

**On several Ramanujan's equations: further mathematical connections with various parameters of Particle Physics, principally the Higgs boson mass,  $\pi$  meson mass 139.57 and Cosmology. X**

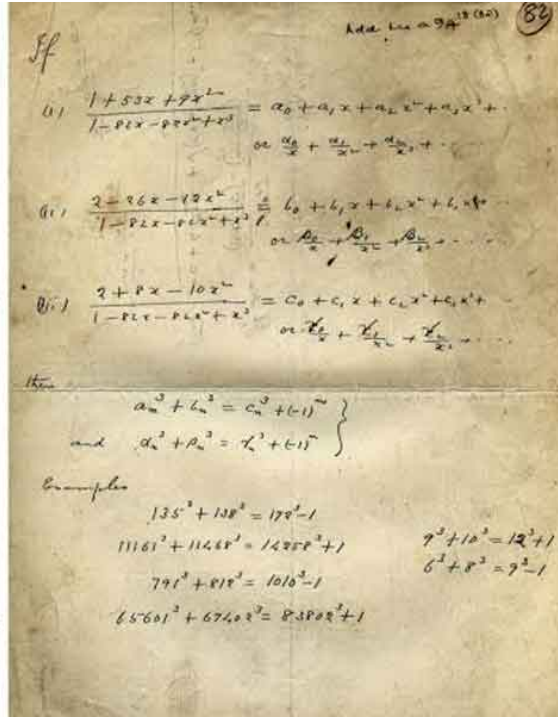
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**Abstract**

*In this research thesis, we have analyzed further Ramanujan formulas and described further possible mathematical connections with some parameters of Particle Physics and Cosmology*

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# Ramanujan

<https://myindiafacts.online/30-ramanujan-random-facts-mathematical-genius/>

## Summary

In this research thesis, we have analyzed the possible and new connections between different formulas of Ramanujan's mathematics and some formulas concerning particle physics and cosmology. In the course of the discussion we describe and highlight the connections between some developments of

**Ramanujan equations and particles type solutions such as the mass of the Higgs boson, and the masses of other baryons and mesons. Moreover solutions of Ramanujan equations, connected with the masses of the  $\pi$  mesons (139.57 and 134.9766 MeV) have been described and highlighted. Furthermore, we have obtained also the values of some black hole entropies and the value of the Cosmological Constant.**

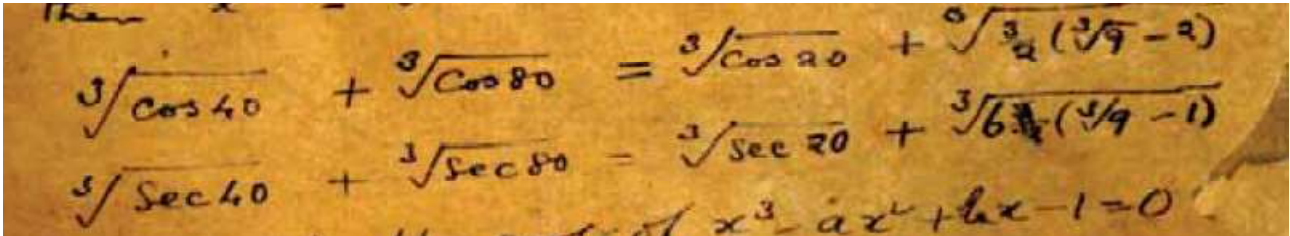
**Is our opinion, that the possible connections between the mathematical developments of some Rogers-Ramanujan continued fractions, the value of the dilaton and that of "the dilaton mass calculated as a type of Higgs boson that is equal about to 125 GeV", the Higgs boson mass itself and the like-particle solutions (masses), are fundamental.**

**All the results of the most important connections are highlighted in blue throughout the drafting of the paper**

**From:**

**MANUSCRIPT BOOK 2 OF SRINIVASA RAMANUJAN**

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$(\sec 40)^{1/3} + (\sec 80)^{1/3}$

**Input:**

$$\sqrt[3]{\sec(40)} + \sqrt[3]{\sec(80)}$$

sec(x) is the secant function

**Decimal approximation:**

1.61458980270085388116938541736391097294862123000919822080... +  
2.79655157166048822158599144501463112640791047258859575026... i

**Polar coordinates:**

$r \approx 3.22918$  (radius),  $\theta \approx 60^\circ$  (angle)

3.22918

**Alternate forms:**

$$\frac{e^{(i\pi)/3}}{\sqrt[3]{-\cos(40)}} + \frac{e^{(i\pi)/3}}{\sqrt[3]{-\cos(80)}}$$

$$\frac{e^{(i\pi)/3}}{\sqrt[3]{\frac{1}{2}(-e^{-40i} - e^{40i})}} + \frac{e^{(i\pi)/3}}{\sqrt[3]{\frac{1}{2}(-e^{-80i} - e^{80i})}}$$

$$\frac{1}{2^{2/3} \sqrt[3]{-(1 + \cos(80)) \sec(40)}} + \frac{1}{2^{2/3} \sqrt[3]{-(1 + \cos(160)) \sec(80)}} +$$

$$i \left( \frac{\sqrt{3}}{2^{2/3} \sqrt[3]{-(1 + \cos(80)) \sec(40)}} + \frac{\sqrt{3}}{2^{2/3} \sqrt[3]{-(1 + \cos(160)) \sec(80)}} \right)$$

**Alternative representations:**

$$\sqrt[3]{\sec(40)} + \sqrt[3]{\sec(80)} = \sqrt[3]{\frac{1}{\cos(40)}} + \sqrt[3]{\frac{1}{\cos(80)}}$$

$$\sqrt[3]{\sec(40)} + \sqrt[3]{\sec(80)} = \sqrt[3]{\frac{1}{\cosh(-40i)}} + \sqrt[3]{\frac{1}{\cosh(-80i)}}$$

$$\sqrt[3]{\sec(40)} + \sqrt[3]{\sec(80)} = \sqrt[3]{\frac{1}{\cosh(40i)}} + \sqrt[3]{\frac{1}{\cosh(80i)}}$$

### Series representations:

$$\sqrt[3]{\sec(40)} + \sqrt[3]{\sec(80)} = \sqrt[3]{\sum_{k=-\infty}^{\infty} (-1)^k e^{40i(1+2k)} (-1 + 2\theta(k))} + \sqrt[3]{\sum_{k=-\infty}^{\infty} (-1)^k e^{80i(1+2k)} (-1 + 2\theta(k))}$$

$$\sqrt[3]{\sec(40)} + \sqrt[3]{\sec(80)} = \sqrt[3]{-\sum_{k=-\infty}^{\infty} (-1)^k e^{-40i(1+2k)} (-1 + 2\theta(k))} + \sqrt[3]{-\sum_{k=-\infty}^{\infty} (-1)^k e^{-80i(1+2k)} (-1 + 2\theta(k))}$$

$$\sqrt[3]{\sec(40)} + \sqrt[3]{\sec(80)} = 2^{2/3} \sqrt[3]{\pi} \left( \sqrt[3]{\sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{-25600 + (\pi + 2k\pi)^2}} + \sqrt[3]{\sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{-6400 + (\pi + 2k\pi)^2}} \right)$$

Or, in degree:

### Input:

$$\sqrt[3]{\sec(40^\circ)} + \sqrt[3]{\sec(80^\circ)}$$

### Exact result:

$$\sqrt[3]{\sec(40^\circ)} + \sqrt[3]{\csc\left(\frac{\pi}{18}\right)}$$

$\csc(x)$  is the cosecant function

### Decimal approximation:

2.885338333237742468085406734012766756045984955872395804746...

2.8853383332377.....

**Alternate forms:**

$$\sqrt[3]{\sec(40^\circ)} + \frac{1}{\sqrt[3]{\sin\left(\frac{\pi}{18}\right)}}$$

$$\sqrt[3]{\csc\left(\frac{\pi}{18}\right)} + \sqrt[3]{\sec\left(\frac{2\pi}{9}\right)}$$

$$\frac{\sqrt[3]{\sin\left(\frac{\pi}{18}\right)\sec(40^\circ)} + 1}{\sqrt[3]{\sin\left(\frac{\pi}{18}\right)}}$$

$$\left(\left(\left(\left(\left(\left(\sec 40\right)^{1/3} + \left(\sec 80\right)^{1/3}\right)\right)\right)\right)\right)^{1/128}$$

**Input:**

$$128\sqrt{\frac{1}{\sqrt[3]{\sec(40^\circ)} + \sqrt[3]{\sec(80^\circ)}}}$$

**Exact result:**

$$\frac{1}{128\sqrt{\sqrt[3]{\sec(40^\circ)} + \sqrt[3]{\csc\left(\frac{\pi}{18}\right)}}}$$

csc(x) is the cosecant function

**Decimal approximation:**

0.991755717617335083291658209017538357942615676386351764406...

0.9917557176173..... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} - \phi + 1 = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 =  $\phi$**

**Alternate forms:**

$$\frac{1}{\sqrt[128]{\sqrt[3]{\csc\left(\frac{\pi}{18}\right)} + \sqrt[3]{\sec\left(\frac{2\pi}{9}\right)}}}$$

$$\frac{384\sqrt{\sin\left(\frac{\pi}{18}\right)}}{\sqrt[128]{\sqrt[3]{\sin\left(\frac{\pi}{18}\right)\sec(40^\circ)} + 1}}$$

$$\frac{1}{\sqrt[128]{\sqrt[3]{\sec(40^\circ)} + \text{root of } x^9 - 6x^6 + 8 \text{ near } x = 1.79243}}}$$

$\sec(x)$  is the secant function

log base 0.991755717617335 (((1/((((sec 40)^1/3 + (sec 80)^1/3)))))))-Pi+1/golden ratio

**Input interpretation:**

$$\log_{0.991755717617335} \left( \frac{1}{\sqrt[3]{\sec(40^\circ)} + \sqrt[3]{\sec(80^\circ)}} \right) - \pi + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

125.4764413352...

125.4764413352.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18

log base 0.991755717617335 (((1/((((sec 40)^1/3 + (sec 80)^1/3)))))))+11+1/golden ratio





$$\frac{1}{\sqrt[64]{\sqrt[3]{\cos(40^\circ)} + \sqrt[3]{\sin\left(\frac{\pi}{18}\right)}}}$$

**Decimal approximation:**

0.993967811865354701118461509936814642851120507784445473081...

0.993967811865..... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{1 + \sqrt[5]{\sqrt{\phi^5 \sqrt[4]{5^3}} - 1}} - \phi + 1 = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 =  $\phi$**

**Alternate forms:**

$$\frac{1}{\sqrt[64]{\sqrt[3]{\sin\left(\frac{\pi}{18}\right)} + \sqrt[3]{\cos\left(\frac{2\pi}{9}\right)}}}$$

$$\frac{1}{\sqrt[64]{\sqrt[3]{\cos(40^\circ)} + \text{root of } 8x^9 - 6x^3 + 1 \text{ near } x = 0.5579}}}$$

$$\frac{1}{\sqrt[64]{\sqrt[3]{\cos(40^\circ)} + \sqrt[3]{\frac{1}{2}i(e^{-i\pi/18} - e^{i\pi/18})}}}}$$

2log base 0.99396781186535 (((1/(((cos 40)^1/3 + (cos 80)^1/3)))))-Pi+1/golden ratio

**Input interpretation:**

$$2 \log_{0.99396781186535} \left( \frac{1}{\sqrt[3]{\cos(40^\circ)} + \sqrt[3]{\cos(80^\circ)}} \right) - \pi + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

125.476441335...

125.476441335.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18

$2 \log_{0.99396781186535} \left( \frac{1}{\sqrt[3]{\cos(40^\circ)} + \sqrt[3]{\cos(80^\circ)}} \right) + 11 + \frac{1}{\phi}$  ratio

**Input interpretation:**

$$2 \log_{0.99396781186535} \left( \frac{1}{\sqrt[3]{\cos(40^\circ)} + \sqrt[3]{\cos(80^\circ)}} \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

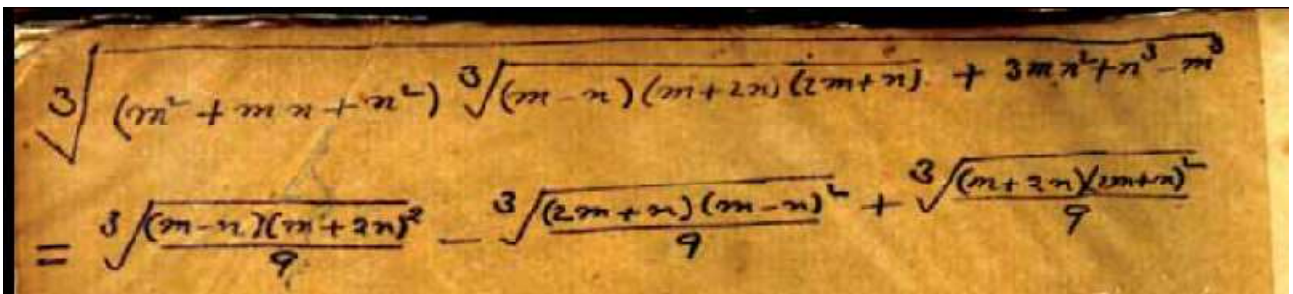
$\phi$  is the golden ratio

**Result:**

139.618033989...

139.61803398.... result practically equal to the rest mass of Pion meson 139.57

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For  $m = 5, n = 3$ , we obtain:

$$\left( \frac{1}{9} * (5-3)(5+3*2)^2 \right)^{1/3} - \left( \frac{1}{9} * (2*5+3)(5-3)^2 \right)^{1/3} + \left( \frac{1}{9} * (5+2*3)(2*5+3)^2 \right)^{1/3}$$

**Input:**

$$\sqrt[3]{\frac{1}{9}(5-3)(5+3 \times 2)^2} - \sqrt[3]{\frac{1}{9}(2 \times 5+3)(5-3)^2} + \sqrt[3]{\frac{1}{9}(5+2 \times 3)(2 \times 5+3)^2}$$

**Result:**

$$\frac{\sqrt[3]{2} 11^{2/3}}{3^{2/3}} - \left(\frac{2}{3}\right)^{2/3} \sqrt[3]{13} + \frac{\sqrt[3]{11} 13^{2/3}}{3^{2/3}}$$

**Decimal approximation:**

7.112719917559886252728237482895165037802526786770790308858...

7.1127199175....

**Alternate forms:**

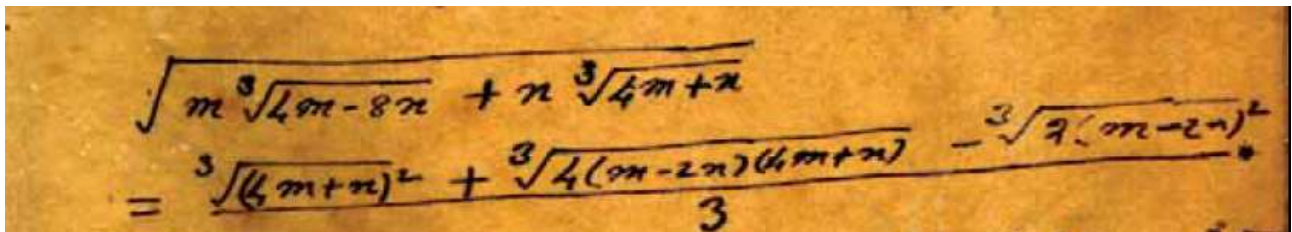
$$\frac{1}{3} \left( \sqrt[3]{6} 11^{2/3} - 2^{2/3} \sqrt[3]{39} + 13^{2/3} \sqrt[3]{33} \right)$$

$$\sqrt[3]{37+49 \sqrt[3]{286}}$$

$$-\frac{-\sqrt[3]{2} 11^{2/3} + 2^{2/3} \sqrt[3]{13} - \sqrt[3]{11} 13^{2/3}}{3^{2/3}}$$

**Minimal polynomial:**

$$x^9 - 111x^6 + 4107x^3 - 33698267$$



For  $m = 5$  and  $n = 3$ , we obtain

$$1/3 * (((4*5+3)^2)^{1/3} + ((4*(5-2*3)(4*5+3)))^{1/3} - ((2*(5-2*3)^2)^{1/3}))$$

**Input:**

$$\frac{1}{3} \left( \sqrt[3]{(4 \times 5 + 3)^2} + \sqrt[3]{4(5 - 2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5 - 2 \times 3)^2} \right)$$

**Result:**

$$\frac{1}{3} \left( -\sqrt[3]{2} + \sqrt[3]{-23} 2^{2/3} + 23^{2/3} \right)$$

**Decimal approximation:**

3.02827902231073061720977558980680967230413925188010251009... +  
 1.30318274029455165944914790942056110508613534995044753900... i

**Polar coordinates:**

$r \approx 3.29678$  (radius),  $\theta \approx 23.284^\circ$  (angle)

3.29678

**Alternate forms:**

$$\frac{1}{3} \left( 23^{2/3} + \sqrt[3]{-92} - \sqrt[3]{2} \right)$$

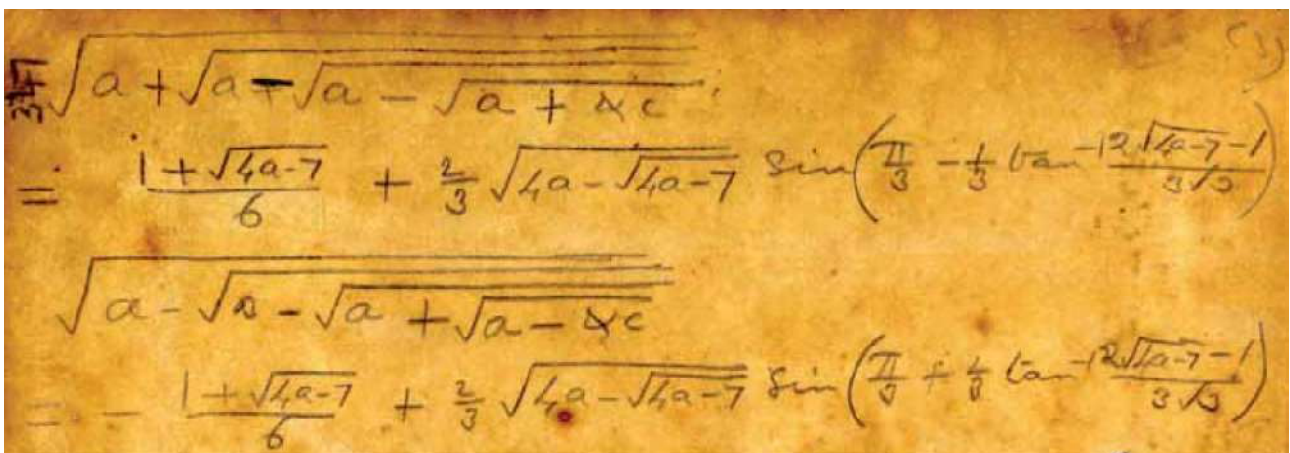
$$\frac{1}{3} \left( \boxed{\text{root of } x^3 + 92 \text{ near } x = 2.25718 + 3.90955 i} - \sqrt[3]{2} + 23^{2/3} \right)$$

$$-\frac{\sqrt[3]{2}}{3} + \frac{1}{3} \sqrt[3]{-23} 2^{2/3} + \frac{23^{2/3}}{3}$$

**Minimal polynomial:**

$19683 x^{18} - 1299078 x^{15} + 131799555 x^{12} - 3549466008 x^9 +$   
 $120930363369 x^6 - 495579761622 x^3 + 981218819953$

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For  $a = 4$

$$-1/6(1+\sqrt{4*4-7}) + 2/3*\sqrt{((4*4-\sqrt{4*4-7}))} * (((\sin((((\text{Pi}/3+1/3 (((\tan^1(2*\sqrt{16-7}-1))/(((3\sqrt{3}))))))))))$$

**Input:**

$$-\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{2}{3} \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} + \frac{1}{3} \times \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3 \sqrt{3}} \right)$$

$\tan^{-1}(x)$  is the inverse tangent function

**Exact Result:**

$$\frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} - \frac{\tan^{-1}(5)}{9 \sqrt{3}} \right) - \frac{2}{3}$$

(result in radians)

**Decimal approximation:**

1.512675874166590932948230949015017085101369986295658176498...

(result in radians)

1.51267587416659....

**Alternate forms:**

$$\frac{2}{3} \left( \sqrt{13} \cos \left( \frac{\pi}{6} - \frac{\tan^{-1}(5)}{9 \sqrt{3}} \right) - 1 \right)$$

$$\frac{2}{3} \left( \sqrt{13} \sin \left( \frac{1}{27} \left( 9\pi + \sqrt{3} \tan^{-1}(5) \right) \right) - 1 \right)$$

$$-\frac{2}{3} + \frac{1}{3} \sqrt{13} \sin \left( \frac{\tan^{-1}(5)}{9 \sqrt{3}} \right) + \sqrt{\frac{13}{3}} \cos \left( \frac{\tan^{-1}(5)}{9 \sqrt{3}} \right)$$

**Addition formula:**

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) (-1) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} + \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3(3 \sqrt{3})} \right) \right) \Bigg|_2 =$$

$$\frac{1}{3} \left( -2 + \sqrt{39} \cos \left( \frac{\tan^{-1}(5)}{9 \sqrt{3}} \right) + \sqrt{13} \sin \left( \frac{\tan^{-1}(5)}{9 \sqrt{3}} \right) \right)$$

### Alternative representations:

$$\frac{1}{6} \left(1 + \sqrt{4 \times 4 - 7}\right) (-1) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} + \frac{\tan^{-1}(2\sqrt{16-7}-1)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$-\frac{1}{6} (1 + \sqrt{9}) + \frac{2}{3} \cos \left( \frac{\pi}{6} - \frac{\tan^{-1}(-1+2\sqrt{9})}{3(3\sqrt{3})} \right) \sqrt{16 - \sqrt{9}}$$

$$\frac{1}{6} \left(1 + \sqrt{4 \times 4 - 7}\right) (-1) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} + \frac{\tan^{-1}(2\sqrt{16-7}-1)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$-\frac{1}{6} (1 + \sqrt{9}) - \frac{2}{3} \cos \left( \frac{5\pi}{6} + \frac{\tan^{-1}(-1+2\sqrt{9})}{3(3\sqrt{3})} \right) \sqrt{16 - \sqrt{9}}$$

$$\frac{1}{6} \left(1 + \sqrt{4 \times 4 - 7}\right) (-1) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} + \frac{\tan^{-1}(2\sqrt{16-7}-1)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$-\frac{1}{6} (1 + \sqrt{9}) +$$

$$\frac{2 \left( -\exp \left( -i \left( \frac{\pi}{3} + \frac{\tan^{-1}(-1+2\sqrt{9})}{3(3\sqrt{3})} \right) \right) + \exp \left( i \left( \frac{\pi}{3} + \frac{\tan^{-1}(-1+2\sqrt{9})}{3(3\sqrt{3})} \right) \right) \right) \sqrt{16 - \sqrt{9}}}{3(2i)}$$

### Series representations:

$$\frac{1}{6} \left(1 + \sqrt{4 \times 4 - 7}\right) (-1) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} + \frac{\tan^{-1}(2\sqrt{16-7}-1)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$-\frac{2}{3} + \frac{2}{3} \sqrt{13} \sum_{k=0}^{\infty} \frac{(-1)^k \left( \frac{1}{54} (9\pi - 2\sqrt{3} \tan^{-1}(5)) \right)^{2k}}{(2k)!}$$

$$\frac{1}{6} \left(1 + \sqrt{4 \times 4 - 7}\right) (-1) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} + \frac{\tan^{-1}(2\sqrt{16-7}-1)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$-\frac{2}{3} + \frac{2}{3} \sqrt{13} \sum_{k=0}^{\infty} \frac{(-1)^k \left( \frac{\pi}{6} - \frac{\tan^{-1}(5)}{9\sqrt{3}} \right)^{2k}}{(2k)!}$$

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) (-1) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} + \frac{\tan^{-1}(2\sqrt{16-7-1})}{3(3\sqrt{3})} \right) \right) 2 =$$

$$-\frac{2}{3} - \frac{2}{3} \sqrt{13} \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{\pi}{3} - \frac{\tan^{-1}(5)}{9\sqrt{3}} \right)^{1+2k}}{(1+2k)!}$$

### Integral representations:

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) (-1) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} + \frac{\tan^{-1}(2\sqrt{16-7-1})}{3(3\sqrt{3})} \right) \right) 2 =$$

$$-\frac{2}{3} - \frac{2\sqrt{13}}{3} \int_{\frac{\pi}{2}}^{\frac{1}{54}(9\pi - 2\sqrt{3}\tan^{-1}(5))} \sin(t) dt$$

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) (-1) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} + \frac{\tan^{-1}(2\sqrt{16-7-1})}{3(3\sqrt{3})} \right) \right) 2 =$$

$$-\frac{2}{3} + \frac{2\sqrt{13}}{3} + \int_0^1 -\frac{1}{81} \sqrt{13} (9\pi - 2\sqrt{3}\tan^{-1}(5)) \sin\left(\frac{1}{54}t(9\pi - 2\sqrt{3}\tan^{-1}(5))\right) dt$$

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) (-1) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} + \frac{\tan^{-1}(2\sqrt{16-7-1})}{3(3\sqrt{3})} \right) \right) 2 =$$

$$-\frac{2}{3} - \frac{1}{3} i \sqrt{\frac{13}{\pi}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{s-(9\pi-2\sqrt{3}\tan^{-1}(5))^2/(11664s)}}{\sqrt{s}} ds \text{ for } \gamma > 0$$

**Continued fraction representations:**

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) (-1) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} + \frac{\tan^{-1}(2\sqrt{16-7-1})}{3(3\sqrt{3})} \right) \right) \Bigg|_2 =$$

$$-\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} - \frac{5}{9\sqrt{3} \left( 1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{25k^2}{1+2k} \right)} \right) =$$

$$-\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} - \frac{5}{9\sqrt{3} \left( 1 + \frac{25}{3 + \frac{100}{5 + \frac{225}{7 + \frac{400}{9 + \dots}}}} \right)} \right)$$

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) (-1) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} + \frac{\tan^{-1}(2\sqrt{16-7-1})}{3(3\sqrt{3})} \right) \right) \Bigg|_2 =$$

$$-\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} - \frac{5 - \frac{125}{3 + \mathop{\text{K}}_{k=1}^{\infty} \frac{25(1+(-1)^{1+k} + k)^2}{3+2k}}}{9\sqrt{3}} \right) =$$

$$-\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} - \frac{5 - \frac{125}{3 + \frac{225}{5 + \frac{100}{7 + \frac{625}{9 + \frac{400}{11 + \dots}}}}} \right)$$



$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) (-1) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} + \frac{\tan^{-1}(2\sqrt{16-7-1})}{3(3\sqrt{3})} \right) \right) \Bigg|_2 =$$

$$-\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} - \frac{5}{9\sqrt{3} \left( 1 + \sum_{k=1}^{\infty} \frac{25(-1+2k)^2}{1+2k-25(-1+2k)} \right)} \right) =$$

$$-\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} - \frac{5}{9\sqrt{3} \left( 1 + \frac{25}{-22 + \frac{225}{-70 + \frac{625}{-118 + \frac{1225}{-166 + \dots}}} \right)} \right)$$

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) (-1) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} + \frac{\tan^{-1}(2\sqrt{16-7-1})}{3(3\sqrt{3})} \right) \right) \Bigg|_2 =$$

$$-\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} - \frac{5}{9\sqrt{3} \left( 26 + \sum_{k=1}^{\infty} \frac{50(1-2\lfloor \frac{1+k}{2} \rfloor) \lfloor \frac{1+k}{2} \rfloor}{(1+\frac{25}{2}(1+(-1)^k))(1+2k)} \right)} \right) =$$

$$-\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} - \frac{5}{9\sqrt{3} \left( 26 + \frac{50}{3 - \frac{50}{130 - \frac{300}{7 - \frac{300}{234 + \dots}}}} \right)} \right)$$

And:

$$\frac{1}{6} * (1 + \sqrt{4 * 4 - 7}) + \frac{2}{3} * \sqrt{4 * 4 - \sqrt{4 * 4 - 7}} * \left( \sin \left( \frac{\pi}{3} - \frac{1}{3} \times \frac{\tan^{-1}(2 \sqrt{16 - 7} - 1)}{3 \sqrt{3}} \right) \right)$$

**Input:**

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{2}{3} \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{1}{3} \times \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3 \sqrt{3}} \right)$$

$\tan^{-1}(x)$  is the inverse tangent function

**Exact Result:**

$$\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} + \frac{\tan^{-1}(5)}{9 \sqrt{3}} \right)$$

(result in radians)

**Decimal approximation:**

2.634508137340150021728933625300885718686913818693674457977...

(result in radians)

2.63450813734....

**Alternate forms:**

$$\frac{2}{3} \left( 1 + \sqrt{13} \cos \left( \frac{\pi}{6} + \frac{\tan^{-1}(5)}{9 \sqrt{3}} \right) \right)$$

$$\frac{2}{3} \left( 1 + \sqrt{13} \cos \left( \frac{1}{54} \left( 9 \pi + 2 \sqrt{3} \tan^{-1}(5) \right) \right) \right)$$

$$\frac{2}{3} - \frac{1}{3} \sqrt{13} \sin \left( \frac{\tan^{-1}(5)}{9 \sqrt{3}} \right) + \sqrt{\frac{13}{3}} \cos \left( \frac{\tan^{-1}(5)}{9 \sqrt{3}} \right)$$

**Addition formula:**

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{1}{3} \left[ \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3 (3 \sqrt{3})} \right) \right] 2 =$$

$$\frac{1}{3} \left( 2 + \sqrt{39} \cos \left( \frac{\tan^{-1}(5)}{9 \sqrt{3}} \right) - \sqrt{13} \sin \left( \frac{\tan^{-1}(5)}{9 \sqrt{3}} \right) \right)$$

**Alternative representations:**

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1}(2\sqrt{16-7}-1)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$\frac{1}{6} \left( 1 + \sqrt{9} \right) + \frac{2}{3} \cos \left( \frac{\pi}{6} + \frac{\tan^{-1}(-1+2\sqrt{9})}{3(3\sqrt{3})} \right) \sqrt{16 - \sqrt{9}}$$

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1}(2\sqrt{16-7}-1)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$\frac{1}{6} \left( 1 + \sqrt{9} \right) - \frac{2}{3} \cos \left( \frac{5\pi}{6} - \frac{\tan^{-1}(-1+2\sqrt{9})}{3(3\sqrt{3})} \right) \sqrt{16 - \sqrt{9}}$$

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1}(2\sqrt{16-7}-1)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$\frac{1}{6} \left( 1 + \sqrt{9} \right) +$$

$$\frac{2 \left( -\exp \left( -i \left( \frac{\pi}{3} - \frac{\tan^{-1}(-1+2\sqrt{9})}{3(3\sqrt{3})} \right) \right) + \exp \left( i \left( \frac{\pi}{3} - \frac{\tan^{-1}(-1+2\sqrt{9})}{3(3\sqrt{3})} \right) \right) \right) \sqrt{16 - \sqrt{9}}}{3(2i)}$$

**Series representations:**

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1}(2\sqrt{16-7}-1)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$\frac{2}{3} + \frac{2}{3} \sqrt{13} \sum_{k=0}^{\infty} \frac{(-1)^k \left( \frac{1}{54} (9\pi + 2\sqrt{3} \tan^{-1}(5)) \right)^{2k}}{(2k)!}$$

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1}(2\sqrt{16-7}-1)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$\frac{2}{3} + \frac{2}{3} \sqrt{13} \sum_{k=0}^{\infty} \frac{(-1)^k \left( \frac{\pi}{6} + \frac{\tan^{-1}(5)}{9\sqrt{3}} \right)^{2k}}{(2k)!}$$

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$\frac{2}{3} - \frac{2}{3} \sqrt{13} \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{\pi}{3} + \frac{\tan^{-1}(5)}{9\sqrt{3}} \right)^{1+2k}}{(1+2k)!}$$

### Integral representations:

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$\frac{2}{3} - \frac{2\sqrt{13}}{3} \int_{\frac{\pi}{2}}^{\frac{1}{54} (9\pi + 2\sqrt{3} \tan^{-1}(5))} \sin(t) dt$$

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$\frac{2}{3} + \frac{2\sqrt{13}}{3} + \int_0^1 -\frac{1}{81} \sqrt{13} \left( 9\pi + 2\sqrt{3} \tan^{-1}(5) \right) \sin \left( \frac{1}{54} t \left( 9\pi + 2\sqrt{3} \tan^{-1}(5) \right) \right) dt$$

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$\frac{2}{3} - \frac{1}{3} i \sqrt{\frac{13}{\pi}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-\left( \frac{\pi}{6} + \frac{\tan^{-1}(5)}{9\sqrt{3}} \right)^2 / (4s)}}{\sqrt{s}} ds \text{ for } \gamma > 0$$

**Continued fraction representations:**

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} + \frac{5}{9\sqrt{3} \left( 1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{25k^2}{1+2k} \right)} \right) =$$

$$\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} + \frac{5}{9\sqrt{3} \left( 1 + \frac{25}{3 + \frac{100}{5 + \frac{225}{7 + \frac{400}{9 + \dots}}}} \right)} \right)$$

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} + \frac{5 - \frac{125}{3 + \mathop{\text{K}}_{k=1}^{\infty} \frac{25(1+(-1)^{1+k+k})^2}{3+2k}}}{9\sqrt{3}} \right) =$$

$$\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} + \frac{5 - \frac{125}{3 + \frac{225}{5 + \frac{100}{7 + \frac{625}{9 + \frac{400}{11 + \dots}}}}} \right)$$

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} + \frac{5}{9\sqrt{3} \left( 1 + \sum_{k=1}^{\infty} \frac{25(-1+2k)^2}{1+2k-25(-1+2k)} \right)} \right) =$$

$$\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} + \frac{5}{9\sqrt{3} \left( 1 + \frac{25}{-22 + \frac{225}{-70 + \frac{625}{-118 + \frac{1225}{-166 + \dots}}} \right)} \right)$$

$$\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3(3\sqrt{3})} \right) \right) 2 =$$

$$\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} + \frac{5}{9\sqrt{3} \left( 26 + \sum_{k=1}^{\infty} \frac{50 \left( 1 - 2 \left\lfloor \frac{1+k}{2} \right\rfloor \right) \left\lfloor \frac{1+k}{2} \right\rfloor}{\left( 1 + \frac{25}{2} (1 + (-1)^k) \right) (1+2k)} \right)} \right) =$$

$$\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} + \frac{5}{9\sqrt{3} \left( 26 + \frac{50}{3 - \frac{50}{130 - \frac{300}{7 - \frac{300}{234 + \dots}}}} \right)} \right)$$

From which, we obtain:

$$\left( \left( \frac{1}{6} * (1 + \sqrt{4*4-7}) \right) + \frac{2}{3} * \sqrt{4*4 - \sqrt{4*4-7}} \right) * \left( \sin \left( \frac{\pi}{3} - \frac{1}{3} * \frac{\tan^{-1}(2\sqrt{16-7}-1)}{3\sqrt{3}} \right) \right)^{1/2}$$

**Input:**

$$\sqrt{\frac{1}{6} \left( 1 + \sqrt{4 \times 4 - 7} \right) + \frac{2}{3} \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{1}{3} \times \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3 \sqrt{3}} \right)}$$

$\tan^{-1}(x)$  is the inverse tangent function

**Exact Result:**

$$\sqrt{\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} + \frac{\tan^{-1}(5)}{9\sqrt{3}} \right)}$$

(result in radians)

**Decimal approximation:**

1.623116797196107517914644397188825082902834043887347364917...

(result in radians)

1.6231167971...

**Alternate forms:**

$$\sqrt{\frac{2}{3} \left( 1 + \sqrt{13} \cos \left( \frac{1}{54} \left( 9\pi + 2\sqrt{3} \tan^{-1}(5) \right) \right) \right)}$$

$$\sqrt{\left( \frac{2}{3} + \frac{1}{3} \sqrt{13} \left( \exp \left( \frac{i\pi}{6} - \frac{\log(1-5i) - \log(1+5i)}{18\sqrt{3}} \right) + \exp \left( \frac{\log(1-5i) - \log(1+5i)}{18\sqrt{3}} - \frac{i\pi}{6} \right) \right) \right)}$$

$$\sqrt{\left( \frac{1}{3} \left( 2 + \left( (-1)^{2/3} \sqrt{13} - \sqrt[3]{-1} \sqrt{13} \right) \sin \left( \frac{\tan^{-1}(5)}{9\sqrt{3}} \right) + \left( \sqrt[6]{-1} \sqrt{13} - (-1)^{5/6} \sqrt{13} \right) \cos \left( \frac{\tan^{-1}(5)}{9\sqrt{3}} \right) \right) \right)}$$

$\log(x)$  is the natural logarithm

**All 2nd roots of  $\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos\left(\frac{\pi}{6} + \frac{\tan^{-1}(5)}{9\sqrt{3}}\right)$ :**

$$e^0 \sqrt{\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos\left(\frac{\pi}{6} + \frac{\tan^{-1}(5)}{9\sqrt{3}}\right)} \approx 1.6231 \text{ (real, principal root)}$$

$$e^{i\pi} \sqrt{\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos\left(\frac{\pi}{6} + \frac{\tan^{-1}(5)}{9\sqrt{3}}\right)} \approx -1.6231 \text{ (real root)}$$

**Addition formulas:**

$$\sqrt{\frac{1}{6} \left(1 + \sqrt{4 \times 4 - 7}\right) + \frac{1}{3} \left[\sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin\left(\frac{\pi}{3} - \frac{\tan^{-1}(2\sqrt{16-7}-1)}{3(3\sqrt{3})}\right)\right]}^2 =$$

$$\sqrt{\frac{2}{3} + \frac{2}{3} \sqrt{13} \left(\frac{1}{2} \sqrt{3} \cos\left(\frac{\tan^{-1}(5)}{9\sqrt{3}}\right) - \frac{1}{2} \sin\left(\frac{\tan^{-1}(5)}{9\sqrt{3}}\right)\right)}$$

$$\sqrt{\frac{1}{6} \left(1 + \sqrt{4 \times 4 - 7}\right) + \frac{1}{3} \left[\sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin\left(\frac{\pi}{3} - \frac{\tan^{-1}(2\sqrt{16-7}-1)}{3(3\sqrt{3})}\right)\right]}^2 =$$

$$\frac{1}{\sqrt{\frac{3}{2 + \sqrt{39} \cos\left(\frac{\tan^{-1}(5)}{9\sqrt{3}}\right) - \sqrt{13} \sin\left(\frac{\tan^{-1}(5)}{9\sqrt{3}}\right)}}$$

**Alternative representations:**

$$\sqrt{\frac{1}{6} \left(1 + \sqrt{4 \times 4 - 7}\right) + \frac{1}{3} \left[\sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin\left(\frac{\pi}{3} - \frac{\tan^{-1}(2\sqrt{16-7}-1)}{3(3\sqrt{3})}\right)\right]}^2 =$$

$$\sqrt{\frac{1}{6} \left(1 + \sqrt{9}\right) + \frac{2}{3} \cos\left(\frac{\pi}{6} + \frac{\tan^{-1}(-1 + 2\sqrt{9})}{3(3\sqrt{3})}\right)} \sqrt{16 - \sqrt{9}}$$

$$\sqrt{\frac{1}{6} \left(1 + \sqrt{4 \times 4 - 7}\right) + \frac{1}{3} \left[\sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin\left(\frac{\pi}{3} - \frac{\tan^{-1}(2\sqrt{16-7}-1)}{3(3\sqrt{3})}\right)\right]}^2 =$$

$$\sqrt{\frac{1}{6} \left(1 + \sqrt{9}\right) - \frac{2}{3} \cos\left(\frac{5\pi}{6} - \frac{\tan^{-1}(-1 + 2\sqrt{9})}{3(3\sqrt{3})}\right)} \sqrt{16 - \sqrt{9}}$$



$$\sqrt{\frac{1}{6}(1+\sqrt{4 \times 4-7})+\frac{1}{3}\left(\sqrt{4 \times 4-\sqrt{4 \times 4-7}} \sin\left(\frac{\pi}{3}-\frac{\tan^{-1}(2 \sqrt{16-7}-1)}{3(3 \sqrt{3})}\right)\right)}_2 =$$

$$\sqrt{\left(\frac{1}{6}(1+\sqrt{9})+\frac{2\left(-\exp\left(-i\left(\frac{\pi}{3}-\frac{\tan^{-1}(-1+2 \sqrt{9})}{3(3 \sqrt{3})}\right)\right)+\exp\left(i\left(\frac{\pi}{3}-\frac{\tan^{-1}(-1+2 \sqrt{9})}{3(3 \sqrt{3})}\right)\right)}{3(2 i)}\right) \sqrt{16-\sqrt{9}}\right)}$$

**Series representations:**

$$\sqrt{\frac{1}{6}(1+\sqrt{4 \times 4-7})+\frac{1}{3}\left(\sqrt{4 \times 4-\sqrt{4 \times 4-7}} \sin\left(\frac{\pi}{3}-\frac{\tan^{-1}(2 \sqrt{16-7}-1)}{3(3 \sqrt{3})}\right)\right)}_2 =$$

$$\sqrt{\frac{2}{3}+\frac{2}{3} \sqrt{13} \sum_{k=0}^{\infty} \frac{(-1)^k\left(\frac{\pi}{6}+\frac{\tan^{-1}(5)}{9 \sqrt{3}}\right)^{2 k}}{(2 k)!}}$$

$$\sqrt{\frac{1}{6}(1+\sqrt{4 \times 4-7})+\frac{1}{3}\left(\sqrt{4 \times 4-\sqrt{4 \times 4-7}} \sin\left(\frac{\pi}{3}-\frac{\tan^{-1}(2 \sqrt{16-7}-1)}{3(3 \sqrt{3})}\right)\right)}_2 =$$

$$\sqrt{\frac{2}{3}-\frac{2}{3} \sqrt{13} \sum_{k=0}^{\infty} \frac{(-1)^k\left(-\frac{\pi}{3}+\frac{\tan^{-1}(5)}{9 \sqrt{3}}\right)^{1+2 k}}{(1+2 k)!}}$$

$$\sqrt{\frac{1}{6}(1+\sqrt{4 \times 4-7})+\frac{1}{3}\left(\sqrt{4 \times 4-\sqrt{4 \times 4-7}} \sin\left(\frac{\pi}{3}-\frac{\tan^{-1}(2 \sqrt{16-7}-1)}{3(3 \sqrt{3})}\right)\right)}_2 =$$

$$\sqrt{\frac{2}{3}+\frac{2}{3} \sqrt{13} \pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{4^s\left(\frac{\pi}{6}+\frac{\tan^{-1}(5)}{9 \sqrt{3}}\right)^{-2 s} \Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}}$$

**Integral representations:**

$$\sqrt{\frac{1}{6} \left(1 + \sqrt{4 \times 4 - 7}\right) + \frac{1}{3} \left[ \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3(3\sqrt{3})} \right) \right]} =$$

$$\sqrt{\frac{2}{3} - \frac{2\sqrt{13}}{3} \int_{\frac{\pi}{2}}^{\frac{\pi}{6} + \frac{\tan^{-1}(5)}{9\sqrt{3}}} \sin(t) dt}$$

$$\sqrt{\frac{1}{6} \left(1 + \sqrt{4 \times 4 - 7}\right) + \frac{1}{3} \left[ \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3(3\sqrt{3})} \right) \right]} =$$

$$\sqrt{\frac{2}{3} + \frac{2}{3} \sqrt{13} \left( 1 - \frac{\pi}{6} + \frac{\tan^{-1}(5)}{9\sqrt{3}} \int_0^1 \sin \left( t \left( \frac{\pi}{6} + \frac{\tan^{-1}(5)}{9\sqrt{3}} \right) \right) dt \right)}$$

$$\sqrt{\frac{1}{6} \left(1 + \sqrt{4 \times 4 - 7}\right) + \frac{1}{3} \left[ \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1} \left( 2 \sqrt{16 - 7} - 1 \right)}{3(3\sqrt{3})} \right) \right]} =$$

$$\sqrt{\frac{2}{3} - \frac{1}{3} i \sqrt{\frac{13}{\pi}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{s - \left( \frac{\pi}{6} + \frac{\tan^{-1}(5)}{9\sqrt{3}} \right)^2 / (4s)}}{\sqrt{s}} ds \quad \text{for } \gamma > 0}$$

**Continued fraction representations:**

$$\sqrt{\frac{1}{6}(1+\sqrt{4 \times 4-7})+\frac{1}{3}\left(\sqrt{4 \times 4-\sqrt{4 \times 4-7}} \sin\left(\frac{\pi}{3}-\frac{\tan^{-1}\left(2 \sqrt{16-7-1}\right)}{3\left(3 \sqrt{3}\right)}\right)\right)}=$$

$$\sqrt{\frac{2}{3}+\frac{2}{3} \sqrt{13} \cos\left(\frac{\pi}{6}+\frac{5}{9 \sqrt{3}\left(1+\sum_{k=1}^{\infty} \frac{25 k^2}{1+2 k}\right)}\right)}=$$

$$\sqrt{\frac{2}{3}+\frac{2}{3} \sqrt{13} \cos\left(\frac{\pi}{6}+\frac{5}{9 \sqrt{3}\left(1+\frac{25}{3+\frac{100}{5+\frac{225}{7+\frac{400}{9+\dots}}}}\right)}\right)}$$

$$\sqrt{\frac{1}{6}(1+\sqrt{4 \times 4-7})+\frac{1}{3}\left(\sqrt{4 \times 4-\sqrt{4 \times 4-7}} \sin\left(\frac{\pi}{3}-\frac{\tan^{-1}\left(2 \sqrt{16-7-1}\right)}{3\left(3 \sqrt{3}\right)}\right)\right)}=$$

$$\sqrt{\frac{2}{3}+\frac{2}{3} \sqrt{13} \cos\left(\frac{\pi}{6}+\frac{5-\frac{125}{3+\sum_{k=1}^{\infty} \frac{25\left(1+(-1)^{1+k}+k\right)^2}{3+2 k}}}{9 \sqrt{3}}\right)}=$$

$$\sqrt{\frac{2}{3}+\frac{2}{3} \sqrt{13} \cos\left(\frac{\pi}{6}+\frac{5-\frac{125}{3+\frac{225}{5+\frac{100}{7+\frac{625}{9+\frac{400}{11+\dots}}}}}}}{9 \sqrt{3}}\right)}$$

$$\sqrt{\frac{1}{6} \left(1 + \sqrt{4 \times 4 - 7}\right) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1}(2\sqrt{16-7-1})}{3(3\sqrt{3})} \right) \right)}^2 =$$

$$\sqrt{\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} + \frac{5}{9\sqrt{3} \left( 1 + \mathbf{K}_{k=1}^{\infty} \frac{25(-1+2k)^2}{1+2k-25(-1+2k)} \right)} \right)} =$$

$$\sqrt{\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} + \frac{5}{9\sqrt{3} \left( 1 + \frac{25}{-22 + \frac{225}{-70 + \frac{625}{-118 + \frac{1225}{-166 + \dots}}} \right)} \right)}$$

$$\sqrt{\frac{1}{6} \left(1 + \sqrt{4 \times 4 - 7}\right) + \frac{1}{3} \left( \sqrt{4 \times 4 - \sqrt{4 \times 4 - 7}} \sin \left( \frac{\pi}{3} - \frac{\tan^{-1}(2\sqrt{16-7-1})}{3(3\sqrt{3})} \right) \right)}^2 =$$

$$\sqrt{\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} + \frac{5}{9\sqrt{3} \left( 26 + \mathbf{K}_{k=1}^{\infty} \frac{50(1-2\lfloor \frac{1+k}{2} \rfloor) \lfloor \frac{1+k}{2} \rfloor}{\left( 1 + \frac{25}{2} (1+(-1)^k) \right) (1+2k)} \right)} \right)} =$$

$$\sqrt{\frac{2}{3} + \frac{2}{3} \sqrt{13} \cos \left( \frac{\pi}{6} + \frac{5}{9\sqrt{3} \left( 26 + \frac{50}{3 - \frac{50}{130 - \frac{300}{7 - \frac{300}{234 + \dots}}} \right)} \right)}$$

From the sum of all four results, adding  $\pi$  and subtracting the value of the golden ratio, we obtain:

$$\begin{aligned} & \left( \left( \left( \left( 37 + 49 \sqrt[3]{286} \right)^{1/3} \right)^{1/3} \right) \right) + \left( \left( \left( \left( \frac{1}{3} * \left( \left( (4*5+3)^2 \right)^{1/3} + \left( (4*(5-2*3)(4*5+3)) \right)^{1/3} - \left( (2*(5-2*3)^2) \right)^{1/3} \right) \right) \right) \right) \right) + 2.63450813734 + 1.51267587416659 \\ & + \text{Pi-golden ratio} \end{aligned}$$

**Input interpretation:**

$$\begin{aligned} & \sqrt[3]{37 + 49 \sqrt[3]{286}} + \frac{1}{3} \left( \sqrt[3]{(4 \times 5 + 3)^2} + \sqrt[3]{4(5 - 2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5 - 2 \times 3)^2} \right) + \\ & 2.63450813734 + 1.51267587416659 + \pi - \phi \end{aligned}$$

$\phi$  is the golden ratio

**Result:**

$$\begin{aligned} & 15.81174161622... + \\ & 1.303182740295... i \end{aligned}$$

**Polar coordinates:**

$$r = 15.86535402040 \text{ (radius), } \theta = 4.711592932784^\circ \text{ (angle)}$$

15.86535402040 result very near to the value of black hole entropy 15.8174

**Alternative representations:**

$$\begin{aligned} & \sqrt[3]{37 + 49 \sqrt[3]{286}} + \frac{1}{3} \left( \sqrt[3]{(4 \times 5 + 3)^2} + \sqrt[3]{4(5 - 2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5 - 2 \times 3)^2} \right) + \\ & 2.634508137340000 + 1.512675874166590000 + \pi - \phi = 4.147184011506590 + \\ & \pi + 2 \cos(216^\circ) + \frac{1}{3} \left( \sqrt[3]{-92} - \sqrt[3]{2} + \sqrt[3]{23^2} \right) + \sqrt[3]{37 + 49 \sqrt[3]{286}} \end{aligned}$$

$$\begin{aligned} & \sqrt[3]{37 + 49 \sqrt[3]{286}} + \frac{1}{3} \left( \sqrt[3]{(4 \times 5 + 3)^2} + \sqrt[3]{4(5 - 2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5 - 2 \times 3)^2} \right) + \\ & 2.634508137340000 + 1.512675874166590000 + \pi - \phi = \\ & 4.147184011506590 + \pi - 2 \cos\left(\frac{\pi}{5}\right) + \frac{1}{3} \left( \sqrt[3]{-92} - \sqrt[3]{2} + \sqrt[3]{23^2} \right) + \sqrt[3]{37 + 49 \sqrt[3]{286}} \end{aligned}$$

$$\begin{aligned} & \sqrt[3]{37 + 49 \sqrt[3]{286}} + \frac{1}{3} \left( \sqrt[3]{(4 \times 5 + 3)^2} + \sqrt[3]{4(5 - 2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5 - 2 \times 3)^2} \right) + \\ & 2.634508137340000 + 1.512675874166590000 + \pi - \phi = 4.147184011506590 + \\ & 180^\circ + 2 \cos(216^\circ) + \frac{1}{3} \left( \sqrt[3]{-92} - \sqrt[3]{2} + \sqrt[3]{23^2} \right) + \sqrt[3]{37 + 49 \sqrt[3]{286}} \end{aligned}$$

### Series representations:

$$\begin{aligned} & \sqrt[3]{37+49\sqrt[3]{286}} + \frac{1}{3} \left( \sqrt[3]{(4 \times 5 + 3)^2} + \sqrt[3]{4(5-2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5-2 \times 3)^2} \right) + \\ & 2.634508137340000 + 1.512675874166590000 + \pi - \phi = \\ & (14.288182951377207 + 1.3031827402945516594491479094206 i) - \\ & \phi + 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \end{aligned}$$

$$\begin{aligned} & \sqrt[3]{37+49\sqrt[3]{286}} + \frac{1}{3} \left( \sqrt[3]{(4 \times 5 + 3)^2} + \sqrt[3]{4(5-2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5-2 \times 3)^2} \right) + \\ & 2.634508137340000 + 1.512675874166590000 + \pi - \phi = \\ & (12.288182951377207 + 1.3031827402945516594491479094206 i) - \\ & \phi + 2 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}} \end{aligned}$$

$$\begin{aligned} & \sqrt[3]{37+49\sqrt[3]{286}} + \frac{1}{3} \left( \sqrt[3]{(4 \times 5 + 3)^2} + \sqrt[3]{4(5-2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5-2 \times 3)^2} \right) + \\ & 2.634508137340000 + 1.512675874166590000 + \pi - \phi = \\ & (14.288182951377207 + 1.3031827402945516594491479094206 i) - \\ & \phi + \sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}} \end{aligned}$$

### Integral representations:

$$\begin{aligned} & \sqrt[3]{37+49\sqrt[3]{286}} + \frac{1}{3} \left( \sqrt[3]{(4 \times 5 + 3)^2} + \sqrt[3]{4(5-2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5-2 \times 3)^2} \right) + \\ & 2.634508137340000 + 1.512675874166590000 + \pi - \phi = \\ & (14.288182951377207 + 1.3031827402945516594491479094206 i) - \\ & \phi + 2 \int_0^{\infty} \frac{1}{1+t^2} dt \end{aligned}$$

$$\begin{aligned} & \sqrt[3]{37+49\sqrt[3]{286}} + \frac{1}{3} \left( \sqrt[3]{(4 \times 5 + 3)^2} + \sqrt[3]{4(5-2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5-2 \times 3)^2} \right) + \\ & 2.634508137340000 + 1.512675874166590000 + \pi - \phi = \\ & (14.288182951377207 + 1.3031827402945516594491479094206 i) - \\ & \phi + 4 \int_0^1 \sqrt{1-t^2} dt \end{aligned}$$

$$\sqrt[3]{37+49\sqrt[3]{286}} + \frac{1}{3} \left( \sqrt[3]{(4 \times 5 + 3)^2} + \sqrt[3]{4(5-2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5-2 \times 3)^2} \right) +$$

$$2.634508137340000 + 1.512675874166590000 + \pi - \phi =$$

$$(14.288182951377207 + 1.3031827402945516594491479094206 i) -$$

$$\phi + 2 \int_0^\infty \frac{\sin(t)}{t} dt$$

Multiplying the results, we obtain:

$$18((((((((((37 + 49 \sqrt[3]{286}^{(1/3)})^{(1/3)})) * (((1/3 * (((4*5+3)^2))^{1/3} * ((4*(5-2*3)(4*5+3)))^{1/3} - ((2*(5-2*3)^2))^{1/3})))) * 2.63450813734 * 1.51267587416659)))))))-76$$

Where 18 and 76 are Lucas numbers

**Input interpretation:**

$$18 \left( \sqrt[3]{37+49\sqrt[3]{286}} \left( \frac{1}{3} \left( \sqrt[3]{(4 \times 5 + 3)^2} + \sqrt[3]{4(5-2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5-2 \times 3)^2} \right) \right) \times \right.$$

$$\left. 2.63450813734 \times 1.51267587416659 \right) - 76$$

**Result:**

$$2814.40322788... +$$

$$5377.46403019... i$$

**Polar coordinates:**

$$r = 6069.43036249 \text{ (radius), } \theta = 62.3737901501^\circ \text{ (angle)}$$

[6069.43036249](#) result very near to the rest mass of bottom Omega baryon [6071](#)

And:

$$\text{golden ratio}^3 + 3((((((((((37 + 49 \sqrt[3]{286}^{(1/3)})^{(1/3)})) * (((1/3 * (((4*5+3)^2))^{1/3} * ((4*(5-2*3)(4*5+3)))^{1/3} - ((2*(5-2*3)^2))^{1/3})))) * 2.63450813734 * 1.51267587416659))))))$$

**Input interpretation:**

$$\phi^3 + 3 \left( \sqrt[3]{37 + 49 \sqrt[3]{286}} \left( \frac{1}{3} \left( \sqrt[3]{(4 \times 5 + 3)^2} \sqrt[3]{4(5 - 2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5 - 2 \times 3)^2} \right) \right) \times \right. \\ \left. 2.63450813734 \times 1.51267587416659 \right)$$

$\phi$  is the golden ratio

### Result:

$$485.969939290... + \\ 896.244005032... i$$

### Polar coordinates:

$$r = 1019.519542946 \text{ (radius), } \theta = 61.5321445626^\circ \text{ (angle)}$$

[1019.519542946](#) result practically equal to the rest mass of Phi meson 1019.445

### Alternative representations:

$$\phi^3 + \frac{3}{3} \sqrt[3]{37 + 49 \sqrt[3]{286}} \left( \left( \sqrt[3]{(4 \times 5 + 3)^2} \sqrt[3]{4(5 - 2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5 - 2 \times 3)^2} \right) \right. \\ \left. 2.634508137340000 \times 1.512675874166590000 \right) =$$

$$3.985156899649779 \left( -\sqrt[3]{2} + \sqrt[3]{-92} \sqrt[3]{23^2} \right) \sqrt[3]{37 + 49 \sqrt[3]{286}} + (2 \sin(54^\circ))^3$$

$$\phi^3 + \frac{3}{3} \sqrt[3]{37 + 49 \sqrt[3]{286}} \left( \left( \sqrt[3]{(4 \times 5 + 3)^2} \sqrt[3]{4(5 - 2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5 - 2 \times 3)^2} \right) \right. \\ \left. 2.634508137340000 \times 1.512675874166590000 \right) =$$

$$(-2 \cos(216^\circ))^3 + 3.985156899649779 \left( -\sqrt[3]{2} + \sqrt[3]{-92} \sqrt[3]{23^2} \right) \sqrt[3]{37 + 49 \sqrt[3]{286}}$$

$$\phi^3 + \frac{3}{3} \sqrt[3]{37 + 49 \sqrt[3]{286}} \left( \left( \sqrt[3]{(4 \times 5 + 3)^2} \sqrt[3]{4(5 - 2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5 - 2 \times 3)^2} \right) \right. \\ \left. 2.634508137340000 \times 1.512675874166590000 \right) =$$

$$3.985156899649779 \left( -\sqrt[3]{2} + \sqrt[3]{-92} \sqrt[3]{23^2} \right) \sqrt[3]{37 + 49 \sqrt[3]{286}} + (-2 \sin(66^\circ))^3$$



golden ratio+21+1/3((((((((((37 + 49 286^(1/3))^(1/3)))) \* (((1/3 \* (((4\*5+3)^2))^1/3 \* ((4\*(5-2\*3)(4\*5+3)))^1/3 - ((2\*(5-2\*3)^2))^1/3)))) \* 2.63450813734 \* 1.512675874166))))))

**Input interpretation:**

$$\phi + 21 + \frac{1}{3} \left( \sqrt[3]{37 + 49 \sqrt[3]{286}} \left( \frac{1}{3} \left( \sqrt[3]{(4 \times 5 + 3)^2} \sqrt[3]{4(5 - 2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5 - 2 \times 3)^2} \right) \right) \times 2.63450813734 \times 1.512675874166 \right)$$

φ is the golden ratio

**Result:**

76.1440196901... +  
99.5826672258... i

**Polar coordinates:**

r = 125.357964830 (radius), θ = 52.5973415807° (angle)

125.357964830 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18

**Alternative representations:**

$$\phi + 21 + \frac{1}{3 \times 3} \sqrt[3]{37 + 49 \sqrt[3]{286}} \left( \sqrt[3]{(4 \times 5 + 3)^2} \sqrt[3]{4(5 - 2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5 - 2 \times 3)^2} \right) \times 2.634508137340000 \times 1.5126758741660000 = 21 + 0.4427952110720250 \left( -\sqrt[3]{2} + \sqrt[3]{-92} \sqrt[3]{23^2} \right) \sqrt[3]{37 + 49 \sqrt[3]{286}} + 2 \sin(54^\circ)$$

$$\phi + 21 + \frac{1}{3 \times 3}$$

$$\sqrt[3]{37 + 49 \sqrt[3]{286}} \left( \sqrt[3]{(4 \times 5 + 3)^2} \sqrt[3]{4(5 - 2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5 - 2 \times 3)^2} \right)$$

$$2.634508137340000 \times 1.5126758741660000 =$$

$$21 - 2 \cos(216^\circ) + 0.4427952110720250 \left( -\sqrt[3]{2} + \sqrt[3]{-92} \sqrt[3]{23^2} \right) \sqrt[3]{37 + 49 \sqrt[3]{286}}$$

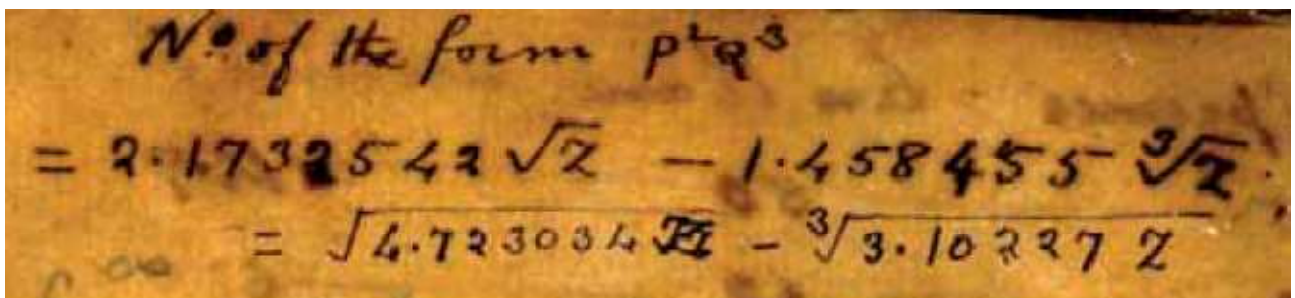
$$\phi + 21 + \frac{1}{3 \times 3}$$

$$\sqrt[3]{37 + 49 \sqrt[3]{286}} \left( \sqrt[3]{(4 \times 5 + 3)^2} \sqrt[3]{4(5 - 2 \times 3)(4 \times 5 + 3)} - \sqrt[3]{2(5 - 2 \times 3)^2} \right)$$

$$2.634508137340000 \times 1.5126758741660000 =$$

$$21 + 0.4427952110720250 \left( -\sqrt[3]{2} + \sqrt[3]{-92} \sqrt[3]{23^2} \right) \sqrt[3]{37 + 49 \sqrt[3]{286}} - 2 \sin(666^\circ)$$

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We have that:

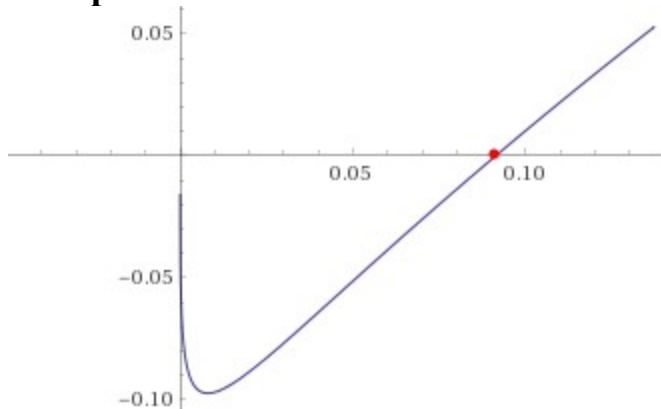
$$2.1732542 \sqrt{x} - 1.458455 (x)^{1/3} = 0$$

**Input interpretation:**

$$2.1732542 \sqrt{x} + \sqrt[3]{x} \times (-1.458455) = 0$$

**Result:**

$$2.17325 \sqrt{x} - 1.45846 \sqrt[3]{x} = 0$$

**Root plot:****Alternate form assuming x is real:**

$$\sqrt[3]{x} = 0.671093 \sqrt[6]{x}$$

**Alternate form:**

$$2.17325 \left( \sqrt[6]{x} - 0.671093 \right) \sqrt[3]{x} = 0$$

**Alternate form assuming x is positive:**

$$\sqrt[6]{x} = 0.671093$$

**Solutions:**

$$x = 0$$

$$x \approx 0.0913472$$

0.0913472

$$2.1732542 \sqrt{0.0913472} - 1.458455 (0.0913472)^{1/3}$$

**Input interpretation:**

$$2.1732542 \sqrt{0.0913472} + \sqrt[3]{0.0913472} \times (-1.458455)$$

**Result:**

$$5.12357... \times 10^{-8}$$

5.12357... \* 10<sup>-8</sup>

$$-\ln(((2.1732542\sqrt{0.0913472})-1.458455(0.0913472)^{1/3})))$$

**Input interpretation:**

$$-\log\left(2.1732542 \sqrt{0.0913472} + \sqrt[3]{0.0913472} \times (-1.458455)\right)$$

log(x) is the natural logarithm

**Result:**

16.78682987177971830435592042565656911895607776705183019409...

16.786829871.... black hole entropy 16.8741

**Alternative representations:**

$$-\log\left(2.17325 \sqrt{0.0913472} - 1.45846 \sqrt[3]{0.0913472}\right) =$$

$$-\log_e\left(-1.45846 \sqrt[3]{0.0913472} + 2.17325 \sqrt{0.0913472}\right)$$

$$-\log\left(2.17325 \sqrt{0.0913472} - 1.45846 \sqrt[3]{0.0913472}\right) =$$

$$-\log(a) \log_a\left(-1.45846 \sqrt[3]{0.0913472} + 2.17325 \sqrt{0.0913472}\right)$$

$$-\log\left(2.17325 \sqrt{0.0913472} - 1.45846 \sqrt[3]{0.0913472}\right) =$$

$$\text{Li}_1\left(1 + 1.45846 \sqrt[3]{0.0913472} - 2.17325 \sqrt{0.0913472}\right)$$

**Series representations:**

$$-\log\left(2.17325 \sqrt{0.0913472} - 1.45846 \sqrt[3]{0.0913472}\right) =$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1.65684 + 2.17325 \sqrt{0.0913472}\right)^k}{k}$$

$$-\log\left(2.17325 \sqrt{0.0913472} - 1.45846 \sqrt[3]{0.0913472}\right) =$$

$$-\log\left(-0.656838 + 2.17325 \sum_{k=0}^{\infty} \frac{(-1)^k (-0.908653)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\begin{aligned}
& -\log\left(2.17325 \sqrt{0.0913472} - 1.45846 \sqrt[3]{0.0913472}\right) = \\
& -2i\pi \left\lfloor \frac{\arg(-0.656838 - x + 2.17325 \sqrt{0.0913472})}{2\pi} \right\rfloor - \log(x) + \\
& \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-0.656838 - x + 2.17325 \sqrt{0.0913472})^k}{k} \quad \text{for } x < 0
\end{aligned}$$

**Integral representation:**

$$\begin{aligned}
& -\log\left(2.17325 \sqrt{0.0913472} - 1.45846 \sqrt[3]{0.0913472}\right) = \\
& -\int_1^{-0.656838+2.17325 \sqrt{0.0913472}} \frac{1}{t} dt
\end{aligned}$$

And:

$$\sqrt{4.723034x} - (3.10227x)^{1/3}$$

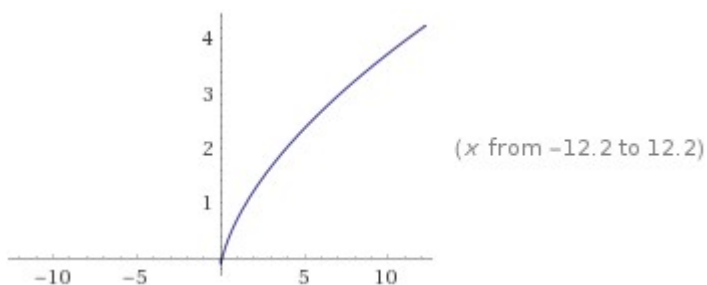
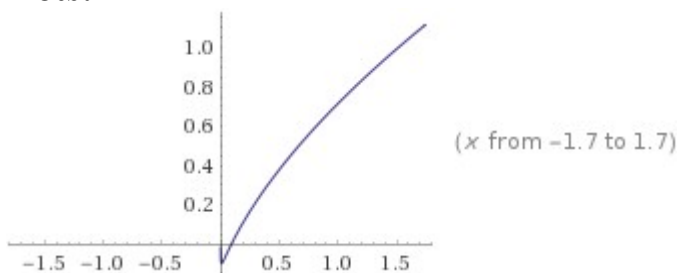
**Input interpretation:**

$$\sqrt{4.723034x} - \sqrt[3]{3.10227x}$$

**Result:**

$$2.17325 \sqrt{x} - 1.45846 \sqrt[3]{x}$$

**Plots:**



**Alternate forms:**

$$2.17325 \left(\sqrt[6]{x} - 0.671093\right) \sqrt[3]{x}$$

$$(2.17325 \sqrt[6]{x} - 1.45846) \sqrt[3]{x}$$

**Roots:**

$$x = 0$$

$$x \approx 0.0913474$$

$$0.0913474$$

**Properties as a real function:**

**Domain**

$\{x \in \mathbb{R} : x \geq 0\}$  (all non-negative real numbers)

**Range**

$$\{y \in \mathbb{R} : y \geq -\frac{757390136718748872655694986769512672141339}{7783320190429684751806492178507995605468750}\}$$

$\mathbb{R}$  is the set of real numbers

**Derivative:**

$$\frac{d}{dx} (2.17325 \sqrt{x} - 1.45846 \sqrt[3]{x}) = \frac{1.08663 \sqrt[6]{x} - 0.486152}{x^{2/3}}$$

**Indefinite integral:**

$$\int (\sqrt{4.723034x} - \sqrt[3]{3.10227x}) dx = 1.44884x^{3/2} - 1.09384x^{4/3} + \text{constant}$$

**Global minimum:**

$$\min\{2.17325 \sqrt{x} - 1.45846 \sqrt[3]{x}\} = \frac{5392513736451350882867243590521543475136}{55416170619709352803091451130045931734407} \text{ at } x = \frac{35252049753829906464031686046932223997891571485322631822}{304915121756586555466817963770574536704} \cdot \frac{4395780098563828079707005004984907610901034384330994242}{005462554224028486649598812920311745182889}$$

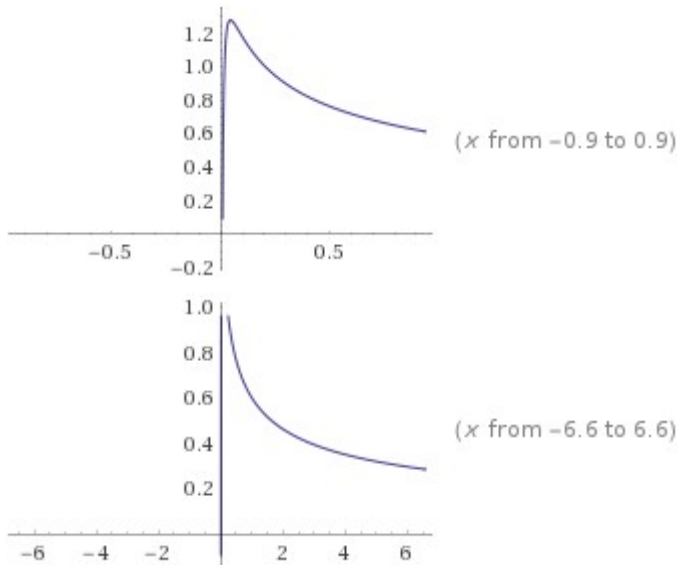
$$(-0.486152 + 1.08663 x^{(1/6)})/x^{(2/3)}$$

**Input interpretation:**

$$\frac{-0.486152 + 1.08663 \sqrt[6]{x}}{x^{2/3}}$$

**Result:**

$$\frac{1.08663 \sqrt[6]{x} - 0.486152}{x^{2/3}}$$

**Plots:****Alternate form:**

$$\frac{1.08663 (\sqrt[6]{x} - 0.447394)}{x^{2/3}}$$

**Expanded form:**

$$\frac{1.08663}{\sqrt{x}} - \frac{0.486152}{x^{2/3}}$$

**Root:**

$$x \approx 0.0080194$$

$$0.0080194$$

**Properties as a real function:****Domain**

$\{x \in \mathbb{R} : x > 0\}$  (all positive real numbers)

**Range**

$$\{y \in \mathbb{R} : y \leq \frac{3764354610070213463547}{2941414273667681484800}\}$$

$\mathbb{R}$  is the set of real numbers

**Derivative:**

$$\frac{d}{dx} \left( \frac{1.08663 \sqrt[6]{x} - 0.486152}{x^{2/3}} \right) = \frac{0.324101 - 0.543315 \sqrt[6]{x}}{x^{5/3}}$$

**Indefinite integral:**

$$\int \frac{-0.486152 + 1.08663 \sqrt[6]{x}}{x^{2/3}} dx = 2.17326 \sqrt{x} - 1.45846 \sqrt[3]{x} + \text{constant}$$

**Global maximum:**

$$\max \left\{ \frac{1.08663 \sqrt[6]{x} - 0.486152}{x^{2/3}} \right\} = \frac{3764354610070213463547}{2941414273667681484800}$$

$$\text{at } x = \frac{844913860286716233189736686127415296}{18751572798939632357787011343993140625}$$

**Limit:**

$$\lim_{x \rightarrow \pm\infty} \frac{-0.486152 + 1.08663 \sqrt[6]{x}}{x^{2/3}} = 0 \approx 0$$

For  $x = 1$ , we obtain:

$$(-0.486152 + 1.08663 \cdot 1^{1/6}) / 1^{2/3}$$

**Input interpretation:**

$$\frac{-0.486152 + 1.08663 \sqrt[6]{1}}{1^{2/3}}$$

**Result:**

0.600478

0.600478

$$(-0.486152 + x \cdot 1^{1/6}) / 1^{2/3} = 0.600478$$

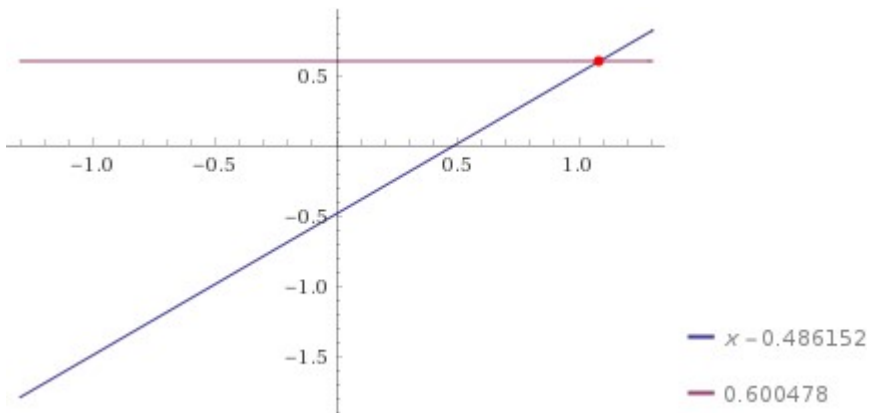
**Input interpretation:**

$$\frac{-0.486152 + x \sqrt[6]{1}}{1^{2/3}} = 0.600478$$



**Result:**

$$x - 0.486152 = 0.600478$$

**Plot:****Alternate forms:**

$$x - 1.08663 = 0$$

$$x - 0.486152 = 0.600478$$

**Solution:**

$$x \approx 1.08663$$

1.08663

From:

$$\frac{1.08663 \sqrt[6]{x} - 0.486152}{x^{2/3}}$$

that is:

$$(-0.486153 + 1.08663 \cdot 0.0913474^{(1/6)}) / 0.0913474^{(2/3)}$$

**Input interpretation:**

$$\frac{-0.486153 + 1.08663 \sqrt[6]{0.0913474}}{0.0913474^{2/3}}$$

**Result:**

1.19843...

1.19843...

$$(-0.486153 + x \sqrt[6]{0.0913474}) / 0.0913474^{2/3} = 1.19843$$

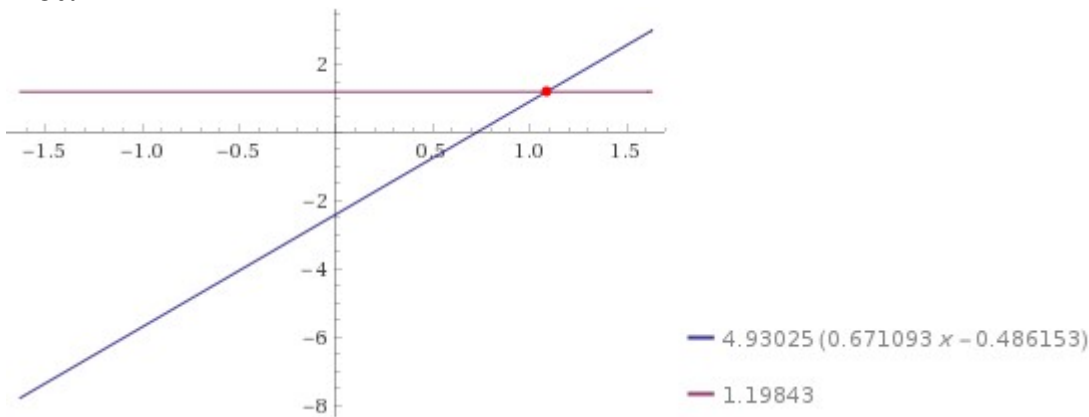
**Input interpretation:**

$$\frac{-0.486153 + x \sqrt[6]{0.0913474}}{0.0913474^{2/3}} = 1.19843$$

**Result:**

$$4.93025 (0.671093 x - 0.486153) = 1.19843$$

**Plot:**



**Alternate forms:**

$$3.30866 (x - 0.72442) = 1.19843$$

$$3.30866 x - 3.59529 = 0$$

$$3.30866 x - 2.39686 = 1.19843$$

**Solution:**

$$x \approx 1.08663$$

**1.08663**

2) We have also that:

$$2.1732542\sqrt{x} - 1.458455(x)^{1/3} = \sqrt{4.723034x} - (3.10227x)^{1/3}$$

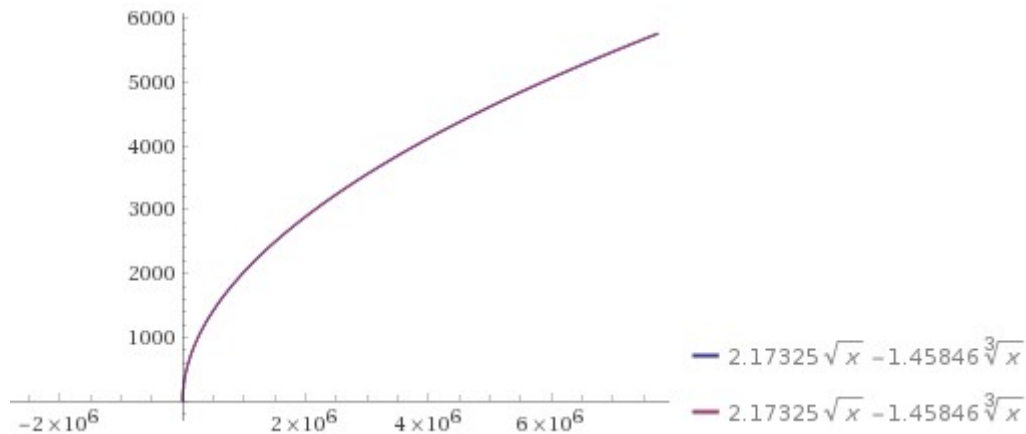
**Input interpretation:**

$$2.1732542 \sqrt{x} + \sqrt[3]{x} \times (-1.458455) = \sqrt{4.723034 x} - \sqrt[3]{3.10227 x}$$

**Result:**

$$2.17325 \sqrt{x} - 1.45846 \sqrt[3]{x} = 2.17325 \sqrt{x} - 1.45846 \sqrt[3]{x}$$

**Plot:**



**Alternate form assuming x is real:**

$$\sqrt[3]{x} = 13.1399 \sqrt[6]{x}$$

**Alternate form:**

$$2.17325 \left( \sqrt[6]{x} - 0.671093 \right) \sqrt[3]{x} = 2.17325 \left( \sqrt[6]{x} - 0.671093 \right) \sqrt[3]{x}$$

**Alternate form assuming x is positive:**

$$\sqrt[6]{x} = 13.1399$$

**Solutions:**

$$x = 0$$

$$x \approx 5.14701 \times 10^6$$

$$5.14701 \times 10^6$$

$$2.17325 \left( -0.671093 + x^{1/6} \right) x^{1/3}$$

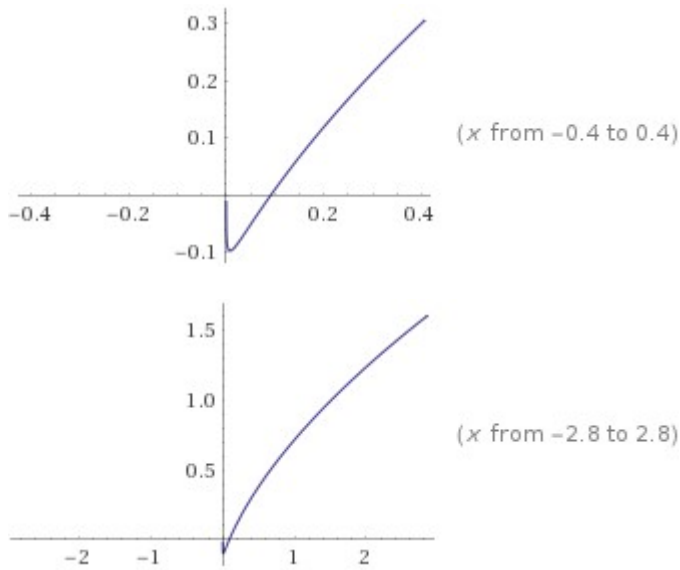
**Input interpretation:**

$$2.17325 \left( -0.671093 + \sqrt[6]{x} \right) \sqrt[3]{x}$$

**Result:**

$$2.17325 \left( \sqrt[6]{x} - 0.671093 \right) \sqrt[3]{x}$$

**Plots:**



**Alternate form:**

$$(2.17325 \sqrt[6]{x} - 1.45845) \sqrt[3]{x}$$

**Expanded form:**

$$2.17325 \sqrt{x} - 1.45845 \sqrt[3]{x}$$

**Roots:**

$$x = 0$$

$$x \approx 0.0913474$$

0.0913474

**Properties as a real function:**

**Domain**

$\{x \in \mathbb{R} : x \geq 0\}$  (all non-negative real numbers)

**Range**

$$\{y \in \mathbb{R} : y \geq -\frac{97\,309\,231\,323\,612\,936\,700\,370\,667\,523\,232\,407\,938\,647\,037\,037\,359}{1\,000}\}$$

$\mathbb{R}$  is the set of real numbers

**Derivative:**

$$\frac{d}{dx} \left( 2.17325 \left( \sqrt[6]{x} - 0.671093 \right) \sqrt[3]{x} \right) = \frac{1.08663 \sqrt[6]{x} - 0.486151}{x^{2/3}}$$

**Indefinite integral:**

$$\int 2.17325 \left( -0.671093 + \sqrt[6]{x} \right) \sqrt[3]{x} \, dx = 1.44883 x^{3/2} - 1.09384 x^{4/3} + \text{constant}$$

**Global minimum:**

$$\min\{2.17325(\sqrt[6]{x} - 0.671093)\sqrt[3]{x}\} = -\frac{2\,627\,349\,245\,737\,548\,116\,401}{27\,000\,000\,000\,000\,000\,000\,000}$$
$$\text{at } x = \frac{91\,347\,413\,105\,703\,468\,120\,568\,152\,353\,201\,449}{11\,390\,625\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

We take:

$$(2.17325\sqrt[6]{x} - 1.45845)\sqrt[3]{x}$$

$$(-1.45845 + 2.17325 x^{(1/6)}) x^{(1/3)}$$

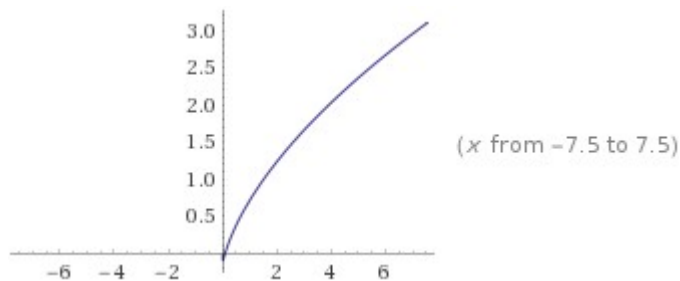
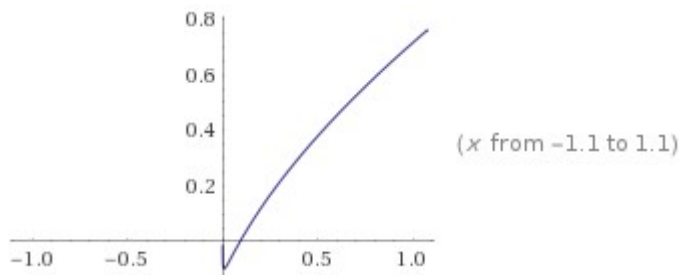
**Input interpretation:**

$$(-1.45845 + 2.17325\sqrt[6]{x})\sqrt[3]{x}$$

**Result:**

$$(2.17325\sqrt[6]{x} - 1.45845)\sqrt[3]{x}$$

**Plots:**



**Alternate form:**

$$2.17325(\sqrt[6]{x} - 0.671092)\sqrt[3]{x}$$

**Expanded form:**

$$2.17325 \sqrt{x} - 1.45845 \sqrt[3]{x}$$

**Roots:**

$$x = 0$$

$$x \approx 0.091346$$

0.091346

**Properties as a real function:****Domain**

$\{x \in \mathbb{R} : x \geq 0\}$  (all non-negative real numbers)

**Range**

$$\{y \in \mathbb{R} : y \geq -\frac{919\,180\,616\,067}{9\,446\,031\,125\,000}\}$$

$\mathbb{R}$  is the set of real numbers

**Derivative:**

$$\frac{d}{dx} \left( (2.17325 \sqrt[6]{x} - 1.45845) \sqrt[3]{x} \right) = \frac{1.08663 \sqrt[6]{x} - 0.48615}{x^{2/3}}$$

**Indefinite integral:**

$$\int (-1.45845 + 2.17325 \sqrt[6]{x}) \sqrt[3]{x} \, dx = 1.44883 x^{3/2} - 1.09384 x^{4/3} + \text{constant}$$

**Global minimum:**

$$\min \left\{ (2.17325 \sqrt[6]{x} - 1.45845) \sqrt[3]{x} \right\} = -\frac{919\,180\,616\,067}{9\,446\,031\,125\,000}$$

$$\text{at } x = \frac{54\,073\,152\,317\,011\,818\,147\,103\,296}{6\,742\,766\,241\,013\,875\,283\,472\,640\,625}$$

We take:

$$\frac{d}{dx} \left( (2.17325 \sqrt[6]{x} - 1.45845) \sqrt[3]{x} \right) = \frac{1.08663 \sqrt[6]{x} - 0.48615}{x^{2/3}}$$

$$(-0.48615 + 1.08663 (0.091346)^{1/6}) / (0.091346)^{2/3}$$

**Input interpretation:**

$$\frac{-0.48615 + 1.08663 \sqrt[6]{0.091346}}{0.091346^{2/3}}$$

**Result:**

1.19845...

1.19845...

$$(-0.48615 + x (0.091346)^{(1/6)})/(0.091346)^{(2/3)} = 1.19845$$

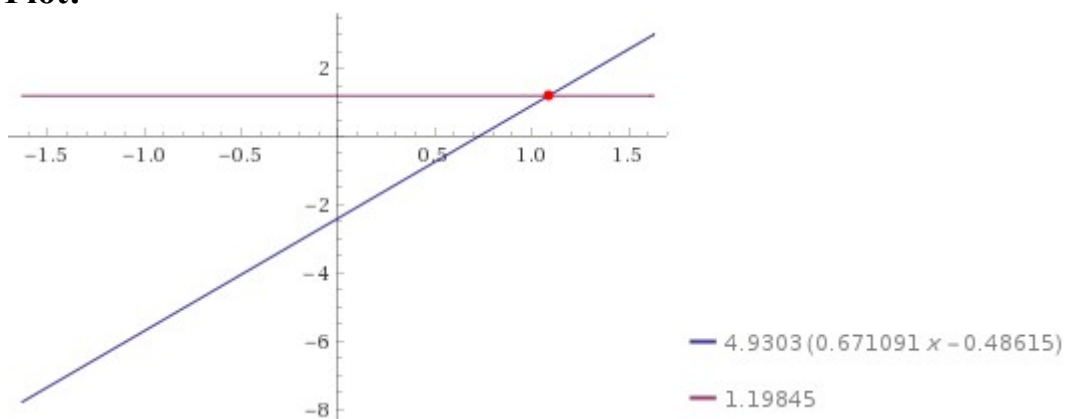
**Input interpretation:**

$$\frac{-0.48615 + x \sqrt[6]{0.091346}}{0.091346^{2/3}} = 1.19845$$

**Result:**

$$4.9303 (0.671091 x - 0.48615) = 1.19845$$

**Plot:**



**Alternate forms:**

$$3.30868 (x - 0.724417) = 1.19845$$

$$3.30868 x - 3.59532 = 0$$

$$3.30868 x - 2.39687 = 1.19845$$

**Solution:**

$$x \approx 1.08663$$

1.08663

From:

$$\frac{-0.48615 + x \sqrt[6]{0.091346}}{0.091346^{2/3}} = 1.19845$$

and:

$$x \approx 0.091346$$

0.091346

we obtain:

$$-11 \times 1/10^{56} + 1/10^{52} \times (((-0.48615 + 1.08663 (0.091346)^{(1/6)}) / (0.091346)^{(2/3)} - 0.091346)))$$

**Input interpretation:**

$$-11 \times \frac{1}{10^{56}} + \frac{1}{10^{52}} \left( \frac{-0.48615 + 1.08663 \sqrt[6]{0.091346}}{0.091346^{2/3}} - 0.091346 \right)$$

**Result:**

$$1.10600... \times 10^{-52}$$

$$1.10600... * 10^{-52}$$

result very near to the value of Cosmological Constant  $1.1056 * 10^{-52} \text{ m}^{-2}$

From the following previous result  $5.12357... * 10^{-8}$ , and the following expression:

$$(((1/24 (28 - 54 e + 45 e^2 - 30 \sqrt{1+e} + 3 \sqrt{1+e^2} - 17 \pi - 28 \pi^2 + 62 \sqrt{1+\pi^2}))))))$$

**Input:**

$$\frac{1}{24} \left( 28 - 54 e + 45 e^2 - 30 \sqrt{1+e} + 3 \sqrt{1+e^2} - 17 \pi - 28 \pi^2 + 62 \sqrt{1+\pi^2} \right)$$

**Decimal approximation:**

$$1.633885091243601871254304990313434242520174971195523935210...$$

$$1.6338850912.... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

**Alternate forms:**

$$\frac{1}{24} \left( 28 - 30 \sqrt{1+e} + 9 e (5 e - 6) + 3 \sqrt{1+e^2} - \pi (17 + 28 \pi) + 62 \sqrt{1+\pi^2} \right)$$



$$-\frac{9e}{4} + \frac{15e^2}{8} + \frac{1}{24} \left( 28 - 30\sqrt{1+e} + 3\sqrt{1+e^2} - 17\pi - 28\pi^2 + 62\sqrt{1+\pi^2} \right)$$

$$\frac{7}{6} - \frac{9e}{4} + \frac{15e^2}{8} - \frac{5\sqrt{1+e}}{4} + \frac{\sqrt{1+e^2}}{8} - \frac{17\pi}{24} - \frac{7\pi^2}{6} + \frac{31\sqrt{1+\pi^2}}{12}$$

**Continued fraction:**

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{5 + \frac{1}{86 + \frac{1}{21 + \frac{1}{1 + \frac{1}{12 + \frac{1}{1 + \frac{1}{76 + \frac{1}{11 + \frac{1}{2 + \frac{1}{2 + \frac{1}{\dots}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}}$$

**Series representations:**

$$\begin{aligned} &\frac{1}{24} \left( 28 - 54e + 45e^2 - 30\sqrt{1+e} + 3\sqrt{1+e^2} - 17\pi - 28\pi^2 + 62\sqrt{1+\pi^2} \right) = \\ &\frac{7}{6} - \frac{9e}{4} + \frac{15e^2}{8} - \frac{17\pi}{24} - \frac{7\pi^2}{6} + \\ &\sum_{k=0}^{\infty} \frac{1}{24} \binom{\frac{1}{2}}{k} \left( -30e^{-k} \sqrt{e} + 3(e^2)^{-k} \sqrt{e^2} + 62(\pi^2)^{-k} \sqrt{\pi^2} \right) \end{aligned}$$

$$\begin{aligned} &\frac{1}{24} \left( 28 - 54e + 45e^2 - 30\sqrt{1+e} + 3\sqrt{1+e^2} - 17\pi - 28\pi^2 + 62\sqrt{1+\pi^2} \right) = \\ &\frac{7}{6} - \frac{9e}{4} + \frac{15e^2}{8} - \frac{17\pi}{24} - \frac{7\pi^2}{6} + \\ &\sum_{k=0}^{\infty} \left( -\frac{5}{4} e^{-k} \binom{\frac{1}{2}}{k} \sqrt{e} + \frac{1}{8} (e^2)^{-k} \binom{\frac{1}{2}}{k} \sqrt{e^2} + \frac{31}{12} (\pi^2)^{-k} \binom{\frac{1}{2}}{k} \sqrt{\pi^2} \right) \end{aligned}$$

$$\begin{aligned} &\frac{1}{24} \left( 28 - 54e + 45e^2 - 30\sqrt{1+e} + 3\sqrt{1+e^2} - 17\pi - 28\pi^2 + 62\sqrt{1+\pi^2} \right) = \\ &\frac{7}{6} - \frac{9e}{4} + \frac{15e^2}{8} - \frac{17\pi}{24} - \frac{7\pi^2}{6} + \sum_{k=0}^{\infty} \frac{1}{24k!} (-1)^k e^{-k} (e^2)^{-k} (\pi^2)^{-k} \\ &\left( -\frac{1}{2} \right)_k \left( -30(e^2)^k (\pi^2)^k \sqrt{e} + e^k \left( 3(\pi^2)^k \sqrt{e^2} + 62(e^2)^k \sqrt{\pi^2} \right) \right) \end{aligned}$$

we obtain:

$$(5.12357e-8)^{\left(\left(\left(\frac{1}{24} (28 - 54 e + 45 e^2 - 30 \sqrt{1+e} + 3 \sqrt{1+e^2} - 17 \pi - 28 \pi^2 + 62 \sqrt{1+\pi^2})\right)\right)\right)^4}$$

**Input interpretation:**

$$(5.12357 \times 10^{-8})^{\left(\frac{1}{24} \left(28 - 54 e + 45 e^2 - 30 \sqrt{1+e} + 3 \sqrt{1+e^2} - 17 \pi - 28 \pi^2 + 62 \sqrt{1+\pi^2}\right)\right)^4}$$

**Result:**

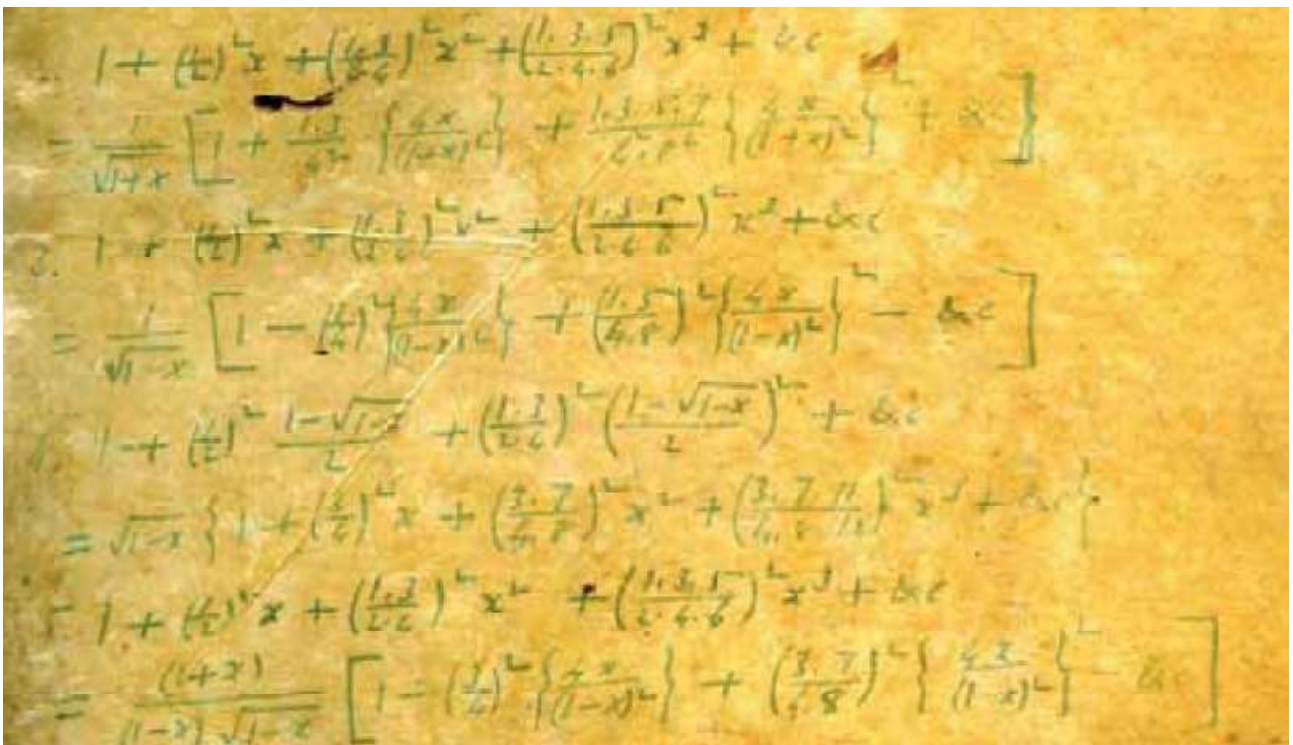
$$1.10561... \times 10^{-52}$$

1.10561... \* 10<sup>-52</sup> result practically equal to the Cosmological Constant

From:

**Manuscript Book I of Srinivasa Ramanujan**

Page 216



$$\frac{1}{\sqrt{1+2}} * \left( \left( \left( \left( 1 + \frac{3}{4^2} * \left( \frac{4*2}{(1+2)^2} \right) + \frac{1*3*5*7}{4^2*8^2} * \left( \frac{4*2}{(1+2)^2} \right)^2 \right) \right) \right) \right)$$

**Input:**

$$\frac{1}{\sqrt{1+2}} \left( 1 + \frac{3}{4^2} * \frac{4*2}{(1+2)^2} + \frac{3*5*7}{4^2*8^2} \left( \frac{4*2}{(1+2)^2} \right)^2 \right)$$

**Result:**

$$\frac{539}{432\sqrt{3}}$$

**Decimal approximation:**

0.720351377530574738588961094191099695819577185033792560588...

0.72035137753....

**Alternate form:**

$$\frac{539\sqrt{3}}{1296}$$

$$\sqrt[2]{\left( \frac{1}{\sqrt{1+2}} * \left( \left( \left( \left( 1 + \frac{3}{4^2} * \left( \frac{4*2}{(1+2)^2} \right) + \frac{1*3*5*7}{4^2*8^2} * \left( \frac{4*2}{(1+2)^2} \right)^2 \right) \right) \right) \right) \right) - \frac{47+2}{10^3}}$$

where 47 and 2 are Lucas numbers

**Input:**

$$\sqrt{\frac{2}{\frac{1}{\sqrt{1+2}} \left( 1 + \frac{3}{4^2} * \frac{4*2}{(1+2)^2} + \frac{3*5*7}{4^2*8^2} \left( \frac{4*2}{(1+2)^2} \right)^2 \right)}} - \frac{47+2}{10^3}$$

**Result:**

$$\frac{12}{7} \sqrt{\frac{2}{11}} 3^{3/4} - \frac{49}{1000}$$

**Decimal approximation:**

1.617260128504324116813239310276394886267364028444305027275...

1.6172601285.... result that is a good approximation to the value of the golden ratio  
1,618033988749...

**Alternate forms:**

$$\frac{12\,000 \times 3^{3/4} \sqrt{22} - 3773}{77\,000}$$

$$\frac{12\,000 \sqrt{\frac{2}{11}} 3^{3/4} - 343}{7\,000}$$

$$\frac{12}{77} \left( 3^{3/4} \sqrt{22} \right) - \frac{49}{1000}$$

$$\frac{1}{\left(\left(\left(\left(\sqrt{2/\left(\left(\left(1/\left(\sqrt{1+2}\right)\right)^2 \times \left(\left(\left(1+3/4^2 \times ((4 \times 2)/(1+2)^2) + (1 \times 3 \times 5 \times 7)/(4^2 \times 8^2) \times ((4 \times 2)/(1+2)^2)^2)\right)\right)\right)\right)\right)\right)\right)} - (47+2)/10^3\right)$$

**Input:**

$$\frac{1}{\sqrt{\frac{2}{\sqrt{1+2} \left( 1 + \frac{3}{4^2} \times \frac{4 \times 2}{(1+2)^2} + \frac{3 \times 5 \times 7}{4^2 \times 8^2} \left( \frac{4 \times 2}{(1+2)^2} \right)^2 \right)}} - \frac{47+2}{10^3}}$$

**Result:**

$$\frac{12}{7} \sqrt{\frac{2}{11}} 3^{3/4} - \frac{49}{1000}$$

**Decimal approximation:**

0.618329718500400333135466209025767512164686555837203513851...

0.6183297185... result that is a very good approximation to the value of the conjugate of golden ratio 0,618033988749...

**Alternate forms:**

$$\frac{12\,000 \times 3^{3/4} \sqrt{22} - 3773}{77\,000}$$

$$\frac{12\,000 \sqrt{\frac{2}{11}} 3^{3/4} - 343}{7\,000}$$

$$-\frac{49}{1000} - \frac{12}{7} \sqrt{\frac{2}{11}} 3^{3/4}$$

$$\frac{2401}{1\,000\,000} - \frac{864 \sqrt{3}}{539}$$

We have also that:

$$\left[ \left( \left( \left( \frac{1}{\sqrt{1+2}} \right)^* \left( \left( \left( \left( \frac{1+3/4^2 * ((4*2)/(1+2)^2) + (1*3*5*7)/(4^2*8^2) * ((4*2)/(1+2)^2)^2 \right) \right) \right) \right) \right) \right) \right]^{1/32}$$

**Input:**

$$\sqrt[32]{\frac{1}{\sqrt{1+2}} \left( 1 + \frac{3}{4^2} \times \frac{4 \times 2}{(1+2)^2} + \frac{3 \times 5 \times 7}{4^2 \times 8^2} \left( \frac{4 \times 2}{(1+2)^2} \right)^2 \right)}$$

**Exact result:**

$$\frac{\sqrt[16]{7} \sqrt[32]{11}}{\sqrt[8]{2} 3^{7/64}}$$

**Decimal approximation:**

0.989801852325518760566781404951068967672174103093435938789...

0.9898018523255....

result practically equal to the dilaton value **0.989117352243 =  $\phi$**

4\*log base 0.989801852325518[(((1/(sqrt(1+2)) \* (((1+3/4^2\*((4\*2)/(1+2)^2)+(1\*3\*5\*7)/(4^2\*8^2)\*((4\*2)/(1+2)^2)^2)))))))]-  
Pi+1/golden ratio

**Input interpretation:**

$$4 \log_{0.989801852325518} \left( \frac{1}{\sqrt{1+2}} \left( 1 + \frac{3}{4^2} \times \frac{4 \times 2}{(1+2)^2} + \frac{3 \times 5 \times 7}{4^2 \times 8^2} \left( \frac{4 \times 2}{(1+2)^2} \right)^2 \right) \right) - \pi + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

125.4764413352...

125.4764413352.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18

**Alternative representation:**

$$4 \log_{0.9898018523255180000} \left( \frac{1 + \frac{3(4 \times 2)}{4^2(1+2)^2} + \frac{(3 \times 5 \times 7) \left( \frac{4 \times 2}{(1+2)^2} \right)^2}{4^2 \times 8^2}}{\sqrt{1+2}} \right) - \pi + \frac{1}{\phi} =$$

$$- \pi + \frac{1}{\phi} + \frac{4 \log \left( \frac{1 + \frac{24}{9 \times 4^2} + \frac{105 \left( \frac{8}{9} \right)^2}{4^2 \times 8^2}}{\sqrt{3}} \right)}{\log(0.9898018523255180000)}$$

**Series representations:**

$$4 \log_{0.9898018523255180000} \left( \frac{1 + \frac{3(4 \times 2)}{4^2(1+2)^2} + \frac{(3 \times 5 \times 7) \left( \frac{4 \times 2}{(1+2)^2} \right)^2}{4^2 \times 8^2}}{\sqrt{1+2}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - \frac{4 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{539}{432 \sqrt{3}} \right)^k}{k}}{\log(0.9898018523255180000)}$$

$$4 \log_{0.9898018523255180000} \left( \frac{1 + \frac{3(4 \times 2)}{4^2(1+2)^2} + \frac{(3 \times 5 \times 7) \left( \frac{4 \times 2}{(1+2)^2} \right)^2}{4^2 \times 8^2}}{\sqrt{1+2}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + 4 \log_{0.9898018523255180000} \left( \frac{539}{432 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}} \right)$$

$$4 \log_{0.9898018523255180000} \left( \frac{1 + \frac{3(4 \times 2)}{4^2(1+2)^2} + \frac{(3 \times 5 \times 7) \left( \frac{4 \times 2}{(1+2)^2} \right)^2}{4^2 \times 8^2}}{\sqrt{1+2}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + 4 \log_{0.9898018523255180000} \left( \frac{539}{432 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left( -\frac{1}{2} \right)^k \left( -\frac{1}{2} \right)_k}{k!}} \right)$$

and:

$4 \cdot \log_{\text{base } 0.989801852325518} [(((1/\sqrt{1+2})) * (((1+3/4^2 * ((4*2)/(1+2)^2) + (1*3*5*7)/(4^2*8^2) * ((4*2)/(1+2)^2)^2)))))] + 11 + 1/\text{golden ratio}$

where 11 is a Lucas number

**Input interpretation:**

$$4 \log_{0.989801852325518} \left( \frac{1}{\sqrt{1+2}} \left( 1 + \frac{3}{4^2} \times \frac{4 \times 2}{(1+2)^2} + \frac{3 \times 5 \times 7}{4^2 \times 8^2} \left( \frac{4 \times 2}{(1+2)^2} \right)^2 \right) \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

139.6180339887...

139.6180339887... result practically equal to the rest mass of Pion meson 139.57

**Alternative representation:**

$$4 \log_{0.9898018523255180000} \left( \frac{1 + \frac{3(4 \times 2)}{4^2(1+2)^2} + \frac{(3 \times 5 \times 7) \left( \frac{4 \times 2}{(1+2)^2} \right)^2}{4^2 \times 8^2}}{\sqrt{1+2}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{4 \log \left( \frac{1 + \frac{24}{9 \times 4^2} + \frac{105 \left( \frac{8}{9} \right)^2}{4^2 \times 8^2}}{\sqrt{3}} \right)}{\log(0.9898018523255180000)}$$

**Series representations:**

$$4 \log_{0.9898018523255180000} \left( \frac{1 + \frac{3(4 \times 2)}{4^2(1+2)^2} + \frac{(3 \times 5 \times 7) \left( \frac{4 \times 2}{(1+2)^2} \right)^2}{4^2 \times 8^2}}{\sqrt{1+2}} \right) + 11 + \frac{1}{\phi} =$$

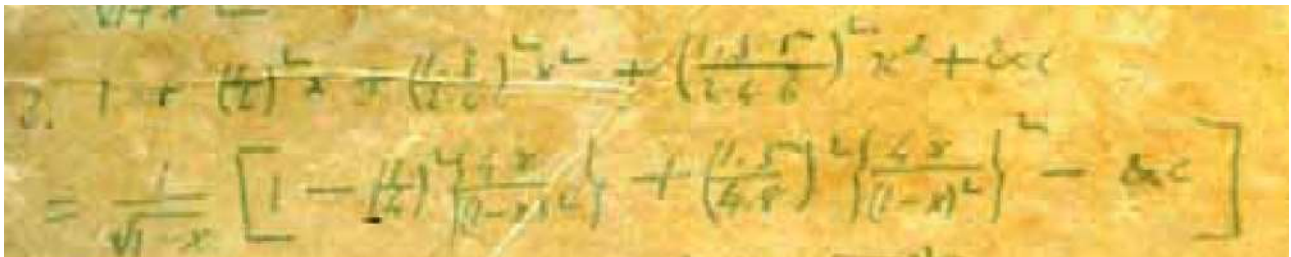
$$11 + \frac{1}{\phi} - \frac{4 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{539}{432\sqrt{3}} \right)^k}{k}}{\log(0.9898018523255180000)}$$

$$4 \log_{0.9898018523255180000} \left( \frac{1 + \frac{3(4 \times 2)}{4^2 (1+2)^2} + \frac{(3 \times 5 \times 7) \left(\frac{4 \times 2}{(1+2)^2}\right)^2}{4^2 \times 8^2}}{\sqrt{1+2}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + 4 \log_{0.9898018523255180000} \left( \frac{539}{432 \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}} \right)$$

$$4 \log_{0.9898018523255180000} \left( \frac{1 + \frac{3(4 \times 2)}{4^2 (1+2)^2} + \frac{(3 \times 5 \times 7) \left(\frac{4 \times 2}{(1+2)^2}\right)^2}{4^2 \times 8^2}}{\sqrt{1+2}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + 4 \log_{0.9898018523255180000} \left( \frac{539}{432 \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \binom{-\frac{1}{2}}{k}}{k!}} \right)$$



$$1/(\text{sqrt}(1-2)) * (((1-(1/4)^2*((4*2)/(1-2)^2)+(((1*5)/(4*8)))^2*((4*2)/(1-2)^2)^2))))$$

**Input:**

$$\frac{1}{\sqrt{1-2}} \left( 1 - \left(\frac{1}{4}\right)^2 \times \frac{4 \times 2}{(1-2)^2} + \left(\frac{1 \times 5}{4 \times 8}\right)^2 \left(\frac{4 \times 2}{(1-2)^2}\right)^2 \right)$$

**Result:**

$$-\frac{33i}{16}$$

**Polar coordinates:**

$$r \approx 2.0625 \text{ (radius), } \theta = -90^\circ \text{ (angle)}$$

2.0625



$$(1+2)/(((1-2)*\text{sqrt}(1-2))) * [1-(3/4)^2*((4*2)/(1-2)^2)+((3*7)/(4*8))^2*(((4*2)/(1-2)^2))^2]$$

**Input:**

$$\frac{1+2}{(1-2)\sqrt{1-2}} \left( 1 - \left(\frac{3}{4}\right)^2 \times \frac{4 \times 2}{(1-2)^2} + \left(\frac{3 \times 7}{4 \times 8}\right)^2 \left(\frac{4 \times 2}{(1-2)^2}\right)^2 \right)$$

**Result:**

$$\frac{1155 i}{16}$$

**Decimal form:**

$$72.1875 i$$

**Polar coordinates:**

$$r \approx 72.1875 \text{ (radius), } \theta = 90^\circ \text{ (angle)}$$

$$72.1875$$

**Alternate form:**

$$\frac{1155 i}{16}$$

**Continued fraction:**

$$[72i; -5i, 3i]$$

(using the Hurwitz expansion)

$$\text{sqrt}(1-2) * (((1+2*(3/4)^2+2^2*((3*7)/(4*8))^2+2^3((3*7*11)/(4*8*12))^2)))$$

**Input:**

$$\sqrt{1-2} \left( 1+2 \left( \frac{3}{4} \right)^2 + 2^2 \left( \frac{3 \times 7}{4 \times 8} \right)^2 + 2^3 \left( \frac{3 \times 7 \times 11}{4 \times 8 \times 12} \right)^2 \right)$$

**Result:**

$$\frac{13809 i}{2048}$$

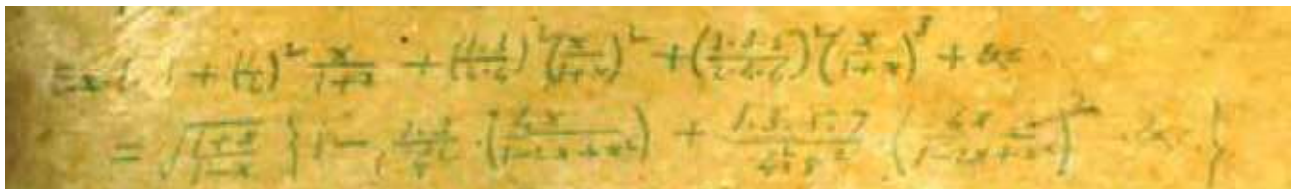
**Decimal form:**

$$6.74267578125 i$$

**Polar coordinates:**

$$r \approx 6.74268 \text{ (radius), } \theta = 90^\circ \text{ (angle)}$$

$$6.74268$$



$$\text{sqrt}(\left(\frac{1+2}{1-2}\right) * \left(\left(\left(\frac{1-(1*3)}{4}\right)^2 * \left(\frac{4*2}{1-2*2+2^2}\right) + \left(\frac{1*3*5*7}{4^2*8^2}\right) * \left(\frac{4*2}{1-2*2+2^2}\right)^2\right)\right))$$

**Input:**

$$\sqrt{\frac{1+2}{1-2} \left( 1 - \frac{1 \times 3}{4^2} \times \frac{4 \times 2}{1 - 2 \times 2 + 2^2} + \frac{3 \times 5 \times 7}{4^2 \times 8^2} \left( \frac{4 \times 2}{1 - 2 \times 2 + 2^2} \right)^2 \right)}$$

**Result:**

$$\frac{97 i \sqrt{3}}{16}$$

**Decimal approximation:**

$$10.50055802088631859201014344537935122459075685122543255758... i$$

$$10.50055802... i$$

**Polar coordinates:**

$$r \approx 10.5006 \text{ (radius), } \theta = 90^\circ \text{ (angle)}$$

$$10.5006$$

**Alternate form:**

$$\frac{1}{16} i \sqrt{3} 97$$



$$2/(1-4)^{1/5} * (((1-1/8 * 2^2/(1-4) + 5^2/(2*4*6*8) * (2^2/(1-4))^2)))$$

**Input:**

$$\frac{2}{\sqrt[5]{1-4}} \left( 1 - \frac{1}{8} \times \frac{2^2}{1-4} + \frac{5^2}{2 \times 4 \times 6 \times 8} \left( \frac{2^2}{1-4} \right)^2 \right)$$

**Result:**

$$-\frac{277(-1)^{4/5}}{108 \sqrt[5]{3}}$$

**Decimal approximation:**

1.66567170054413256023744471976347199183303467998566449844... -  
 1.21018132814032252180795053575087558592044061830491417446... i

**Polar coordinates:**

$r \approx 2.05888$  (radius),  $\theta = -36^\circ$  (angle)

2.05888

**Alternate forms:**

$$-\frac{277}{324} (-3)^{4/5}$$

$$\frac{277}{432 \sqrt[5]{3}} + \frac{277 \sqrt{5}}{432 \sqrt[5]{3}} - \frac{277 i \sqrt{\frac{5}{8} - \frac{\sqrt{5}}{8}}}{108 \sqrt[5]{3}}$$

$$-\frac{277 e^{(4i\pi)/5}}{108 \sqrt[5]{3}}$$

Results

0.72035137753; 2.0625; 72.1875; 6.74268; 10.5006; 2.05888

From the sum of these results, multiplied by 11, subtracting 18 (that are Lucas numbers) and adding the golden ratio conjugate, we obtain:

$$(0.72035137753 + 2.0625 + 72.1875 + 6.74268 + 10.5006 + 2.05888) \times 11 - 18 + 1/\text{golden ratio}$$

**Input interpretation:**

$$(0.72035137753 + 2.0625 + 72.1875 + 6.74268 + 10.5006 + 2.05888) \times 11 - 18 + \frac{1}{\phi}$$

$\phi$  is the golden ratio

**Result:**

1019.62...

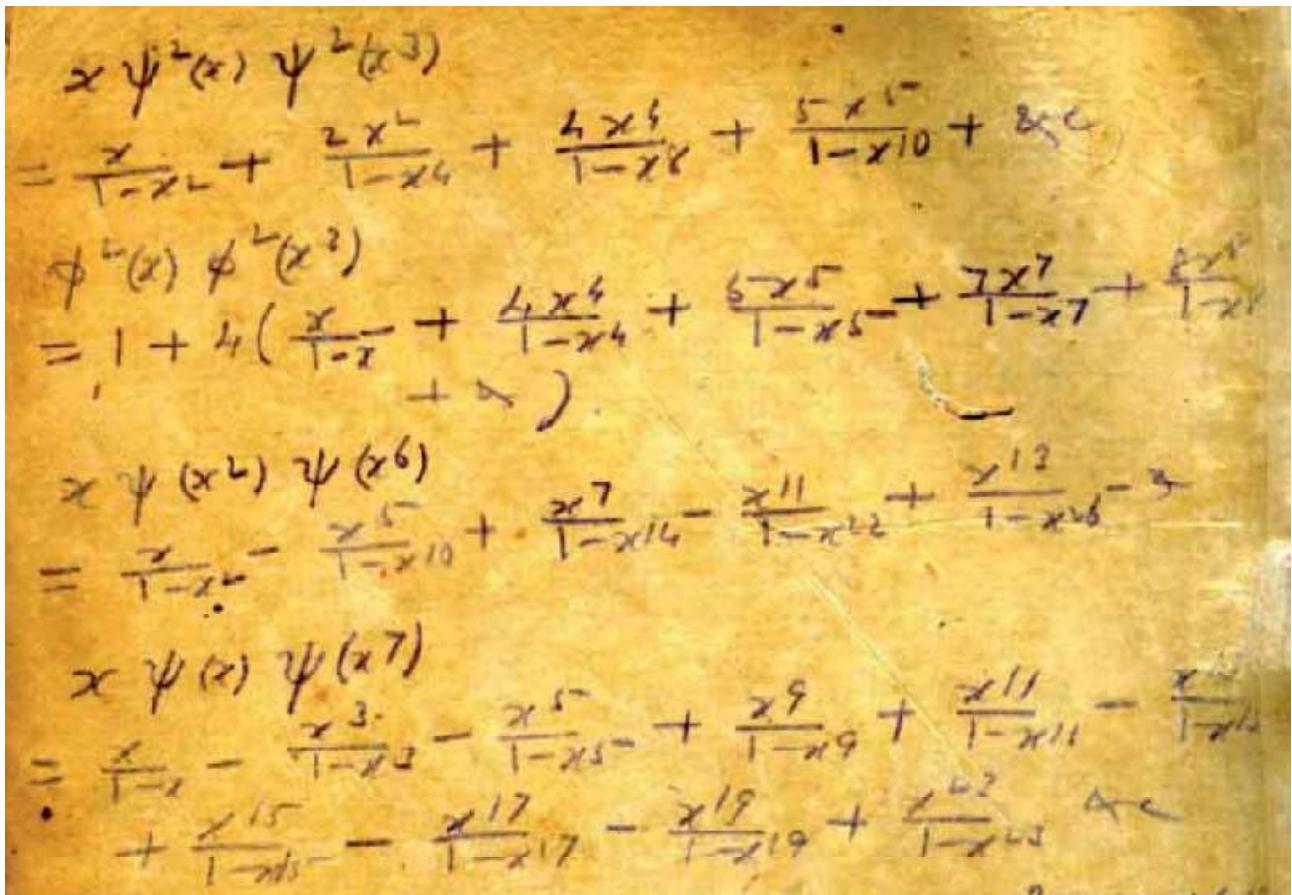
1019.62... result practically equal to the rest mass of Phi meson 1019.445...

**Alternative representations:**

$$(0.720351377530000 + 2.0625 + 72.1875 + 6.74268 + 10.5006 + 2.05888) 11 - 18 + \frac{1}{\phi} = 1019. + \frac{1}{2 \sin(54^\circ)}$$

$$(0.720351377530000 + 2.0625 + 72.1875 + 6.74268 + 10.5006 + 2.05888) 11 - 18 + \frac{1}{\phi} = 1019. + - \frac{1}{2 \cos(216^\circ)}$$

$$(0.720351377530000 + 2.0625 + 72.1875 + 6.74268 + 10.5006 + 2.05888) 11 - 18 + \frac{1}{\phi} = 1019. + - \frac{1}{2 \sin(666^\circ)}$$

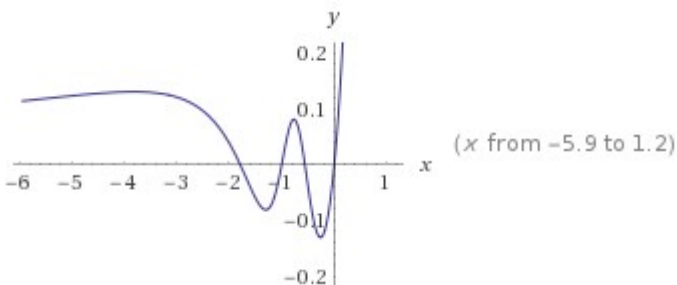


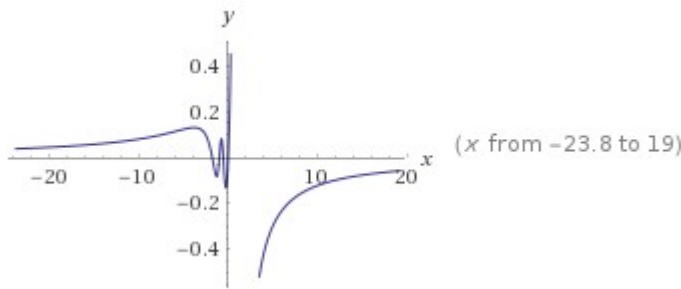
$$x/(1-x^2) + (2x^2)/(1-x^4) + (4x^4)/(1-x^8) + (5x^5)/(1-x^{10}) \dots$$

**Input:**

$$\frac{x}{1-x^2} + \frac{2x^2}{1-x^4} + \frac{4x^4}{1-x^8} + \frac{5x^5}{1-x^{10}}$$

**Plots:**





### Alternate forms:

$$x \left( -\frac{2x}{x^4-1} + \frac{1}{1-x^2} - \frac{5x^4}{x^{10}-1} - \frac{4x^3}{x^8-1} \right)$$

$$= \frac{x(x+1)(x^{10} + 4x^7 - x^6 + 2x^5 - x^4 + 4x^3 + 1)}{(x-1)(x^4+1)(x^4-x^3+x^2-x+1)(x^4+x^3+x^2+x+1)}$$

(ignoring removable singularities)

### Partial fraction expansion:

$$-\frac{2}{x^4+1} + \frac{x^3-2x^2+3x-4}{2(x^4-x^3+x^2-x+1)} + \frac{x^3+2x^2+3x+4}{2(x^4+x^3+x^2+x+1)} - \frac{2}{x-1}$$

(ignoring removable singularities)

### Series expansion at $x = 0$ :

$$x + 2x^2 + x^3 + 4x^4 + 6x^5 + O(x^6)$$

(Taylor series)

### Series expansion at $x = \infty$ :

$$-\frac{1}{x} - \frac{2}{x^2} - \left(\frac{1}{x}\right)^3 - \frac{4}{x^4} + O\left(\left(\frac{1}{x}\right)^5\right)$$

(Laurent series)

### Derivative:

$$\frac{d}{dx} \left( \frac{x}{1-x^2} + \frac{2x^2}{1-x^4} + \frac{4x^4}{1-x^8} + \frac{5x^5}{1-x^{10}} \right) =$$

$$-\frac{4x}{x^4-1} + \frac{2x^2}{(1-x^2)^2} + \frac{1}{1-x^2} + \frac{50x^{14}}{(1-x^{10})^2} + \frac{32x^{11}}{(1-x^8)^2} - \frac{25x^4}{x^{10}-1} - \frac{16x^3}{x^8-1} + \frac{8x^5}{(1-x^4)^2}$$

**Indefinite integral:**

$$\int \left( \frac{x}{1-x^2} + \frac{2x^2}{1-x^4} + \frac{4x^4}{1-x^8} + \frac{5x^5}{1-x^{10}} \right) dx =$$

$$\frac{1}{8} \left( -4 \log(1-x^2) + 2\sqrt{2} \log(x^2 - \sqrt{2}x + 1) - 2\sqrt{2} \log(x^2 + \sqrt{2}x + 1) + \right.$$

$$(1 + \sqrt{5}) \log(2x^2 + (-1 - \sqrt{5})x + 2) - (\sqrt{5} - 1) \log(2x^2 + (1 - \sqrt{5})x + 2) -$$

$$(\sqrt{5} - 1) \log(2x^2 + (\sqrt{5} - 1)x + 2) + (1 + \sqrt{5}) \log(2x^2 + (1 + \sqrt{5})x + 2) -$$

$$12 \log(1-x) + 4 \log(x+1) + 2\sqrt{10-2\sqrt{5}} \tan^{-1} \left( \frac{-4x + \sqrt{5} + 1}{\sqrt{10-2\sqrt{5}}} \right) +$$

$$2\sqrt{2(5+\sqrt{5})} \tan^{-1} \left( \frac{4x - \sqrt{5} + 1}{\sqrt{2(5+\sqrt{5})}} \right) - 2\sqrt{2(5+\sqrt{5})}$$

$$\tan^{-1} \left( \frac{4x + \sqrt{5} - 1}{\sqrt{2(5+\sqrt{5})}} \right) + 2\sqrt{10-2\sqrt{5}} \tan^{-1} \left( \frac{4x + \sqrt{5} + 1}{\sqrt{10-2\sqrt{5}}} \right) +$$

$$\left. 4\sqrt{2} \tan^{-1}(1 - \sqrt{2}x) - 4\sqrt{2} \tan^{-1}(\sqrt{2}x + 1) \right) + \text{constant}$$

(assuming a complex-valued logarithm)

Or:

**Indefinite integral:**

$$\int \left( \frac{x}{1-x^2} + \frac{2x^2}{1-x^4} + \frac{4x^4}{1-x^8} + \frac{5x^5}{1-x^{10}} \right) dx \approx$$

$$0.125 (4.70228 \tan^{-1}(0.425325 (3.23607 - 4x)) - 4 \log(1-x^2) +$$

$$2.82843 \log(x^2 - 1.41421x + 1) - 2.82843 \log(x^2 + 1.41421x + 1) +$$

$$3.23607 \log(2x^2 - 3.23607x + 2) - 1.23607 \log(2x^2 - 1.23607x + 2) -$$

$$1.23607 \log(2x^2 + 1.23607x + 2) + 3.23607 \log(2x^2 + 3.23607x + 2) -$$

$$12 \log(1-x) + 4 \log(x+1) + 5.65685 \tan^{-1}(1 - 1.41421x) -$$

$$5.65685 \tan^{-1}(1.41421x + 1) + 7.60845 \tan^{-1}(0.262866 (4x - 1.23607)) -$$

$$7.60845 \tan^{-1}(0.262866 (4x + 1.23607)) +$$

$$4.70228 \tan^{-1}(0.425325 (4x + 3.23607))) + \text{constant}$$

(assuming a complex-valued logarithm)

For  $x = 1$ , we obtain:

$$7.60845 \tan^{-1}(0.262866 (3.23607)) - 7.60845 \tan^{-1}(0.262866 (5.23607)) + 4.70228 \tan^{-1}(0.425325 (7.23607))$$

**Input interpretation:**

$$7.60845 \tan^{-1}(0.262866 \times 3.23607) + \tan^{-1}(0.262866 \times 5.23607) \times (-7.60845) + 4.70228 \tan^{-1}(0.425325 \times 7.23607)$$

$\tan^{-1}(x)$  is the inverse tangent function

**Result:**

4.10125...

(result in radians)

4.10125...

**Alternative representations:**

$$7.60845 \tan^{-1}(0.262866 \times 3.23607) - 7.60845 \tan^{-1}(0.262866 \times 5.23607) + 4.70228 \tan^{-1}(0.425325 \times 7.23607) = 7.60845 \operatorname{sc}^{-1}(0.850653 | 0) - 7.60845 \operatorname{sc}^{-1}(1.37638 | 0) + 4.70228 \operatorname{sc}^{-1}(3.07768 | 0)$$

$$7.60845 \tan^{-1}(0.262866 \times 3.23607) - 7.60845 \tan^{-1}(0.262866 \times 5.23607) + 4.70228 \tan^{-1}(0.425325 \times 7.23607) = 7.60845 \cot^{-1}\left(\frac{1}{0.850653}\right) - 7.60845 \cot^{-1}\left(\frac{1}{1.37638}\right) + 4.70228 \cot^{-1}\left(\frac{1}{3.07768}\right)$$

$$7.60845 \tan^{-1}(0.262866 \times 3.23607) - 7.60845 \tan^{-1}(0.262866 \times 5.23607) + 4.70228 \tan^{-1}(0.425325 \times 7.23607) = 7.60845 \tan^{-1}(1, 0.850653) - 7.60845 \tan^{-1}(1, 1.37638) + 4.70228 \tan^{-1}(1, 3.07768)$$

**Series representations:**



$$\begin{aligned}
& 7.60845 \tan^{-1}(0.262866 \times 3.23607) - \\
& 7.60845 \tan^{-1}(0.262866 \times 5.23607) + 4.70228 \tan^{-1}(0.425325 \times 7.23607) = \\
& \sum_{k=0}^{\infty} \left( \frac{7.60845 \left(-\frac{1}{5}\right)^k 1.70131^{1+2k} F_{1+2k} \left(\frac{1}{1+\sqrt{1.57889}}\right)^{1+2k}}{1+2k} - \right. \\
& \left. \frac{7.60845 \left(-\frac{1}{5}\right)^k 2.75277^{1+2k} F_{1+2k} \left(\frac{1}{1+\sqrt{2.51555}}\right)^{1+2k}}{1+2k} + \right. \\
& \left. \frac{4.70228 \left(-\frac{1}{5}\right)^k 6.15536^{1+2k} F_{1+2k} \left(\frac{1}{1+\sqrt{8.5777}}\right)^{1+2k}}{1+2k} \right)
\end{aligned}$$

$$\begin{aligned}
& 7.60845 \tan^{-1}(0.262866 \times 3.23607) - 7.60845 \tan^{-1}(0.262866 \times 5.23607) + \\
& 4.70228 \tan^{-1}(0.425325 \times 7.23607) = 0 + 4.70228 \tan^{-1}(z_0) + \\
& \sum_{k=1}^{\infty} \frac{1}{k} i \left( 3.80423 (0.850653 - z_0)^k - 3.80423 (1.37638 - z_0)^k + \right. \\
& \left. 2.35114 (3.07768 - z_0)^k \right) \left( -(-i - z_0)^{-k} + (i - z_0)^{-k} \right) \\
& \text{for } (i z_0 \notin \mathbb{R} \text{ or } (\text{not } (1 \leq i z_0 < \infty) \text{ and } \text{not } (-\infty < i z_0 \leq -1)))
\end{aligned}$$

$$\begin{aligned}
& 7.60845 \tan^{-1}(0.262866 \times 3.23607) - \\
& 7.60845 \tan^{-1}(0.262866 \times 5.23607) + 4.70228 \tan^{-1}(0.425325 \times 7.23607) = \\
& 0 + 4.70228 \tan^{-1}(x) + 7.60845 \pi \left[ \frac{\arg(i(0.850653 - x))}{2\pi} \right] - \\
& 7.60845 \pi \left[ \frac{\arg(i(1.37638 - x))}{2\pi} \right] + 4.70228 \pi \left[ \frac{\arg(i(3.07768 - x))}{2\pi} \right] + \\
& \sum_{k=1}^{\infty} \frac{1}{k} i \left( 3.80423 (0.850653 - x)^k - 3.80423 (1.37638 - x)^k + \right. \\
& \left. 2.35114 (3.07768 - x)^k \right) \left( -(-i - x)^{-k} + (i - x)^{-k} \right) \text{ for } (ix \in \mathbb{R} \text{ and } ix < -1)
\end{aligned}$$

### Integral representations:

$$\begin{aligned}
& 7.60845 \tan^{-1}(0.262866 \times 3.23607) - \\
& 7.60845 \tan^{-1}(0.262866 \times 5.23607) + 4.70228 \tan^{-1}(0.425325 \times 7.23607) = \\
& \int_0^1 \frac{0.806497 + 0.360688 t^2 + 4.94426 t^4}{0.0770138 + 0.931109 t^2 + 2.01539 t^4 + t^6} dt
\end{aligned}$$

$$\begin{aligned}
& 7.60845 \tan^{-1}(0.262866 \times 3.23607) - \\
& 7.60845 \tan^{-1}(0.262866 \times 5.23607) + 4.70228 \tan^{-1}(0.425325 \times 7.23607) = \\
& \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{\pi^{3/2}} e^{-3.95593s} \left( -3.61803 e^{1.60721s} + 2.61804 e^{2.89314s} - 1.61804 e^{3.41151s} \right) \\
& i \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s)^2 ds \text{ for } 0 < \gamma < \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
& 7.60845 \tan^{-1}(0.262866 \times 3.23607) - 7.60845 \tan^{-1}(0.262866 \times 5.23607) + \\
& 4.70228 \tan^{-1}(0.425325 \times 7.23607) = \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{i\pi \Gamma\left(\frac{3}{2} - s\right)} e^{-2.88727s} \\
& \left( 3.61803 e^{0.638921s} - 2.61804 e^{2.24835s} + 1.61804 e^{3.21078s} \right) \\
& \Gamma\left(\frac{1}{2} - s\right) \Gamma(1-s) \Gamma(s) ds \text{ for } 0 < \gamma < \frac{1}{2}
\end{aligned}$$

### Continued fraction representations:

$$\begin{aligned}
& 7.60845 \tan^{-1}(0.262866 \times 3.23607) - \\
& \frac{7.60845 \tan^{-1}(0.262866 \times 5.23607) + 4.70228 \tan^{-1}(0.425325 \times 7.23607)}{6.47215} = \\
& \frac{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{0.72361 k^2}{1+2k}}{6.47215} - \frac{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{1.89444 k^2}{1+2k}}{10.4722} + \frac{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{9.47212 k^2}{1+2k}}{14.4721} = \\
& \frac{1 + \frac{0.72361}{3 + \frac{2.89444}{5 + \frac{6.51249}{7 + \frac{11.5778}{9 + \dots}}}}}{6.47215} - \frac{1 + \frac{1.89444}{3 + \frac{7.57774}{5 + \frac{17.0499}{7 + \frac{30.311}{9 + \dots}}}}}{10.4722} + \frac{1 + \frac{9.47212}{3 + \frac{37.8885}{5 + \frac{85.2491}{7 + \frac{151.554}{9 + \dots}}}}}{14.4721}
\end{aligned}$$

$$\begin{aligned}
& 7.60845 \tan^{-1}(0.262866 \times 3.23607) - \\
& \frac{7.60845 \tan^{-1}(0.262866 \times 5.23607) + 4.70228 \tan^{-1}(0.425325 \times 7.23607)}{6.47215} = \\
& \frac{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{0.72361(1-2k)^2}{1.72361+0.55278k}}{6.47215} - \frac{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{1.89444(1-2k)^2}{2.89444-1.78887k}}{10.4722} + \frac{1 + \mathop{\text{K}}_{k=1}^{\infty} \frac{9.47212(1-2k)^2}{10.4721-16.9442k}}{14.4721} = \\
& \frac{1 + \frac{0.72361}{2.27639 + \frac{6.51249}{2.82917 + \frac{18.0903}{3.38195 + \frac{35.4569}{3.93473 + \dots}}}}}{6.47215} - \\
& \frac{1 + \frac{1.89444}{1.10556 + \frac{17.0499}{-0.683305 + \frac{47.3609}{-2.47218 + \frac{92.8273}{-4.26105 + \dots}}}}}{10.4722} + \\
& \frac{1 + \frac{9.47212}{-6.47212 + \frac{85.2491}{-23.4164 + \frac{236.803}{-40.3606 + \frac{464.134}{-57.3049 + \dots}}}}}{14.4721}
\end{aligned}$$

$$\begin{aligned}
& 7.60845 \tan^{-1}(0.262866 \times 3.23607) - 7.60845 \tan^{-1}(0.262866 \times 5.23607) + \\
& 4.70228 \tan^{-1}(0.425325 \times 7.23607) = 10.4721 - \frac{4.68331}{3 + \sum_{k=1}^{\infty} \frac{0.72361(1+(-1)^{1+k+k})^2}{3+2k}} + \\
& \frac{19.8388}{3 + \sum_{k=1}^{\infty} \frac{1.89444(1+(-1)^{1+k+k})^2}{3+2k}} - \frac{137.082}{3 + \sum_{k=1}^{\infty} \frac{9.47212(1+(-1)^{1+k+k})^2}{3+2k}} = \\
& 10.4721 - \frac{4.68331}{3 + \frac{6.51249}{5 + \frac{2.89444}{7 + \frac{18.0903}{9 + \frac{11.5778}{11 + \dots}}}}} + \frac{19.8388}{3 + \frac{17.0499}{5 + \frac{7.57774}{7 + \frac{47.3609}{9 + \frac{30.311}{11 + \dots}}}}} - \frac{137.082}{3 + \frac{85.2491}{5 + \frac{37.8885}{7 + \frac{236.803}{9 + \frac{151.554}{11 + \dots}}}}}
\end{aligned}$$

$$\begin{aligned}
& 7.60845 \tan^{-1}(0.262866 \times 3.23607) - 7.60845 \tan^{-1}(0.262866 \times 5.23607) + \\
& 4.70228 \tan^{-1}(0.425325 \times 7.23607) = \frac{6.47215}{1.72361 + \sum_{k=1}^{\infty} \frac{1.44722(1-2\lfloor \frac{1+k}{2} \rfloor)\lfloor \frac{1+k}{2} \rfloor}{(1.36181+0.361805(-1)^k)(1+2k)}} - \\
& \frac{10.4722}{2.89444 + \sum_{k=1}^{\infty} \frac{3.78887(1-2\lfloor \frac{1+k}{2} \rfloor)\lfloor \frac{1+k}{2} \rfloor}{(1.94722+0.947218(-1)^k)(1+2k)}} + \\
& \frac{14.4721}{10.4721 + \sum_{k=1}^{\infty} \frac{18.9442(1-2\lfloor \frac{1+k}{2} \rfloor)\lfloor \frac{1+k}{2} \rfloor}{(5.73606+4.73606(-1)^k)(1+2k)}} = \\
& 1.72361 + - \frac{1.44722}{3 - \frac{8.61805 - \frac{8.68332}{7 - \frac{8.68332}{15.5125 + \dots}}}}{1.44722} - \\
& \frac{10.4722}{2.89444 + - \frac{3.78887}{3 - \frac{14.4722 - \frac{22.7332}{7 - \frac{22.7332}{26.0499 + \dots}}}}{3.78887}} + \\
& \frac{14.4721}{10.4721 + - \frac{18.9442}{3 - \frac{52.3606 - \frac{113.665}{7 - \frac{113.665}{94.2491 + \dots}}}}{18.9442}}
\end{aligned}$$

For x = 2, also

$$\begin{aligned}
& 1/8 [(-4 \log(1 - 4) + 2 \sqrt{2} \log(4 - \sqrt{2})^2 + 1) - 2 \sqrt{2} \log(4 + \sqrt{2})^2 + 1) + \\
& (1 + \sqrt{5}) \log(8 + (-1 - \sqrt{5})^2 + 2) - (\sqrt{5} - 1) \log(8 + (1 - \sqrt{5})^2 + 2) - \\
& (\sqrt{5} - 1) \log(8 + (\sqrt{5} - 1)^2 + 2) + (1 + \sqrt{5}) \log(8 + (1 + \sqrt{5})^2 + 2) -
\end{aligned}$$

$$12 \log(-1) + 4 \log(3) + 2 \sqrt{10 - 2 \sqrt{5}} \tan^{-1}\left(\frac{-8 + \sqrt{5} + 1}{\sqrt{10 - 2 \sqrt{5}}}\right) + 2 \sqrt{2(5 + \sqrt{5})} \tan^{-1}\left(\frac{8 - \sqrt{5} + 1}{\sqrt{2(5 + \sqrt{5})}}\right) - 2 \sqrt{2(5 + \sqrt{5})} \tan^{-1}\left(\frac{8 + \sqrt{5} - 1}{\sqrt{2(5 + \sqrt{5})}}\right) + 2 \sqrt{10 - 2 \sqrt{5}} \tan^{-1}\left(\frac{8 + \sqrt{5} + 1}{\sqrt{10 - 2 \sqrt{5}}}\right) + 4 \sqrt{2} \tan^{-1}(1 - \sqrt{2}) - 4 \sqrt{2} \tan^{-1}(\sqrt{2} + 1)]$$

a)

$$(-4 \log(1 - 4) + 2 \sqrt{2} \log(4 - \sqrt{2} + 1) - 2 \sqrt{2} \log(4 + \sqrt{2} + 1) + (1 + \sqrt{5}) \log(8 + (-1 - \sqrt{5}) + 2))$$

$$-4 \log(1 - 4) + 2 \sqrt{2} \log(4 - \sqrt{2} + 1) - 2 \sqrt{2} \log(4 + \sqrt{2} + 1) + (1 + \sqrt{5}) \log(8 + (-1 - \sqrt{5}) + 2)$$

$$-4(\log(3) + i\pi) + 2 \sqrt{2} \log(5 - 2\sqrt{2}) - 2 \sqrt{2} \log(5 + 2\sqrt{2}) + (1 + \sqrt{5}) \log(10 + 2(-1 - \sqrt{5}))$$

$$-3.9416822431930270659401964932975393089516331352147330489... - 12.566370614359172953850573533118011536788677597500423283... i$$

$r \approx 13.1701$  (radius),  $\theta \approx -107.415^\circ$  (angle)

13.1701

b)

$$-(\sqrt{5} - 1) \log(8 + (1 - \sqrt{5}) + 2) - (\sqrt{5} - 1) \log(8 + (\sqrt{5} - 1) + 2) + (1 + \sqrt{5}) \log(8 + (1 + \sqrt{5}) + 2)$$

$$-(\sqrt{5} - 1) \log(8 + (1 - \sqrt{5}) + 2) - (\sqrt{5} - 1) \log(8 + (\sqrt{5} - 1) + 2) + (1 + \sqrt{5}) \log(8 + (1 + \sqrt{5}) + 2)$$

$$-(\sqrt{5} - 1) \log(10 + 2(1 - \sqrt{5})) - (\sqrt{5} - 1) \log(10 + 2(\sqrt{5} - 1)) + (1 + \sqrt{5}) \log(10 + 2(1 + \sqrt{5}))$$

3.452040581111875270859068819714559114804957185053154150551...

3.452040581...

c)

$$-12 \log(-1) + 4 \log(3) + 2 \sqrt{10 - 2 \sqrt{5}} \tan^{-1}\left(\frac{-8 + \sqrt{5} + 1}{\sqrt{10 - 2 \sqrt{5}}}\right) + 2 \sqrt{2(5 + \sqrt{5})} \tan^{-1}\left(\frac{8 - \sqrt{5} + 1}{\sqrt{2(5 + \sqrt{5})}}\right)$$

$$-12 \log(-1) + 4 \log(3) + 2 \sqrt{10 - 2 \sqrt{5}} \tan^{-1}\left(\frac{-8 + \sqrt{5} + 1}{\sqrt{10 - 2 \sqrt{5}}}\right) +$$

$$2 \sqrt{2(5 + \sqrt{5})} \tan^{-1}\left(\frac{8 - \sqrt{5} + 1}{\sqrt{2(5 + \sqrt{5})}}\right)$$

$$-12 i \pi + 4 \log(3) + 2 \sqrt{10 - 2 \sqrt{5}} \tan^{-1}\left(\frac{\sqrt{5} - 7}{\sqrt{10 - 2 \sqrt{5}}}\right) +$$

$$2 \sqrt{2(5 + \sqrt{5})} \tan^{-1}\left(\frac{9 - \sqrt{5}}{\sqrt{2(5 + \sqrt{5})}}\right)$$

(result in radians)

7.21717284178781349901818172918281695482839113281501969193... -  
37.6991118430775188615517205993540346103660327925012698516... *i*

(result in radians)

$r \approx 38.3837$  (radius),  $\theta \approx -79.1623^\circ$  (angle)

38.3837

d)

$$-2 \sqrt{2(5 + \sqrt{5})} \tan^{-1}\left(\frac{8 + \sqrt{5} - 1}{\sqrt{2(5 + \sqrt{5})}}\right) + 2 \sqrt{10 - 2 \sqrt{5}} \tan^{-1}\left(\frac{8 + \sqrt{5} + 1}{\sqrt{10 - 2 \sqrt{5}}}\right)$$

$$-2\sqrt{2(5+\sqrt{5})} \tan^{-1}\left(\frac{8+\sqrt{5}-1}{\sqrt{2(5+\sqrt{5})}}\right) + 2\sqrt{10-2\sqrt{5}} \tan^{-1}\left(\frac{8+\sqrt{5}+1}{\sqrt{10-2\sqrt{5}}}\right)$$

$$2\sqrt{10-2\sqrt{5}} \tan^{-1}\left(\frac{9+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right) - 2\sqrt{2(5+\sqrt{5})} \tan^{-1}\left(\frac{7+\sqrt{5}}{\sqrt{2(5+\sqrt{5})}}\right)$$

(result in radians)

-2.56223886262039834557339289027171407738727835210221986001...

(result in radians)

-2.562238862...

e)

$$4\sqrt{2} \tan^{-1}(1 - \sqrt{2} \times 2) - 4\sqrt{2} \tan^{-1}(\sqrt{2} \times 2 + 1)$$

$$4\sqrt{2} \tan^{-1}(1 - \sqrt{2} \times 2) - 4\sqrt{2} \tan^{-1}(\sqrt{2} \times 2 + 1)$$

$$4\sqrt{2} \tan^{-1}(1 - 2\sqrt{2}) - 4\sqrt{2} \tan^{-1}(1 + 2\sqrt{2})$$

(result in radians)

-13.4951229807907334893190893660401176478898557291179671568...

(result in radians)

-13.49512298...

Thence, the final result of integral is the following:

$$\frac{1}{8}(13.1701 + 3.452040581 + 38.3837 - 2.562238862 - 13.49512298)$$

**Input interpretation:**

$$\frac{1}{8}(13.1701 + 3.452040581 + 38.3837 - 2.562238862 - 13.49512298)$$

**Result:**

4.868559842375

4.868559842375

From:

$$\int \left( \frac{x}{1-x^2} + \frac{2x^2}{1-x^4} + \frac{4x^4}{1-x^8} + \frac{5x^5}{1-x^{10}} \right) dx =$$

$$\frac{1}{8} \left( -4 \log(1-x^2) + 2\sqrt{2} \log(x^2 - \sqrt{2}x + 1) - 2\sqrt{2} \log(x^2 + \sqrt{2}x + 1) + \right.$$

$$(1 + \sqrt{5}) \log(2x^2 + (-1 - \sqrt{5})x + 2) - (\sqrt{5} - 1) \log(2x^2 + (1 - \sqrt{5})x + 2) -$$

$$(\sqrt{5} - 1) \log(2x^2 + (\sqrt{5} - 1)x + 2) + (1 + \sqrt{5}) \log(2x^2 + (1 + \sqrt{5})x + 2) -$$

$$12 \log(1-x) + 4 \log(x+1) + 2\sqrt{10-2\sqrt{5}} \tan^{-1} \left( \frac{-4x + \sqrt{5} + 1}{\sqrt{10-2\sqrt{5}}} \right) +$$

$$2\sqrt{2(5+\sqrt{5})} \tan^{-1} \left( \frac{4x - \sqrt{5} + 1}{\sqrt{2(5+\sqrt{5})}} \right) - 2\sqrt{2(5+\sqrt{5})}$$

$$\tan^{-1} \left( \frac{4x + \sqrt{5} - 1}{\sqrt{2(5+\sqrt{5})}} \right) + 2\sqrt{10-2\sqrt{5}} \tan^{-1} \left( \frac{4x + \sqrt{5} + 1}{\sqrt{10-2\sqrt{5}}} \right) +$$

$$\left. 4\sqrt{2} \tan^{-1}(1 - \sqrt{2}x) - 4\sqrt{2} \tan^{-1}(\sqrt{2}x + 1) \right) + \text{constant}$$

(assuming a complex-valued logarithm)

Multiplying by 1/3 all the integral, we obtain:

$$\frac{1}{3} * [\text{integral}(x/(1-x^2) + (2x^2)/(1-x^4) + (4x^4)/(1-x^8) + (5x^5)/(1-x^{10}))] dx =$$

$$\frac{1}{24} [(-4 \log(1-x^2) + 2\sqrt{2} \log(x^2 - \sqrt{2}x + 1) - 2\sqrt{2} \log(x^2 + \sqrt{2}x + 1) + (1 + \sqrt{5}) \log(2x^2 + (-1 - \sqrt{5})x + 2) - (\sqrt{5} - 1) \log(2x^2 + (1 - \sqrt{5})x + 2) - (\sqrt{5} - 1) \log(2x^2 + (\sqrt{5} - 1)x + 2) + (1 + \sqrt{5}) \log(2x^2 + (1 + \sqrt{5})x + 2) - 12 \log(1-x) + 4 \log(x+1) + 2\sqrt{10-2\sqrt{5}} \tan^{-1}((-4x + \sqrt{5} + 1)/\sqrt{10-2\sqrt{5}}) + 2\sqrt{2(5+\sqrt{5})} \tan^{-1}((4x - \sqrt{5} + 1)/\sqrt{2(5+\sqrt{5})}) - 2\sqrt{2(5+\sqrt{5})} \tan^{-1}((4x + \sqrt{5} - 1)/\sqrt{2(5+\sqrt{5})}) + 2\sqrt{10-2\sqrt{5}} \tan^{-1}((4x + \sqrt{5} + 1)/\sqrt{10-2\sqrt{5}}) + 4\sqrt{2} \tan^{-1}(1 - \sqrt{2}x) - 4\sqrt{2} \tan^{-1}(\sqrt{2}x + 1)]$$

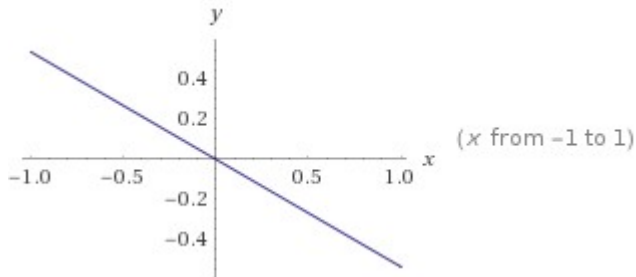




**Indefinite integral:**

$$\frac{1}{3} \int \left( \frac{2}{1-2^2} + \frac{2 \times 2^2}{1-2^4} + \frac{4 \times 2^4}{1-2^8} + \frac{5 \times 2^5}{1-2^{10}} \right) dx = -\frac{3106 x}{5797} + \text{constant}$$

**Plot:**



For  $x = 2$  and constant = 0.551264527988492

**Input interpretation:**

$$\frac{3106 \times 2}{5797} + 0.551264527988492$$

**Result:**

1.622853280791666055545972054510953941693979644643781266172...

1.622853280791666....

Or:

$$(3106 \times 2) / 5797 + 0.551285598$$

Where 0.551285598 is the mathematical constant concerning the vertex solid angle

**Input interpretation:**

$$\frac{3106 \times 2}{5797} + 0.551285598$$

**Result:**

1.622874350803174055545972054510953941693979644643781266172...

1.622874350803....

We have that:

$$x/(1-x^2)+(2x^2)/(1-x^4)+(4x^4)/(1-x^8)+(5x^5)/(1-x^{10})...$$

for  $x = 2$ , we obtain:

$$2/(1-2^2)+(2*2^2)/(1-2^4)+(4*2^4)/(1-2^8)+(5*2^5)/(1-2^{10})$$

**Input:**

$$\frac{2}{1-2^2} + \frac{2 \times 2^2}{1-2^4} + \frac{4 \times 2^4}{1-2^8} + \frac{5 \times 2^5}{1-2^{10}}$$

**Exact result:**

$$-\frac{9318}{5797}$$

**Decimal approximation:**

$$-1.60738312920476108331895808176643091254096946696567189925...$$

$$-1.6073831292....$$

$$-1/(((2/(1-2^2)+(2*2^2)/(1-2^4)+(4*2^4)/(1-2^8)+(5*2^5)/(1-2^{10}))))$$

**Input:**

$$-\frac{1}{\frac{2}{1-2^2} + \frac{2 \times 2^2}{1-2^4} + \frac{4 \times 2^4}{1-2^8} + \frac{5 \times 2^5}{1-2^{10}}}$$

**Exact result:**

$$\frac{5797}{9318}$$

**Decimal approximation:**

$$0.622129212277312728053230306932818201330757673320455033268...$$

$$0.622129212...$$

$$(((1/(((2/(1-2^2)+(2*2^2)/(1-2^4)+(4*2^4)/(1-2^8)+(5*2^5)/(1-2^{10})))))))^{1/64}$$

**Input:**

$$\sqrt[64]{-\frac{1}{\frac{2}{1-2^2} + \frac{2 \times 2^2}{1-2^4} + \frac{4 \times 2^4}{1-2^8} + \frac{5 \times 2^5}{1-2^{10}}}}$$

**Result:**

$$\sqrt[64]{\frac{5797}{9318}}$$

**Decimal approximation:**

0.992611687034350897401848108483231712689657415351271870065...

0.992611687... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 =  $\phi$**

**Alternate form:**

$$\frac{\sqrt[64]{5797} \cdot 9318^{63/64}}{9318}$$

2\*log base 0.99261168703435 (((-1/(((2/(1-2^2)+(2\*2^2)/(1-2^4)+(4\*2^4)/(1-2^8)+(5\*2^5)/(1-2^10)))))))+11+1/golden ratio

**Input interpretation:**

$$2 \log_{0.99261168703435} \left( -\frac{1}{\frac{2}{1-2^2} + \frac{2 \times 2^2}{1-2^4} + \frac{4 \times 2^4}{1-2^8} + \frac{5 \times 2^5}{1-2^{10}}} \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

139.618033989...

139.618033989... result practically equal to the rest mass of Pion meson 139.57

**Alternative representation:**

$$2 \log_{0.992611687034350000} \left( -\frac{1}{\frac{2}{1-2^2} + \frac{2 \times 2^2}{1-2^4} + \frac{4 \times 2^4}{1-2^8} + \frac{5 \times 2^5}{1-2^{10}}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{2 \log \left( -\frac{1}{-\frac{2}{3} + \frac{8}{1-2^4} + \frac{4 \times 2^4}{1-2^8} + \frac{5 \times 2^5}{1-2^{10}}} \right)}{\log(0.992611687034350000)}$$

**Series representations:**

$$2 \log_{0.992611687034350000} \left( -\frac{1}{\frac{2}{1-2^2} + \frac{2 \times 2^2}{1-2^4} + \frac{4 \times 2^4}{1-2^8} + \frac{5 \times 2^5}{1-2^{10}}} \right) + 11 + \frac{1}{\phi} =$$

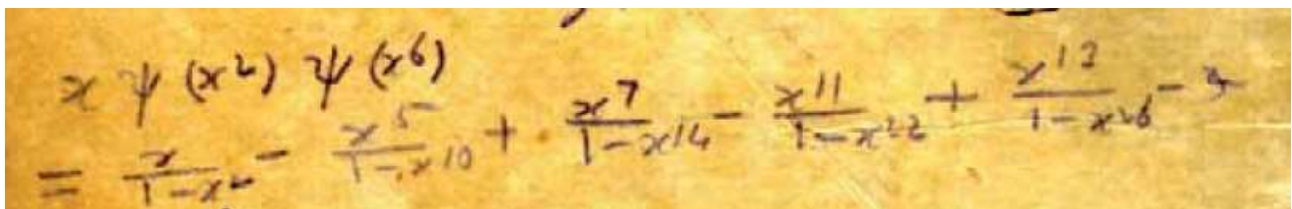
$$11 + \frac{1}{\phi} - \frac{2 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -\frac{3521}{9318} \right)^k}{k}}{\log(0.992611687034350000)}$$

$$2 \log_{0.992611687034350000} \left( -\frac{1}{\frac{2}{1-2^2} + \frac{2 \times 2^2}{1-2^4} + \frac{4 \times 2^4}{1-2^8} + \frac{5 \times 2^5}{1-2^{10}}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 269.6977911328972 \log \left( \frac{5797}{9318} \right) -$$

$$2 \log \left( \frac{5797}{9318} \right) \sum_{k=0}^{\infty} (-0.007388312965650000)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$



For  $x = 2$ , we obtain:

$$2/(1-2^2) - 2^5/(1-2^{10}) + 2^7/(1-2^{14}) - 2^{11}/(1-2^{22}) + 2^{13}/(1-2^{26})$$

**Input:**

$$\frac{2}{1-2^2} - \frac{2^5}{1-2^{10}} + \frac{2^7}{1-2^{14}} - \frac{2^{11}}{1-2^{22}} + \frac{2^{13}}{1-2^{26}}$$

**Exact result:**

$$\frac{112\,316\,372\,440\,473\,074\,242}{174\,720\,950\,108\,243\,807\,763}$$

**Decimal approximation:**

-0.64283288507125444891761669607510143990132564600064508054...

**-0.64283288507125...**

**Alternate form:**

$$\frac{112\,316\,372\,440\,473\,074\,242}{174\,720\,950\,108\,243\,807\,763}$$

$$\left(\left(\frac{-2}{1-2^2} - \frac{2^5}{1-2^{10}} + \frac{2^7}{1-2^{14}} - \frac{2^{11}}{1-2^{22}} + \frac{2^{13}}{1-2^{26}}\right)\right)^{1/32}$$

**Input:**

$$\sqrt[32]{-\frac{2}{1-2^2} - \frac{2^5}{1-2^{10}} + \frac{2^7}{1-2^{14}} - \frac{2^{11}}{1-2^{22}} + \frac{2^{13}}{1-2^{26}}}$$

**Result:**

$$\sqrt[32]{\frac{40\,214\,964\,790\,172\,889\,814}{58\,240\,316\,702\,747\,935\,921}}$$

**Decimal approximation:**

0.988493627498942626382411128459586468294061421849699448776...

**0.988493627...** result practically equal to the dilaton value **0.989117352243 =  $\phi$**

**Alternate form:**

$$\frac{\sqrt[32]{40\,214\,964\,790\,172\,889\,814 \cdot 58\,240\,316\,702\,747\,935\,921^{31/32}}}{58\,240\,316\,702\,747\,935\,921}$$

$4 \cdot \log_{0.988493627} \left( \left( \left( \left( \frac{-2}{1-2^2} - \frac{2^5}{1-2^{10}} + \frac{2^7}{1-2^{14}} - \frac{2^{11}}{1-2^{22}} + \frac{2^{13}}{1-2^{26}} \right) \right) \right) \right) + 11 + \frac{1}{\phi}$

**Input interpretation:**

$$4 \log_{0.988493627} \left( -\frac{2}{1-2^2} - \frac{2^5}{1-2^{10}} + \frac{2^7}{1-2^{14}} - \frac{2^{11}}{1-2^{22}} + \frac{2^{13}}{1-2^{26}} \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

139.6180...

139.618... result practically equal to the rest mass of Pion meson 139.57

**Alternative representation:**

$$4 \log_{0.988494} \left( -\frac{2}{1-2^2} - \frac{2^5}{1-2^{10}} + \frac{2^7}{1-2^{14}} - \frac{2^{11}}{1-2^{22}} + \frac{2^{13}}{1-2^{26}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{4 \log_{-3} \left( \frac{-2}{-3} - \frac{2^5}{1-2^{10}} + \frac{2^7}{1-2^{14}} - \frac{2^{11}}{1-2^{22}} + \frac{2^{13}}{1-2^{26}} \right)}{\log(0.988494)}$$

**Series representations:**

$$4 \log_{0.988494} \left( -\frac{2}{1-2^2} - \frac{2^5}{1-2^{10}} + \frac{2^7}{1-2^{14}} - \frac{2^{11}}{1-2^{22}} + \frac{2^{13}}{1-2^{26}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - \frac{4 \sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{-18025351912575046107}{58240316702747935921} \right)^k}{k}}{\log(0.988494)}$$

$$4 \log_{0.988494} \left( -\frac{2}{1-2^2} - \frac{2^5}{1-2^{10}} + \frac{2^7}{1-2^{14}} - \frac{2^{11}}{1-2^{22}} + \frac{2^{13}}{1-2^{26}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 345.633 \log \left( \frac{40214964790172889814}{58240316702747935921} \right) -$$

$$4 \log \left( \frac{40214964790172889814}{58240316702747935921} \right) \sum_{k=0}^{\infty} (-0.0115064)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$$-((-1+2/(1-2^2))-2^5/(1-2^{10})+2^7/(1-2^{14})-2^{11}/(1-2^{22})+2^{13}/(1-2^{26})))$$

**Input:**

$$-\left(-1 + \frac{2}{1-2^2} - \frac{2^5}{1-2^{10}} + \frac{2^7}{1-2^{14}} - \frac{2^{11}}{1-2^{22}} + \frac{2^{13}}{1-2^{26}}\right)$$

**Exact result:**

$$\frac{287037322548716882005}{174720950108243807763}$$

**Decimal approximation:**

1.642832885071254448917616696075101439901325646000645080543...

$$1.64283288507... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

**Alternate form:**

$$\frac{287037322548716882005}{174720950108243807763}$$

$$-24 \times \frac{1}{10^3} - ((-1+2/(1-2^2))-2^5/(1-2^{10})+2^7/(1-2^{14})-2^{11}/(1-2^{22})+2^{13}/(1-2^{26})))$$

where 24 corresponding to the physical vibrations of a bosonic string.

**Input:**

$$-24 \times \frac{1}{10^3} - \left(-1 + \frac{2}{1-2^2} - \frac{2^5}{1-2^{10}} + \frac{2^7}{1-2^{14}} - \frac{2^{11}}{1-2^{22}} + \frac{2^{13}}{1-2^{26}}\right)$$

**Exact result:**

$$\frac{35355502468264878827336}{21840118763530475970375}$$

**Decimal approximation:**

1.618832885071254448917616696075101439901325646000645080543...

1.61883288507... result that is a very good approximation to the value of the golden ratio 1,618033988749...

**Alternate form:**

$$\frac{35\ 355\ 502\ 468\ 264\ 878\ 827\ 336}{21\ 840\ 118\ 763\ 530\ 475\ 970\ 375}$$

$$-1 * 1/10^{27} * (((-29 * 1/10^3 + ((-1 + 2/(1-2^2) - 2^5/(1-2^{10}) + 2^7/(1-2^{14}) - 2^{11}/(1-2^{22}) + 2^{13}/(1-2^{26}))))))$$

Where 29 is a Lucas number

**Input:**

$$-\frac{1}{10^{27}} \left( -29 \times \frac{1}{10^3} + \left( -1 + \frac{2}{1-2^2} - \frac{2^5}{1-2^{10}} + \frac{2^7}{1-2^{14}} - \frac{2^{11}}{1-2^{22}} + \frac{2^{13}}{1-2^{26}} \right) \right)$$

**Exact result:**

$$\frac{292\ 104\ 230\ 101\ 855\ 952\ 430\ 127}{174\ 720\ 950\ 108\ 243\ 807\ 763\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000}$$

**Decimal approximation:**

1.6718328850712544489176166960751014399013256460006450... × 10<sup>-27</sup>

1.67183288507... \* 10<sup>-27</sup> result practically equal to the value of the formula:

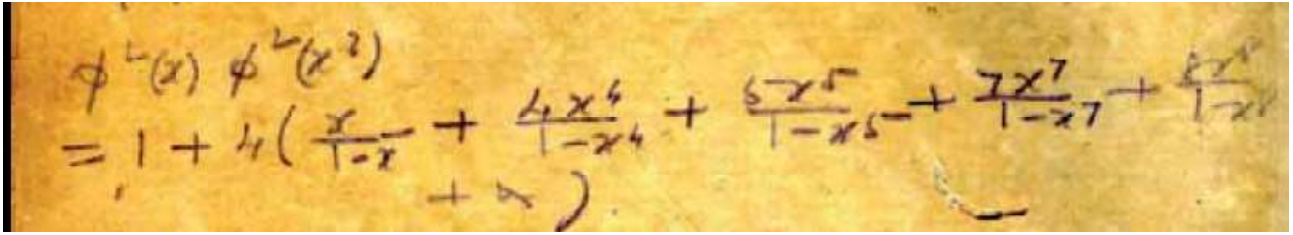
$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-27} \text{ kg}$$

that is the holographic proton mass (N. Hamein)

**Alternate form:**

$$\frac{292\ 104\ 230\ 101\ 855\ 952\ 430\ 127}{174\ 720\ 950\ 108\ 243\ 807\ 763\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000}$$





For  $x = 2$ , we obtain:

$$1+4(2/(1-2)+(4*2^4)/(1-2^4)+(5*2^5)/(1-2^5)+(7*2^7)/(1-2^7)+(8*2^8)/(1-2^8))$$

**Input:**

$$1+4\left(\frac{2}{1-2} + \frac{4 \times 2^4}{1-2^4} + \frac{5 \times 2^5}{1-2^5} + \frac{7 \times 2^7}{1-2^7} + \frac{8 \times 2^8}{1-2^8}\right)$$

**Exact result:**

$$\frac{105471193}{1003935}$$

**Decimal approximation:**

-105.057790594012560574140756124649504200969186252097994392...

**-105.057790594...**

$$-((1+4(2/(1-2)+(4*2^4)/(1-2^4)+(5*2^5)/(1-2^5)+(7*2^7)/(1-2^7)+(8*2^8)/(1-2^8))))+34+1/\text{golden ratio}$$

**Input:**

$$-\left(1+4\left(\frac{2}{1-2} + \frac{4 \times 2^4}{1-2^4} + \frac{5 \times 2^5}{1-2^5} + \frac{7 \times 2^7}{1-2^7} + \frac{8 \times 2^8}{1-2^8}\right)\right)+34 + \frac{1}{\phi}$$

$\phi$  is the golden ratio

**Result:**

$$\frac{1}{\phi} + \frac{139604983}{1003935}$$

**Decimal approximation:**

139.6758245827624554223453429590151423186894954319037572542...

139.6758245827... result practically equal to the rest mass of Pion meson 139.57

**Alternate forms:**

$$\frac{278\,206\,031 + 1\,003\,935\sqrt{5}}{2\,007\,870}$$

$$\frac{139\,604\,983\phi + 1\,003\,935}{1\,003\,935\phi}$$

$$\frac{\sqrt{5}}{2} + \frac{278\,206\,031}{2\,007\,870}$$

**Alternative representations:**

$$-\left(1 + 4\left(\frac{2}{1-2} + \frac{4 \times 2^4}{1-2^4} + \frac{5 \times 2^5}{1-2^5} + \frac{7 \times 2^7}{1-2^7} + \frac{8 \times 2^8}{1-2^8}\right)\right) + 34 + \frac{1}{\phi} =$$

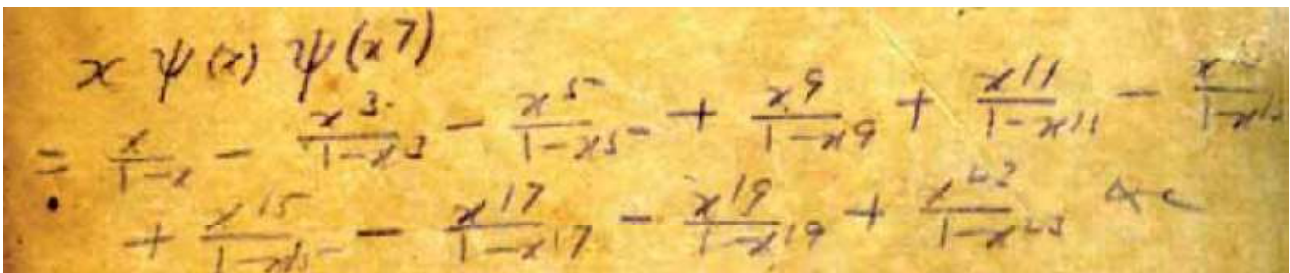
$$33 - 4\left(-2 + \frac{4 \times 2^4}{1-2^4} + \frac{5 \times 2^5}{1-2^5} + \frac{7 \times 2^7}{1-2^7} + \frac{8 \times 2^8}{1-2^8}\right) + \frac{1}{2 \sin(54^\circ)}$$

$$-\left(1 + 4\left(\frac{2}{1-2} + \frac{4 \times 2^4}{1-2^4} + \frac{5 \times 2^5}{1-2^5} + \frac{7 \times 2^7}{1-2^7} + \frac{8 \times 2^8}{1-2^8}\right)\right) + 34 + \frac{1}{\phi} =$$

$$33 + \frac{1}{2 \cos(216^\circ)} - 4\left(-2 + \frac{4 \times 2^4}{1-2^4} + \frac{5 \times 2^5}{1-2^5} + \frac{7 \times 2^7}{1-2^7} + \frac{8 \times 2^8}{1-2^8}\right)$$

$$-\left(1 + 4\left(\frac{2}{1-2} + \frac{4 \times 2^4}{1-2^4} + \frac{5 \times 2^5}{1-2^5} + \frac{7 \times 2^7}{1-2^7} + \frac{8 \times 2^8}{1-2^8}\right)\right) + 34 + \frac{1}{\phi} =$$

$$33 - 4\left(-2 + \frac{4 \times 2^4}{1-2^4} + \frac{5 \times 2^5}{1-2^5} + \frac{7 \times 2^7}{1-2^7} + \frac{8 \times 2^8}{1-2^8}\right) + \frac{1}{2 \sin(666^\circ)}$$



For  $x = 2$ , we obtain:

$$2/(1-2) - 2^3/(1-2^3) - 2^5/(1-2^5) + 2^9/(1-2^9) + 2^{11}/(1-2^{11}) - 2^{13}/(1-2^{13}) + 2^{15}/(1-2^{15}) - 2^{17}/(1-2^{17}) - 2^{19}/(1-2^{19}) + 2^{23}/(1-2^{23})$$

**Input:**

$$\frac{\frac{2}{1-2} - \frac{2^3}{1-2^3} - \frac{2^5}{1-2^5} + \frac{2^9}{1-2^9} + \frac{2^{11}}{1-2^{11}} - \frac{2^{13}}{1-2^{13}} + \frac{2^{15}}{1-2^{15}} - \frac{2^{17}}{1-2^{17}} - \frac{2^{19}}{1-2^{19}} + \frac{2^{23}}{1-2^{23}}}$$

**Exact result:**

$$\frac{2\,732\,169\,985\,131\,547\,176\,255\,351\,845\,238}{3\,302\,796\,536\,343\,024\,015\,629\,782\,184\,239}$$

**Decimal approximation:**

-0.82722927527249521991289446356244269699122343402130683324...

**-0.82722927527...**

**Alternate form:**

$$\frac{2\,732\,169\,985\,131\,547\,176\,255\,351\,845\,238}{3\,302\,796\,536\,343\,024\,015\,629\,782\,184\,239}$$

$$1/10^{27} * (((18 * 1/10^3) + (((-2 * [2/(1-2) - 2^3/(1-2^3) - 2^5/(1-2^5) + 2^9/(1-2^9) + 2^11/(1-2^{11}) - 2^13/(1-2^{13}) + 2^15/(1-2^{15}) - 2^17/(1-2^{17}) - 2^19/(1-2^{19}) + 2^23/(1-2^{23})])))$$

Where 18 is a Lucas number

**Input:**

$$\frac{1}{10^{27}} \left( 18 \times \frac{1}{10^3} - 2 \left( \frac{2}{1-2} - \frac{2^3}{1-2^3} - \frac{2^5}{1-2^5} + \frac{2^9}{1-2^9} + \frac{2^{11}}{1-2^{11}} - \frac{2^{13}}{1-2^{13}} + \frac{2^{15}}{1-2^{15}} - \frac{2^{17}}{1-2^{17}} - \frac{2^{19}}{1-2^{19}} + \frac{2^{23}}{1-2^{23}} \right) \right)$$

**Exact result:**

$$\frac{2\,761\,895\,153\,958\,634\,392\,396\,019\,884\,896\,151}{1\,651\,398\,268\,171\,512\,007\,814\,891\,092\,119\,500\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

**Decimal approximation:**

1.6724585505449904398257889271248853939824468680426136... × 10<sup>-27</sup>

**1.67245855054... \* 10<sup>-27</sup> result practically equal to the proton mass**

**Alternate form:**

$$\frac{2\,761\,895\,153\,958\,634\,392\,396\,019\,884\,896\,151}{1\,651\,398\,268\,171\,512\,007\,814\,891\,092\,119\,500\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\left( \left( \frac{2}{1-2} - \frac{2^3}{1-2^3} - \frac{2^5}{1-2^5} + \frac{2^9}{1-2^9} + \frac{2^{11}}{1-2^{11}} - \frac{2^{13}}{1-2^{13}} + \frac{2^{15}}{1-2^{15}} - \frac{2^{17}}{1-2^{17}} - \frac{2^{19}}{1-2^{19}} + \frac{2^{23}}{1-2^{23}} \right) \right)^{1/16}$$

**Input:**

$$\left( \frac{2}{1-2} - \frac{2^3}{1-2^3} - \frac{2^5}{1-2^5} + \frac{2^9}{1-2^9} + \frac{2^{11}}{1-2^{11}} - \frac{2^{13}}{1-2^{13}} + \frac{2^{15}}{1-2^{15}} - \frac{2^{17}}{1-2^{17}} - \frac{2^{19}}{1-2^{19}} + \frac{2^{23}}{1-2^{23}} \right)^{(1/16)}$$

**Result:**

$$\sqrt[16]{\frac{2\ 732\ 169\ 985\ 131\ 547\ 176\ 255\ 351\ 845\ 238}{3\ 302\ 796\ 536\ 343\ 024\ 015\ 629\ 782\ 184\ 239}}$$

**Decimal approximation:**

0.96922712035530468628594044770166718266374810278768567320... +  
0.19279126103844237291409098070979525706816183456591372135... i

**Polar coordinates:**

$r \approx 0.988215$  (radius),  $\theta \approx 11.25^\circ$  (angle)

0.988215 result practically equal to the dilaton value **0.989117352243 =  $\phi$**

**Alternate forms:**

$$\left( \sqrt[16]{\frac{2\ 732\ 169\ 985\ 131\ 547\ 176\ 255\ 351\ 845\ 238}{3\ 302\ 796\ 536\ 343\ 024\ 015\ 629\ 782\ 184\ 239}} \right)^{15/16} //$$

$$\sqrt[16]{\frac{2\ 732\ 169\ 985\ 131\ 547\ 176\ 255\ 351\ 845\ 238}{3\ 302\ 796\ 536\ 343\ 024\ 015\ 629\ 782\ 184\ 239}} \cos\left(\frac{\pi}{16}\right) +$$

$$i \sqrt[16]{\frac{2\ 732\ 169\ 985\ 131\ 547\ 176\ 255\ 351\ 845\ 238}{3\ 302\ 796\ 536\ 343\ 024\ 015\ 629\ 782\ 184\ 239}} \sin\left(\frac{\pi}{16}\right)$$

$8 \cdot \log_{0.988215} - \left( \left( \left( \left( \left( \frac{2}{1-2} - \frac{2^3}{1-2^3} - \frac{2^5}{1-2^5} + \frac{2^9}{1-2^9} + \frac{2^{11}}{1-2^{11}} - \frac{2^{13}}{1-2^{13}} + \frac{2^{15}}{1-2^{15}} - \frac{2^{17}}{1-2^{17}} - \frac{2^{19}}{1-2^{19}} + \frac{2^{23}}{1-2^{23}} \right) \right) \right) \right) \right) + 11 + 1/\text{golden ratio}$

**Input interpretation:**

$$8 \log_{0.988215} \left( \left( \left( \left( \left( \frac{2}{1-2} - \frac{2^3}{1-2^3} - \frac{2^5}{1-2^5} + \frac{2^9}{1-2^9} + \frac{2^{11}}{1-2^{11}} - \frac{2^{13}}{1-2^{13}} + \frac{2^{15}}{1-2^{15}} - \frac{2^{17}}{1-2^{17}} - \frac{2^{19}}{1-2^{19}} + \frac{2^{23}}{1-2^{23}} \right) \right) \right) \right) \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$  is the base- $b$  logarithm

$\phi$  is the golden ratio

**Result:**

139.614...

139.614... result practically equal to the rest mass of Pion meson 139.57

**Alternative representation:**

$$8 \log_{0.988215} \left( - \left( \frac{2}{1-2} - \frac{2^3}{1-2^3} - \frac{2^5}{1-2^5} + \frac{2^9}{1-2^9} + \frac{2^{11}}{1-2^{11}} - \frac{2^{13}}{1-2^{13}} + \frac{2^{15}}{1-2^{15}} - \frac{2^{17}}{1-2^{17}} - \frac{2^{19}}{1-2^{19}} + \frac{2^{23}}{1-2^{23}} \right) \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{8 \log \left( 2 + \frac{8}{7} + \frac{2^5}{1-2^5} - \frac{2^9}{1-2^9} - \frac{2^{11}}{1-2^{11}} + \frac{2^{13}}{1-2^{13}} - \frac{2^{15}}{1-2^{15}} + \frac{2^{17}}{1-2^{17}} + \frac{2^{19}}{1-2^{19}} - \frac{2^{23}}{1-2^{23}} \right)}{\log(0.988215)}$$

**Series representations:**

$$8 \log_{0.988215} \left( - \left( \frac{2}{1-2} - \frac{2^3}{1-2^3} - \frac{2^5}{1-2^5} + \frac{2^9}{1-2^9} + \frac{2^{11}}{1-2^{11}} - \frac{2^{13}}{1-2^{13}} + \frac{2^{15}}{1-2^{15}} - \frac{2^{17}}{1-2^{17}} - \frac{2^{19}}{1-2^{19}} + \frac{2^{23}}{1-2^{23}} \right) \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - \frac{8 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -\frac{570626551211476839374430339001}{3302796536343024015629782184239} \right)^k}{k}}{\log(0.988215)}$$

$$8 \log_{0.988215} \left( - \left( \frac{2}{1-2} - \frac{2^3}{1-2^3} - \frac{2^5}{1-2^5} + \frac{2^9}{1-2^9} + \frac{2^{11}}{1-2^{11}} - \frac{2^{13}}{1-2^{13}} + \frac{2^{15}}{1-2^{15}} - \frac{2^{17}}{1-2^{17}} - \frac{2^{19}}{1-2^{19}} + \frac{2^{23}}{1-2^{23}} \right) \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - 674.829 \log \left( \frac{2732169985131547176255351845238}{3302796536343024015629782184239} \right) -$$

$$8 \log \left( \frac{2732169985131547176255351845238}{3302796536343024015629782184239} \right) \sum_{k=0}^{\infty} (-0.011785)^k G(k)$$

for  $\left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$



From the results of above expression

-1.6073831292 -0.64283288507125 -105.057790594 -0.82722927527  
 -6.161894349459

we obtain:

$-(-1+1/-1.6073831292 *1/ -0.64283288507125 *1/ -105.057790594 *1/ -0.82722927527 *1/ -6.161894349459)$

**Input interpretation:**

$$-\left(-1 + \frac{1}{1.6073831292} \left( -\frac{1}{0.64283288507125} \left( \frac{1}{-105.057790594} \left( -\frac{1}{0.82722927527} \left( -\frac{1}{6.161894349459} \right) \right) \right) \right) \right)$$

**Result:**

1.001807232808681385922392462130095367937323337620356209257...

1.001807232808... result practically equal to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}} - \varphi} = 1 + \frac{e^{-2\pi}}{1 + \frac{e^{-4\pi}}{1 + \frac{e^{-6\pi}}{1 + \frac{e^{-8\pi}}{1 + \dots}}}} \approx 1.0018674362$$

<http://www.bitman.name/math/article/102/109/>

Note that:

$1/[-(-1+1/-1.6073831292 *1/ -0.64283288507125 *1/ -105.057790594 *1/ -0.82722927527 *1/ -6.161894349459)]^6$

**Input interpretation:**

$$1/\left(-\left(-1 + \frac{1}{1.6073831292} \left( -\frac{1}{0.64283288507125} \left( \frac{1}{-105.057790594} \left( -\frac{1}{0.82722927527} \left( -\frac{1}{6.161894349459} \right) \right) \right) \right) \right)\right)^6$$

**Result:**

0.989224861841272302472237075291828264418396430748362794653...

0.9892248618... result practically equal to the dilaton value **0.989117352243 =  $\phi$**

$$21 \times 1 / \log_{\text{base } 0.9892248618} \left( \left( \frac{1}{-1 + \frac{1}{-1.6073831292 + \frac{1}{-0.64283288507125 + \frac{1}{-105.057790594 + \frac{1}{-0.82722927527 + \frac{1}{-6.161894349459}}}}} \right) \right) - \pi + \phi^2$$

Where 21 is a Fibonacci number

**Input interpretation:**

$$21 \times 1 / \log_{0.9892248618} \left( \frac{1}{-1 + \frac{1}{1.6073831292 + \frac{1}{0.64283288507125 + \frac{1}{105.057790594 + \frac{1}{0.82722927527 + \frac{1}{6.161894349459}}}}} \right) - \pi + \phi^2$$

$\log_b(x)$  is the base-  $b$  logarithm

$\phi$  is the golden ratio

**Result:**

125.47644...

125.47644... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18

**Alternative representation:**

$$\pi + \phi^2 = -\pi + \phi^2 + 21 / \frac{1}{\log(0.989225) \left( 1 - \frac{1}{105.0577905940000 + \frac{1}{-6.1618943494590000} + \frac{1}{-1.60738312920000} + \frac{1}{-0.827229275270000} + \frac{1}{-0.642832885071250000} \right)}$$



**Series representations:**

$$21 / \log_{0.989225}(-1 / (-1 + -(1 / (0.642832885071250000 (-1.60738312920000) (-105.0577905940000) (-0.827229275270000 (-6.1618943494590000)))))) - \pi + \phi^2 = \phi^2 - \pi - \frac{21 \log(0.989225)}{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.00180397261019428)^k}{k}}$$

$$21 / \log_{0.989225}(-1 / (-1 + -(1 / (0.642832885071250000 (-1.60738312920000) (-105.0577905940000) (-0.827229275270000 (-6.1618943494590000)))))) - \pi + \phi^2 = \frac{\phi^2 - \pi - \frac{21}{\log(0.99819602738980572) (92.3062 + \sum_{k=0}^{\infty} (-0.0107751)^k G(k))}}{21}$$

for

$$G(0) = 0$$

and

$$G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j}$$

21\*1/log base 0.9892248618((1/[-(-1+1/-1.6073831292 \*1/ -0.64283288507125 \*1/ -105.057790594 \*1/ -0.82722927527 \*1/ -6.161894349459)])))-5+18+1/golden ratio

Where 21 and 5 are Fibonacci numbers, while 18 is a Lucas number

**Input interpretation:**

$$21 \times 1 / \log_{0.9892248618} \left( - \left( 1 / \left( -1 + - \frac{1}{1.6073831292} \left( - \frac{1}{0.64283288507125} \right) \left( - \frac{1}{105.057790594} \right) \left( - \frac{1}{0.82722927527} \right) \left( - \frac{1}{6.161894349459} \right) \right) \right) \right) - 5 + 18 + \frac{1}{\phi}$$

log<sub>b</sub>(x) is the base- b logarithm

φ is the golden ratio

**Result:**

139.61803...

139.61803... result practically equal to the rest mass of Pion meson 139.57

**Alternative representation:**

$$21 / \log_{0.989225}(-1 / (-1 + -(1 / (0.642832885071250000 (-1.60738312920000) (-105.0577905940000) (-0.827229275270000) (-6.1618943494590000)))))) -$$

$$5 + 18 + \frac{1}{\phi} = 13 + \frac{1}{\phi} + 21 / \frac{1}{\log(0.989225)}$$

$$\log(1 / (1 - -(1 / (105.0577905940000 (-6.1618943494590000) (-1.60738312920000) (-0.827229275270000) (-0.642832885071250000))))))$$

**Series representations:**

$$21 / \log_{0.989225}(-1 / (-1 + -(1 / (0.642832885071250000 (-1.60738312920000) (-105.0577905940000) (-0.827229275270000) (-6.1618943494590000)))))) -$$

$$5 + 18 + \frac{1}{\phi} = 13 + \frac{1}{\phi} - \frac{21 \log(0.989225)}{\sum_{k=1}^{\infty} \frac{(-1)^k (-0.00180397261019428)^k}{k}}$$

$$21 / \log_{0.989225}(-1 / (-1 + -(1 / (0.642832885071250000 (-1.60738312920000) (-105.0577905940000) (-0.827229275270000) (-6.1618943494590000)))))) - 5 + 18 + \frac{1}{\phi} =$$

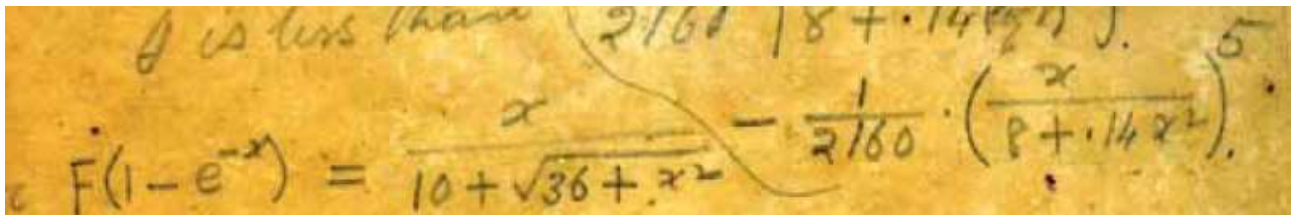
$$13 + \frac{1}{\phi} - \frac{21}{\log(0.99819602738980572) (92.3062 + \sum_{k=0}^{\infty} (-0.0107751)^k G(k))}$$

for

$$\left( G(0) = 0 \right.$$

and

$$G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j}$$



For  $x = 18$ , we obtain:

$$18/(10+\sqrt{36+18^2}) - 1/2160 * (18/(8+14*18^3))^5$$

**Input:**

$$\frac{18}{10 + \sqrt{36 + 18^2}} - \frac{1}{2160} \left( \frac{18}{8 + 14 \times 18^3} \right)^5$$

**Result:**

$$\frac{18}{10 + 6\sqrt{10}} - \frac{2187}{9075708061687780786749440}$$

**Decimal approximation:**

0.621253797300711414830068653214948104631976939406456177439...

0.6212537973...

**Alternate forms:**

$$\frac{9(5445424837012668472049664\sqrt{10} - 9075708061687780786752599)}{117984204801941150227742720}$$

$$\frac{27\sqrt{10}}{65} - \frac{81681372555190027080773391}{117984204801941150227742720}$$

$$\frac{49008823533114016248446976\sqrt{10} - 81681372555190027080773391}{117984204801941150227742720}$$

**Minimal polynomial:**

$$1070790198672799474772222730814213662594057424076800x^2 + 1482632582777722349685132151731484146490560272302080x - 1334369324499950114715797205614156009131066472446683$$

$$1/(((18/(10+\sqrt{36+18^2})) - 1/2160 * (18/(8+14*18^3))^5)))$$

**Input:**

$$\frac{1}{\frac{18}{10+\sqrt{36+18^2}} - \frac{1}{2160} \left(\frac{18}{8+14 \times 18^3}\right)^5}$$

**Result:**

$$\frac{1}{\frac{18}{10+6\sqrt{10}} - \frac{2187}{9075708061687780786749440}}$$

**Decimal approximation:**

1.609648108945015332889144423294764565669127214084266903221...

1.6096481089...

From which, we can to obtain:

$$(-x/9075708061687780786749440 + 18/(10 + 6\sqrt{10}))=0.621253797300711414830068653214948104631976939406456177439$$

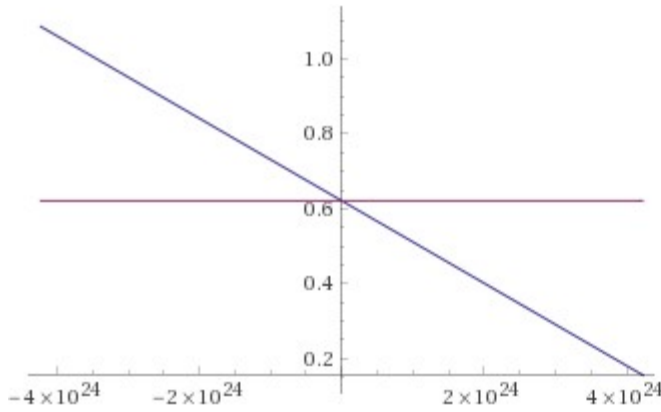
**Input interpretation:**

$$-\frac{x}{9075708061687780786749440} + \frac{18}{10+6\sqrt{10}} = 0.621253797300711414830068653214948104631976939406456177439$$

**Result:**

$$\frac{18}{10+6\sqrt{10}} - \frac{x}{9075708061687780786749440} = 0.621253797300711414830068653214948104631976939406456177439$$

**Plot:**



$$\frac{18}{10+6\sqrt{10}} - \frac{x}{9075708061687780786749440}$$

$$0.621253797300711414830068653214948104631976939406456177439$$

**Alternate forms:**

$$\frac{2.40972934027286297684426159403119871 \times 10^{-22} - x}{9075708061687780786749440} = 0$$

$$\frac{1}{65} \left( 27\sqrt{10} - 45 \right) - \frac{x}{9075708061687780786749440} = 0.621253797300711414830068653214948104631976939406456177439$$

$$\frac{9}{5 + 3\sqrt{10}} - \frac{x}{9075708061687780786749440} = 0.621253797300711414830068653214948104631976939406456177439$$

**Solution:**

$$x \approx 2187.00000000000000000000000000000001$$

**Integer solution:**

$$x = 2187$$

2187

And:

$$\left( \frac{-(x+76)}{9075708061687780786749440} + \frac{18}{10 + 6\sqrt{10}} \right) = 0.621253797300711414830068653214948104631976939406456177439$$

Where 76 is a Lucas number

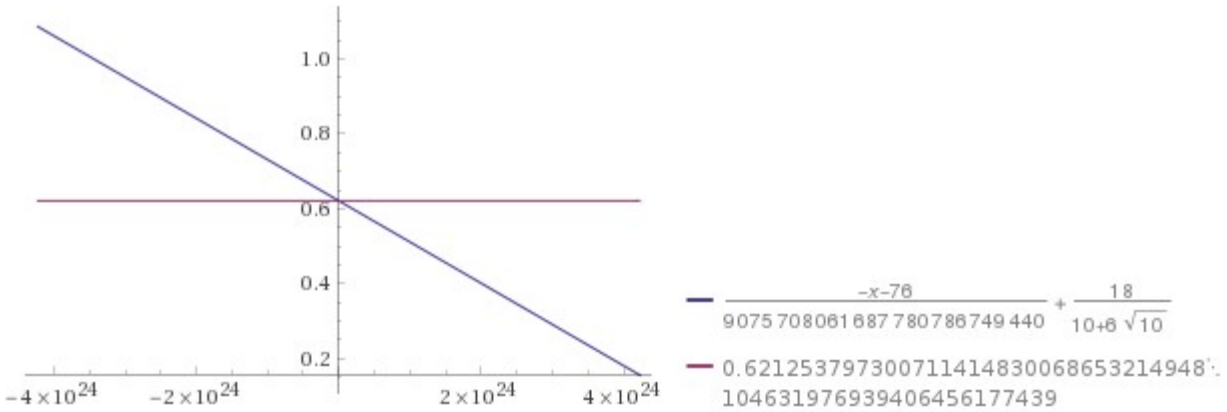
**Input interpretation:**

$$\frac{-(x+76)}{9075708061687780786749440} + \frac{18}{10 + 6\sqrt{10}} = 0.621253797300711414830068653214948104631976939406456177439$$

**Result:**

$$\frac{-x - 76}{9075708061687780786749440} + \frac{18}{10 + 6\sqrt{10}} = 0.621253797300711414830068653214948104631976939406456177439$$

**Plot:**



**Alternate forms:**

$$\frac{2.32598931747417180801016745541831755 \times 10^{-22}}{x} - \frac{9075708061687780786749440}{9075708061687780786749440} = 0$$

$$\frac{-x - 76}{9075708061687780786749440} + \frac{1}{65} (27\sqrt{10} - 45) = 0.621253797300711414830068653214948104631976939406456177439$$

$$\frac{-x - 76}{9075708061687780786749440} + \frac{9}{5 + 3\sqrt{10}} = 0.621253797300711414830068653214948104631976939406456177439$$

**Expanded form:**

$$\frac{-\frac{9075708061687780786749440}{18} + \frac{18}{19}}{10 + 6\sqrt{10}} - \frac{2268927015421945196687360}{2268927015421945196687360} = 0.621253797300711414830068653214948104631976939406456177439$$

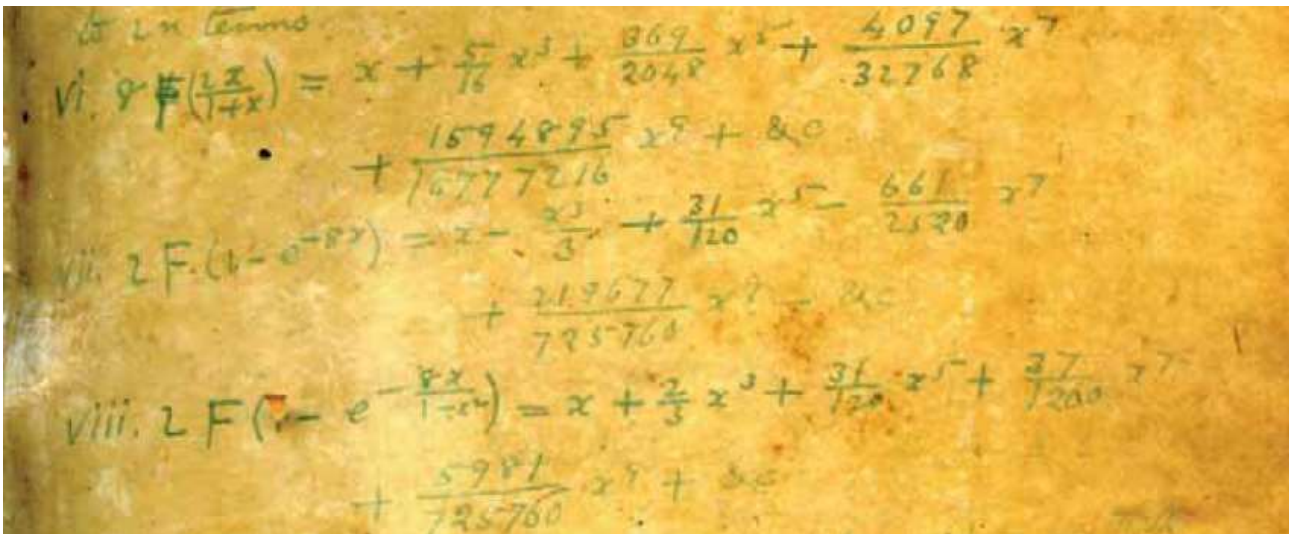
**Solution:**

$$x \approx 2111.0000000000000000000000000000001$$

**Integer solution:**

$$x = 2111$$

2111 result very near to the rest mass of strange D meson 2112.3



For  $x = 2$ , we obtain:

$$2 + (5/16) \cdot 2^3 + (369/2048) \cdot 2^5 + (4097/32268) \cdot 2^7 + (1594895/16777216) \cdot 2^9$$

**Input:**

$$2 + \frac{5}{16} \times 2^3 + \frac{369}{2048} \times 2^5 + \frac{4097}{32268} \times 2^7 + \frac{1594895}{16777216} \times 2^9$$

**Exact result:**

$$\frac{19875643565}{264339456}$$

**Decimal approximation:**

75.18984818142320758956241477624891533407710425189041775133...  
75.18984818142320.....

$$2 - (2^3)/3 + (31/120) \cdot 2^5 - (661/2520) \cdot 2^7 + (219677/725760) \cdot 2^9$$

**Input:**

$$2 - \frac{2^3}{3} + \frac{31}{120} \times 2^5 - \frac{661}{2520} \times 2^7 + \frac{219677}{725760} \times 2^9$$

**Exact result:**

$$\frac{365716}{2835}$$

**Decimal approximation:**

129.0003527336860670194003527336860670194003527336860670194...

129.000352733686067.....

$$2 + (2/3) \cdot 2^3 + (31/120) \cdot 2^5 + (37/1260) \cdot 2^7 + (5981/725760) \cdot 2^9$$

**Input:**

$$2 + \frac{2}{3} \times 2^3 + \frac{31}{120} \times 2^5 + \frac{37}{1260} \times 2^7 + \frac{5981}{725760} \times 2^9$$

**Exact result:**

$$\frac{66844}{2835}$$

**Decimal approximation:**

23.57813051146384479717813051146384479717813051146384479717...

23.5781305114638..... result very near to the black hole entropy 23.6954

(19875643565/264339456 + 365716/2835 + 66844/2835) - 76 - 11 - golden ratio

where 76 and 11 are Lucas number

**Input:**

$$\left( \frac{19875643565}{264339456} + \frac{365716}{2835} + \frac{66844}{2835} \right) - 76 - 11 - \phi$$

$\phi$  is the golden ratio

**Result:**

$$\frac{7032807964601}{49960157184} - \phi$$

**Decimal approximation:**

139.1502974378232245579363111870331890329352783172345667057...

139.1502974 result practically equal to the rest mass of Pion meson 139.57

**Alternate forms:**

$$\frac{7007827886009 - 24980078592\sqrt{5}}{49960157184}$$

$$\frac{7032807964601 - 49960157184\phi}{49960157184}$$



$$\frac{7007827886009}{49960157184} - \frac{\sqrt{5}}{2}$$

**Alternative representations:**

$$\left( \frac{19875643565}{264339456} + \frac{365716}{2835} + \frac{66844}{2835} \right) - 76 - 11 - \phi =$$

$$-87 + \frac{432560}{2835} + \frac{19875643565}{264339456} - 2 \sin(54^\circ)$$

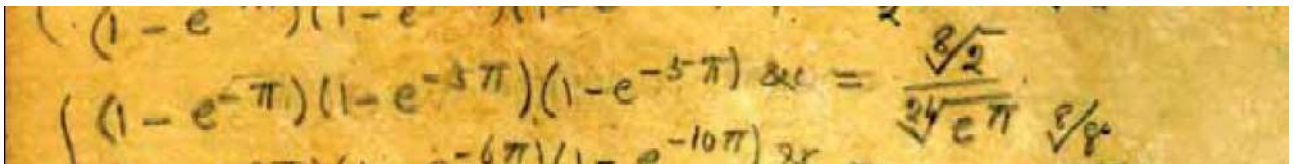
$$\left( \frac{19875643565}{264339456} + \frac{365716}{2835} + \frac{66844}{2835} \right) - 76 - 11 - \phi =$$

$$-87 + 2 \cos(216^\circ) + \frac{432560}{2835} + \frac{19875643565}{264339456}$$

$$\left( \frac{19875643565}{264339456} + \frac{365716}{2835} + \frac{66844}{2835} \right) - 76 - 11 - \phi =$$

$$-87 + \frac{432560}{2835} + \frac{19875643565}{264339456} + 2 \sin(666^\circ)$$

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$$(1 - e^{-\pi})(1 - e^{-3\pi})(1 - e^{-5\pi})$$

**Input:**

$$(1 - e^{-\pi})(1 - e^{-3\pi})(1 - e^{-5\pi})$$

**Decimal approximation:**

0.956708725383334259887083150002997516798687988267252736507...

0.9567087253.... result very near to the spectral index  $n_s$ , to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 (see Appendix) and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

From:

Astronomy & Astrophysics manuscript no. ms c ESO 2019 - September 24, 2019  
**Planck 2018 results. VI. Cosmological parameters**

*The primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a spectral index  $n_s = 0.965 \pm 0.004$ , consistent with the predictions of slow-roll, single-field, inflation.*

We know that  $\alpha'$  is the Regge slope (string tension). With regard the Omega mesons, the values are:

$$\omega \quad | \quad 6 \quad | \quad m_{u/d} = 0 - 60 \quad | \quad 0.910 - 0.918$$

$$\omega/\omega_3 \quad | \quad 5 + 3 \quad | \quad m_{u/d} = 255 - 390 \quad | \quad 0.988 - 1.18$$

$$\omega/\omega_3 \quad | \quad 5 + 3 \quad | \quad m_{u/d} = 240 - 345 \quad | \quad 0.937 - 1.000$$

**Property:**

$(1 - e^{-5\pi})(1 - e^{-3\pi})(1 - e^{-\pi})$  is a transcendental number

**Alternate forms:**

$$(e^{-5\pi} - 1)(e^{-3\pi} - 1)(1 - e^{-\pi})$$

$$(1 - e^{-5\pi})(1 - e^{-3\pi})(1 + \sinh(\pi) - \cosh(\pi))$$

$$e^{-9\pi} (e^{\pi} - 1)^3 (1 + e^{\pi} + e^{2\pi})(1 + e^{\pi} + e^{2\pi} + e^{3\pi} + e^{4\pi})$$

$\cosh(x)$  is the hyperbolic cosine function

$\sinh(x)$  is the hyperbolic sine function

### Alternative representations:

$$(1 - e^{-\pi})(1 - e^{-3\pi})(1 - e^{-5\pi}) = (1 - e^{i \log(-1)})(1 - e^{3i \log(-1)})(1 - e^{5i \log(-1)})$$

$$(1 - e^{-\pi})(1 - e^{-3\pi})(1 - e^{-5\pi}) = (1 - e^{-900^\circ})(1 - e^{-540^\circ})(1 - e^{-180^\circ})$$

$$(1 - e^{-\pi})(1 - e^{-3\pi})(1 - e^{-5\pi}) = (1 - \exp^{-\pi}(z))(1 - \exp^{-3\pi}(z))(1 - \exp^{-5\pi}(z)) \text{ for } z = 1$$

### Series representations:

$$(1 - e^{-\pi})(1 - e^{-3\pi})(1 - e^{-5\pi}) = 1 - e^{-36 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + e^{-32 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + e^{-24 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} - e^{-20 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + e^{-16 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} - e^{-12 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} - e^{-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$(1 - e^{-\pi})(1 - e^{-3\pi})(1 - e^{-5\pi}) = \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{-9\pi} \left( -1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{\pi} \right)^3 \left( 1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{\pi} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{2\pi} \right) \left( 1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{\pi} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{2\pi} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{3\pi} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{4\pi} \right)$$

$$(1 - e^{-\pi})(1 - e^{-3\pi})(1 - e^{-5\pi}) = \left( -1 + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{\pi} \right)^3 \left( 1 + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{\pi} + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{2\pi} \right) \left( 1 + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{\pi} + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{2\pi} + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{3\pi} + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{4\pi} \right) \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{-9\pi}$$

### Integral representations:

$$(1 - e^{-\pi})(1 - e^{-3\pi})(1 - e^{-5\pi}) = 1 - e^{-36 \int_0^1 \sqrt{1-t^2} dt} + e^{-32 \int_0^1 \sqrt{1-t^2} dt} + e^{-24 \int_0^1 \sqrt{1-t^2} dt} - e^{-20 \int_0^1 \sqrt{1-t^2} dt} + e^{-16 \int_0^1 \sqrt{1-t^2} dt} - e^{-12 \int_0^1 \sqrt{1-t^2} dt} - e^{-4 \int_0^1 \sqrt{1-t^2} dt}$$

$$(1 - e^{-\pi})(1 - e^{-3\pi})(1 - e^{-5\pi}) = 1 - e^{-24 \int_0^{\infty} \sin^3(t)/t^3 dt} + e^{-64/3 \int_0^{\infty} \sin^3(t)/t^3 dt} + e^{-16 \int_0^{\infty} \sin^3(t)/t^3 dt} - e^{-40/3 \int_0^{\infty} \sin^3(t)/t^3 dt} + e^{-32/3 \int_0^{\infty} \sin^3(t)/t^3 dt} - e^{-8 \int_0^{\infty} \sin^3(t)/t^3 dt} - e^{-8/3 \int_0^{\infty} \sin^3(t)/t^3 dt}$$

$$\begin{aligned}
(1 - e^{-\pi})(1 - e^{-3\pi})(1 - e^{-5\pi}) &= 1 - \exp\left(-\frac{3\sqrt{3}}{4} - 24 \int_0^{\frac{1}{4}} \sqrt{-(-1+t)t} dt\right) + \\
&\exp\left(-8\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{-(-1+t)t} dt\right)\right) - \\
&\exp\left(-5\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{-(-1+t)t} dt\right)\right) - \\
&\exp\left(-3\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{-(-1+t)t} dt\right)\right) - \\
&\exp\left(-\frac{3\sqrt{3}}{4} - 24 \int_0^{\frac{1}{4}} \sqrt{-(-1+t)t} dt - 8\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{-(-1+t)t} dt\right)\right) + \\
&\exp\left(-\frac{3\sqrt{3}}{4} - 24 \int_0^{\frac{1}{4}} \sqrt{-(-1+t)t} dt - 5\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{-(-1+t)t} dt\right)\right) + \\
&\exp\left(-\frac{3\sqrt{3}}{4} - 24 \int_0^{\frac{1}{4}} \sqrt{-(-1+t)t} dt - 3\left(\frac{3\sqrt{3}}{4} + 24 \int_0^{\frac{1}{4}} \sqrt{-(-1+t)t} dt\right)\right)
\end{aligned}$$

$$(((2)^{1/8})/((e^{\pi})^{1/24}))$$

**Input:**

$$\frac{\sqrt[8]{2}}{\sqrt[24]{e^{\pi}}}$$

**Exact result:**

$$\sqrt[8]{2} e^{-\pi/24}$$

**Decimal approximation:**

0.956708725113587003449038717361890724715615702454393013400...

[0.956708725... as above described](#)

**Property:**

$\sqrt[8]{2} e^{-\pi/24}$  is a transcendental number

**Alternative representations:**

$$\frac{\sqrt[8]{2}}{\sqrt[24]{e^{\pi}}} = \frac{\sqrt[8]{2}}{\sqrt[24]{e^{180^\circ}}}$$

$$\frac{\sqrt[8]{2}}{\sqrt[24]{e^{\pi}}} = \frac{\sqrt[8]{2}}{\sqrt[24]{\exp^{\pi}(z)}} \text{ for } z = 1$$

$$\frac{\sqrt[8]{2}}{\sqrt[24]{e^\pi}} = \frac{\sqrt[8]{2}}{\sqrt[24]{e^{-i \log(-1)}}}$$

**Series representations:**

$$\frac{\sqrt[8]{2}}{\sqrt[24]{e^\pi}} = \sqrt[8]{2} e^{-1/6 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$\frac{\sqrt[8]{2}}{\sqrt[24]{e^\pi}} = \sqrt[8]{2} \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{-\pi/24}$$

$$\frac{\sqrt[8]{2}}{\sqrt[24]{e^\pi}} = \sqrt[8]{2} \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{-\pi/24}$$

**Integral representations:**

$$\frac{\sqrt[8]{2}}{\sqrt[24]{e^\pi}} = \sqrt[8]{2} e^{-1/6 \int_0^1 \sqrt{1-t^2} dt}$$

$$\frac{\sqrt[8]{2}}{\sqrt[24]{e^\pi}} = \sqrt[8]{2} e^{-1/12 \int_0^1 1/\sqrt{1-t^2} dt}$$

$$\frac{\sqrt[8]{2}}{\sqrt[24]{e^\pi}} = \sqrt[8]{2} e^{-1/12 \int_0^{\infty} 1/(1+t^2) dt}$$

Thence:

$$\sqrt[8]{2} e^{-\pi/24}$$

0.956708725113587003449038717361890724715615702454393013400...

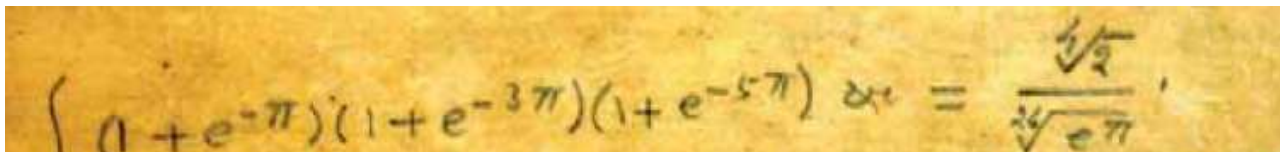
0.95670872511.....

$$(1 - e^{-\pi})(1 - e^{-3\pi})(1 - e^{-5\pi})$$

0.956708725383334259887083150002997516798687988267252736507...

0.95670872583...

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}-\varphi+1}} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$



$$\left( (1 + e^{-\pi})(1 + e^{-3\pi})(1 + e^{-5\pi}) \dots \right)^{-1} = \frac{\sqrt[4]{2}}{\sqrt[24]{e^{\pi}}}$$

$$\left( (2)^{1/4} \right) / \left( (e^{\pi})^{1/24} \right)$$

**Input:**

$$\frac{\sqrt[4]{2}}{\sqrt[24]{e^{\pi}}}$$

**Exact result:**

$$\sqrt[4]{2} e^{-\pi/24}$$

**Decimal approximation:**

1.043298262644687012527875688815591456103311209998752645741...

1.043298262644.....

**Property:**

$\sqrt[4]{2} e^{-\pi/24}$  is a transcendental number

**Alternative representations:**

$$\frac{\sqrt[4]{2}}{\sqrt[24]{e^{\pi}}} = \frac{\sqrt[4]{2}}{\sqrt[24]{e^{180^\circ}}}$$

$$\frac{\sqrt[4]{2}}{\sqrt[24]{e^\pi}} = \frac{\sqrt[4]{2}}{\sqrt[24]{\exp^\pi(z)}} \text{ for } z = 1$$

$$\frac{\sqrt[4]{2}}{\sqrt[24]{e^\pi}} = \frac{\sqrt[4]{2}}{\sqrt[24]{e^{-i \log(-1)}}}$$

### Series representations:

$$\frac{\sqrt[4]{2}}{\sqrt[24]{e^\pi}} = \sqrt[4]{2} e^{-1/6 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

$$\frac{\sqrt[4]{2}}{\sqrt[24]{e^\pi}} = \sqrt[4]{2} \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{-\pi/24}$$

$$\frac{\sqrt[4]{2}}{\sqrt[24]{e^\pi}} = \sqrt[4]{2} \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{-\pi/24}$$

### Integral representations:

$$\frac{\sqrt[4]{2}}{\sqrt[24]{e^\pi}} = \sqrt[4]{2} e^{-1/6 \int_0^1 \sqrt{1-t^2} dt}$$

$$\frac{\sqrt[4]{2}}{\sqrt[24]{e^\pi}} = \sqrt[4]{2} e^{-1/12 \int_0^1 1/\sqrt{1-t^2} dt}$$

$$\frac{\sqrt[4]{2}}{\sqrt[24]{e^\pi}} = \sqrt[4]{2} e^{-1/12 \int_0^{\infty} 1/(1+t^2) dt}$$

$$(1+e^{(-\pi)})(1+e^{(-3\pi)})(1+e^{(-5\pi)})$$

### Input:

$$(1 + e^{-\pi})(1 + e^{-3\pi})(1 + e^{-5\pi})$$

## Decimal approximation:

1.043298262350525543682520473748670633841480414792812410071...

1.04329826235.....

## Property:

$(1 + e^{-5\pi})(1 + e^{-3\pi})(1 + e^{-\pi})$  is a transcendental number

## Alternate forms:

$$1 + e^{-9\pi} + e^{-8\pi} + e^{-6\pi} + e^{-5\pi} + e^{-4\pi} + e^{-3\pi} + e^{-\pi}$$

$$e^{-9\pi} (1 + e^{\pi})^3 (1 - e^{\pi} + e^{2\pi}) (1 - e^{\pi} + e^{2\pi} - e^{3\pi} + e^{4\pi})$$

$$e^{-9\pi} (1 + e^{\pi} + e^{3\pi} + e^{4\pi} + e^{5\pi} + e^{6\pi} + e^{8\pi} + e^{9\pi})$$

## Alternative representations:

$$(1 + e^{-\pi})(1 + e^{-3\pi})(1 + e^{-5\pi}) = (1 + e^{i \log(-1)})(1 + e^{3i \log(-1)})(1 + e^{5i \log(-1)})$$

$$(1 + e^{-\pi})(1 + e^{-3\pi})(1 + e^{-5\pi}) = (1 + e^{-900^\circ})(1 + e^{-540^\circ})(1 + e^{-180^\circ})$$

$$(1 + e^{-\pi})(1 + e^{-3\pi})(1 + e^{-5\pi}) = (1 + \exp^{-\pi}(z))(1 + \exp^{-3\pi}(z))(1 + \exp^{-5\pi}(z)) \text{ for } z = 1$$

## Series representations:

$$(1 + e^{-\pi})(1 + e^{-3\pi})(1 + e^{-5\pi}) = e^{-36 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \left(1 + e^{4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}\right)^3 \left(1 - e^{4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + e^{8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}\right) \left(1 - e^{4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + e^{8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} - e^{12 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + e^{16 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}\right)$$

$$(1 + e^{-\pi})(1 + e^{-3\pi})(1 + e^{-5\pi}) = \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{-9\pi} \left(1 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{\pi}\right)^3 \left(1 - \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{\pi} + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2\pi}\right) \left(1 - \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{\pi} + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2\pi} - \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{3\pi} + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4\pi}\right)$$



$$\begin{aligned}
& (1 + e^{-\pi})(1 + e^{-3\pi})(1 + e^{-5\pi}) = \\
& \left( 1 + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{\pi} \right)^3 \left( 1 - \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{\pi} + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{2\pi} \right) \\
& \left( 1 - \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{\pi} + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{2\pi} - \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{3\pi} + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{4\pi} \right) \\
& \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{-9\pi}
\end{aligned}$$

**Integral representations:**

$$\begin{aligned}
(1 + e^{-\pi})(1 + e^{-3\pi})(1 + e^{-5\pi}) &= \left( 1 + e^{-\left(3\sqrt{3}\right)/4 - 24 \int_0^{1/4} \sqrt{t-t^2} dt} \right) \\
& \left( 1 + e^{-5 \left( \frac{3\sqrt{3}}{4} + 24 \int_0^{1/4} \sqrt{t-t^2} dt \right)} \right) \left( 1 + e^{-3 \left( \frac{3\sqrt{3}}{4} + 24 \int_0^{1/4} \sqrt{t-t^2} dt \right)} \right)
\end{aligned}$$

$$\begin{aligned}
(1 + e^{-\pi})(1 + e^{-3\pi})(1 + e^{-5\pi}) &= \\
& 1 + e^{-24 \int_0^{\infty} \sin^3(t)/t^3 dt} + e^{-64/3 \int_0^{\infty} \sin^3(t)/t^3 dt} + e^{-16 \int_0^{\infty} \sin^3(t)/t^3 dt} + \\
& e^{-40/3 \int_0^{\infty} \sin^3(t)/t^3 dt} + e^{-32/3 \int_0^{\infty} \sin^3(t)/t^3 dt} + e^{-8 \int_0^{\infty} \sin^3(t)/t^3 dt} + e^{-8/3 \int_0^{\infty} \sin^3(t)/t^3 dt}
\end{aligned}$$

$$\begin{aligned}
(1 + e^{-\pi})(1 + e^{-3\pi})(1 + e^{-5\pi}) &= \\
& e^{-18 \int_0^{\infty} \sin(t)/t dt} \left( 1 + e^{2 \int_0^{\infty} \sin(t)/t dt} \right)^3 \left( 1 - e^{2 \int_0^{\infty} \sin(t)/t dt} + e^{4 \int_0^{\infty} \sin(t)/t dt} \right) \\
& \left( 1 - e^{2 \int_0^{\infty} \sin(t)/t dt} + e^{4 \int_0^{\infty} \sin(t)/t dt} - e^{6 \int_0^{\infty} \sin(t)/t dt} + e^{8 \int_0^{\infty} \sin(t)/t dt} \right)
\end{aligned}$$

From which, we obtain:

$$1/(((1+e^{(-\pi)})(1+e^{(-3\pi)})(1+e^{(-5\pi)})))$$

**Input:**

$$\frac{1}{(1 + e^{-\pi})(1 + e^{-3\pi})(1 + e^{-5\pi})}$$

### Decimal approximation:

0.958498672994072074405294544580159111029190412076885988463...

0.958498672994... result very near to the spectral index  $n_s$ , to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 (see Appendix) and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}-\varphi+1}} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}} \approx 0.9568666373$$

From:

Astronomy & Astrophysics manuscript no. ms c ESO 2019 - September 24, 2019  
**Planck 2018 results. VI. Cosmological parameters**

*The primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a spectral index  $n_s = 0.965 \pm 0.004$ , consistent with the predictions of slow-roll, single-field, inflation.*

We know that  $\alpha'$  is the Regge slope (string tension). With regard the Omega mesons, the values are:

$$\omega \quad | \quad 6 \quad | \quad m_{u/d} = 0 - 60 \quad | \quad 0.910 - 0.918$$

$$\omega/\omega_3 \quad | \quad 5 + 3 \quad | \quad m_{u/d} = 255 - 390 \quad | \quad 0.988 - 1.18$$

$$\omega/\omega_3 \quad | \quad 5 + 3 \quad | \quad m_{u/d} = 240 - 345 \quad | \quad 0.937 - 1.000$$

### Property:

$\frac{1}{(1+e^{-5\pi})(1+e^{-3\pi})(1+e^{-\pi})}$  is a transcendental number

**Alternate forms:**

$$\frac{e^{9\pi}}{(1+e^\pi)^3(1-e^\pi+e^{2\pi})(1-e^\pi+e^{2\pi}-e^{3\pi}+e^{4\pi})}$$

$$1 - \frac{1}{15(1+e^\pi)^3} + \frac{2}{5(1+e^\pi)^2} - \frac{41}{45(1+e^\pi)} + \frac{e^\pi - 2}{9(1-e^\pi+e^{2\pi})} + \frac{-1-e^{3\pi}}{5(1-e^\pi+e^{2\pi}-e^{3\pi}+e^{4\pi})}$$

**Alternative representations:**

$$\frac{1}{(1+e^{-\pi})(1+e^{-3\pi})(1+e^{-5\pi})} = \frac{1}{(1+e^{-900^\circ})(1+e^{-540^\circ})(1+e^{-180^\circ})}$$

$$\frac{1}{(1+e^{-\pi})(1+e^{-3\pi})(1+e^{-5\pi})} = \frac{1}{(1+e^{i \log(-1)})(1+e^{3i \log(-1)})(1+e^{5i \log(-1)})}$$

$$\frac{1}{(1+e^{-\pi})(1+e^{-3\pi})(1+e^{-5\pi})} = \frac{1}{(1+\exp^{-\pi}(z))(1+\exp^{-3\pi}(z))(1+\exp^{-5\pi}(z))} \text{ for } z = 1$$

**Series representations:**

$$\frac{1}{(1+e^{-\pi})(1+e^{-3\pi})(1+e^{-5\pi})} = e^{36 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} / \left( \left( 1 + e^{4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \right)^3 \left( 1 - e^{4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + e^{8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \right) \left( 1 - e^{4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + e^{8 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} - e^{12 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} + e^{16 \sum_{k=0}^{\infty} (-1)^k / (1+2k)} \right) \right)$$

$$\frac{1}{(1+e^{-\pi})(1+e^{-3\pi})(1+e^{-5\pi})} = \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{9\pi} / \left( \left( 1 + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^\pi \right)^3 \left( 1 - \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^\pi + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{2\pi} \right) \left( 1 - \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^\pi + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{2\pi} - \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{3\pi} + \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{4\pi} \right) \right)$$

$$\frac{1}{(1+e^{-\pi})(1+e^{-3\pi})(1+e^{-5\pi})} = \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{\pi} / \left( \left( 1 + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{\pi} \right)^3 \left( 1 - \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{\pi} + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{2\pi} \right) \left( 1 - \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{\pi} + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{2\pi} - \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{3\pi} + \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{4\pi} \right)$$

### Integral representations:

$$\frac{1}{(1+e^{-\pi})(1+e^{-3\pi})(1+e^{-5\pi})} = \frac{1}{\left( 1 + e^{-15 \int_0^{\infty} \sin^4(t)/t^4 dt} \right) \left( 1 + e^{-9 \int_0^{\infty} \sin^4(t)/t^4 dt} \right) \left( 1 + e^{-3 \int_0^{\infty} \sin^4(t)/t^4 dt} \right)}$$

$$\frac{1}{(1+e^{-\pi})(1+e^{-3\pi})(1+e^{-5\pi})} = \frac{1}{\left( 1 + e^{-\left( \frac{3\sqrt{3}}{4} + 24 \int_0^1 \sqrt{t-t^2} dt \right)} \right) \left( 1 + e^{-\left( \frac{3\sqrt{3}}{4} + 24 \int_0^4 \sqrt{t-t^2} dt \right)} \right) \left( 1 + e^{-\left( \frac{3\sqrt{3}}{4} + 24 \int_0^1 \sqrt{t-t^2} dt \right)} \right)}$$

$$\frac{1}{(1+e^{-\pi})(1+e^{-3\pi})(1+e^{-5\pi})} = \frac{e^{18 \int_0^{\infty} \sin(t)/t dt}}{\left( \left( 1 + e^{2 \int_0^{\infty} \sin(t)/t dt} \right)^3 \left( 1 - e^{2 \int_0^{\infty} \sin(t)/t dt} + e^{4 \int_0^{\infty} \sin(t)/t dt} \right) \left( 1 - e^{2 \int_0^{\infty} \sin(t)/t dt} + e^{4 \int_0^{\infty} \sin(t)/t dt} - e^{6 \int_0^{\infty} \sin(t)/t dt} + e^{8 \int_0^{\infty} \sin(t)/t dt} \right) \right)}$$

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