PROPERTIES OF IDEAL BITOPOLOGICAL α-OPEN SETS

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Dedicated to Professor Valeriu Popa on the Occasion of His 80th Birthday

Abstract. The aim of this paper is to introduced and characterized the concepts of α-open sets and their related notions in ideal bitopological spaces.

1. Introduction

The concept of ideals in topological spaces has been introduced and studied by Kuratowski [10] and Vaidyanathasamy [11]. An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies (i) $A \in \mathcal{I}$ and $B \subset A$ implies $B \in \mathcal{I}$ and (ii) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$. Given a bitopological space (X, τ_1, τ_2) with an ideal $\mathcal{I}$ on X and if $\mathcal{P}(X)$ is the set of all subsets of X, a set operator $(.)^*_i$: $\mathcal{P}(X) \to \mathcal{P}(X)$, called the local function [11] of A with respect to τ_i and $\mathcal{I}$, is defined as follows: for $A \subset X$, $A^*_i(\tau_i, \mathcal{I}) = \{x \in X | U \cap A \notin \mathcal{I}$ for every $U \in \tau_i(x)\}$, where $\tau_i(x) = \{U \in \tau_i | x \in U\}$. For every ideal topological space $(X, \tau, \mathcal{I})$, there exists topology $\tau^*(I)$, finer than τ, generated by the base $\beta(\mathcal{I}, \tau) = \{U \backslash I \mid U \in \tau$ and $I \in \mathcal{I}\}$, but in general $\beta(\mathcal{I}, \tau)$ is not always a topology [7]. Observe additionally that τ_i-Cl$^*(A) = A \cup A^*_i(\tau_i, \mathcal{I})$ defines a Kuratowski closure operator for $\tau^*(\mathcal{I})$, when there is no chance of confusion, $A^*_i(\mathcal{I})$ is denoted by A^*_i and τ_i-Int$^*(A)$ denotes the interior of A in $\tau^*_i(\mathcal{I})$. The aim of this paper is to introduced and characterized the concepts of α-open sets and their related notions in ideal bitopological spaces.

2. Preiliminaries

Let A be a subset of a bitopological space (X, τ_1, τ_2). We denote the closure of A and the interior of A with respect to τ_i by τ_i-Cl(A) and τ_i-Int(A), respectively.

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Definition 2.1. *A subset A of a bitopological space (X, τ_1, τ_2) is said to be (i, j)-α-open* [9] *if $A \subset \tau_i$-Int$(\tau_j$-Cl$(\tau_i$-Int$(A)))$, where $i, j = 1, 2$ and $i \neq j$.*

Definition 2.2. *A subset S of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be α-$\mathcal{I}$-open* [8] *if $S \subset$ Int(Cl*(Int(S))). The family of all α-$\mathcal{I}$-open sets of $(X, \tau, \mathcal{I})$ is denoted by $\alpha\mathcal{I}O(X, \tau)$.*

Definition 2.3. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (i, j)-α-continuous* [9] *if the inverse image of every σ_j-open set in (Y, σ_1, σ_2) is (i, j)-α-open in $(X, \tau_1, \tau_2, \mathcal{I})$, where $i \neq j$, i, j=1, 2.*

Definition 2.4. *A subset A of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ is said to be*

(i) *(i, j)-R-$\mathcal{I}$-open* [1] *if $A = \tau_i$-Int$(\tau_j$-Cl$^*(A))$.*
(ii) *(i, j)-semi-$\mathcal{I}$-open* [3] *if $A \subset \tau_j$-Cl$^*(\tau_i$-Int$(A))$.*
(iii) *(i, j)-pre-$\mathcal{I}$-open* [2] *if $A \subset \tau_i$-Int$(\tau_j$-Cl$^*(A))$.*
(iv) *(i, j)-b-$\mathcal{I}$-open* [4] *if $A \subset \tau_i$-Int$(\tau_j$-Cl$^*(A)) \cup \tau_j$-Cl$^*(\tau_i$-Int$(A))$.*
(v) *(i, j)-β-$\mathcal{I}$-open* [5] *if $A \subset \tau_j$-Cl$(\tau_i$-Int$(\tau_j$-Cl$^*(A)))$.*
(vi) *(i, j)-δ-$\mathcal{I}$-open* [1] *if τ_i-Int$(\tau_j$-Cl$^*(A) \subset \tau_j$-Cl$^*(\tau_i$-Int$(A))$.*

The complemenet of an (i, j)-pre-$\mathcal{I}$-open (resp. (i, j)-β-$\mathcal{I}$-open) set is called an (i, j)-pre-$\mathcal{I}$-closed (resp. (i, j)-β-$\mathcal{I}$-closed) set.

Lemma 2.5. *Let $(X, \tau_1, \tau_2, \mathcal{I})$ be an ideal bitopological space. Then*

(i) *A subset A is (i, j)-pre-$\mathcal{I}$-closed if and only if τ_i-Cl$(\tau_j$-Int$^*(A)) \subset A$* [2];
(i) *A subset A is (i, j)-β-$\mathcal{I}$-closed if and only if τ_j-Int$(\tau_i$-Cl$(\tau_j$-Int$^*(A))) \subset A$* [5].

Definition 2.6. *A function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ is said to be*

(i) *pairwise pre-$\mathcal{I}$-continuous* [2] *if the inverse image of every σ_i-open set of Y is (i, j)-pre-$\mathcal{I}$-open in X, where $i \neq j$, i, j=1, 2.*
(i) *pairwise semi-$\mathcal{I}$-continuous* [3] *if the inverse image of every σ_i-open set of Y is (i, j)-semi-$\mathcal{I}$-open in X, where $i \neq j$, i, j=1, 2.*
(i) *pairwise b-$\mathcal{I}$-continuous* [4] *if the inverse image of every σ_i-open set of Y is (i, j)-b-$\mathcal{I}$-open in X, where $i \neq j$, i, j=1, 2.*
(i) *pairwise β-$\mathcal{I}$-continuous* [5] *if the inverse image of every σ_i-open set of Y is (i, j)-β-$\mathcal{I}$-open in X, where $i \neq j$, i, j=1, 2.*
(i) *pairwise δ-$\mathcal{I}$-continuous* [3] *if the inverse image of every σ_i-open set of Y is (i, j)-δ-$\mathcal{I}$-open in X, where $i \neq j$, i, j=1, 2.*
(i) *pairwise strongly β-$\mathcal{I}$-continuous* [5] *if the inverse image of every σ_i-open set of Y is strongly (i, j)-β-$\mathcal{I}$-open in X, where $i \neq j$, i, j=1, 2.*

3. (i,j)-α-$\mathcal{I}$-OPEN SETS

Definition 3.1. *A subset A of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ is said to be (i,j)-α-$\mathcal{I}$-open if and only if $A \subset \tau_i\text{-Int}(\tau_j\text{-Cl}^*(\tau_i\text{-Int}(A)))$, where $i, j = 1, 2$ and $i \neq j$.*
The family of all (i,j)-α-$\mathcal{I}$-open sets of $(X, \tau_1, \tau_2, \mathcal{I})$ is denoted by $\alpha\mathcal{I}O(X, \tau_1, \tau_2)$ or (i,j)-$\alpha\mathcal{I}O(X)$. Also, The family of all (i,j)-α-$\mathcal{I}$-open sets of $(X, \tau_1, \tau_2, \mathcal{I})$ containing x is denoted by (i,j)-$\alpha\mathcal{I}O(X, x)$.

Remark 3.2. *Let $\mathcal{I}$ and $\mathcal{J}$ be two ideals on (X, τ_1, τ_2). If $\mathcal{I} \subset \mathcal{J}$, then $\alpha\mathcal{J}O(X, \tau_1, \tau_2) \subset \alpha\mathcal{I}O(X, \tau_1, \tau_2)$.*

Proposition 3.3. (i) *Every (i,j)-α-$\mathcal{I}$-open set is (i,j)-semi-$\mathcal{I}$-open.*
(ii) *Every (i,j)-α-$\mathcal{I}$-open set is (i,j)-α-open.*
(iii) *Every (i,j)-α-$\mathcal{I}$-open set is (i,j)-pre-$\mathcal{I}$-open.*
(iv) *Every (i,j)-α-$\mathcal{I}$-open set is (i,j)-b-$\mathcal{I}$-open.*
(v) *Every (i,j)-α-$\mathcal{I}$-open set is (i,j)-β-$\mathcal{I}$-open.*

Proof. The proof follows from the definitions. □

The following example show that the converses of Proposition 3.3 is not true in general.

Example 3.4. *Let $X = \{a, b, c\}$ $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau_2 = \{\emptyset, \{a\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then the set $\{a, c\}$ is (i,j)-b-$\mathcal{I}$-open but not (i,j)-α-$\mathcal{I}$-open. Also, the set $\{b, c\}$ is (i,j)-semi-$\mathcal{I}$-open but not (i,j)-α-$\mathcal{I}$-open and the set $\{a, c\}$ is (i,j)-pre-$\mathcal{I}$-open and (i,j)-α-open but not (i,j)-α-$\mathcal{I}$-open.*

Proposition 3.5. *For an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ and $A \subset X$ we have:*
(i) *If $\mathcal{I} = \{\emptyset\}$, then A is (i,j)-α-$\mathcal{I}$-open if and only if A is (i,j)-α-open.*
(ii) *If $\mathcal{I} = \mathcal{P}(X)$, then A is (i,j)-α-$\mathcal{I}$-open if and only if A is τ_i-open.*

Proof. The proof follows from the fact that
(i) If $\mathcal{I} = \{\emptyset\}$, then $A^* = \text{Cl}(A)$.
(ii) If $\mathcal{I} = \mathcal{P}(X)$, then $A^* = \emptyset$ for every subset A of X.

□

Proposition 3.6. *Let A be a subset of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$. If B is an (i,j)-semi-$\mathcal{I}$-open set of X such that $B \subset A \subset \tau_i\text{-Int}(\tau_j\text{-Cl}^*(B))$, then A is an (i,j)-α-$\mathcal{I}$-open set of X.*

Proof. Since B is an (i,j)-semi-$\mathcal{I}$-open set of X, we have $B \subset \tau_j\text{-Cl}^*(\tau_i\text{-Int}(B))$. Thus, $A \subset \tau_i\text{-Int}(\tau_j\text{-Cl}^*(B)) \subset \tau_i\text{-Int}(\tau_j\text{-Cl}^*(\tau_j\text{-Cl}^*(\tau_i\text{-Int}(B)))) = \tau_i\text{-Int}(\tau_j\text{-Cl}^*(\tau_i\text{-Int}(B))) \subset \tau_i\text{-Int}(\tau_j\text{-Cl}^*(\tau_i\text{-Int}(A)))$, and so A is an (i,j)-α-$\mathcal{I}$-open set of X. □

Proposition 3.7. *Let $(X, \tau_1, \tau_2, \mathcal{I})$ be an ideal bitopological space. Then a subset ofX is (i, j)-α-$\mathcal{I}$-open if and only if it is both δ-$\mathcal{I}$-open and pre-$\mathcal{I}$-open.*

Proof. Let A be an (i, j)-α-$\mathcal{I}$-open set. Since every (i, j)-α-$\mathcal{I}$-open set is (i, j)-semi-$\mathcal{I}$-open, by Proposition 3.3 A is an (i, j)-δ-$\mathcal{I}$-open. Now we prove that $A \subset \tau_i$-Int$(\tau_j$-Cl$^*(A))$. Since A is an (i, j)-α-$\mathcal{I}$-open, we have $A \subset \tau_i$-Int$(\tau_j$-Cl$^*(\tau_i$-Int$(A))) \subset \tau_i$-Int$(\tau_j$-Cl$^*(A))$. Hence A is (i, j)-pre-$\mathcal{I}$-open. Conversely, let A be an (i, j)-δ-$\mathcal{I}$-open and (i, j)-pre-$\mathcal{I}$-open set. Then we have τ_i-Int$(\tau_j$-Cl$^*(A)) \subset \tau_j$-Cl$^*(\tau_i$-Int$(A))$ and hence τ_i-Int$(\tau_j$-Cl$^*(A)) \subset \tau_i$-Int$(\tau_j$-Cl$^*(\tau_i$-Int$(A)))$. Since A is (i, j)-pre-$\mathcal{I}$-open, we have $A \subset \tau_i$-Int$(\tau_j$-Cl$^*(A))$. Therefore, we obtain that $A \subset \tau_i$-Int$(\tau_j$-Cl$^*(\tau_i$-Int$(A)))$; hence A is (i, j)-α-$\mathcal{I}$-open. □

Lemma 3.8. *A subset A is (i, j)-α-$\mathcal{I}$-open if and only if (i, j)-semi-$\mathcal{I}$-open and (i, j)-pre-$\mathcal{I}$-open.*

Proof. Let A be (i, j)-semi-$\mathcal{I}$-open and (i, j)-pre-$\mathcal{I}$-open. Then, $A \subset \tau_i$-Int$(\tau_j$-Cl$^*(A)) \subset \tau_i$-Int$(\tau_j$-Cl$^*(\tau_j$-Cl$^*(\tau_i$-Int$(A)))) = \tau_i$-Int$(\tau_j$-Cl$^*(\tau_i$-Int$(A)))$. This shows that A is (i, j)-α-$\mathcal{I}$-open. The converse is obvious. □

Corollary 3.9. *The following properties are equivqlent for subsets of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$:*

(i) *Every (i, j)-pre-$\mathcal{I}$-open set is (i, j)-semi-$\mathcal{I}$-open.*
(ii) *A subset A of X is (i, j)-α-$\mathcal{I}$-open if and only if it is (i, j)-pre-$\mathcal{I}$-open.*

Corollary 3.10. *The following properties are equivqlent for subsets of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$:*

(i) *Every (i, j)-semi-$\mathcal{I}$-open set is (i, j)-pre-$\mathcal{I}$-open.*
(ii) *A subset A of X is (i, j)-α-$\mathcal{I}$-open if and only if it is (i, j)-semi-$\mathcal{I}$-open.*

Proposition 3.11. *Let A be a subset of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$. If A is (i, j)-pre-$\mathcal{I}$-closed and (i, j)-α-$\mathcal{I}$-open, then it is τ_i-open.*

Proof. Suppose A is (i, j)-pre-$\mathcal{I}$-closed and (i, j)-α-$\mathcal{I}$-open. Then by Lemma 2.5 τ_i-Cl$(\tau_j$-Int$^*(A)) \subset A$ and $A \subset \tau_i$-Int$(\tau_j$-Cl$^*(\tau_i$-Int$(A)))$. Now τ_i-Cl$(\tau_i$-Int$(A)) \subset \tau_i$-Cl$(\tau_i$-Int$(A)) \subset \tau_i$-Cl$(\tau_i$-Int$^*(A)) \subset A$ and so $A \subset \tau_i$-Int$(\tau_i$-Cl$^*(tau_i - \text{Int}(A)) \subset A \subset \tau_i$-Int$(A)$. Therefore, A is τ_i-open. □

Lemma 3.12. [1] *If A is any subset of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$, then τ_i-*Int$(\tau_j$-Cl$^*(A))$ *is (i, j)-R-$\mathcal{I}$-open.*

Proposition 3.13. *Let A be a subset of an ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$. If A is (i, j)-α-$\mathcal{I}$-open and (i, j)-β-$\mathcal{I}$-closed, then it is (i, j)-R-$\mathcal{I}$-open.*

Proof. Let A be (i,j)-α-$\mathcal{I}$-open and (i,j)-β-$\mathcal{I}$-closed. We have by Lemma 2.5, $A \subset \tau_j$-Int(τ_i-Cl*(τ_j-Int(A))) and τ_j-Int(τ_i-Cl*(τ_j-Int(A))) $\subset$ τ_j-Int(τ_i-Cl(τ_j-Int*(A))) $\subset A$; hence $A = \tau_j$-Int(τ_i-Cl*(τ_j-Int(A))). Thus, by Lemma 3.12, A is (i,j)-R-$\mathcal{I}$-open. □

An ideal bitopological space is said to satisfy the condition $(\mathcal{A})$ if $U \cap \tau_j$-Cl*$(A) \subset \tau_j$-Cl*$(U \cap A)$ for every $U \in \tau_i$.

Theorem 3.14. *Let $(X, \tau_1, \tau_2, \mathcal{I})$ be an ideal bitopological space that satisfies the condition $(\mathcal{A})$. Then we have the following*

(i) *If $V \in (i,j)$-$\alpha\mathcal{I}O(X)$ and $A \in (i,j)$-$\alpha\mathcal{I}O(X)$, then $V \cap A \in (i,j)$-$\alpha\mathcal{I}O(X)$.*
(ii) *If $V \in (i,j)$-$\alpha\mathcal{I}O(X)$ and $A \in (i,j)$-$\alpha\mathcal{I}O(X)$, then $V \cap A \in (i,j)$-$\alpha\mathcal{I}O(X)$.*

Proof. (*i*). Let $V \in (i,j)$-$\alpha\mathcal{I}O(X)$ and $A \in (i,j)$-$\alpha\mathcal{I}O(X)$. Then $V \cap A \subset \tau_j$-Cl*(τ_i-Int(V))$\cap\tau_i$-Int(τ_j-Cl*(τ_i-Int A))) $\subset \tau_j$-Cl*(τ_i-Int(V)$\cap$ τ_i-Int(τ_j-Cl*(τ_i-Int(A)))) $\subset \tau_j$-Cl*(τ_i-Int(V) $\cap$ τ_j-Cl*(τ_i-Int(A))) $\subset \tau_j$-Cl*(τ_j-Cl*(τ_i-Int(V) $\cap$ τ_i-Int(A))) $\subset \tau_j$-Cl*(τ_i-Int(V)). This shows that $V \cap A \in (i,j)$-$\alpha\mathcal{I}O(X)$.
(*ii*). Let $V \in (i,j)$-$P\mathcal{I}O(X)$ and $A \in (i,j)$-$\alpha\mathcal{I}O(X)$. Then $V \cap A \subset \tau_i$-Int(τ_j-Cl*(A)) $\cap$ τ_i-Int(τ_j-Cl*(τ_i-Int(A)))) $=$ τ_i-Int(τ_i-Int(τ_j-Cl*(V)) $\cap$ τ_j-Cl*(τ_i-Int(A))) $\subset \tau_i$-Int(τ_j-Cl*(τ_i-Int(τ_j-Cl*(V)) $\cap$ τ_i-Int(A))) $\subset \tau_i$-Int(τ_j-Cl*(τ_j-Cl*(V)$\cap\tau_i$-Int(A))) $\subset \tau_i$-Int(τ_j-Cl*(τ_j-Cl*($V\cap\tau_i$-Int(A)))) $\subset$ τ_i-Int(τ_i-Cl*($V \cap V$)). This shows that $V \cap A \in (i,j)$-$P\mathcal{I}O(X)$. □

Remark 3.15. *The intersection of two (i,j)-α-$\mathcal{I}$-open sets need not be (i,j)-α-$\mathcal{I}$-open as it can be seen from the following example.*

Example 3.16. *Let $X = \{a,b,c,d\}$, $\tau_1 = \{\varnothing, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, X\}$, $\tau_2 = \{\varnothing, X\}$ and $\mathcal{I} = \{\varnothing, \{c\}, \{d\}, \{c,d\}\}$. Then the sets $\{a,c\}$ and $\{b,c\}$ are $(1,2)$-α-$\mathcal{I}$-open sets of $(X, \tau_1, \tau_2, \mathcal{I})$ but their intersection $\{c\}$ is not an $(1,2)$-α-$\mathcal{I}$-open set of $(X, \tau_1, \tau_2, \mathcal{I})$.*

Theorem 3.17. *If $\{A_\alpha\}_{\alpha\in\Omega}$ be a family of (i,j)-α-$\mathcal{I}$-open sets in $(X, \tau_1, \tau_2, \mathcal{I})$, then $\bigcup_{\alpha\in\Omega} A_\alpha$ is (i,j)-α-$\mathcal{I}$-open in $(X, \tau_1, \tau_2, \mathcal{I})$.*

Proof. Since $\{A_\alpha : \alpha \in \Omega\} \subset (i,j)$-$\alpha\mathcal{I}O(X)$, then $A_\alpha \subset \tau_i$-Int(τ_j-Cl*(τ_i-Int(A_α))) for every $\alpha \in \Omega$. Thus, $\cup_{\alpha\in\Omega} A_\alpha \subset \cup_{\alpha\in\Omega} \tau_i$-Int($\tau_j$-Cl*($\tau_i$-Int($A_\alpha$))) $\subset \tau_i$-Int(τ_j-Cl*($\cup_{\alpha\in\Omega} \tau_i$-Int($A_\alpha$)) $= \tau_i$-Int(τ_j-Cl*(τ_i-Int($\cup_{\alpha\in\Omega} A_\alpha$)). Therefore, we obtain $\cup_{\alpha\in\Omega} A_\alpha \subset \tau_i$-Int($\tau_j$-Cl*($\tau_i$-Int($\cup_{\alpha\in\Omega} A_\alpha$)). Hence any union of (i,j)-α-$\mathcal{I}$-open sets is (i,j)-α-$\mathcal{I}$-open. □

If $(X, \tau, \mathcal{I})$ is an ideal topological space and A is a subset of X, we denote by $\tau_{|A}$, the relative topology on A and $\mathcal{I}_{|A} = \{A \cap I \in \mathcal{I}\}$ is obviously an ideal on A.

Theorem 3.18. *Let $(X,\tau_1,\tau_2,\mathcal{I})$ be an ideal bitopological space satisfies the condition $(\mathcal{A})$. If $A\in(i,j)$-$\alpha\mathcal{I}O(X)$ and $A\subset B\in(i,j)$-$\alpha\mathcal{I}O(X)$, Then $A\in(i,j)$-$\alpha\mathcal{I}_{|B}O(B)$.*

Proof. By definition, $A\subset\tau_i$-Int$(\tau_j$-Cl$^*(\tau_i$-Int$(A\cap B)))\cap B=\tau_i$-Int$_B(\tau_i$-Int$(\tau_j$-Cl$^*(A\cap B)))\cap B)\subset\tau_i$-Int$_B(\tau_j$-Cl$^*(A\cap B))\cap B)=\tau_i$-Int$_B(\tau_j$-Cl$^*_B(\tau_i$-Int$(A)\cap\tau_i$-Int$(B)))\subset\tau_i$-Int$_B(\tau_j$-Cl$^*_B(\tau_i$-Int$(A)\cap B))=\tau_i$-Int$_B(\tau_i$-Cl$^*_B(\tau_i$-Int$_B(\tau_i$-Int$(A)\cap B)))\subset\tau_i$-Int$_B(\tau_j$-Cl$^*_B(\tau_i$-Int$_B(A)))$. This shows that $A\in(i,j)$-$\alpha\mathcal{I}_{|B}O(B)$. □

Definition 3.19. *In an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$, $A\subset X$ is said to be (i,j)-α-$\mathcal{I}$-closed if $X\backslash A$ is (i,j)-α-$\mathcal{I}$-open in X, $i,j=1,2$ and $i\neq j$.*

Theorem 3.20. *If A is an (i,j)-α-$\mathcal{I}$-closed set in an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ if and only if τ_i-Cl$(\tau_j$-Int$^*(\tau_i$-Cl$(A)))\subset A$.*

Proof. The proof follows from the definitions. □

Theorem 3.21. *If A is an (i,j)-α-$\mathcal{I}$-closed set in an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$, then τ_i-Cl$(\tau_j$-Int$(\tau_i$-Cl$^*(A)))\subset A$.*

Proof. Since $A\in(i,j)$-$\alpha\mathcal{I}C(X)$, $X\backslash A\in(i,j)$-$\alpha\mathcal{I}O(X)$. Hence, $X\backslash A\subset\tau_i$-Int$(\tau_j$-Cl$^*(\tau_i$-Int$(X\backslash A))\subset\tau_i$-Int$(\tau_j$-Cl$(\tau_i$-Int$(X\backslash A)))=X\backslash\tau_i$-Cl$(\tau_j$-Int$(\tau_i$-Cl$(A)))\subset X\backslash(\tau_i$-Cl$(\tau_j$-Int$(\tau_i$-Cl$^*(A)))$. Therefore, we obtain τ_i-Cl$(\tau_i$-Int$(\tau_i$-Cl$^*(A))\subset A$. □

Proposition 3.22. *Let $(X,\tau_1,\tau_2,\mathcal{I})$ be an ideal bitopological space. If a subset of X is (i,j)-β-$\mathcal{I}$-closed and (i,j)-δ-$\mathcal{I}$-open, then it is (i,j)-α-$\mathcal{I}$-closed.*

Proof. The proof follows from the definitions. □

Theorem 3.23. *Arbitrary intersection of (i,j)-α-$\mathcal{I}$-closed sets is always (i,j)-α-$\mathcal{I}$-closed.*

Proof. Follows from Theorems 3.17 and 3.21. □

Definition 3.24. *Let $(X,\tau_1,\tau_2,\mathcal{I})$ be an ideal bitopological space, S a subset of X and x be a point of X. Then*

(i) *x is called an (i,j)-α-$\mathcal{I}$-interior point of S if there exists $V\in(i,j)$-$\alpha\mathcal{I}O(X,\tau_1,\tau_2)$ such that $x\in V\subset S$.*
ii) *the set of all (i,j)-α-$\mathcal{I}$-interior points of S is called (i,j)-α-$\mathcal{I}$-interior of S and is denoted by (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(S)$.*

Theorem 3.25. *Let A and B be subsets of $(X,\tau_1,\tau_2,\mathcal{I})$. Then the following properties hold:*

(i) *(i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A)=\cup\{T:T\subset A$ and $A\in(i,j)$-$\alpha\mathcal{I}O(X)\}$.*
(ii) *(i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A)$ is the largest (i,j)-α-$\mathcal{I}$-open subset of X contained in A.*
(iii) *A is (i,j)-α-$\mathcal{I}$-open if and only if $A=(i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A)$.*

(iv) (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}((i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A)) = (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A)$.
(v) *If* $A \subset B$, *then* (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(B)$.
(vi) (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A\cap B) = (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A) \cap (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(B)$.
(vii) (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A\cup B) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A) \cup (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(B)$.

Proof. (i). Let $x \in \cup\{T : T \subset A$ and $A \in (i,j)$-$\alpha\mathcal{I}O(X)\}$. Then, there exists $T \in (i,j)$-$\alpha\mathcal{I}O(X,x)$ such that $x \in T \subset A$ and hence $x \in (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A)$. This shows that $\cup\{T : T \subset A$ and $A \in (i,j)$-$\alpha\mathcal{I}O(X)\}$ $\subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A)$. For the reverse inclusion, let $x \in (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A)$. Then there exists $T \in (i,j)$-$\alpha\mathcal{I}O(X,x)$ such that $x \in T \subset A$. we obtain $x \in \cup\{T : T \subset A$ and $A \in (i,j)$-$\alpha\mathcal{I}O(X)\}$. This shows that (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A) \subset \cup\{T : T \subset A$ and $A \in (i,j)$-$\alpha\mathcal{I}O(X)\}$. Therefore, we obtain (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A) = \cup\{T : T \subset A$ and $A \in (i,j)$-$\alpha\mathcal{I}O(X)\}$.
The proof of $(ii) - (v)$ are obvious.
(vi). By (v), we have (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A\cup B)$ and (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(B) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A\cup B)$. Then we obtain (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A) \cup (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(B) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A\cup B)$ Since (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A) \subset A$ and (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(B) \subset B$, we obtain (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A\cup B) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A)$ $\cup\, (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(B)$. It follows that (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A\cap B) = (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A)$ $\cap\, (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(B)$.
(vii). Since $A\cap B \subset A$ and $A\cap B \subset B$, by (v), we have (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A\cap B) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A)$ and (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A\cap B) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(B)$. Therefore, (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A) \cup (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(B) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A\cap B)$. □

Theorem 3.26. *If* $(X,\tau_1,\tau_2,\mathcal{I})$ *is an ideal bitopological space satisfying the condition* $(\mathcal{A})$, *then* (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A) = A\cap\tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A)))$ *holds for every subset* A *of* X.

Proof. Since $A\cap\tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A))) \subset \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A))) =$ τ_i-$\mathrm{Int}(\tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A)))) = \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A)) \cap (\tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A)))) \subset \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A)\cap\tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A))))) =$ τ_i-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A\cap\tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A))))))$, $A\cap\tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A)))$ is an (i,j)-α-$\mathcal{I}$-open set contained in A and so $A\cap\tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A))) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A)$. Since (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A)$ is (i,j)-α-$\mathcal{I}$-open, (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A) \subset \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}((i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A)))) \subset \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A)))$ and so (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A) \subset A\cap\tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\mathrm{Int}(A)))$. Hence (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A) = A\cap\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(A)))$. □

Definition 3.27. *The union of all* (i,j)*-pre-*$\mathcal{I}$*-open sets of* $(X,\tau_1,\tau_2,\mathcal{I})$ *containing* A *is called the* (i,j)*-pre-*$\mathcal{I}$*-interior of* A *and is denote by* (i,j)-$p\mathcal{I}\,\mathrm{Int}(A)$.

Lemma 3.28. *If* $(X,\tau_1,\tau_2,\mathcal{I})$ *is an ideal bitopological space satisfies the condition* $(\mathcal{A})$, *then* (i,j)-$p\mathcal{I}\,\mathrm{Int}(A) = A\cap\tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(A))$ *holds for every subset* A *of* X.

Theorem 3.29. *If* $(X,\tau_1,\tau_2,\mathcal{I})$ *is an ideal bitopological space satisfies the condition* $(\mathcal{A})$, *then* (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(A) = (i,j)$-$p\mathcal{I}\,\mathrm{Int}(A)$ *holds for every* (i,j)-δ-$\mathcal{I}$*-open subset* A *of* X.

Proof. Since every (i,j)-α-$\mathcal{I}$-open set is (i,j)-pre-$\mathcal{I}$-open, (i,j)-$\alpha\mathcal{I}\operatorname{Int}(A) \subset (i,j)$-$p\mathcal{I}\operatorname{Int}(A)$. By Theorem 3.26, $\alpha\mathcal{I}\operatorname{Int}(A) = A \cap \tau_i$-$\operatorname{Int}(\tau_j$-$\operatorname{Cl}^*(\tau_i$-$\operatorname{Int}(A)))$. Since A is (i,j)-δ-$\mathcal{I}$-open, (i,j)-$\alpha\mathcal{I}\operatorname{Int}(A) \supset A \cap \tau_i$-$\operatorname{Int}(\tau_i$-$\operatorname{Int}(\tau_j$-$\operatorname{Cl}^*(A))) = A \cap \tau_i$-$\operatorname{Int}(\tau_j$-$\operatorname{Cl}^*(A)) = (i,j)$-$p\mathcal{I}\operatorname{Int}(A)$ by Lemma 3.28 and so (i,j)-$\alpha\mathcal{I}\operatorname{Int}(A) \supset (i,j)$-$p\mathcal{I}\operatorname{Int}(A)$. Therefore, (i,j)-$\alpha\mathcal{I}\operatorname{Int}(A) = (i,j)$-$p\mathcal{I}\operatorname{Int}(A)$. □

Definition 3.30. *Let $(X,\tau_1,\tau_2,\mathcal{I})$ be an ideal bitopological space, S a subset of X and x be a point of X. Then*

(i) *x is called an (i,j)-α-$\mathcal{I}$-cluster point of S if $V \cap S \neq \emptyset$ for every $V \in (i,j)$-$\alpha\mathcal{I}O(X,x)$.*
(ii) *the set of all (i,j)-α-$\mathcal{I}$-cluster points of S is called (i,j)-α-$\mathcal{I}$-closure of S and is denoted by (i,j)-$\alpha\mathcal{I}\operatorname{Cl}(S)$.*

Theorem 3.31. *Let A and B be subsets of $(X,\tau_1,\tau_2,\mathcal{I})$. Then the following properties hold:*

(i) (i,j)-$\alpha\mathcal{I}\operatorname{Cl}(A) = \cap\{F : A \subset F$ *and* $F \in (i,j)$-$\alpha\mathcal{I}C(X)\}$.
(ii) (i,j)-$\alpha\mathcal{I}\operatorname{Cl}(A)$ *is the smallest (i,j)-α-$\mathcal{I}$-closed subset of X containing A.*
(iii) *A is (i,j)-α-$\mathcal{I}$-closed if and only if $A = (i,j)$-$\alpha\mathcal{I}\operatorname{Cl}(A)$.*
(iv) (i,j)-$\alpha\mathcal{I}\operatorname{Cl}((i,j)$-$\alpha\mathcal{I}\operatorname{Cl}(A) = (i,j)$-$\alpha\mathcal{I}\operatorname{Cl}(A)$.
(v) *If $A \subset B$, then (i,j)-$\alpha\mathcal{I}\operatorname{Cl}(A) \subset (i,j)$-$\alpha\mathcal{I}\operatorname{Cl}(B)$.*
(vi) (i,j)-$\alpha\mathcal{I}\operatorname{Cl}(A \cup B) = (i,j)$-$\alpha\mathcal{I}\operatorname{Cl}(A) \cup (i,j)$-$\alpha\mathcal{I}\operatorname{Cl}(B)$.
(vii) (i,j)-$\alpha\mathcal{I}\operatorname{Cl}(A \cap B) \subset (i,j)$-$\alpha\mathcal{I}\operatorname{Cl}(A) \cap (i,j)$-$\alpha\mathcal{I}\operatorname{Cl}(B)$.

Proof. (*i*). Suppose that $x \notin (i,j)$-$\alpha\mathcal{I}\operatorname{Cl}(A)$. Then there exists $F \in (i,j)$-$\alpha\mathcal{I}O(X)\}$ such that $V \cap S \neq \emptyset$. Since $X \backslash V$ is (i,j)-α-$\mathcal{I}$-closed set containing A and $x \notin X\backslash V$, we obtain $x \notin \cap\{F : A \subset F$ and $F \in (i,j)$-$\alpha\mathcal{I}C(X)\}$. Then there exists $F \in (i,j)$-$\alpha\mathcal{I}C(X)$ such that $A \subset F$ and $x \notin F$. Since $X\backslash V$ is (i,j)-α-$\mathcal{I}$-closed set containing x, we obtain $(X\backslash F) \cap A = \emptyset$. This shows that $x \notin (i,j)$-$\alpha\mathcal{I}\operatorname{Cl}(A)$. Therefore, we obtain (i,j)-$\alpha\mathcal{I}\operatorname{Cl}(A) = \cap\{F : A \subset F$ and $F \in (i,j)$-$\alpha\mathcal{I}C(X)\}$.
The other proofs are obvious. □

Theorem 3.32. *Let $(X,\tau_1,\tau_2,\mathcal{I})$ be an ideal bitopological space and $A \subset X$. A point $x \in (i,j)$-$\alpha\mathcal{I}\operatorname{Cl}(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in (i,j)$-$\alpha\mathcal{I}O(X,x)$.*

Proof. Suppose that $x \in (i,j)$-$\alpha\mathcal{I}\operatorname{Cl}(A)$. We shall show that $U \cap A \neq \emptyset$ for every $U \in (i,j)$-$\alpha\mathcal{I}O(X,x)$. Suppose that there exists $U \in (i,j)$-$\alpha\mathcal{I}O(X,x)$ such that $U \cap A = \emptyset$. Then $A \subset X\backslash U$ and $X\backslash U$ is (i,j)-α-$\mathcal{I}$-closed. since $A \subset X\backslash U$, (i,j)-$\alpha\mathcal{I}\operatorname{Cl}(A) \subset (i,j)$-$\alpha\mathcal{I}\operatorname{Cl}(X\backslash U)$. Since $x \in (i,j)$-$\alpha\mathcal{I}\operatorname{Cl}(A)$, we have $x \in (i,j)$-$\alpha\mathcal{I}\operatorname{Cl}(X\backslash U)$. Since $X\backslash U$ is (i,j)-α-$\mathcal{I}$-closed, we have $x \in X\backslash U$; hence $x \notin U$, which is a contradicition that $x \in U$. Therefore, $U \cap A \neq \emptyset$. Conversely, suppose that $U \cap A \neq \emptyset$ for every $U \in (i,j)$-$\alpha\mathcal{I}O(X,x)$. We shall show that

$x \in (i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(A)$. Suppose that $x \notin (i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(A)$. Then there exists $U \in (i,j)$-$\alpha\mathcal{I}O(X,x)$ such that $U \cap A = \emptyset$. This is a contradicition to $U \cap A \neq \emptyset$; hence $x \in (i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(A)$. □

Theorem 3.33. *Let $(X,\tau_1,\tau_2,\mathcal{I})$ be an ideal bitopological space and $A \subset X$. Then the following propeties hold:*

(i) *(i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(X\backslash A) = X\backslash(i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(A)$;*
(i) *(i,j)-$\alpha\mathcal{I}\,\mathrm{Cl}(X\backslash A) = X\backslash(i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(A)$.*

Proof. (i). Let $x \in (i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(A)$. Since $x \notin (i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(A)$, there exists $V \in (i,j)$-$\alpha\mathcal{I}O(X,x)$ such that $V \cap A \neq \emptyset$; hence we obtain $x \in (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(X\backslash A)$. This shows that $X\backslash(i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(A) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(X\backslash A)$. Let $x \in (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(X\backslash A)$. Since (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(X\backslash A) \cap A = \emptyset$, we obtain $x \notin (i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(A)$; hence $x \in X\backslash(i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(A)$. Therefore, we obtain (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(X\backslash A) = X\backslash(i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(A)$.
(ii). Follows from (i). □

Theorem 3.34. *If $(X,\tau_1,\tau_2,\mathcal{I})$ is an ideal bitopological space satisfies the condition $(\mathcal{A})$, then (i,j)-$\alpha\mathcal{I}\,\mathrm{Cl}(A) = A \cup \tau_i$-$\mathrm{Cl}(\tau_j$-$\mathrm{Int}^*(\tau_i$-$\mathrm{Cl}(A)))$ holds for every subset A of X.*

Proof. The proof follows from the definitions. □

Definition 3.35. *A subset B_x of an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is said to be an (i,j)-α-$\mathcal{I}$-neighbourhood of a point $x \in X$ if there exists an (i,j)-α-$\mathcal{I}$-open set U such that $x \in U \subset B_x$.*

Theorem 3.36. *A subset of an ideal bitopological space $(X,\tau_1,\tau_2,\mathcal{I})$ is (i,j)-α-$\mathcal{I}$-open if and only if it is an (i,j)-α-$\mathcal{I}$-neighbourhood of each of its points.*

Proof. Let G be an (i,j)-α-$\mathcal{I}$-open set of X. Then by definition, it is clear that G is an (i,j)-α-$\mathcal{I}$-neighbourhood of each of its points, since for every $x \in G$, $x \in G \subset G$ and G is (i,j)-α-$\mathcal{I}$-open. Conversely, suppose G is an (i,j)-α-$\mathcal{I}$-neighbourhood of each of its points. Then for each $x \in G$, there exists $S_x \in (i,j)$-$\alpha\mathcal{I}O(X)$ such that $S_x \subset G$. Then $G = \bigcup\{S_x : x \in G\}$. Since each S_x is (i,j)-α-$\mathcal{I}$-open, G is (i,j)-α-$\mathcal{I}$-open in $(X,\tau_1,\tau_2,\mathcal{I})$. □

Proposition 3.37. *The product of two (i,j)-α-$\mathcal{I}$-open sets is (i,j)-α-$\mathcal{I}$-open.*

Proof. The proof follows from Lemma 3.3 of [12]. □

4. PAIRWISE α-$\mathcal{I}$-CONTINUOUS FUNCTIONS

Definition 4.1. *A function $f : (X,\tau_1,\tau_2,\mathcal{I}) \to (Y,\sigma_1,\sigma_2)$ is said to be (i,j)-α-$\mathcal{I}$-continuous if the inverse image of every σ_i-open set of Y is (i,j)-α-$\mathcal{I}$-open in X, where $i \neq j$, i,j=1, 2.*

Proposition 4.2. (i) *Every (i,j)-α-$\mathcal{I}$-continuous function is (i,j)-semi-$\mathcal{I}$-continuous but not conversely.*

(ii) *Every (i,j)-α-$\mathcal{I}$-continuous function is (i,j)-α-continuous but not conversely.*
(iii) *Every (i,j)-α-$\mathcal{I}$-continuous function is (i,j)-pre-$\mathcal{I}$-continuous but not conversely.*
(iv) *Every (i,j)-α-$\mathcal{I}$-continuous function is (i,j)-b-$\mathcal{I}$-continuous but not conversely.*
(v) *Every (i,j)-α-$\mathcal{I}$-continuous function is (i,j)-β-$\mathcal{I}$-continuous but not conversely.*

Proof. The proof follows from Proposition 3.3 and Example 3.4. □

Theorem 4.3. *A function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ is (i,j)-α-$\mathcal{I}$-continuous if and only if it is (i,j)-semi-$\mathcal{I}$-continuous and (i,j)-pre-$\mathcal{I}$-continuous.*

Proof. This is an immediate consequence of Lemma 3.8. □

Theorem 4.4. *For a function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$, the following statements are equivalent:*

(i) *f is pairwise α-$\mathcal{I}$-continuous;*
(ii) *For each point x in X and each σ_i-open set F in Y such that $f(x) \in F$, there is a (i,j)-α-$\mathcal{I}$-open set A in X such that $x \in A$, $f(A) \subset F$;*
(iii) *The inverse image of each σ_i-closed set in Y is (i,j)-α-$\mathcal{I}$-closed in X;*
(iv) *For each subset A of X, $f((i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Cl}(A)) \subset \sigma_i\text{-}\mathrm{Cl}(f(A))$;*
(v) *For each subset B of Y, $(i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(B)) \subset f^{-1}(\sigma_i\text{-}\mathrm{Cl}(B))$;*
(vi) *For each subset C of Y, $f^{-1}(\sigma_i\text{-}\mathrm{Int}(C)) \subset (i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}(C))$.*
(vii) *$\tau_i\text{-}\mathrm{Cl}(\tau_j\text{-}\mathrm{Int}^*(\tau_i\text{-}\mathrm{Cl}(f^{-1}(B)))) \subset f^{-1}(\tau_i\text{-}\mathrm{Cl}(B))$ for each subset B of Y.*
(viii) *$f(\tau_i\text{-}\mathrm{Cl}(\tau_j\text{-}\mathrm{Int}^*(\tau_i\text{-}\mathrm{Cl}(A)))) \subset \tau_i\text{-}\mathrm{Cl}(f(A))$ for each subset A of X.*

Proof. *(i) ⇒ (ii)*: Let $x \in X$ and F be a σ_j-open set of Y containing $f(x)$. By (i), $f^{-1}(F)$ is (i,j)-α-$\mathcal{I}$-open in X. Let $A = f^{-1}(F)$. Then $x \in A$ and $f(A) \subset F$.
(ii) ⇒ (i): Let F be σ_j-open in Y and let $x \in f^{-1}(F)$. Then $f(x) \in F$. By (ii), there is an (i,j)-α-$\mathcal{I}$-open set U_x in X such that $x \in U_x$ and $f(U_x) \subset F$. Then $x \in U_x \subset f^{-1}(F)$. Hence $f^{-1}(F)$ is (i,j)-α-$\mathcal{I}$-open in X.
(i) ⇔ (iii): This follows due to the fact that for any subset B of Y, $f^{-1}(Y \backslash B) = X \backslash f^{-1}(B)$.
(iii) ⇒ (iv): Let A be a subset of X. Since $A \subset f^{-1}(f(A))$ we have $A \subset f^{-1}(\sigma_j\text{-}\mathrm{Cl}(f(A)))$. Now, $(i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Cl}(f(A))$ is σ_j-closed in Y and hence $f^{-1}(\sigma_j\text{-}\mathrm{Cl}(A)) \subset f^{-1}(\sigma_j\text{-}\mathrm{Cl}(f(A)))$, for $(i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Cl}(A)$ is the smallest (i,j)-α-$\mathcal{I}$-closed set containing A. Then $f((i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Cl}(A)) \subset \sigma_j\text{-}\mathrm{Cl}(f(A))$.

$(iv) \Rightarrow (iii)$: Let F be any (i,j)-α-$\mathcal{I}$-closed subset of Y. Then $f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(F))) \subset (i,j)$-$\sigma_i$-$\mathrm{Cl}(f(f^{-1}(F))) = (i,j)$-$\sigma_i$-$\mathrm{Cl}(F) = F$. Therefore, (i,j)-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(F)) \subset f^{-1}(F)$. Consequently, $f^{-1}(F)$ is (i,j)-α-$\mathcal{I}$-closed in X.
$(iv) \Rightarrow (v)$: Let B be any subset of Y. Now, $f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(B))) \subset (i,j)$-$\sigma_i$-$\mathrm{Cl}(f(f^{-1}(B))) \subset \sigma_i$-$\mathrm{Cl}(B)$. Consequently, (i,j)-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(B)) \subset f^{-1}(\sigma_i$-$\mathrm{Cl}(B))$.
$(v) \Rightarrow (iv)$: Let $B = f(A)$ where A is a subset of X. Then, (i,j)-$\alpha\mathcal{I}\,\mathrm{Cl}(A) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(B)) \subset f^{-1}(\sigma_i$-$\mathrm{Cl}(B)) = f^{-1}(\sigma_i$-$\mathrm{Cl}(f(A)))$. This shows that $f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(A)) \subset \sigma_i$-$\mathrm{Cl}(f(A))$.
$(i) \Rightarrow (vi)$: Let B be a σ_j-open set in Y. Clearly, $f^{-1}(\sigma_i$-$\mathrm{Int}(B)$ is (i,j)-α-$\mathcal{I}$-open and we have $f^{-1}(\sigma_i$-$\mathrm{Int}(B)) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}\sigma_i$-$\mathrm{Int}(B)) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}B)$.
$(vi) \Rightarrow (i)$: Let B be a σ_j-open set in Y. Then σ_i-$\mathrm{Int}(B) = B$ and $f^{-1}(B) \backslash f^{-1}(\sigma_i$-$\mathrm{Int}(B)) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}(B))$. Hence we have $f^{-1}(B) = (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}(B))$. This shows that $f^{-1}(B)$ is (i,j)-α-$\mathcal{I}$-open in X.
$(iii) \Rightarrow (vii)$: Let B be any subset os Y. Since τ_i-$\mathrm{Cl}(B)$ is τ_i-closed in Y, by (iii), $f^{-1}(\tau_i$-$\mathrm{Cl}(B))$ is α-$\mathcal{I}$-closed and $X \backslash f^{-1}(\tau_i$-$\mathrm{Cl}(B))$ is α-$\mathcal{I}$-open. Then $X \backslash f^{-1}(\tau_i$-$\mathrm{Cl}(B)) \subset \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(f^{-1}(\tau_i$-$\mathrm{Cl}(B))))) = X \backslash \tau_i$-$\mathrm{Cl}(\tau_j$-$\mathrm{Int}^*(\tau_i$-$\mathrm{Cl}(f^{-1}(\tau_i$-$\mathrm{Cl}(B)))))$. Hence we obtain τ_i-$\mathrm{Cl}(\tau_j$-$\mathrm{Int}^*(\tau_i$-$\mathrm{Cl}(f^{-1}(B)))) \subset f^{-1}(\tau_i$-$\mathrm{Cl}(B))$.
$(vii) \Rightarrow (viii)$: Let A be any ubset of X. By(iv), we have $\mathrm{Cl}(\tau_j$-$\mathrm{Int}^*(\tau_i$-$\mathrm{Cl}(A)) \subset \tau_i$-$\mathrm{Cl}(\tau_j$-$\mathrm{Int}^*(\tau_i$-$\mathrm{Cl}(f^{-1}(f(A))))) \subset f^{-1}(\tau_i$-$\mathrm{Cl}(f(A)))$ and hence $f(\tau_i$-$\mathrm{Cl}(\tau_j$-$\mathrm{Int}^*(\tau_i$-$\mathrm{Cl}(A)))) \subset \tau_i$-$\mathrm{Cl}(f(A))$.
$(viii) \Rightarrow (i)$: Let V be any open set of Y. Then by (v), $f(\tau_i$-$\mathrm{Cl}(\tau_j$-$\mathrm{Int}^*(\tau_i$-$\mathrm{Cl}(f^{-1}(Y \backslash V))))) \subset \tau_i$-$\mathrm{Cl}(f(f^{-1}(Y \backslash V))) \subset \tau_i$-$\mathrm{Cl}(Y \backslash V) = Y \backslash V$. Therefore, we have τ_i-$\mathrm{Cl}(\tau_j$-$\mathrm{Int}^*(\tau_i$-$\mathrm{Cl}(f^{-1}(Y \backslash V)))) \subset f^{-1}(Y \backslash V) \subset X \backslash f^{-1}(V)$. Consequently, we obtain $f^{-1}(V) \subset \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(f^{-1}(V))))$. This shows that $f^{-1}(V)$ is α-$\mathcal{I}$-open. Thus, f is α-$\mathcal{I}$-continuous. □

Corollary 4.5. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$ be an (i,j)-α-$\mathcal{I}$-continuous function, then*

(i) $f(\tau_j$-$\mathrm{Cl}^*(U)) \subset \tau_j$-$\mathrm{Cl}(f(U))$ *for every (i,j)-pre-$\mathcal{I}$-open set U of X,*
(ii) τ_j-$\mathrm{Cl}^*(f^{-1}(V)) \subset f^{-1}(\tau_j$-$\mathrm{Cl}(V))$ *for every (i,j)-pre-$\mathcal{I}$-open set V of Y.*

Proof. (1). Let U be any (i,j)-pre-$\mathcal{I}$-open set of X, then $U \subset \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(U))$. Therefore, by Theorem 4.4, we have $f(\tau_j$-$\mathrm{Cl}^*(U)) \subset f(\tau_j$-$\mathrm{Cl}(U)) \subset f(\tau_j$-$\mathrm{Cl}(\tau_j$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(U)))) \subset f(\tau_j$-$\mathrm{Cl}(\tau_j$-$\mathrm{Int}^*(\tau_j$-$\mathrm{Cl}(U)))) \subset \tau_j$-$\mathrm{Cl}(f(U))$.

(2). Let V be any (i,j)-pre-$\mathcal{I}$-open set of Y. By Theorem 4.4, τ_j-$\mathrm{Cl}^*(f^{-1}(V)) \subset \tau_j$-$\mathrm{Cl}(f^{-1}(V)) \subset \tau_j$-$\mathrm{Cl}(f^{-}(\tau_j$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(V)))) \subset \tau_j$-$\mathrm{Cl}(\tau_j$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_j$-$\mathrm{Int}(f^{-1}(\tau_j$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^((V)))))))) \subset \tau_j$-$\mathrm{Cl}(\tau_j$-$\mathrm{Int}^*(\tau_j$-$\mathrm{Cl}(f^{-1}(\tau_j$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^((V))))))) \subset f^{-1}(\tau_j$-$\mathrm{Cl}(\tau_j$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^(V)))) \subset f^{-1}(\tau_j$-$\mathrm{Cl}(V))$. □

Theorem 4.6. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ be a pairwise α-$\mathcal{I}$-continuous function. Then for each subset V of Y, $f^{-1}(\sigma_i$-$\mathrm{Int}(V)) \subset \tau_j$-$\mathrm{Cl}^*(f^{-1}(V))$.*

Proof. Let V be any subset of Y. Then σ_i-$\mathrm{Int}(V)$ is σ_i-open in Y and so $f^{-1}(\sigma_i$-$\mathrm{Int}(V))$ is (i,j)-α-$\mathcal{I}$-open in X. Hence $f^{-1}(\sigma_i$-$\mathrm{Int}(V)) \subset \tau_i$-$\mathrm{Int}(\tau_j$-$\mathrm{Cl}^*(\tau_i$-$\mathrm{Int}(f^{-1}(\sigma_i$-$\mathrm{Int}(V))))) \subset \tau_j$-$\mathrm{Cl}^*(f^{-1}(V))$. □

Theorem 4.7. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ be a bijective. Then f is pairwise α-$\mathcal{I}$-continuous if and only if σ_i-$\mathrm{Int}(f(U)) \subset f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(U))$ for each subset U of X.*

Proof. Let U be any subset of X. Then by Theorem 4.4, $f^{-1}(\sigma_i$-$\mathrm{Int}(f(U))) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}(f(U)))$. Since f is bijection, σ_i-$\mathrm{Int}(f(U)) = f(f^{-1}(\sigma_i$-$\mathrm{Int}(f(U))) \subset f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(U))$. Conversely, let V be any subset of Y. Then σ_i-$\mathrm{Int}(f(f^{-1}(V))) \subset f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}(V)))$. Since f is bijection, σ_i-$\mathrm{Int}(V) = \sigma_i$-$\mathrm{Int}(f(f^{-1}(V))) \subset f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}(V)))$; hence $f^{-1}(\sigma_i$-$\mathrm{Int}(V) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}(V))$. Therefore, by Theorem 4.4, f is pairwise α-$\mathcal{I}$-continuous. □

Theorem 4.8. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2)$ be a function. If $g : (X, \tau_1, \tau_2, \mathcal{I}) \to (X \times Y, \sigma_1 \times \sigma_2)$ defined by $g(x) = (x, f(x))$ is a pairwise α-$\mathcal{I}$-continuous function, then f is pairwise α-$\mathcal{I}$-continuous.*

Proof. Let V be a σ_i-open set of Y. Then $f^{-1}(V) = X \cap f^{-1}(V) = g^{-1}(X \times V)$. Since g is a pairwise α-$\mathcal{I}$-continuous function and $X \times V$ is a $\tau_i \times \sigma_i$-open set of $X \times Y$, $f^{-1}(V)$ is a (i,j)-α-$\mathcal{I}$-open set of X. Hence f is pairwise α-$\mathcal{I}$-continuous. □

Definition 4.9. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$ is said to be:*

(i) *pairwise α-$\mathcal{I}$-open (resp. pairwise semi-$\mathcal{I}$-open* [3]*, pairwise pre-$\mathcal{I}$-open* [6]*) if $f(U)$ is a (i,j)-α-$\mathcal{I}$-open (resp. (i,j)-semi-$\mathcal{I}$-open, (i,j)-pre-$\mathcal{I}$-open) set of Y for every τ_i-open set U of X.*

(ii) *pairwise α-$\mathcal{I}$-closed (resp. pairwise semi-$\mathcal{I}$-closed* [3]*, pairwise pre-$\mathcal{I}$-closed* [6]*) if $f(U)$ is a (i,j)-α-$\mathcal{I}$-closed set of Y for every τ_i-closed set U of X.*

Theorem 4.10. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$ is (i,j)-α-$\mathcal{I}$-open if and only if it is (i,j)-semi-$\mathcal{I}$-open and (i,j)-pre-$\mathcal{I}$-open.*

Proof. This is an immediate consequence of Lemma 3.8. □

Theorem 4.11. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$, the following statements are equivalent:*

(i) *f is pairwise α-$\mathcal{I}$-open;*
(ii) *$f(\tau_i\text{-Int}(U)) \subset (i,j)\text{-}\alpha\mathcal{I}\,\text{Int}(f(U))$ for each subset U of X;*
(iii) *$\tau_i\text{-Int}(f^{-1}(V)) \subset f^{-1}((i,j)\text{-}\alpha\mathcal{I}\,\text{Int}(V))$ for each subset V of Y.*

Proof. *(i)* $\Rightarrow$ *(ii)*: Let U be any subset of X. Then τ_i-Int(U) is a τ_i-open set of X. Then $f(\tau_i\text{-Int}(U))$ is a (i,j)-α-$\mathcal{I}$-open set of Y. Since $f(\tau_i\text{-Int}(U)) \subset f(U)$, $f(\tau_i\text{-Int}(U)) = (i,j)\text{-}\alpha\mathcal{I}\,\text{Int}(f(\tau_i\text{-Int}(U))) \subset (i,j)\text{-}\alpha\mathcal{I}\,\text{Int}(f(U))$.
(ii) $\Rightarrow$ *(iii)*: Let V be any subset of Y. Then $f^{-1}(V)$ is a subset of X. Hence $f(\tau_i\text{-Int}(f^{-1}(V))) \subset (i,j)\text{-}\alpha\mathcal{I}\,\text{Int}(f(f^{-1}(V))) \subset (i,j)\text{-}\alpha\mathcal{I}\,\text{Int}(V))$. Then $\tau_i\text{-Int}(f^{-1}(V)) \subset f^{-1}(f(\tau_i\text{-Int}(f^{-1}(V)))) \subset f^{-1}((i,j)\text{-}\alpha\mathcal{I}\,\text{Int}(V))$.
(iii) $\Rightarrow$ *(i)*: Let U be any τ_i-open set of X. Then $\tau_i\text{-Int}(U) = U$ and $f(U)$ is a subset of Y. Now, $V = \tau_i\text{-Int}(V) \subset \tau_i\text{-Int}(f^{-1}(f(V))) \subset f^{-1}((i,j)\text{-}\alpha\mathcal{I}\,\text{Int}(f(V)))$. Then $f(V) \subset f(f^{-1}((i,j)\text{-}\alpha\mathcal{I}\,\text{Int}(f(V)))) \subset (i,j)\text{-}\alpha\mathcal{I}\,\text{Int}(f(V))$ and $(i,j)\text{-}\alpha\mathcal{I}\,\text{Int}(f(V)) \subset f(V)$. Hence $f(V)$ is a (i,j)-α-$\mathcal{I}$-open set of Y; hence f is pairwise α-$\mathcal{I}$-open. □

Theorem 4.12. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$ be a function. Then f is a pairwise α-$\mathcal{I}$-closed function if and only if for each subset V of X, $(i,j)\text{-}\alpha\mathcal{I}\,\text{Cl}(f(V)) \subset f(\tau_i\text{-Cl}(V))$.*

Proof. Let f be a pairwise α-$\mathcal{I}$-closed function and V any subset of X. Then $f(V) \subset f(\tau_i\text{-Cl}(V))$ and $f(\tau_i\text{-Cl}(V))$ is a (i,j)-α-$\mathcal{I}$-closed set of Y. We have $(i,j)\text{-}\alpha\mathcal{I}\,\text{Cl}(f(V)) \subset (i,j)\text{-}\alpha\mathcal{I}\,\text{Cl}(f(\tau_i\text{-Cl}(V))) = f(\tau_i\text{-Cl}(V))$. Conversely, let V be a τ_i-open set of X. Then $f(V) \subset (i,j)\text{-}\alpha\mathcal{I}\,\text{Cl}(f(V)) \subset f(\tau_i\text{-Cl}(V)) = f(V)$; hence $f(V)$ is a (i,j)-α-$\mathcal{I}$-closed subset of Y. Therefore, f is a pairwise α-$\mathcal{I}$-closed function. □

Theorem 4.13. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$ be a function. Then f is a pairwise α-$\mathcal{I}$-closed function if and only if for each subset V of Y, $f^{-1}((i,j)\text{-}\alpha\mathcal{I}\,\text{Cl}(V)) \subset \tau_i\text{-Cl}(f^{-1}(V))$.*

Proof. Let V be any subset of Y. Then by Theorem 4.12, $(i,j)\text{-}\alpha\mathcal{I}\,\text{Cl}(V) \subset f(\tau_i\text{-Cl}(f^{-1}(V)))$. Since f is bijection, $f^{-1}((i,j)\text{-}\alpha\mathcal{I}\,\text{Cl}(V)) = f^{-1}((i,j)\text{-}\alpha\mathcal{I}\,\text{Cl}(f(f^{-1}(V)))) \subset f^{-1}(f(\tau_i\text{-Cl}(f^{-1}(V)))) = \tau_i\text{-Cl}(f^{-1}(V))$. Conversely, let U be any subset of X. Since f is bijection, $(i,j)\text{-}\alpha\mathcal{I}\,\text{Cl}(f(U)) = f(f^{-1}((i,j)\text{-}\alpha\mathcal{I}\,\text{Cl}(f(U))) \subset f(\tau_i\text{-Cl}(f^{-1}(f(U)))) = f(\tau_i\text{-Cl}(U))$. Therefore, by Theorem 4.12, f is a pairwise α-$\mathcal{I}$-closed function. □

Theorem 4.14. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$ be a pairwise α-$\mathcal{I}$-open function. If V is a subset of Y and U is a τ_i-closed subset of X containing $f^{-1}(V)$, then there exists a (i,j)-α-$\mathcal{I}$-closed set F of Y containing V such that $f^{-1}(F) \subset U$.*

Proof. Let V be any subset of Y and U a τ_i-closed subset of X containing $f^{-1}(V)$, and let $F = Y\backslash(f(X\backslash V))$. Then $f(X\backslash V) \subset f(f^{-1}(X\backslash V)) \subset X\backslash V$ and $X\backslash U$ is a τ_i-open set of X. Since f is pairwise α-$\mathcal{I}$-open, $f(X\backslash U)$ is a (i,j)-α-$\mathcal{I}$-open set of Y. Hence F is an (i,j)-α-$\mathcal{I}$-closed set of Y and $f^{-1}(F) = f^{-1}(Y\backslash(f(X\backslash U)) \subset U$. □

Theorem 4.15. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$ be a pairwise α-$\mathcal{I}$-closed function. If V is a subset of Y and U is a open subset of X containing $f^{-1}(V)$, then there exists (i,j)-α-$\mathcal{I}$-open set F of Y containing V such that $f^{-1}(F) \subset U$.*

Proof. The proof is similar to the Theorem 4.14. □

Theorem 4.16. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2, \mathcal{I})$ be a pairwise α-$\mathcal{I}$-open function. Then for each subset V of Y, $f^{-1}(\tau_i\text{-Cl}(\tau_j\text{-Int}^*(\tau_i\text{-Cl}(V))) \subset \tau_i\text{-Cl}(f^{-1}(V)$.*

Proof. Let V be any subset of Y. Then $\tau_i\text{-Cl}(f^{-1}(V)$ is a τ_i-closed set of X. Then by Theorem 4.14, there exists an (i,j)-α-$\mathcal{I}$-closed set F of Y containing V such that $f^{-1}(F) \subset \tau_i\text{-Cl}(f^{-1}(V)$. Since $Y\backslash F$ is (i,j)-α-$\mathcal{I}$-open, $f^{-1}(Y\backslash F) \subset f^{-1}(\tau_j\text{-Int}(\tau_i\text{-Cl}^*(\tau_j\text{-Int}(Y\backslash F))))$ and $X\backslash f^{-1}(F) \subset f^{-1}(Y\backslash(\tau_i\text{-Cl}(\tau_j\text{-Int}^*(\tau_i\text{-Cl}(F))))) = X\backslash f^{-1}(\tau_i\text{-Cl}(\tau_j\text{-Int}^*(\tau_i\text{-Cl}(F))))$. Thus we obtain that $f^{-1}(\tau_i\text{-Cl}(\tau_j\text{-Int}^*(\tau_i\text{-Cl}(V)))) \subset \tau_i\text{-Cl}(\tau_j\text{-Int}^*(\tau_i\text{-Cl}(F))) \subset f^{-1}(F) \subset \tau_i\text{-Cl}(f^{-1}(V))$. Therefore, we have $f^{-1}(\tau_i\text{-Cl}(\tau_j\text{-Int}^*(\tau_i\text{-Cl}(V))) \subset \tau_i\text{-Cl}(f^{-1}(V))$. □

Definition 4.17. *A function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ is said to be:*

(i) *pairwise α-$(\mathcal{I},\mathcal{J})$-open if $f(U)$ is a (i,j)-α-$\mathcal{J}$-open set of Y for every (i,j)-α-$\mathcal{I}$-open set U of X.*

(ii) *pairwise α-$(\mathcal{I},\mathcal{J})$-closed if $f(U)$ is a (i,j)-α-$\mathcal{J}$-closed set of Y for every (i,j)-α-$\mathcal{I}$-closed set U of X.*

It is clear that every pairwise α-$(\mathcal{I},\mathcal{J})$-open (resp. pairwise α-$(\mathcal{I},\mathcal{J})$-closed) function is pairwise α-$\mathcal{J}$-open (resp. pairwise α-$\mathcal{J}$-closed) function. But the converse is not true in general.

Example 4.18. *Let $X = \{a, b, c\}$ $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$, $\tau_2 = \{\emptyset, \{a\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then the identity function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (X, \tau_1, \tau_2, \mathcal{I})$ is pairwise α-$\mathcal{J}$-open but not pairwise α-$(\mathcal{I},\mathcal{I})$-open.*

Theorem 4.19. *For a function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$, the following statements are equivalent:*

(i) *f is pairwise α-$(\mathcal{I},\mathcal{J})$-open;*

(ii) *$f((i,j)\text{-}\alpha\mathcal{I}\,\text{Int}(U)) \subset (i,j)\text{-}\alpha\mathcal{J}\,\text{Int}(f(U))$ for each subset U of X;*

(iii) *$(i,j)\text{-}\alpha\mathcal{I}\,\text{Int}(f^{-1}(V)) \subset f^{-1}((i,j)\text{-}\alpha\mathcal{J}\,\text{Int}(V))$ for each subset V of Y.*

Proof. $(i) \Rightarrow (ii)$: Let U be any subset of X. Then (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(U)$ is a (i,j)-α-$\mathcal{I}$-open set of X. Then $f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(U))$ is a (i,j)-α-$\mathcal{I}$-open set of Y. Since $f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(U)) \subset f(U)$, $f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(U)) = (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(U))) \subset (i,j)$-$s\mathcal{J}\,\mathrm{Int}(f(U))$.
$(ii) \Rightarrow (iii)$: Let V be any subset of Y. Then $f^{-1}(V)$ is a subset of X. Hence $f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}(V))) \subset (i,j)$-$\alpha\mathcal{J}\,\mathrm{Int}(f(f^{-1}(V))) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(V))$. Then (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}(V)) \subset f^{-1}(f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}(V)))) \subset f^{-1}((i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(V))$.
$(iii) \Rightarrow (i)$: Let U be any (i,j)-α-$\mathcal{I}$-open set of X. Then (i,j)-$\alpha\mathcal{I}\,\mathrm{Int}(U) = U$ and $f(U)$ is a subset of Y. Now, $U = (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(U) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}(f(U))) \subset f^{-1}((i,j)$-$\alpha\mathcal{J}\,\mathrm{Int}(f(U)))$. Then $f(U) \subset f(f^{-1}((i,j)$-$\alpha\mathcal{J}\,\mathrm{Int}(f(U))) \subset (i,j)$-$\alpha\mathcal{J}\,\mathrm{Int}(f(U))$ and (i,j)-$\alpha\mathcal{J}\,\mathrm{Int}(f(U)) \subset f(U)$. Hence $f(U)$ is a (i,j)-α-$\mathcal{J}$-closed set of Y; hence f is pairwise α-$(\mathcal{I},\mathcal{J})$-open. □

Theorem 4.20. *Let $f : (X,\tau_1,\tau_2,\mathcal{I}) \to (Y,\sigma_1,\sigma_2,\mathcal{J})$ be a function. Then f is a pairwise α-$(\mathcal{I},\mathcal{J})$-closed function if and only if for each subset U of X, (i,j)-$\alpha\mathcal{J}\,\mathrm{Cl}(f(U)) \subset f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(U))$.*

Proof. Let f be a pairwise α-$(\mathcal{I},\mathcal{J})$-closed function and U any subset of X. Then $f(U) \subset f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(U))$ and $f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(U))$ is a (i,j)-α-$\mathcal{J}$-closed set of Y. We have (i,j)-$\alpha\mathcal{J}\,\mathrm{Cl}(f(U)) \subset (i,j)$-$\alpha\mathcal{J}\,\mathrm{Cl}(f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(U))) = f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(U))$. Conversely, let U be a (i,j)-α-$\mathcal{I}$-open set of X. Then $f(U) \subset (i,j)$-$\alpha\mathcal{J}\,\mathrm{Cl}(f(U)) \subset f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(U)) = f(U)$; hence $f(U)$ α-$\mathcal{J}$-closed subset of Y. Therefore, f is a pairwise α-$(\mathcal{I},\mathcal{J})$-closed function. □

Theorem 4.21. *Let $f : (X,\tau_1,\tau_2,\mathcal{I}) \to (Y,\sigma_1,\sigma_2,\mathcal{J})$ be a function. Then f is a pairwise α-$(\mathcal{I},\mathcal{J})$-closed function if and only if for each subset V of Y, $f^{-1}((i,j)$-$\alpha\mathcal{J}\,\mathrm{Cl}(V)) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(V))$.*

Proof. Let V be any subset of Y. Then by Theorem 4.20, (i,j)-$\alpha\mathcal{J}\,\mathrm{Cl}(f(f^{-1}(V))) \subset f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(V)))$. Since f is bijection, $f^{-1}((i,j)$-$\alpha\mathcal{J}\,\mathrm{Cl}(V)) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(V))$. Conversely, let U be any subset of X. Then $f^{-1}((i,j)$-$\alpha\mathcal{J}\,\mathrm{Cl}(f(U))) \subset (i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(f(U)))$. Hence (i,j)-$\alpha\mathcal{J}\,\mathrm{Cl}(f(U)) \subset f((i,j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(f(U))))$. Therefore, by Theorem 4.20 f is a pairwise α-$(\mathcal{I},\mathcal{J})$-closed function. □

Theorem 4.22. *Let $f : (X,\tau_1,\tau_2,\mathcal{I}) \to (Y,\sigma_1,\sigma_2,\mathcal{J})$ be a pairwise α-$(\mathcal{I},\mathcal{J})$-open function. If V is a subset of Y and U is a (i,j)-α-$\mathcal{I}$-closed subset of X containing $f^{-1}(V)$, then there exists (i,j)-α-$\mathcal{I}$-closed set F of Y containing V such that $f^{-1}(F) \subset V$.*

Proof. The proof is similar to the Theorem 4.14. □

Theorem 4.23. *Let $f : (X,\tau_1,\tau_2,\mathcal{I}) \to (Y,\sigma_1,\sigma_2,\mathcal{J})$ be a pairwise α-$(\mathcal{I},\mathcal{J})$-closed function. If V is a subset of Y and U is a (i,j)-α-$\mathcal{I}$-open subset of X containing $f^{-1}(V)$, then there exists (i,j)-α-$\mathcal{J}$-open set F of Y containing V such that $f^{-1}(F) \subset V$.*

Proof. The proof is similar to the Theorem 4.14. □

Theorem 4.24. *For a bijective function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$, the following statements are equivalent:*

(i) *f is pairwise α-$(\mathcal{I}, \mathcal{J})$-closed;*
(ii) *f is pairwise α-$(\mathcal{I}, \mathcal{J})$-open.*

Proof. The proof is clear. □

5. PAIRWISE α-$\mathcal{I}$-IRRESOLUTE FUNCTIONS

Definition 5.1. *A function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ is said to be (i, j)-α-$\mathcal{I}$-irresolute if the inverse image of every (i, j)-α-$\mathcal{J}$-open set of Y is (i, j)-α-$\mathcal{I}$-open in X, where $i \neq j$, i, j=1, 2.*

Proposition 5.2. *Every pairwise α-$\mathcal{I}$-irresolute function is pairwise α-$\mathcal{I}$-continuous but not conversely.*

Proof. Straigtforward. □

Theorem 5.3. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ be a function, then*

(1) *f is pairwise α-$\mathcal{I}$-irresolute;*
(2) *the inverse image of each (i, j)-α-$\mathcal{J}$-closed subset of Y is (i, j)-α-$\mathcal{I}$-closed in X;*
(3) *for each $x \in X$ and each $V \in S\mathcal{J}O(Y)$ containing $f(x)$, there exists $U \in \alpha\mathcal{I}O(X)$ containing x such that $f(U) \subset V$.*

Proof. The proof is obvious from that fact that the arbitrary union of (i, j)-α-$\mathcal{I}$-open subsets is (i, j)-α-$\mathcal{I}$-open. □

Theorem 5.4. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ be a function, then*

(i) *f is pairwise α-$\mathcal{I}$-irresolute;*
(ii) *(i, j)-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(V)) \subset f^{-1}((i, j)$-$\alpha\mathcal{J}\,\mathrm{Cl}(V))$ for each subset V of Y;*
(iii) *$f((i, j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(U) \subset (i, j)$-$\alpha\mathcal{J}\,\mathrm{Cl}(f(U))$ for each subset U of X.*

Proof. *(i)* $\Rightarrow$ *(ii)*: Let V be any subset of Y. Then $V \subset (i, j)$-$\alpha\mathcal{J}\,\mathrm{Cl}(V)$ and $f^{-1}(V) \subset f^{-1}((i, j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(V))$. Since f is pairwisr α-$\mathcal{I}$-irresolute, $f^{-1}((i, j)$-$\alpha\mathcal{J}\,\mathrm{Cl}(V))$ is a (i, j)-α-$\mathcal{I}$-closed subset of X. Hence (i, j)-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(V)) \subset (i, j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}((i, j)$-$\alpha\mathcal{J}\,\mathrm{Cl}(V))) = f^{-1}((i, j)$-$\alpha\mathcal{J}\,\mathrm{Cl}(V))$.
(ii) $\Rightarrow$ *(iii)*: Let U be any subset of X. Then $f(U) \subset (i, j)$-$\alpha\mathcal{J}\,\mathrm{Cl}(f(U))$ and (i, j)-$\alpha\mathcal{I}\,\mathrm{Cl}(U) \subset (i, j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(f(U))) \subset f^{-1}((i, j)$-$\alpha\mathcal{J}\,\mathrm{Cl}(f(U)))$. This implies that $f((i, j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(U)) \subset f(f^{-1}((i, j)$-$\alpha\mathcal{J}\,\mathrm{Cl}(f(U)))) \subset (i, j)$-$\alpha\mathcal{J}\,\mathrm{Cl}(f(U))$.
(iii) $\Rightarrow$ *(i)*: Let V be a (i, j)-α-$\mathcal{J}$-closed subset of Y. Then $f((i, j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(V)) \subset (i, j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(f(V))) \subset (i, j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(V) = V$. This implies that (i, j)-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(V)) \subset f^{-1}(f((i, j)$-$\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(V)))) \subset$

$f^{-1}(V)$. Therefore, $f^{-1}(V)$ is a (i,j)-α-$\mathcal{I}$-closed subset of X and consequently f is a pairwise α-$\mathcal{I}$-irresolute function. □

Theorem 5.5. *A function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ is a pairwise α-$\mathcal{I}$-irresolute if and only if $f^{-1}((i,j)\text{-}\alpha\mathcal{J}\operatorname{Int}(V)) \subset (i,j)\text{-}\alpha\mathcal{I}\operatorname{Int}(f^{-1}(V))$ for each subset V of Y.*

Proof. Let V be any subset of Y. Then (i,j)-$\alpha\mathcal{J}\operatorname{Int}(V) \subset V$. Since f is pairwise α-$\mathcal{I}$-irresolute, $f^{-1}((i,j)\text{-}\alpha\mathcal{J}\operatorname{Int}(V))$ is a (i,j)-α-$\mathcal{I}$-open subset of X. Hence $f^{-1}((i,j)\text{-}\alpha\mathcal{J}\operatorname{Int}(V)) = (i,j)\text{-}\alpha\mathcal{I}\operatorname{Int}(f^{-1}((i,j)\text{-}\alpha\mathcal{J}\operatorname{Int}(V))) \subset (i,j)\text{-}\alpha\mathcal{I}\operatorname{Int}(f^{-1}(V))$. Conversely, let V be a (i,j)-α-$\mathcal{J}$-open subset of Y. Then $f^{-1}(V) = f^{-1}((i,j)\text{-}\alpha\mathcal{J}\operatorname{Int}(V)) \subset (i,j)\text{-}\alpha\mathcal{I}\operatorname{Int}(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is a (i,j)-α-$\mathcal{I}$-open subset of X and consequently f is a pairwisr α-$\mathcal{I}$-irresolute function. □

Corollary 5.6. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ be a function. Then f is pairwise α-$\mathcal{I}$-closed and pairwise α-$\mathcal{I}$-irresolute if and only if $f((i,j)\text{-}\alpha\mathcal{I}\operatorname{Cl}(V)) = (i,j)\text{-}\alpha\mathcal{J}\operatorname{Cl}(f(V)))$ for evey subset V of X.*

Definition 5.7. *An ideal bitopological space $(X, \tau_1, \tau_2, \mathcal{I})$ is called pairwise α-$\mathcal{I}$-Hausdorff if for each two distinct points $x \neq y$, there exist disjoint (i,j)-α-$\mathcal{I}$-open sets U and V containing x and y, respectively.*

Theorem 5.8. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ be a pairwise α-$\mathcal{I}$-irresolute function. If Y is pairwise α-$\mathcal{J}$-Hausdorff, then X is pairwise α-$\mathcal{I}$-Hausdorff.*

Proof. The proof is clear. □

Corollary 5.9. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ be a function. Then f is pairwise α-$\mathcal{I}$-open and pairwise α-$\mathcal{I}$-irresolute if and only if $f^{-1}(((i,j)\text{-}\alpha\mathcal{J}\operatorname{Cl}(V))) = (i,j)\text{-}\alpha\mathcal{I}\operatorname{Cl}(f^{-1}((V)))$ for evey subset V of Y.*

Definition 5.10. *A function $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ is said to be pairwise α-$\mathcal{I}$-homeomorphism if f and f^{-1} are pairwise α-$\mathcal{I}$-irresolute.*

Theorem 5.11. *Let $f : (X, \tau_1, \tau_2, \mathcal{I}) \to (Y, \sigma_1, \sigma_2, \mathcal{J})$ be a bijection. Then the following statements are equivalent:*

(i) *f is pairwise α-$\mathcal{I}$-homeomorphism;*
(ii) *f^{-1} is pairwise α-$\mathcal{I}$-homeomorphism;*
(iii) *f and f^{-1} are pairwise α-$(\mathcal{I},\mathcal{J})$-open (pairwise α-$(\mathcal{J},\mathcal{I})$-closed);*
(1) *f is pairwise α-$\mathcal{I}$-irresolute and pairwise α-$(\mathcal{I},\mathcal{J})$-open (pairwise α-$(\mathcal{J},\mathcal{I})$-closed);*
(2) *$f((i,j)\text{-}\alpha\mathcal{I}\operatorname{Cl}(V)) = (i,j)\text{-}\alpha\mathcal{J}\operatorname{Cl}(f(V))$ for each subset V of X;*
(3) *$f((i,j)\text{-}\alpha\mathcal{I}\operatorname{Int}(V)) = (i,j)\text{-}\alpha\mathcal{J}\operatorname{Int}(f(V))$ for each subset V of X;*

(4) $f^{-1}((i,j)\text{-}\alpha\mathcal{J}\,\mathrm{Int}(V)) = (i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}(V))$ *for each subset* V *of* Y*;*

(5) $(i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(V)) = f^{-1}((i,j)\text{-}\alpha\mathcal{J}\,\mathrm{Cl}(V))$ *for each subset* V *of* Y*;*

Proof. (1) $\Rightarrow$ (2): It follows immediately from the definition of a pairwise α-$\mathcal{I}$-homeomorphism.
(2) $\Rightarrow$ (3) $\Rightarrow$ (4): It follows from Theorem 4.24.
(4) $\Rightarrow$ (5): It follows from Theorem 4.21 and Corollary 5.6.
(5) $\Rightarrow$ (6): Let U be a subset of X. Then by Theorem 3.33, $f((i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Int}(U)) = X\backslash f((i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Cl}(X\backslash U)) = X\backslash (i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Cl}(f(X\backslash U)) = (i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Int}(f(U))$.
(6) $\Rightarrow$ (7): Let V be a subset of Y. Then $f((i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}(V))) = (i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Int}(f(f^{-1}(V))) = (i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Int}(f(V))$. Hence $f^{-1}(f((i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}(V)))) = f^{-1}((i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Int}(V))$. Therefore, $f^{-1}((i,j)\text{-}\alpha\mathcal{J}\,\mathrm{Int}(V)) = (i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}(V))$.
(7) $\Rightarrow$ (8): Let V be a subset of Y. Then by Theorem 3.33, $(i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Cl}(f^{-1}(V)) = X\backslash(f^{-1}((i,j)\text{-}\alpha\mathcal{J}\,\mathrm{Int}(Y\backslash V))) = X\backslash((i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Int}(f^{-1}((X\backslash V)))) = f^{-1}((i,j)\text{-}\alpha\mathcal{I}\,\mathrm{Cl}(V))$.
(8) $\Rightarrow$ (1): It follows from Theorem 4.21 and Corollary 5.9. □

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