On proofs of the Poincaré Conjecture

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Abstract

On December 22, 2006, the journal Science honored Perelman's proof of the Poincaré conjecture as the scientific "Breakthrough of the Year", the first time this honor was bestowed in the area of mathematics. However, I have critical questions about Perelman's proof of Poincare Conjecture. The conjecture states, that "Every simply connected, closed 3-manifold is homeomorphic to the 3-sphere." The "homeomorphic" means that by non-singular deformation one produces perfect sphere - the equivalent of initial space. However, pasting in foreign caps will not make such deformation. My short proofs are given.

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Quote from 2019 Wikipedia "Poincaré conjecture"

Perelman proved [4] the conjecture by deforming the manifold M using the Ricci (NB! PROBLEM D) flow (which behaves similarly to the heat equation that describes the diffusion of heat through an object). The Ricci flow usually deforms the manifold towards a rounder shape, except for some cases where it stretches the manifold apart from itself towards what is known as singularities. Perelman and Hamilton then chop the manifold at the singularities (a process called "surgery") causing the separate pieces to form into ball-like shapes.

Hamilton created a list of possible singularities that could form, but he was concerned that some singularities might lead to difficulties. He wanted to cut the manifold at the singularities and paste in caps (NB! PROBLEM A), and then run the Ricci flow again, so he needed to understand the singularities and show that certain kinds of singularities do not occur.

In essence, Perelman showed that all the strands that form can be cut and capped (NB! PROBLEM A) and none stick out on one side only.

Completing the proof, Perelman takes any compact, simply connected, three-dimensional manifold without boundary and starts to run the Ricci flow. This deforms the manifold into round pieces with strands (NB! PROBLEM B) running between them. He cuts the strands and continues deforming the manifold until eventually he is left with a collection of round three-dimensional spheres. Then he rebuilds the original manifold by connecting the spheres together with three-dimensional cylinders (NB! PROBLEM C), morphs them into a round shape and sees that, despite all the initial confusion, the manifold was, in fact, homeomorphic to a sphere S.

Problems I have noticed

PROBLEM A: the caps, which do not belong to the original manifold M, are going into the final sphere S. These caps K are not present in initial manifold M; and their size is not exactly zero, neither it becomes zero. Thus, it holds the M+K = Final Sphere. Thus, M is not homeomorphically equivalent to the final sphere. More exactly: $M_1 - Singularities = M_2$, $Singularities \equiv spheres$ (the "ball-like shapes"), $M_2 + caps \rightarrow spheres$, $spheres \equiv Final\,Sphere$, thus the initial manifold is not homeomorphically equivalent to Final Sphere:

$M_1 \neq Final\,Sphere.$

PROBLEM B: the strands, which belong to the original manifold M, are not going into the final sphere S.

PROBLEM C: the cylinders, which do not belong to the original manifold M, are going into the final sphere S.

PROBLEM D: it is not sufficient to demand non-singularity of Ricci Scalar, we need to keep non-singularity of Riemann Curvature Tensor because the latter completely describes the curvature of space; whereby any singularity of space destroys the homeomorphic transition from the original manifold to the final sphere, even if the Ricci-flow equation is not violated.

My own proof

Lets us allow non-smooth deformation of the initial manifold. Namely, we can look at the most curved place of the initial manifold and draw a bordering line around it. Then, we can reduce the curvature inside this line even to zero, while producing non-smooth edge at this line. This way, the maximum curvature of the manifold becomes reduced. Then, looking at the maximum of the produced manifold, we can draw a bordering line around this maximum of curvature and apply the procedure again. The aim of this is to freely change the shape of the manifold by transforming small areas of the manifold into curvature-less areas. Recall, that the final sphere S is like a polyhedron with the infinite number of edges.

My own second proof

Let us prove: "Homeomorphism from simply connected manifold makes only the simply connected one."

Let we have to starting facts: manifold A is simple (i.e. simply connected) and the transformation from A is the homeomorphism. Suppose we are wrong and the manifold B is not simple. Lets take a curve (loop) in B, the one which can not be contracted. Because of homeomorphism it is the loop in A. The loop in A can be contracted; because of homeomorphism, it can be contracted in B. I came to logical contradiction. Thus, the B is simple. Proof ends.

Let us prove: "two simple connected manifolds can always have the homeomorphism."

The A is simply connected manifold. The B is simply connected manifold. Suppose they are not homeomorphic. Then the neighboring areas in A are not neighboring in B. Therefore the curve, which can be contracted in A, can not be contracted in B as the loop. Latter means, that B is not simple. I came to logical contradiction, thus, the A and B are homeomorphic. Proof ends.

[1] Colin McLarty. What does it take to prove Fermat's last theorem? Grothendieck and the logic of number theory. Bulletin of Symbolic Logic 16(3): 359–377 (2010).

[2] Andrew Wiles, "Modular elliptic curves and Fermat's last theorem", Annals of Mathematics. 141 (3): 443–551 (1995).

[3] J. J. Macys, "On Euler's hypothetical proof", Mathematical Notes 82 (3–4): 352–356 (2007).

[4] Dana Mackenzie, "The Poincaré Conjecture–Proved". Science 314 (5807): 1848–1849 (2006)