Self-Organized Critical Markets - Implied Volatility and Avalanche Intensity

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Abstract

Assuming self-organized criticality to characterize capital markets, this paper seeks to explain why equity implied volatility is a relevant proxy for avalanche intensity. Historical data analysis of the CBOE Volatility Index (VIX) shows that implied volatility spikes are distributed following a power law, making financial stress similar to earthquakes as anticipated long ago by Bak.

Keywords: Implied Volatility, Power Law, Self-Organized Criticality

JEL Classification Codes: G100, G110, G120, G130

1. Introduction

How do capital markets work? And why do they crash? When Per Bak introduced *self-organized criticality* (SOC) - a common characteristic of various natural phenomena such as earthquakes, floods, solar flares, epidemics and biological evolution - he stated that SOC also applies to the financial system [1][2]. Since then, interesting work has been accomplished to explain the formation of speculative bubbles, searching for indicators of an imminent crash like log-periodicity [3]. However, empirical studies concluded that such warning models might fail on the long run [4].

Assuming SOC to characterize capital markets, the focus of this paper is set on a more accurate definition of an *avalanche* in the specific case of investing activity, studying the evolution of asset price volatility rather than price variations or returns. Indeed, higher volatility would be equivalent to lower visibility on the price other participants are willing to pay. Since options enable investors to protect their net worth when uncertainty arises, implied volatility could be treated as a relevant proxy for markets avalanche intensity.

Historical analysis of daily spikes of the Chicago Board Options Exchange Volatility Index (VIX) shows that implied volatility spikes are distributed following a power law. Therefore, capital markets exhibit the same transition pattern as seismic activity [5], as anticipated by Bak.

2. Assumptions and Methodology

Human interactions are a necessary condition for asset valuation. Without interaction between investors, the market value of any asset would be null. Considering the case of Robinson Crusoe economy enables us to illustrate this simple idea: if Robinson Crusoe owns an asset (e.g. a gold bullion) but Man Friday is not interested in buying it, even at a very low price, then the asset's value drops to zero. Even though it might provide some form of personal utility to Robinson, there is no utility recognition at the level of the community. In other words, asset prices exist if economic agents are willing to trade it. One consequence of that is the well-documented popularity factor on investment markets: 'investors demand more of an asset, the more desirable the asset's characteristics' [6].

This leads to the following hypothesis: there might be a relationship between price volatility and the degree of market interactions. A high degree of interactions between agents would imply a higher visibility on price, and therefore a lower price fluctuation. Conversely, low visibility would be associated with high price fluctuation.

Going further, capital markets are assumed to display SOC, meaning that they naturally self-organize toward a critical point and do not reach an equilibrium. An important characteristic of SOC is the fact that such systems are subject to brutal organizational changes, also known as avalanches, that can be seen as a form of energy dissipation [1][2]. In the present case, the previous hypothesis would imply that volatility is a relevant proxy for avalanche intensity. From this perspective, an avalanche is defined by a higher price fluctuation due to a sudden change in the interactions between participants, analysis of past crises showing that such change can be triggered by various types of events like the failure of an issuer, the release of major economic data, a psychological shift reversing a consensual trade, etc. [3], [7].

When the market enters such a transition phase, investors may try to protect their net worth, either by selling liquid assets or by buying derivatives like forward contracts or options [8]. For that purpose, equity options have become a popular instrument, allowing to hedge different types of securities during periods of financial distress. Indeed, when the share price of an issuer drops toward zero, the market value of its related securities such as bonds becomes highly sensitive to the share price variation (i.e. the equity delta tends toward 1). In the specific case of convex instruments such as convertible bonds, this phenomenon is known as the 'double-signed gamma' and can be exacerbated by reset features [9].

An interesting concept is the so-called implied volatility of the underlying shares that derives from option prices [10]. Although dependent on pricing assumptions, implied volatility provides useful information on what investors are anticipating in terms of asset price fluctuation.

In this paper, we focus on the VIX, which is a measure of the implied volatility of the S&P 500 index [11]. An increase of the VIX value indicates that investors are anticipating higher stocks volatility and are willing to protect all or part of their net worth. Conversely, a decrease of the VIX means that agents are less worried about the future and that the aggregate demand of S&P 500 options is shrinking.

Given the influence of U.S. financial markets worldwide, the VIX can be seen as a relevant proxy for avalanche intensity on a global scale.

3. Results

Daily spikes of implied volatility are defined as:

$$S_t = \max(\sigma_t - \sigma_{t-1}, 0) \tag{1}$$

where σ_t is the closing level of the VIX at the end of day t. Data are sourced from the Chicago Board Options Exchange, ranging from January 2^{nd} 1991 to October 1^{st} 2019.

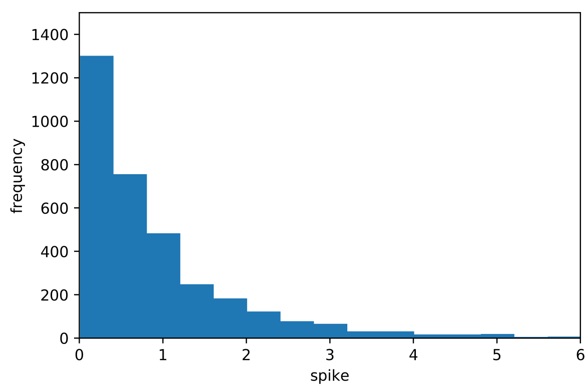

Figure 1: S&P 500 Implied Volatility Daily Spikes Distribution

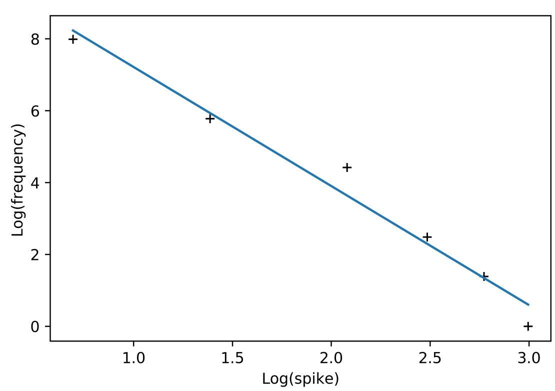

Figure 2: Logarithm Representation of the Daily Spikes Distribution

The results show that historical VIX spikes are distributed following a power law (Figure 1), implying a linear relationship between the logarithms of both spikes and frequencies (Figure 2). Thus, with N the number of events implying an implied volatility spike S, the model reads:

$$N = AS^{-b} \tag{2}$$

or

$$\log N = a - b \log S \tag{3}$$

where a and b are constants, and $A = e^a$. In the case of the VIX, we find that $a \approx 10.53$ and $b \approx 3.31$.

Equations (2) and (3) characterize a power law and are similar to the relationship between the number of earthquakes and their seismic magnitude, also known as the Gutenberg–Richter law [5]. Such a similarity should not come as a surprise and was anticipated by Bak with the concept of SOC [2].

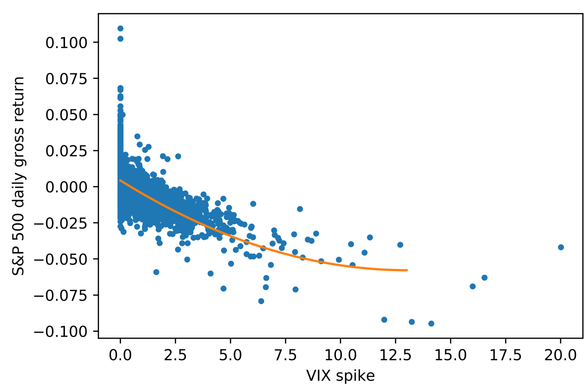

Figure 3: S&P 500 daily returns vs Volatility Spikes

Juxtaposing VIX spikes with S&P 500 daily returns (excluding dividends) enables us to visualize a negative non-linear relationship between both variables (Figure 3), higher spikes leading to higher index drops. The strength of this effect tends to diminish as spikes amplitude increases. However, the theoretical relationship becomes weak for high volatility days. Indeed, a 5% S&P 500 decrease can occur simultaneously with a 20 points VIX spike, but also with a mere 2 points spike. One possible explanation may be the fact that in some cases financial stress is not directly related to the equity market (e.g. a credit event involving unlisted issuers). As already mentioned before, equity options can be used to hedge several types of assets.

4. Conclusion

This short paper provides a piece of evidence that implied volatility can be regarded as a proxy measure of avalanche intensity, assuming capital markets to be self-organized and critical. However, our work might look simplistic since the focus is in only on VIX daily changes, while historical data show that volatility spikes can last more than one single day. Although studying spikes on a monthly basis would also lead to a power law distribution, a more accurate approach should rely on a more complex definition of implied volatility spikes taking into account the problem of non-linear temporal duration of such events. Nevertheless, the primary goal of this paper is to emphasize the importance of instantaneous volatility for capital markets understanding. Assuming that the system can be modeled as a network of interactions connecting investors to each other, financial stress reflects a sudden rise in uncertainty about those interactions, thus leading to larger asset price fluctuation in a similar way to other natural phenomena such as earthquakes.

5. Acknowledgement

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