

Letter N°1: Integrals

Edgar Valdebenito

August 18 , 2019

ABSTRACT. We give three integrals.

Keywords: definite integrals , number Pi.

I. The number Pi .

Recall that:

$$\pi = 4 \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots \right) = 3.1415926535 \dots \quad (1)$$

this note presents three integrals involving π .

II. Integrals .

$$\int_0^{3-2\sqrt{2}} \left(\sqrt[4]{1-x+\sqrt{1-6x+x^2}} - \sqrt[4]{1-x-\sqrt{1-6x+x^2}} \right) dx = \sqrt[4]{2} \left(\frac{9}{5} - \frac{\pi}{2\sqrt{2}} - \frac{1}{\sqrt{2}} \ln(1+\sqrt{2}) \right) \quad (2)$$

$$\int_0^{\sqrt{2}-1} \left(\sqrt[4]{1-x^2+\sqrt{1-6x^2+x^4}} - \sqrt[4]{1-x^2-\sqrt{1-6x^2+x^4}} \right) x dx = \frac{1}{\sqrt[4]{8}} \left(\frac{9}{5} - \frac{\pi}{2\sqrt{2}} - \frac{1}{\sqrt{2}} \ln(1+\sqrt{2}) \right) \quad (3)$$

$$\int_{\sqrt{2}-1}^{1/2} \left(\sqrt[4]{x+\sqrt{x^2+2x-1}} - \sqrt[4]{x-\sqrt{x^2+2x-1}} \right) dx = \frac{1}{2} \left(\frac{9}{5} - \frac{\pi}{2\sqrt{2}} - \frac{1}{\sqrt{2}} \ln(1+\sqrt{2}) \right) \quad (4)$$

III. References .

[1] Gradshteyn, I.S., and Ryzhik, I.M.: Table of Integrals, Series and Products, 7th ed., edited by Alan Jeffrey and Daniel Zwillinger, Academic Press, 2007.