The Einstein-Seiberg-Witten equations

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Abstract

We define the ESW equations like the SW equations with a riemannian metric.

1 The Seiberg-Witten equations

The Seiberg-Witten equations $SW(A,\psi)$ over a spin-c manifold M are depending on (A,ψ), A a connection and ψ a spinor, and are defined as:

$$\mathcal{D}_A(\psi) = 0$$

$$F(A)_+(X,Y) = < X.Y.\psi, \psi > + g(X,Y) < \psi, \psi >$$

with $\mathcal{D}_A$ the Dirac operator of the spin-c structure and $F(A)_+$ the self-dual part of the curvature of A.

2 The Einstein-Seiberg-Witten equations

The Einstein-Seiberg-Witten equations are depending on (g, A, ψ), g a riemannian metric and are defined as:

$$\mathcal{D}_A(\psi) = 0$$

$$F(A)_+(X,Y) = < Ric(X).Ric(Y).\psi, \psi > + g(Ric(X), Ric(Y)) < \psi, \psi >$$

with Ric the Ricci curvature viewed as an endomorphism of the tangent bundle. If the manifold is Einstein, then $Ric = \lambda.Id$ and we have:

$$ESW(g, A, \psi) = SW(A, \psi)$$

The gauge group is:

$$\mathcal{G} = \mathcal{C}^\infty(S^1).Diff(M)$$

with $Diff(M)$ the group of diffeomorphisms of the manifold M acting on the ESW structures. The ESW moduli space is then:

$$\mathcal{M}(M) = ESW(g, A, \psi)/\mathcal{G}$$

References

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