

# **On some Ramanujan's equations of Manuscript Book 2. Further new possible mathematical connections with some parameters of Particle Physics and Cosmology. V**

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## **Abstract**

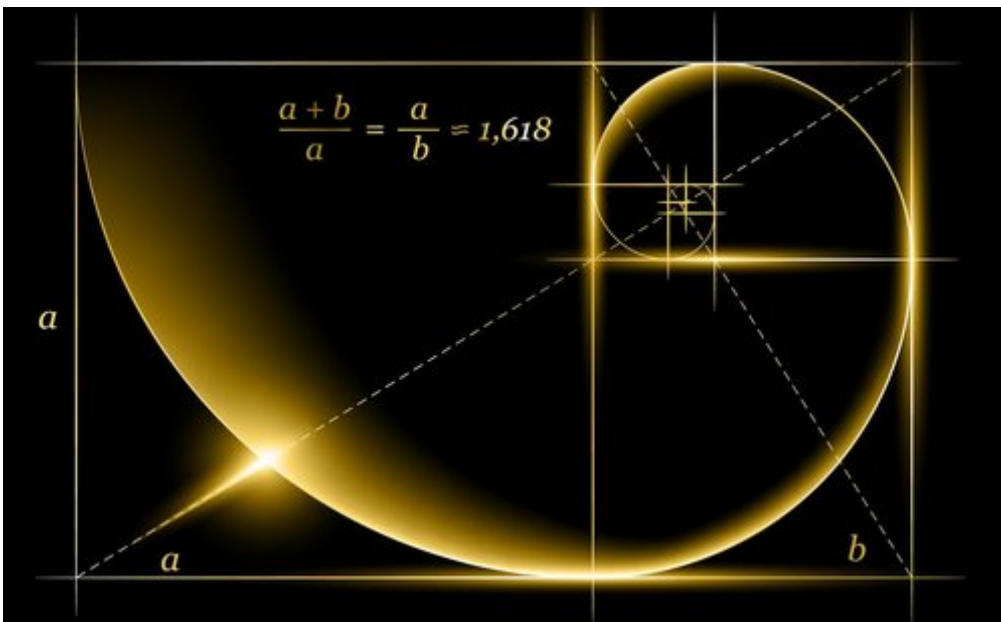
*In this research thesis, we continue to analyze and deepen further Ramanujan's equations of Manuscript Book 2 and describe new possible mathematical connections with some parameters of Particle Physics and Cosmology.*

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From : <http://scienceofhindu.blogspot.com/2016/04/man-who-knew-infinity-by-ramana.html> (modified by A. Nardelli)

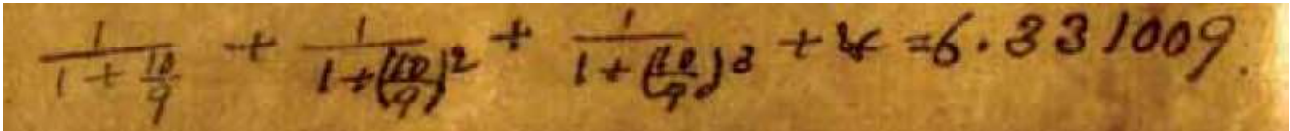


<https://kindtrainer.com/fractalbliss>

From: Manuscript Book 2 of Srinivasa Ramanujan

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Examples of infinite sum



$$1/(1+(10/9)) + 1/(1+(10/9)^2) + 1/(1+(10/9)^3) + \dots$$

**Input interpretation:**

$$\frac{1}{1 + \frac{10}{9}} + \frac{1}{1 + \left(\frac{10}{9}\right)^2} + \frac{1}{1 + \left(\frac{10}{9}\right)^3} + \dots$$

**Infinite sum:**

$$\sum_{n=1}^{\infty} \frac{1}{\left(\frac{10}{9}\right)^n + 1} = \frac{i \operatorname{Im}\left(\psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)}{\log\left(\frac{10}{9}\right)} + \frac{\operatorname{Re}\left(\psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)}{\log\left(\frac{10}{9}\right)} - \frac{\log(10)}{\log\left(\frac{10}{9}\right)}$$

$\log(x)$  is the natural logarithm

$\psi_q(z)$  gives the  $q$ -digamma function

$\operatorname{Im}(z)$  is the imaginary part of  $z$

$\operatorname{Re}(z)$  is the real part of  $z$

**Decimal approximation:**

6.331008692864745537718386879838180649341260412564743295777...

6.331008692...

**Convergence tests:**

By the ratio test, the series converges.

**Partial sum formula:**

$$\sum_{n=1}^m \frac{1}{1 + \left(\frac{10}{9}\right)^n} = \frac{\psi_{\frac{9}{10}}^{(0)}\left(-\frac{i\pi - \log\left(\frac{10}{9}\right)}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)} - \frac{\psi_{\frac{9}{10}}^{(0)}\left(-\frac{i\pi - (m+1)\log\left(\frac{10}{9}\right)}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)}$$

**Alternate forms:**

$$\frac{\log(10) - \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right)}{\log\left(\frac{10}{9}\right)}$$

$$-\frac{\log(10)}{\log\left(\frac{10}{9}\right)} + \frac{\psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right)}{\log\left(\frac{10}{9}\right)}$$

$$\frac{-\log(10) + \psi_{\frac{10}{9}}^{(0)} \left( \frac{-i\pi - 2\log(3) + \log(10)}{-2\log(3) + \log(10)} \right)}{\log(10) - 2\log(3)}$$

**Series representations:**

$$\frac{i \operatorname{Im} \left( \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right) \right)}{\log\left(\frac{10}{9}\right)} - \frac{\log(10)}{\log\left(\frac{10}{9}\right)} + \frac{\operatorname{Re} \left( \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right) \right)}{\log\left(\frac{10}{9}\right)} =$$

$$-\left( \left( 2\pi \left\lfloor \frac{\arg(10-x)}{2\pi} \right\rfloor - \operatorname{Im} \left( \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i\pi}{2i\pi \left\lfloor \frac{\arg\left(\frac{10}{9}-x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9}-x\right)^k x^{-k}}{k} \right) \right) \right) - \right.$$

$$\left. i \log(x) + i \operatorname{Re} \left( \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i\pi}{2i\pi \left\lfloor \frac{\arg\left(\frac{10}{9}-x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9}-x\right)^k x^{-k}}{k} \right) \right) \right) +$$

$$\left. i \sum_{k=1}^{\infty} \frac{(-1)^k (10-x)^k x^{-k}}{k} \right) /$$

$$\left( 2\pi \left\lfloor \frac{\arg\left(\frac{10}{9}-x\right)}{2\pi} \right\rfloor - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9}-x\right)^k x^{-k}}{k} \right) \text{ for } x < 0$$

$$\begin{aligned}
& \frac{i \operatorname{Im} \left( \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i \pi}{\log \left( \frac{10}{9} \right)} \right) \right)}{\log \left( \frac{10}{9} \right)} - \frac{\log(10)}{\log \left( \frac{10}{9} \right)} + \frac{\operatorname{Re} \left( \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i \pi}{\log \left( \frac{10}{9} \right)} \right) \right)}{\log \left( \frac{10}{9} \right)} = \\
& - \left( \left( 2 \pi \left[ \frac{\pi - \arg \left( \frac{1}{z_0} \right) - \arg(z_0)}{2 \pi} \right] - \operatorname{Im} \left( \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i \pi}{\log \left( \frac{10}{9} \right)} \right) \right) \right. \right. \\
& \quad \left. \left. \frac{i \pi}{2 i \pi \left[ \frac{\pi - \arg \left( \frac{1}{z_0} \right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{10}{9} - z_0 \right)^k z_0^{-k}}{k} \right)} - i \log(z_0) + \right. \\
& \quad \left. i \operatorname{Re} \left( \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i \pi}{2 i \pi \left[ \frac{\pi - \arg \left( \frac{1}{z_0} \right) - \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{10}{9} - z_0 \right)^k z_0^{-k}}{k} \right)} \right) \right) + \\
& \quad \left. i \sum_{k=1}^{\infty} \frac{(-1)^k (10 - z_0)^k z_0^{-k}}{k} \right) / \\
& \quad \left( 2 \pi \left[ \frac{\pi - \arg \left( \frac{1}{z_0} \right) - \arg(z_0)}{2 \pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{10}{9} - z_0 \right)^k z_0^{-k}}{k} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{i \operatorname{Im} \left( \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i \pi}{\log \left( \frac{10}{9} \right)} \right) \right)}{\log \left( \frac{10}{9} \right)} - \frac{\log(10)}{\log \left( \frac{10}{9} \right)} + \frac{\operatorname{Re} \left( \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i \pi}{\log \left( \frac{10}{9} \right)} \right) \right)}{\log \left( \frac{10}{9} \right)} = \\
& \left( i \operatorname{Im} \left( \psi_{\frac{10}{9}}^{(0)} \left[ 1 - \frac{i \pi}{\log(z_0) + \left\lfloor \frac{\arg \left( \frac{10}{9} - z_0 \right)}{2 \pi} \right\rfloor \left( \log \left( \frac{1}{z_0} \right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{10}{9} - z_0 \right)^k z_0^{-k}}{k}} \right] \right) - \right. \\
& \quad \left. \left\lfloor \frac{\arg(10 - z_0)}{2 \pi} \right\rfloor \log \left( \frac{1}{z_0} \right) - \log(z_0) - \left\lfloor \frac{\arg(10 - z_0)}{2 \pi} \right\rfloor \log(z_0) + \right. \\
& \quad \left. \operatorname{Re} \left( \psi_{\frac{10}{9}}^{(0)} \left[ 1 - \frac{i \pi}{\log(z_0) + \left\lfloor \frac{\arg \left( \frac{10}{9} - z_0 \right)}{2 \pi} \right\rfloor \left( \log \left( \frac{1}{z_0} \right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{10}{9} - z_0 \right)^k z_0^{-k}}{k}} \right] \right) + \right. \\
& \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^k (10 - z_0)^k z_0^{-k}}{k} \right) / \\
& \quad \left( \left\lfloor \frac{\arg \left( \frac{10}{9} - z_0 \right)}{2 \pi} \right\rfloor \log \left( \frac{1}{z_0} \right) + \log(z_0) + \left\lfloor \frac{\arg \left( \frac{10}{9} - z_0 \right)}{2 \pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{10}{9} - z_0 \right)^k z_0^{-k}}{k} \right)
\end{aligned}$$

$$34/10^2 + (((1/(1+(10/9))+1/(1+(10/9)^2)+1/(1+(10/9)^3)+...)))$$

**Input interpretation:**

$$\frac{34}{10^2} + \left( \frac{1}{1 + \frac{10}{9}} + \frac{1}{1 + \left( \frac{10}{9} \right)^2} + \frac{1}{1 + \left( \frac{10}{9} \right)^3} + \dots \right)$$

**Result:**

$$\frac{17}{50} + \frac{-\log(10) + \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i \pi}{\log \left( \frac{10}{9} \right)} \right)}{\log \left( \frac{10}{9} \right)}$$

$\log(x)$  is the natural logarithm

$\psi_q(z)$  gives the  $q$ -digamma function

**Alternate forms:**

$$\frac{50 \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i \pi}{\log \left( \frac{10}{9} \right)} \right) + 17 \log \left( \frac{10}{9} \right) - 50 \log(10)}{50 \log \left( \frac{10}{9} \right)}$$

$$\frac{17 \log\left(\frac{10}{q}\right) - 50 \log(10)}{50 \log\left(\frac{10}{q}\right)} + \frac{\psi_{\frac{q}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{q}\right)}\right)}{\log\left(\frac{10}{q}\right)}$$

$$\frac{50 \psi_{\frac{q}{10}}^{(0)}\left(\frac{-i\pi - 2 \log(3) + \log(10)}{-2 \log(3) + \log(10)}\right) - 34 \log(3) - 33 \log(10)}{50 (\log(10) - 2 \log(3))}$$

From which:

**Input interpretation:**

$$\frac{\frac{34}{10^2} + \left(\frac{1}{1+\frac{10}{q}} + \frac{1}{1+\left(\frac{10}{q}\right)^2} + \frac{1}{1+\left(\frac{10}{q}\right)^3} + \dots\right)}{10^{11}}$$

**Result:**

$$\frac{\frac{17}{50} + \frac{-\log(10) + \psi_{\frac{q}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{q}\right)}\right)}{\log\left(\frac{10}{q}\right)}}{100\,000\,000\,000}$$

$\log(x)$  is the natural logarithm  
 $\psi_q(z)$  gives the  $q$ -digamma function

**Input:**

$$\frac{\frac{17}{50} + \frac{-\log(10) + \psi_{\frac{q}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{q}\right)}\right)}{\log\left(\frac{10}{q}\right)}}{100\,000\,000\,000}$$

**Decimal approximation:**

$$6.6710086928647455377183868798381806493412604125647432... \times 10^{-11}$$

[6.671008692... \\* 10<sup>-11</sup>](#) result practically to the value of Gravitational Constant

**Alternate forms:**

$$\frac{50 \psi_{\frac{q}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{q}\right)}\right) + 17 \log\left(\frac{10}{q}\right) - 50 \log(10)}{5\,000\,000\,000\,000 \log\left(\frac{10}{q}\right)}$$

$$\frac{17 \log\left(\frac{10}{9}\right) - 50 \log(10)}{5\,000\,000\,000\,000 \log\left(\frac{10}{9}\right)} + \frac{\psi\left(\frac{0}{10}\right) \left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{100\,000\,000\,000 \log\left(\frac{10}{9}\right)}$$

$$\frac{50 \psi\left(\frac{0}{10}\right) \left(\frac{-i\pi - 2 \log(3) + \log(10)}{-2 \log(3) + \log(10)}\right) - 34 \log(3) - 33 \log(10)}{5\,000\,000\,000\,000 (\log(10) - 2 \log(3))}$$

**Alternative representations:**

$$\frac{\frac{17}{50} + \frac{-\log(10) + \psi\left(\frac{0}{9}\right) \left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)}}{100\,000\,000\,000} = \frac{\frac{17}{50} + \frac{-\log(a) \log_a(10) + \psi\left(\frac{0}{10}\right) \left(1 - \frac{i\pi}{\log(a) \log_a\left(\frac{10}{9}\right)}\right)}{\log(a) \log_a\left(\frac{10}{9}\right)}}{100\,000\,000\,000}$$

$$\frac{\frac{17}{50} + \frac{-\log(10) + \psi\left(\frac{0}{9}\right) \left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)}}{100\,000\,000\,000} = \frac{\frac{17}{50} + \frac{-\log_e(10) + \psi\left(\frac{0}{10}\right) \left(1 - \frac{i\pi}{\log_e\left(\frac{10}{9}\right)}\right)}{\log_e\left(\frac{10}{9}\right)}}{100\,000\,000\,000}$$

$$\frac{\frac{17}{50} + \frac{-\log(10) + \psi\left(\frac{0}{9}\right) \left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)}}{100\,000\,000\,000} = \frac{\frac{17}{50} + \frac{\text{Li}_1(-9) + \psi\left(\frac{0}{10}\right) \left(1 - \frac{i\pi}{\text{Li}_1\left(1 - \frac{10}{9}\right)}\right)}{\text{Li}_1\left(1 - \frac{10}{9}\right)}}{100\,000\,000\,000}$$

**Series representations:**



$$\begin{aligned}
& \frac{\frac{17}{50} + \frac{-\log(10) + \psi\left(\frac{0}{9}\right) \left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)} = \\
& \frac{100\,000\,000\,000}{\left( \left( -34\pi \left\lfloor \frac{\arg\left(\frac{10}{9} - x\right)}{2\pi} \right\rfloor + 100\pi \left\lfloor \frac{\arg(10 - x)}{2\pi} \right\rfloor - 33i \log(x) + 50i \right. \right. \\
& \quad \left. \left. \psi\left(\frac{0}{9}\right) \frac{1}{10} \left( 1 - \frac{i\pi}{2i\pi \left\lfloor \frac{\arg\left(\frac{10}{9} - x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - x\right)^k x^{-k}}{k} \right) - \right. \right. \\
& \quad \left. \left. 17i \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - x\right)^k x^{-k}}{k} + 50i \sum_{k=1}^{\infty} \frac{(-1)^k (10 - x)^k x^{-k}}{k} \right) / \right. \\
& \quad \left. \left( 5\,000\,000\,000\,000 \left( 2\pi \left\lfloor \frac{\arg\left(\frac{10}{9} - x\right)}{2\pi} \right\rfloor - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - x\right)^k x^{-k}}{k} \right) \right) \right)
\end{aligned}$$

for  $x < 0$

$$\begin{aligned}
& \frac{\frac{17}{50} + \frac{-\log(10) + \psi\left(\frac{10}{9}\right) \left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{100\,000\,000\,000} = - \left( \left( 66\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - 33i \log(z_0) + \right. \right. \\
& \left. \left. 50i\psi\left(\frac{10}{9}\right) \left( 1 - \frac{i\pi}{2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - z_0\right)^k z_0^{-k}}{k} \right) - \right. \right. \\
& \left. \left. 17i \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - z_0\right)^k z_0^{-k}}{k} + \right. \right. \\
& \left. \left. 50i \sum_{k=1}^{\infty} \frac{(-1)^k (10 - z_0)^k z_0^{-k}}{k} \right) / \left( 5\,000\,000\,000\,000 \right. \right. \\
& \left. \left. \left( 2\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - z_0\right)^k z_0^{-k}}{k} \right) \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{\frac{17}{50} + \frac{-\log(10) + \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right)}{\log\left(\frac{10}{9}\right)}}{100\,000\,000\,000} = \\
& - \left( \left( -17 \left[ \frac{\arg\left(\frac{10}{9} - z_0\right)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + 50 \left[ \frac{\arg(10 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + 33 \log(z_0) - \right. \right. \\
& \quad \left. \left. 17 \left[ \frac{\arg\left(\frac{10}{9} - z_0\right)}{2\pi} \right] \log(z_0) + 50 \left[ \frac{\arg(10 - z_0)}{2\pi} \right] \log(z_0) - 50 \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right) \right. \right. \\
& \quad \left. \left. 1 - \frac{i\pi}{\log(z_0) + \left[ \frac{\arg\left(\frac{10}{9} - z_0\right)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - z_0\right)^k z_0^{-k}}{k} \right)} \right) + \\
& \quad \left. \left. 17 \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - z_0\right)^k z_0^{-k}}{k} - 50 \sum_{k=1}^{\infty} \frac{(-1)^k (10 - z_0)^k z_0^{-k}}{k} \right) / \right. \\
& \quad \left. \left( 5\,000\,000\,000\,000 \left( \left[ \frac{\arg\left(\frac{10}{9} - z_0\right)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \right. \right. \right. \\
& \quad \left. \left. \left. \left[ \frac{\arg\left(\frac{10}{9} - z_0\right)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - z_0\right)^k z_0^{-k}}{k} \right) \right) \right) \right)
\end{aligned}$$

### Integral representations:

$$\frac{\frac{17}{50} + \frac{-\log(10) + \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right)}{\log\left(\frac{10}{9}\right)}}{100\,000\,000\,000} = \frac{17 \int_1^{\frac{10}{9}} \frac{1}{t} dt - 50 \int_1^{10} \frac{1}{t} dt + 50 \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right)}{5\,000\,000\,000\,000 \int_1^{\frac{10}{9}} \frac{1}{t} dt}$$

$$\frac{\frac{17}{50} + \frac{-\log(10) + \psi_{\frac{10}{9}}^{(0)} \left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log\left(\frac{10}{9}\right)}}{100\,000\,000\,000} =$$

$$-\left(50 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{9^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds - 17 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{9^s \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds - \right.$$

$$\left. 100 i \pi \psi_{\frac{10}{9}}^{(0)} \left(1 + \frac{2\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{9^s \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}\right) \right) /$$

$$\left(5\,000\,000\,000\,000 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{9^s \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right) \text{ for } -1 < \gamma < 0$$

$$((((1/(1+(10/9))+1/(1+(10/9)^2)+1/(1+(10/9)^3)+...)))i)^4$$

**Input interpretation:**

$$\left(\left(\frac{1}{1+\frac{10}{9}} + \frac{1}{1+\left(\frac{10}{9}\right)^2} + \frac{1}{1+\left(\frac{10}{9}\right)^3} + \dots\right)i\right)^4$$

$i$  is the imaginary unit

**Result:**

$$\frac{\left(-\log(10) + \psi_{\frac{10}{9}}^{(0)} \left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\log^4\left(\frac{10}{9}\right)}$$

**Alternate forms:**

$$\frac{\left(\log(10) - \psi_{\frac{10}{9}}^{(0)} \left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\log^4\left(\frac{10}{9}\right)}$$

$$\frac{\left(-\log(10) + \psi_{\frac{10}{9}}^{(0)} \left(\frac{-i\pi - 2\log(3) + \log(10)}{-2\log(3) + \log(10)}\right)\right)^4}{(\log(10) - 2\log(3))^4}$$

$$-\frac{4\log(10)\psi_{\frac{10}{9}}^{(0)} \left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^3}{\log^4\left(\frac{10}{9}\right)} + \frac{\psi_{\frac{10}{9}}^{(0)} \left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^4}{\log^4\left(\frac{10}{9}\right)}$$

$$-\frac{4\log^3(10)\psi_{\frac{10}{9}}^{(0)} \left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log^4\left(\frac{10}{9}\right)} + \frac{6\log^2(10)\psi_{\frac{10}{9}}^{(0)} \left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^2}{\log^4\left(\frac{10}{9}\right)} + \frac{\log^4(10)}{\log^4\left(\frac{10}{9}\right)}$$

From:

$$\frac{\left(\log(10) - \psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\log^4\left(\frac{10}{9}\right)}$$

that is:

$$(\log(10) - \text{QPolyGamma}(0, 1 - (i\pi)/\log(10/9), 9/10))^4/(\log^4(10/9))$$

we obtain:

**Input:**

$$\frac{\left(\log(10) - \psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\log^4\left(\frac{10}{9}\right)}$$

$\log(x)$  is the natural logarithm

$\psi_q(z)$  gives the  $q$ -digamma function

$i$  is the imaginary unit

**Decimal approximation:**

1606.540355693850581318901798130314953989878211993232234372...

1606.54....

**Alternate forms:**

$$\frac{\left(-\log(10) + \psi_{\frac{9}{10}}^{(0)}\left(\frac{-i\pi - 2\log(3) + \log(10)}{-2\log(3) + \log(10)}\right)\right)^4}{(\log(10) - 2\log(3))^4}$$

$$-\frac{4\log(10)\psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^3}{\log^4\left(\frac{10}{9}\right)} + \frac{\psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^4}{\log^4\left(\frac{10}{9}\right)}$$

$$\frac{4\log^3(10)\psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)}{\log^4\left(\frac{10}{9}\right)} + \frac{6\log^2(10)\psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)^2}{\log^4\left(\frac{10}{9}\right)} + \frac{\log^4(10)}{\log^4\left(\frac{10}{9}\right)}$$

$$\frac{\left(-\psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right) + \log(2) + \log(5)\right)^4}{\log^4\left(\frac{10}{9}\right)}$$

**Alternative representations:**

$$\frac{\left(\log(10) - \psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\log^4\left(\frac{10}{9}\right)} = \frac{\left(\log_e(10) - \psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log_e\left(\frac{10}{9}\right)}\right)\right)^4}{\log_e^4\left(\frac{10}{9}\right)}$$

$$\frac{\left(\log(10) - \psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\log^4\left(\frac{10}{9}\right)} = \frac{\left(\log(a) \log_a(10) - \psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log(a) \log_a\left(\frac{10}{9}\right)}\right)\right)^4}{\left(\log(a) \log_a\left(\frac{10}{9}\right)\right)^4}$$

$$\frac{\left(\log(10) - \psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\log^4\left(\frac{10}{9}\right)} = \frac{\left(-\text{Li}_1(-9) - \psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\text{Li}_1\left(1 - \frac{10}{9}\right)}\right)\right)^4}{\left(-\text{Li}_1\left(1 - \frac{10}{9}\right)\right)^4}$$

**Series representations:**

$$\frac{\left(\log(10) - \psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\log^4\left(\frac{10}{9}\right)} = \left(2\pi \left\lfloor \frac{\arg(10-x)}{2\pi} \right\rfloor - i \log(x) + \right. \\ \left. i \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{2i\pi \left\lfloor \frac{\arg\left(\frac{10}{9}-x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9}-x\right)^k x^{-k}}{k} \right) + \right. \\ \left. i \sum_{k=1}^{\infty} \frac{(-1)^k (10-x)^k x^{-k}}{k} \right)^4 / \\ \left( 2\pi \left\lfloor \frac{\arg\left(\frac{10}{9}-x\right)}{2\pi} \right\rfloor - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9}-x\right)^k x^{-k}}{k} \right)^4 \text{ for } x < 0$$

$$\begin{aligned}
& \frac{\left(\log(10) - \psi_{\frac{10}{9}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\log^4\left(\frac{10}{9}\right)} = \left( 2\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - i \log(z_0) + \right. \\
& \left. i \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i\pi}{2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - z_0\right)^k z_0^{-k}}{k}} \right) + \right. \\
& \left. i \sum_{k=1}^{\infty} \frac{(-1)^k (10 - z_0)^k z_0^{-k}}{k} \right)^4 / \\
& \left( 2\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - z_0\right)^k z_0^{-k}}{k} \right)^4 \\
& \frac{\left(\log(10) - \psi_{\frac{10}{9}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\log^4\left(\frac{10}{9}\right)} = \\
& \left( \left[ \frac{\arg(10 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\arg(10 - z_0)}{2\pi} \right] \log(z_0) - \right. \\
& \left. \psi_{\frac{10}{9}}^{(0)} \left( 1 - \frac{i\pi}{\log(z_0) + \left[ \frac{\arg\left(\frac{10}{9} - z_0\right)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - z_0\right)^k z_0^{-k}}{k}} \right) - \right. \\
& \left. \sum_{k=1}^{\infty} \frac{(-1)^k (10 - z_0)^k z_0^{-k}}{k} \right)^4 / \\
& \left( \left[ \frac{\arg\left(\frac{10}{9} - z_0\right)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\arg\left(\frac{10}{9} - z_0\right)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - z_0\right)^k z_0^{-k}}{k} \right)^4
\end{aligned}$$

**Integral representations:**

$$\frac{\left(\log(10) - \psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\log^4\left(\frac{10}{9}\right)} = \frac{\left(\int_1^{10} \frac{1}{t} dt - \psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\left(\int_1^{\frac{10}{9}} \frac{1}{t} dt\right)^4}$$

$$\frac{\left(\log(10) - \psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\log^4\left(\frac{10}{9}\right)} = \frac{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{9^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds - 2i\pi \psi_{\frac{9}{10}}^{(0)}\left(1 + \frac{2\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{9^s \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}\right)\right)^4}{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{9^s \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right)^4} \text{ for } -1 < \gamma < 0$$

From which:

$(\log(10) - \text{QPolyGamma}(0, 1 - (i\pi)/\log(10/9), 9/10))^4 / (\log^4(10/9)) + 64 + \text{golden ratio}$

**Input:**

$$\frac{\left(\log(10) - \psi_{\frac{9}{10}}^{(0)}\left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\log^4\left(\frac{10}{9}\right)} + 64 + \phi$$

$\log(x)$  is the natural logarithm

$\psi_q(z)$  gives the  $q$ -digamma function

$i$  is the imaginary unit

$\phi$  is the golden ratio

**Decimal approximation:**

1672.158389682600476167106384964680592107598521173037997234...

1672.1583896.... result practically equal to the rest mass of Omega baryon 1672.45



**Alternate forms:**

$$\begin{aligned} & \frac{1}{2} (129 + \sqrt{5}) + \frac{\left( \log(10) - \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right) \right)^4}{\log^4\left(\frac{10}{9}\right)} \\ & \frac{1}{2 \log^4\left(\frac{10}{9}\right)} \left( -8 \log^3(10) \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right) + \right. \\ & \quad 12 \log^2(10) \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right)^2 - 8 \log(10) \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right)^3 + \\ & \quad \left. 2 \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right)^4 + 129 \log^4\left(\frac{10}{9}\right) + \sqrt{5} \log^4\left(\frac{10}{9}\right) + 2 \log^4(10) \right) \\ & - \frac{4 \log(10) \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right)^3}{\log^4\left(\frac{10}{9}\right)} + \frac{\psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right)^4}{\log^4\left(\frac{10}{9}\right)} - \frac{4 \log^3(10) \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right)}{\log^4\left(\frac{10}{9}\right)} + \\ & \frac{6 \log^2(10) \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right)^2}{\log^4\left(\frac{10}{9}\right)} + 64 + \frac{1}{2} (1 + \sqrt{5}) + \frac{\log^4(10)}{\log^4\left(\frac{10}{9}\right)} \end{aligned}$$

**Alternative representations:**

$$\begin{aligned} & \frac{\left( \log(10) - \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right) \right)^4}{\log^4\left(\frac{10}{9}\right)} + 64 + \phi = 64 + \phi + \frac{\left( \log_e(10) - \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log_e\left(\frac{10}{9}\right)} \right) \right)^4}{\log_e^4\left(\frac{10}{9}\right)} \\ & \frac{\left( \log(10) - \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right) \right)^4}{\log^4\left(\frac{10}{9}\right)} + 64 + \phi = \\ & \quad 64 + \phi + \frac{\left( \log(a) \log_a(10) - \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log(a) \log_a\left(\frac{10}{9}\right)} \right) \right)^4}{\left( \log(a) \log_a\left(\frac{10}{9}\right) \right)^4} \\ & \frac{\left( \log(10) - \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right) \right)^4}{\log^4\left(\frac{10}{9}\right)} + 64 + \phi = 64 + \phi + \frac{\left( -\text{Li}_1(-9) - \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\text{Li}_1\left(1 - \frac{10}{9}\right)} \right) \right)^4}{\left( -\text{Li}_1\left(1 - \frac{10}{9}\right) \right)^4} \end{aligned}$$

**Series representations:**

$$\frac{\left(\log(10) - \psi_{\frac{10}{9}}^{(0)} \left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\log^4\left(\frac{10}{9}\right)} + 64 + \phi = 64 + \phi + \left(2i\pi \left\lfloor \frac{\arg(10-x)}{2\pi} \right\rfloor + \log(x) - \right.$$

$$\left. \psi_{\frac{10}{9}}^{(0)} \left(1 - \frac{i\pi}{2i\pi \left\lfloor \frac{\arg\left(\frac{10}{9}-x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9}-x\right)^k x^{-k}}{k}}\right) - \right.$$

$$\left. \sum_{k=1}^{\infty} \frac{(-1)^k (10-x)^k x^{-k}}{k} \right)^4 /$$

$$\left(2i\pi \left\lfloor \frac{\arg\left(\frac{10}{9}-x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9}-x\right)^k x^{-k}}{k} \right)^4 \text{ for } x < 0$$

$$\frac{\left(\log(10) - \psi_{\frac{10}{9}}^{(0)} \left(1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)}\right)\right)^4}{\log^4\left(\frac{10}{9}\right)} + 64 + \phi =$$

$$64 + \phi + \left( \log(z_0) + \left\lfloor \frac{\arg(10-z_0)}{2\pi} \right\rfloor \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \psi_{\frac{10}{9}}^{(0)} \left( \right. \right.$$

$$\left. \left. 1 - \frac{i\pi}{\log(z_0) + \left\lfloor \frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi} \right\rfloor \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k}} \right) - \right.$$

$$\left. \sum_{k=1}^{\infty} \frac{(-1)^k (10-z_0)^k z_0^{-k}}{k} \right)^4 /$$

$$\left( \log(z_0) + \left\lfloor \frac{\arg\left(\frac{10}{9}-z_0\right)}{2\pi} \right\rfloor \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9}-z_0\right)^k z_0^{-k}}{k} \right)^4$$

$$\begin{aligned}
& \frac{\left( \log(10) - \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right) \right)^4}{\log^4\left(\frac{10}{9}\right)} + 64 + \phi = \\
& 64 + \phi + \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right) \right) \\
& \left( 1 - \frac{i\pi}{2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - z_0\right)^k z_0^{-k}}{k}} \right) \\
& \left. \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (10 - z_0)^k z_0^{-k}}{k}}{\left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - z_0\right)^k z_0^{-k}}{k} \right)^4} \right)
\end{aligned}$$

### Integral representations:

$$\frac{\left( \log(10) - \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right) \right)^4}{\log^4\left(\frac{10}{9}\right)} + 64 + \phi = 64 + \phi + \frac{\left( \int_1^{10} \frac{1}{t} dt - \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right) \right)^4}{\left( \int_1^{\frac{10}{9}} \frac{1}{t} dt \right)^4}$$

$$\begin{aligned}
& \frac{\left( \log(10) - \psi_{\frac{9}{10}}^{(0)} \left( 1 - \frac{i\pi}{\log\left(\frac{10}{9}\right)} \right) \right)^4}{\log^4\left(\frac{10}{9}\right)} + 64 + \phi = 64 + \phi + \\
& \frac{\left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{9^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds - 2i\pi \psi_{\frac{9}{10}}^{(0)} \left( 1 + \frac{2\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{9^s \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \right) \right)^4}{\left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{9^s \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)^4} \quad \text{for } -1 < \gamma < 0
\end{aligned}$$

$\Gamma(x)$  is the gamma function



and:

$$\sqrt{\left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \left( \frac{11\pi}{199+7} \right)^2} \times \frac{1}{2.980893 \times 10^{-27}} \right)} \sqrt{-\frac{4.116898 \times 10^{49} \times 4\pi (4.426184 \times 10^{-54})^3 - (4.426184 \times 10^{-54})^2}{6.67 \times 10^{-11}}} \right]}$$

**Input interpretation:**

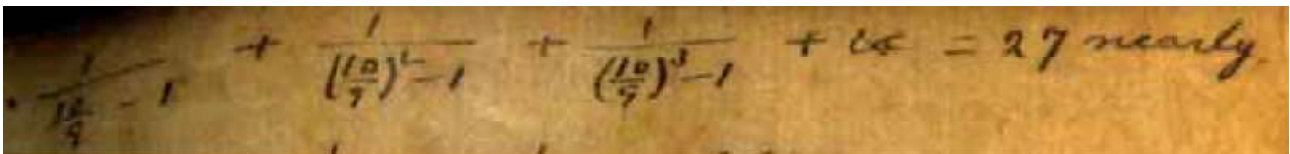
$$\sqrt{\left( \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \left( \frac{11\pi}{199+7} \right)^2} \times \frac{1}{2.980893 \times 10^{-27}} \right)} \sqrt{-\frac{4.116898 \times 10^{49} \times 4\pi (4.426184 \times 10^{-54})^3 - (4.426184 \times 10^{-54})^2}{6.67 \times 10^{-11}}} \right)}$$

**Result:**

3.141805805878682075280841709346146312458152935922769297567...

[3.1418058058....](#)

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$$1/((10/9)-1)+1/((10/9)^2-1)+1/((10/9)^3-1)+...$$

**Input interpretation:**

$$\frac{1}{\frac{10}{9} - 1} + \frac{1}{\left(\frac{10}{9}\right)^2 - 1} + \frac{1}{\left(\frac{10}{9}\right)^3 - 1} + \dots$$

**Infinite sum:**

$$\sum_{n=1}^{\infty} \frac{1}{\left(\frac{10}{9}\right)^n - 1} = \frac{\log(10) - \psi_{\frac{9}{10}}^{(0)}(1)}{\log\left(\frac{10}{9}\right)}$$

$\log(x)$  is the natural logarithm

$\psi_q(z)$  gives the  $q$ -digamma function

### Decimal approximation:

27.08648503406816780327872576570091022140786017495536508019...

27.08648503...

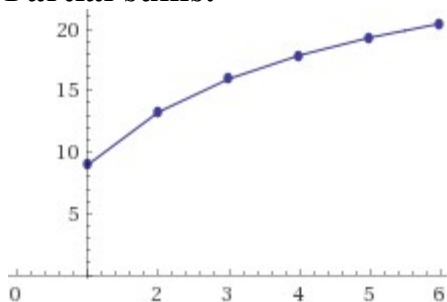
### Convergence tests:

By the ratio test, the series converges.

### Partial sum formula:

$$\sum_{n=1}^m \frac{1}{-1 + \left(\frac{10}{9}\right)^n} = \frac{\psi_{\frac{10}{9}}^{(0)}(m+1)}{\log\left(\frac{10}{9}\right)} - \frac{\psi_{\frac{10}{9}}^{(0)}(1)}{\log\left(\frac{10}{9}\right)}$$

### Partial sums:



### Alternate forms:

$$\frac{\psi_{\frac{10}{9}}^{(0)}(1) - \log(10)}{\log(10) - 2 \log(3)}$$

$$\frac{\log(10)}{\log\left(\frac{10}{9}\right)} - \frac{\psi_{\frac{10}{9}}^{(0)}(1)}{\log\left(\frac{10}{9}\right)}$$

$$-\frac{\psi_{\frac{10}{9}}^{(0)}(1)}{\log(2) - 2 \log(3) + \log(5)} + \frac{\log(2)}{\log(2) - 2 \log(3) + \log(5)} + \frac{\log(5)}{\log(2) - 2 \log(3) + \log(5)}$$

### Series representations:

$$\frac{\log(10) - \psi_{\frac{10}{9}}^{(0)}(1)}{\log\left(\frac{10}{9}\right)} = \frac{2\pi \left[ \frac{\arg(10-x)}{2\pi} \right] - i \log(x) + i \psi_{\frac{10}{9}}^{(0)}(1) + i \sum_{k=1}^{\infty} \frac{(-1)^k (10-x)^k x^{-k}}{k}}{2\pi \left[ \frac{\arg\left(\frac{10}{9}-x\right)}{2\pi} \right] - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9}-x\right)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{\log(10) - \psi_{\frac{10}{9}}^{(0)}(1)}{\log\left(\frac{10}{9}\right)} = \frac{2\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - i \log(z_0) + i \psi_{\frac{10}{9}}^{(0)}(1) + i \sum_{k=1}^{\infty} \frac{(-1)^k (10-z_0)^k z_0^{-k}}{k}}{2\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - z_0\right)^k z_0^{-k}}{k}}$$

$$\frac{\log(10) - \psi_{\frac{10}{9}}^{(0)}(1)}{\log\left(\frac{10}{9}\right)} = \frac{\left[ \frac{\arg(10-z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\arg(10-z_0)}{2\pi} \right] \log(z_0) - \psi_{\frac{10}{9}}^{(0)}(1) - \sum_{k=1}^{\infty} \frac{(-1)^k (10-z_0)^k z_0^{-k}}{k}}{\left[ \frac{\arg\left(\frac{10}{9} - z_0\right)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\arg\left(\frac{10}{9} - z_0\right)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{10}{9} - z_0\right)^k z_0^{-k}}{k}}$$

$$\left( \left( \left( \left( \left( \left( \frac{1}{(10/9)-1} + \frac{1}{(10/9)^2-1} + \frac{1}{(10/9)^3-1} + \dots \right) \right) \right) \right) \right) \right)^2$$

**Input interpretation:**

$$\left( \frac{1}{\frac{10}{9}-1} + \frac{1}{\left(\frac{10}{9}\right)^2-1} + \frac{1}{\left(\frac{10}{9}\right)^3-1} + \dots \right)^2$$

**Result:**

$$\frac{\left( \log(10) - \psi_{\frac{10}{9}}^{(0)}(1) \right)^2}{\log^2\left(\frac{10}{9}\right)}$$

$\log(x)$  is the natural logarithm  
 $\psi_q(z)$  gives the  $q$ -digamma function

**Alternate forms:**

$$\frac{\left( \psi_{\frac{10}{9}}^{(0)}(1) - \log(10) \right)^2}{(\log(10) - 2 \log(3))^2} - \frac{2 \psi_{\frac{10}{9}}^{(0)}(1) \log(10)}{\log^2\left(\frac{10}{9}\right)} + \frac{\psi_{\frac{10}{9}}^{(0)}(1)^2}{\log^2\left(\frac{10}{9}\right)} + \frac{\log^2(10)}{\log^2\left(\frac{10}{9}\right)}$$

$$\frac{\left( -\psi_{\frac{10}{9}}^{(0)}(1) + \log(2) + \log(5) \right)^2}{(\log(2) - 2 \log(3) + \log(5))^2}$$

From:

$$\frac{\left(-\psi_{\frac{9}{10}}^{(0)}(1) + \log(2) + \log(5)\right)^2}{(\log(2) - 2 \log(3) + \log(5))^2}$$

that is

$$(\log(2) + \log(5) - \text{QPolyGamma}(0, 1, 9/10))^2 / (\log(2) - 2 \log(3) + \log(5))^2$$

we obtain:

**Input:**

$$\frac{\left(\log(2) + \log(5) - \psi_{\frac{9}{10}}^{(0)}(1)\right)^2}{(\log(2) - 2 \log(3) + \log(5))^2}$$

$\log(x)$  is the natural logarithm

$\psi_q(z)$  gives the  $q$ -digamma function

**Decimal approximation:**

733.6776715007988335226243700158996375199355977400390292164...

733.6776715...

**Alternate forms:**

$$\frac{\left(\log(10) - \psi_{\frac{9}{10}}^{(0)}(1)\right)^2}{\log^2\left(\frac{10}{9}\right)}$$

$$\frac{\left(\psi_{\frac{9}{10}}^{(0)}(1) - \log(10)\right)^2}{(\log(10) - 2 \log(3))^2}$$

$$\begin{aligned} & -\frac{2 \psi_{\frac{9}{10}}^{(0)}(1) \log(2)}{(\log(2) - 2 \log(3) + \log(5))^2} - \frac{2 \psi_{\frac{9}{10}}^{(0)}(1) \log(5)}{(\log(2) - 2 \log(3) + \log(5))^2} + \\ & \frac{\psi_{\frac{9}{10}}^{(0)}(1)^2}{(\log(2) - 2 \log(3) + \log(5))^2} + \frac{\log^2(2)}{(\log(2) - 2 \log(3) + \log(5))^2} + \\ & \frac{\log^2(5)}{(\log(2) - 2 \log(3) + \log(5))^2} + \frac{2 \log(2) \log(5)}{(\log(2) - 2 \log(3) + \log(5))^2} \end{aligned}$$



### Alternative representations:

$$\frac{\left(\log(2) + \log(5) - \psi_{\frac{9}{10}}^{(0)}(1)\right)^2}{(\log(2) - 2\log(3) + \log(5))^2} = \frac{\left(\log(10) - \psi_{\frac{9}{10}}^{(0)}(1)\right)^2}{(-2\log(3) + \log(10))^2}$$

$$\frac{\left(\log(2) + \log(5) - \psi_{\frac{9}{10}}^{(0)}(1)\right)^2}{(\log(2) - 2\log(3) + \log(5))^2} = \frac{\left(\log_e(2) + \log_e(5) - \psi_{\frac{9}{10}}^{(0)}(1)\right)^2}{(\log_e(2) - 2\log_e(3) + \log_e(5))^2}$$

$$\frac{\left(\log(2) + \log(5) - \psi_{\frac{9}{10}}^{(0)}(1)\right)^2}{(\log(2) - 2\log(3) + \log(5))^2} = \frac{\left(\log(a)\log_a(2) + \log(a)\log_a(5) - \psi_{\frac{9}{10}}^{(0)}(1)\right)^2}{(\log(a)\log_a(2) - 2\log(a)\log_a(3) + \log(a)\log_a(5))^2}$$

### Series representations:

$$\frac{\left(\log(2) + \log(5) - \psi_{\frac{9}{10}}^{(0)}(1)\right)^2}{(\log(2) - 2\log(3) + \log(5))^2} = \frac{\left(4\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - 2i \log(z_0) + i \psi_{\frac{9}{10}}^{(0)}(1) - i \sum_{k=1}^{\infty} \frac{(-1)^{1+k} \left( (2-z_0)^k + (5-z_0)^k \right) z_0^{-k}}{k} \right)^2}{\left( \sum_{k=1}^{\infty} \frac{(-1)^{1+k} \left( (2-z_0)^k + (5-z_0)^k \right) z_0^{-k}}{k} + 2 \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right)^2}$$

$$\frac{\left(\log(2) + \log(5) - \psi_{\frac{9}{10}}^{(0)}(1)\right)^2}{(\log(2) - 2\log(3) + \log(5))^2} = \left( 2\pi \left[ \frac{\arg(2-x)}{2\pi} \right] + 2\pi \left[ \frac{\arg(5-x)}{2\pi} \right] - \right. \\ \left. 2i \log(x) + i \psi_{\frac{9}{10}}^{(0)}(1) - i \sum_{k=1}^{\infty} \frac{(-1)^{1+k} \left( (2-x)^k + (5-x)^k \right) x^{-k}}{k} \right)^2 / \\ \left( 2\pi \left[ \frac{\arg(2-x)}{2\pi} \right] - 4\pi \left[ \frac{\arg(3-x)}{2\pi} \right] + 2\pi \left[ \frac{\arg(5-x)}{2\pi} \right] - \right. \\ \left. i \sum_{k=1}^{\infty} \frac{(-1)^{1+k} \left( (2-x)^k + (5-x)^k \right) x^{-k}}{k} - 2i \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right)^2 \text{ for } x < 0$$

$$\frac{\left(\log(2) + \log(5) - \psi_{\frac{9}{10}}^{(0)}(1)\right)^2}{(\log(2) - 2 \log(3) + \log(5))^2} = \frac{\left(\left[\frac{\arg(2 - z_0)}{2\pi}\right] \log\left(\frac{1}{z_0}\right) + \left[\frac{\arg(5 - z_0)}{2\pi}\right] \log\left(\frac{1}{z_0}\right) + 2 \log(z_0) + \left[\frac{\arg(2 - z_0)}{2\pi}\right] \log(z_0) + \left[\frac{\arg(5 - z_0)}{2\pi}\right] \log(z_0) - \psi_{\frac{9}{10}}^{(0)}(1) + \sum_{k=1}^{\infty} \frac{(-1)^{1+k} \left((2 - z_0)^k + (5 - z_0)^k\right) z_0^{-k}}{k}\right)^2}{\left(\left[\frac{\arg(2 - z_0)}{2\pi}\right] \log\left(\frac{1}{z_0}\right) - 2 \left[\frac{\arg(3 - z_0)}{2\pi}\right] \log\left(\frac{1}{z_0}\right) + \left[\frac{\arg(5 - z_0)}{2\pi}\right] \log\left(\frac{1}{z_0}\right) + \left[\frac{\arg(2 - z_0)}{2\pi}\right] \log(z_0) - 2 \left[\frac{\arg(3 - z_0)}{2\pi}\right] \log(z_0) + \left[\frac{\arg(5 - z_0)}{2\pi}\right] \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^{1+k} \left((2 - z_0)^k + (5 - z_0)^k\right) z_0^{-k}}{k} + 2 \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k}\right)^2}$$

From which:

$$(\log(2) + \log(5) - \text{QPolyGamma}(0, 1, 9/10))^2 / (\log(2) - 2 \log(3) + \log(5))^2 + 47 + \text{golden ratio}$$

**Input:**

$$\frac{\left(\log(2) + \log(5) - \psi_{\frac{9}{10}}^{(0)}(1)\right)^2}{(\log(2) - 2 \log(3) + \log(5))^2} + 47 + \phi$$

$\log(x)$  is the natural logarithm

$\psi_q(z)$  gives the  $q$ -digamma function

$\phi$  is the golden ratio

**Decimal approximation:**

782.2957054895487283708289568502652756376559069198447920785...

782.2957054.... result practically equal to the rest mass of Omega meson 782.65

**Alternate forms:**

$$\frac{\left(\log(10) - \psi_{\frac{10}{9}}^{(0)}(1)\right)^2}{\log^2\left(\frac{10}{9}\right)} + \phi + 47$$

$$\frac{\left(-\psi_{\frac{10}{9}}^{(0)}(1) + \log(2) + \log(5)\right)^2}{(\log(2) - 2\log(3) + \log(5))^2} + \frac{1}{2}(95 + \sqrt{5})$$

$$\frac{\left(\psi_{\frac{10}{9}}^{(0)}(1)^2 - 2\psi_{\frac{10}{9}}^{(0)}(1)(\log(2) + \log(5)) + \phi(\log(2) - 2\log(3) + \log(5))^2 + 4(12\log^2(2) + 47\log^2(3) + 12\log^2(5) - 47\log(3)\log(5) + \log(2)(24\log(5) - 47\log(3)))\right)}{(\log(2) - 2\log(3) + \log(5))^2}$$

**Alternative representations:**

$$\frac{\left(\log(2) + \log(5) - \psi_{\frac{10}{9}}^{(0)}(1)\right)^2}{(\log(2) - 2\log(3) + \log(5))^2} + 47 + \phi = 47 + \phi + \frac{\left(\log(10) - \psi_{\frac{10}{9}}^{(0)}(1)\right)^2}{(-2\log(3) + \log(10))^2}$$

$$\frac{\left(\log(2) + \log(5) - \psi_{\frac{10}{9}}^{(0)}(1)\right)^2}{(\log(2) - 2\log(3) + \log(5))^2} + 47 + \phi = 47 + \phi + \frac{\left(\log_e(2) + \log_e(5) - \psi_{\frac{10}{9}}^{(0)}(1)\right)^2}{(\log_e(2) - 2\log_e(3) + \log_e(5))^2}$$

$$\frac{\left(\log(2) + \log(5) - \psi_{\frac{10}{9}}^{(0)}(1)\right)^2}{(\log(2) - 2\log(3) + \log(5))^2} + 47 + \phi = 47 + \phi + \frac{\left(\log(a)\log_a(2) + \log(a)\log_a(5) - \psi_{\frac{10}{9}}^{(0)}(1)\right)^2}{(\log(a)\log_a(2) - 2\log(a)\log_a(3) + \log(a)\log_a(5))^2}$$

**Series representations:**

$$\begin{aligned}
& \frac{\left(\log(2) + \log(5) - \psi_{\frac{9}{10}}^{(0)}(1)\right)^2}{(\log(2) - 2\log(3) + \log(5))^2} + 47 + \phi = \\
& 47 + \phi + \left( 4i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + 2\log(z_0) - \psi_{\frac{9}{10}}^{(0)}(1) + \right. \\
& \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^{1+k} \left( (2-z_0)^k + (5-z_0)^k \right) z_0^{-k}}{k} \right)^2 / \\
& \left( 4i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + 2\log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^{1+k} \left( (2-z_0)^k + (5-z_0)^k \right) z_0^{-k}}{k} - \right. \\
& \quad \left. 2 \left( 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k} \right) \right)^2
\end{aligned}$$

$$\begin{aligned}
& \frac{\left(\log(2) + \log(5) - \psi_{\frac{9}{10}}^{(0)}(1)\right)^2}{(\log(2) - 2\log(3) + \log(5))^2} + 47 + \phi = \\
& 47 + \phi + \left( 2i\pi \left[ \frac{\arg(2-x)}{2\pi} \right] + 2i\pi \left[ \frac{\arg(5-x)}{2\pi} \right] + 2\log(x) - \psi_{\frac{9}{10}}^{(0)}(1) + \right. \\
& \quad \left. \sum_{k=1}^{\infty} \frac{(-1)^{1+k} \left( (2-x)^k + (5-x)^k \right) x^{-k}}{k} \right)^2 / \left( 2i\pi \left[ \frac{\arg(2-x)}{2\pi} \right] + \right. \\
& \quad \left. 2i\pi \left[ \frac{\arg(5-x)}{2\pi} \right] + 2\log(x) + \sum_{k=1}^{\infty} \frac{(-1)^{1+k} \left( (2-x)^k + (5-x)^k \right) x^{-k}}{k} - \right. \\
& \quad \left. 2 \left( 2i\pi \left[ \frac{\arg(3-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k} \right) \right)^2 \text{ for } x < 0
\end{aligned}$$



**Result:**

1.618249203738314188466133990871260931282050873179293907582...

1.6182492....

And:

$$\sqrt{\left[ \frac{1}{\left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \left( \frac{11\pi}{199+7} \right)^2} \times \frac{1}{1.394569 \times 10^{-27}} \right)} \right]} \times \sqrt{\left[ \frac{-\left( \left( 8.799876 \times 10^{49} \times 4\pi (2.070728 \times 10^{-54})^3 - (2.070728 \times 10^{-54})^2 \right) \right)}{6.67 \times 10^{-11}} \right]}$$

**Input interpretation:**

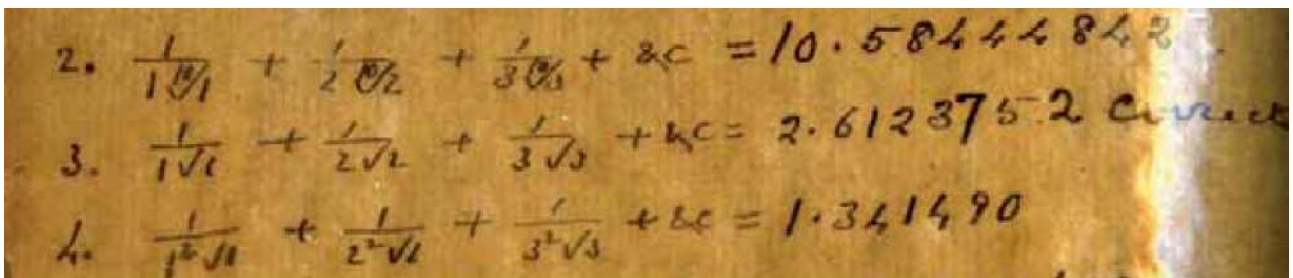
$$\sqrt{\left( 1 / \left( \frac{4 \times 1.962364415 \times 10^{19}}{5 \left( \frac{11\pi}{199+7} \right)^2} \times \frac{1}{1.394569 \times 10^{-27}} \right) \right)} \sqrt{\left( \frac{8.799876 \times 10^{49} \times 4\pi (2.070728 \times 10^{-54})^3 - (2.070728 \times 10^{-54})^2}{6.67 \times 10^{-11}} \right)}$$

**Result:**

3.141805906456086499777048095402389029165681664688965129415...

3.141805906456.....

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$$\left( \frac{1}{10^1} + \frac{1}{20^2} + \frac{1}{30^3} + \dots \right) = 10.58444842$$

**Input interpretation:**

$$\left( \frac{1}{10^1} + \frac{1}{20^2} + \frac{1}{30^3} \right) + \dots = 10.58444842$$

**Result:**

$$\sum_{n=1}^{\infty} \left( 1 + \frac{1}{2^{10\sqrt{2}}} + \frac{1}{3^{10\sqrt{3}}} \right) = 10.5844$$

10.5844

$$\left( \frac{1}{\sqrt{1}} + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} \right) + \dots = 2.6123752$$

**Input interpretation:**

$$\left( \frac{1}{\sqrt{1}} + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} \right) + \dots = 2.6123752$$

**Result:**

$$\sum_{n=1}^{\infty} \left( 1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} \right) = 2.61238$$

2.61238 (about equal Planck Area)

$$\frac{1}{(1^2\sqrt{1})} + \frac{1}{(2^2\sqrt{2})} + \frac{1}{(3^2\sqrt{3})} + \dots$$

**Input interpretation:**

$$\frac{1}{1^2\sqrt{1}} + \frac{1}{2^2\sqrt{2}} + \frac{1}{3^2\sqrt{3}} + \dots$$

**Infinite sum:**

$$\sum_{n=1}^{\infty} \frac{1}{n^{5/2}} = \zeta\left(\frac{5}{2}\right)$$

$\zeta(s)$  is the Riemann zeta function

**Decimal approximation:**

1.341487257250917179756769693348612136623037629505986511253...

1.3414872572...

**Convergence tests:**

The ratio test is inconclusive.

The root test is inconclusive.

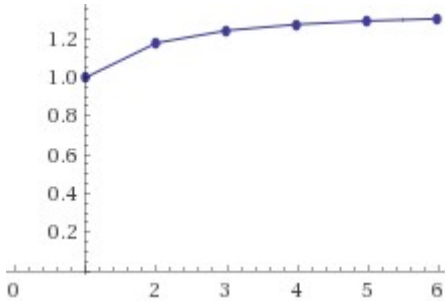
By the comparison test, the series converges.

**Partial sum formula:**

$$\sum_{n=1}^m \frac{1}{n^{5/2}} = H_m^{(5/2)}$$

$H_n^{(r)}$  is the generalized harmonic number

**Partial sums:**



**Series representations:**

$$\zeta\left(\frac{5}{2}\right) = \sum_{k=1}^{\infty} \frac{1}{k^{5/2}}$$

$$\zeta\left(\frac{5}{2}\right) = \frac{4 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^{5/2}}}{-4 + \sqrt{2}}$$

$$\zeta\left(\frac{5}{2}\right) = -\frac{8 \sum_{k=0}^{\infty} \frac{1}{(1+2k)^{5/2}}}{-8 + \sqrt{2}}$$

$$\zeta\left(\frac{5}{2}\right) = e^{\sum_{k=1}^{\infty} P\left(\frac{5k}{2}\right)/k}$$

$$\frac{1}{1^2 \sqrt{1}} + \frac{1}{2^2 \sqrt{2}} + \frac{1}{3^2 \sqrt{3}} + \frac{1}{4^2 \sqrt{4}} + \frac{1}{5^2 \sqrt{5}} + \frac{1}{6^2 \sqrt{6}} + \frac{1}{7^2 \sqrt{7}} + \frac{1}{8^2 \sqrt{8}} + \frac{1}{9^2 \sqrt{9}} + \frac{1}{10^2 \sqrt{10}}$$

**Input:**

$$\frac{1}{1^2 \sqrt{1}} + \frac{1}{2^2 \sqrt{2}} + \frac{1}{3^2 \sqrt{3}} + \frac{1}{4^2 \sqrt{4}} + \frac{1}{5^2 \sqrt{5}} + \frac{1}{6^2 \sqrt{6}} + \frac{1}{7^2 \sqrt{7}} + \frac{1}{8^2 \sqrt{8}} + \frac{1}{9^2 \sqrt{9}} + \frac{1}{10^2 \sqrt{10}}$$

**Result:**

$$\frac{8051}{7776} + \frac{33}{128 \sqrt{2}} + \frac{1}{9 \sqrt{3}} + \frac{1}{25 \sqrt{5}} + \frac{1}{36 \sqrt{6}} + \frac{1}{49 \sqrt{7}} + \frac{1}{100 \sqrt{10}}$$



**Decimal approximation:**

1.321920835716551018567751309087971926680964238192181767849...

1.3219208357...

**Alternate forms:**

$$\frac{1}{2667168000} \left( 2761493000 + 343814625\sqrt{2} + 98784000\sqrt{3} + 21337344\sqrt{5} + 12348000\sqrt{6} + 7776000\sqrt{7} + 2667168\sqrt{10} \right)$$

$$\frac{1}{49\sqrt{7}} +$$

$$\frac{8051000 + 1002375\sqrt{2} + 288000\sqrt{3} + 62208\sqrt{5} + 36000\sqrt{6} + 7776\sqrt{10}}{7776000}$$

$$\frac{33\sqrt{2}}{256} + \frac{\sqrt{3}}{27} + \frac{\sqrt{5}}{125} + \frac{\sqrt{6}}{216} + \frac{\sqrt{7}}{343} + \frac{\sqrt{10}}{1000} + \frac{8051}{7776}$$

We can calculate also:

$$\left( \left( \frac{1}{1^{1/10}} + \frac{1}{2^{1/10}} + \frac{1}{3^{1/10}} \right) \right) + \left( \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \right) + \left( \frac{1}{1^2\sqrt{1}} + \frac{1}{2^2\sqrt{2}} + \frac{1}{3^2\sqrt{3}} \right)$$

**Input:**

$$\left( \frac{1}{1^{10}\sqrt{1}} + \frac{1}{2^{10}\sqrt{2}} + \frac{1}{3^{10}\sqrt{3}} \right) + \left( \frac{1}{1\sqrt{1}} + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} \right) + \left( \frac{1}{1^2\sqrt{1}} + \frac{1}{2^2\sqrt{2}} + \frac{1}{3^2\sqrt{3}} \right)$$

**Result:**

$$3 + \frac{3}{4\sqrt{2}} + \frac{1}{2^{10}\sqrt{2}} + \frac{4}{9\sqrt{3}} + \frac{1}{3^{10}\sqrt{3}}$$

**Decimal approximation:**

4.552099521245068755915584481957471514940738892243578367035...

4.552099521...

**Alternate forms:**

$$\frac{1}{216} \left( 648 + 81\sqrt{2} + 54 \times 2^{9/10} + 32\sqrt{3} + 24 \times 3^{9/10} \right)$$

$$\frac{4}{9\sqrt{3}} + \frac{1}{3^{10}\sqrt{3}} + \frac{1}{8} \left( 24 + 3\sqrt{2} + 2 \times 2^{9/10} \right)$$

$$\frac{3\sqrt{2}}{8} + \frac{2^{9/10}}{4} + \frac{4\sqrt{3}}{27} + \frac{3^{9/10}}{9} + 3$$

$$4 * ((((((1/(1(1)^{(1/10)})) + 1/(2(2)^{(1/10)})) + 1/(3(3)^{(1/10)})))))) + ((1/(1\sqrt{1}) + 1/(2\sqrt{2}) + 1/(3\sqrt{3}))) + (((1/(1^2\sqrt{1}) + 1/(2^2\sqrt{2}) + 1/(3^2\sqrt{3}))))))^{4+11}$$

**Input:**

$$4 \left( \left( \frac{1}{1^{10}\sqrt{1}} + \frac{1}{2^{10}\sqrt{2}} + \frac{1}{3^{10}\sqrt{3}} \right) + \left( \frac{1}{1\sqrt{1}} + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} \right) + \left( \frac{1}{1^2\sqrt{1}} + \frac{1}{2^2\sqrt{2}} + \frac{1}{3^2\sqrt{3}} \right) \right)^4 + 11$$

**Exact result:**

$$11 + 4 \left( 3 + \frac{3}{4\sqrt{2}} + \frac{1}{2^{10}\sqrt{2}} + \frac{4}{9\sqrt{3}} + \frac{1}{3^{10}\sqrt{3}} \right)^4$$

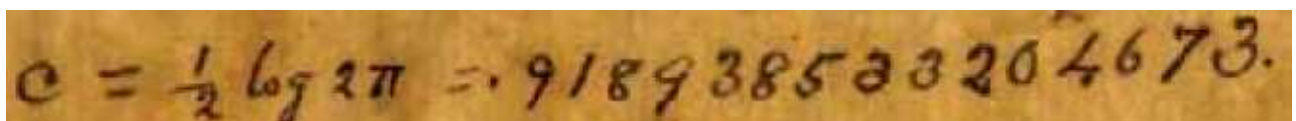
**Decimal approximation:**

1728.540492475795279443106679782349021660859345491293082454...

1728.5404924...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

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$1/2 \ln(2\pi)$

**Input:**

$$\frac{1}{2} \log(2\pi)$$

$\log(x)$  is the natural logarithm

**Decimal approximation:**

0.918938533204672741780329736405617639861397473637783412817...

0.9189385332046...

**Alternate forms:**

$$\frac{1}{2} (\log(2) + \log(\pi))$$

$$\frac{\log(2)}{2} + \frac{\log(\pi)}{2}$$

**Alternative representations:**

$$\frac{1}{2} \log(2\pi) = \frac{\log_e(2\pi)}{2}$$

$$\frac{1}{2} \log(2\pi) = \frac{1}{2} \log(a) \log_a(2\pi)$$

$$\frac{1}{2} \log(2\pi) = -\frac{1}{2} \text{Li}_1(1 - 2\pi)$$

**Series representations:**

$$\frac{1}{2} \log(2\pi) = \frac{1}{2} \log(-1 + 2\pi) - \frac{1}{2} \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-2\pi}\right)^k}{k}$$

$$\frac{1}{2} \log(2\pi) = i\pi \left[ \frac{\arg(2\pi - x)}{2\pi} \right] + \frac{\log(x)}{2} - \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (2\pi - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\frac{1}{2} \log(2\pi) = i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \frac{\log(z_0)}{2} - \frac{1}{2} \sum_{k=1}^{\infty} \frac{(-1)^k (2\pi - z_0)^k z_0^{-k}}{k}$$

### Integral representations:

$$\frac{1}{2} \log(2\pi) = \frac{1}{2} \int_1^{2\pi} \frac{1}{t} dt$$

$$\frac{1}{2} \log(2\pi) = -\frac{i}{4\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1+2\pi)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$$\left(\frac{1}{2} \ln(2\pi)\right)^{1/8}$$

### Input:

$$\sqrt[8]{\frac{1}{2} \log(2\pi)}$$

$\log(x)$  is the natural logarithm

### Decimal approximation:

0.989488629253083908737611692221593277240610962414002716072...

0.989488629253... result practically equal to the dilaton value **0.989117352243 =  $\phi$**

### Alternate form:

$$\sqrt[8]{\frac{1}{2} (\log(2) + \log(\pi))}$$

### All 8th roots of $\frac{1}{2} \log(2\pi)$ :

$$e^0 \sqrt[8]{\frac{1}{2} \log(2\pi)} \approx 0.98949 \quad (\text{real, principal root})$$

$$e^{(i\pi)/4} \sqrt[8]{\frac{1}{2} \log(2\pi)} \approx 0.69967 + 0.69967 i$$

$$e^{(i\pi)/2} \sqrt[8]{\frac{1}{2} \log(2\pi)} \approx 0.98949 i$$

$$e^{(3i\pi)/4} \sqrt[8]{\frac{1}{2} \log(2\pi)} \approx -0.6997 + 0.69967 i$$

$$e^{i\pi} \sqrt[8]{\frac{1}{2} \log(2\pi)} \approx -0.9895 \quad (\text{real root})$$

### Alternative representations:

$$\sqrt[8]{\frac{1}{2} \log(2\pi)} = \sqrt[8]{\frac{\log_e(2\pi)}{2}}$$

$$\sqrt[8]{\frac{1}{2} \log(2\pi)} = \sqrt[8]{\frac{1}{2} \log(a) \log_a(2\pi)}$$

$$\sqrt[8]{\frac{1}{2} \log(2\pi)} = \sqrt[8]{-\frac{1}{2} \text{Li}_1(1-2\pi)}$$

### Series representations:

$$\sqrt[8]{\frac{1}{2} \log(2\pi)} = \frac{\sqrt[8]{\log(-1+2\pi) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-2\pi}\right)^k}{k}}}{\sqrt[8]{2}}$$

$$\sqrt[8]{\frac{1}{2} \log(2\pi)} = \frac{\sqrt[8]{2i\pi \left[ \frac{\text{arg}(2\pi-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2\pi-x)^k x^{-k}}{k}}}{\sqrt[8]{2}} \quad \text{for } x < 0$$

$$\sqrt[8]{\frac{1}{2} \log(2\pi)} = \frac{\sqrt[8]{\log(z_0) + \left[ \frac{\text{arg}(2\pi-z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2\pi-z_0)^k z_0^{-k}}{k}}}{\sqrt[8]{2}}$$

### Integral representations:

$$\sqrt[8]{\frac{1}{2} \log(2\pi)} = \frac{\sqrt[8]{\int_1^{2\pi} \frac{1}{t} dt}}{\sqrt[8]{2}}$$

$$\sqrt[8]{\frac{1}{2} \log(2\pi)} = \frac{\sqrt[8]{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1+2\pi)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}}{\sqrt[4]{2} \sqrt[8]{\pi}} \quad \text{for } -1 < \gamma < 0$$

16\*log base 0.98948862925((1/2 ln(2Pi)))-Pi+1/golden ratio

**Input interpretation:**

$$16 \log_{0.98948862925} \left( \frac{1}{2} \log(2 \pi) \right) - \pi + \frac{1}{\phi}$$

$\log(x)$  is the natural logarithm

$\log_b(x)$  is the base-  $b$  logarithm

$\phi$  is the golden ratio

**Result:**

125.476441...

125.476441... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2 \pi) \right) - \pi + \frac{1}{\phi} = -\pi + 16 \log_{0.989488629250000} \left( \frac{\log_e(2 \pi)}{2} \right) + \frac{1}{\phi}$$

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2 \pi) \right) - \pi + \frac{1}{\phi} = -\pi + \frac{1}{\phi} + \frac{16 \log\left(\frac{1}{2} \log(2 \pi)\right)}{\log(0.989488629250000)}$$

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2 \pi) \right) - \pi + \frac{1}{\phi} = -\pi + 16 \log_{0.989488629250000} \left( \frac{1}{2} \log(a) \log_a(2 \pi) \right) + \frac{1}{\phi}$$

**Series representations:**

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2 \pi) \right) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi - \frac{16 \sum_{k=1}^{\infty} \frac{\left(\frac{-1}{2}\right)^k (-2 + \log(2 \pi))^k}{k}}{\log(0.989488629250000)}$$

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2 \pi) \right) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 16 \log_{0.989488629250000} \left( \frac{1}{2} \left( \log(-1 + 2 \pi) - \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + 2 \pi)^{-k}}{k} \right) \right)$$

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2\pi) \right) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 16 \log_{0.989488629250000} \left( \frac{1}{2} \left( 2i\pi \left\lfloor \frac{\arg(2\pi - x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2\pi - x)^k x^{-k}}{k} \right) \right) \text{ for } x < 0$$

**Integral representations:**

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2\pi) \right) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 16 \log_{0.989488629250000} \left( \frac{1}{2} \int_1^{2\pi} \frac{1}{t} dt \right)$$

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2\pi) \right) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 16 \log_{0.989488629250000} \left( \frac{1}{4i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1+2\pi)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \text{ for } -1 < \gamma < 0$$

16\*log base 0.98948862925((1/2 ln(2Pi)))+11+1/golden ratio

**Input interpretation:**

$$16 \log_{0.98948862925} \left( \frac{1}{2} \log(2\pi) \right) + 11 + \frac{1}{\phi}$$

$\log(x)$  is the natural logarithm

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

139.618034...

139.618034... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2\pi) \right) + 11 + \frac{1}{\phi} = 11 + 16 \log_{0.989488629250000} \left( \frac{\log_e(2\pi)}{2} \right) + \frac{1}{\phi}$$

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2\pi) \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} + \frac{16 \log\left(\frac{1}{2} \log(2\pi)\right)}{\log(0.989488629250000)}$$

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2\pi) \right) + 11 + \frac{1}{\phi} =$$

$$11 + 16 \log_{0.989488629250000} \left( \frac{1}{2} \log(a) \log_a(2\pi) \right) + \frac{1}{\phi}$$

### Series representations:

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2\pi) \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} - \frac{16 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k (-2+\log(2\pi))^k}{k}}{\log(0.989488629250000)}$$

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2\pi) \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + 16 \log_{0.989488629250000} \left( \frac{1}{2} \left( \log(-1+2\pi) - \sum_{k=1}^{\infty} \frac{(-1)^k (-1+2\pi)^{-k}}{k} \right) \right)$$

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2\pi) \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} + 16 \log_{0.989488629250000} \left( \frac{1}{2} \left( 2i\pi \left\lfloor \frac{\arg(2\pi-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2\pi-x)^k x^{-k}}{k} \right) \right) \text{ for } x < 0$$

### Integral representations:

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2\pi) \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + 16 \log_{0.989488629250000} \left( \frac{1}{2} \int_1^{2\pi} \frac{1}{t} dt \right)$$

$$16 \log_{0.989488629250000} \left( \frac{1}{2} \log(2\pi) \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} +$$

$$16 \log_{0.989488629250000} \left( \frac{1}{4i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1+2\pi)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \text{ for } -1 < \gamma < 0$$



$$(64 \times 7) \left( \frac{1}{2} \ln(2\pi) \right) \phi^3 - 18 + \pi$$

**Input:**

$$(64 \times 7) \left( \frac{1}{2} \log(2\pi) \right) \phi^3 - 18 + \pi$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Exact result:**

$$224 \phi^3 \log(2\pi) - 18 + \pi$$

**Decimal approximation:**

1729.064962675515539878238424424381187084359484818054507853...

1729.0649626...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternate forms:**

$$-18 + \pi + 28 \left( 1 + \sqrt{5} \right)^3 (\log(2) + \log(\pi))$$

$$-18 + \pi + 448 \log(2\pi) + 224 \sqrt{5} \log(2\pi)$$

$$\pi + 2 \left( -9 + 224 \log(2\pi) + 112 \sqrt{5} \log(2\pi) \right)$$

**Alternative representations:**

$$\frac{1}{2} (\log(2\pi) \phi^3) 64 \times 7 - 18 + \pi = -18 + \pi + 224 \log_e(2\pi) \phi^3$$

$$\frac{1}{2} (\log(2\pi) \phi^3) 64 \times 7 - 18 + \pi = -18 + \pi + 224 \log(a) \log_a(2\pi) \phi^3$$

$$\frac{1}{2} (\log(2\pi) \phi^3) 64 \times 7 - 18 + \pi = -18 + \pi - 224 \text{Li}_1(1 - 2\pi) \phi^3$$

**Series representations:**

$$\frac{1}{2} (\log(2\pi) \phi^3) 64 \times 7 - 18 + \pi = -18 + \pi + 224 \phi^3 \log(-1 + 2\pi) - 224 \phi^3 \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-2\pi}\right)^k}{k}$$

$$\frac{1}{2} (\log(2\pi) \phi^3) 64 \times 7 - 18 + \pi = -18 + \pi + 448 i \phi^3 \pi \left[ \frac{\arg(2\pi - x)}{2\pi} \right] + 224 \phi^3 \log(x) - 224 \phi^3 \sum_{k=1}^{\infty} \frac{(-1)^k (2\pi - x)^k x^{-k}}{k} \text{ for } x < 0$$

$$\frac{1}{2} (\log(2\pi) \phi^3) 64 \times 7 - 18 + \pi = -18 + \pi + 448 i \phi^3 \pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + 224 \phi^3 \log(z_0) - 224 \phi^3 \sum_{k=1}^{\infty} \frac{(-1)^k (2\pi - z_0)^k z_0^{-k}}{k}$$

**Integral representations:**

$$\frac{1}{2} (\log(2\pi) \phi^3) 64 \times 7 - 18 + \pi = -18 + \pi + 224 \phi^3 \int_1^{2\pi} \frac{1}{t} dt$$

$$\frac{1}{2} (\log(2\pi) \phi^3) 64 \times 7 - 18 + \pi = -18 + \pi - \frac{112 i \phi^3}{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1 + 2\pi)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

$$(64 \times 7) \left( \frac{1}{2} \ln(2\pi) \right) \phi^3 + \pi + 29 + 7$$

**Input:**

$$(64 \times 7) \left( \frac{1}{2} \log(2\pi) \right) \phi^3 + \pi + 29 + 7$$

$\log(x)$  is the natural logarithm

$\phi$  is the golden ratio

**Exact result:**

$$224 \phi^3 \log(2\pi) + 36 + \pi$$

### Decimal approximation:

1783.064962675515539878238424424381187084359484818054507853...

1783.0649626...

result in the range of the hypothetical mass of Gluino (gluino = 1785.16 GeV).

### Alternate forms:

$$36 + \pi + 28 \left(1 + \sqrt{5}\right)^3 (\log(2) + \log(\pi))$$

$$36 + \pi + 448 \log(2\pi) + 224 \sqrt{5} \log(2\pi)$$

$$\pi + 4 \left(9 + 112 \log(2\pi) + 56 \sqrt{5} \log(2\pi)\right)$$

### Alternative representations:

$$\frac{1}{2} (\log(2\pi) \phi^3) 64 \times 7 + \pi + 29 + 7 = 36 + \pi + 224 \log_e(2\pi) \phi^3$$

$$\frac{1}{2} (\log(2\pi) \phi^3) 64 \times 7 + \pi + 29 + 7 = 36 + \pi + 224 \log(\alpha) \log_\alpha(2\pi) \phi^3$$

$$\frac{1}{2} (\log(2\pi) \phi^3) 64 \times 7 + \pi + 29 + 7 = 36 + \pi - 224 \text{Li}_1(1 - 2\pi) \phi^3$$

### Series representations:

$$\frac{1}{2} (\log(2\pi) \phi^3) 64 \times 7 + \pi + 29 + 7 = 36 + \pi + 224 \phi^3 \log(-1 + 2\pi) - 224 \phi^3 \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-2\pi}\right)^k}{k}$$

$$\frac{1}{2} (\log(2\pi) \phi^3) 64 \times 7 + \pi + 29 + 7 = 36 + \pi + 448 i \phi^3 \pi \left[ \frac{\arg(2\pi - x)}{2\pi} \right] + 224 \phi^3 \log(x) - 224 \phi^3 \sum_{k=1}^{\infty} \frac{(-1)^k (2\pi - x)^k x^{-k}}{k} \text{ for } x < 0$$

$$\frac{1}{2} (\log(2\pi) \phi^3) 64 \times 7 + \pi + 29 + 7 = 36 + \pi + 448 i \phi^3 \pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + 224 \phi^3 \log(z_0) - 224 \phi^3 \sum_{k=1}^{\infty} \frac{(-1)^k (2\pi - z_0)^k z_0^{-k}}{k}$$









## Decimal approximation:

203.4226135479069615779272039359980621708749898980454152305...

203.42261354...

## Property:

$$\frac{e^{\sqrt{22/3} \pi} \sqrt{\frac{\phi}{22}}}{2 \times 5^{3/4}} + \phi^2 \text{ is a transcendental number}$$

## Alternate forms:

$$\frac{1}{2} (3 + \sqrt{5}) + \frac{1}{20} \sqrt{\frac{1}{11} (5 + \sqrt{5})} e^{\sqrt{22/3} \pi}$$

$$\frac{1}{220} \left( 330 + 110 \sqrt{5} + \sqrt[4]{5} \sqrt{11 (1 + \sqrt{5})} e^{\sqrt{22/3} \pi} \right)$$

$$\frac{\sqrt{\frac{\phi}{22}} \left( 2 \times 5^{3/4} \sqrt{22} \phi^{3/2} + e^{\sqrt{22/3} \pi} \right)}{2 \times 5^{3/4}}$$

## Series representations:

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{110}{15}}\right)}{2 \sqrt[4]{5} \sqrt{110}} + \phi^2 = \left( 10 \phi^2 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (110 - z_0)^k z_0^{-k}}{k!} + \right. \\ \left. 5^{3/4} \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{22}{3} - z_0\right)^k z_0^{-k}}{k!}\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!} \right) / \\ \left( 10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (110 - z_0)^k z_0^{-k}}{k!} \right) \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$



$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{110}{15}}\right)}{2 \sqrt[4]{5} \sqrt{110}} + \phi^2 = \left( 10 \phi^2 \exp\left(i \pi \left[ \frac{\arg(110-x)}{2\pi} \right] \right) \sum_{k=0}^{\infty} \frac{(-1)^k (110-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \\ \left. 5^{3/4} \exp\left(i \pi \left[ \frac{\arg(\phi-x)}{2\pi} \right] \right) \exp\left(\pi \exp\left(i \pi \left[ \frac{\arg\left(\frac{22}{3}-x\right)}{2\pi} \right] \right) \sqrt{x} \right. \right. \\ \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{22}{3}-x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k (\phi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \\ \left( 10 \exp\left(i \pi \left[ \frac{\arg(110-x)}{2\pi} \right] \right) \sum_{k=0}^{\infty} \frac{(-1)^k (110-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)$$

for  $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{110}{15}}\right)}{2 \sqrt[4]{5} \sqrt{110}} + \phi^2 = \\ \left( \left( \frac{1}{z_0} \right)^{-1/2 [\arg(110-z_0)/(2\pi)]} z_0^{-1/2 [\arg(110-z_0)/(2\pi)]} \left( 10 \phi^2 \left( \frac{1}{z_0} \right)^{1/2 [\arg(110-z_0)/(2\pi)]} \right. \right. \\ \left. \left. z_0^{1/2 [\arg(110-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (110-z_0)^k z_0^{-k}}{k!} + \right. \right. \\ \left. \left. 5^{3/4} \exp\left(\pi \left( \frac{1}{z_0} \right)^{1/2 [\arg\left(\frac{22}{3}-z_0\right)/(2\pi)]} z_0^{1/2 (1+[\arg\left(\frac{22}{3}-z_0\right)/(2\pi)])} \right. \right. \right. \\ \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{22}{3}-z_0\right)^k z_0^{-k}}{k!} \right) \left( \frac{1}{z_0} \right)^{1/2 [\arg(\phi-z_0)/(2\pi)]} z_0^{1/2 [\arg(\phi-z_0)/(2\pi)]} \right. \\ \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi-z_0)^k z_0^{-k}}{k!} \right) \right) / \left( 10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (110-z_0)^k z_0^{-k}}{k!} \right)$$

We note that, with the measures, after some calculations, we obtain:

**Result:**

203.1792 kg s/m (kilogram seconds per meter)

203.1792 Kg s/m

Inserting the mass in kg in the Hawking radiation calculator, considering this mass as a quantum black hole, we obtain:

$$\text{Mass} = 203.1792$$

$$\text{Radius} = 3.016909e-25$$

$$\text{Temperature} = 6.040004e+20$$

From the Ramanujan-Nardelli mock formula, we obtain:

$$\text{sqrt}[\text{[[[1/(((((((4*1.962364415e+19)/(5*0.0864055^2))))*1/(203.1792)* \text{sqrt}[-(((6.040004e+20 * 4*\text{Pi}*(3.016909e-25)^3-(3.016909e-25)^2)))] / ((6.67*10^-11)))]]]]]]$$

**Input interpretation:**

$$\sqrt{\frac{1}{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^2} \times \frac{1}{203.1792} \sqrt{-\frac{6.040004 \times 10^{20} \times 4 \pi (3.016909 \times 10^{-25})^3 - (3.016909 \times 10^{-25})^2}{6.67 \times 10^{-11}}}}}$$

**Result:**

1.618249360560957963462419751385261137337036408185774860860...

[1.61824936056095.....](#)

We have also:

$$(1.10563853320467274178 \times 10^{-52} / 9 \times 10^{16}) * 1 / (2.6123752 * 6.331009)^{-59}$$

**Input interpretation:**

$$\frac{1.10563853320467274178 \times 10^{-52}}{9 \times 10^{16}} \times \frac{1}{\frac{1}{(2.6123752 \times 6.331009)^{59}}}$$

**Result:**

958.0124688937313321811071525838001425727656747213008038039...

[958.012468.... result very near to the rest mass of Eta prime meson 957.78](#)



We have that:

For generic choices of  $\psi_1(r)$ ,  $\omega_n$ ,  $i_n$ ,  $r_0$  and  $\alpha$

$\Upsilon$  is an undetermined constant

Here  $\alpha$  is proportional to the angular momentum and is in the range  $0 \leq \alpha \leq 1$ . The Kerr black hole is stationary and axisymmetric.

For:

**Input interpretation:**

$5.9 \times 10^{-4}$  pc (parsecs)

**Unit conversions:**

0.001924 ly (light years)

121.7 au (astronomical units)

$1.821 \times 10^{10}$  km (kilometers)

$$r_0 = 1.821e+10$$

$$r = 1.63161e+20$$

$$\alpha = J/(GM^2)$$

$$0.90 / (6.674e-11 * (13.12806e+39)^2)$$

**Input interpretation:**

$$\frac{0.9}{6.674 \times 10^{-11} (13.12806 \times 10^{39})^2}$$

**Result:**

$$7.8244748915763397308510282300423879248157617983986097... \times 10^{-71}$$

$$7.8244748915... * 10^{-71}$$







We know that:

$$\alpha = J/(GM^2)$$

$$\frac{0.9}{6.674 \times 10^{-11} (13.12806 \times 10^{39})^2}$$

$$7.8244748915763397308510282300423879248157617983986097... \times 10^{-71}$$

$$7.8244748915... * 10^{-71}$$

and  $r_0 = 1.821e+10$

Now, we have that:

$$1/10^{27}(((((((((((1.821e+10 * 1/7.8244748915e-71 * [(1/12)^2 * (1+1)*(((6/7*7.82447489e-71 * 5 * ((1.821e+10)/(1.63161e+20)) \ln((1.63161e+20)/(1.821e+10)))))))])))))^{1/2} + 7/10^3))))))$$

**Input interpretation:**

$$\frac{1}{10^{27}} \left( \sqrt{ \left( 1.821 \times 10^{10} \times \frac{1}{7.8244748915 \times 10^{-71}} \left( \left( \frac{1}{12} \right)^2 (1+1) \left( \frac{6}{7} \times 7.82447489 \times 10^{-71} \times 5 \times \frac{1.821 \times 10^{10}}{1.63161 \times 10^{20}} \log \left( \frac{1.63161 \times 10^{20}}{1.821 \times 10^{10}} \right) \right) \right) + \frac{7}{10^3} \right) } \right)$$

log(x) is the natural logarithm

**Result:**

$$1.6720099737536994425065337723316150604342393680263729... \times 10^{-27}$$

1.67200997375... \* 10<sup>-27</sup> result practically equal to the proton mass in kg



## Observations

The reason why inserting any mass, temperature and radius of a black hole, from the quantum to the supermassive one, is ALWAYS the golden ratio as a result, would seem to lie in the intrinsic spiral rotation in the black holes. The novelty in the calculations carried out in this paper is that with the same formula (Ramanujan-Nardelli mock formula), we obtain always by entering the above parameters, the value of  $\pi$ . Note that in this formula there are numbers belonging to the succession of Lucas and / or to that of Fibonacci, both linked to  $\phi$ . These constants are connected to black holes:  $\pi$  and "e" are related to the geometry of these celestial bodies, Planck's length to their quantum nature and the Cosmological Constant is connected to dark energy which, according to some studies, it would also be related to black holes. Finally, we remember that black holes are the central and fundamental part in the formation and evolution of a galaxy. The galaxies themselves are connected to  $\pi$  and  $\phi$ , being of elliptical or spiral form (logarithmic-golden spiral) and also in the black holes in the center of them, as can be seen from the figure, the trace of the two fundamental physical-mathematical constants  $\pi$  and  $\phi$ , is evident.

Finally, it should be highlighted how all Ramanujan's expressions are developed using ALWAYS numbers belonging to the Lucas and / or Fibonacci sequences connected strictly to the golden ratio, in addition to  $\pi$  and the golden ratio itself.

<https://www.cambridgesciencefestival.org/event/photographing-black-holes-first-results-from-the-event-horizon-telescope/>



Fig. Black Hole (SMBH87)

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## **References**

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