Stellar Aberration Revisited

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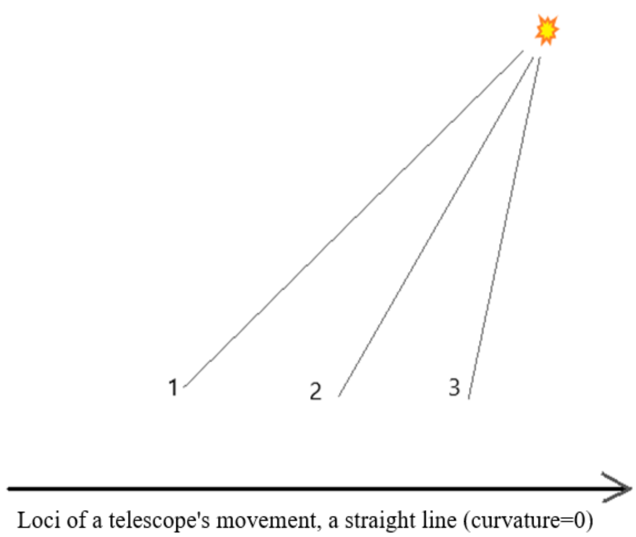
Abstract It appears that the reasoning in calculating stellar aberration proposed by James Bradley [1] cannot satisfy a big group of people, particularly those people who believe only special relativity can present such calculation that is accurate to the point [2]. This means that, in case relativity happens to be invalid, the argument on the calculation about stellar aberration has not yet truly settled in the science world these days. The problem is that relativity does show evidence that it must lead itself to conclude speed of light c=0 and thus refutes itself [3]. So, history still leaves open the opportunity for us to approach the calculation from a new angle.

The Calculation



Ray 1, 2 and 3 from a star cannot be parallel to each other

Fig. 1

Conventionally, in analyzing stellar aberration, opinions from different camps all seem agreeing to assume that the rays from the star in concern are parallel to each other before they are arrested by the telescope in the aberration observation. If we must truly pursue a calculation precise to the point, maybe we must face the fact that the star in concern is a *point* source for all the rays of light fed to the telescope. As suggested in Fig. 1, a ray from the star found by an observer at location 1 cannot be parallel to the ray found at location 2 or location 3 if the loci for the observer's movement are restrictively on a straight line (curvature=0).

One of the objectives emphasized in the aberration analysis is a "clear image" of the star to be seen by the observer. To realize this objective, a ray must be intercepted at the dead center of the objective lens and leaves the telescope at the dead center of the eye lens. If the ray leaves the eye lens anywhere else other than this dead center, the image would be less clear, or even blurry, or unseen. Therefore, if a telescope during its movement is to arrest one of the rays in Fig-1 and produce a clear image, its angle must be adjusted according to different rays. In other words, 3 different rays in Fig 1 require 3 different tilting angles to produce a clear image. This would further demand that, to produce a clear image, the front tip of a ray after entering the telescope through the objective lens must move along the axial line of the telescope in the entire journey it passes though the telescope barrel; only zero deviation from the axial line is allowed for the tip's trip. Any point from the ray, upon its entrance at the objective lens, can be considered as a tip for the light beam coming after it.

From Fig. 2, we can judge that in order for the ray's tip to move along the axial line, the telescope cannot take the same angle α as the incident angle that is for the light ray, but must take an angle $\beta < \alpha$. If we take $\beta = \alpha$, due to the speed v of the telescope with respect to the background, the exiting point for the light tip must be behind the dead center of the eye lens.

When the ray's propagation direction is not coinciding with the axial line AB but form an angle, say, $\emptyset$, the speed of the ray's tip projected on AB would not be c, but be $c' = c\ \cos\emptyset$. In our situation, we just have $\emptyset = (\alpha - \beta)$, therefore we have the following equation:

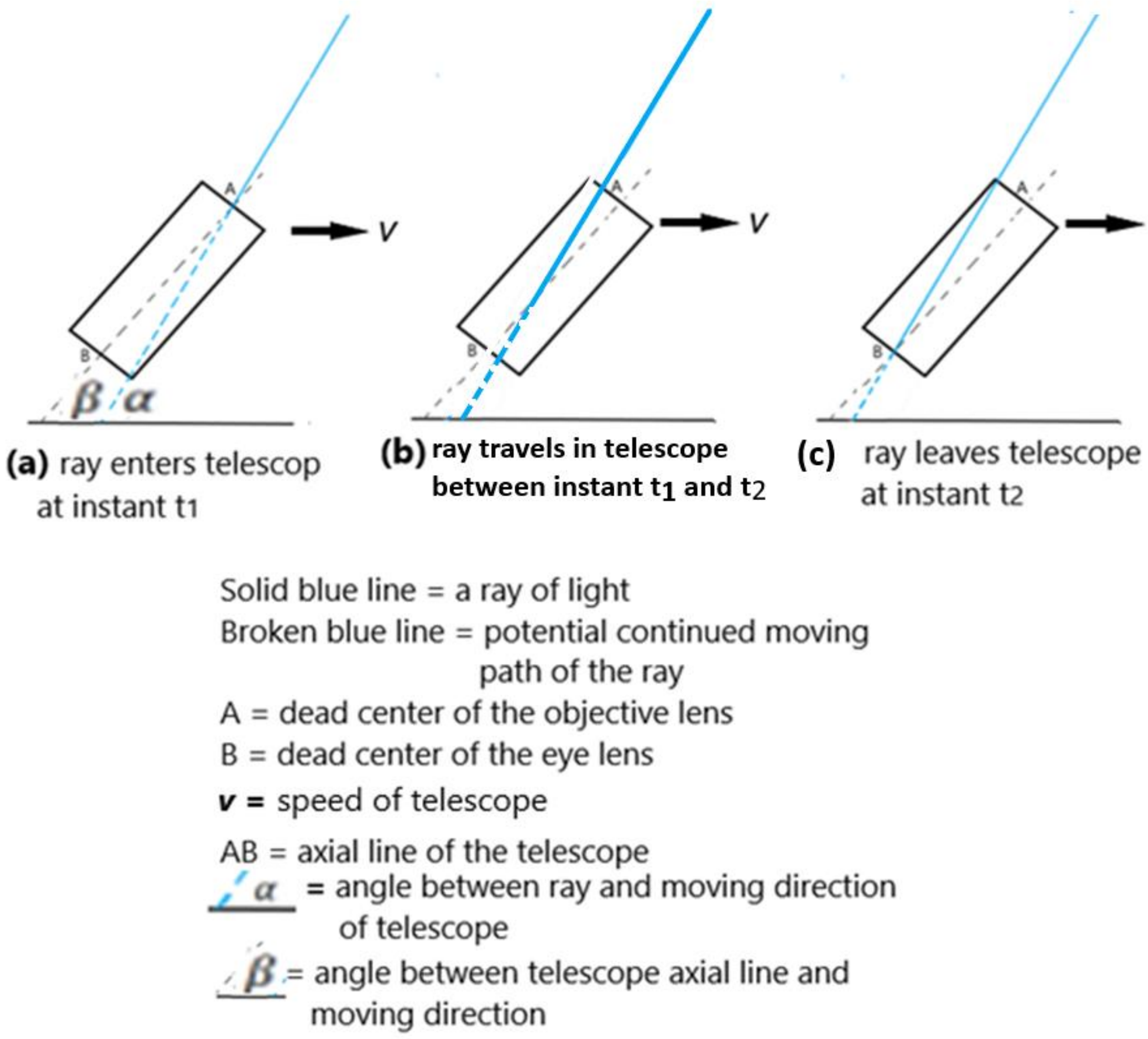


Fig 2

$$c' = c\ \cos(\alpha - \beta) \qquad\qquad (Eq.-1)$$

However, c' is a value obtained only when the telescope stays at rest with respect to the background, with respect to which c is concluded. The telescope's movement must add another component to change the ray tip's speed to become another value *c"* on the axial line. With the situation given, the component so added should be $(v \cos \beta)$. Therefore, we have:

$$c" \ = c\ \cos\,(\alpha - \beta) + v \cos\ \beta \qquad\qquad (Eq.-2)$$

Let the length of the axial line AB be h. With *c"*, the time Δt required for the ray's tip to move from A to B on the axial line can be found as

$$\Delta t = \frac{h}{c"} = \frac{h}{c\ \cos\,(\alpha - \beta) + v \cos\ \beta} \qquad\qquad (Eq.-3)$$

At the completion of Δt , the telescope would have moved a distancer of l with respect to the background, where

$$l = \Delta t \cdot v = \frac{hv}{(c \cos(\alpha - \beta) + v \cos\ \beta})} \qquad\qquad (EQ.-4)$$

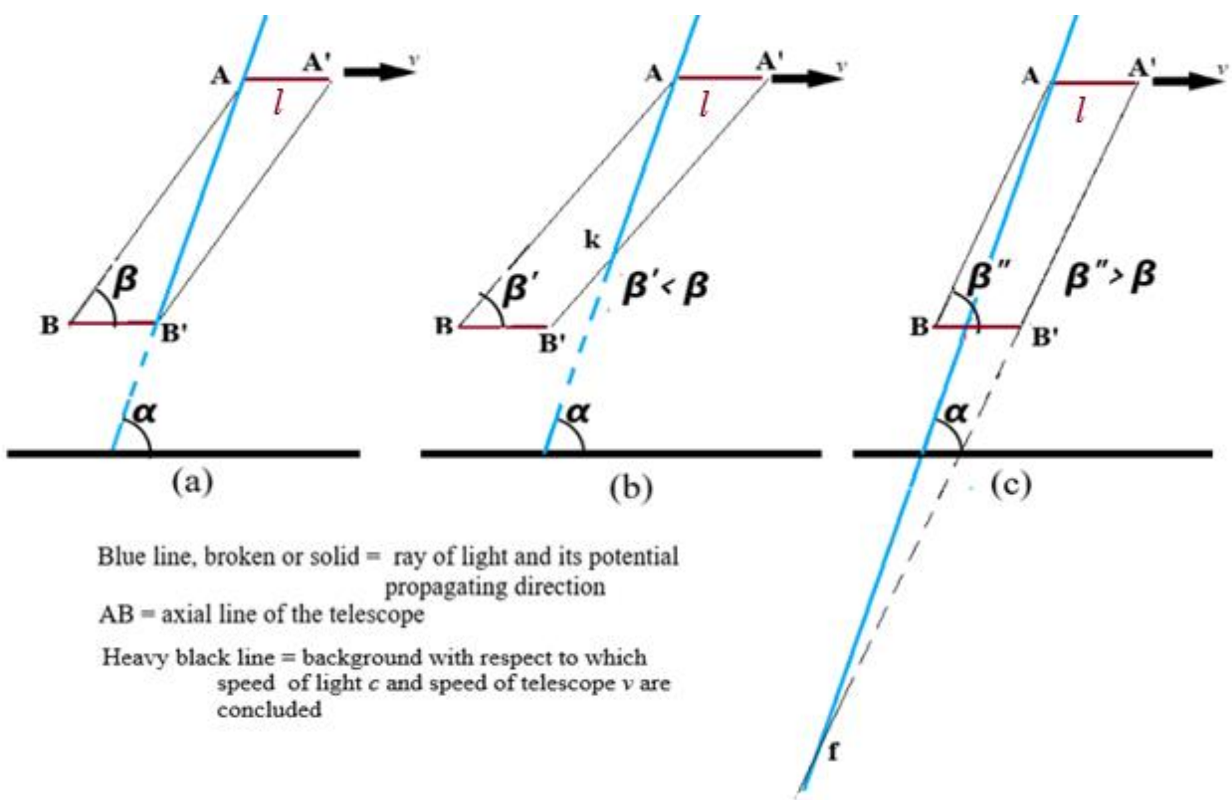


Fig. 3

Fig 3(a) shows the relative position between the axial line AB and the light ray at the instant the ray leaves the eye lens. With respect to the background, the ray always stays on the same track, but a moment ago its tip entered the telescope at A. By the time the tip leaves the telescope, the axial line AB has moved to the place marked as A'B', where B' is the point the tip leaves the dead center of the eye lens. At this precise instant, we have the following equation

$$(AB')^2 = (AA')^2 + (A'B')^2 - 2(AA')(A'B')\cos\beta \qquad (Eq.-5)$$

Or, further, because all the liner segments in $(Eq.-5)$ are covered by some traveling during the same time interval Δt, we have

$$c^2 = v^2 + c"^2 - 2vc"\cos\beta \qquad (Eq.-6)$$

$(Eq.-6)$ tells us that, given any specific speed *v*, there is only one tilting angle β for the telescope to detect the aberration for one unique ray that has incident angle α among the numerous rays from the same point source. Had the tilting angle been chosen as β'< β like what Fig. 3(b) shows, the tip of the ray will cross the axial line at k before it reaches the dead center of the eye lens, unable to give a clear image to the observer. Had the tilting angle been chosen as β''> β like what Fig. 3(c) shows, the tip of the ray will cross the axial line at f , a point behind the dead center of the eye lens, and no clear aberration image can be delivered to the observer either. Therefore, if the telescope continues moving on a restrictive straight line and to detect the aberration caused by other rays, the tilting angle of the axial line must be continuously adjusted accordingly.

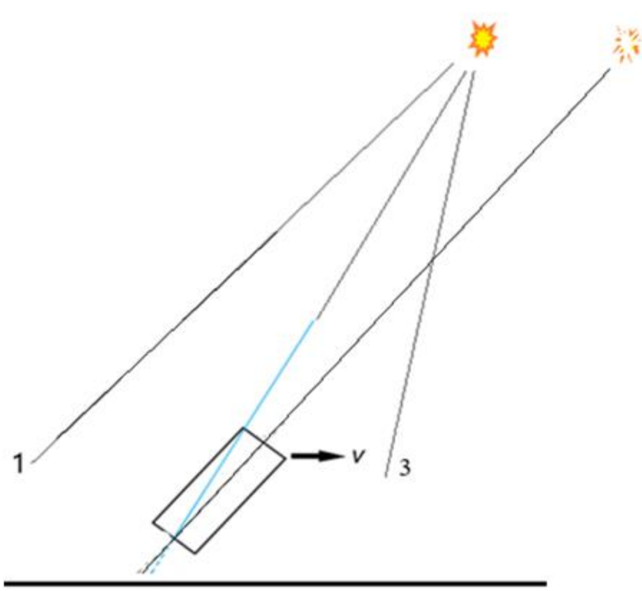


Fig. 4

With the situation given by Fig. 2 for aberration detection, the image detected should be behind the actual light source like what Fig 4 suggests. If the telescope moves backward, the image should be in front of the source, just like what Fig. 5 suggests.

Compared to the straight-line movement for the telescope mentioned above, the telescope on Earth with which we found stellar aberration is moving on a *circular* (almost) orbit. This orbit and the star we are observing form a cone in effect with the star being at the cone's apex. For any point on the periphery of the circular base, a line connecting it from the apex and a line tangential to the circle at this point must be

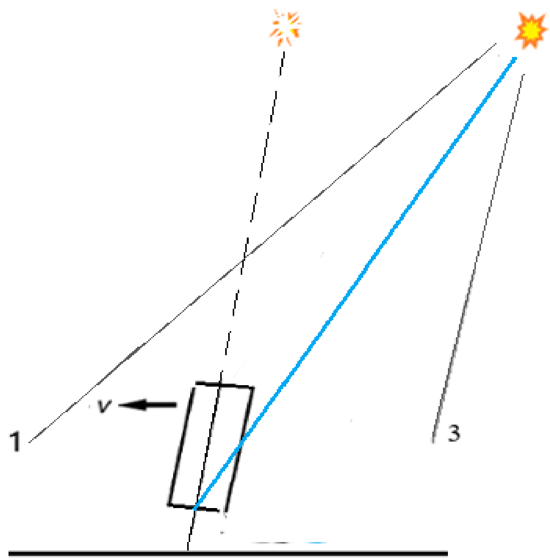


Fig 5

perpendicular to each other (Fig 6). If the blue line in Fig 6 is to represent a ray of light from a star striking at the dead center of the object lens of our telescope, this makes $\alpha = 90°$ in $(Eq.-1)$. Obviously, if we also tilt the telescope with $\beta = 90°$, we will make both $(Eq.-1)$ and $(Eq.-2)$ meaningless to us. The meaninglessness is not mathematical but physical. Of course, to arrest the ray, the telescope needs to lie on the surface of the cone. Lying on such a surface, the tilting angle of the axial line of the telescope can then be found with $(Eq.-6)$, which, with $\alpha = 90°$, enables us to have the following calculation:

$$c^2 = v^2 + [c\cos(90° - \beta) + v\cos\beta]^2 - 2v\,[c\ \cos(90° - \beta) + v\cos\beta]\cos\beta$$

$$c^2 = v^2 + [c\sin\beta + v\cos\beta]^2 - 2v(c\sin\beta + v\cos\beta)\cos\beta$$

$$c^2 = v^2 + c^2\,(\sin\beta)^2 - v^2\,(\cos\beta)^2$$

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$$c^2 = v^2\,(\sin\beta)^2 + c^2\,(\sin\beta)^2 \qquad (Eq.-7)$$

$(Eq.-7)$ therefore leads to

$$\sin\beta = \frac{c}{\sqrt{c^2 + v^2}} \qquad (Eq.-8)$$

$(Eq.-8)$ clearly tells us that angle β is a precise product of nature presented with a straight triangle shown as in Fig. 7. Therefore, with the reasoning presented in the "classic" analysis, the word "approximation" for the calculation result should not have been necessary. In the classic approach, instead of tackling the problem with a sine function, a tangent function is found. The reason leading to the introduction of a tangent function is the assumption that the rays from the star in concern are all parallel to each other before they are arrested by the telescope. However, the more accurate fact is that, among all rays from a point source, only one has the incident angle to be precisely α ; any other ray must have angle of $\alpha \mp \delta$, where δ is any nonzero value.

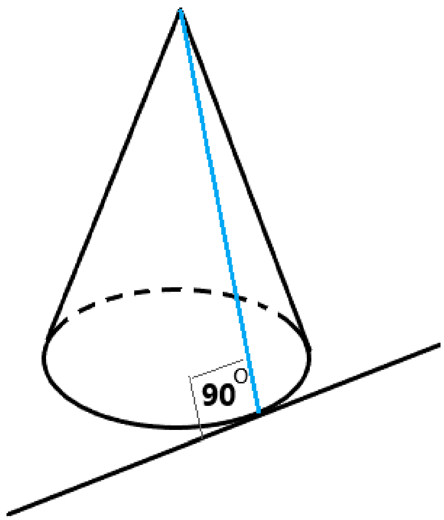

Fig. 6

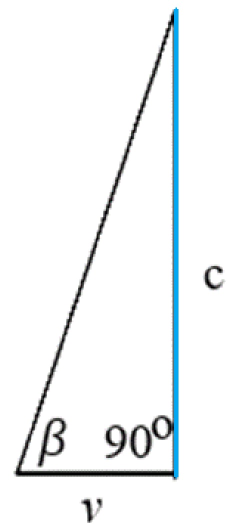

Fig. 7

If we must consider the accuracy that is good to every instant of the observation, the result may still need to be "approximated" even if a sine function is more to the point in explaining the true nature of the aberration. The reason for this is that the Earth's equatorial plane forms an angle of 23.5^{o} with the ecliptic. The linear speed of the telescope and its moving direction with respect to the background is not exactly as what the orbital movement presents to us. However, whatever interference the equatorial plane may bring into our observation would be reversed in every other 12 hours or every other 2 seasons. So, for a yearlong observation, we can only talk about an average aberration value.

According to $(Eq.-8)$, the average aberration observed from Earth is 20.6 arcseconds for an average orbital speed of 30 km/sec for the Earth. This value of aberration must also vary according to different location on the orbit where the Earth travels at different speed, such as, typically, 30.28 km/sec at the perihelion and 29.30 km/sec at the aphelion.

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