An Analysis of the State-Transition Function of a Self-Reproducing Structure in Cellular Automata Space.

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Abstract

A cellular automata structure described by J Byl (1989) self-replicates under a corresponding state-transition function. Subsequent work has established that replication of this and related structures given by other researchers is homochiral. This work describes a detailed analysis of the state-transition function for replication of J Byl's structure, so the work serves as an Appendix to accompany the preceding work *viXra:1904.0225*.

Keywords: cellular automata, self-reproduction, artificial life, biological homochirality.

Introduction

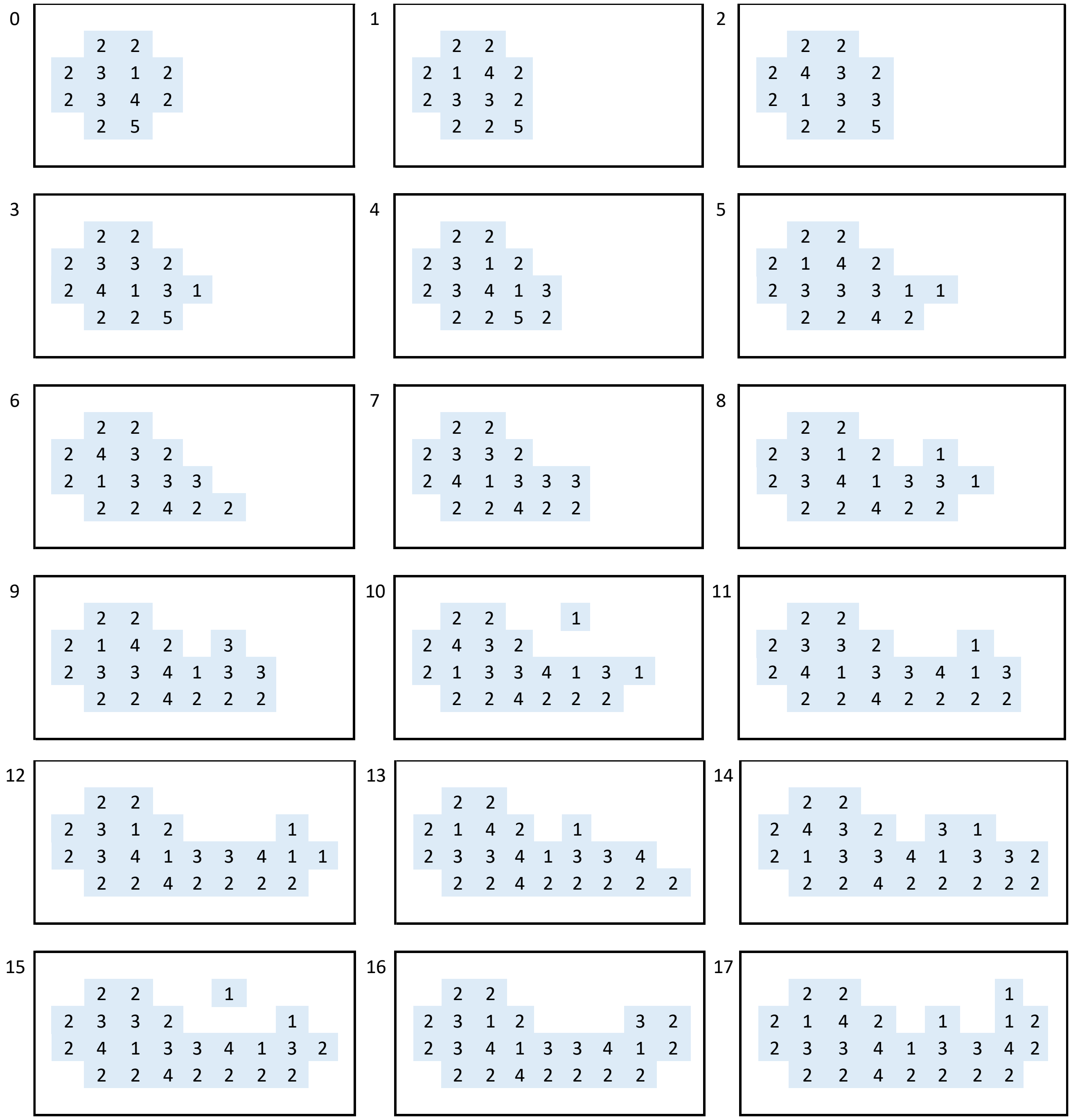
In a previous work [2], it was shown that self-reproduction of loops in cellular automata (CA) spaces is homochiral. The work [2] was intended for a readership already familiar with CA in general, and the CA works cited in it specifically. Subsequent to feedback from some readers, I considered that a more-detailed analysis of one of the systems cited in the prior work [1] could be of interest to readers requiring more detail. For purposes not concerned with chirality questions, only 43 state-transition rules (36 + 7 default rules) need to be explicitly specified for replication of J Byl's structure [1], but by enlarging the state-transition rules list by identifying and explicitly listing all of the default and state-conserving transitions, subsequent analysis comprehensively describes homochirality of CA loop self-reproduction.

The state-transition rules for loop replication sort into three subsets: ***achiral*** rules, rule and corresponding mirror-rule pairs common to both R- and L- loop (mirror of R-loop) self-reproduction, and ***chiral*** rules. Mirrors of the R-loop-only chiral rules apply only to L-loop replication. A notable observation is that the chiral rules subset contains rules which contradict rules within the corresponding mirror-chiral rules list, proving that loop self-reproduction is ***homochiral***, *i.e.* R-loop replication and L-loop replication cannot coexist under a pooling of the state-transition function with its corresponding mirror-function, because the pooled list is unworkable due to the contradictions [2]. The parallel with homochirality of real biology has been noted [2], but the question of the relevance of this work to organic biological homochirality remains open at the current time.

J Byl's loop, with the state-transition function under which it replicates.

Figure 1 below shows one replication of J Byl's structure within 27 recursions [1]. The cell-state set is {0,1,2,3,4,5}, state 0 being the "quiescent" state. The state of each cell **C** at time t+1 is a function of its time t state and time t states of its immediate neighbour-cells above, to the right, below, and to the left (respectively, **N**orth, **E**ast**,** **S**outh and **W**est). Rules within the state-transition function are notated in the format **CNESW → C'** expressing the state transition from the state of **C** (at time t) to **C'** (state of **C** at time t+1), given the specific **N,E,S,W** neighbour-states. Importantly, strong rotational symmetry applies, *i.e.* all 90^{o} rotations of neighbourhoods are equivalent, so for example, the rule notations **CNESW → C' =** 42531 → 3, 45312 → 3, 43125 → 3 and 41253 → 3 are all equivalently one rule.

It is obvious without demonstration that mirror-reflections of the state-transition rules applied to a mirror-configuration of the initial structure will deliver an exact mirror of the development shown in Figure 1 below. I refer to the development of J Byl's structure shown in Figure 1 as "R-loop" reproduction to distinguish it from corresponding mirror "L-loop" reproduction. (As an example of notation of a rule with its corresponding mirror, the mirror of rule 42531 → 3 is 42135 → 3).



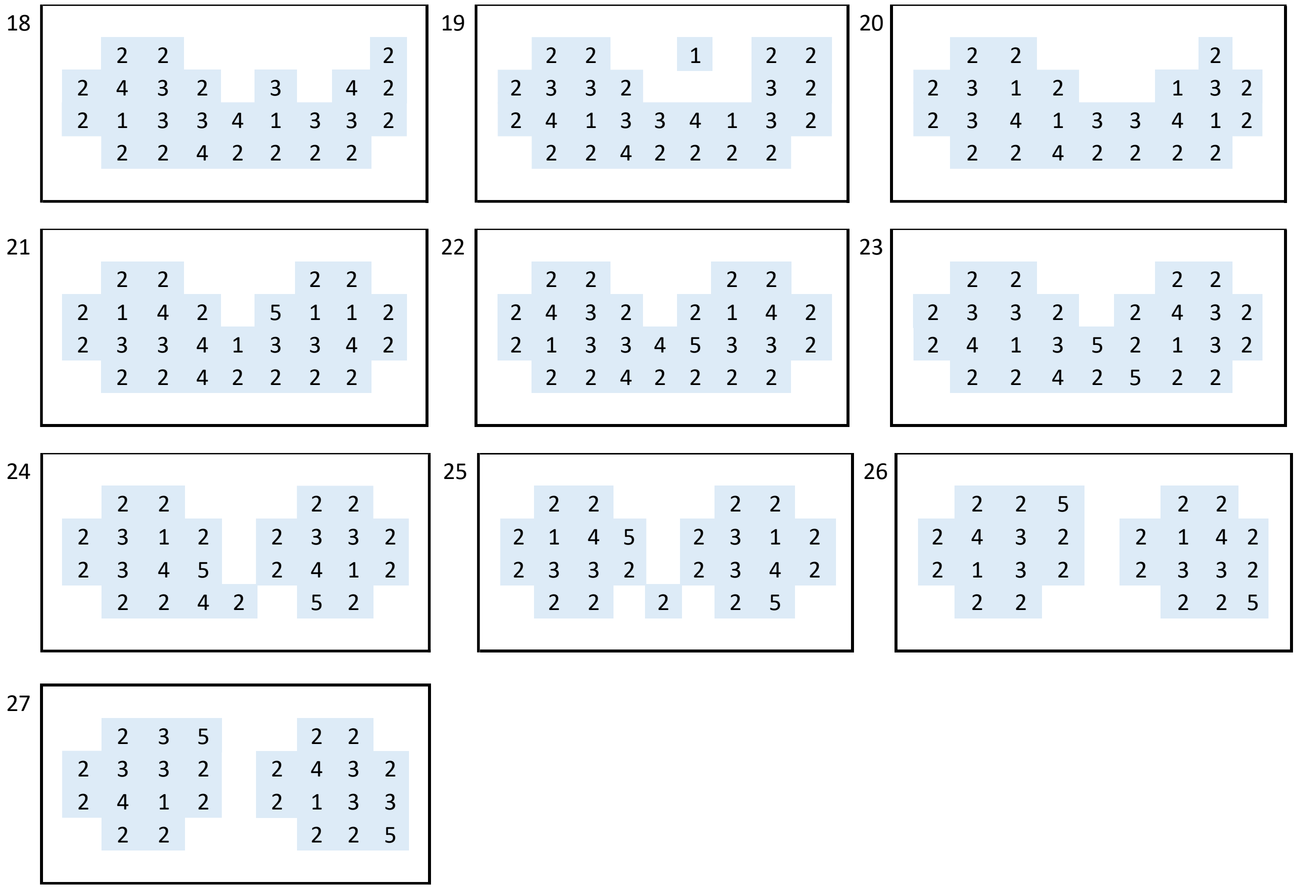


Figure 1, from [1]. Self-reproduction of the structure shown at time 0 (the first frame above, labelled 0), under the corresponding state-transition function [1] subsequently studied in this work. The structure develops to two separated structures (parent (left) and daughter) at recursion 26. The white space in all frames corresponds to the 0 quiescent state.

The rules of the state-transition function sort into three categories: **achiral** rules which are not changed under mirror-transformation and apply to both R-loop and L-loop replication, **mutual-mirror pairs of rules** which also apply to both R- and L-loop replication, and **chiral** rules which apply only to one ***or*** the other of R- and L-loop replication (the corresponding mirrors of the R-loop-only chiral rules apply only to L-loop replication). The three categories of rules are tabulated below in Tables 1, 2 and 3.

Table 1 below lists the achiral rules applying to replication of J Byl's structure.

Table 1. ***Achiral*** subset of rules of the state-transition function for self-reproduction of the structure shown at time 0 in Figure 1. (Achiral rules are the rules unchanged by mirror-transformation.) The highlighted rules are illustrated in Figures 2 to 4.

CNESW → C'

00000 --> 0	00252 --> 0	10003 --> 3	20252 --> 5	31215 --> 1
00003 --> 1	01010 --> 0	10033 --> 0	21202 --> 2	34323 --> 3
00033 --> 0	01040 --> 0	20000 --> 0	23202 --> 2	40212 --> 4
00100 --> 0	02000 --> 0	20004 --> 2	24202 --> 2	40232 --> 4
00110 --> 0	02200 --> 0	20011 --> 2	30001 --> 0	40242 --> 4
00131 --> 0	02202 --> 0	20020 --> 2	30003 --> 0	40252 --> 0
00201 --> 0	04000 --> 0	20022 --> 0	30011 --> 0	50022 --> 5
00202 --> 0	05000 --> 0	20030 --> 2	30121 --> 1	50212 → 4
00205 --> 0	10000 --> 0	20033 --> 2	30323 --> 3	50222 --> 0
00242 --> 0	10001 --> 0	20044 --> 2	31122 --> 1	52324 --> 2

Figures 2 to 4 below illustrate the achiral rules highlighted in Table 1. The frames on the left correspond to R-loop replication (structures reproduced from Figure 1), and the frames on the right correspond to structures from L-Loop mirror-development.

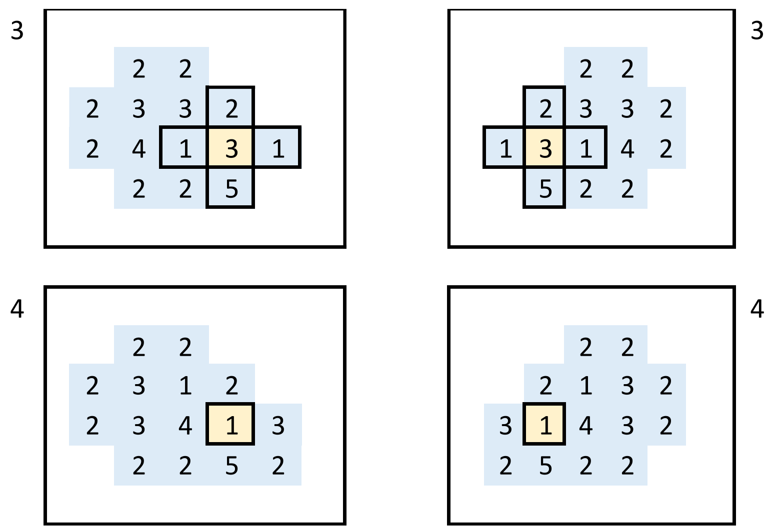


Figure 2. R-loop structures at t = 3 to 4 (left frames) illustrating the **achiral** state-transition rule 31215 → 1 (as equivalent rotation 32151 → 1). The rule's mirror is the same rule shown acting in the corresponding mirror-recursion from t = 3 to 4 (right frames).

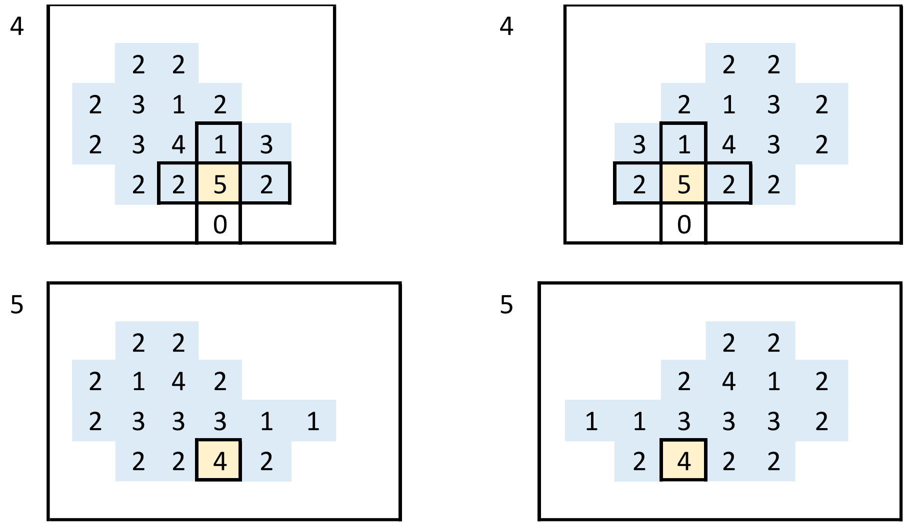


Figure 3. R-loop structures at t = 4 to 5 (left frames) illustrating the **achiral** state-transition rule 50212 → 4 (as equivalent rotation 51202 → 4). The rule's mirror is the same rule shown acting in the corresponding mirror-recursion from t = 4 to 5 (right frames). The quiescent state 0 is explicitly shown in the rule neighbourhoods at t = 4 (all white space within the frames corresponds to state 0).

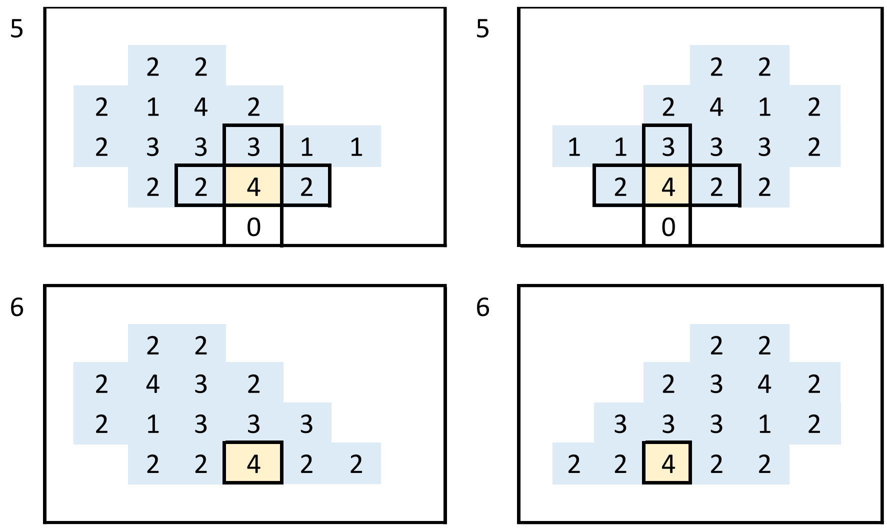


Figure 4. R-loop structures at t = 5 to 6 (left frames) illustrating the **achiral** state-transition rule 40232 → 4 (as equivalent rotation 43202 → 4). The rule's mirror is the same rule shown acting in the corresponding mirror-recursion from t = 5 to 6 (right frames). The quiescent state 0 is explicitly shown in the rule neighbourhood at t = 5 (all white space within the frames corresponds to state 0).

Table 2. The nine ***mirror-pairs*** of rules in the state-transition function. Each mutual-mirror rule-pair of the nine pairs listed below applies to both R- and L-loop replication. The two highlighted pairs are illustrated in Figure 5.

CNESW → C'

21204 --> 2	23005 --> 2	24204 --> 2
21402 --> 2	25003 --> 2	24402 --> 2

22001 --> 2	23204 --> 2	32443 --> 3
22100 --> 2	23402 --> 2	34423 --> 3

23002 --> 2	24002 --> 2	54002 --> 2
23200 --> 2	24200 --> 2	54200 --> 2

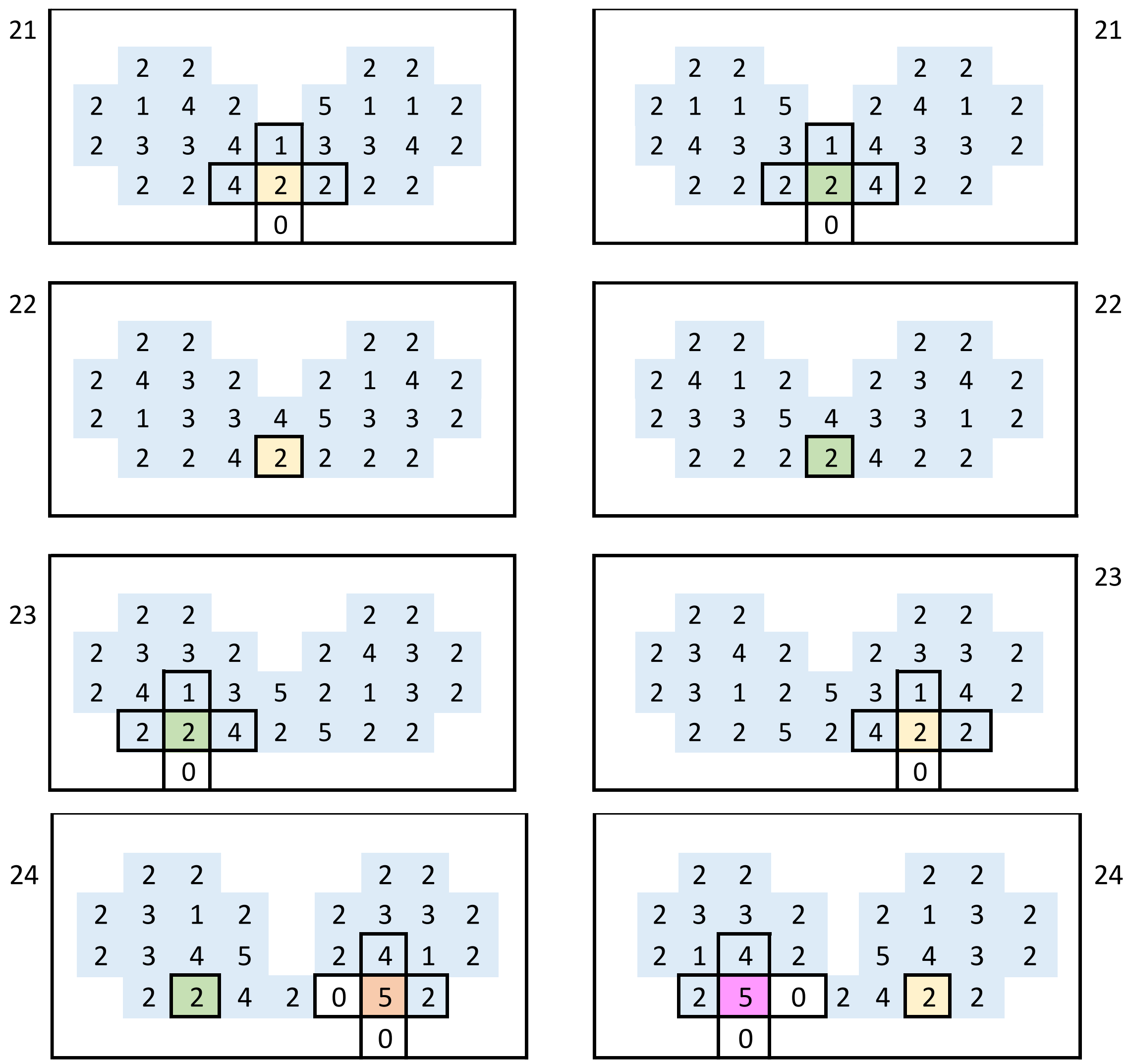


Figure 5 continues next page:

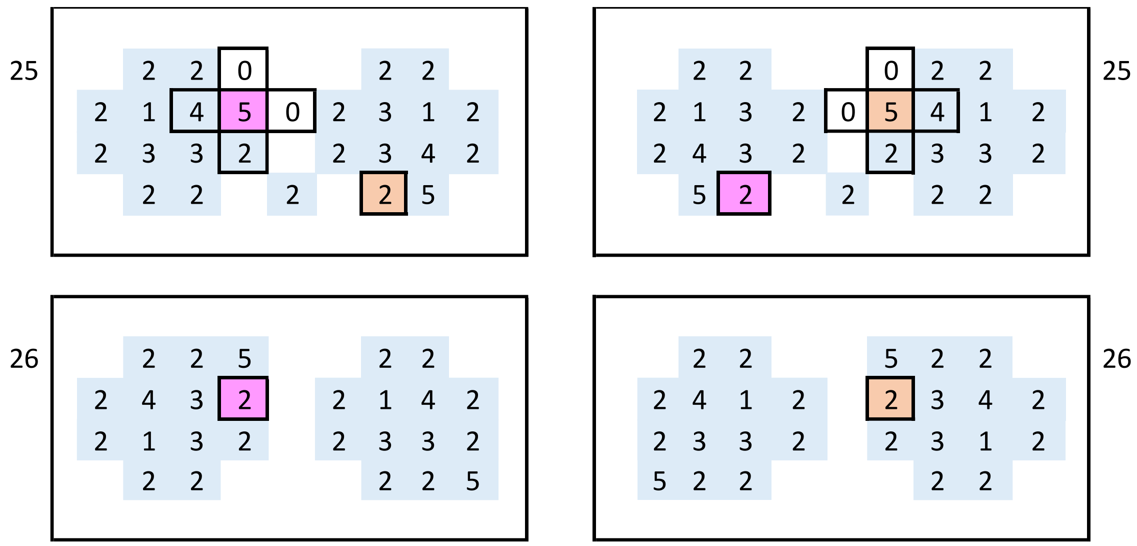


Figure 5. Rule 21204 → 2 and its mirror 21402 → 2 both apply to R-loop development (left frames), as shown here in the state-transitions over t = 21 to 24. This mutual mirror-pair of rules applies also to L-loop development (right frames) over t = 21 to 24, but in opposite order. Like many of the rules in the state-transition function, these rules serve to conserve a cell-state (this rule-pair conserves sheathing-state 2). Rule 54200 → 2 and its mirror 54002 → 2 both apply to R-loop development as shown here in the state-transitions over t = 24 to 26. This mirror-pair of rules applies also to L-loop development over t = 24 to 26, but in opposite order. These rules establish a sheathing-state 2 on the boundary of a developing loop (state transition 5 → 2).

The rules listed in Tables 1 and 2 together form a subset of the complete state-transition function that is closed under mirror-transformation. These rules are therefore common to both R- and L-loop replication.

Table 3 below completes the list of rules comprising the state-transition function for replication of J Byl's structure. Unlike the rules listed in Tables 1 and 2, they are **chiral** rules which apply only to one **or** the other of R- and L-loop replication. Table 3 shows the contradictions which prove replication of J Byl's structure is homochiral.

Table 3. List of 72 **chiral** rules within the state-transition function for self-reproduction of J. Byl's structure [1]. All contradictions between rules in the R-loop-only list and the corresponding list of L-loop-only mirror-rules are shown highlighted in blue.

CNESW → C'

Chiral rule index number 1 to 72	**72 R-loop-only rules**	**L-loop-only mirror of R-loop-only rules**	**Mirror contradicts R-rule (by index)**
1	00012 --> 0	00210 --> 0	3
2	00013 --> 5	00310 --> 5	6
3	00021 --> 2	00120 --> 2	1
4	00023 --> 3	00320 --> 3	7
5	00024 --> 2	00420 --> 2	8

6	00031 --> 1	00130 --> 1	2
7	00032 --> 0	00230 --> 0	4
8	00042 --> 0	00240 --> 0	5
9	00051 --> 2	00150 --> 2	
10	00052 --> 5	00250 --> 5	14
11	00112 --> 0	00211 --> 0	
12	00132 --> 0	00231 --> 0	
13	00140 --> 0	00041 --> 0	
14	00250 --> 0	00052 --> 0	10
15	00312 --> 0	00213 --> 0	
16	00340 --> 0	00043 --> 0	
17	00342 --> 0	00243 --> 0	
18	00433 --> 0	00334 --> 0	
19	00512 --> 0	00215 --> 0	
20	02502 --> 0	02205 --> 0	
21	10034 --> 1	10430 --> 1	
22	10123 --> 3	10321 --> 3	
23	10324 --> 4	10423 --> 4	
24	11124 --> 4	11421 --> 4	
25	11240 --> 4	11042 --> 4	
26	11324 --> 4	11423 --> 4	
27	11352 --> 1	11253 --> 1	
28	12241 --> 4	12142 --> 4	
29	12344 --> 4	12443 --> 4	
30	12354 --> 3	12453 --> 3	
31	13324 --> 4	13423 --> 4	
32	14322 --> 4	14223 --> 4	
33	20034 --> 2	20430 --> 2	
34	20051 --> 5	20150 --> 5	35
35	20150 --> 2	20051 --> 2	34
36	20512 --> 5	20215 --> 5	39
37	20532 --> 3	20235 --> 3	42
38	21004 --> 2	21400 --> 2	
39	21502 --> 2	21205 --> 2	36
40	22155 --> 2	22551 --> 2	
41	24502 --> 2	24205 --> 2	
42	25023 --> 2	25320 --> 2	37
43	25504 --> 2	25405 --> 2	
44	30021 --> 1	30120 --> 1	
45	30023 --> 3	30320 --> 3	
46	30321 --> 1	30123 --> 1	
47	30423 --> 3	30324 --> 3	
48	31123 --> 3	31321 --> 3	49
49	31132 --> 1	31231 --> 1	48
50	31234 --> 1	31432 --> 1	58
51	31254 --> 5	31452 --> 5	
52	31322 --> 1	31223 --> 1	
53	31325 --> 1	31523 --> 1	

54	31332 --> 1	31233 --> 1	
55	31423 --> 3	31324 --> 3	
56	31532 --> 5	31235 --> 5	
57	32053 --> 3	32350 --> 3	
58	32143 --> 3	32341 --> 3	50
59	32230 --> 3	32032 --> 3	
60	34223 --> 3	34322 --> 3	
61	40123 --> 3	40321 --> 3	
62	40230 --> 3	40032 --> 3	
63	40523 --> 5	40325 --> 5	
64	41123 --> 3	41321 --> 3	
65	42143 --> 3	42341 --> 3	
66	42531 --> 3	42135 --> 3	
67	43122 --> 3	43221 --> 3	
68	43152 --> 3	43251 --> 3	
69	50023 --> 5	50320 --> 5	
70	50130 --> 2	50031 --> 2	
71	50223 --> 0	50322 --> 0	
72	52044 --> 2	52440 --> 2	

A selection of the contradictions shown in Table 3 above is illustrated in Figures 6 to 9 below. R-loop replication corresponds to the left-frames. L-loop replication corresponds to the right-frames.

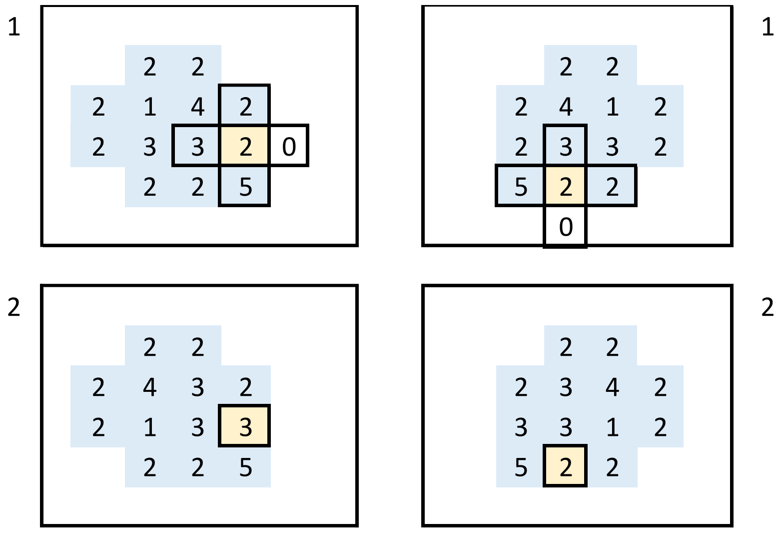


Figure 6. Left frames show the R-loop-only rule **22053 → 3** (equivalent to rotation 20532 → 3 listed at index line 37, Table 3). In the right frames, the contradiction is L-loop-only rule 23205 → 2, equivalent to **22053 → 2 (not → 3)**. Rotated, this is 25320 → 2 listed in the L-loop-only column of Table 3 at index line 42.

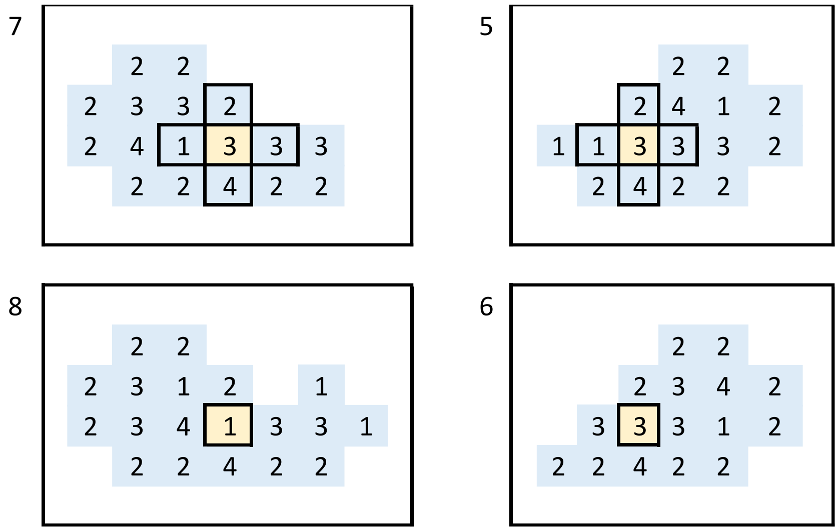


Figure 7. Left frames show the R-loop-only rule **32341 → 1** for transition t = 7 to 8 (equivalent to rotation 31234 → 1 listed at index line 50, Table 3). In the right frames (t = 5 to 6), the contradiction is L-loop-only rule **32341 → 3, (not → 1)** listed in the L-loop-only column of Table 3 at index line 58.

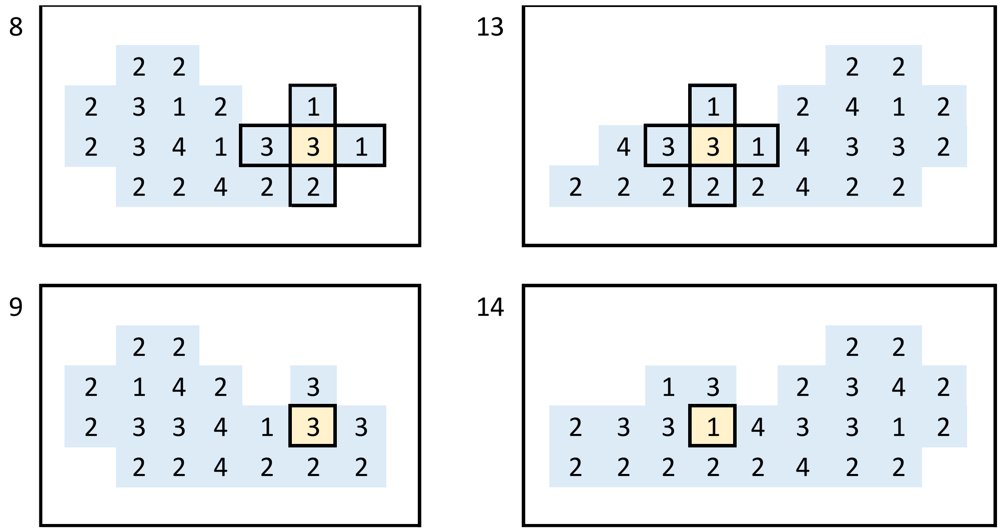


Figure 8. Left frames (t = 8 to 9) show the R-loop-only rule **31123 → 3**, listed in Table 3 at index line 48. In the right frames (t = 13 to 14), the contradiction is L-loop-only rule **31123 → 1, (not → 3)**. Rotated, this is 31231 → 1 listed in the L-loop-only column of Table 3 at index line 49.

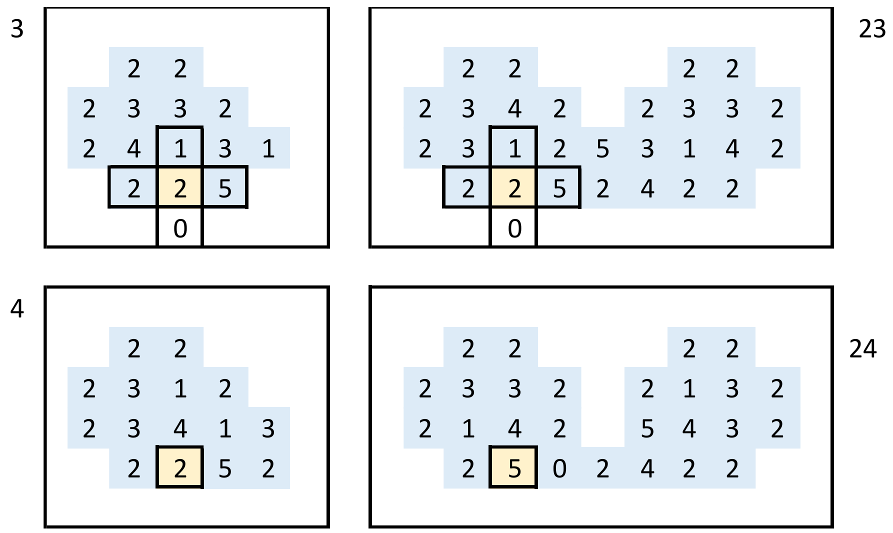


Figure 9. Left frames (t = 3 to 4) show the R-loop-only rule **21502 → 2**, listed in Table 3 at index line 39. In the right frames (t = 23 to 24), the contradiction is L-loop-only rule **21502 → 5, (not → 2)** listed as equivalent rotation 20215 → 5 in the L-loop-only column of Table 3 at index line 36.

Conclusion.

As for self-reproducing loops in cellular automata spaces generally, the state-transition function corresponding to replication of J Byl's structure [1] consists of a subset of rules which is closed under mirror transformation, and so is common to both right- and left-handed replication. There are also state-transition chiral rules and their mirrors which correspond respectively to right-handed-only and left-handed-only replication. Contradictions between right-handed-only and left-handed-only chiral state-transition rules prevent heterochiral self-reproduction of loops under a pooling of the state-transition function with its corresponding mirror function. Homochirality of loop self-reproduction in CA spaces parallels homochirality observed in real biology, but the question of relevance of this work to real biology is left open at the present time.

References

[1] J Byl, Self-reproduction in small cellular automata, *Physica D* **34** (1989) 295-299.

[2] PW Swanborough, Chiral Asymmetry of Self-Reproduction in Cellular Automata Spaces. *viXra:1904.0225* (2019).