The 2 Goldbach's Conjecture by Proof

Every even integer> 2 is the sum of two primes

and the Equivalent

Every odd integer> 5 is the sum of three primes

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An introduction to the conjecture of Goldbach's

The 1741 Goldbach [1] made his most famous contribution to mathematics with the conjecture that all even numbers can be expressed as the sum of the two primes (currently Conjecture referred to as "all even numbers greater than 2 can be expressed as the sum- two primes) .Yet, no proof of Goldbach's Conjecture has been found. This assumption seems to be right for a large amount of numbers using numerical calculations. Some examples are 10 = 3 + 7, 18 = 7 + 11, 100 + 97 = 3, and so on. But there are a multitude sums primes meeting an even number which I would say is irregular, but increases with the size of the even number.To same happens and with an odd number as example 25, ie. 25 = 3 + 3 + 19 = 3 + 5 + 17 = = 3 + 11 + 11 = 5 + 7 + 13 = 7 + 7 + 11. We say only 5 cases and only those that meet the constant sum of 25 .First turns in Theorem 3 that there may be at least a pair of primes, such that their sum is equal to every even number. But at the same time reveals the method of finding all pairs that satisfy this condition. According to Theorem 4, the second guess switches to form first guess and this is primarily elementary, to prove the truth of 1.The alternative and the side stream assistant, to prove Goldbach's Conjecture in this research was the Mathematica program which is the main tool for data collection, but also for finding all the steps in order to demonstrate each part of the proof. We must mention that Vinogradov proved to1937, that for every sufficiently large number can be expressed, that is the sum of the three primes. And finally the Chinese mathematician Chen Jing Run proved big first for a constant number that is the sum of the first three in 1966 [2]. Finally, the investigation of Goldbach's Conjecture has acted as a catalyst for the creation and development of many methods that are useful, with many theorems that help and other areas of mathematics.

~ Basic theory ~

Definition. A natural number p > 1 is called prime number if and only if the only divisors of is (+/-) 1 and (+/–) p. A natural number n > 1 which is not prime, will be called composite. Therefore a positive integer p > 1 is Prime if and only if for each $(p,n)=1$,or $(p,n)=p$ i.e the related prime to the n or p divides n. The 2 is first but outside of 2 is odd numbers. The 25 first primes numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 61, 67, 71, 73, 79, 83, 89, 97.We note that the number 1 is not considered neither prime nor composite.

Lemma 1.
Every prime number greater than 1, it is odd and is written in the form $2k+1$ and therefore the sum of 2 primes is an even number.

Proof .. Let p, prime in N. Suppose p_k , $p_{k+\lambda}$ by $\kappa,\lambda \in \mathrm{N}$, $p_{k+\lambda} > p_k$ then we will have $p_k + p_{k+\lambda} = 2k+1+2(k+\lambda)+1 = 2(2\kappa+\lambda+1)$ to be even as easily seen.

Lemma 2. The sum of 3 odd numbers it is an odd number.

Proof.. If p, prime in N ,natural numbers. Assume, p_k , $p_{k+\lambda}$, $p_{k+\rho}$ $\kappa,\lambda,\rho \in \mathrm{N}$, $p_{k+\rho} > p_{k+\lambda} > p_k$ then we'll have $p_k + p_{k+\lambda} + p_{k+\rho} = 2k+1+2k+2\lambda+1+2\kappa+2\rho+1 =$

$= 2(3\kappa+\lambda+\rho+1)+1$ that's odd as easy it seems.

Theorem.1 If we have a integer $n=2h, h \in N$ even number, asking to find the upper and lower limit of the sum of two odd, which equal with n.

Proof..

From **Lemma 1** we have:

i). $n = 2(2h-\lambda+1) \Rightarrow 2h-\lambda = n/2-1 \Rightarrow 2h > n/2-1 \Rightarrow h > \dfrac{n-2}{4}, \lambda > 0, h \in N$.

Ii).The largest odd should be less than $n-3$. Suppose therefore that $2h+1 \le n-3 \Rightarrow h \le \dfrac{n-4}{2}$. So the limits of the number of all cases, it will be

$$\frac{n-2}{4} < h \le \frac{n-4}{2}.$$

The Existence Theorems 2 or 3 primes, constant sum.

Theorem 2. For $n > 2, n = 2h, h, w \in N$ and $n = p_1 + p_2$,which p_1, p_2 are prime numbers, and there is a positive integer j with $j = 2w$ if $n/2$ is **odd** and $j = 2w+1$ if $n/2$ **is even**, then $n/2 - j$ and $n/2 + j$ **is odd** and possibly also prime numbers.[8].

Proof. ..

i) We assume that we have 2 odd and possibly primes $p_1 = n/2 - \varepsilon$ and $p_2 = n/2 + \mu$ with ε,μ to be positive numbers. Because $n = p_1 + p_2 = n/2 + \mu + n/2 - \varepsilon \Rightarrow \mu = \varepsilon$.
ii) Previously by (i), we assume that there is an integer, $j > 0$ so $j = \mu = \varepsilon$, and then implies that $n = p_1 + p_2$ with $p_1 = n/2 - j$ and $p_1 = n/2 + j$. We can now transform the relationship $n = p_1 + p_2$ in accordance with previously to $n/2 - (p_2 - n/2) = n - p_2 = p_1$ and also $n/2 + (p_2 - n/2) = p_2$. From the mathematical logic is clear that if $j = 2w$ with $n/2$ odd and $j = 2w+1$ if $n/2$ is even, the p_1, p_2 results always odd numbers. Of course if may be primes, as you know it will always be odd numbers. Therefore there is always a positive j integer such that $n/2 - j$ and $n/2 + j$ is odd numbers and possibly primes numbers and apply that $n = (n/2 - j) + (n/2 + j)$.

We define as π (x) is the function that determines the number of primes that are less than the equal by x. Specifically applicable for the sieve of Eratosthenes that $k = \pi[\sqrt{2x}]$ **integer part of** $\sqrt{2x}$. We prove then that there may be at least a couple of primes such that there the sum is equal to n.

The First Conjecture of Goldbach's.

Theorem 3. Given an even, $n > 2, n = 2h, h, w \in N$, $k = \pi(\sqrt{n}), k \geq 2$ and p_k is prime corresponding to an index k.Then there are positive integers j so in order to apply in each case conditions:

a. If $n/2$ is **odd.** then $j = 2w$ and

b. If $n/2$ is **even**, then $j = 2w+1$

so that there is at least a pair, and for which each of the pair $n/2 - j$ και $n/2 + j$ to be prime and not divisible by any of the primes 2, 3, 5, 7,..,. p_k.

Proof..

We start with the $n > 2, n = 2h, h >= 0, h, w \in N$ from Theorem.1, and Theorem.2 occurs that there then $n/2 - j$ and $n/2 + j$ is odd and possibly also prime numbers with $j = 2w$. In this case the maximum number of values $w \in N$ will occur from inequality... $n/2 + 2w \leq n - 3 \Rightarrow w \leq \frac{1}{2} \cdot (n/2 - 3) \wedge 2w \leq n/2 - 3$. It is obvious as total solutions that represent all cases for an n/2 = odd is $S = \{0,2,4,6,,,n/2-3\}$ while when n/2 = even is $S = \{1,3,5,7,,,n/2-3\}$. The set that we get and the inequality mod 2, i.e. Accordance with the sieve of Eratosthenes is $n/2 \pm j \neq 0(\mathrm{mod}2)$ that so $k = \pi[\sqrt{2n/2}] = \pi(\sqrt{n})$. Must will first need from the set S, remove or reject all cases of individual systems modulo ...

$$s_1 \left\langle \begin{array}{l} n/2 \pm j = 0(\mathrm{mod}\ p_2) \\ n/2 \pm j \neq 0(\mathrm{mod}\ 2) \end{array} \right.$$

$$s_2 \left\langle \begin{array}{l} n/2 \pm j = 0(\mathrm{mod}\,p_3) \\ n/2 \pm j \neq 0(\mathrm{mod}2) \end{array} \right.$$

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$$s_f \left\langle \begin{array}{l} n/2 \pm j = 0(\mathrm{mod}\,p_k) \\ n/2 \pm j \neq 0(\mathrm{mod}2) \end{array} \right.$$

The comprehensive solution of all systems, is from the Union of the individual solutions of each system $s_1, s_2, s_3,,, s_f$, $1 \leq f \leq k-1$ and is the unique solution of systems, with the respective sets S_f , i.e $R = \bigcup_{i=1}^{f} S_i$. The clear solution such as we seek will be the difference of sets S and R , i.e $\Omega = S - R$. So the solution Ω that we are interested coincides with the solution of the system unequally mod i.e….

$$\langle MJ \rangle \quad \begin{array}{l} n/2 \pm j \neq 0(\mathrm{mod}\,p_1) \\ n/2 \pm j \neq 0(\mathrm{mod}\,p_2) \\ \dots\dots\dots \\ \dots\dots\dots \\ \dots\dots\dots \\ n/2 \pm j \neq 0(\mathrm{mod}\,p_v) \\ \dots\dots\dots \\ \dots\dots\dots \\ n/2 \pm j \neq 0(\mathrm{mod}\,p_k) \end{array}$$

for which each of the pair $n/2 - j$ and $n/2 + j$ is primes and not divisible by any of the primes 2,3,5,7, …, p_v ,..., p_k ,with $1 \leq v \leq k$. By this logic therefore determine that the possibility that $n/2 - j$ and $n/2 + j$ be prime numbers, it is now safe to say there may be at least a pair (p_1, p_2) of primes, such that their sum equals n , because the set Ω **is never empty** set because S **is never empty set** and $R \neq S$ with $R \subset S$.
If $N(\Omega\} = \{x_\iota : x_\iota \in \Omega\}$ with {i = 1,2,3, …, λ} where λ the cardinality of the set Ω, then the $N(\Omega)$ define the elements of the set Ω .The pairs of primes arising from the set Ω, given by the relations ...

$${}_1p_i = n/2 - x_i$$
$${}_1p'_i = n/2 + x_i \quad \text{με } {}_1p_i + {}_1p'_i = n$$

Further to previous we symbolized to $P(\Omega) = \{({}_1p_1, {}_1p'_1), ({}_1p_2, {}_1p'_2), \ldots ({}_1p_\lambda, {}_1p'_\lambda)\}$ the set of pairs that correspond to the set Ω ,which is derived from the system unequally mod (MJ) and constitutes the first part of the set of solutions of couples $({}_1p_i, {}_1p'_i)$ with {i = 1, 2,..., λ} so that the sum to equals n and cardinality λ. Finally we define as T = {2,5,7, ..., p_k } to set the of primes so that the couples $n/2-j$ and $n/2+j$ who are primes not divisible by any of the primes 2,3,5, ..., p_k , as defined multitude k .More specifically define the set of primes $P(T) = \{({}_2p_v, {}_2p'_v): {}_2p_v \in T$ and ${}_2p_v/({}_2p_v \cdot m + \upsilon_v)$, m $\in Z$: $\upsilon_v \in \{n/2 \pm j = 0(\text{mod } p_v) \wedge n/2 \pm j \neq 0(\text{mod}2)\}\}$ with $1 < v \leq k, k \geq 2$,where υ_v the residues arising from the system $\{n/2 \pm j = 0(\text{mod } p_v) \wedge n/2 \pm j \neq 0(\text{mod } 2)\}$ with cardinality σ .Finally the complete set of pairs of the primes to sum n will be the set $L_2 = P(\Omega) \cup P(T)$ as defined above, with cardinality $\sigma + \lambda$.

The Second Conjecture of Goldbach's

Theorem 4. For $n > 2, n = 2h+1, h \in N$ and $n = p_1 + p_2 + p_3$,with p_1, p_2, p_3 prime numbers. Namely every odd number is obtained as the sum of 3 prime numbers.

Proof .. According to the relationship $n = p_1 + p_2 + p_3 \Rightarrow p_1 + p_2 = n - p_3$ .If call $n = {}_1p_i + {}_2p_i + {}_3p_i$ and if we have many different values ${}_3p_i$ and other primes, then we can call ${}_2S_i = {}_1p_i + {}_2p_i = n - {}_3p_i$ the sums of 2 primes ${}_2S_i$. According to **Theorem 3,** we know the cardinality of cases are ${}_2S_i$,with $\sigma_i + \lambda_i$, in each case $n - {}_3p_i$ of different values ${}_3p_i$ according to each preset selection .The method of finding all the cases is precisely from selected values that are ${}_3p_i$ which is the first of the primes less of n, to the greatest prime of the two is closer to the value $n/3$ .The fact leads the order of the Triad p_1, p_2, p_3 in relationship to $n/3$.

I) If $p_1 < n/3 < p_2 < p_3$ In this case, if the value of the lower limit of the selected values ${}_3p_i$ will be the value p_3 of that is obviously the second greater price series during the first of prime numbers after i.e $Round(n/3)$. In the mathematica language will define this number as $NextPrime[NextPrime[Round[n/3]]]$.
ii) If $p_1 < p_2 < n/3 < p_3 \vee p_1 \leq p_2 < n/3 < p_3 \vee p_1 < p_2 < n/3 \leq p_3$. In this case the value of the lower limit of the selected values ${}_3p_i$ will be the value p_3 of which is obviously a next-largest number first value after $Round(n/3)$ .In language of mathematica will define this figure as $NextPrime[Round[n/3]]$.
iii) If $p_1 = p_2 = n/3 = p_3$. The value of the lower limit of the selected values ${}_3p_i$ will be the value of p_3 which is obviously the $n/3 = Round(n/3)$.
And for the three cases in relation to each device, we symbolized as the prime minimum $\delta_{\min}$. Finally, the biggest ${}_3p_i$ being the first of the first below, will be denoted by the

language of mathematica as $NextPrime[n,-\mu] \le n-6, \mu \ge 1$ and will be the largest prime number before n , less than or equal to n-6, which symbolizes $\delta_{\max}$.If you now call the cardinality of the primes, $\pi([\delta_{\min}, \delta_{\max}]) = \varepsilon$ we will know how many sums of 2 will have, i.e {i = 1,2,3, ..., ε}.Therefore the set of summation of total solutions of 2 primes will be... $L_3 = \{_2S_1, _2S_2, ..., _2S_\varepsilon\}$. If we now call $\kappa_i = \sigma_i + \lambda_i$ with {i = 1,2,3, ..., ε} by **Theorem 3**, the final number of the individual triads κ_t ie. the final number $\kappa_t = \kappa_1 + \kappa_2 + ... + \kappa_\varepsilon$ will be the total number of pairs of primes, for sums of 2 , $_2S_i = _1p_i + _2p_i = n - _3p_i$, with cardinality δ and hence the total number of triads equal κ_t ,using ε multitude sums with two primes, making ε-multiple use of the method Theorem 3. .The permissible cases $_2S_i, i = 1,2,..., \varepsilon$ therefore the will is the total .. $L_3 = \{_2S_1, _2S_2, ..., _2S_\varepsilon\}$. Namely every odd number is obtained as the sum of 3 of primes numbers after respectively switched to multiple totals 2 primes, δ multitude,the existence of which fully ensures the Theorem 3.

a) For example sum 2 primes, let's say when m = n / 2=159 is odd and $\sqrt{n} = 17.83$ therefore i.e get to mod, the first 2,3,5,7,11,13,17 .The corresponding program with mathematica and according to the theory would be:

<u># Program 1 #...</u>

```
n/2 := 159;m:=n/2;
Cases[Reduce[ Mod[m + j, 17] ≠0 && Mod[m - j, 17] ≠ 0 &&
Mod[m + j, 13] ≠ 0 && Mod[m - j, 13] ≠ 0 &&
Mod[m + j, 11] ≠ 0 && Mod[m - j, 11] ≠ 0 &&
Mod[m + j, 7] ≠0 && Mod[m - j, 7] ≠ 0 &&
Mod[m + j, 5] ≠ 0 && Mod[m - j, 5] ≠ 0 &&
Mod[m + j, 3] ≠ 0 && Mod[m - j, 3] ≠ 0 &&
Mod[m + j, 2] ≠ 0 && Mod[m - j, 2] ≠ 0 &&
0 ≤ j ≤ (m - 3), {j}, Integers], Except[j]]
```

Results: j=8,j=20,j=22,j=32,j=52,j=70,j=80,j=92,j=98,j=112,j=118,j=122} are the values for j. The $N(\Omega) = \{x_t : x_t \in \Omega\} = \{8,20,22,32,52,70,80,92,98,112,118,122\}$ with λ=12. Therefore the set of pairs will be $P(\Omega) = \{(_1p_1, _1p'_1), (_1p_2, _1p'_2), ..., (_1p_\lambda, _1p'_\lambda)\} =$ {(37,281),(41,277),(47,271),(61,257), (67,251),(79,239),(89,229),(107,211),(127,191),(137,181),(139,179),(151,167)}. with Thereafter the T = {2,5,7, ..., 17} with k = 7 and shuts off after looking at how many primes values we receive from all T. The corresponding program will be mathematica according to relevant theory

<u># Program 2 #..</u>

```
n:= 2·159 ; m:=n;c:=IntegerPart[√m];Cases[Table[Reduce[ Mod[m-p',k]==0
&& Mod[m+p',2] ≠0 && Mod[m-p',2] ≠0 && p=<p' && p=<c && 0<=p'<=(m-
3) && p==m-p',{p,p'},Primes],{k,1,c}],Except[False]].
```

Final result: {(5,313) (7,311) (11, 307)} with cardinality σ = 3. So all of the pairs of primes will be λ + σ = 12 + 3 = 15, according to the theorem 3 & 4 mentioned above. Something similar happens for example if n = 2 * 158 i.e when the n / 2 = m =even then we have $N(\Omega)=\{x_t\ :x_t\in\Omega\}=\{9,21,69,75,99,105,111,135\}$ and λ= 8 using the same programs 1,2 in language mathematica. Therefore the set of pairs is {(23,293), (47,269), (53,263), (59,257), (83,233), (89,227), (137,179), (149,167)}, then T = {2,5 , 7, ..., 17} with k = 7 the primes value, for which accept to be with cardinality σ = 2, which is {(5,311), (3,313)}. So all of the pairs of primes will be λ + σ = 8 + 2 = 10.

b)For example of sum 3 primes, consider the case when n = 111. According to Theorem 4, and iii case, that the $\delta_{\min}=n/3=37$. with the language mathematica as $NextPrime[n,-\mu]\le n-6,\mu\ge 1$ with n = 111 and μ = 3, and therefore the largest prime number before $n=111$, less than or equal to n-6, will be $\delta_{\max}=103$. So the respectively $\pi([\delta_{\min},\delta_{\max}])=16$, that 16 is the number of summations, which will look to find all triads first verifying the relationship ${}_2S_i={}_1p_i+{}_2p_i=n-{}_3p_i, i=1,2,...,16$ .The permissible cases ${}_2S_i, i=1,2,...,16$ and ε=16, therefore the will is the total ...

$$L_3=\{{}_2S_1,{}_2S_2,...,{}_2S_\varepsilon\}=\{111-103,111-101,111-97,111-89,111-83,111-79,111-73,111-71,111-67,$$
$$111-61,111-59,111-53,111-47,111-43,111-41,111-37\}=\{8,10,14,22,28,32,40,44,50,52,58,64,68,70,74\}.$$

by Theorem 4, the final number of the individual triads, the multitude κ_t which would occur with multiple use of Theromem.3, without repetitions is ...

$$\kappa_t=\kappa_1+\kappa_2+...+\kappa_\varepsilon=1+2+2+3+3+2+2+3+3+4+3+4+2+1+1+1=36.,$$

thus the relevant full program in mathematica will be:

<u># Program 3 #..</u>

```
n:=111;
Reduce[p1+p2+p3==n && p1>2 && p2>2 && p3>2 && p1<=p2 &&
p2<=p3,{p1,p2,p3},Primes]
Count[Reduce[p1+p2+p3==n && p1>2 && p2>2 && p3>2 && p1<=p2 &&
p2<=p3,{p1,p2,p3},Primes],Except[False]]
```

Total 36 Triads will therefore be in ascending order:

$(p_1=3,p_2=5,p_3=103),(p_1=3,p_2=7,p_3=101),(p_1=3,p_2=11,p_3=97),(p_1=3,p_2=19,p_3=89),$
$(p_1=3,p_2=29,p_3=79),(p_1=3,p_2=37,p_3=71),(p_1=3,p_2=41,p_3=67),(p_1=3,p_2=47,p_3=61),$
$(p_1=5,p_2=5,p_3=101),(p_1=5,p_2=17,p_3=89),(p_1=5,p_2=23,p_3=83),(p_1=5,p_2=47,p_3=59),$
$(p_1=5,p_2=53,p_3=53),(p_1=7,p_2=7,p_3=97),(p_1=7,p_2=31,p_3=73),(p_1=7,p_2=37,p_3=67),$
$(p_1=7,p_2=43,p_3=61),(p_1=11,p_2=11,p_3=89),(p_1=11,p_2=17,p_3=83),(p_1=11,p_2=29,p_3=67)$
$(p_1=11,p_2=41,p_3=59),(p_1=11,p_2=47,p_3=53),(p_1=13,p_2=19,p_3=79),(p_1=13,p_2=31,p_3=67),$
$(p_1=13,p_2=37,p_3=61),(p_1=17,p_2=23,p_3=71),(p_1=17,p_2=41,p_3=53),(p_1=17,p_2=47,p_3=47),$
$(p_1=19,p_2=19,p_3=73),(p_1=19,p_2=31,p_3=61),(p_1=23,p_2=29,p_3=59),(p_1=23,p_2=41,p_3=47),$
$(p_1=29,p_2=29,p_3=53),(p_1=29,p_2=41,p_3=41),(p_1=31,p_2=37,p_3=43),(p_1=37,p_2=37,p_3=37).$

with Count=36.

Conclusions ..

The purpose of this project was to solve the Goldbach Conjecture using the same statement: For every integer n ≥ 2, there is an integer j such that n / 2 + j and n / 2 - j are prime numbers. The two approaches were made using this statement, which resulted in the development and resolution of the two assumptions. Hopefully these speculations reveal new approaches to solve the problems and make solving this problem of old, a much easier task Since the Goldbach put the two speculations, have passed 259 years. Illustrating this "unfathomable enigma", many mathematicians and amateurs from many countries have expended great energy. But until now, no one could find a suitable method to perform this research, since the opportunity to demonstrate this problem depends only on the available theories. Obviously, the available theories are not very effective, because they could not fully take to prove this very complex problem of existence and distribution and therefore can not be used to prove the conjecture of Goldbach [3 4,5,6,7]. Therefore, we must actively open a new path, find a new way and method to analyze again and methodically finding and selective union of those sets that comprise this seemingly simple but so complex problem. Thus, the new technique is clearly shown that success is easier to understand and thus were practically more possible.

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