

Pi , A Short Letter

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abstract. We give a formula for Pi

keywords: number Pi, bernoulli numbers.

I. Introduction:

Recall that:

$$(1) \text{ Number Pi: } \pi = 4 \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 3.141592 \dots$$

$$(2) \text{ Bernoulli numbers: } n \geq 1, B_n = \left\{ \frac{1}{6}, \frac{1}{30}, \frac{1}{42}, \frac{1}{30}, \frac{5}{66}, \frac{691}{2730}, \frac{7}{6}, \dots \right\}$$

II. Pi Formula:

$$(3) \pi = 8 \sum_{n=0}^{\infty} \frac{e^{-(2n+1)} \sin(2n+1)}{2n+1} + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} 2^{2n+1} (2^{4n-3}-1) B_{2n-1}}{(2n-1)(4n-2)!}$$

$$(4) B_{2n-1} = \sum_{k=2}^{4n-1} \frac{(-1)^{k-1}}{k} \binom{4n-1}{k} \sum_{m=1}^{k-1} m^{4n-2}, \quad n \geq 1$$

III. References:

[1] Arndt, J., and Haenel, C.: Pi unleashed, Springer-Verlag, 2001.

[2] Blatner, D.: The Joy of Pi, Walker Publ., 1999.

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