

# **On some formulas of Manuscript Book 1 of Srinivasa Ramanujan: new possible mathematical connections with various parameters of Particle Physics and Cosmology part II.**

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## **Abstract**

*In this research thesis, we have analyzed further formulas of Manuscript Book 1 of Srinivasa Ramanujan and described new possible mathematical connections with various parameters of Particle Physics and Cosmology (Cosmological Constant, some parameters of Dark Energy)*

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*I have not trodden through a conventional university course, but I am striking out a new path for myself. I have made a special investigation of divergent series in general and the results I get are termed by the local mathematicians as 'startling.'* Srinivasa Ramanujan



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## Summary

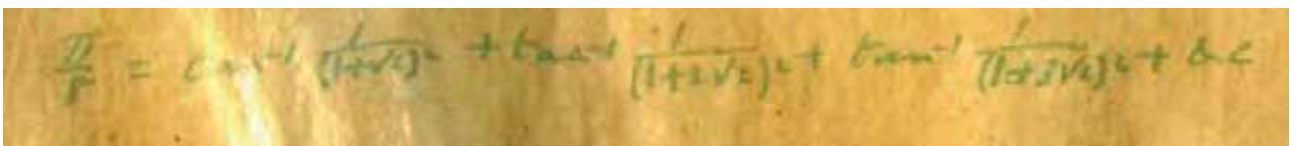
In this research thesis, we have analyzed the possible and new connections between different formulas of Manuscript Book 1 of Srinivasa Ramanujan and some parameters concerning Particle Physics and Cosmology. In the course of the discussion we describe and highlight the connections between some developments of Ramanujan equations and particles type solutions such as the mass of the Higgs boson, and the masses of other baryons and mesons, principally  $f_0(1710)$  scalar meson candidate “glueball”. Moreover, solutions of Ramanujan equations, connected with the mass of the  $\pi$  meson 139.57 have been described and highlighted. Furthermore, we have obtained also the values of some black hole entropies, the value of the Cosmological Constant and some parameters of Dark Energy.

All the results of the most important connections are highlighted in blue throughout the drafting of the paper

**From:**

**MANUSCRIPT BOOK 1 OF SRINIVASA RAMANUJAN**

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$$\tan^{-1}\left(\frac{1}{(1+\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+2\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+3\sqrt{2})^2}\right) + \dots$$

**Input interpretation:**

$$\tan^{-1}\left(\frac{1}{(1+\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+2\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+3\sqrt{2})^2}\right) + \dots$$

$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

$$\sum_{n=0}^{\infty} \tan^{-1}\left(\frac{1}{(\sqrt{2}n + \sqrt{2} + 1)^2}\right) \approx \sum_{n=0}^{\infty} \tan^{-1}\left(\frac{1}{(1.41421n + 2.41421)^2}\right)$$

**Approximated sum:**

$$\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{1}{(1 + \sqrt{2}n)^2}\right) \approx 0.390214$$

$$0.390214 \approx \frac{\pi}{8} = 0.392699081698$$

If we take

$$\tan^{-1}\left(\frac{1}{(1+\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+2\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+3\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+4\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+5\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+6\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+12\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+24\sqrt{2})^2}\right)$$

we obtain:

**Input:**

$$\begin{aligned} & \tan^{-1}\left(\frac{1}{(1+\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+2\sqrt{2})^2}\right) + \\ & \tan^{-1}\left(\frac{1}{(1+3\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+4\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+5\sqrt{2})^2}\right) + \\ & \tan^{-1}\left(\frac{1}{(1+6\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+12\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+24\sqrt{2})^2}\right) \end{aligned}$$

$\tan^{-1}(x)$  is the inverse tangent function

**Exact Result:**

$$\begin{aligned} & \tan^{-1}\left(\frac{1}{(1+\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+2\sqrt{2})^2}\right) + \\ & \tan^{-1}\left(\frac{1}{(1+3\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+4\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+5\sqrt{2})^2}\right) + \\ & \tan^{-1}\left(\frac{1}{(1+6\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+12\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+24\sqrt{2})^2}\right) \end{aligned}$$

(result in radians)

### Decimal approximation:

0.327349706812720645444438112798192882967536628173104216641...

(result in radians)

0.327349706...

### Alternate forms:

$$\tan^{-1}\left(\frac{190\,969\,859\,251\sqrt{2} - 216\,845\,894\,344}{156\,747\,794\,826}\right)$$

$$\begin{aligned} & \cot^{-1}\left((1 + \sqrt{2})^2\right) + \cot^{-1}\left((1 + 2\sqrt{2})^2\right) + \\ & \cot^{-1}\left((1 + 3\sqrt{2})^2\right) + \cot^{-1}\left((1 + 4\sqrt{2})^2\right) + \cot^{-1}\left((1 + 5\sqrt{2})^2\right) + \\ & \cot^{-1}\left((1 + 6\sqrt{2})^2\right) + \cot^{-1}\left((1 + 12\sqrt{2})^2\right) + \cot^{-1}\left((1 + 24\sqrt{2})^2\right) \end{aligned}$$

$$\begin{aligned} & \tan^{-1}\left(\frac{1153 - 48\sqrt{2}}{1\,324\,801}\right) + \tan^{-1}\left(\frac{289 - 24\sqrt{2}}{82\,369}\right) + \\ & \tan^{-1}\left(\frac{73 - 12\sqrt{2}}{5041}\right) + \tan^{-1}\left(\frac{51 - 10\sqrt{2}}{2401}\right) + \tan^{-1}\left(\frac{1}{961}(33 - 8\sqrt{2})\right) + \\ & \tan^{-1}\left(\frac{1}{289}(19 - 6\sqrt{2})\right) + \tan^{-1}\left(\frac{1}{49}(9 - 4\sqrt{2})\right) + \tan^{-1}(3 - 2\sqrt{2}) \end{aligned}$$

$\cot^{-1}(x)$  is the inverse cotangent function

### Alternative representations:

$$\begin{aligned} & \tan^{-1}\left(\frac{1}{(1 + \sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1 + 2\sqrt{2})^2}\right) + \\ & \tan^{-1}\left(\frac{1}{(1 + 3\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1 + 4\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1 + 5\sqrt{2})^2}\right) + \\ & \tan^{-1}\left(\frac{1}{(1 + 6\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1 + 12\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1 + 24\sqrt{2})^2}\right) = \\ & \operatorname{sc}^{-1}\left(\frac{1}{(1 + \sqrt{2})^2} \middle| 0\right) + \operatorname{sc}^{-1}\left(\frac{1}{(1 + 2\sqrt{2})^2} \middle| 0\right) + \operatorname{sc}^{-1}\left(\frac{1}{(1 + 3\sqrt{2})^2} \middle| 0\right) + \\ & \operatorname{sc}^{-1}\left(\frac{1}{(1 + 4\sqrt{2})^2} \middle| 0\right) + \operatorname{sc}^{-1}\left(\frac{1}{(1 + 5\sqrt{2})^2} \middle| 0\right) + \operatorname{sc}^{-1}\left(\frac{1}{(1 + 6\sqrt{2})^2} \middle| 0\right) + \\ & \operatorname{sc}^{-1}\left(\frac{1}{(1 + 12\sqrt{2})^2} \middle| 0\right) + \operatorname{sc}^{-1}\left(\frac{1}{(1 + 24\sqrt{2})^2} \middle| 0\right) \end{aligned}$$

$$\begin{aligned} & \tan^{-1}\left(\frac{1}{(1+\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+2\sqrt{2})^2}\right) + \\ & \tan^{-1}\left(\frac{1}{(1+3\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+4\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+5\sqrt{2})^2}\right) + \\ & \tan^{-1}\left(\frac{1}{(1+6\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+12\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+24\sqrt{2})^2}\right) = \\ & \tan^{-1}\left(1, \frac{1}{(1+\sqrt{2})^2}\right) + \tan^{-1}\left(1, \frac{1}{(1+2\sqrt{2})^2}\right) + \tan^{-1}\left(1, \frac{1}{(1+3\sqrt{2})^2}\right) + \\ & \tan^{-1}\left(1, \frac{1}{(1+4\sqrt{2})^2}\right) + \tan^{-1}\left(1, \frac{1}{(1+5\sqrt{2})^2}\right) + \tan^{-1}\left(1, \frac{1}{(1+6\sqrt{2})^2}\right) + \\ & \tan^{-1}\left(1, \frac{1}{(1+12\sqrt{2})^2}\right) + \tan^{-1}\left(1, \frac{1}{(1+24\sqrt{2})^2}\right) \end{aligned}$$

$$\begin{aligned} & \tan^{-1}\left(\frac{1}{(1+\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+2\sqrt{2})^2}\right) + \\ & \tan^{-1}\left(\frac{1}{(1+3\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+4\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+5\sqrt{2})^2}\right) + \\ & \tan^{-1}\left(\frac{1}{(1+6\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+12\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+24\sqrt{2})^2}\right) = \\ & i \tanh^{-1}\left(-\frac{i}{(1+\sqrt{2})^2}\right) + i \tanh^{-1}\left(-\frac{i}{(1+2\sqrt{2})^2}\right) + i \tanh^{-1}\left(-\frac{i}{(1+3\sqrt{2})^2}\right) + \\ & i \tanh^{-1}\left(-\frac{i}{(1+4\sqrt{2})^2}\right) + i \tanh^{-1}\left(-\frac{i}{(1+5\sqrt{2})^2}\right) + i \tanh^{-1}\left(-\frac{i}{(1+6\sqrt{2})^2}\right) + \\ & i \tanh^{-1}\left(-\frac{i}{(1+12\sqrt{2})^2}\right) + i \tanh^{-1}\left(-\frac{i}{(1+24\sqrt{2})^2}\right) \end{aligned}$$

From the previous expression, we obtain:

$$1/(((\tan^{-1}(1/(1+\sqrt{2})^2)+\tan^{-1}(1/(1+2\sqrt{2})^2)+\tan^{-1}(1/(1+3\sqrt{2})^2)+\dots)))$$

**Input interpretation:**

$$\frac{1}{\tan^{-1}\left(\frac{1}{(1+\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+2\sqrt{2})^2}\right) + \tan^{-1}\left(\frac{1}{(1+3\sqrt{2})^2}\right) + \dots}$$

$\tan^{-1}(x)$  is the inverse tangent function

**Results:**

$$\frac{1}{\sum_{n=0}^{\infty} \tan^{-1}\left(\frac{1}{(\sqrt{2}n+\sqrt{2}+1)^2}\right)}$$

$$1/\left(\sum_{n=0}^{\infty} \tan^{-1}\left(\frac{1}{(\sqrt{2}n + \sqrt{2} + 1)^2}\right)\right)$$

**Input interpretation:**

$$\frac{1}{\sum_{n=0}^{\infty} \tan^{-1}\left(\frac{1}{(\sqrt{2}n + \sqrt{2} + 1)^2}\right)}$$

$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

2.54648

2.54648

$$\left(\left(\frac{1}{\sum_{n=0}^{\infty} \tan^{-1}\left(\frac{1}{(\sqrt{2}n + \sqrt{2} + 1)^2}\right)}\right)\right)^5 + 29 + \pi$$

Where 29 is a Lucas number

**Input interpretation:**

$$\left(\frac{1}{\sum_{n=0}^{\infty} \tan^{-1}\left(\frac{1}{(\sqrt{2}n + \sqrt{2} + 1)^2}\right)}\right)^5 + 29 + \pi$$

$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

139.22

139.22 result practically equal to the rest mass of Pion meson 139.57 MeV

And:

$$\left(\left(\frac{1}{\sum_{n=0}^{\infty} \tan^{-1}\left(\frac{1}{(\sqrt{2}n + \sqrt{2} + 1)^2}\right)}\right)\right)^5 + 18$$

Where 18 is a Lucas number

**Input interpretation:**

$$\left( \frac{1}{\sum_{n=0}^{\infty} \tan^{-1} \left( \frac{1}{(\sqrt{2} n + \sqrt{2} + 1)^2} \right)} \right)^5 + 18$$

$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

125.078

125.078 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

From the following expression:

$$\tan^{-1}(1/(1+\sqrt{2})^2) + \tan^{-1}(1/(1+2\sqrt{2})^2) + \tan^{-1}(1/(1+3\sqrt{2})^2) + \tan^{-1}(1/(1+4\sqrt{2})^2) + \tan^{-1}(1/(1+5\sqrt{2})^2) + \tan^{-1}(1/(1+6\sqrt{2})^2) + \tan^{-1}(1/(1+12\sqrt{2})^2) + \tan^{-1}(1/(1+24\sqrt{2})^2)$$

we obtain:

$$\left[ 1 / \left( \left( \tan^{-1}(1/(1+\sqrt{2})^2) + \tan^{-1}(1/(1+2\sqrt{2})^2) + \tan^{-1}(1/(1+3\sqrt{2})^2) + \tan^{-1}(1/(1+4\sqrt{2})^2) + \tan^{-1}(1/(1+5\sqrt{2})^2) + \tan^{-1}(1/(1+6\sqrt{2})^2) + \tan^{-1}(1/(1+12\sqrt{2})^2) + \tan^{-1}(1/(1+24\sqrt{2})^2) \right) \right) \right]^5$$

**Input:**

$$\left( 1 / \left( \tan^{-1} \left( \frac{1}{(1+\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+2\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+3\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+4\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+5\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+6\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+12\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+24\sqrt{2})^2} \right) \right) \right)^5$$

$\tan^{-1}(x)$  is the inverse tangent function

**Exact Result:**



$$1 / \left( \tan^{-1} \left( \frac{1}{(1 + \sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1 + 2\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1 + 3\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1 + 4\sqrt{2})^2} \right) + \right. \\ \left. \tan^{-1} \left( \frac{1}{(1 + 5\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1 + 6\sqrt{2})^2} \right) + \right. \\ \left. \tan^{-1} \left( \frac{1}{(1 + 12\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1 + 24\sqrt{2})^2} \right) \right)^5$$

(result in radians)

### Decimal approximation:

266.0358831303531310292452360902628407164242284220609033941...

(result in radians)

266.03588313....

### Alternate forms:

$$\frac{1}{\tan^{-1} \left( \frac{190969859251\sqrt{2} - 216845894344}{156747794826} \right)^5}$$

$$1 / \left( \cot^{-1} \left( (1 + \sqrt{2})^2 \right) + \cot^{-1} \left( (1 + 2\sqrt{2})^2 \right) + \right. \\ \left. \cot^{-1} \left( (1 + 3\sqrt{2})^2 \right) + \cot^{-1} \left( (1 + 4\sqrt{2})^2 \right) + \cot^{-1} \left( (1 + 5\sqrt{2})^2 \right) + \right. \\ \left. \cot^{-1} \left( (1 + 6\sqrt{2})^2 \right) + \cot^{-1} \left( (1 + 12\sqrt{2})^2 \right) + \cot^{-1} \left( (1 + 24\sqrt{2})^2 \right) \right)^5$$

$$1 / \left( \tan^{-1} \left( \frac{1153 - 48\sqrt{2}}{1324801} \right) + \tan^{-1} \left( \frac{289 - 24\sqrt{2}}{82369} \right) + \right. \\ \left. \tan^{-1} \left( \frac{73 - 12\sqrt{2}}{5041} \right) + \tan^{-1} \left( \frac{51 - 10\sqrt{2}}{2401} \right) + \tan^{-1} \left( \frac{1}{961} (33 - 8\sqrt{2}) \right) + \right. \\ \left. \tan^{-1} \left( \frac{1}{289} (19 - 6\sqrt{2}) \right) + \tan^{-1} \left( \frac{1}{49} (9 - 4\sqrt{2}) \right) + \tan^{-1} (3 - 2\sqrt{2}) \right)^5$$

$\cot^{-1}(x)$  is the inverse cotangent function

### Alternative representations:

$$\begin{aligned}
& \left( 1 / \left( \tan^{-1} \left( \frac{1}{(1+\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+2\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+3\sqrt{2})^2} \right) + \right. \right. \\
& \quad \left. \tan^{-1} \left( \frac{1}{(1+4\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+5\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+6\sqrt{2})^2} \right) + \right. \\
& \quad \left. \left. \tan^{-1} \left( \frac{1}{(1+12\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+24\sqrt{2})^2} \right) \right) \right)^5 = \\
& \left( 1 / \left( \operatorname{sc}^{-1} \left( \frac{1}{(1+\sqrt{2})^2} \middle| 0 \right) + \operatorname{sc}^{-1} \left( \frac{1}{(1+2\sqrt{2})^2} \middle| 0 \right) + \operatorname{sc}^{-1} \left( \frac{1}{(1+3\sqrt{2})^2} \middle| 0 \right) + \right. \right. \\
& \quad \left. \operatorname{sc}^{-1} \left( \frac{1}{(1+4\sqrt{2})^2} \middle| 0 \right) + \operatorname{sc}^{-1} \left( \frac{1}{(1+5\sqrt{2})^2} \middle| 0 \right) + \operatorname{sc}^{-1} \left( \frac{1}{(1+6\sqrt{2})^2} \middle| 0 \right) + \right. \\
& \quad \left. \left. \operatorname{sc}^{-1} \left( \frac{1}{(1+12\sqrt{2})^2} \middle| 0 \right) + \operatorname{sc}^{-1} \left( \frac{1}{(1+24\sqrt{2})^2} \middle| 0 \right) \right) \right)^5
\end{aligned}$$

$$\begin{aligned}
& \left( 1 / \left( \tan^{-1} \left( \frac{1}{(1+\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+2\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+3\sqrt{2})^2} \right) + \right. \right. \\
& \quad \left. \tan^{-1} \left( \frac{1}{(1+4\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+5\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+6\sqrt{2})^2} \right) + \right. \\
& \quad \left. \left. \tan^{-1} \left( \frac{1}{(1+12\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+24\sqrt{2})^2} \right) \right) \right)^5 = \\
& \left( 1 / \left( \tan^{-1} \left( 1, \frac{1}{(1+\sqrt{2})^2} \right) + \tan^{-1} \left( 1, \frac{1}{(1+2\sqrt{2})^2} \right) + \tan^{-1} \left( 1, \frac{1}{(1+3\sqrt{2})^2} \right) + \right. \right. \\
& \quad \left. \tan^{-1} \left( 1, \frac{1}{(1+4\sqrt{2})^2} \right) + \tan^{-1} \left( 1, \frac{1}{(1+5\sqrt{2})^2} \right) + \tan^{-1} \left( 1, \frac{1}{(1+6\sqrt{2})^2} \right) + \right. \\
& \quad \left. \left. \tan^{-1} \left( 1, \frac{1}{(1+12\sqrt{2})^2} \right) + \tan^{-1} \left( 1, \frac{1}{(1+24\sqrt{2})^2} \right) \right) \right)^5
\end{aligned}$$

$$\begin{aligned}
& \left( 1 / \left( \tan^{-1} \left( \frac{1}{(1+\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+2\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+3\sqrt{2})^2} \right) + \right. \right. \\
& \quad \left. \tan^{-1} \left( \frac{1}{(1+4\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+5\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+6\sqrt{2})^2} \right) + \right. \\
& \quad \left. \left. \tan^{-1} \left( \frac{1}{(1+12\sqrt{2})^2} \right) + \tan^{-1} \left( \frac{1}{(1+24\sqrt{2})^2} \right) \right) \right)^5 = \\
& \left( 1 / \left( \cot^{-1} \left( \frac{1}{(1+\sqrt{2})^2} \right) + \cot^{-1} \left( \frac{1}{(1+2\sqrt{2})^2} \right) + \cot^{-1} \left( \frac{1}{(1+3\sqrt{2})^2} \right) + \right. \right. \\
& \quad \left. \cot^{-1} \left( \frac{1}{(1+4\sqrt{2})^2} \right) + \cot^{-1} \left( \frac{1}{(1+5\sqrt{2})^2} \right) + \cot^{-1} \left( \frac{1}{(1+6\sqrt{2})^2} \right) + \right. \\
& \quad \left. \left. \cot^{-1} \left( \frac{1}{(1+12\sqrt{2})^2} \right) + \cot^{-1} \left( \frac{1}{(1+24\sqrt{2})^2} \right) \right) \right)^5
\end{aligned}$$

From the following alternate form

$$\frac{1}{\tan^{-1}\left(\frac{190969859251\sqrt{2}-216845894344}{156747794826}\right)^5}$$

We obtain:

$$1/2((1/\tan^{-1}((-216845894344 + 190969859251\sqrt{2})/156747794826))^5)+5+\text{golden ratio}$$

Where 5 is a Fibonacci number

**Input:**

$$\frac{1}{2} \times \frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 5 + \phi$$

$\tan^{-1}(x)$  is the inverse tangent function  
 $\phi$  is the golden ratio

**Exact Result:**

$$\phi + 5 + \frac{1}{2 \tan^{-1}\left(\frac{190969859251\sqrt{2}-216845894344}{156747794826}\right)^5}$$

(result in radians)

**Decimal approximation:**

$$139.6359755539264603628272048794970584759324233908362145591\dots$$

(result in radians)

139.635975... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternate forms:**

$$\frac{1}{2}\left(11 + \sqrt{5}\right) + \frac{1}{2 \tan^{-1}\left(\frac{190969859251\sqrt{2}-216845894344}{156747794826}\right)^5}$$

$$5 + \frac{1}{2}\left(1 + \sqrt{5}\right) + \frac{1}{2 \tan^{-1}\left(\frac{190969859251\sqrt{2}-216845894344}{156747794826}\right)^5}$$

$$\phi + 5 + \frac{16}{\left(\tan^{-1}\left(\frac{190969859251\sqrt{2}-216845894344}{156747794826}\right) - \tan^{-1}\left(\frac{216845894344-190969859251\sqrt{2}}{156747794826}\right)\right)^5}$$

### Alternative representations:

$$\frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 5 + \phi =$$

$$5 + \phi + \frac{1}{2 \operatorname{sc}^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826} \mid 0\right)^5}$$

$$\frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 5 + \phi =$$

$$5 + \phi + \frac{1}{2 \tan^{-1}\left(1, \frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5}$$

$$\frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 5 + \phi =$$

$$5 + \phi + \frac{1}{2 \cot^{-1}\left(\frac{1}{\frac{-216845894344+190969859251\sqrt{2}}{156747794826}}\right)^5}$$

### Series representations:

$$\frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 5 + \phi =$$

$$5 + \phi + \frac{1}{2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k 156747794826^{-1-2k} (-216845894344+190969859251\sqrt{2})^{1+2k}}{1+2k}\right)^5}$$

$$\frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 5 + \phi =$$

$$5 + \phi - (16i) / \left( \log(2) - \log\left(i \left(-i + \frac{-216845894344 + 190969859251\sqrt{2}}{156747794826}\right)\right) \right) -$$

$$\sum_{k=1}^{\infty} \frac{\left(\frac{i}{2}\right)^k \left(-i + \frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^k}{k} \Big)^5$$

$$\frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 5 + \phi =$$

$$5 + \phi - (16i) / \left( -\log(2) + \log\left(-i\left(i + \frac{-216845894344 + 190969859251\sqrt{2}}{156747794826}\right)\right) \right) +$$

$$\sum_{k=1}^{\infty} \frac{\left(-\frac{i}{2}\right)^k \left(i + \frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^k}{k}$$

**Continued fraction representations:**

$$\frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 5 + \phi = 5 + \phi +$$

$$\left( 47312642310909092068219567484966973846403528990332452688 \right.$$

$$\left. \left( 1 + \prod_{k=1}^{\infty} \frac{\left(-216845894344+190969859251\sqrt{2}\right)^2 k^2}{24569871182813792370276} \right) \right) /$$

$$\left(-216845894344 + 190969859251\sqrt{2}\right)^5 = 5 + \phi +$$

$$\left( 47312642310909092068219567484966973846403528990332452688 \right.$$

$$\left. \left( 1 + \left(-216845894344 + 190969859251\sqrt{2}\right)^2 / \right.$$

$$\left. \left( 24569871182813792370276 \right.$$

$$\left. \left( 3 + \left(-216845894344 + 190969859251\sqrt{2}\right)^2 / \right.$$

$$\left. \left( 6142467795703448092569 \right.$$

$$\left. \left( 5 + \left(-216845894344 + 190969859251\sqrt{2}\right)^2 / \right.$$

$$\left. \left( 2729985686979310263364 \left( 7 + \right.$$

$$\left. \left( 4 \left(-216845894344 + 190969859251\sqrt{2}\right)^2 / \left( 6142467795703448092569 \right.$$

$$\left. \left( 9 + \dots \right) \right) \right) \right) \right) \right) /$$

$$\left(-216845894344 + 190969859251\sqrt{2}\right)^5$$

$$\begin{aligned}
& \frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 5 + \phi = 5 + \phi + \\
& \left( 47312642310909092068219567484966973846403528990332452688 \right. \\
& \quad \left. \left( 1 + \prod_{k=1}^{\infty} \frac{(216845894344-190969859251\sqrt{2})^2 k^2}{24569871182813792370276(1+2k)} \right)^5 \right) / \\
& \quad (-216845894344 + 190969859251\sqrt{2})^5 = 5 + \phi + \\
& \quad (47312642310909092068219567484966973846403528990332452688 \\
& \quad \quad (1 + (216845894344 - 190969859251\sqrt{2})^2 / \\
& \quad \quad \quad (24569871182813792370276 \\
& \quad \quad \quad \quad (3 + (216845894344 - 190969859251\sqrt{2})^2) / \\
& \quad \quad \quad \quad \quad (6142467795703448092569 \\
& \quad \quad \quad \quad \quad \quad (5 + (216845894344 - 190969859251\sqrt{2})^2) / \\
& \quad \quad \quad \quad \quad \quad \quad (2729985686979310263364 \\
& \quad \quad \quad \quad \quad \quad \quad \quad (7 + (4(216845894344 - 190969859251 \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \sqrt{2})^2) / (6142467795703448092569 \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (9 + \dots)))))))))) / \\
& \quad (-216845894344 + 190969859251\sqrt{2})^5
\end{aligned}$$



$$1/2((1/\tan^{(-1)}((-216845894344 + 190969859251 \sqrt{2}))/156747794826)^5)-11+\pi$$

Where 11 is a Lucas number

**Input:**

$$\frac{1}{2} \times \frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} - 11 + \pi$$

$\tan^{-1}(x)$  is the inverse tangent function

**Exact Result:**

$$-11 + \pi + \frac{1}{2 \tan^{-1}\left(\frac{190969859251\sqrt{2}-216845894344}{156747794826}\right)^5}$$

(result in radians)

**Decimal approximation:**

125.1595342187663587530852614284109232424092836104055575180...

(result in radians)

125.159534... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternate forms:**

$$-11 + \pi + \frac{16}{\left(\tan^{-1}\left(\frac{190969859251\sqrt{2}-216845894344}{156747794826}\right) - \tan^{-1}\left(\frac{216845894344-190969859251\sqrt{2}}{156747794826}\right)\right)^5}$$

$$-11 + \pi - (16i) / \left( \log\left(1 - \frac{i(190969859251\sqrt{2}-216845894344)}{156747794826}\right) - \log\left(1 + \frac{i(190969859251\sqrt{2}-216845894344)}{156747794826}\right) \right)^5$$

$$\left(1 - 22 \tan^{-1}\left(\frac{190969859251\sqrt{2}-216845894344}{156747794826}\right)\right)^5 + \frac{2\pi \tan^{-1}\left(\frac{190969859251\sqrt{2}-216845894344}{156747794826}\right)^5}{\left(2 \tan^{-1}\left(\frac{190969859251\sqrt{2}-216845894344}{156747794826}\right)\right)^5}$$

$\log(x)$  is the natural logarithm



### Alternative representations:

$$\frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5 2} - 11 + \pi =$$

$$-11 + \pi + \frac{1}{2 \operatorname{sc}^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826} \mid 0\right)^5}$$

$$\frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5 2} - 11 + \pi =$$

$$-11 + \pi + \frac{1}{2 \tan^{-1}\left(1, \frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5}$$

$$\frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5 2} - 11 + \pi =$$

$$-11 + \pi + \frac{1}{2 \cot^{-1}\left(\frac{1}{\frac{-216845894344+190969859251\sqrt{2}}{156747794826}}\right)^5}$$

### Series representations:

$$\frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5 2} - 11 + \pi =$$

$$-11 + \pi + \frac{1}{2 \left( \sum_{k=0}^{\infty} \frac{(-1)^k 156747794826^{-1-2k} \left(-216845894344+190969859251\sqrt{2}\right)^{1+2k}}{1+2k} \right)^5}$$

$$\frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5 2} - 11 + \pi =$$

$$-11 + \pi - (16i) / \left( \log(2) - \log\left(i \left(-i + \frac{-216845894344 + 190969859251\sqrt{2}}{156747794826}\right)\right) \right) -$$

$$\sum_{k=1}^{\infty} \frac{\left(\frac{i}{2}\right)^k \left(-i + \frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^k}{k} \Big)^5$$

$$\frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} - 11 + \pi =$$

$$-11 + \pi - (16i) / \left[ -\log(2) + \log\left(-i \left( i + \frac{-216845894344+190969859251\sqrt{2}}{156747794826} \right) \right) \right] +$$

$$\sum_{k=1}^{\infty} \frac{\left( \frac{-i}{2} \right)^k \left( i + \frac{-216845894344+190969859251\sqrt{2}}{156747794826} \right)^k}{k} \Big)^5$$

**Continued fraction representations:**

$$\frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} - 11 + \pi = -11 + \pi +$$

$$\left( 47312642310909092068219567484966973846403528990332452688 \right.$$

$$\left. \left( 1 + \mathbf{K}_{k=1}^{\infty} \frac{\left( \frac{-216845894344+190969859251\sqrt{2}}{156747794826} \right)^2 k^2}{1+2k} \right)^5 \right) /$$

$$\left( -216845894344 + 190969859251\sqrt{2} \right)^5 = -11 + \pi +$$

$$\left( 47312642310909092068219567484966973846403528990332452688 \right.$$

$$\left( 1 + \left( -216845894344 + 190969859251\sqrt{2} \right)^2 / \right.$$

$$\left. \left( 24569871182813792370276 \right.$$

$$\left. \left( 3 + \left( -216845894344 + 190969859251\sqrt{2} \right)^2 / \right.$$

$$\left. \left( 6142467795703448092569 \right.$$

$$\left. \left( 5 + \left( -216845894344 + 190969859251\sqrt{2} \right)^2 / \right.$$

$$\left. \left( 2729985686979310263364 \left( 7 + \right.$$

$$\left. \left( 4 \left( -216845894344 + 190969859251\sqrt{2} \right)^2 / \left( 6142467795703448092569 \right. \right. \right.$$

$$\left. \left. \left( 9 + \dots \right) \right) \right) \right) \right) \right) \right) \right) /$$

$$\left( -216845894344 + 190969859251\sqrt{2} \right)^5$$



$$\begin{aligned}
& \frac{1}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} - 11 + \pi = -11 + \pi + \\
& \left( 47312642310909092068219567484966973846403528990332452688 \right. \\
& \left. \left( 1 + \sum_{k=1}^{\infty} \frac{\left(\frac{216845894344-190969859251\sqrt{2}}{24569871182813792370276}\right)^2 (1-2k)^2}{1+2k - \frac{\left(\frac{216845894344-190969859251\sqrt{2}}{24569871182813792370276}\right)^2 (-1+2k)}{24569871182813792370276}} \right)^5 \right) / \\
& \left( -216845894344 + 190969859251\sqrt{2} \right)^5 = -11 + \pi + \\
& \left( 47312642310909092068219567484966973846403528990332452688 \right. \\
& \left. \left( 1 + \left( 216845894344 - 190969859251\sqrt{2} \right)^2 / \right. \right. \\
& \left. \left( 24569871182813792370276 \right. \right. \\
& \left. \left( 3 - \frac{\left( 216845894344 - 190969859251\sqrt{2} \right)^2}{24569871182813792370276} + \right. \right. \\
& \left. \left. \left( 216845894344 - 190969859251\sqrt{2} \right)^2 / \right. \right. \\
& \left. \left( 2729985686979310263364 \right. \right. \\
& \left. \left( 5 - \frac{\left( 216845894344 - 190969859251\sqrt{2} \right)^2}{8189957060937930790092} + \right. \right. \\
& \left. \left. \left( 25 \left( 216845894344 - 190969859251\sqrt{2} \right)^2 \right) / \left( 24569871182813792370276 \right. \right. \\
& \left. \left( 7 - \left( 5 \left( 216845894344 - 190969859251\sqrt{2} \right)^2 \right) / 24569871182813792370276 + \right. \right. \\
& \left. \left( 216845894344 - 190969859251\sqrt{2} \right)^2 / \left( 501425942506403925924 \left( 9 - \right. \right. \right. \\
& \left. \left. \left. \frac{\left( 216845894344 - 190969859251\sqrt{2} \right)^2}{3509981597544827481468} \right. \right. \right. \\
& \left. \left. \left. + \left( \dots \right) \right) \right) / \right. \\
& \left. \left( -216845894344 + 190969859251\sqrt{2} \right)^5 \right)
\end{aligned}$$

$2\pi * ((1/\tan^{-1})((-216845894344 + 190969859251 \sqrt{2})/156747794826)^5) + 47 + 11 - 1/\text{golden ratio}$

Where 47 and 11 are Lucas numbers

**Input:**

$$2\pi \times \frac{1}{\tan^{-1}\left(\frac{-216845894344 + 190969859251\sqrt{2}}{156747794826}\right)^5} + 47 + 11 - \frac{1}{\phi}$$

$\tan^{-1}(x)$  is the inverse tangent function

$\phi$  is the golden ratio

**Exact Result:**

$$-\frac{1}{\phi} + 58 + \frac{2\pi}{\tan^{-1}\left(\frac{190969859251\sqrt{2} - 216845894344}{156747794826}\right)^5}$$

(result in radians)

**Decimal approximation:**

1728.934718078430510743282317395182371646810220521659841850...

(result in radians)

1728.934718....

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternate forms:**

$$58 - \frac{2}{1 + \sqrt{5}} + \frac{2\pi}{\tan^{-1}\left(\frac{190969859251\sqrt{2} - 216845894344}{156747794826}\right)^5}$$

$$\frac{1}{2}(117 - \sqrt{5}) + \frac{2\pi}{\tan^{-1}\left(\frac{190969859251\sqrt{2} - 216845894344}{156747794826}\right)^5}$$

$$\frac{2(28 + 29\sqrt{5})}{1 + \sqrt{5}} + \frac{2\pi}{\tan^{-1}\left(\frac{190969859251\sqrt{2} - 216845894344}{156747794826}\right)^5}$$

### Alternative representations:

$$\frac{2\pi}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 47 + 11 - \frac{1}{\phi} =$$

$$58 - \frac{1}{\phi} + \frac{2\pi}{\operatorname{sc}^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826} \mid 0\right)^5}$$

$$\frac{2\pi}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 47 + 11 - \frac{1}{\phi} =$$

$$58 - \frac{1}{\phi} + \frac{2\pi}{\tan^{-1}\left(1, \frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5}$$

$$\frac{2\pi}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 47 + 11 - \frac{1}{\phi} =$$

$$58 - \frac{1}{\phi} + \frac{2\pi}{\cot^{-1}\left(\frac{1}{\frac{-216845894344+190969859251\sqrt{2}}{156747794826}}\right)^5}$$

### Series representations:

$$\frac{2\pi}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 47 + 11 - \frac{1}{\phi} =$$

$$58 - \frac{1}{\phi} + \frac{2\pi}{\left(\sum_{k=0}^{\infty} \frac{(-1)^k 156747794826^{-1-2k} \left(-216845894344+190969859251\sqrt{2}\right)^{1+2k}}{1+2k}\right)^5}$$

$$\frac{2\pi}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 47 + 11 - \frac{1}{\phi} =$$

$$58 - \frac{1}{\phi} - (64i\pi) \left/ \log\left(2\right) - \log\left(i\left(-i + \frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)\right)\right. -$$

$$\left.\sum_{k=1}^{\infty} \frac{\left(\frac{i}{2}\right)^k \left(-i + \frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^k}{k}\right)^5$$

$$\frac{2\pi}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 47 + 11 - \frac{1}{\phi} =$$

$$58 - \frac{1}{\phi} - (64i\pi) / \left( -\log(2) + \log\left(-i\left(i + \frac{-216845894344 + 190969859251\sqrt{2}}{156747794826}\right)\right) \right) +$$

$$\sum_{k=1}^{\infty} \frac{\left(\frac{-i}{2}\right)^k \left(i + \frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^k}{k} \Big)^5$$

**Continued fraction representations:**

$$\frac{2\pi}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 47 + 11 - \frac{1}{\phi} = 58 - \frac{1}{\phi} +$$

$$\left( 189250569243636368272878269939867895385614115961329810752 \right.$$

$$\left. \pi \left( 1 + \mathbf{K}_{k=1}^{\infty} \frac{\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^2 k^2}{24569871182813792370276} \right) \right) /$$

$$\left( -216845894344 + 190969859251\sqrt{2} \right)^5 = 58 - \frac{1}{\phi} +$$

$$\left( 189250569243636368272878269939867895385614115961329810752 \right.$$

$$\pi \left( 1 + \left( -216845894344 + 190969859251\sqrt{2} \right)^2 / \right.$$

$$\left. \left( 24569871182813792370276 \right. \right.$$

$$\left. \left. \left( 3 + \left( -216845894344 + 190969859251\sqrt{2} \right)^2 / \right. \right.$$

$$\left. \left. \left( 6142467795703448092569 \right. \right.$$

$$\left. \left. \left( 5 + \left( -216845894344 + 190969859251\sqrt{2} \right)^2 / \right. \right.$$

$$\left. \left. \left( 2729985686979310263364 \left( 7 + \right. \right. \right.$$

$$\left. \left. \left( 4 \left( -216845894344 + 190969859251\sqrt{2} \right)^2 \right) / \left( 6142467795703448092569 \right. \right.$$

$$\left. \left. \left( 9 + \dots \right) \right) \right) \right) \right) \right) \right) /$$

$$\left( -216845894344 + 190969859251\sqrt{2} \right)^5$$

$$\begin{aligned}
& \frac{2\pi}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 47 + 11 - \frac{1}{\phi} = 58 - \frac{1}{\phi} + \\
& \left( 189250569243636368272878269939867895385614115961329810752 \right. \\
& \left. \pi \left( 1 + \sum_{k=1}^{\infty} \frac{\left(\frac{216845894344-190969859251\sqrt{2}}{24569871182813792370276}\right)^2 k^2}{1+2k} \right)^5 \right) / \\
& \left( -216845894344 + 190969859251\sqrt{2} \right)^5 = 58 - \frac{1}{\phi} + \\
& \left( 189250569243636368272878269939867895385614115961329810752 \right. \\
& \left. \pi \left( 1 + \left( \frac{216845894344 - 190969859251\sqrt{2}}{24569871182813792370276} \right)^2 / \right. \right. \\
& \left. \left. \left( 3 + \left( \frac{216845894344 - 190969859251\sqrt{2}}{6142467795703448092569} \right)^2 / \right. \right. \right. \\
& \left. \left. \left( 5 + \left( \frac{216845894344 - 190969859251\sqrt{2}}{2729985686979310263364} \right)^2 / \right. \right. \right. \\
& \left. \left. \left( 7 + \left( \frac{216845894344 - 190969859251\sqrt{2}}{(6142467795703448092569)} \right)^2 / \right. \right. \right. \\
& \left. \left. \left( 9 + \dots \right) \right) \right) \right) \right) / \\
& \left( -216845894344 + 190969859251\sqrt{2} \right)^5
\end{aligned}$$



$$\begin{aligned}
& \frac{2\pi}{\tan^{-1}\left(\frac{-216845894344+190969859251\sqrt{2}}{156747794826}\right)^5} + 47 + 11 - \frac{1}{\phi} = 58 - \frac{1}{\phi} + \\
& \left( 189250569243636368272878269939867895385614115961329810752 \right. \\
& \left. \pi \left( 1 + \sum_{k=1}^{\infty} \frac{\left( \frac{216845894344-190969859251\sqrt{2}}{24569871182813792370276} \right)^2 (1-2k)^2}{1+2k - \frac{\left( \frac{216845894344-190969859251\sqrt{2}}{24569871182813792370276} \right)^2 (-1+2k)}{24569871182813792370276}} \right)^5 \right) / \\
& \left( -216845894344 + 190969859251\sqrt{2} \right)^5 = 58 - \frac{1}{\phi} + \\
& \left( 189250569243636368272878269939867895385614115961329810752 \right. \\
& \left. \pi \left( 1 + \left( \frac{216845894344 - 190969859251\sqrt{2}}{24569871182813792370276} \right)^2 / \right. \right. \\
& \left. \left. \left( 3 - \frac{\left( \frac{216845894344 - 190969859251\sqrt{2}}{24569871182813792370276} \right)^2}{\left( \frac{216845894344 - 190969859251\sqrt{2}}{24569871182813792370276} \right)^2} + \right. \right. \\
& \left. \left. \left( 5 - \frac{\left( \frac{216845894344 - 190969859251\sqrt{2}}{8189957060937930790092} \right)^2}{\left( 25 \left( \frac{216845894344 - 190969859251\sqrt{2}}{24569871182813792370276} \right)^2 \right) / \left( \frac{216845894344 - 190969859251\sqrt{2}}{24569871182813792370276} \right)^2} + \right. \right. \\
& \left. \left. \left( 7 - \left( 5 \left( \frac{216845894344 - 190969859251\sqrt{2}}{24569871182813792370276} \right)^2 \right) / \left( \frac{216845894344 - 190969859251\sqrt{2}}{24569871182813792370276} \right)^2 + \right. \right. \\
& \left. \left. \left( 9 - \frac{\left( \frac{216845894344 - 190969859251\sqrt{2}}{3509981597544827481468} \right)^2}{\left( \frac{216845894344 - 190969859251\sqrt{2}}{3509981597544827481468} \right)^2} \right. \right. \\
& \left. \left. + \dots \right) \right)^5 / \\
& \left( -216845894344 + 190969859251\sqrt{2} \right)^5
\end{aligned}$$

















With regard  $\pi/8$ , we obtain:

$\pi/8$

**Input:**

$$\frac{\pi}{8}$$

**Decimal approximation:**

0.392699081698724154807830422909937860524646174921888227621...  
0.392699081....

**Property:**

$\frac{\pi}{8}$  is a transcendental number

**Alternative representations:**

$$\frac{\pi}{8} = \frac{180^\circ}{8}$$

$$\frac{\pi}{8} = -\frac{1}{8} i \log(-1)$$

$$\frac{\pi}{8} = \frac{1}{8} \cos^{-1}(-1)$$

**Series representations:**

$$\frac{\pi}{8} = \frac{1}{2} \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{\pi}{8} = \sum_{k=0}^{\infty} -\frac{(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{2(1+2k)}$$

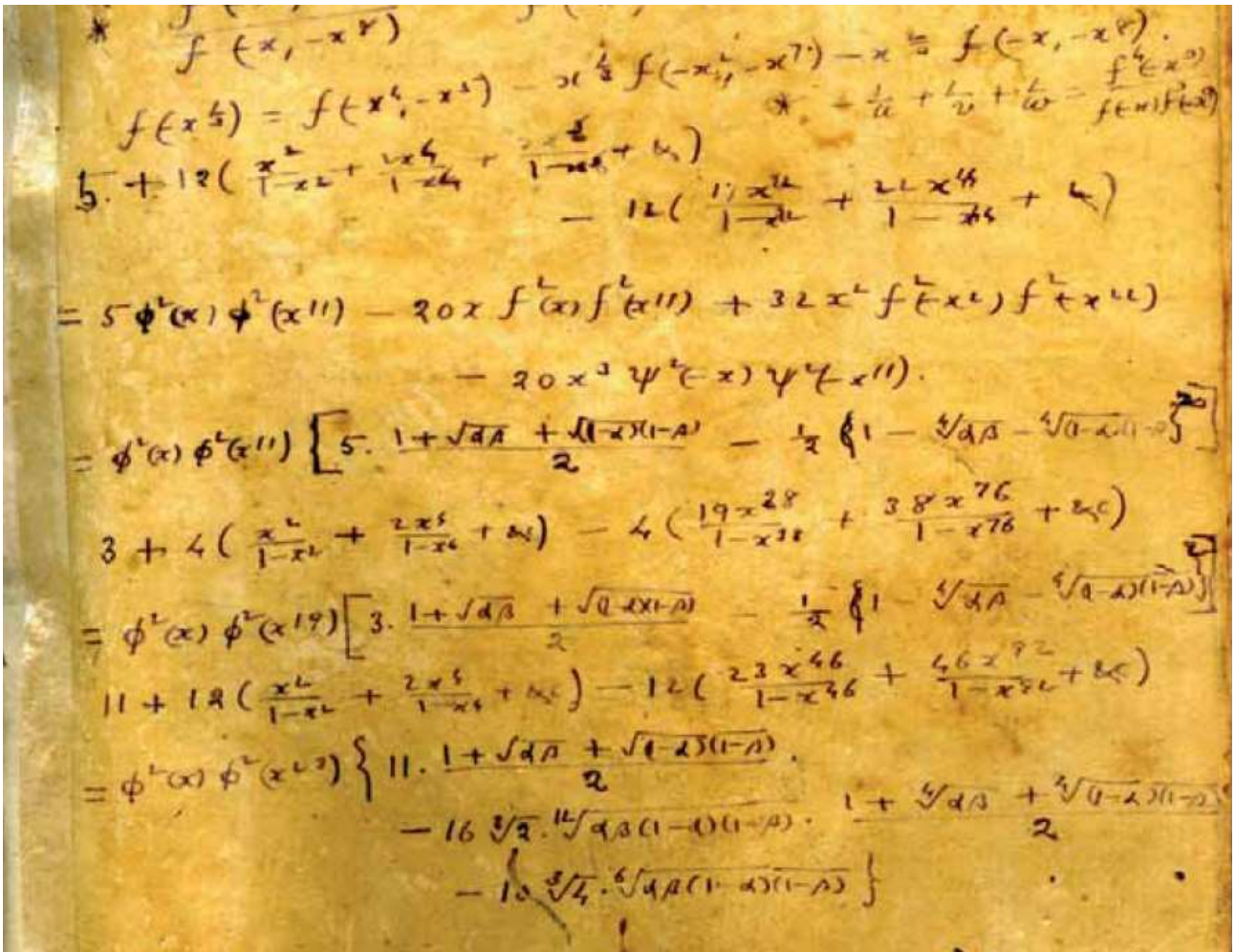
$$\frac{\pi}{8} = \frac{1}{8} \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

**Integral representations:**

$$\frac{\pi}{8} = \frac{1}{2} \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{\pi}{8} = \frac{1}{4} \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{\pi}{8} = \frac{1}{4} \int_0^{\infty} \frac{1}{1+t^2} dt$$



$$11+12(((2^2/(1-2^2))+(2*2^4)/(1-2^4)))-12((((23*2^46)/(1-2^46)+(46*2^92)/(1-2^92))))$$

**Input:**

$$11+12\left(\frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4}\right) - 12\left(\frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}}\right)$$

**Exact result:**

263 235 569 953 644 556 439 442 644 011  
 330 117 343 809 434 739 973 099 793

**Decimal approximation:**

797.40000000000039221959013959202392769638452537006676432338...

797.4...

**Alternate form:**

$$\frac{263\ 235\ 569\ 953\ 644\ 556\ 439\ 442\ 644\ 011}{330\ 117\ 343\ 809\ 434\ 739\ 973\ 099\ 793}$$

**Mixed fraction:**

$$797 \frac{132046937525068680882108990}{330117343809434739973099793}$$

**Continued fraction:**

$$797 + \frac{1}{2 + \frac{1}{2 + \frac{1}{10\ 198\ 368\ 721 + \frac{1}{6899 + \frac{1}{1 + \frac{1}{1\ 274\ 796\ 089 + \frac{1}{5 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{7}}}}}}}}}}}}}}}}$$

$$2((11+12(((2^2/(1-2^2)))+(2*2^4)/(1-2^4))))-12((((23*2^46)/(1-2^46)+(46*2^92)/(1-2^92)))))))+123+11$$

Where 123 and 11 are Lucas numbers

**Input:**

$$2 \left( 11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) + 123 + 11$$

**Exact result:**

$$\frac{570\ 706\ 863\ 977\ 753\ 368\ 035\ 280\ 660\ 660\ 284}{330\ 117\ 343\ 809\ 434\ 739\ 973\ 099\ 793}$$

**Decimal approximation:**

1728.8000000000007844391802791840478553927690507401335286467...

1728.8....

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternate form:**

$$\frac{570\,706\,863\,977\,753\,368\,035\,280\,660\,284}{330\,117\,343\,809\,434\,739\,973\,099\,793}$$

**Mixed fraction:**

$$1728 \frac{264093875050137361764217980}{330117343809434739973099793}$$

**Continued fraction:**

$$1728 + \frac{1}{1 + \frac{1}{4 + \frac{1}{5099\,184\,360 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3449 + \frac{1}{2 + \frac{1}{637\,398\,044 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{2}}}}}}}}}}}}}}}}$$

$$((11+12(((2^2/(1-2^2))+(2*2^4)/(1-2^4)))-12((((23*2^46)/(1-2^46)+(46*2^92)/(1-2^92)))))))-18+\pi$$

Where 18 is a Lucas number

**Input:**

$$\left( 11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) - 18 + \pi$$

**Result:**

$$\frac{257\,293\,457\,765\,074\,731\,119\,926\,847\,737}{330\,117\,343\,809\,434\,739\,973\,099\,793} + \pi$$

**Decimal approximation:**

782.5415926535937154343640393035187798480424231000427490548...

782.541592.... result practically equal to the rest mass of Omega meson 782.65

**Property:**

$$\frac{257\,293\,457\,765\,074\,731\,119\,926\,847\,737}{330\,117\,343\,809\,434\,739\,973\,099\,793} + \pi$$
 is a transcendental number
**Alternate forms:**

$$\frac{257\,293\,457\,765\,074\,731\,119\,926\,847\,737 + 330\,117\,343\,809\,434\,739\,973\,099\,793\,\pi}{330\,117\,343\,809\,434\,739\,973\,099\,793}$$

$$\frac{257\,293\,457\,765\,074\,731\,119\,926\,847\,737 + 330\,117\,343\,809\,434\,739\,973\,099\,793\,\pi}{330\,117\,343\,809\,434\,739\,973\,099\,793}$$

**Continued fraction:**

$$782 + \frac{1}{1 + \frac{1}{1 + \frac{1}{5 + \frac{1}{1 + \frac{1}{1 + \frac{1}{22 + \frac{1}{1 + \frac{1}{10 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{31 + \frac{1}{1 + \frac{1}{1 + \frac{1}{6 + \frac{1}{2 + \frac{1}{1 + \frac{1}{\dots}}$$

**Alternative representations:**

$$\left( 11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) - 18 + \pi =$$

$$-7 + 180^\circ + 12 \left( -\frac{4}{3} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right)$$

$$\left(11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) - 18 + \pi =$$

$$-7 - i \log(-1) + 12 \left( -\frac{4}{3} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right)$$

$$\left(11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) - 18 + \pi =$$

$$-7 + \cos^{-1}(-1) + 12 \left( -\frac{4}{3} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right)$$

### Series representations:

$$\left(11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) - 18 + \pi =$$

$$\frac{257\,293\,457\,765\,074\,731\,119\,926\,847\,737}{330\,117\,343\,809\,434\,739\,973\,099\,793} + 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\left(11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) - 18 + \pi =$$

$$\frac{257\,293\,457\,765\,074\,731\,119\,926\,847\,737}{330\,117\,343\,809\,434\,739\,973\,099\,793} +$$

$$\sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}$$

$$\left(11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) - 18 + \pi =$$

$$\frac{257\,293\,457\,765\,074\,731\,119\,926\,847\,737}{330\,117\,343\,809\,434\,739\,973\,099\,793} + \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right)$$

### Integral representations:

$$\left(11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) - 18 + \pi =$$

$$\frac{257\,293\,457\,765\,074\,731\,119\,926\,847\,737}{330\,117\,343\,809\,434\,739\,973\,099\,793} + 4 \int_0^1 \sqrt{1-t^2} \, dt$$

$$\left(11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) - 18 + \pi =$$

$$\frac{257\,293\,457\,765\,074\,731\,119\,926\,847\,737}{330\,117\,343\,809\,434\,739\,973\,099\,793} + 2 \int_0^1 \frac{1}{\sqrt{1-t^2}} \, dt$$

$$\left(11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) - 18 + \pi = \frac{257293457765074731119926847737}{330117343809434739973099793} + 2 \int_0^\infty \frac{1}{1+t^2} dt$$

$$1/6((11+12(((2^2/(1-2^2))+(2*2^4)/(1-2^4)))-12((((23*2^46)/(1-2^46)+(46*2^92)/(1-2^92)))))))+2\pi+1/\text{golden ratio}$$

**Input:**

$$\frac{1}{6} \left( 11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) + 2\pi + \frac{1}{\phi}$$

$\phi$  is the golden ratio

**Result:**

$$\frac{1}{\phi} + \frac{263235569953644556439442644011}{1980704062856608439838598758} + 2\pi$$

**Decimal approximation:**

139.8012192959301350244467729209645233800888569286672483763...

139.80121929.... result practically equal to the rest mass of Pion meson 139.57 MeV

**Property:**

$$\frac{263235569953644556439442644011}{1980704062856608439838598758} + \frac{1}{\phi} + 2\pi \text{ is a transcendental number}$$

**Alternate forms:**

$$\left( \frac{262245217922216252219523344632 + 990352031428304219919299379\sqrt{5} + 3961408125713216879677197516\pi}{1980704062856608439838598758} \right)$$

$$\left( \frac{263235569953644556439442644011\phi + 3961408125713216879677197516\pi\phi + 1980704062856608439838598758}{(1980704062856608439838598758\phi)} \right)$$

$$\frac{263235569953644556439442644011}{1980704062856608439838598758} + \frac{2}{1+\sqrt{5}} + 2\pi$$

**Alternative representations:**

$$\frac{1}{6} \left( 11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) + 2\pi + \frac{1}{\phi} =$$

$$2\pi + -\frac{1}{2 \cos(216^\circ)} + \frac{1}{6} \left( 11 + 12 \left( -\frac{4}{3} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right)$$

$$\frac{1}{6} \left( 11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) + 2\pi + \frac{1}{\phi} =$$

$$360^\circ + -\frac{1}{2 \cos(216^\circ)} + \frac{1}{6} \left( 11 + 12 \left( -\frac{4}{3} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right)$$

$$\frac{1}{6} \left( 11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) + 2\pi + \frac{1}{\phi} =$$

$$2\pi + \frac{1}{2 \cos\left(\frac{\pi}{5}\right)} + \frac{1}{6} \left( 11 + 12 \left( -\frac{4}{3} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right)$$

**Series representations:**

$$\frac{1}{6} \left( 11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) + 2\pi + \frac{1}{\phi} =$$

$$\frac{263\,235\,569\,953\,644\,556\,439\,442\,644\,011}{1\,980\,704\,062\,856\,608\,439\,838\,598\,758} + \frac{1}{\phi} + 8 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{1}{6} \left( 11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) + 2\pi + \frac{1}{\phi} =$$

$$\frac{263\,235\,569\,953\,644\,556\,439\,442\,644\,011}{1\,980\,704\,062\,856\,608\,439\,838\,598\,758} +$$

$$\frac{1}{\phi} + \sum_{k=0}^{\infty} -\frac{8(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}$$

$$\frac{1}{6} \left( 11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) + 2\pi + \frac{1}{\phi} =$$

$$\frac{263\,235\,569\,953\,644\,556\,439\,442\,644\,011}{1\,980\,704\,062\,856\,608\,439\,838\,598\,758} +$$

$$\frac{1}{\phi} + 2 \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right)$$



**Integral representations:**

$$\frac{1}{6} \left( 11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) + 2\pi + \frac{1}{\phi} = \frac{263\,235\,569\,953\,644\,556\,439\,442\,644\,011}{1\,980\,704\,062\,856\,608\,439\,838\,598\,758} + \frac{1}{\phi} + 8 \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{1}{6} \left( 11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) + 2\pi + \frac{1}{\phi} = \frac{263\,235\,569\,953\,644\,556\,439\,442\,644\,011}{1\,980\,704\,062\,856\,608\,439\,838\,598\,758} + \frac{1}{\phi} + 4 \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{1}{6} \left( 11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) + 2\pi + \frac{1}{\phi} = \frac{263\,235\,569\,953\,644\,556\,439\,442\,644\,011}{1\,980\,704\,062\,856\,608\,439\,838\,598\,758} + \frac{1}{\phi} + 4 \int_0^\infty \frac{1}{1+t^2} dt$$

$$\frac{1}{6} \left( 11 + 12 \left( \left( \frac{2^2}{1-2^2} \right) + \left( \frac{2 \times 2^4}{1-2^4} \right) \right) - 12 \left( \left( \frac{23 \times 2^{46}}{1-2^{46}} \right) + \left( \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) \right) - 7 - \frac{1}{\phi}$$

where 7 is a Lucas number

**Input:**

$$\frac{1}{6} \left( 11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 12 \left( \frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}} \right) \right) - 7 - \frac{1}{\phi}$$

φ is the golden ratio

**Result:**

$$\frac{249\,370\,641\,513\,648\,297\,360\,572\,452\,705}{1\,980\,704\,062\,856\,608\,439\,838\,598\,758} - \frac{1}{\phi}$$

**Decimal approximation:**

125.2819660112507588511123124856742413762538997703055110101...

125.281966.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternate forms:**









$$\frac{1416793898045596676147421}{10074381830121909789218} - \frac{\sqrt{5}}{2}$$

**Alternative representations:**

$$3 + 4 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 4 \left( \frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}} \right) - 76 - \phi =$$

$$-73 + 4 \left( -\frac{4}{3} + \frac{2 \times 2^4}{1-2^4} \right) - 4 \left( \frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}} \right) - 2 \sin(54^\circ)$$

$$3 + 4 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 4 \left( \frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}} \right) - 76 - \phi =$$

$$-73 + 2 \cos(216^\circ) + 4 \left( -\frac{4}{3} + \frac{2 \times 2^4}{1-2^4} \right) - 4 \left( \frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}} \right)$$

$$3 + 4 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 4 \left( \frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}} \right) - 76 - \phi =$$

$$-73 + 4 \left( -\frac{4}{3} + \frac{2 \times 2^4}{1-2^4} \right) - 4 \left( \frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}} \right) + 2 \sin(666^\circ)$$

$$3 + 4 \left( \left( \frac{2^2}{1-2^2} \right) + \left( \frac{2 \times 2^4}{1-2^4} \right) \right) - 4 \left( \left( \frac{19 \times 2^{38}}{1-2^{38}} \right) + \left( \frac{38 \times 2^{76}}{1-2^{76}} \right) \right) - 89 - \text{golden ratio}^2$$

**Input:**

$$3 + 4 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 4 \left( \frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}} \right) - 89 - \phi^2$$

$\phi$  is the golden ratio

**Result:**

$$\frac{645432062584536401891098}{5037190915060954894609} - \phi^2$$

**Decimal approximation:**

125.5152993448599248744707342381684641948378494281213687353...

125.515299... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

**Alternate forms:**

$$\frac{1\ 275\ 752\ 552\ 423\ 889\ 939\ 098\ 369 - 5\ 037\ 190\ 915\ 060\ 954\ 894\ 609\ \sqrt{5}}{10\ 074\ 381\ 830\ 121\ 909\ 789\ 218}$$

$$\frac{1\ 275\ 752\ 552\ 423\ 889\ 939\ 098\ 369}{10\ 074\ 381\ 830\ 121\ 909\ 789\ 218} - \frac{\sqrt{5}}{2}$$

$$\frac{645\ 432\ 062\ 584\ 536\ 401\ 891\ 098 - 5\ 037\ 190\ 915\ 060\ 954\ 894\ 609\ \phi^2}{5\ 037\ 190\ 915\ 060\ 954\ 894\ 609}$$

**Alternative representations:**

$$3 + 4 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 4 \left( \frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}} \right) - 89 - \phi^2 =$$

$$-86 + 4 \left( -\frac{4}{3} + \frac{2 \times 2^4}{1-2^4} \right) - 4 \left( \frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}} \right) - (2 \sin(54^\circ))^2$$

$$3 + 4 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 4 \left( \frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}} \right) - 89 - \phi^2 =$$

$$-86 - (-2 \cos(216^\circ))^2 + 4 \left( -\frac{4}{3} + \frac{2 \times 2^4}{1-2^4} \right) - 4 \left( \frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}} \right)$$

$$3 + 4 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 4 \left( \frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}} \right) - 89 - \phi^2 =$$

$$-86 + 4 \left( -\frac{4}{3} + \frac{2 \times 2^4}{1-2^4} \right) - 4 \left( \frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}} \right) - (-2 \sin(666^\circ))^2$$

$$8 * (((3+4(((2^2/(1-2^2)))+(2*2^4)/(1-2^4)))-4((((19*2^38)/(1-2^38)+(38*2^76)/(1-2^76)))))))-8$$

Where 8 is a Fibonacci number

**Input:**

$$8 \left( 3 + 4 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 4 \left( \frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}} \right) \right) - 8$$

**Exact result:**

$$\frac{8\ 709\ 638\ 904\ 879\ 203\ 460\ 933\ 520}{5\ 037\ 190\ 915\ 060\ 954\ 894\ 609}$$

**Decimal approximation:**

1729.066666668878557781402568580272818500465268863417052779...

1729.066...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternate form:**

$$\frac{8\,709\,638\,904\,879\,203\,460\,933\,520}{5\,037\,190\,915\,060\,954\,894\,609}$$

$$2 * (((3+4(((2^2/(1-2^2))+(2*2^4)/(1-2^4))))-4((((19*2^38)/(1-2^38)+(38*2^76)/(1-2^76)))))))+47+\text{golden ratio}$$

Where 47 is a Lucas number

**Input:**

$$2 \left( 3 + 4 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 4 \left( \frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}} \right) \right) + 47 + \phi$$

$\phi$  is the golden ratio

**Result:**

$$\phi + \frac{2\,424\,232\,081\,057\,787\,655\,069\,221}{5\,037\,190\,915\,060\,954\,894\,609}$$

**Decimal approximation:**

482.8847006559695342935552289794338427428366263956600260571...

482.8847006... result very near to Holographic Ricci dark energy model, where

$$\chi_{\text{RDE}}^2 = 483.130.$$



**Alternate forms:**

$$\frac{4853501353030636265033051 + 5037190915060954894609\sqrt{5}}{10074381830121909789218}$$

$$\frac{5037190915060954894609\phi + 2424232081057787655069221}{5037190915060954894609}$$

$$\frac{4853501353030636265033051}{10074381830121909789218} + \frac{\sqrt{5}}{2}$$

**Alternative representations:**

$$2\left(3+4\left(\frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4}\right) - 4\left(\frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}}\right)\right) + 47 + \phi =$$

$$47 + 2\left(3+4\left(-\frac{4}{3} + \frac{2 \times 2^4}{1-2^4}\right) - 4\left(\frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}}\right)\right) + 2 \sin(54^\circ)$$

$$2\left(3+4\left(\frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4}\right) - 4\left(\frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}}\right)\right) + 47 + \phi =$$

$$47 - 2 \cos(216^\circ) + 2\left(3+4\left(-\frac{4}{3} + \frac{2 \times 2^4}{1-2^4}\right) - 4\left(\frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}}\right)\right)$$

$$2\left(3+4\left(\frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4}\right) - 4\left(\frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}}\right)\right) + 47 + \phi =$$

$$47 + 2\left(3+4\left(-\frac{4}{3} + \frac{2 \times 2^4}{1-2^4}\right) - 4\left(\frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}}\right)\right) - 2 \sin(666^\circ)$$

We observe also that from the sum of the two results, adding 5 that is a Fibonacci number, we obtain:

$$11+12\left(\left(\frac{2^2}{1-2^2}\right)+\left(\frac{2 \times 2^4}{1-2^4}\right)\right)-12\left(\left(\frac{23 \times 2^{46}}{1-2^{46}}\right)+\left(\frac{46 \times 2^{92}}{1-2^{92}}\right)\right) + \left(\left(3+4\left(\frac{2^2}{1-2^2}\right)+\left(\frac{2 \times 2^4}{1-2^4}\right)\right)-4\left(\frac{19 \times 2^{38}}{1-2^{38}}\right)+\left(\frac{38 \times 2^{76}}{1-2^{76}}\right)\right)+5$$

**Input:**

$$11 + 12\left(\frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4}\right) - 12\left(\frac{23 \times 2^{46}}{1-2^{46}} + \frac{46 \times 2^{92}}{1-2^{92}}\right) +$$

$$\left(3+4\left(\frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4}\right) - 4\left(\frac{19 \times 2^{38}}{1-2^{38}} + \frac{38 \times 2^{76}}{1-2^{76}}\right)\right) + 5$$

**Exact result:**

$$\frac{1\ 695\ 345\ 363\ 604\ 491\ 043\ 436\ 883\ 743\ 614\ 143\ 568\ 419\ 476\ 425\ 677\ 491}{1\ 662\ 864\ 085\ 140\ 938\ 431\ 378\ 392\ 734\ 842\ 904\ 212\ 377\ 354\ 715\ 937}$$

**Decimal approximation:**

1019.533333333613741918576716992773379276403412308594774831...

1019.5333... result practically equal to the rest mass of Phi meson 1019.445

**Alternate form:**

$$\frac{1\ 695\ 345\ 363\ 604\ 491\ 043\ 436\ 883\ 743\ 614\ 143\ 568\ 419\ 476\ 425\ 677\ 491}{1\ 662\ 864\ 085\ 140\ 938\ 431\ 378\ 392\ 734\ 842\ 904\ 212\ 377\ 354\ 715\ 937}$$

**Mixed fraction:**

$$1019 \frac{886860845874781862301546809224176006951970137688}{1662864085140938431378392734842904212377354715937}$$

**Continued fraction:**

$$1019 + \frac{1}{1 + \frac{1}{1 + \frac{1}{6 + \frac{1}{1 + \frac{1}{15\ 849\ 886 + \frac{1}{3 + \frac{1}{19 + \frac{1}{2 + \frac{1}{4 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{5 + \frac{1}{1 + \frac{1}{5 + \frac{1}{1 + \frac{1}{4}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

$$= \phi^2(x) \phi^2(x^2) \left\{ 11 \cdot \frac{1 + \sqrt{4\beta} + \sqrt{(1-2\beta)(1-\beta)}}{2} - 16 \sqrt[3]{\beta} \cdot \sqrt[3]{4\beta(1-\beta)(1-\beta)} \cdot \frac{1 + \sqrt{4\beta} + \sqrt{(1-2\beta)(1-\beta)}}{2} - 10 \sqrt[3]{\frac{\beta}{4}} \cdot \sqrt[3]{4\beta(1-\beta)(1-\beta)} \right\}$$

$$\begin{aligned} & 11 + 12 \left( \frac{x^2}{1-x^2} + \frac{2x^4}{1-x^4} + 4x \right) - 12 \left( \frac{15x^{10}}{1-x^{10}} + \frac{30x^{20}}{1-x^{20}} + 2x \right) \\ &= \phi^2(x) \phi^2(x^2) \left\{ 7 \cdot \frac{1 + \sqrt{4\beta} + \sqrt{(1-2\beta)(1-\beta)}}{2} - 2 \sqrt[3]{4\beta(1-\beta)(1-\beta)} \left( 1 + \sqrt{4\beta} + \sqrt{(1-2\beta)(1-\beta)} \right) \right\} \\ &= \frac{11}{2 + \frac{2+1}{2+1} - \frac{8 \sqrt[3]{4\beta(1-\beta)(1-\beta)}}{(1 + \sqrt{4\beta} + \sqrt{(1-2\beta)(1-\beta)})^2}} \\ &= \phi^2(x) \phi^2(x^2) \left\{ 5 \cdot \frac{1 + \sqrt{4\beta} + \sqrt{(1-2\beta)(1-\beta)}}{2} - 6 \sqrt[3]{4\beta(1-\beta)(1-\beta)} \left( 1 + \sqrt{4\beta} + \sqrt{(1-2\beta)(1-\beta)} \right) \right\} \\ &= \phi^2(x) \phi^2(x^2) \left\{ 1 + 6 \left( \frac{x^2}{1-x^2} + \frac{2x^4}{1-x^4} + 4x \right) - 6 \left( \frac{15x^{10}}{1-x^{10}} + \frac{30x^{20}}{1-x^{20}} + 2x \right) \right\} \\ &= \phi^2(x) \phi^2(x^2) \sqrt{\frac{1 + \sqrt{4\beta} + \sqrt{(1-2\beta)(1-\beta)}}{2}} \left\{ \frac{1 + \sqrt{4\beta} + \sqrt{(1-2\beta)(1-\beta)}}{2} + \frac{1 - \sqrt{4\beta} - \sqrt{(1-2\beta)(1-\beta)}}{4} \right\} \\ &= \phi^2(x) \phi^2(x^2) \sqrt{\frac{1 + \sqrt{4\beta} + \sqrt{(1-2\beta)(1-\beta)}}{2}} - \frac{3}{\sqrt[3]{4}} \sqrt[3]{4\beta(1-\beta)(1-\beta)} \end{aligned}$$

Now, we have that:

$$11 + 12 \left( \left( \frac{2^2}{1-2^2} + \frac{2 \cdot 2^4}{1-2^4} \right) + \left( \frac{15 \cdot 2^{10}}{1-2^{10}} + \frac{30 \cdot 2^{20}}{1-2^{20}} \right) \right) - 12 \left( \left( \frac{15 \cdot 2^{10}}{1-2^{10}} + \frac{30 \cdot 2^{20}}{1-2^{20}} \right) + 2 \right)$$

**Input:**

$$11 + 12 \left( \frac{2^2}{1-2^2} + \frac{2 \cdot 2^4}{1-2^4} \right) - 12 \left( \frac{15 \cdot 2^{10}}{1-2^{10}} + \frac{30 \cdot 2^{20}}{1-2^{20}} \right)$$

**Exact result:**

$$\frac{35\,621\,931}{69\,905}$$

**Decimal approximation:**

509.5762964022602102853873113511193762964022602102853873113...

509.5762964...

**Mixed fraction:**

$$509 \frac{40286}{69905}$$

**Continued fraction:**

$$509 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3 + \frac{1}{2 + \frac{1}{10 + \frac{1}{1 + \frac{1}{19 + \frac{1}{1 + \frac{1}{4}}}}}}}}}}}}$$

$$5 + 4 \left( \left( \frac{2^2}{1 - 2^2} \right) + \left( \frac{2 \times 2^4}{1 - 2^4} \right) \right) - 4 \left( \left( \frac{31 \times 2^{62}}{1 - 2^{62}} \right) + \left( \frac{62 \times 2^{124}}{1 - 2^{124}} \right) \right)$$

**Input:**

$$5 + 4 \left( \frac{2^2}{1 - 2^2} + \frac{2 \times 2^4}{1 - 2^4} \right) - 4 \left( \frac{31 \times 2^{62}}{1 - 2^{62}} + \frac{62 \times 2^{124}}{1 - 2^{124}} \right)$$

**Exact result:**

$$\frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{141\,784\,319\,503\,910\,264\,430\,727\,530\,965\,700\,881}$$

**Decimal approximation:**

363.1333333333333333602215472109738433142095187351435258965...

363.1333...

**Alternate form:**

$$\frac{514866\ 125\ 727\ 319\ 947\ 395\ 068\ 128\ 497\ 011\ 253\ 285}{1417843\ 195\ 503\ 910\ 264430\ 727530\ 965\ 700\ 881}$$

**Mixed fraction:**

$$363 \frac{189045759400521406714034756461833482}{1417843195503910264430727530965700881}$$

**Continued fraction:**

$$363 + \frac{1}{7 + \frac{1}{2 + \frac{1}{165\ 293\ 405\ 678\ 400 + \frac{1}{1 + \frac{1}{1 + \frac{1}{6 + \frac{1}{1 + \frac{1}{123 + \frac{1}{7 + \frac{1}{1 + 1 \times \frac{1}{1}}}}}}}}}}}}$$

$$1 + 6 \left( \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) \right) - 6 \left( \left( \frac{5 \times 2^{10}}{1-2^{10}} + \frac{10 \times 2^{20}}{1-2^{20}} \right) \right)$$

**Input:**

$$1 + 6 \left( \frac{2^2}{1-2^2} + \frac{2 \times 2^4}{1-2^4} \right) - 6 \left( \frac{5 \times 2^{10}}{1-2^{10}} + \frac{10 \times 2^{20}}{1-2^{20}} \right)$$

**Exact result:**

$$\frac{981\ 877}{13\ 981}$$

**Decimal approximation:**

70.22938273371003504756455189185322938273371003504756455189...

70.2293827...

**Mixed fraction:**

$$70 \frac{3207}{13981}$$

**Continued fraction:**

$$70 + \frac{1}{4 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{4 + \frac{1}{2 + \frac{1}{3}}}}}}}}}}}}$$

We observe that, from the sum of the three results:

$$35621931/69905 + 514866125727319947395068128497011253285/1417843195503910264430727530965700881 + 981877/13981$$

We obtain:

$$(35621931/69905) + (514866125727319947395068128497011253285/1417843195503910264430727530965700881) + (981877/13981) - 4$$

Where 4 is a Lucas number

**Input:**

$$\frac{35621931}{69905} + \frac{514866125727319947395068128497011253285}{1417843195503910264430727530965700881} + \frac{981877}{13981} - 4$$

**Exact result:**

$$\frac{93062309800060263697797495157845549949832101}{99114328581700847035030008052157320086305}$$

**Decimal approximation:**

938.9390124693035786931734104539464489933454889804764777597...

938.93901246... result practically equal to the rest mass of neutron mass in MeV

$2 * (((((35621931/69905) + (514866125727319947395068128497011253285/1417843195503910264430727530965700881) + (981877/13981)))) - 123 - 29 - \pi - \text{golden ratio}$

Where 123 and 29 are Lucas numbers

**Input:**

$$2 \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{141\,784\,319\,550\,391\,026\,443\,072\,753\,096\,570\,0881} + \frac{981\,877}{13\,981} \right) - 123 - 29 - \pi - \phi$$

$\phi$  is the golden ratio

**Result:**

$$-\phi + \frac{171\,852\,156\,284\,355\,605\,422\,550\,669\,156\,180\,445\,807\,236\,282}{99\,114\,328\,581\,700\,847\,035\,030\,008\,052\,157\,320\,086\,305} - \pi$$

**Decimal approximation:**

1729.118398296267469299679590690247756984773499381772086836...

1729.118398...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Property:**

$$\frac{171\,852\,156\,284\,355\,605\,422\,550\,669\,156\,180\,445\,807\,236\,282}{99\,114\,328\,581\,700\,847\,035\,030\,008\,052\,157\,320\,086\,305} - \phi - \pi$$

is a transcendental number

**Alternate forms:**

$$\left( \frac{343\,605\,198\,240\,129\,509\,998\,066\,308\,304\,308\,734\,294\,386\,259 - 99\,114\,328\,581\,700\,847\,035\,030\,008\,052\,157\,320\,086\,305 \sqrt{5} - 198\,228\,657\,163\,401\,694\,070\,060\,016\,104\,314\,640\,172\,610 \pi}{198\,228\,657\,163\,401\,694\,070\,060\,016\,104\,314\,640\,172\,610} \right) / (-99\,114\,328\,581\,700\,847\,035\,030\,008\,052\,157\,320\,086\,305 \phi + 171\,852\,156\,284\,355\,605\,422\,550\,669\,156\,180\,445\,807\,236\,282 - 99\,114\,328\,581\,700\,847\,035\,030\,008\,052\,157\,320\,086\,305 \pi) / 99\,114\,328\,581\,700\,847\,035\,030\,008\,052\,157\,320\,086\,305$$

$$\frac{343\,605\,198\,240\,129\,509\,998\,066\,308\,304\,308\,734\,294\,386\,259}{198\,228\,657\,163\,401\,694\,070\,060\,016\,104\,314\,640\,172\,610} - \frac{\sqrt{5}}{2} - \pi$$

**Alternative representations:**

$$2 \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) -$$

$$123 - 29 - \pi - \phi = -152 - \pi + 2 \cos(216^\circ) +$$

$$2 \left( \frac{981\,877}{13\,981} + \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} \right)$$

$$2 \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) -$$

$$123 - 29 - \pi - \phi = -152 - 180^\circ + 2 \cos(216^\circ) +$$

$$2 \left( \frac{981\,877}{13\,981} + \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} \right)$$

$$2 \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) -$$

$$123 - 29 - \pi - \phi = -152 - \pi - 2 \cos\left(\frac{\pi}{5}\right) +$$

$$2 \left( \frac{981\,877}{13\,981} + \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} \right)$$

**Series representations:**

$$2 \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) -$$

$$123 - 29 - \pi - \phi =$$

$$\frac{171\,852\,156\,284\,355\,605\,422\,550\,669\,156\,180\,445\,807\,236\,282}{99\,114\,328\,581\,700\,847\,035\,030\,008\,052\,157\,320\,086\,305} - \phi - 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$2 \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) -$$

$$123 - 29 - \pi - \phi =$$

$$\frac{171\,852\,156\,284\,355\,605\,422\,550\,669\,156\,180\,445\,807\,236\,282}{99\,114\,328\,581\,700\,847\,035\,030\,008\,052\,157\,320\,086\,305} -$$

$$\phi + \sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}$$



$$2 \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) -$$

$$\frac{123 - 29 - \pi - \phi = 171\,852\,156\,284\,355\,605\,422\,550\,669\,156\,180\,445\,807\,236\,282}{99\,114\,328\,581\,700\,847\,035\,030\,008\,052\,157\,320\,086\,305} -$$

$$\phi - \sum_{k=0}^{\infty} \left( -\frac{1}{4} \right)^k \left( \frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right)$$

**Integral representations:**

$$2 \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) -$$

$$\frac{123 - 29 - \pi - \phi = 171\,852\,156\,284\,355\,605\,422\,550\,669\,156\,180\,445\,807\,236\,282}{99\,114\,328\,581\,700\,847\,035\,030\,008\,052\,157\,320\,086\,305} -$$

$$\phi - 4 \int_0^1 \sqrt{1-t^2} \, dt$$

$$2 \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) -$$

$$\frac{123 - 29 - \pi - \phi = 171\,852\,156\,284\,355\,605\,422\,550\,669\,156\,180\,445\,807\,236\,282}{99\,114\,328\,581\,700\,847\,035\,030\,008\,052\,157\,320\,086\,305} -$$

$$\phi - 2 \int_0^1 \frac{1}{\sqrt{1-t^2}} \, dt$$

$$2 \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) -$$

$$\frac{123 - 29 - \pi - \phi = 171\,852\,156\,284\,355\,605\,422\,550\,669\,156\,180\,445\,807\,236\,282}{99\,114\,328\,581\,700\,847\,035\,030\,008\,052\,157\,320\,086\,305} - \phi - 2 \int_0^{\infty} \frac{1}{1+t^2} \, dt$$

$$1/7((((35621931/69905)+(514866125727319947395068128497011253285/1417843195503910264430727530965700881)+(981877/13981))))+5$$

**Input:**

$$\frac{1}{7} \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) + 5$$

**Exact result:**

$$\frac{96\,927\,768\,614\,746\,596\,732\,163\,665\,471\,879\,685\,433\,197\,996}{693\,800\,300\,071\,905\,929\,245\,210\,056\,365\,101\,240\,604\,135}$$

**Decimal approximation:**

139.7055732099005112418819157791352069990493555686394968228...

139.705573... result practically equal to the rest mass of Pion meson 139.57 MeV

**Mixed fraction:**

$$139 \frac{489526904751672567079467637130612989223231}{693800300071905929245210056365101240604135}$$

**Continued fraction:**

$$139 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{10 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{34 + \cfrac{1}{1 + \cfrac{1}{3\ 020\ 096 + \cfrac{1}{1 + \cfrac{1}{20}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

$$1/7((((35621931/69905)+(514866125727319947395068128497011253285/1417843195503910264430727530965700881)+(981877/13981))))-11+\text{golden ratio}$$

Where 11 is a Lucas number

**Input:**

$$\frac{1}{7} \left( \frac{35\ 621\ 931}{69\ 905} + \frac{514\ 866\ 125\ 727\ 319\ 947\ 395\ 068\ 128\ 497\ 011\ 253\ 285}{1\ 417\ 843\ 195\ 503\ 910\ 264\ 430\ 727\ 530\ 965\ 700\ 881} + \frac{981\ 877}{13\ 981} \right) - 11 + \phi$$

φ is the golden ratio

**Result:**

$$\phi + \frac{85\,826\,963\,813\,596\,101\,864\,240\,304\,570\,038\,065\,583\,531\,836}{693\,800\,300\,071\,905\,929\,245\,210\,056\,365\,101\,240\,604\,135}$$

**Decimal approximation:**

125.3236071986504060900865026135008451167696647484452596849...

125.323607... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternate forms:**

$$\left( \frac{172\,347\,727\,927\,264\,109\,657\,725\,819\,196\,441\,232\,407\,667\,807 + 693\,800\,300\,071\,905\,929\,245\,210\,056\,365\,101\,240\,604\,135 \sqrt{5}}{1\,387\,600\,600\,143\,811\,858\,490\,420\,112\,730\,202\,481\,208\,270} \right)$$

$$\frac{(693\,800\,300\,071\,905\,929\,245\,210\,056\,365\,101\,240\,604\,135 \phi + 85\,826\,963\,813\,596\,101\,864\,240\,304\,570\,038\,065\,583\,531\,836)}{693\,800\,300\,071\,905\,929\,245\,210\,056\,365\,101\,240\,604\,135}$$

$$\frac{172\,347\,727\,927\,264\,109\,657\,725\,819\,196\,441\,232\,407\,667\,807}{1\,387\,600\,600\,143\,811\,858\,490\,420\,112\,730\,202\,481\,208\,270} + \frac{\sqrt{5}}{2}$$

**Alternative representations:**

$$\frac{1}{7} \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) - 11 + \phi = -11 + \frac{1}{7} \left( \frac{981\,877}{13\,981} + \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} \right) + 2 \sin(54^\circ)$$

$$\frac{1}{7} \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) - 11 + \phi = -11 - 2 \cos(216^\circ) + \frac{1}{7} \left( \frac{981\,877}{13\,981} + \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} \right)$$

$$\frac{1}{7} \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) - 11 + \phi = -11 + \frac{1}{7} \left( \frac{981\,877}{13\,981} + \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} \right) - 2 \sin(666^\circ)$$

$1/2((((35621931/69905)+(514866125727319947395068128497011253285/1417843195503910264430727530965700881)+(981877/13981))))-7-1/\text{golden ratio}$

Where 7 is a Lucas number

**Input:**

$$\frac{1}{2} \left( \frac{35621931}{69905} + \frac{514866125727319947395068128497011253285}{1417843195503910264430727530965700881} + \frac{981877}{13981} \right) - 7 - \frac{1}{\phi}$$

$\phi$  is the golden ratio

**Result:**

$$\frac{92071166514243255227447195077323976748969051}{198228657163401694070060016104314640172610} - \frac{1}{\phi}$$

**Decimal approximation:**

463.8514722459018944983821183926075863789524353104324760177...

463.8514722459... result very near to Holographic Dark Energy model, where

$$\chi_{\text{HDE}}^2 = 465.912.$$

**Alternate forms:**

$$\left( \frac{92170280842824956074482225085376134069055356 - 99114328581700847035030008052157320086305\sqrt{5}}{198228657163401694070060016104314640172610} \right) /$$

$$\frac{-(198228657163401694070060016104314640172610 - 92071166514243255227447195077323976748969051\phi)}{(198228657163401694070060016104314640172610\phi)}$$

$$\frac{(92071166514243255227447195077323976748969051\phi - 198228657163401694070060016104314640172610)}{(198228657163401694070060016104314640172610\phi)}$$

**Alternative representations:**

$$\frac{1}{2} \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) - \frac{1}{2 \sin(54^\circ)}$$

$$7 - \frac{1}{\phi} = -7 + \frac{1}{2} \left( \frac{981\,877}{13\,981} + \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} \right) - \frac{1}{2 \sin(54^\circ)}$$

$$\frac{1}{2} \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) - \frac{1}{2 \cos(216^\circ)}$$

$$7 - \frac{1}{\phi} = -7 + \frac{1}{2} \left( \frac{981\,877}{13\,981} + \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} \right) - \frac{1}{2 \cos(216^\circ)}$$

$$\frac{1}{2} \left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right) - \frac{1}{2 \sin(666^\circ)}$$

$$7 - \frac{1}{\phi} = -7 + \frac{1}{2} \left( \frac{981\,877}{13\,981} + \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} \right) - \frac{1}{2 \sin(666^\circ)}$$

And:

$$\frac{1}{10^{52}} \left( \left( \left( \left( 1 + \frac{1}{\left( \frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981} \right)} \right) + \frac{7+3}{10^2} + \frac{47-2}{10^4} \right) \right) \right)$$

Where 2, 3, 7 and 47 are Lucas numbers

**Input:**

$$\frac{1}{10^{52}} \left( 1 + \frac{1}{\frac{35\,621\,931}{69\,905} + \frac{514\,866\,125\,727\,319\,947\,395\,068\,128\,497\,011\,253\,285}{1\,417\,843\,195\,503\,910\,264\,430\,727\,530\,965\,700\,881} + \frac{981\,877}{13\,981}} + \frac{7+3}{10^2} + \frac{47-2}{10^4} \right)$$

**Exact result:**

206 648 645 212 844 432 886 906 251 970 933 996 559 634 312 089 /  
 1 869 175 342 287 741 341 718 752 303 801 083 584 603 546 420 000 000 000 000 :  
 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000

**Decimal approximation:**

1.1055605139746856682141664262726504850514268403606160... × 10<sup>-52</sup>

1.1055605... \* 10<sup>-52</sup> result practically equal to the value of Cosmological Constant

1.1056 \* 10<sup>-52</sup> m<sup>-2</sup>

**Alternate form:**

206 648 645 212 844 432 886 906 251 970 933 996 559 634 312 089 /

1 869 175 342 287 741 341 718 752 303 801 083 584 603 546 420 000 000 000 000 :

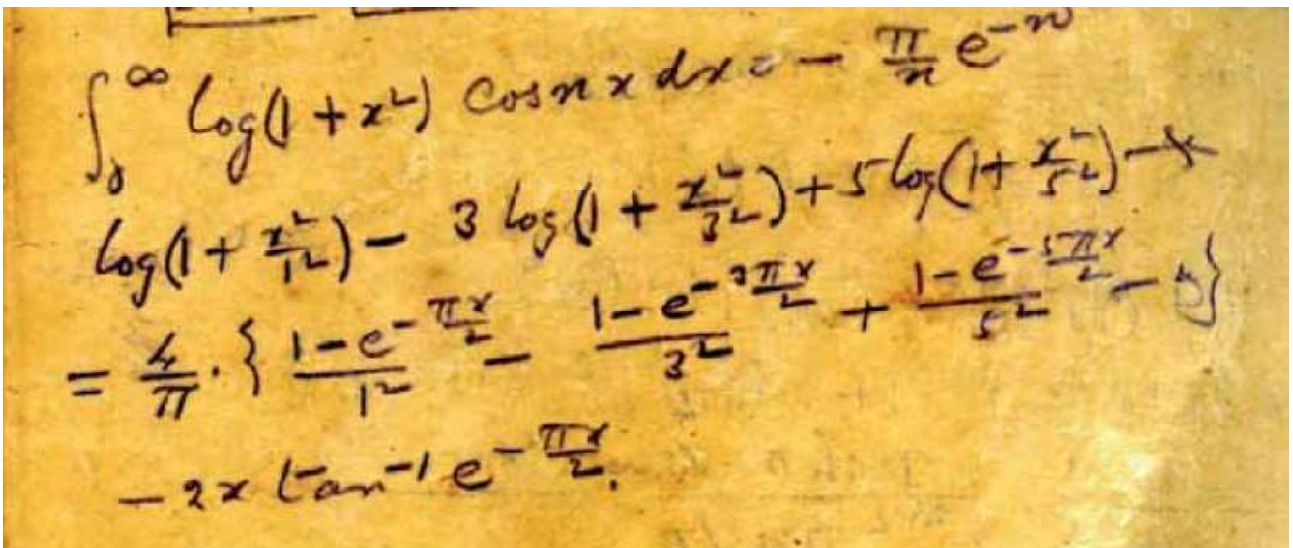
000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000

**Continued fraction:**

1

$$9045\ 185\ 562\ 975\ 861\ 517\ 070\ 790\ 391\ 337\ 765\ 558\ 947\ 244\ 523\ 684\ 128 + \frac{1}{3}$$

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integrate [ ln(1+x<sup>2</sup>) cos2x] dx – Pi/2\*e<sup>^(-2)</sup>

**Input:**

$$\int \log(1+x^2) \cos(2x) \, dx - \frac{\pi}{e^2}$$

log(x) is the natural logarithm

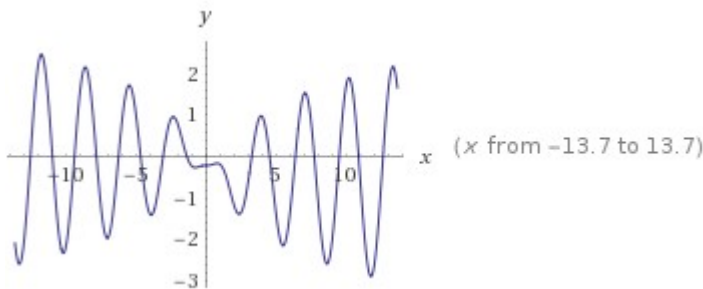
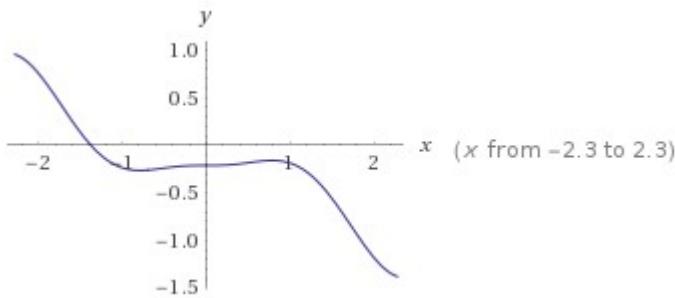
**Exact result:**

$$-\frac{\pi}{2e^2} + \frac{1}{4e^2} (-i(e^4 - 1) \text{Ci}(2i - 2x) + i(e^4 - 1) \text{Ci}(2x + 2i) + e^4 \text{Si}(2i - 2x) + \text{Si}(2i - 2x) - e^4 \text{Si}(2x + 2i) - \text{Si}(2x + 2i) + 2e^2 \log(x^2 + 1) \sin(2x))$$

Ci(x) is the cosine integral

Si(x) is the sine integral

**Plots:**



**Series expansion of the integral at x = 0:**

$$-\frac{\pi}{2e^2} + \frac{x^3}{3} - \frac{x^5}{2} + \frac{2x^7}{7} + O(x^9)$$

(Taylor series)

**Series expansion of the integral at x = -i:**

$$\frac{1}{4e^2} i(-e^4 \text{Ci}(4i) + \text{Ci}(4i) + e^4 \text{Shi}(4) + \text{Shi}(4) - \frac{(e^4 - 1) \log(1 - ix) + (e^4 - 1) \log(x + i) + 2i\pi + e^4 \gamma - \gamma}{(1 + e^4)(x + i)(-1 + \log(2 - 2ix))} + \frac{i(x + i)^2 (4(e^4 - 1) \log(2 - 2ix) - e^4 + 3)}{8e^2} + \frac{(x + i)^3 (-48(1 + e^4) \log(2 - 2ix) - 5e^4 + 43)}{144e^2} + O((x + i)^4))$$

(generalized Puiseux series)

**Series expansion of the integral at  $x = i$ :**

$$\begin{aligned} & \frac{1}{2e^2} \\ & \left( \left( -\frac{1}{2} i (-e^4 \log(ix+1) + \log(2ix+2) + e^4 \log(x-i) - \log(2(x-i)) + e^4 \operatorname{Shi}(4) + \operatorname{Shi}(4) - \right. \right. \\ & \quad \left. \left. e^4 \operatorname{Ci}(4i) + \operatorname{Ci}(4i) + i e^4 \pi - 3i\pi + e^4 \gamma - \gamma) + (1+e^4) \right. \right. \\ & \quad \left. \left. (\log(2ix+2) - 1)(x-i) - \frac{1}{4} i (4(-1+e^4) \log(2ix+2) - e^4 + 3)(x-i)^2 + \right. \right. \\ & \quad \left. \left. \frac{1}{72} (-48(1+e^4) \log(2ix+2) - 5e^4 + 43)(x-i)^3 + O((x-i)^4) \right) + \right. \\ & \quad \left. \pi \left[ \frac{\arg(x-i)}{2\pi} \right] + e^4 \pi \left[ -\frac{\arg(x-i)}{2\pi} \right] \right) \end{aligned}$$

**Series expansion of the integral at  $x = \infty$ :**

$$\begin{aligned} & \sin(2x) \left( \log(x) + \frac{1}{2x^2} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \left( -\frac{i(-8i\pi - 2\log(2) + 2e^4 \log(2) + \log(4) - e^4 \log(4))}{8e^2} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \sinh(2-2ix) \left( \frac{i}{16e^2 x^2} + \frac{1}{8e^2 x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \sinh(2-2ix) \left( \frac{ie^2}{16x^2} + \frac{e^2}{8x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \sinh(2-2ix) \left( \frac{1}{8e^2 x} - \frac{i}{8e^2 x^2} - \frac{3}{16e^2 x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \sinh(2-2ix) \left( -\frac{e^2}{8x} + \frac{ie^2}{8x^2} + \frac{3e^2}{16x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \sinh(2+2ix) \left( -\frac{i}{16e^2 x^2} + \frac{1}{8e^2 x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \sinh(2+2ix) \left( -\frac{ie^2}{16x^2} + \frac{e^2}{8x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \sinh(2+2ix) \left( \frac{1}{8e^2 x} + \frac{i}{8e^2 x^2} - \frac{3}{16e^2 x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \sinh(2+2ix) \left( -\frac{e^2}{8x} - \frac{ie^2}{8x^2} + \frac{3e^2}{16x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \cosh(2+2ix) \left( -\frac{i}{16e^2 x^2} + \frac{1}{8e^2 x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \cosh(2-2ix) \left( \frac{i}{16e^2 x^2} + \frac{1}{8e^2 x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \cosh(2-2ix) \left( -\frac{ie^2}{16x^2} - \frac{e^2}{8x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \cosh(2+2ix) \left( \frac{ie^2}{16x^2} - \frac{e^2}{8x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \cosh(2-2ix) \left( \frac{1}{8e^2 x} - \frac{i}{8e^2 x^2} - \frac{3}{16e^2 x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \cosh(2+2ix) \left( \frac{1}{8e^2 x} + \frac{i}{8e^2 x^2} - \frac{3}{16e^2 x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \cosh(2-2ix) \left( \frac{e^2}{8x} - \frac{ie^2}{8x^2} - \frac{3e^2}{16x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) + \\ & \cosh(2+2ix) \left( \frac{e^2}{8x} + \frac{ie^2}{8x^2} - \frac{3e^2}{16x^3} + O\left(\left(\frac{1}{x}\right)^4\right) \right) \end{aligned}$$



**Indefinite integral:**

$$\int \log(1+x^2) \cos(2x) dx - \frac{\pi}{2e^2} = -\frac{\pi}{2e^2} + \left( \text{constant} + \frac{1}{4e^2} (-i(e^4-1) \text{Ci}(2i-2x) + i(e^4-1) \text{Ci}(2x+2i) + e^4 \text{Si}(2i-2x) + \text{Si}(2i-2x) - e^4 \text{Si}(2x+2i) - \text{Si}(2x+2i) + 2e^2 \log(x^2+1) \sin(2x)) \right)$$

From

$$-\frac{\pi}{2e^2} + \frac{1}{4e^2} (-i(e^4-1) \text{Ci}(2i-2x) + i(e^4-1) \text{Ci}(2x+2i) + e^4 \text{Si}(2i-2x) + \text{Si}(2i-2x) - e^4 \text{Si}(2x+2i) - \text{Si}(2x+2i) + 2e^2 \log(x^2+1) \sin(2x))$$

For  $x = 2$ , we obtain:

$$-\pi/(2e^2) + (-i(e^4-1) \text{Ci}(2i-4) + i(e^4-1) \text{Ci}(4+2i) + e^4 \text{Si}(2i-4) + \text{Si}(2i-4) - e^4 \text{Si}(4+2i) - \text{Si}(4+2i) + 2e^2 \log(4+1) \sin(4))/(4e^2)$$

**Input:**

$$-\frac{\pi}{2e^2} + \frac{1}{4e^2} (-i(e^4-1) \text{Ci}(2i-4) + i(e^4-1) \text{Ci}(4+2i) + e^4 \text{Si}(2i-4) + \text{Si}(2i-4) - e^4 \text{Si}(4+2i) - \text{Si}(4+2i) + 2e^2 \log(4+1) \sin(4))$$

$\text{Ci}(x)$  is the cosine integral

$\text{Si}(x)$  is the sine integral

$\log(x)$  is the natural logarithm

$i$  is the imaginary unit

**Exact result:**

$$-\frac{\pi}{2e^2} + \frac{1}{4e^2} (-i(e^4-1) \text{Ci}(-4+2i) + i(e^4-1) \text{Ci}(4+2i) + \text{Si}(-4+2i) + e^4 \text{Si}(-4+2i) - \text{Si}(4+2i) - e^4 \text{Si}(4+2i) + 2e^2 \log(5) \sin(4))$$

**Decimal approximation:**

-1.18321332402796601454275392981085658958037709205874608141...

-1.18321332...

**Alternate forms:**

$$\frac{1}{4e^2} (e^4 (-i \text{Ci}(-4 + 2i) + i \text{Ci}(4 + 2i) + \text{Si}(-4 + 2i) - \text{Si}(4 + 2i)) + i \text{Ci}(-4 + 2i) - i \text{Ci}(4 + 2i) + \text{Si}(-4 + 2i) - \text{Si}(4 + 2i) - 2\pi + e^2 \log(25) \sin(4))$$

$$\frac{1}{4e^2} (i \text{Ci}(-4 + 2i) - i e^4 \text{Ci}(-4 + 2i) - i \text{Ci}(4 + 2i) + i e^4 \text{Ci}(4 + 2i) + \text{Si}(-4 + 2i) + e^4 \text{Si}(-4 + 2i) - \text{Si}(4 + 2i) - e^4 \text{Si}(4 + 2i) - 2\pi + 2e^2 \log(5) \sin(4))$$

$$\frac{\frac{1}{4} i \text{Ci}(-4 + 2i) - \frac{1}{4} i \text{Ci}(4 + 2i) + \frac{1}{4} \text{Si}(-4 + 2i) - \frac{\text{Si}(4 + 2i)}{4} - \frac{\pi}{2}}{e^2} + e^2 \left( -\frac{1}{4} i \text{Ci}(-4 + 2i) + \frac{1}{4} i \text{Ci}(4 + 2i) + \frac{1}{4} \text{Si}(-4 + 2i) - \frac{\text{Si}(4 + 2i)}{4} \right) + \frac{1}{2} \log(5) \sin(4)$$

**Expanded form:**

$$\frac{i \text{Ci}(-4 + 2i)}{4e^2} - \frac{1}{4} i e^2 \text{Ci}(-4 + 2i) - \frac{i \text{Ci}(4 + 2i)}{4e^2} + \frac{1}{4} i e^2 \text{Ci}(4 + 2i) + \frac{\text{Si}(-4 + 2i)}{4e^2} + \frac{1}{4} e^2 \text{Si}(-4 + 2i) - \frac{\text{Si}(4 + 2i)}{4e^2} - \frac{1}{4} e^2 \text{Si}(4 + 2i) - \frac{\pi}{2e^2} + \frac{1}{2} \log(5) \sin(4)$$

**Alternative representations:**

$$\begin{aligned} & -\frac{\pi}{2e^2} + \frac{1}{4e^2} (-i((e^4 - 1) \text{Ci}(2i - 4)) + i(e^4 - 1) \text{Ci}(4 + 2i) + e^4 \text{Si}(2i - 4) + \text{Si}(2i - 4) - e^4 \text{Si}(4 + 2i) - \text{Si}(4 + 2i) + 2e^2 \log(4 + 1) \sin(4)) = \\ & -\frac{\pi}{2e^2} + \frac{1}{4e^2} \left( -i((\text{Chi}(i(-4 + 2i)) + \log(-4 + 2i) - \log(i(-4 + 2i)))(-1 + e^4)) + i(\text{Chi}(i(4 + 2i)) + \log(4 + 2i) - \log(i(4 + 2i)))(-1 + e^4) + \frac{2 \log(a) \log_a(5) e^2 (-e^{-4i} + e^{4i})}{2i} - i \text{Shi}(i(-4 + 2i)) - i(e^4 \text{Shi}(i(-4 + 2i))) + i \text{Shi}(i(4 + 2i)) + i e^4 \text{Shi}(i(4 + 2i)) \right) \end{aligned}$$

$$\begin{aligned}
& -\frac{\pi}{2e^2} + \frac{1}{4e^2} (-i((e^4 - 1) \text{Ci}(2i - 4)) + i(e^4 - 1) \text{Ci}(4 + 2i) + e^4 \text{Si}(2i - 4) + \\
& \quad \text{Si}(2i - 4) - e^4 \text{Si}(4 + 2i) - \text{Si}(4 + 2i) + 2e^2 \log(4 + 1) \sin(4)) = -\frac{\pi}{2e^2} + \\
& \quad \frac{1}{4e^2} \left( -i \left( \left( \log(-4 + 2i) + \frac{1}{2} (-\Gamma(0, -i(-4 + 2i)) - \Gamma(0, i(-4 + 2i)) - \log(-i(-4 + 2i)) - \right. \right. \right. \\
& \quad \left. \left. \left. \log(i(-4 + 2i)) \right) \right) (-1 + e^4) \right) + i \left( \log(4 + 2i) + \right. \\
& \quad \left. \frac{1}{2} (-\Gamma(0, -i(4 + 2i)) - \Gamma(0, i(4 + 2i)) - \log(-i(4 + 2i)) - \log(i(4 + 2i))) \right) \\
& \quad \left. (-1 + e^4) + \frac{2 \log(a) \log_a(5) e^2 (-e^{-4i} + e^{4i})}{2i} - i \text{Shi}(i(-4 + 2i)) - \right. \\
& \quad \left. i(e^4 \text{Shi}(i(-4 + 2i))) + i \text{Shi}(i(4 + 2i)) + i e^4 \text{Shi}(i(4 + 2i)) \right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{\pi}{2e^2} + \frac{1}{4e^2} (-i((e^4 - 1) \text{Ci}(2i - 4)) + i(e^4 - 1) \text{Ci}(4 + 2i) + e^4 \text{Si}(2i - 4) + \\
& \quad \text{Si}(2i - 4) - e^4 \text{Si}(4 + 2i) - \text{Si}(4 + 2i) + 2e^2 \log(4 + 1) \sin(4)) = -\frac{\pi}{2e^2} + \\
& \quad \frac{1}{4e^2} \left( \frac{1}{2} i (\Gamma(0, -i(-4 + 2i)) - \Gamma(0, i(-4 + 2i)) + \log(-i(-4 + 2i)) - \log(i(-4 + 2i))) - \right. \\
& \quad \frac{1}{2} i (\Gamma(0, -i(4 + 2i)) - \Gamma(0, i(4 + 2i)) + \log(-i(4 + 2i)) - \log(i(4 + 2i))) + \\
& \quad 2 \cos\left(-4 + \frac{\pi}{2}\right) \log(a) \log_a(5) e^2 - \\
& \quad i \left( \left( \log(-4 + 2i) + \frac{1}{2} (-\Gamma(0, -i(-4 + 2i)) - \Gamma(0, i(-4 + 2i)) - \right. \right. \\
& \quad \left. \left. \log(-i(-4 + 2i)) - \log(i(-4 + 2i))) \right) \right) (-1 + e^4) \right) + \\
& \quad i \left( \log(4 + 2i) + \frac{1}{2} (-\Gamma(0, -i(4 + 2i)) - \Gamma(0, i(4 + 2i)) - \log(-i(4 + 2i)) - \right. \\
& \quad \left. \log(i(4 + 2i))) \right) (-1 + e^4) + \frac{1}{2} i \\
& \quad \left. (\Gamma(0, -i(-4 + 2i)) - \Gamma(0, i(-4 + 2i)) + \log(-i(-4 + 2i)) - \log(i(-4 + 2i))) e^4 - \right. \\
& \quad \left. \frac{1}{2} i (\Gamma(0, -i(4 + 2i)) - \Gamma(0, i(4 + 2i)) + \log(-i(4 + 2i)) - \log(i(4 + 2i))) e^4 \right)
\end{aligned}$$

**Series representations:**

$$\begin{aligned}
 & -\frac{\pi}{2e^2} + \frac{1}{4e^2} (-i((e^4 - 1) \text{Ci}(2i - 4)) + i(e^4 - 1) \text{Ci}(4 + 2i) + e^4 \text{Si}(2i - 4) + \\
 & \quad \text{Si}(2i - 4) - e^4 \text{Si}(4 + 2i) - \text{Si}(4 + 2i) + 2e^2 \log(4 + 1) \sin(4)) = \frac{1}{4e^2} \\
 & \left( -2\pi + i \log(-4 + 2i) - ie^4 \log(-4 + 2i) - i \log(4 + 2i) + ie^4 \log(4 + 2i) + \right. \\
 & \quad 4e^2 \sum_{k=1}^{\infty} -\frac{i(-1)^k 2^{-3+2k} ((-2+i)^{2k} - (2+i)^{2k})(-1+e^4)}{e^2 k (2k)!} + \\
 & \quad 4e^2 \sum_{k=0}^{\infty} \left( (-1)^k 2^{-1+2k} ((-2+i)^{1+2k} - (2+i)^{1+2k} + \right. \\
 & \quad \quad \left. \left. ((-2+i)^{1+2k} - (2+i)^{1+2k}) e^4 + 4^{1+k} e^2 (1+2k) \log(4) \right) \right) / \\
 & \quad \left. \left( e^2 (1+2k)(1+2k)! - 2e^2 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 4^{1-k_1+2k_2}}{(1+2k_2)! k_1} \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{\pi}{2e^2} + \frac{1}{4e^2} (-i((e^4 - 1) \text{Ci}(2i - 4)) + i(e^4 - 1) \text{Ci}(4 + 2i) + e^4 \text{Si}(2i - 4) + \\
 & \quad \text{Si}(2i - 4) - e^4 \text{Si}(4 + 2i) - \text{Si}(4 + 2i) + 2e^2 \log(4 + 1) \sin(4)) = \frac{1}{4e^2} \\
 & \left( -2\pi + i \log(-4 + 2i) - ie^4 \log(-4 + 2i) - i \log(4 + 2i) + ie^4 \log(4 + 2i) + \right. \\
 & \quad 4e^2 \sum_{k=1}^{\infty} -\frac{i(-1)^k 2^{-3+2k} ((-2+i)^{2k} - (2+i)^{2k})(-1+e^4)}{e^2 k (2k)!} + \\
 & \quad 4e^2 \sum_{k=0}^{\infty} \left( \frac{(-2+i)^{1+2k} (-1)^k 2^{-1+2k}}{e^2 (1+2k)^2 (2k)!} + \frac{(-1)^{1+k} 2^{-1+2k} (2+i)^{1+2k}}{e^2 (1+2k)^2 (2k)!} + \right. \\
 & \quad \quad \frac{(-2+i)^{1+2k} (-1)^k 2^{-1+2k} e^2}{(1+2k)^2 (2k)!} + \frac{(-1)^{1+k} 2^{-1+2k} (2+i)^{1+2k} e^2}{(1+2k)^2 (2k)!} + \\
 & \quad \quad \left. \left. \frac{(-1)^k 2^{1+4k} \log(4)}{(1+2k)!} \right) - 2e^2 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 4^{1-k_1+2k_2}}{(1+2k_2)! k_1} \right)
 \end{aligned}$$

$$\begin{aligned}
& -\frac{\pi}{2e^2} + \frac{1}{4e^2} (-i((e^4 - 1) \text{Ci}(2i - 4)) + i(e^4 - 1) \text{Ci}(4 + 2i) + e^4 \text{Si}(2i - 4) + \\
& \quad \text{Si}(2i - 4) - e^4 \text{Si}(4 + 2i) - \text{Si}(4 + 2i) + 2e^2 \log(4 + 1) \sin(4)) = \frac{1}{4e^2} \\
& \left( -2\pi + i \log(-4 + 2i) - ie^4 \log(-4 + 2i) - i \log(4 + 2i) + ie^4 \log(4 + 2i) + \right. \\
& \quad 4e^2 \sum_{k=1}^{\infty} -\frac{i(-1)^k 2^{-3+2k} ((-2+i)^{2k} - (2+i)^{2k})(-1+e^4)}{e^2 k (2k)!} + \\
& \quad 4e^2 \sum_{k=0}^{\infty} \left( \frac{(-2+i)^{1+2k} (-1)^k 2^{-1+2k}}{e^2 (1+2k)^2 (2k)!} + \frac{(-1)^{1+k} 2^{-1+2k} (2+i)^{1+2k}}{e^2 (1+2k)^2 (2k)!} + \right. \\
& \quad \left. \frac{(-2+i)^{1+2k} (-1)^k 2^{-1+2k} e^2}{(1+2k)^2 (2k)!} + \frac{(-1)^{1+k} 2^{-1+2k} (2+i)^{1+2k} e^2}{(1+2k)^2 (2k)!} + \right. \\
& \quad \left. \frac{(-1)^k (4 - \frac{\pi}{2})^{2k} \log(4)}{2(2k)!} \right) - 2e^2 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 4^{-k_1} (4 - \frac{\pi}{2})^{2k_2}}{(2k_2)! k_1} \Big)
\end{aligned}$$

$$\ln(1+4) - 3 \ln(1+4/9) + 5 \ln(1+4/25)$$

**Input:**

$$\log(1 + 4) - 3 \log\left(1 + \frac{4}{9}\right) + 5 \log\left(1 + \frac{4}{25}\right)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5)$$

**Decimal approximation:**

1.248363597649514704720535259613067685211291893852936258545...

1.2483635...

**Property:**

$5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5)$  is a transcendental number

**Alternate forms:**

$$5 \log\left(\frac{29}{25}\right) + \log\left(\frac{3645}{2197}\right)$$

$$6 \log(3) - 9 \log(5) - 3 \log(13) + 5 \log(29)$$

$$-9 \log(5) - 3 \log(13) + 5 \log(29) + \log(729)$$

### Alternative representations:

$$\log(1+4) - 3 \log\left(1 + \frac{4}{9}\right) + 5 \log\left(1 + \frac{4}{25}\right) =$$

$$\log(a) \log_a(5) - 3 \log(a) \log_a\left(1 + \frac{4}{9}\right) + 5 \log(a) \log_a\left(1 + \frac{4}{25}\right)$$

$$\log(1+4) - 3 \log\left(1 + \frac{4}{9}\right) + 5 \log\left(1 + \frac{4}{25}\right) = \log_e(5) - 3 \log_e\left(1 + \frac{4}{9}\right) + 5 \log_e\left(1 + \frac{4}{25}\right)$$

$$\log(1+4) - 3 \log\left(1 + \frac{4}{9}\right) + 5 \log\left(1 + \frac{4}{25}\right) = -\text{Li}_1(-4) + 3 \text{Li}_1\left(-\frac{4}{9}\right) - 5 \text{Li}_1\left(-\frac{4}{25}\right)$$

### Series representations:

$$\log(1+4) - 3 \log\left(1 + \frac{4}{9}\right) + 5 \log\left(1 + \frac{4}{25}\right) = 6 i \pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right] +$$

$$3 \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^{1+k} \left( 5 \left(\frac{29}{25} - z_0\right)^k - 3 \left(\frac{13}{9} - z_0\right)^k + (5 - z_0)^k \right) z_0^{-k}}{k}$$

$$\log(1+4) - 3 \log\left(1 + \frac{4}{9}\right) + 5 \log\left(1 + \frac{4}{25}\right) =$$

$$10 i \pi \left[ \frac{\arg\left(\frac{29}{25} - x\right)}{2 \pi} \right] - 6 i \pi \left[ \frac{\arg\left(\frac{13}{9} - x\right)}{2 \pi} \right] + 2 i \pi \left[ \frac{\arg(5 - x)}{2 \pi} \right] + 3 \log(x) +$$

$$\sum_{k=1}^{\infty} \frac{(-1)^{1+k} \left( 5 \left(\frac{29}{25} - x\right)^k - 3 \left(\frac{13}{9} - x\right)^k + (5 - x)^k \right) x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(1+4) - 3 \log\left(1 + \frac{4}{9}\right) + 5 \log\left(1 + \frac{4}{25}\right) =$$

$$5 \left[ \frac{\arg\left(\frac{29}{25} - z_0\right)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) - 3 \left[ \frac{\arg\left(\frac{13}{9} - z_0\right)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) + \left[ \frac{\arg(5 - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) +$$

$$3 \log(z_0) + 5 \left[ \frac{\arg\left(\frac{29}{25} - z_0\right)}{2 \pi} \right] \log(z_0) - 3 \left[ \frac{\arg\left(\frac{13}{9} - z_0\right)}{2 \pi} \right] \log(z_0) +$$

$$\left[ \frac{\arg(5 - z_0)}{2 \pi} \right] \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^{1+k} \left( 5 \left(\frac{29}{25} - z_0\right)^k - 3 \left(\frac{13}{9} - z_0\right)^k + (5 - z_0)^k \right) z_0^{-k}}{k}$$

**Integral representations:**

$$\log(1 + 4) - 3 \log\left(1 + \frac{4}{9}\right) + 5 \log\left(1 + \frac{4}{25}\right) = \int_1^{\frac{29}{25}} 5 \left( \frac{1}{t} + 5 \left( \frac{3}{16 - 25t} + \frac{1}{-24 + 25t} \right) \right) dt$$

$$\log(1 + 4) - 3 \log\left(1 + \frac{4}{9}\right) + 5 \log\left(1 + \frac{4}{25}\right) = \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{i 2^{-1-2s} (-1 + 3^{1+2s} - 5^{1+2s}) \Gamma(-s)^2 \Gamma(1+s)}{\pi \Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$$4/\text{Pi}(((((((1-e^{((-2\text{Pi})/2)})/(1)^2))-((1-e^{((-6\text{Pi})/2)})/(3)^2))+((1-e^{((-10\text{Pi})/2)})/(5)^2)))))-2*2 \tan^{-1}(e^{((-2\text{Pi})/2)})$$

**Input:**

$$\frac{4}{\pi} \left( \left( \frac{1 - e^{1/2(-2\pi)}}{1^2} - \frac{1 - e^{1/2(-6\pi)}}{3^2} \right) + \frac{1 - e^{1/2(-10\pi)}}{5^2} \right) - (2 \times 2) \tan^{-1}(e^{1/2(-2\pi)})$$

$\tan^{-1}(x)$  is the inverse tangent function

**Exact Result:**

$$\frac{4 \left( 1 - e^{-\pi} + \frac{1}{25} (1 - e^{-5\pi}) + \frac{1}{9} (e^{-3\pi} - 1) \right)}{\pi} - 4 \tan^{-1}(e^{-\pi})$$

(result in radians)

**Decimal approximation:**

0.954939611254082249939094312747766773216649377749888300192...

(result in radians)

0.954939611254.... result very near to the spectral index  $n_s$  , to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 (see Appendix) and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}} \approx 0.9568666373$$

**Alternate forms:**

$$\frac{4(-209 + 9e^{-5\pi} - 25e^{-3\pi} + 225e^{-\pi} + 225\pi \tan^{-1}(e^{-\pi}))}{225\pi}$$

$$\frac{836 - 4e^{-5\pi}(9 - 25e^{2\pi} + 225e^{4\pi})}{225\pi} - 4 \cot^{-1}(e^\pi)$$

$$\frac{4\left(1 - e^{-\pi} + \frac{1}{25}(1 - e^{-5\pi}) + \frac{1}{9}(e^{-3\pi} - 1)\right)}{\pi} - 4 \cot^{-1}(e^\pi)$$

$\cot^{-1}(x)$  is the inverse cotangent function

**Continued fraction:**

$$1 + \frac{1}{21 + \frac{1}{5 + \frac{1}{5 + \frac{1}{10 + \frac{1}{1 + \frac{1}{15 + \frac{1}{2 + \frac{1}{2 + \frac{1}{1 + \frac{1}{5 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3 + \frac{1}{53 + \frac{1}{3 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

**Alternative representations:**

$$\frac{\left(\left(\frac{1-e^{-(2\pi)/2}}{1^2} - \frac{1-e^{-(6\pi)/2}}{3^2}\right) + \frac{1-e^{-(10\pi)/2}}{5^2}\right) 4}{\pi} - \tan^{-1}(e^{-(2\pi)/2}) 2 \times 2 =$$
$$-4 \operatorname{sc}^{-1}(e^{-\pi} | 0) + \frac{4\left(-\frac{1}{9}(1 - e^{-3\pi}) + \frac{1}{1}(1 - e^{-\pi}) + \frac{1-e^{-5\pi}}{5^2}\right)}{\pi}$$

$$\frac{\left(\left(\frac{1-e^{-(2\pi)/2}}{1^2} - \frac{1-e^{-(6\pi)/2}}{3^2}\right) + \frac{1-e^{-(10\pi)/2}}{5^2}\right) 4}{\pi} - \tan^{-1}(e^{-(2\pi)/2}) 2 \times 2 =$$
$$-4 \tan^{-1}(1, e^{-\pi}) + \frac{4\left(-\frac{1}{9}(1 - e^{-3\pi}) + \frac{1}{1}(1 - e^{-\pi}) + \frac{1-e^{-5\pi}}{5^2}\right)}{\pi}$$



$$\left( \frac{1-e^{-(2\pi)/2}}{1^2} - \frac{1-e^{-(6\pi)/2}}{3^2} + \frac{1-e^{-(10\pi)/2}}{5^2} \right) 4 - \tan^{-1}(e^{-(2\pi)/2}) 2 \times 2 =$$

$$-4 \cot^{-1}\left(\frac{1}{e^{-\pi}}\right) + \frac{4\left(-\frac{1}{9}(1-e^{-3\pi}) + \frac{1}{1}(1-e^{-\pi}) + \frac{1-e^{-5\pi}}{5^2}\right)}{\pi}$$

### Series representations:

$$\left( \frac{1-e^{-(2\pi)/2}}{1^2} - \frac{1-e^{-(6\pi)/2}}{3^2} + \frac{1-e^{-(10\pi)/2}}{5^2} \right) 4 - \tan^{-1}(e^{-(2\pi)/2}) 2 \times 2 =$$

$$\frac{836}{225\pi} - \frac{4e^{-5\pi}}{25\pi} + \frac{4e^{-3\pi}}{9\pi} - \frac{4e^{-\pi}}{\pi} - 4 \sum_{k=0}^{\infty} \frac{e^{(-1-(2-i)k)\pi}}{1+2k}$$

$$\left( \frac{1-e^{-(2\pi)/2}}{1^2} - \frac{1-e^{-(6\pi)/2}}{3^2} + \frac{1-e^{-(10\pi)/2}}{5^2} \right) 4 - \tan^{-1}(e^{-(2\pi)/2}) 2 \times 2 =$$

$$\frac{836}{225\pi} - \frac{4e^{-5\pi}}{25\pi} + \frac{4e^{-3\pi}}{9\pi} - \frac{4e^{-\pi}}{\pi} - 2i \log(2) + 2i \log(i(-i+e^{-\pi})) + 2i \sum_{k=1}^{\infty} \frac{\left(\frac{1}{2} + \frac{ie^{-\pi}}{2}\right)^k}{k}$$

$$\left( \frac{1-e^{-(2\pi)/2}}{1^2} - \frac{1-e^{-(6\pi)/2}}{3^2} + \frac{1-e^{-(10\pi)/2}}{5^2} \right) 4 - \tan^{-1}(e^{-(2\pi)/2}) 2 \times 2 = \frac{836}{225\pi} - \frac{4e^{-5\pi}}{25\pi} +$$

$$\frac{4e^{-3\pi}}{9\pi} - \frac{4e^{-\pi}}{\pi} + 2i \log(2) - 2i \log(-i(i+e^{-\pi})) - 2i \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}i(i+e^{-\pi})\right)^k}{k}$$

### Integral representations:

$$\left( \frac{1-e^{-(2\pi)/2}}{1^2} - \frac{1-e^{-(6\pi)/2}}{3^2} + \frac{1-e^{-(10\pi)/2}}{5^2} \right) 4 - \tan^{-1}(e^{-(2\pi)/2}) 2 \times 2 =$$

$$\frac{836}{225\pi} - \frac{4e^{-5\pi}}{25\pi} + \frac{4e^{-3\pi}}{9\pi} - \frac{4e^{-\pi}}{\pi} - 4e^{-\pi} \int_0^1 \frac{1}{1+e^{-2\pi}t^2} dt$$

$$\left( \frac{1-e^{-(2\pi)/2}}{1^2} - \frac{1-e^{-(6\pi)/2}}{3^2} + \frac{1-e^{-(10\pi)/2}}{5^2} \right) 4 - \tan^{-1}(e^{-(2\pi)/2}) 2 \times 2 = \frac{836}{225\pi} - \frac{4e^{-5\pi}}{25\pi} +$$

$$\frac{4e^{-3\pi}}{9\pi} - \frac{4e^{-\pi}}{\pi} + \frac{ie^{-\pi}}{\pi^{3/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} (1+e^{-2\pi})^{-s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)^2 ds \text{ for } 0 < \gamma < \frac{1}{2}$$

$$\left( \frac{1-e^{-(2\pi)/2}}{1^2} - \frac{1-e^{-(6\pi)/2}}{3^2} + \frac{1-e^{-(10\pi)/2}}{5^2} \right) 4 - \tan^{-1}(e^{-(2\pi)/2}) 2 \times 2 = \frac{836}{225\pi} - \frac{4e^{-5\pi}}{25\pi} +$$

$$\frac{4e^{-3\pi}}{9\pi} - \frac{4e^{-\pi}}{\pi} + \frac{ie^{-\pi}}{\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{2\pi s} \Gamma\left(\frac{1}{2}-s\right) \Gamma(1-s) \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} ds \text{ for } 0 < \gamma < \frac{1}{2}$$

**Continued fraction representations:**

$$\frac{\left(\frac{1-e^{-(2\pi)/2}}{1^2} - \frac{1-e^{-(6\pi)/2}}{3^2}\right) + \frac{1-e^{-(10\pi)/2}}{5^2}}{\pi} 4 - \tan^{-1}\left(e^{-(2\pi)/2}\right) 2 \times 2 =$$

$$\frac{4\left(1 - e^{-\pi} + \frac{1}{25}(1 - e^{-5\pi}) + \frac{1}{9}(-1 + e^{-3\pi})\right)}{\pi} - \frac{4e^{-\pi}}{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{e^{-2\pi} k^2}{1+2k}} =$$

$$\frac{4\left(1 - e^{-\pi} + \frac{1}{25}(1 - e^{-5\pi}) + \frac{1}{9}(-1 + e^{-3\pi})\right)}{\pi} - \frac{4e^{-\pi}}{1 + \frac{e^{-2\pi}}{3 + \frac{4e^{-2\pi}}{5 + \frac{9e^{-2\pi}}{7 + \frac{16e^{-2\pi}}{9 + \dots}}}}}$$

$$\frac{\left(\frac{1-e^{-(2\pi)/2}}{1^2} - \frac{1-e^{-(6\pi)/2}}{3^2}\right) + \frac{1-e^{-(10\pi)/2}}{5^2}}{\pi} 4 - \tan^{-1}\left(e^{-(2\pi)/2}\right) 2 \times 2 =$$

$$\frac{4\left(1 - e^{-\pi} + \frac{1}{25}(1 - e^{-5\pi}) + \frac{1}{9}(-1 + e^{-3\pi})\right)}{\pi} - 4 \left( e^{-\pi} - \frac{e^{-3\pi}}{3 + \mathop{\text{K}}_{k=1}^{\infty} \frac{e^{-2\pi} (1+(-1)^{1+k} + k)^2}{3+2k}} \right) =$$

$$\frac{4\left(1 - e^{-\pi} + \frac{1}{25}(1 - e^{-5\pi}) + \frac{1}{9}(-1 + e^{-3\pi})\right)}{\pi} - 4 \left( e^{-\pi} - \frac{e^{-3\pi}}{3 + \frac{9e^{-2\pi}}{5 + \frac{4e^{-2\pi}}{7 + \frac{25e^{-2\pi}}{9 + \frac{16e^{-2\pi}}{11 + \dots}}}}} \right)$$

$$\begin{aligned} & \left( \left( \frac{1-e^{-(2\pi)/2}}{1^2} - \frac{1-e^{-(6\pi)/2}}{3^2} \right) + \frac{1-e^{-(10\pi)/2}}{5^2} \right) 4 - \tan^{-1}(e^{-(2\pi)/2}) 2 \times 2 = \\ & \frac{4 \left( 1 - e^{-\pi} + \frac{1}{25} (1 - e^{-5\pi}) + \frac{1}{9} (-1 + e^{-3\pi}) \right)}{\pi} - \frac{4 e^{-\pi}}{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{e^{-2\pi} (-1+2k)^2}{1+2k-e^{-2\pi} (-1+2k)}} = \\ & \frac{4 \left( 1 - e^{-\pi} + \frac{1}{25} (1 - e^{-5\pi}) + \frac{1}{9} (-1 + e^{-3\pi}) \right)}{\pi} - \frac{4 e^{-\pi}}{1 + \frac{e^{-2\pi}}{3-e^{-2\pi} + \frac{9 e^{-2\pi}}{5-3 e^{-2\pi} + \frac{25 e^{-2\pi}}{7-5 e^{-2\pi} + \frac{49 e^{-2\pi}}{9+\dots-7 e^{-2\pi}}}}} \end{aligned}$$

$$\begin{aligned} & \left( \left( \frac{1-e^{-(2\pi)/2}}{1^2} - \frac{1-e^{-(6\pi)/2}}{3^2} \right) + \frac{1-e^{-(10\pi)/2}}{5^2} \right) 4 - \tan^{-1}(e^{-(2\pi)/2}) 2 \times 2 = \\ & \frac{4 \left( 1 - e^{-\pi} + \frac{1}{25} (1 - e^{-5\pi}) + \frac{1}{9} (-1 + e^{-3\pi}) \right)}{\pi} - \frac{4 e^{-\pi}}{1 + e^{-2\pi} + \mathop{\text{K}}_{k=1}^{\infty} \frac{2 e^{-2\pi} \left( 1 - 2 \left| \frac{1+k}{2} \right| \right) \left| \frac{1+k}{2} \right|}{\left( 1 + \frac{1}{2} (1+(-1)^k) e^{-2\pi} \right) (1+2k)}} = \\ & \frac{4 \left( 1 - e^{-\pi} + \frac{1}{25} (1 - e^{-5\pi}) + \frac{1}{9} (-1 + e^{-3\pi}) \right)}{\pi} - \frac{4 e^{-\pi}}{1 + e^{-2\pi} + \frac{2 e^{-2\pi}}{3 - \frac{2 e^{-2\pi}}{5 (1 + e^{-2\pi})} - \frac{12 e^{-2\pi}}{7 - \frac{12 e^{-2\pi}}{9 (1 + e^{-2\pi})} + \dots}} \end{aligned}$$

$\mathop{\text{K}}_{k=k_1}^{k_2} a_k / b_k$  is a continued fraction

We obtain also:

$$\left( (5 \log(29/25) - 3 \log(13/9) + \log(5)) \right) x = \left( \left( 4 (1 - e^{-\pi}) + \frac{1}{25} (1 - e^{-5\pi}) + \frac{1}{9} (-1 + e^{-3\pi}) \right) / \pi - 4 \tan^{-1}(e^{-\pi}) \right)$$

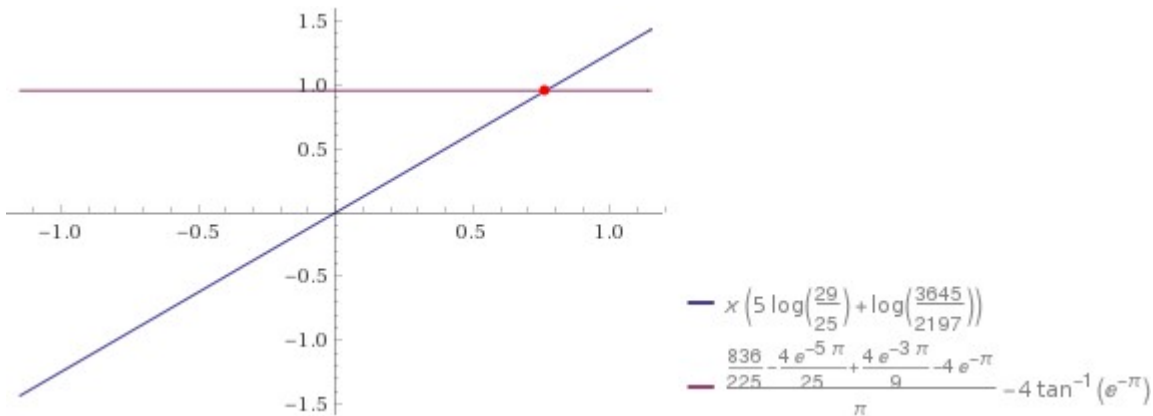
**Input:**

$$\left(5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5)\right)x = \frac{4\left(1 - e^{-\pi} + \frac{1}{25}(1 - e^{-5\pi}) + \frac{1}{9}(-1 + e^{-3\pi})\right)}{\pi} - 4 \tan^{-1}(e^{-\pi})$$

$\log(x)$  is the natural logarithm

$\tan^{-1}(x)$  is the inverse tangent function

**Plot:**



**Alternate forms:**

$$x \left(5 \log\left(\frac{29}{25}\right) + \log\left(\frac{3645}{2197}\right)\right) = \frac{836 - 4 e^{-5\pi} (9 - 25 e^{2\pi} + 225 e^{4\pi})}{225 \pi} - 4 \cot^{-1}(e^{\pi})$$

$$x \left(5 \log\left(\frac{29}{25}\right) + \log\left(\frac{3645}{2197}\right)\right) = \frac{\frac{836}{225} - \frac{4 e^{-5\pi}}{25} + \frac{4 e^{-3\pi}}{9} - 4 e^{-\pi}}{\pi} - 4 \tan^{-1}(e^{-\pi})$$

$$x \left(5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5)\right) = \frac{4\left(1 - e^{-\pi} + \frac{1}{25}(1 - e^{-5\pi}) + \frac{1}{9}(e^{-3\pi} - 1)\right)}{\pi} - 4 \cot^{-1}(e^{\pi})$$

$\cot^{-1}(x)$  is the inverse cotangent function

**Expanded form:**

$$x \log(5) - 3 x \log\left(\frac{13}{9}\right) + 5 x \log\left(\frac{29}{25}\right) = \frac{836}{225 \pi} - \frac{4 e^{-5\pi}}{25 \pi} + \frac{4 e^{-3\pi}}{9 \pi} - \frac{4 e^{-\pi}}{\pi} - 4 \tan^{-1}(e^{-\pi})$$

**Alternate forms assuming  $x > 0$ :**

$$x(-9 \log(5) - 3 \log(13) + 5 \log(29) + \log(729)) = \frac{\frac{836}{225} - \frac{4e^{-5\pi}}{25} + \frac{4e^{-3\pi}}{9} - 4e^{-\pi}}{\pi} - 4 \tan^{-1}(e^{-\pi})$$

$$5x(\log(29) - 2 \log(5)) - 3x(\log(13) - 2 \log(3)) + x \log(5) = \frac{4\left(1 - e^{-\pi} + \frac{1}{25}(1 - e^{-5\pi}) + \frac{1}{9}(e^{-3\pi} - 1)\right)}{\pi} - 4 \tan^{-1}(e^{-\pi})$$

**Solution:**

$$x \approx 0.76495$$

$$0.76495$$

Thence:

$$(((5 \log(29/25) - 3 \log(13/9) + \log(5)))) \times 0.76495$$

**Input:**

$$\left(5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5)\right) \times 0.76495$$

$\log(x)$  is the natural logarithm

**Result:**

$$0.954935734021996273375973446841016125802377734202803590974...$$

0.954935734... result very near to the spectral index  $n_s$ , to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 (see Appendix) and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}} \approx 0.9568666373$$

**Alternative representations:**

$$\left(5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5)\right) 0.76495 =$$

$$0.76495 \left(\log(a) \log_a(5) - 3 \log(a) \log_a\left(\frac{13}{9}\right) + 5 \log(a) \log_a\left(\frac{29}{25}\right)\right)$$

$$\left(5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5)\right) 0.76495 = 0.76495 \left(\log_e(5) - 3 \log_e\left(\frac{13}{9}\right) + 5 \log_e\left(\frac{29}{25}\right)\right)$$

$$\left(5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5)\right) 0.76495 =$$

$$0.76495 \left(-\text{Li}_1(-4) + 3 \text{Li}_1\left(1 - \frac{13}{9}\right) - 5 \text{Li}_1\left(1 - \frac{29}{25}\right)\right)$$

**Series representations:**

$$\left(5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5)\right) 0.76495 = 7.6495 i \pi \left[ \frac{\arg\left(\frac{29}{25} - x\right)}{2 \pi} \right] -$$

$$4.5897 i \pi \left[ \frac{\arg\left(\frac{13}{9} - x\right)}{2 \pi} \right] + 1.5299 i \pi \left[ \frac{\arg(5 - x)}{2 \pi} \right] + 2.29485 \log(x) +$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left(-3.82475 \left(\frac{29}{25} - x\right)^k + 2.29485 \left(\frac{13}{9} - x\right)^k - 0.76495 (5 - x)^k\right) x^{-k}}{k} \text{ for } x <$$

$$0$$

$$\left(5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5)\right) 0.76495 =$$

$$3.82475 \left[ \frac{\arg\left(\frac{29}{25} - z_0\right)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) - 2.29485 \left[ \frac{\arg\left(\frac{13}{9} - z_0\right)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) +$$

$$0.76495 \left[ \frac{\arg(5 - z_0)}{2 \pi} \right] \log\left(\frac{1}{z_0}\right) + 2.29485 \log(z_0) + 3.82475 \left[ \frac{\arg\left(\frac{29}{25} - z_0\right)}{2 \pi} \right] \log(z_0) -$$

$$2.29485 \left[ \frac{\arg\left(\frac{13}{9} - z_0\right)}{2 \pi} \right] \log(z_0) + 0.76495 \left[ \frac{\arg(5 - z_0)}{2 \pi} \right] \log(z_0) +$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left(-3.82475 \left(\frac{29}{25} - z_0\right)^k + 2.29485 \left(\frac{13}{9} - z_0\right)^k - 0.76495 (5 - z_0)^k\right) z_0^{-k}}{k}$$

$$\begin{aligned} & \left(5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5)\right) 0.76495 = \\ & 7.6495 i \pi \left[ \frac{\pi - \arg\left(\frac{29}{25 z_0}\right) - \arg(z_0)}{2 \pi} \right] - 4.5897 i \pi \left[ \frac{\pi - \arg\left(\frac{13}{9 z_0}\right) - \arg(z_0)}{2 \pi} \right] + \\ & 1.5299 i \pi \left[ \frac{\pi - \arg\left(\frac{5}{z_0}\right) - \arg(z_0)}{2 \pi} \right] + 2.29485 \log(z_0) + \\ & \sum_{k=1}^{\infty} \frac{(-1)^k \left( -3.82475 \left(\frac{29}{25} - z_0\right)^k + 2.29485 \left(\frac{13}{9} - z_0\right)^k - 0.76495 (5 - z_0)^k \right) z_0^{-k}}{k} \end{aligned}$$

**Integral representations:**

$$\left(5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5)\right) 0.76495 = \int_1^{\frac{29}{25}} \frac{2.34993 - 4.40611 t + 2.29485 t^2}{0.6144 t - 1.6 t^2 + t^3} dt$$

$$\begin{aligned} & \left(5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5)\right) 0.76495 = \\ & \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{4^{-s} (0.382475 - 1.14743 \times 9^s + 1.91238 \times 25^s) \Gamma(-s)^2 \Gamma(1+s)}{i \pi \Gamma(1-s)} ds \text{ for } \\ & -1 < \gamma < 0 \end{aligned}$$

$$\left(\left(\left(\left(\left(5 \log(29/25) - 3 \log(13/9) + \log(5)\right)\right)\right)\right)\right) * 0.76495)^{1/64}$$

**Input:**

$$\sqrt[64]{\left(5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5)\right) \times 0.76495}$$

log(x) is the natural logarithm

**Result:**

0.99927977...

0.99927977... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 =  $\phi$**

2\*log base 0.99927977 ((((((5 log(29/25) - 3 log(13/9) + log(5)))))\*0.76495)))-  
Pi+1/golden ratio

**Input interpretation:**

$$2 \log_{0.99927977} \left( \left( 5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5) \right) \times 0.76495 \right) - \pi + \frac{1}{\phi}$$

$\log(x)$  is the natural logarithm

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

125.476...

125.476... result very near to the dilaton mass calculated as a type of Higgs boson:  
125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$2 \log_{0.99928} \left( \left( 5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5) \right) 0.76495 \right) - \pi + \frac{1}{\phi} =$$

$$-\pi + \frac{1}{\phi} + \frac{2 \log(0.76495 (\log(5) - 3 \log(\frac{13}{9}) + 5 \log(\frac{29}{25})))}{\log(0.99928)}$$



$$2 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) - \pi + \frac{1}{\phi} =$$

$$-\pi + 2 \log_{0.99928} \left( 0.76495 \left( \log(a) \log_a(5) - 3 \log(a) \log_a \left( \frac{13}{9} \right) + 5 \log(a) \log_a \left( \frac{29}{25} \right) \right) \right) + \frac{1}{\phi}$$

$$2 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) - \pi + \frac{1}{\phi} =$$

$$-\pi + 2 \log_{0.99928} \left( 0.76495 \left( \log_e(5) - 3 \log_e \left( \frac{13}{9} \right) + 5 \log_e \left( \frac{29}{25} \right) \right) \right) + \frac{1}{\phi}$$

### Series representations:

$$2 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - \frac{2 \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + 3.82475 \log \left( \frac{29}{25} \right) - 2.29485 \log \left( \frac{13}{9} \right) + 0.76495 \log(5))^k}{k}}{\log(0.99928)}$$

$$2 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - 2775.89 \log \left( 0.76495 \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) \right) -$$

$$2 \log \left( 0.76495 \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) \right) \sum_{k=0}^{\infty} (-0.00072023)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$$2 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - 2775.89 \log \left( 0.76495 \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) \right) -$$

$$2 \log \left( 0.76495 \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) \right) \sum_{k=0}^{\infty} (-0.00072023)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

### Integral representations:

$$2 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + 2 \log_{0.99928} \left( 0.76495 \int_1^{\frac{29}{25}} 5 \left( \frac{1}{t} + 5 \left( \frac{3}{16-25t} + \frac{1}{-24+25t} \right) \right) dt \right)$$

$$2 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) - \pi + \frac{1}{\phi} = \frac{1}{\phi} - \pi + 2 \log_{0.99928} \left( \frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{4^{-s} (0.382475 - 1.14743 \times 9^s + 1.91238 \times 25^s) \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)$$

for  $-1 < \gamma < 0$

2\*log base 0.99927977 ((((((5 log(29/25) - 3 log(13/9) + log(5))))\*0.76495)))+11+1/golden ratio

**Input interpretation:**

$$2 \log_{0.99927977} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) \times 0.76495 \right) + 11 + \frac{1}{\phi}$$

$\log(x)$  is the natural logarithm

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

139.618...

139.618... result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$2 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} + \frac{2 \log \left( 0.76495 \left( \log(5) - 3 \log \left( \frac{13}{9} \right) + 5 \log \left( \frac{29}{25} \right) \right) \right)}{\log(0.99928)}$$

$$2 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) + 11 + \frac{1}{\phi} = 11 + 2 \log_{0.99928} \left( 0.76495 \left( \log(a) \log_a(5) - 3 \log(a) \log_a \left( \frac{13}{9} \right) + 5 \log(a) \log_a \left( \frac{29}{25} \right) \right) \right) + \frac{1}{\phi}$$

$$2 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) + 11 + \frac{1}{\phi} = 11 + 2 \log_{0.99928} \left( 0.76495 \left( \log_e(5) - 3 \log_e \left( \frac{13}{9} \right) + 5 \log_e \left( \frac{29}{25} \right) \right) \right) + \frac{1}{\phi}$$

### Series representations:

$$2 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - \frac{2 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + 3.82475 \log \left( \frac{29}{25} \right) - 2.29485 \log \left( \frac{13}{9} \right) + 0.76495 \log(5) \right)^k}{k}}{\log(0.99928)}$$

$$2 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 2775.89 \log \left( 0.76495 \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) \right) -$$

$$2 \log \left( 0.76495 \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) \right) \sum_{k=0}^{\infty} (-0.00072023)^k G(k)$$

$$\text{for } \left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

$$2 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 2775.89 \log \left( 0.76495 \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) \right) -$$

$$2 \log \left( 0.76495 \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) \right) \sum_{k=0}^{\infty} (-0.00072023)^k G(k)$$

$$\text{for } \left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

### Integral representations:

$$2 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + 2 \log_{0.99928} \left( 0.76495 \int_1^{29} 5 \left( \frac{1}{t} + 5 \left( \frac{3}{16-25t} + \frac{1}{-24+25t} \right) \right) dt \right)$$

$$2 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} + 2 \log_{0.99928} \left( \right.$$

$$\left. \frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{4^{-s} (0.382475 - 1.14743 \times 9^s + 1.91238 \times 25^s) \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)$$

$$\text{for } -1 < \gamma < 0$$

$27 \cdot \log_{0.99927977} \left( \left( \left( \left( \left( 5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5) \right) \right) \right) \right) \right) \cdot 0.76495 \right) + \frac{1}{\phi}$

**Input interpretation:**

$$27 \log_{0.99927977} \left( \left( 5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5) \right) \times 0.76495 \right) + \frac{1}{\phi}$$

$\log(x)$  is the natural logarithm

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

1728.61...

1728.61...

This result is very near to the mass of candidate glueball  $f_0(1710)$  meson. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

**Alternative representations:**

$$27 \log_{0.99928} \left( \left( 5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5) \right) 0.76495 \right) + \frac{1}{\phi} = \frac{1}{\phi} + \frac{27 \log\left(0.76495 \left( \log(5) - 3 \log\left(\frac{13}{9}\right) + 5 \log\left(\frac{29}{25}\right) \right)\right)}{\log(0.99928)}$$

$$27 \log_{0.99928} \left( \left( 5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5) \right) 0.76495 \right) + \frac{1}{\phi} = 27 \log_{0.99928} \left( 0.76495 \left( \log(a) \log_a(5) - 3 \log(a) \log_a\left(\frac{13}{9}\right) + 5 \log(a) \log_a\left(\frac{29}{25}\right) \right) \right) + \frac{1}{\phi}$$

$$27 \log_{0.99928} \left( \left( 5 \log\left(\frac{29}{25}\right) - 3 \log\left(\frac{13}{9}\right) + \log(5) \right) 0.76495 \right) + \frac{1}{\phi} = 27 \log_{0.99928} \left( 0.76495 \left( \log_e(5) - 3 \log_e\left(\frac{13}{9}\right) + 5 \log_e\left(\frac{29}{25}\right) \right) \right) + \frac{1}{\phi}$$

### Series representations:

$$27 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \frac{27 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + 3.82475 \log \left( \frac{29}{25} \right) - 2.29485 \log \left( \frac{13}{9} \right) + 0.76495 \log(5) \right)^k}{k}}{\log(0.99928)}$$

$$27 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - 37474.5 \log \left( 0.76495 \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) \right) -$$

$$27 \log \left( 0.76495 \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) \right) \sum_{k=0}^{\infty} (-0.00072023)^k G(k)$$

$$\text{for } \left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

$$27 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - 37474.5 \log \left( 0.76495 \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) \right) -$$

$$27 \log \left( 0.76495 \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) \right) \sum_{k=0}^{\infty} (-0.00072023)^k G(k)$$

$$\text{for } \left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

### Integral representations:

$$27 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) + \frac{1}{\phi} =$$

$$\frac{1}{\phi} + 27 \log_{0.99928} \left( 0.76495 \int_1^{29} 5 \left( \frac{1}{t} + 5 \left( \frac{3}{16-25t} + \frac{1}{-24+25t} \right) \right) dt \right)$$

$$27 \log_{0.99928} \left( \left( 5 \log \left( \frac{29}{25} \right) - 3 \log \left( \frac{13}{9} \right) + \log(5) \right) 0.76495 \right) + \frac{1}{\phi} = \frac{1}{\phi} + 27 \log_{0.99928} \left( \right.$$

$$\left. \frac{1}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{4^{-s} (0.382475 - 1.14743 \times 9^s + 1.91238 \times 25^s) \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right)$$

$$\text{for } -1 < \gamma < 0$$

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## **References**

**MANUSCRIPT BOOK 1 OF SRINIVASA RAMANUJAN**