt{\tan^{-1}\left(1, \frac{4096}{2304}\right)^2+\log_e^2\left(\sqrt{2304^2+4096^2}\right)}}$$

Series representation:

$$\frac{48}{\tan^{-1}\left(\frac{4096}{2304}\right)} + 13 =$$

$$\frac{\log\left(\sqrt{2304^2+4096^2}\right)+\sqrt{\log^2\left(\sqrt{2304^2+4096^2}\right)+\tan^{-1}\left(\frac{4096}{2304}\right)^2}}{\left(26 \tan^{-1}(z_0) + 96 \log(-1 + 256 \sqrt{337}) - 96 \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256 \sqrt{337}}\right)^k}{k} + 96 \sqrt{\left(\log(-1 + 256 \sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256 \sqrt{337}}\right)^k}{k}\right)^2 + \left(\tan^{-1}(z_0) + \frac{1}{2} i \sum_{k=1}^{\infty} \frac{(-(-i-z_0)^{-k} + (i-z_0)^{-k})\left(\frac{16}{9} - z_0\right)^k}{k}\right)^2}\right) + 13 i \sum_{k=1}^{\infty} \frac{(-(-i-z_0)^{-k} + (i-z_0)^{-k})\left(\frac{16}{9} - z_0\right)^k}{k}\right)}{\left(2 \tan^{-1}(z_0) + i \sum_{k=1}^{\infty} \frac{(-(-i-z_0)^{-k} + (i-z_0)^{-k})\left(\frac{16}{9} - z_0\right)^k}{k}\right)}$$

for ($i z_0 \notin \mathbb{R}$ or ($\text{not } (1 \leq i z_0 < \infty)$ and $\text{not } (-\infty < i z_0 \leq -1)$))

We have also, from:

$$\frac{dy}{dx} = \frac{\arctan\left(\frac{y}{x}\right)}{\log\sqrt{x^2+y^2} + \sqrt{\left(\log\sqrt{x^2+y^2}\right)^2 + \left(\arctan\left(\frac{y}{x}\right)\right)^2}}, \quad (4.259)$$

and

$$\left[\operatorname{atan}\left(\frac{4096}{2304}\right) * \frac{1}{\left(\left(\ln\sqrt{2304^2+4096^2}\right)+\left(\left(\ln\sqrt{2304^2+4096^2}\right)^2+\left(\operatorname{atan}\left(\frac{4096}{2304}\right)\right)^2\right)^{1/2}\right)}\right]$$

$$\tan^{-1}\left(\frac{4096}{2304}\right) \times \frac{1}{\log\left(\sqrt{2304^2+4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2+4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}$$

$$= 0.06234568652$$

$$\frac{dy}{dx} = \frac{\arctan\left(\frac{y}{x}\right)}{\log\sqrt{x^2+y^2} + \sqrt{\left(\log\sqrt{x^2+y^2}\right)^2 + \left(\arctan\left(\frac{y}{x}\right)\right)^2}}, \quad (4.259)$$

$$\left[\left(\left(\ln\sqrt{2304^2+4096^2}\right)^2+\left(\operatorname{atan}\left(\frac{4096}{2304}\right)\right)^2\right)^{1/4}\right]/\left[\sqrt{1+0.06234568652^2}\right]$$

One can actually turn the (complex) integrable system into the set of coupled first-order differential equations

$$\frac{dx}{dt} = \frac{\sqrt[4]{\left(\log\sqrt{x^2+y^2}\right)^2 + \left(\arctan\left(\frac{y}{x}\right)\right)^2}}{\sqrt{1 + \left(\frac{\arctan\left(\frac{y}{x}\right)}{\log\sqrt{x^2+y^2} + \sqrt{\left(\log\sqrt{x^2+y^2}\right)^2 + \left(\arctan\left(\frac{y}{x}\right)\right)^2}}\right)^2}}, \quad (4.261)$$

$$\frac{dy}{dt} = \frac{\arctan\left(\frac{y}{x}\right) \sqrt[4]{\left(\log\sqrt{x^2+y^2}\right)^2 + \left(\arctan\left(\frac{y}{x}\right)\right)^2}}{\sqrt{\left(\log\sqrt{x^2+y^2} + \sqrt{\left(\log\sqrt{x^2+y^2}\right)^2 + \left(\arctan\left(\frac{y}{x}\right)\right)^2}\right)^2 + \left(\arctan\left(\frac{y}{x}\right)\right)^2}}$$

the second of which is obtained combining the first with eq. (4.259), or alternatively one can recast this system in terms of the cosmic time t_c .

For $y = 4096$, $x = 2304$ and

$$\frac{dy}{dx} = \frac{\arctan\left(\frac{y}{x}\right)}{\log \sqrt{x^2 + y^2} + \sqrt{\left(\log \sqrt{x^2 + y^2}\right)^2 + \left(\arctan\left(\frac{y}{x}\right)\right)^2}}, \quad (4.259)$$

We obtain:

$$\left[\left(\left(\ln \sqrt{2304^2 + 4096^2}\right)^2 + \left(\arctan\left(\frac{4096}{2304}\right)\right)^2\right)^{1/4}\right] / \left[\sqrt{1 + 0.06234568652^2}\right]$$

Input interpretation:

$$\frac{\sqrt[4]{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{\sqrt{1 + 0.06234568652^2}}$$

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Result:

2.91345369982...

(result in radians)

2.91345369982...

Alternative representations:

$$\frac{\sqrt[4]{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{\sqrt{1 + 0.0623457^2}} =$$

$$\frac{\sqrt[4]{\tan^{-1}\left(1, \frac{4096}{2304}\right)^2 + \log^2\left(\sqrt{2304^2 + 4096^2}\right)}}{\sqrt{1 + 0.0623457^2}}$$

$$\frac{\sqrt[4]{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{\sqrt{1 + 0.0623457^2}} = \frac{\sqrt[4]{\tan^{-1}\left(\frac{4096}{2304}\right)^2 + \log_e^2\left(\sqrt{2304^2 + 4096^2}\right)}}{\sqrt{1 + 0.0623457^2}}$$

$$\frac{\sqrt[4]{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{\sqrt{1 + 0.0623457^2}} = \frac{\sqrt[4]{\tan^{-1}\left(1, \frac{4096}{2304}\right)^2 + \log_e^2\left(\sqrt{2304^2 + 4096^2}\right)}}{\sqrt{1 + 0.0623457^2}}$$

Series representations:

$$\frac{\sqrt[4]{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{\sqrt{1 + 0.0623457^2}} = \left(\left(\left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{5}\right)^k 32^{1+2k} F_{1+2k}\left(\frac{1}{9+9\sqrt{\frac{1429}{405}}}\right)^{1+2k}}{1+2k} \right)^2 + \left(\log\left(-1 + \sqrt{22085632}\right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \sqrt{22085632}\right)^{-k}}{k} \right)^2 \right)^{\wedge} (1/4) \right) / \left(\sum_{k=0}^{\infty} \frac{(-0.00388698)^k \left(-\frac{1}{2}\right)_k}{k!} \right)$$

$$\frac{\sqrt[4]{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{\sqrt{1 + 0.0623457^2}} =$$

$$\left(\left(\tan^{-1}(x) + \pi \left[\frac{\arg\left(i\left(\frac{16}{9} - x\right)\right)}{2\pi} \right] + \frac{1}{2} i \sum_{k=1}^{\infty} \frac{-(-i-x)^{-k} + (i-x)^{-k} \left(\frac{16}{9} - x\right)^k}{k} \right)^2 + \right. \\ \left. \left(\log\left(-1 + \sqrt{22085632}\right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \sqrt{22085632}\right)^{-k}}{k} \right)^2 \right)^{\wedge(1/4)} /$$

$$\left(\sum_{k=0}^{\infty} \frac{(-0.00388698)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (ix \in \mathbb{R} \text{ and } ix < -1)$$

$$\frac{\sqrt[4]{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{\sqrt{1 + 0.0623457^2}} =$$

$$\left(\left(\log^2\left(-1 + \sqrt{22085632}\right) + \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{5}\right)^k 32^{1+2k} F_{1+2k} \left(\frac{1}{9+9\sqrt{\frac{1429}{405}}}\right)^{1+2k}}{1+2k} \right)^2 - \right. \\ \left. 2 \log\left(-1 + \sqrt{22085632}\right) \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \sqrt{22085632}\right)^{-k}}{k} + \right. \\ \left. \left(\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \sqrt{22085632}\right)^{-k}}{k} \right)^2 \right)^{\wedge(1/4)} /$$

$$\left(\exp\left(i\pi \left[\frac{\arg(1.00389 - x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (1.00389 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$n!$ is the factorial function

$(\alpha)_n$ is the Pochhammer symbol (rising factorial)

F_n is the n^{th} Fibonacci number

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

\mathbb{R} is the set of real numbers

Continued fraction representations:

$$\frac{\sqrt[4]{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{\sqrt{1 + 0.0623457^2}} =$$

$$\frac{\sqrt[4]{81 \left(1 + \sum_{k=1}^{\infty} \frac{256k^2}{1+2k}\right)^2 + \left(1 + \sum_{k=1}^{\infty} \frac{\left(\frac{1+k}{2}\right)^2 (-1 + \sqrt{22085632})}{1+k}\right)^2}}{\sqrt{1.00389}} = 0.998062$$

$$\frac{\sqrt[4]{\left(81 \left(1 + \frac{256}{1 + \frac{256}{81 \left(3 + \frac{1024}{81 \left(5 + \frac{256}{9 \left(7 + \frac{4096}{81(9+\dots)}\right)}\right)}\right)}\right)}\right)^2 + \left(-1 + \frac{256\sqrt{337}}{1 + \frac{-1+256\sqrt{337}}{2 + \frac{-1+256\sqrt{337}}{3 + \frac{4(-1+256\sqrt{337})}{4 + \frac{4(-1+256\sqrt{337})}{5+\dots}}}\right)}\right)^2}}{\sqrt{1.00389}} = 0.998062$$

$$\begin{aligned}
& \sqrt[4]{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2} = \\
& \quad \sqrt{1 + 0.0623457^2} \\
& \sqrt[4]{\frac{256}{81 \left(1 + \sum_{k=1}^{\infty} \frac{256 k^2}{1+2k}\right)^2} + \frac{(-1 + \sqrt{22085632})^2}{\left(1 + \sum_{k=1}^{\infty} \frac{\lfloor \frac{1+k}{2} \rfloor (-1 + \sqrt{22085632})}{\frac{1}{2}(3+(-1)^k(-1+k)+k)}\right)^2}} = 0.998062 \\
& \quad \sqrt{1.00389} \\
& \sqrt[4]{\frac{256}{81 \left(1 + \frac{256}{81 \left(3 + \frac{1024}{81 \left(5 + \frac{256}{9 \left(7 + \frac{4096}{81(9+\dots)}\right)}\right)}\right)}\right)^2} + \frac{(-1 + 256 \sqrt{337})^2}{\left(1 + \frac{-1 + 256 \sqrt{337}}{2 + \frac{-1 + 256 \sqrt{337}}{3 + \frac{2(-1 + 256 \sqrt{337})}{2 + \frac{2(-1 + 256 \sqrt{337})}{5+\dots}}}\right)^2}}
\end{aligned}$$

$$\begin{aligned}
& \sqrt[4]{\frac{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}{\sqrt{1 + 0.0623457^2}}} = \\
& \sqrt[4]{\frac{256}{81 \left(1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{256k^2}{1+2k}\right)^2} + \frac{(-1 + \sqrt{22085632})^2}{\left(1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{\left(\frac{1+k}{2}\right)^2 (-1 + \sqrt{22085632})}{1+k}\right)^2}} = \\
& 1 + 4 \left(\mathop{\text{K}}_{k=1}^{\infty} \frac{0.000242937}{\frac{1}{2}} \right) \\
& \sqrt[4]{\frac{256}{81 \left(1 + \frac{256}{81 \left(3 + \frac{1024}{81 \left(5 + \frac{256}{81 \left(7 + \frac{4096}{81(9+\dots)}\right)}\right)}\right)}\right)^2} + \frac{(-1 + 256\sqrt{337})^2}{\left(1 + \frac{-1 + 256\sqrt{337}}{2 + \frac{-1 + 256\sqrt{337}}{4 \left(-1 + 256\sqrt{337}\right)} + \frac{4 \left(-1 + 256\sqrt{337}\right)}{4 + \frac{4 \left(-1 + 256\sqrt{337}\right)}{5+\dots}}\right)^2}} = \\
& 1 + 4 \frac{0.000242937}{\frac{1}{2} + \frac{0.000242937}{\frac{1}{2} + \frac{0.000242937}{\frac{1}{2} + \frac{0.000242937}{\frac{1}{2} + \dots}}}
\end{aligned}$$

$\mathop{\text{K}}_{k=k_1}^{k_2} a_k / b_k$ is a continued fraction

And:

$$\begin{aligned}
& \text{atan}(4096/2304) * [((((\ln \sqrt{2304^2 + 4096^2}))^2 + (\text{atan}(4096/2304))^2))]^{1/4} / [\\
& ((((((\ln \sqrt{2304^2 + 4096^2})) + (\sqrt{(\ln \sqrt{2304^2 + 4096^2}))^2 + (\text{atan}(4096/2304))^2))))^2))] + (\text{atan}(4096/2304))^2]^{1/2}
\end{aligned}$$

Input:

$$\tan^{-1}\left(\frac{4096}{2304}\right) \times \left(4 \sqrt{\log^2(\sqrt{2304^2 + 4096^2}) + \tan^{-1}\left(\frac{4096}{2304}\right)^2} \right) /$$

$$\left(\sqrt{\left(\left(\log(\sqrt{2304^2 + 4096^2}) + \sqrt{\log^2(\sqrt{2304^2 + 4096^2}) + \tan^{-1}\left(\frac{4096}{2304}\right)^2} \right)^2 + \tan^{-1}\left(\frac{4096}{2304}\right)^2 \right) \right)$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Exact Result:

$$\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^2}}$$

(result in radians)

Decimal approximation:

0.181641271070908116703099520422878314716906317546232500491...

(result in radians)

0.18164127107...

Alternate forms:

$$\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\left(\log(256) + \frac{\log(337)}{2}\right)^2 + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256) + \frac{\log(337)}{2} + \sqrt{\left(\log(256) + \frac{\log(337)}{2}\right)^2 + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}}$$

$$\left(\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{(16 \log(2) + \log(337))^2 + 4 \tan^{-1}\left(\frac{16}{9}\right)^2} \right) /$$

$$\left(\sqrt{\left(4 \tan^{-1}\left(\frac{16}{9}\right)^2 + (16 \log(2) + \log(337)) \left(16 \log(2) + \log(337) + \sqrt{256 \log^2(2) + \log^2(337) + 32 \log(2) \log(337) + 4 \tan^{-1}\left(\frac{16}{9}\right)^2} \right) \right) \right)$$

$$\begin{aligned}
& \left(i \left(\log \left(1 - \frac{16i}{9} \right) - \log \left(1 + \frac{16i}{9} \right) \right) \right. \\
& \quad \left. \sqrt[4]{\log^2(256\sqrt{337}) - \frac{1}{4} \left(\log \left(1 - \frac{16i}{9} \right) - \log \left(1 + \frac{16i}{9} \right) \right)^2} \right) / \\
& \left(2 \sqrt{\left(\left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) - \frac{1}{4} \left(\log \left(1 - \frac{16i}{9} \right) - \log \left(1 + \frac{16i}{9} \right) \right)^2} \right)^2 \right. \right. \\
& \quad \left. \left. - \frac{1}{4} \left(\log \left(1 - \frac{16i}{9} \right) - \log \left(1 + \frac{16i}{9} \right) \right)^2 \right) \right)
\end{aligned}$$

Alternative representations:

$$\begin{aligned}
& \frac{\tan^{-1}\left(\frac{4096}{2304}\right) \sqrt[4]{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{\sqrt{\left(\log\left(\sqrt{2304^2 + 4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}\right)^2 + \tan^{-1}\left(\frac{4096}{2304}\right)^2}} \\
& = \left(\tan^{-1}\left(1, \frac{4096}{2304}\right) \sqrt[4]{\tan^{-1}\left(1, \frac{4096}{2304}\right)^2 + \log^2\left(\sqrt{2304^2 + 4096^2}\right)} \right) / \\
& \left(\sqrt{\left(\tan^{-1}\left(1, \frac{4096}{2304}\right)^2 + \left(\log\left(\sqrt{2304^2 + 4096^2}\right) + \right. \right. \right. \\
& \quad \left. \left. \left. \sqrt{\tan^{-1}\left(1, \frac{4096}{2304}\right)^2 + \log^2\left(\sqrt{2304^2 + 4096^2}\right)} \right)^2 \right) \right) \right) \\
& \frac{\tan^{-1}\left(\frac{4096}{2304}\right) \sqrt[4]{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{\sqrt{\left(\log\left(\sqrt{2304^2 + 4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}\right)^2 + \tan^{-1}\left(\frac{4096}{2304}\right)^2}} \\
& = \left(\tan^{-1}\left(\frac{4096}{2304}\right) \sqrt[4]{\tan^{-1}\left(\frac{4096}{2304}\right)^2 + \log_e^2\left(\sqrt{2304^2 + 4096^2}\right)} \right) / \left(\sqrt{\left(\tan^{-1}\left(\frac{4096}{2304}\right)^2 + \right. \right. \\
& \quad \left. \left. \left(\log_e\left(\sqrt{2304^2 + 4096^2}\right) + \sqrt{\tan^{-1}\left(\frac{4096}{2304}\right)^2 + \log_e^2\left(\sqrt{2304^2 + 4096^2}\right)} \right)^2 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\tan^{-1}\left(\frac{4096}{2304}\right) \sqrt[4]{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{\sqrt{\left(\log\left(\sqrt{2304^2 + 4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}\right)^2 + \tan^{-1}\left(\frac{4096}{2304}\right)^2}} \\
&= \left(\tan^{-1}\left(1, \frac{4096}{2304}\right) \sqrt[4]{\tan^{-1}\left(1, \frac{4096}{2304}\right)^2 + \log_e^2\left(\sqrt{2304^2 + 4096^2}\right)} \right) / \\
&\left(\sqrt{\left(\tan^{-1}\left(1, \frac{4096}{2304}\right)^2 + \left(\log_e\left(\sqrt{2304^2 + 4096^2}\right) + \right. \right. \right. \\
&\quad \left. \left. \left. \sqrt{\tan^{-1}\left(1, \frac{4096}{2304}\right)^2 + \log_e^2\left(\sqrt{2304^2 + 4096^2}\right)} \right)^2 \right) \right) \right)
\end{aligned}$$

Series representation:

$$\begin{aligned}
& \frac{\tan^{-1}\left(\frac{4096}{2304}\right) \sqrt{4 \log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}}{\sqrt{\left(\log\left(\sqrt{2304^2 + 4096^2}\right) + \sqrt{\log^2\left(\sqrt{2304^2 + 4096^2}\right) + \tan^{-1}\left(\frac{4096}{2304}\right)^2}\right)^2 + \tan^{-1}\left(\frac{4096}{2304}\right)^2}} \\
&= \left(\left(\log\left(-1 + 256 \sqrt{337}\right) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2 + \right. \\
&\quad \left. \left(\tan^{-1}(z_0) + \frac{1}{2} i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right) \left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right)^{1/4} \\
&\quad \left(2 \tan^{-1}(z_0) + i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right) \left(\frac{16}{9} - z_0\right)^k}{k} \right) / \\
&\quad \sqrt{\sqrt{2} \left(4 \tan^{-1}(z_0)^2 + 4 \log^2\left(-1 + 256 \sqrt{337}\right) - \right. \\
&\quad 8 \log\left(-1 + 256 \sqrt{337}\right) \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} + 4 \left(\sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2 + \\
&\quad 4 i \tan^{-1}(z_0) \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right) \left(\frac{16}{9} - z_0\right)^k}{k} - \\
&\quad \left. \left(\sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right) \left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 + 2 \log\left(-1 + 256 \sqrt{337}\right) \right) \\
&\quad \sqrt{\left(4 \tan^{-1}(z_0)^2 + 4 \log^2\left(-1 + 256 \sqrt{337}\right) - 8 \log\left(-1 + 256 \sqrt{337}\right) \right. \\
&\quad \left. \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} + 4 \left(\sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2 + \right. \\
&\quad \left. 4 i \tan^{-1}(z_0) \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right) \left(\frac{16}{9} - z_0\right)^k}{k} - \right. \\
&\quad \left. \left(\sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right) \left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right) - \\
&\quad 2 \left(\sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right) \sqrt{\left(4 \tan^{-1}(z_0)^2 + 4 \log^2\left(-1 + 256 \sqrt{337}\right) - \right. \\
&\quad 8 \log\left(-1 + 256 \sqrt{337}\right) \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} + \\
&\quad \left. 4 \left(\sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2 \right) +
\end{aligned}$$

From the exact result

$$\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}}$$

(result in radians)

we obtain:

$$1/\left(\left(\left(\left(\left(\tan^{-1}\left(\frac{16}{9}\right) \left(\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2\right)^{1/4}\right)/\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}\right)\right)^3 - 29 + \text{golden ratio}\right)$$

where 29 is a Lucas number

Input:

$$\frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}}\right)^3 - 29 + \phi}$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Exact Result:

$$\phi - 29 + \frac{\left(\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2\right)^{3/2}}{\tan^{-1}\left(\frac{16}{9}\right)^3 \left(\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2\right)^{3/4}}$$

(result in radians)

Decimal approximation:

139.4796337855778235756250292055734928253628941874126208740...

(result in radians)

139.4796337... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternate forms:

$$\begin{aligned}
 & -29 + \frac{1}{2} (1 + \sqrt{5}) + \\
 & \frac{\left(\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256) + \frac{\log(337)}{2} + \sqrt{\left(\log(256) + \frac{\log(337)}{2} \right)^2 + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^2 \right)^{3/2}}{\tan^{-1}\left(\frac{16}{9}\right)^3 \left(\left(\log(256) + \frac{\log(337)}{2} \right)^2 + \tan^{-1}\left(\frac{16}{9}\right)^2 \right)^{3/4}} \\
 \\
 & \phi - 29 + \left(4 \tan^{-1}\left(\frac{16}{9}\right)^2 + (16 \log(2) + \log(337)) \left(16 \log(2) + \log(337) + \right. \right. \\
 & \quad \left. \left. \sqrt{256 \log^2(2) + \log^2(337) + 32 \log(2) \log(337) + 4 \tan^{-1}\left(\frac{16}{9}\right)^2} \right) \right)^{3/2} / \\
 & \left(\tan^{-1}\left(\frac{16}{9}\right)^3 \left((16 \log(2) + \log(337))^2 + 4 \tan^{-1}\left(\frac{16}{9}\right)^2 \right)^{3/4} \right) \\
 \\
 & \phi - 29 + \\
 & \left(8i \left(\left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) - \frac{1}{4} \left(\log\left(1 - \frac{16i}{9}\right) - \log\left(1 + \frac{16i}{9}\right) \right)^2} \right)^2 - \right. \right. \\
 & \quad \left. \left. \frac{1}{4} \left(\log\left(1 - \frac{16i}{9}\right) - \log\left(1 + \frac{16i}{9}\right) \right)^2 \right)^{3/2} \right) / \\
 & \left(\left(\log\left(1 - \frac{16i}{9}\right) - \log\left(1 + \frac{16i}{9}\right) \right)^3 \right. \\
 & \quad \left. \left(\log^2(256 \sqrt{337}) - \frac{1}{4} \left(\log\left(1 - \frac{16i}{9}\right) - \log\left(1 + \frac{16i}{9}\right) \right)^2 \right)^{3/4} \right)
 \end{aligned}$$

Alternative representations:

$$\begin{aligned}
 & \frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) 4\sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3} - 29 + \phi = \\
 & -29 + \phi + \frac{1}{\left(\frac{\tan^{-1}\left(1, \frac{16}{9}\right) 4\sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log^2(256\sqrt{337})}}{\sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log^2(256\sqrt{337})}\right)^2}} \right)^3}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) 4\sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3} - 29 + \phi = \\
 & -29 + \phi + \frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) 4\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log_e(256\sqrt{337}) + \sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}\right)^2}} \right)^3}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) 4\sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3} - 29 + \phi = \\
 & -29 + \phi + \frac{1}{\left(\frac{\tan^{-1}\left(1, \frac{16}{9}\right) 4\sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}}{\sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \left(\log_e(256\sqrt{337}) + \sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}\right)^2}} \right)^3}
 \end{aligned}$$

Series representation:

$$\begin{aligned}
 & \frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3 - 29 + \phi = \\
 & -29 + \phi + \left(\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} + \sqrt{\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2 + \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2} \right)^2 + \right. \\
 & \left. \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right)^{3/2} / \\
 & \left(\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2 + \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right)^{3/4} \\
 & \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^3 \Bigg)
 \end{aligned}$$

for $(i z_0 \notin \mathbb{R}$ or $(\text{not } (1 \leq i z_0 < \infty)$ and $\text{not } (-\infty < i z_0 \leq -1))$)

\mathbb{R} is the set of real numbers

And:

$$1 / \left(\left(\left(\left(\left(\left(\tan^{-1}\left(\frac{16}{9}\right) \left(\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2 \right)^{1/4} \right) / \sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^2} \right)^3 - 34 - 8 + 1 / \text{golden ratio} \right) \right) \right) \right) \right) \right)$$

where 34 and 8 are Fibonacci numbers

Input:

$$\frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{\phi}\right) \sqrt[4]{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{\phi}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{\phi}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{\phi}\right)^2}\right)^2}} \right)^3 - 34 - 8 + \frac{1}{\phi}}$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Exact Result:

$$\frac{1}{\phi} - 42 + \frac{\left(\tan^{-1}\left(\frac{16}{\phi}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{\phi}\right)^2} \right)^2 \right)^{3/2}}{\tan^{-1}\left(\frac{16}{\phi}\right)^3 \left(\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{\phi}\right)^2 \right)^{3/4}}$$

(result in radians)

Decimal approximation:

125.4796337855778235756250292055734928253628941874126208740...

(result in radians)

125.4796337... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV

Alternate forms:

$$-42 + \frac{2}{1 + \sqrt{5}} + \frac{\left(\tan^{-1}\left(\frac{16}{\phi}\right)^2 + \left(\log(256) + \frac{\log(337)}{2} + \sqrt{\left(\log(256) + \frac{\log(337)}{2} \right)^2 + \tan^{-1}\left(\frac{16}{\phi}\right)^2} \right)^2 \right)^{3/2}}{\tan^{-1}\left(\frac{16}{\phi}\right)^3 \left(\left(\log(256) + \frac{\log(337)}{2} \right)^2 + \tan^{-1}\left(\frac{16}{\phi}\right)^2 \right)^{3/4}}$$

$$\frac{1}{\phi} - 42 + \left(4 \tan^{-1}\left(\frac{16}{9}\right)^2 + (16 \log(2) + \log(337)) \left(16 \log(2) + \log(337) + \sqrt{256 \log^2(2) + \log^2(337) + 32 \log(2) \log(337) + 4 \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^{3/2} \right) / \left(\tan^{-1}\left(\frac{16}{9}\right)^3 \left((16 \log(2) + \log(337))^2 + 4 \tan^{-1}\left(\frac{16}{9}\right)^2 \right)^{3/4} \right)$$

$$\frac{1}{\phi} - 42 + \left(8i \left(\left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) - \frac{1}{4} \left(\log\left(1 - \frac{16i}{9}\right) - \log\left(1 + \frac{16i}{9}\right) \right)^2} \right)^2 - \frac{1}{4} \left(\log\left(1 - \frac{16i}{9}\right) - \log\left(1 + \frac{16i}{9}\right) \right)^2 \right)^{3/2} \right) / \left(\left(\log\left(1 - \frac{16i}{9}\right) - \log\left(1 + \frac{16i}{9}\right) \right)^3 \left(\log^2(256 \sqrt{337}) - \frac{1}{4} \left(\log\left(1 - \frac{16i}{9}\right) - \log\left(1 + \frac{16i}{9}\right) \right)^2 \right)^{3/4} \right)$$

Alternative representations:

$$\frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) 4 \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^2}} \right)^3 - 34 - 8 + \frac{1}{\phi} = -42 + \frac{1}{\phi} + \frac{1}{\left(\frac{\tan^{-1}\left(1, \frac{16}{9}\right) 4 \sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log^2(256 \sqrt{337})}}{\sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log^2(256 \sqrt{337})} \right)^2}} \right)^3$$

$$\begin{aligned}
& \frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) 4\sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3 - 34 - 8 + \frac{1}{\phi} = \\
& -42 + \frac{1}{\phi} + \frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) 4\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log_e(256\sqrt{337}) + \sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}\right)^2}} \right)^3}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) 4\sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3 - 34 - 8 + \frac{1}{\phi} = \\
& -42 + \frac{1}{\phi} + \frac{1}{\left(\frac{\tan^{-1}\left(1, \frac{16}{9}\right) 4\sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}}{\sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \left(\log_e(256\sqrt{337}) + \sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}\right)^2}} \right)^3}
\end{aligned}$$

Series representation:

$$\begin{aligned}
 & \frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) 4 \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3 - 34 - 8 + \frac{1}{\phi} = \\
 & -42 + \frac{1}{\phi} + \left(\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} + \sqrt{\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2 + \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2} \right)^2 + \right. \\
 & \left. \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right)^{3/2} / \\
 & \left(\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2 + \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right)^{3/4} \\
 & \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^3 \Big)
 \end{aligned}$$

for ($i z_0 \notin \mathbb{R}$ or (not ($1 \leq i z_0 < \infty$) and not ($-\infty < i z_0 \leq -1$)))

\mathbb{R} is the set of real numbers

$$13 * [1 / ((((((\tan^{-1}(16/9) (\log^2(256 \sqrt{337})) + \tan^{-1}(16/9)^2)^{(1/4)}) / \sqrt{\tan^{-1}(16/9)^2 + (\log(256 \sqrt{337})) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}(16/9)^2}})^2))))))^3 - 34 - 8 + \pi] + 64$$

where 13 is a Fibonacci number

Input:

$$13 \left(\frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3 - 34 - 8 + \pi \right) + 64$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Exact Result:

$$64 + 13 \left(-42 + \pi + \frac{\left(\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^2 \right)^{3/2}}{\tan^{-1}\left(\frac{16}{9}\right)^3 \left(\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2 \right)^{3/4}} \right)$$

(result in radians)

Decimal approximation:

1728.041501855430385556480114808335648693916807290765529828...

(result in radians)

1728.041501855...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Alternate forms:

$$64 + 13 \left(-42 + \pi + \frac{\left(\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256) + \frac{\log(337)}{2} + \sqrt{\left(\log(256) + \frac{\log(337)}{2} \right)^2 + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^2 \right)^{3/2}}{\tan^{-1}\left(\frac{16}{9}\right)^3 \left(\left(\log(256) + \frac{\log(337)}{2} \right)^2 + \tan^{-1}\left(\frac{16}{9}\right)^2 \right)^{3/4}} \right)$$

$$-482 + 13 \pi + \left(13 \left(4 \tan^{-1}\left(\frac{16}{9}\right)^2 + (16 \log(2) + \log(337)) \right. \right. \\ \left. \left. \frac{\left(16 \log(2) + \log(337) + \sqrt{256 \log^2(2) + \log^2(337) + 32 \log(2) \log(337) + 4 \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^{3/2}}{\left(\tan^{-1}\left(\frac{16}{9}\right)^3 \left((16 \log(2) + \log(337))^2 + 4 \tan^{-1}\left(\frac{16}{9}\right)^2 \right)^{3/4} \right)} \right) \right)$$

$$-482 + 13 \pi + \left(104 i \left(\left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) - \frac{1}{4} \left(\log\left(1 - \frac{16 i}{9}\right) - \log\left(1 + \frac{16 i}{9}\right) \right)^2} \right)^2 - \frac{1}{4} \left(\log\left(1 - \frac{16 i}{9}\right) - \log\left(1 + \frac{16 i}{9}\right) \right)^2 \right)^{3/2} \right) / \\ \left(\left(\log\left(1 - \frac{16 i}{9}\right) - \log\left(1 + \frac{16 i}{9}\right) \right)^3 \left(\log^2(256 \sqrt{337}) - \frac{1}{4} \left(\log\left(1 - \frac{16 i}{9}\right) - \log\left(1 + \frac{16 i}{9}\right) \right)^2 \right)^{3/4} \right)$$

Alternative representations:

$$13 \left(\frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3 - 34 - 8 + \pi} \right) + 64 =$$

$$64 + 13 \left(-42 + \pi + \frac{1}{\left(\frac{\tan^{-1}\left(1, \frac{16}{9}\right) \sqrt[4]{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log^2(256\sqrt{337})}}{\sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log^2(256\sqrt{337})}\right)^2}} \right)^3} \right)$$

$$13 \left(\frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3 - 34 - 8 + \pi} \right) + 64 =$$

$$64 + 13 \left(-42 + \pi + \frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\tan^{-1}\left(\frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log_e(256\sqrt{337}) + \sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}\right)^2}} \right)^3} \right)$$

$$13 \left(\frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3 - 34 - 8 + \pi + 64 =$$

$$64 + 13 \left(-42 + \pi + \frac{1}{\left(\frac{\tan^{-1}\left(1, \frac{16}{9}\right) \sqrt[4]{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}}{\sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \left(\log_e(256\sqrt{337}) + \sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}\right)^2}} \right)^3 \right)$$

Series representation:

$$\begin{aligned}
 & 13 \left(\frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3 - 34 - 8 + \pi + 64 = \right. \\
 & 64 + 13 \left(-42 + \pi + \right. \\
 & \left. \left(\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} + \sqrt{\left(\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2 + \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{(-(-i-z_0)^{-k} + (i-z_0)^{-k})\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right)} \right)^2 + \right. \\
 & \left. \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{(-(-i-z_0)^{-k} + (i-z_0)^{-k})\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right)^{3/2} / \right. \\
 & \left. \left(\left(\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2 + \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{(-(-i-z_0)^{-k} + (i-z_0)^{-k})\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right)^{3/4} \right. \right. \\
 & \left. \left. \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{(-(-i-z_0)^{-k} + (i-z_0)^{-k})\left(\frac{16}{9} - z_0\right)^k}{k} \right)^3 \right) \right) \right)
 \end{aligned}$$

for ($i z_0 \notin \mathbb{R}$ or (not ($1 \leq i z_0 < \infty$) and not ($-\infty < i z_0 \leq -1$)))

\mathbb{R} is the set of real numbers

We have also that:

$$24 * [1 / ((((((\tan^{-1}(16/9) (\log^2(256 \sqrt{337})) + \tan^{-1}(16/9)^2)^{1/4}) / \sqrt{\tan^{-1}(16/9)^2 + (\log(256 \sqrt{337})) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}(16/9)^2}})^2))))))^{3/4}] + 29 + \text{golden ratio}$$

where 29 is a Lucas number

Input:

$$24 \times \frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3} + 29 + \phi$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Exact Result:

$$\phi + 29 + \frac{24 \left(\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^2 \right)^{3/2}}{\tan^{-1}\left(\frac{16}{9}\right)^3 \left(\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2 \right)^{3/4}}$$

(result in radians)

Decimal approximation:

4035.296429112620184306295203743354151101142349362370355148...

(result in radians)

4035.296429...

Alternate forms:

$$29 + \frac{1}{2} (1 + \sqrt{5}) + \frac{24 \left(\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256) + \frac{\log(337)}{2} + \sqrt{\left(\log(256) + \frac{\log(337)}{2}\right)^2 + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^2 \right)^{3/2}}{\tan^{-1}\left(\frac{16}{9}\right)^3 \left(\left(\log(256) + \frac{\log(337)}{2}\right)^2 + \tan^{-1}\left(\frac{16}{9}\right)^2 \right)^{3/4}}$$

$$\phi + 29 + \left(24 \left(4 \tan^{-1} \left(\frac{16}{9} \right)^2 + (16 \log(2) + \log(337)) \right. \right. \\ \left. \left. \left(16 \log(2) + \log(337) + \sqrt{256 \log^2(2) + \log^2(337) + 32 \log(2) \log(337) + 4 \tan^{-1} \left(\frac{16}{9} \right)^2} \right)^{3/2} \right) \right) / \\ \left(\tan^{-1} \left(\frac{16}{9} \right) \right)^3 \left((16 \log(2) + \log(337))^2 + 4 \tan^{-1} \left(\frac{16}{9} \right)^2 \right)^{3/4}$$

$$\phi + 29 + \\ \left(192 i \left(\left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) - \frac{1}{4} \left(\log \left(1 - \frac{16 i}{9} \right) - \log \left(1 + \frac{16 i}{9} \right) \right)^2} \right)^2 - \right. \right. \\ \left. \left. \frac{1}{4} \left(\log \left(1 - \frac{16 i}{9} \right) - \log \left(1 + \frac{16 i}{9} \right) \right) \right)^{3/2} \right) / \\ \left(\left(\log \left(1 - \frac{16 i}{9} \right) - \log \left(1 + \frac{16 i}{9} \right) \right) \right)^3 \\ \left(\log^2(256 \sqrt{337}) - \frac{1}{4} \left(\log \left(1 - \frac{16 i}{9} \right) - \log \left(1 + \frac{16 i}{9} \right) \right)^2 \right)^{3/4}$$

Alternative representations:

$$\frac{24}{\left(\frac{\tan^{-1} \left(\frac{16}{9} \right) \sqrt[4]{\log^2(256 \sqrt{337}) + \tan^{-1} \left(\frac{16}{9} \right)^2}}{\sqrt{\tan^{-1} \left(\frac{16}{9} \right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1} \left(\frac{16}{9} \right)^2} \right)^2}} \right)^3 + 29 + \phi = \\ 29 + \phi + \frac{24}{\left(\frac{\tan^{-1} \left(1, \frac{16}{9} \right) \sqrt[4]{\tan^{-1} \left(1, \frac{16}{9} \right)^2 + \log^2(256 \sqrt{337})}}{\sqrt{\tan^{-1} \left(1, \frac{16}{9} \right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\tan^{-1} \left(1, \frac{16}{9} \right)^2 + \log^2(256 \sqrt{337})} \right)^2}} \right)^3}$$

$$\frac{24}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3 + 29 + \phi =$$

$$29 + \phi + \frac{24}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\tan^{-1}\left(\frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log_e(256\sqrt{337}) + \sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}\right)^2}} \right)^3}$$

$$\frac{24}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3 + 29 + \phi =$$

$$29 + \phi + \frac{24}{\left(\frac{\tan^{-1}\left(1, \frac{16}{9}\right) \sqrt[4]{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}}{\sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \left(\log_e(256\sqrt{337}) + \sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}\right)^2}} \right)^3}$$

Series representations:

$$\begin{aligned}
 & \frac{24}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) 4 \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3 + 29 + \phi = \\
 & 29 + \phi + \left(24 \left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} + \sqrt{\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2} \right. \right. \\
 & \quad \left. \left. + \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right) \right)^2 + \\
 & \quad \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right)^{3/2} \Bigg/ \\
 & \left(\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2 + \right. \\
 & \quad \left. \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right)^{3/4} \\
 & \quad \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^3 \Bigg)
 \end{aligned}$$

for ($i z_0 \notin \mathbb{R}$ or (not ($1 \leq i z_0 < \infty$) and not ($-\infty < i z_0 \leq -1$)))

$$\begin{aligned}
& \left(\frac{\tan^{-1}\left(\frac{16}{9}\right) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3 + 29 + \phi = \\
& 29 + \phi + \left(24 \left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} + \sqrt{\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2} \right. \right. \\
& \quad \left. \left. + \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right) \right)^2 + \\
& \quad \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right)^{3/2} \Bigg/ \\
& \left(\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2 + \right. \\
& \quad \left. \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right)^{3/4} \\
& \quad \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^3 \Bigg)
\end{aligned}$$

for (not $((i z_0 \in \mathbb{R} \text{ and } -\infty < i z_0 \leq -1))$

and

not

$((i z_0 \in \mathbb{R} \text{ and } 1 \leq i z_0 < \infty))$)

From:

The interaction of glueball and heavy-light flavoured meson in holographic QCD

Si-wen Li - arXiv:1809.10379v1 [hep-th] 27 Sep 2018

In order to match the experimental value of the ρ meson mass $m_\rho = \sqrt{\lambda_1} M_{KK} \simeq 776 \text{MeV}$ where λ_1 is numerically evaluated as $\lambda_1 = 0.669314\dots$, we first need to fix the Kaluza-Klein mass to $M_{KK} = 949 \text{MeV}$. Then let us identify the lowest mode of the heavy-light meson field $Q_{\mu,n=0}, Q_{n=0}$ as the vector meson D_μ^0 and the pseudoscalar meson D^0 whose mass is given as $M(D_\mu^0) \simeq 2007 \text{MeV}$, $M(D^0) \simeq 1865 \text{MeV}$ by experiments. In this sense, we further consider the case of $N_f = 2$ for D-meson, so the parameters in (3.20) (3.21) can be numerically fixed as

$$M_S \simeq 1754 \text{MeV}, M_V \simeq 2007 \text{MeV}, M_{KK} R \simeq 1.048, v \simeq 35195.4 \lambda^{-1} N_c^{-1} M_{KK}. \quad (5.4)$$

On the other hand, the glueball mass in holography is determined by solving the eigen equations (2.11) (2.16) for exotic, dilatonic and tensor glueball respectively. For the reader's convenience, we collect the mass spectrum of various glueballs in Table 2. It is clear that the glueball decay occurs when the mass relation $m_{\text{glueball}}/m_{\text{D-meson}} \simeq 2$ is satisfied because the effective holographic action we discussed is quadratic of the heavy-light meson fields. Accordingly we will choose the excited mass of $n = 3$ as $M_E = \sqrt{154.963/9} M_{KK} \simeq 3938 \text{MeV}$, $M_D = M_T = \sqrt{162.699/9} M_{KK} \simeq 4035 \text{MeV}$ for the exotic, dilatonic and tensor glueball respectively. With these values for the parameters, the boundary condition for the eigen equations (2.11) (2.16) is numerically evaluated as,

$$H_E(r_{KK})^{-1} \simeq 0.00692402 \lambda^{1/2} N_c M_{KK}, \quad H_{D,T}(r_{KK})^{-1} \simeq 0.0211777 \lambda^{1/2} N_c M_{KK}. \quad (5.5)$$

$$M_D = M_T = \sqrt{162.699/9} M_{KK} \simeq 4035 \text{MeV}$$

value of the dilatonic glueball.

We note the following mathematical connection:

Exact Result:

$$\phi + 29 + \frac{24 \left(\tan^{-1}\left(\frac{16}{\phi}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{\phi}\right)^2} \right)^2 \right)^{3/2}}{\tan^{-1}\left(\frac{16}{\phi}\right)^3 \left(\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{\phi}\right)^2 \right)^{3/4}}$$

(result in radians)

Decimal approximation:

4035.296429112620184306295203743354151101142349362370355148...

(result in radians)

4035.296429...

Furthermore, we have also:

$$\frac{1}{e} * [24 / ((((((\tan^{-1}(16/9) (\log^2(256 \sqrt{337})) + \tan^{-1}(16/9)^2)^{1/4}) / \sqrt{\tan^{-1}(16/9)^2 + (\log(256 \sqrt{337})) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}(16/9)^2}})}))^3] + 256$$

where $256 = 64 * 4$

Input:

$$\frac{1}{e} \times \frac{24}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3} + 256$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Exact Result:

$$256 + \frac{24 \left(\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^2 \right)^{3/2}}{e \tan^{-1}\left(\frac{16}{9}\right)^3 \left(\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2 \right)^{3/4}}$$

(result in radians)

Decimal approximation:

1729.238850069517889145823150151776428910233634101026225911...

(result in radians)

1729.23885...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Alternate forms:

$$256 + \frac{24 \left(\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256) + \frac{\log(337)}{2} + \sqrt{\left(\log(256) + \frac{\log(337)}{2}\right)^2 + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^2 \right)^{3/2}}{e \tan^{-1}\left(\frac{16}{9}\right)^3 \left(\left(\log(256) + \frac{\log(337)}{2}\right)^2 + \tan^{-1}\left(\frac{16}{9}\right)^2 \right)^{3/4}}$$

$$256 + \left(24 \left(4 \tan^{-1}\left(\frac{16}{9}\right)^2 + (16 \log(2) + \log(337)) \right. \right. \\ \left. \left. \left(16 \log(2) + \log(337) + \sqrt{256 \log^2(2) + \log^2(337) + 32 \log(2) \log(337) + 4 \tan^{-1}\left(\frac{16}{9}\right)^2} \right) \right)^{3/2} \right) \\ \left(e \tan^{-1}\left(\frac{16}{9}\right)^3 \left((16 \log(2) + \log(337))^2 + 4 \tan^{-1}\left(\frac{16}{9}\right)^2 \right)^{3/4} \right)$$

$$\left(16 \left(16 e \tan^{-1}\left(\frac{16}{9}\right)^3 \left(\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2 \right)^{3/4} + \right. \right. \\ \left. \left. 3 \sqrt{2} \left(\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2 + \right. \right. \right. \\ \left. \left. \left. \log(256 \sqrt{337}) \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^{3/2} \right) \right) \right) / \\ \left(e \tan^{-1}\left(\frac{16}{9}\right)^3 \left(\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2 \right)^{3/4} \right)$$

Alternative representations:

$$\frac{24}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^2}} \right)^3} + 256 = \\ 256 + \frac{24}{\left(\frac{\tan^{-1}\left(1, \frac{16}{9}\right) \sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log^2(256 \sqrt{337})}}{\sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log^2(256 \sqrt{337})} \right)^2}} \right)^3} e$$

$$\frac{24}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^e} + 256 =$$

$$256 + \frac{24}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\tan^{-1}\left(\frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log_e(256\sqrt{337}) + \sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}\right)^2}} \right)^e}$$

$$\frac{24}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^e} + 256 =$$

$$256 + \frac{24}{\left(\frac{\tan^{-1}\left(1, \frac{16}{9}\right) \sqrt[4]{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}}{\sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \left(\log_e(256\sqrt{337}) + \sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}\right)^2}} \right)^e}$$

Series representation:

$$\begin{aligned}
 & \frac{24}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3 + 256 = \\
 & 256 + \left(24 \left(\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} + \sqrt{\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2 + \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right)^2 + \right. \right. \\
 & \left. \left. \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right)^{3/2} \right) / \\
 & \left(e \left(\left(\log(-1 + 256\sqrt{337}) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-256\sqrt{337}}\right)^k}{k} \right)^2 + \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^2 \right)^{3/4} \right) \\
 & \left(\tan^{-1}(z_0) + \frac{1}{2}i \sum_{k=1}^{\infty} \frac{\left(-(-i-z_0)^{-k} + (i-z_0)^{-k}\right)\left(\frac{16}{9} - z_0\right)^k}{k} \right)^3 \Big)
 \end{aligned}$$

for ($i z_0 \notin \mathbb{R}$ or (not ($1 \leq i z_0 < \infty$) and not ($-\infty < i z_0 \leq -1$)))

From the formula of coefficients of the '5th order' mock theta function $\psi_1(q)$:
 (A053261 OEIS Sequence)

$$\sqrt{\phi} \times \exp(\pi \sqrt{n/15}) / (2 \cdot 5^{1/4} \sqrt{n})$$

for $n = 213$, we obtain:

$$\sqrt{\phi} \times \exp(\pi \sqrt{213/15}) / (2 \cdot 5^{1/4} \sqrt{213})$$

Input:

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{213}{15}}\right)}{2 \sqrt[4]{5} \sqrt{213}}$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{\sqrt{71/5} \pi} \sqrt{\frac{\phi}{213}}}{2 \sqrt[4]{5}}$$

Decimal approximation:

4035.483425247420442884076501190630859556030569888526687828...

4035.483425...

Property:

$$\frac{e^{\sqrt{71/5} \pi} \sqrt{\frac{\phi}{213}}}{2 \sqrt[4]{5}} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{1}{2} \sqrt{\frac{5 + \sqrt{5}}{2130}} e^{\sqrt{71/5} \pi}$$

$$\frac{\sqrt{\frac{1}{426} (1 + \sqrt{5})} e^{\sqrt{71/5} \pi}}{2 \sqrt[4]{5}}$$

Series representations:

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{213}{15}}\right)}{2 \sqrt[4]{5} \sqrt{213}} = \frac{\exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{71}{5} - z_0\right)^k z_0^{-k}}{k!}\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!}}{2 \sqrt[4]{5} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (213 - z_0)^k z_0^{-k}}{k!}}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{213}{15}}\right)}{2 \sqrt[4]{5} \sqrt{213}} = \frac{\left(\exp\left(i\pi \left\lfloor \frac{\arg(\phi - x)}{2\pi} \right\rfloor\right) \exp\left(\pi \exp\left(i\pi \left\lfloor \frac{\arg\left(\frac{71}{5} - x\right)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{71}{5} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right) \sum_{k=0}^{\infty} \frac{(-1)^k (\phi - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)}{\left(2 \sqrt[4]{5} \exp\left(i\pi \left\lfloor \frac{\arg(213 - x)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (213 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{213}{15}}\right)}{2 \sqrt[4]{5} \sqrt{213}} = \frac{\left(\exp\left(\pi \left(\frac{1}{z_0}\right)^{1/2} \left\lfloor \frac{\arg\left(\frac{71}{5} - z_0\right)}{2\pi} \right\rfloor \right) \frac{1}{z_0} \left(1 + \left\lfloor \frac{\arg\left(\frac{71}{5} - z_0\right)}{2\pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{71}{5} - z_0\right)^k z_0^{-k}}{k!}\right) \left(\frac{1}{z_0}\right)^{-1/2} \left\lfloor \frac{\arg(213 - z_0)}{2\pi} \right\rfloor + 1/2 \left\lfloor \frac{\arg(\phi - z_0)}{2\pi} \right\rfloor \frac{-1/2 \left\lfloor \frac{\arg(213 - z_0)}{2\pi} \right\rfloor + 1/2 \left\lfloor \frac{\arg(\phi - z_0)}{2\pi} \right\rfloor}{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!}\right)}{\left(2 \sqrt[4]{5} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (213 - z_0)^k z_0^{-k}}{k!}\right)}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

\mathbb{R} is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Now, from

$$\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}}$$

(result in radians)

0.181641271070908116703099520422878314716906317546232500491...

(result in radians)

= 0.18164127107.....

We obtain:

$$12 * \left[\frac{1}{\left(\left(\left(\left(\left(\tan^{-1}\left(\frac{16}{9}\right) \left(\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2 \right)^{1/4} \right) / \sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^2} \right)^2 \right)^3 + 34 - 288 - 21 + 1.618034} \right]}$$

where 34 and 21 are Fibonacci numbers and 288 is equal to 144*2

Input interpretation:

$$12 \times \frac{1}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256 \sqrt{337}) + \sqrt{\log^2(256 \sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2} \right)^2}} \right)^3 + 34 - 288 - 21 + 1.618034}$$

$\tan^{-1}(x)$ is the inverse tangent function

$\log(x)$ is the natural logarithm

Result:

1728.957232...

(result in radians)

1728.957232...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Alternative representations:

$$\begin{aligned}
 & \frac{12}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) 4\sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3} + 34 - 288 - 21 + 1.61803 = \\
 & -273.382 + \frac{12}{\left(\frac{\tan^{-1}\left(1, \frac{16}{9}\right) 4\sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log^2(256\sqrt{337})}}{\sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log^2(256\sqrt{337})}\right)^2}} \right)^3}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{12}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) 4\sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3} + 34 - 288 - 21 + 1.61803 = \\
 & -273.382 + \frac{12}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) 4\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log_e(256\sqrt{337}) + \sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}\right)^2}} \right)^3}
 \end{aligned}$$

$$\begin{aligned}
& \frac{12}{\left(\frac{\tan^{-1}\left(\frac{16}{9}\right) \sqrt[4]{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}}{\sqrt{\tan^{-1}\left(\frac{16}{9}\right)^2 + \left(\log(256\sqrt{337}) + \sqrt{\log^2(256\sqrt{337}) + \tan^{-1}\left(\frac{16}{9}\right)^2}\right)^2}} \right)^3} + 34 - 288 - 21 + 1.61803 = \\
& -273.382 + \frac{12}{\left(\frac{\tan^{-1}\left(1, \frac{16}{9}\right) \sqrt[4]{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}}{\sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \left(\log_e(256\sqrt{337}) + \sqrt{\tan^{-1}\left(1, \frac{16}{9}\right)^2 + \log_e^2(256\sqrt{337})}\right)^2}} \right)^3}
\end{aligned}$$

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References

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S. Ramanujan to G.H. Hardy 12 January 1920

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