Numbers are three dimensional, as nature

Surajit Ghosh, Kolkata, India

30th June, 2019

Abstract

Riemann hypothesis stands proved in three different ways.To prove Riemann hypothesis from the functional equation concept of Delta function is introduced similar to Gamma and Pi function. Other two proofs are derived using Eulers formula and elementary algebra. Analytically continuing gamma and zeta function to an extended domain, poles and zeros of zeta values are redefined.

Keywords— Primes, zeta function, gamma function, analytic continuation of zeta function, Riemann hypothesis

$$R iemannHypothesis$$

$$\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s)$$

Contents

1 Introduction

In this section let us have a short introduction to zeta function and riemann hypothesis on zeta function.

1.1 Euler the grandfather of zeta function

In 1737, Leonard Euler published a paper where he derived a tricky formula that pointed to a wonderful connection between the infinite sum of the reciprocals of all natural integers (zeta function in its simplest form) and all prime numbers.

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+...=\frac{2.3.5.7.11....}{1.2.4.6.8....}$$

Now:

$$1+\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^4...=\frac{2}{1}$$

$$1+\left(\frac{1}{3}\right)+\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^3+\left(\frac{1}{3}\right)^4...=\frac{3}{2}$$

$\vdots$

Euler product form of zeta function when $s>1$:

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}=\prod_p\left(1+\frac{1}{p^s}+\frac{1}{p^{2s}}+\frac{1}{p^{3s}}+\frac{1}{p^{4s}}...\right)$$

Equivalent to:

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}=\prod_p\frac{1}{1-P^{-s}}$$

To carry out the multiplication on the right, we need to pick up exactly one term from every sum that is a factor in the product and, since every integer admits a unique prime factorization, the reciprocal of every integer will be obtained in this manner - each exactly once.

1.2 Riemann the father of zeta function

Riemann might had seen the following relation between zeta function and eta function (also known as alternate zeta function) which converges for all values $\mathrm{Re}(s)>0$.

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}$$

$$\sum_{n=1}^{\infty}\frac{2}{(2n)^s}=\frac{1}{2^{s-1}}\zeta(s)$$

Now subtracting the latter from the former we get:

$$\left(1-\frac{1}{2^{s-1}}\right)\zeta(s)=\frac{1}{1^s}-\frac{1}{2^s}+\frac{1}{3^s}-\ldots=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^s}=:\eta(s)\implies\zeta(s)=\left(1-2^{1-s}\right)^{-1}\eta(s)$$

Then Riemann might had realised that he could analytically continue zeta function from the above equation for $1\neq\ \mathrm{Re}\,(s)>0$ after re-normalizing the potential problematic points. In his seminal paper Riemann showed that zeta function have the property of analytic continuation in the whole complex plane except for s=1 where the zeta function has its pole. Zeta function satisfies Riemann's functional equation.

$$\zeta(s)=2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s)$$

Riemann Hypothesis is all about non trivial zeros of zeta function. There are trivial zeros which occur at every negative even integer. There are no zeros for $s > 1$. All other zeros lies at a critical strip $0 < s < 1$. In this critical strip he conjectured that all non trivial zeros lies on a critical line of the form of $z = \frac{1}{2} \pm iy$ i.e. the real part of all those complex numbers equals $\frac{1}{2}$.

Showing that there are no zeros with real part 1 - Jacques Hadamard and Charles Jean de la Valle-Poussin independently prove the prime number theorem which essentially says that if there exists a limit to the ratio of primes upto a given number and that numbers natural logarithm, that should be equal to 1. When I started reading about number theory I wondered that if prime number theorem is proved then what is left. The biggest job is done. I questioned myself why zeta function cannot be defined at 1. Calculus has got set of rules for checking convergence of any infinite series, sometime especially when we are encapsulating infinities into unity, those rules may fall short to check the convergence of infinite series. In spite of that Euler was successful proving sum to product form and calculated zeta values for some numbers by hand only. Leopold Kronecker proved and interpreted Euler's formulas is the outcome of passing to the right-sided limit as $s \to 1^+$. I decided I will stick to Grandpa Eulers approach in attacking the problem.

2 Proof of Riemann Hypothesis

In this section we shall prove Riemann Hypothesis in different ways.

2.1 An exhaustive proof using Riemanns functional equation

Multiplying both side of Riemanns functional equation by $(s-1)$ we get

$$(1-s)\zeta(s) = 2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)(1-s)\Gamma(1-s)\zeta(1-s)$$

Putting $(1-s)\Gamma(1-s) = \Gamma(2-s)$ we get:

$$\zeta(1-s) = \frac{(1-s)\zeta(s)}{2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(2-s)}$$

$s \to 1$we get: $\because \lim_{s\to 1}(s-1)\zeta(s) = 1 \therefore (1-s)\zeta(s) = -1$ and $\Gamma(2-1) = \Gamma(1) = 1$

$$\zeta(0) = \frac{-1}{2^1 \pi^0 \sin\left(\frac{\pi}{2}\right)} = -\frac{1}{2}$$

Examining the functional equation we shall observe that the pole of zeta function at $Re(s) = 1$ is attributable to the pole of Gamma function. In the critical strip $0 < s < 1$ Delta function (see explanation) holds equally good if not better for factorial function. As zeta function have got the holomorphic property the act of stretching or squeezing preserves the holomorphic character. Using this property we can remove the pole of zeta function. Introducing Delta function for factorial we can remove the poles of Gamma and Pi function and rewrite the functional equation in terms of its harmonic conjugate function as follows(see explanation below):

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Delta(4-s)\zeta(1-s)$$

Which can be rewritten in terms of Gamma function as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$$

Which again can be rewritten in terms of Pi function as follows:

$$\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Pi(2-s)\zeta(1-s)$$

Now Putting $s = 1$we get:

$$\zeta(1) = -2^1 \pi^{(1-1)} \sin\left(\frac{\pi}{2}\right) \Gamma(3-1)\zeta(0) = 1$$

zeta function is now defined on entire $\mathbb{C}$, and as such it becomes an entire function. In complex analysis, Liouville's theorem states that every bounded entire function must be constant. That is, every holomorphic function f for which there exists a positive number M such that $|f(z)| \leq M$ for all z in $\mathbb{C}$ is constant. Being an entire function zeta function is constant as none of the values of zeta function do not exceed $M = \zeta(2) = \frac{\pi^2}{6}$.Maximum modulus principle further requires that non constant holomorphic functions attain maximum modulus on the boundary of the unit circle. Being a constant function zeta function duly complies with maximum modulus principle as it reaches maximum modulus $\frac{\pi^2}{6}$ outside the unit circle i.e. on the boundary of the double unit circle. Gauss's mean value theorem requires that in case a function is bounded in some neighborhood, then its mean value shall occur at the center of the unit circle drawn on the neighborhood. $|\zeta(0)| = \frac{1}{2}$ is the mean modulus of entire zeta function. Inverse of maximum modulus principle implies points on half unit circle give the minimum modulus or zeros of zeta function. Minimum modulus principle requires holomorphic functions having all non zero values shall attain minimum modulus on the boundary of the unit circle. Having lots of zero values holomorphic zeta function do not attain minimum modulus on the boundary of the unit circle rather points on half unit circle gives the minimum modulus or zeros of zeta function. Everything put together it implies that points on the half unit circle will mostly be the zeros of the zeta function which all have $\pm\frac{1}{2}$ as real part as Riemann rightly hypothesized.

Putting $s = \frac{1}{2}$ in $\zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \Gamma(3-s)\zeta(1-s)$

$$\boxed{\begin{aligned}
\zeta\left(\frac{1}{2}\right) &= -2^{\frac{1}{2}} \pi^{(1-\frac{1}{2})} \sin\left(\frac{\pi}{2.2}\right) \Gamma\left(\frac{5}{2}\right) \zeta\left(\frac{1}{2}\right) \\
\zeta\left(\frac{1}{2}\right)\left(1 + \frac{3\sqrt{2.\pi.\pi}}{4.\sqrt{2}}\right) &= 0 \\
\zeta\left(\frac{1}{2}\right)\left(1 + \frac{3\pi}{4}\right) &= 0 \\
\zeta\left(\frac{1}{2}\right) &= 0
\end{aligned}}$$

Therefore principal value of $\zeta(\frac{1}{2})$ is zero and Riemann Hypothesis holds good.

2.1.1 Introduction of Delta function

Explanation 1 *Euler in the year 1730 proved that the following indefinite integral gives the factorial of x for all real positive numbers,*

$$x! = \Pi(x) = \int_0^\infty t^x e^{-t} dt, x > 1$$

Eulers Pi function satisfies the following recurrence relation for all positive real numbers.

$$\Pi(x+1) = (x+1)\Pi(x), x > 0$$

In 1768, Euler defined Gamma function, $\Gamma(x)$, and extended the concept of factorials to all real negative numbers, except zero and negative integers. $\Gamma(x)$, is an extension of the Pi function, with its argument shifted down by 1 *unit.*

$$\Gamma(x)=\int_0^{\infty} t^{x-1}e^{-t}dt$$

Eulers Gamma function is related to Pi function as follows:

$$\Gamma(x+1)=\Pi(x)=x!$$

Now let us extend factorials of negative integers by way of shifting the argument of Gamma function further down by 1 unit.Let us define Delta function as follows:

$$\Delta(x)=\int_0^{\infty} t^{x-2}e^{-t}dt$$

The extended Delta function shall have the following recurrence relation.

$$\Delta(x+2)=(x+2)\Delta(x+1)=(x+2)(x+1)\Delta(x)=x!$$

Newly defined Delta function is related to Eulers Gamma function and Pi function as follows:

$$\Delta(x+2)=\Gamma(x+1)=\Pi(x)$$

Plugging into $x=2$ above

$$\Delta(4)=\Gamma(3)=\Pi(2)=2$$

Plugging into $x=1$ above

$$\Delta(3)=\Gamma(2)=\Pi(1)=1$$

Plugging into $x=0$ above

$$\Delta(2)=\Gamma(1)=\Pi(0)=1$$

Plugging into $x=-1$ above we can remove poles of Gamma and Pi function as follows:

$$\Delta(1)=\Gamma(0)=\Pi(-1)=1.\Delta(0)=-1.\Delta(-1)=\int_0^{\infty} t^{1-1}e^{-t}dt=\Big[-e^{-x}\Big]_0^{\infty}=\lim_{x\to\infty} -e^{-x}-e^{-0}=0+1=1$$

Therefore we can say $\Delta(-1)=-1$. Similarly plugging into $x=-2$ above

$$\Delta(0)=\Gamma(-1)=\Pi(-2)=-1.\Delta(-1)=-2.\Delta(-2)=\int_0^{\infty} t^{0}e^{-t}dt=\Big[-e^{-x}\Big]_0^{\infty}=\lim_{x\to\infty} -e^{-x}-e^{-0}=0+1=1$$

Therefore we can say $\Delta(-2)=-\frac{1}{2}$. Continuing further we can remove poles of Gamma and Pi function:

Plugging into $x=-3$ above and equating with result found above

$$\Delta(-1)=\Gamma(-2)=\Pi(-3)=-2.-1.\Delta(-3)=-1\implies\Delta(-3)=-\frac{1}{2}$$

Plugging into $x=-4$ above and equating with result found above

$$\Delta(-2)=\Gamma(-3)=\Pi(-4)=-3.-2.\Delta(-4)=-\frac{1}{2}\implies\Delta(-4)=-\frac{1}{12}$$

Plugging into $x=-5$ above and equating with result found above

$$\Delta(-3)=\Gamma(-4)=\Pi(-5)=-4.-3.\Delta(-5)=-\frac{1}{2}\implies\Delta(-5)=-\frac{1}{24}$$

Plugging into $x=-6$ above and equating with result found above

$$\Delta(-4)=\Gamma(-5)=\Pi(-6)=-5.-4.\Delta(-6)=-\frac{1}{12}\implies\Delta(-6)=-\frac{1}{240}$$

Plugging into $x = -7$ above and equating with result found above

$$\Delta(-5) = \Gamma(-6) = \Pi(-7) = -6. -5.\Delta(-7) = -\frac{1}{24} \implies \Delta(-7) = -\frac{1}{720}$$

Plugging into $x = -8$ above and equating with result found above

$$\Delta(-6) = \Gamma(-7) = \Pi(-8) = -7. -6.\Delta(-8) = -\frac{1}{240} \implies \Delta(-8) = -\frac{1}{10080}$$

$\vdots$

And the pattern continues upto infinity.

2.1.2 Alternate functional equation

Explanation 2 *Multiplying both side of Riemanns functional equation by $(s-1)$ we get*

$$(1-s)\zeta(s) = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)(1-s)\Gamma(1-s)\zeta(1-s)$$

Putting $(1-s)\Gamma(1-s) = \Gamma(2-s)$ we get:

$$(1-s)\zeta(s) = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(2-s)\zeta(1-s)$$

$s \to 1$*we get:* $\because \lim_{s\to 1}(s-1)\zeta(s) = 1 \therefore (1-s)\zeta(s) = -1$

$$\boxed{2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(2-s)\zeta(1-s) = -1}$$

Similarly multiplying both numerator and denominator right hand side of Riemanns functional equation by $(1-s)(2-s)$ before applying any limit we get :

$$\zeta(s) = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\frac{(1-s)(2-s)\Gamma(1-s)\zeta(1-s)}{(1-s)(2-s)}$$

Putting $(1-s)(2-s)\Gamma(1-s) = \Gamma(3-s)$ we get:

$$\zeta(s) = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\frac{\Gamma(3-s)\zeta(1-s)}{(1-s)(2-s)}$$

Multiplying both side of the above equation by $(1-s)$ we get

$$(1-s)\zeta(s) = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\frac{\Gamma(3-s)\zeta(1-s)}{(2-s)}$$

$s \to 1$*we get:* $\because \lim_{s\to 1}(s-1)\zeta(s) = 1 \therefore (1-s)\zeta(s) = -1$

$$-1 = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\frac{\Gamma(3-s)\zeta(1-s)}{(2-s)}$$

Multiplying both side of the above equation further by $(2-s)$ we get:

$$(s-2) = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$$

Multiplying both side of the above equation by $\zeta(s-1)$ we get

$$(s-2)\zeta(s-1) = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)\zeta(s-1)$$

$s \to 2$ *we get:* $\because \lim_{s\to 2}(s-2)\zeta(s-1) = 1$

$$2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)\zeta(s-1) = 1$$

To manually define zeta function such a way that it takes value 1 or mathematically $\exists! s \in \mathbb{N}; \zeta(s-1) = 1$, Euler's induction approach was applied and it was observed that zeta function have the potential unit value as demonstrated in the section 3.1 & 3.3.So we can set $\zeta(s-1) = 1$ and we can write

$$2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s) = 1$$

Multiplying above equation by -1 we get

$$\boxed{-2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s) = -1}$$

Both the above boxed forms are equivalent to Riemann's original functional equation therefore Riemann's original functional equation can be analytically continued as:

$$\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Delta(4-s)\zeta(1-s)$$

Which can be rewritten in terms of Gamma function as follows:

$$\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$$

Which again can be rewritten in terms of Pi function as follows:

$$\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Pi(2-s)\zeta(1-s)$$

Justification of the definition we set for $\zeta(3-2) = 1$ and consistency of the above forms of functional equation have been cross checked in the main proof and also it was found that the proposition complies with all the theorems used in complex analysis.Justification of the definition we set for $\zeta(-1) = \frac{1}{2}$ and consistency of the above forms of functional equation have been cross checked in the in the section 3.2. $\zeta(-1) = \frac{1}{2}$ must be the second solution to $\zeta(-1)$ apart from the known Ramanujan's proof $\zeta(-1) = \frac{-1}{12}$. One has to accept that following the zeta functions analytic behavior zeta values can be multivalued.

2.2 An elegant proof using Eulers original product form

Eulers Product form of zeta Function in Eulers exponential form of complex numbers is as follows:

$$\zeta(s) = \sum_{n=1}^{\infty}\frac{1}{n^s} = \prod_p\left(1 + re^{i\theta} + r^2e^{i2\theta} + r^3e^{i3\theta}...\right)$$

Now any such factor $\left(1 + re^{i\theta} + r^2e^{i2\theta} + r^3e^{i3\theta}...\right)$ will be zero if

$$\left(re^{i\theta} + r^2e^{i2\theta} + r^3e^{i3\theta}...\right) = -1 = e^{i\pi}$$

Comparing both side of the equation and equating left side to right side on the unit circle we can say: *

$$\boxed{\theta + 2\theta + 3\theta + 4\theta... = \pi}$$

$$\boxed{r + r^2 + r^3 + r^4.... = 1}$$

We can solve θ and r as follows:

$$\boxed{\begin{aligned} \theta + 2\theta + 3\theta + 4\theta... &= \pi \\ \theta(1 + 2 + 3 + 4...) &= \pi \\ \theta.\zeta(-1) &= \pi \\ \theta.\frac{-1}{12} &= \pi \\ \theta &= -12\pi \end{aligned} \qquad \begin{aligned} r + r^2 + r^3 + r^4.... &= 1 \\ r(1 + r + r^2 + r^3 + r^4....) &= 1 \\ r\frac{1}{1-r} &= 1 \\ r &= 1 - r \\ r &= \frac{1}{2} \end{aligned}}$$

We can determine the real part of the non trivial zeros of zeta function as follows:

$$\boxed{r\cos\theta = \frac{1}{2}\cos(-12\pi) = \frac{1}{2}}$$

Therefore Principal value of $\zeta(\frac{1}{2})$ will be zero, hence Riemann Hypothesis is proved.

Explanation 3 ** We can try back the trigonometric form then the algebraic form of complex numbers do the summation algebraically and then come back to exponential form as follows:*

$$\begin{aligned} & re^{i\theta} + r^2e^{i2\theta} + r^3e^{i3\theta}... \\ &= (r\cos\theta + ir\sin\theta) + (r^2\cos 2\theta + ir^2\sin 2\theta) + (r^3\cos 3\theta + ir^3\sin 3\theta) + (r^4\cos 4\theta + ir^4\sin 4\theta).... \\ &= (x_1 + iy_1) + (x_2 + iy_2) + (x_3 + iy_3) + (x_4 + iy_4) + (x_5 + iy_5).... \\ &= (x_1 + x_2 + x_3 + x_4 + x_5 + ...) + i(y_1 + y_2 + y_3 + y_4 + y_5 + ...) \\ &= R\cos\Theta + iR\sin\Theta \\ &= (r + r^2 + r^3 + r^4....)e^{i(\theta+2\theta+3\theta+4\theta...)} \end{aligned}$$

Explanation 4 *One may attempt to show that* $(re^{i\theta}+r^2e^{i2\theta}+r^3e^{i3\theta}...) = -1$ *actually results* $\frac{re^{i\theta}}{1-re^{i\theta}}$ *which implies in absurdity of* $0 = -1$*. Correct way to evaluate* $\frac{re^{i\theta}}{1-re^{i\theta}}$ *is to apply the complex conjugate of denominator before reaching any conclusion.* $\frac{re^{i\theta}(1+re^{i\theta})}{(1-re^{i\theta})(1+re^{i\theta})}$ *then shall result to* $re^{i\theta} = -1$ *which points towards the unit circle. In the present proof we need to go deeper into the d-unit circle and come up with the interpretation which can explain the Riemann Hypothesis.*

Explanation 5 *One may attempt to show inequality of the reverse calculation* $\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}... = 1 \neq -1$*.* $re^{i\pi} = -1$ *need to be interpreted as the exponent which then satisfies* $1^{-1} = 1$ *or* $2.2^{-1} = 1$ *on the unit or d-unit circle. There is nothing called t-unit circle satisfying* $3.3^{-1} = 1$*.*

2.3 An elementary proof using alternate product form

Eulers alternate Product form of zeta Function in Eulers exponential form of complex numbers is as follows:

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(\frac{1}{1 - \frac{1}{re^{i\theta}}} \right) = \prod_p \left(\frac{re^{i\theta}}{re^{i\theta} - 1} \right)$$

Multiplying both numerator and denominator by $re^{i\theta} + 1$we get:

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_p \left(\frac{re^{i\theta}(re^{i\theta} + 1)}{(re^{i\theta} - 1)(re^{i\theta} + 1)} \right)$$

Now any such factor $\left(\frac{re^{i\theta}(re^{i\theta}+1)}{(r^2e^{i2\theta}-1)} \right)$ will be zero if $re^{i\theta}(re^{i\theta} + 1)$ is zero:

$$\boxed{\begin{aligned} re^{i\theta}(re^{i\theta} + 1) &= 0 \\ re^{i\theta}(re^{i\theta} - e^{i\pi}) &= 0 \\ r^2e^{i2\theta} - re^{i(\pi-\theta)*} &= 0 \\ r^2e^{i2\theta} &= re^{i(\pi-\theta)} \end{aligned}}$$

We can solve θ and r as follows:

$$\boxed{\begin{array}{rrrr} 2\theta = & (\pi - \theta) & r^2 = & r \\ 3\theta = & \pi & \frac{r^2}{r} = & \frac{r}{r} \\ \theta = & \frac{\pi}{3} & r = & 1 \end{array}}$$

We can determine the real part of the non trivial zeros of zeta function as follows:

$$\boxed{r\cos\theta = 1.\cos(\tfrac{\pi}{3}) = \tfrac{1}{2}}$$

Therefore Principal value of $\zeta(\frac{1}{2})$ will be zero, and Riemann Hypothesis is proved.

Explanation 6 ** $e^{i(-\theta)}$ is arrived as follows:*

$$e^{i\theta} = \left(e^{i\theta}\right)^1 = \left(e^{i\theta}\right)^{1^{-1}} = \left(e^{i\theta}\right)^{-1^1} = \left(\left(e^{i\theta}\right)^{i^2}\right)^1 = \left(e^{i\theta}\right)^{i^2} = e^{i^3(\theta)} = e^{-i\theta}$$

Explanation 7 *Essentially proving* $\log_2(\frac{1}{2}) = -1$ *in a complex turned simple way is equivalent of saying* $\log(1) = 0$ *in real way. Primes other than 2 satisfy* $\log_p(\frac{1}{2}) = e^{i\theta}$ *also in a pure complex way.*

3 Infinite product of zeta values

3.1 Infinite product of positive zeta values converges

$$\zeta(1) = 1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1}... = \left(1 + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}...\right)\left(1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right)\left(1 + \frac{1}{5^1} + \frac{1}{5^2}...\right)...$$

$$\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}... = \left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6}...\right)\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6}...\right)\left(1 + \frac{1}{5^2} + \frac{1}{5^4}...\right)...$$

$$\zeta(3) = 1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3}... = \left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9}...\right)\left(1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9}...\right)\left(1 + \frac{1}{5^3} + \frac{1}{5^6}...\right)...$$

$$\vdots$$

From the side of infinite sum of negative exponents of all natural integers:

$$\begin{aligned}
&\zeta(1)\zeta(2)\zeta(3)...\\
&= \left(1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1}...\right)\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}...\right)\left(1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3}...\right)...\\
&= 1 + \left(\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}...\right) + \left(\frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right) + \left(\frac{1}{4^1} + \frac{1}{4^2} + \frac{1}{4^3}...\right)...\\
&= 1 + 1 + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1} + \frac{1}{5^1} + \frac{1}{6^1} + \frac{1}{7^1} + \frac{1}{8^1} + \frac{1}{9^1}...\\
&= 1 + \zeta(1)
\end{aligned}$$

$$\vdots$$

From the side of infinite product of sum of negative exponents of all primes:

$$\begin{aligned}
&\zeta(1)\zeta(2)\zeta(3)... =\\
&\left(1 + \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3}...\right)\left(1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right)\left(1 + \frac{1}{5^1} + \frac{1}{5^2} + \frac{1}{5^3}...\right)...\\
&\left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6}...\right)\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6}...\right)\left(1 + \frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6}...\right)...\\
&\left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9}...\right)\left(1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9}...\right)\left(1 + \frac{1}{5^3} + \frac{1}{5^6} + \frac{1}{5^9}...\right)...
\end{aligned}$$

$$\vdots$$

$$\begin{aligned}
&= \left(1 + 1\right)\left(1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right)\left(1 + \frac{1}{5^1} + \frac{1}{5^2} + \frac{1}{5^3}...\right)...\\
&\left(1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6}...\right)\left(1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6}...\right)\left(1 + \frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6}...\right)...\\
&\left(1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9}...\right)\left(1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9}...\right)\left(1 + \frac{1}{5^3} + \frac{1}{5^6} + \frac{1}{5^9}...\right)...
\end{aligned}$$

$$\vdots$$

continued to next page....

continued from last page....

Simultaneously halfing and doubling each factor and writing it sum of two equivalent forms

$$= 2\left(\frac{1}{2}\left(1 + \frac{\frac{1}{3}}{1-\frac{1}{3}} + 1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right)\right)\left(\frac{1}{2}\left(1 + \frac{\frac{1}{5}}{1-\frac{1}{5}} + 1 + \frac{1}{5^1} + \frac{1}{5^2} + \frac{1}{5^3}...\right)\right)...$$

$$\left(\frac{1}{2}\left(1 + \frac{\frac{1}{4}}{1-\frac{1}{4}} + 1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6}...\right)\right)\left(\frac{1}{2}\left(1 + \frac{\frac{1}{9}}{1-\frac{1}{9}} + 1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6}...\right)\right)...$$

$$\left(\frac{1}{2}\left(1 + \frac{\frac{1}{8}}{1-\frac{1}{8}} + 1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9}...\right)\left(\frac{1}{2}\left(1 + \frac{\frac{1}{27}}{1-\frac{1}{27}} + 1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9}...\right)\right)...$$

$$\vdots$$

$$= 2\left(\frac{1}{2}\left(1 + \frac{1}{2} + 1 + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right)\right)\left(\frac{1}{2}\left(1 + \frac{1}{4} + 1 + \frac{1}{5^1} + \frac{1}{5^2} + \frac{1}{5^3}...\right)\right)...$$

$$\left(\frac{1}{2}\left(1 + \frac{1}{3} + 1 + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6}...\right)\left(\frac{1}{2}\left(1 + \frac{1}{8} + 1 + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6}...\right)\right)...$$

$$\left(\frac{1}{2}\left(1 + \frac{1}{7} + 1 + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9}...\right)\left(\frac{1}{2}\left(1 + \frac{1}{26} + 1 + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9}...\right)\right)...$$

$$\vdots$$

$$= 2\left(1 + \frac{1}{2}\left(\frac{1}{2} + \frac{1}{3^1} + \frac{1}{3^2} + \frac{1}{3^3}...\right)\right)\left(1 + \frac{1}{2}\left(\frac{1}{4} + \frac{1}{5^1} + \frac{1}{5^2} + \frac{1}{5^3}...\right)\right)...$$

$$\left(1 + \frac{1}{2}\left(\frac{1}{3} + \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6}...\right)\right)\left(1 + \frac{1}{2}\left(\frac{1}{8} + \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6}...\right)\right)...$$

$$\left(1 + \frac{1}{2}\left(\frac{1}{7} + \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9}...\right)\right)\left(1 + \frac{1}{2}\left(\frac{1}{26} + \frac{1}{3^3} + \frac{1}{3^6} + \frac{1}{3^9}...\right)\right)...$$

$$\vdots$$

$$= 2\left(1 + \frac{1}{2}\left(\frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1}... + \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{4^1}...\right)\right)$$

$$= 2\left(1 + \frac{1}{2}\left(2\zeta(1) - 2\right)\right)$$

$$= 2(1 - 1 + \zeta(1))$$

$$= 2\zeta(1)$$

Now comparing two identities:

$$\boxed{1 + \zeta(1) = 2\zeta(1))}$$

$$\boxed{\zeta(1) = 1}$$

Hence Infinite product of positive zeta values converges to 2

3.2 Infinite product of negative zeta values converges

$$\zeta(-1) = 1 + 2^1 + 3^1 + 4^1 + 5^1 ... = \Big(1 + 2 + 2^2 + 2^3 ...\Big)\Big(1 + 3 + 3^2 + 3^3 ...\Big)\Big(1 + 5 + 5^2 + 5^3 ...\Big)...$$

$$\zeta(-2) = 1 + 2^2 + 3^2 + 4^2 + 5^2 ... = \Big(1 + 2^2 + 2^4 + 2^6 ...\Big)\Big(1 + 3^2 + 3^4 + 3^6 ...\Big)\Big(1 + 5^2 + 5^4 + 5^6 ...\Big)...$$

$$\zeta(-3) = 1 + 2^3 + 3^3 + 4^3 + 5^3 ... = \Big(1 + 2^3 + 2^6 + 2^9 ...\Big)\Big(1 + 3^3 + 3^6 + 3^9 ...\Big)\Big(1 + 5^3 + 5^6 + 5^9 ...\Big)...$$

$\vdots$

From the side of infinite sum of negative exponents of all natural integers:

$$\begin{aligned}
&\zeta(-1)\zeta(-2)\zeta(-3)...\\
&= \Big(1 + 2^1 + 3^1 + 4^1 + 5^1 ...\Big)\Big(1 + 2^2 + 3^2 + 4^2 + 5^2 ...\Big)\Big(1 + 2^3 + 3^3 + 4^3 + 5^3 ...\Big)...\\
&= 1 + \Big(2 + 2^2 + 2^3 ...\Big) + \Big(3 + 3^2 + 3^3 ...\Big) + \Big(4 + 4^2 + 4^3 ...\Big)...\\
&= 1 + \Big(1 + 2 + 2^2 + 2^3 ... - 1\Big) + \Big(1 + 3 + 3^2 + 3^3 ... - 1\Big) + \Big(1 + 4 + 4^2 + 4^3 ... - 1\Big)...\\
&= 1 + \Big(-\frac{1}{1} - 1\Big) + \Big(-\frac{1}{2} - 1\Big) + \Big(-\frac{1}{3} - 1\Big) + \Big(-\frac{1}{4} - 1\Big)...\\
&= 1 - \Big(\Big(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} ...\Big) + 1 + 1 + 1 + 1 ...\Big)\\
&= 1 - \Big(\zeta(1) + \zeta(0)\Big)\\
&= 1 - \Big(1 - \frac{1}{2}\Big)\\
&= \frac{1}{2}
\end{aligned}$$

From the side of infinite product of sum of negative exponents of all primes:

$$\begin{aligned}
&\zeta(-1)\zeta(-2)\zeta(-3)... =\\
&\Big(1 + 2 + 2^2 + 2^3 ...\Big)\Big(1 + 3 + 3^2 + 3^3 ...\Big)\Big(1 + 5 + 5^2 + 5^3 ...\Big)...\\
&\Big(1 + 2^2 + 2^4 + 2^6 ...\Big)\Big(1 + 3^2 + 3^4 + 3^6 ...\Big)\Big(1 + 5^2 + 5^4 + 5^6 ...\Big)...\\
&\Big(1 + 2^3 + 2^6 + 2^9 ...\Big)\Big(1 + 3^3 + 3^6 + 3^9 ...\Big)\Big(1 + 5^3 + 5^6 + 5^9 ...\Big)...\\
&\vdots\\
&= 1 + 2^1 + 3^1 + 4^1 + 5^1 ...\\
&= \zeta(-1)
\end{aligned}$$

Therefore $\boxed{\zeta(-1)=\frac{1}{2}}$ must be the second solution of $\zeta(-1)$ apart from the known one $\zeta(-1)=\frac{-1}{12}$.

Using Delta function instead of Gamma function we can rewrite the functional equation applicable as follows:

$$\zeta(s)=-2^{s}\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Delta(4-s)\zeta(1-s)$$

Which can be rewritten in terms of Gamma function as follows:

$$\zeta(s)=-2^{s}\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$$

Which again can be rewritten in terms of Pi function as follows:

$$\zeta(s)=-2^{s}\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Pi(2-s)\zeta(1-s)$$

Putting $s=-1$we get:

$$\zeta(-1)=-2^{-1}\pi^{(-1-1)}\sin\left(\frac{-\pi}{2}\right)\Gamma(3-s)\zeta(2)=\frac{1}{2}$$

To proof Ramanujans Way

$$\sigma=\boxed{1+2+3+4+5+6+7+8+9.....}$$
$$2\sigma=\boxed{0+1+2+3+4+5+6+7+8+9...}+1+1+1+1+1+1+1...*$$

Subtracting the bottom from the top one we get:

$$-\sigma=0+1+1+1+1+1+1+1+1...+1+1+1+1+1+1+1...$$
$$\sigma=-(1+1+1+1+1+1+1+1+1+1........)$$
$$\sigma=\frac{1}{2}$$

*The second part is calculated subtracting bottom from the top before doubling.

3.3 Counter proof on Nicole Oresme's proof of divergent harmonic series

Nicole Oresme in around 1350 proved divergence of harmonic series by comparing the harmonic series with another divergent series. He replaced each denominator with the next-largest power of two.

$$\Rightarrow 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}...$$
$$>1+\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}...$$
$$>1+\left(\frac{1}{2}\right)+\left(\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right)+...$$
$$>1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}...$$

He then concluded that the harmonic series must diverge as the above series diverges.

It was too quick to conclude as we can go ahead and show:

$$R.H.S=1+\frac{1}{2}\left(1+1+1+1+1+1+1+1+1+1+1+1+1+...\right)$$
$$=1+\frac{1}{2}.\frac{-1}{2}$$
$$=1-\frac{1}{4}$$

If we consider $\zeta(1)=1$ then also it passes the comparison test.

Therefore We need to come out of the belief that harmonic series diverges.Continuing further we can show

$$R.H.S=1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\left(1+1+1...\right)$$
$$=1+\frac{3}{2}+\frac{1}{2}.\frac{-1}{2}$$
$$=1+\frac{3}{2}-\frac{1}{4}$$
$$=1+\frac{3}{2}-\left(1-2+3-4+...\right)$$
$$=1+\frac{3}{2}-\left(\left(1+2+3...\right)-2\left(1+2+4...\right)\right)$$
$$=1+\frac{3}{2}-\left(\frac{1}{2}-2\left(1+1+1...\right)\right)$$
$$=1+\frac{3}{2}-\left(\frac{1}{2}-2\frac{-1}{2}\right)$$
$$=1+\frac{3}{2}-\left(\frac{1}{2}+1\right)$$
$$=1+\frac{3}{2}-\frac{3}{2}$$
$$=1$$

$$R.H.S=1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\left(1+1+1...\right)$$
$$=1+\frac{5}{2}+\frac{1}{2}.\frac{-1}{2}$$
$$=1+\frac{5}{2}-\frac{1}{4}$$
$$=1+\frac{5}{2}-\left(1-2+3-4+...\right)$$
$$=1+\frac{5}{2}-\left(\left(1+2+3...\right)-2\left(1+2+4...\right)\right)$$
$$=1+\frac{5}{2}-\left(\frac{1}{2}-2\left(\frac{1}{1-2}\right)\right)$$
$$=1+\frac{5}{2}-\left(\frac{1}{2}+2\right)$$
$$=1+\frac{5}{2}-\left(\frac{1+4}{2}\right)$$
$$=1+\frac{5}{2}-\frac{5}{2}$$
$$=1$$

According residue theorem we can have a Laurent expansion of an analytic function at the pole $f(s)=\sum_{n=-\infty}^{\infty}a_n(s-s_0)^n$ of f in a punctured disk around s_0, and therefrom we can have $\operatorname{Res}(f(s);s_0)=a_{-1}$, i.e. the residue is the coefficient of $(s-s_0)^{-1}$ in that expansion. For the pole $\zeta(1)$, we know the start of the Laurent series (since it is a simple pole, there is only one term with a negative exponent), namely $\zeta(s)=\frac{1}{s-1}+\gamma+\ldots$ so we have $\operatorname{Res}(\zeta(s);1)=1$. At the pole zeta function have zero radius of convergence. Interpreting zeta function at the pole to be divergent is extreme arbitrary, contradictory and void of rationality. The pole neither falls outside the radius of convergence resulting $\zeta(1) = \infty$ nor inside the radius of convergence resulting $\zeta(1) = 1$, its just on the zero radius of convergence suggesting both values to be equally good. Since none of the above value is more natural than the others, all of them can be incorporated into a multivalued zeta function which is again totally consistent with harmonic conjugate theorem and allows us to interpret $\Rightarrow 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} ... = 1$

4 Zeta results confirms Cantors theory

Cantors theorem, in set theory, the theorem that the cardinality (numerical size) of a set is strictly less than the cardinality of its power set, or collection of subsets. In symbols, a finite set S with n elements contains $2n$ subsets, so that the cardinality of the set S is n and its power set $P(S)$ is $2n$. While this is clear for finite sets, no one had seriously considered the case for infinite sets before the German mathematician George Cantor who is universally recognized as the founder of modern set theorybegan working in this area toward the end of the 19th century.The 1891 proof of Cantors theorem for infinite sets rested on a version of his so-called diagonalization argument, which he had earlier used to prove that the cardinality of the rational numbers is the same as the cardinality of the integers by putting them into a one-to-one correspondence.

We have seen both sum and product of positive Zeta values are greater than sum and product of negative Zeta values respectively. This actually proves a different flavor of Cantors theory numerically. If negative Zeta values are associated with the set of rational numbers and positive Zeta values are associated with the set of natural numbers then the numerical inequality between sum and product of both proves that there are more ordinal numbers in the form of rational numbers than cardinal numbers in the form of natural numbers in spite of having one to one correlation among them. This actually happens because of dual nature of reality. Every unit fractions can be written in two different ways i.e. one upon the integer or two upon the double of the integer as they are equivalent. But the number of integer representation being unique will always fall short of the former. Even if we bring into products,factors,sum,partitions etc. then also the result remain same. So there are more rational numbers than natural numbers. Stepping down the ladder we can say there are more ordinal numbers than cardinal numbers.

5 Zeta results confirms Prime Number theorem

In number theory, the prime number theorem (PNT) describes the asymptotic distribution of the prime numbers among the positive integers. It formalizes the intuitive idea that primes become less common as they become larger by precisely quantifying the rate at which this occurs. The theorem was proved independently by Jacques Hadamard and Charles Jean de la Valle Poussin in 1896 using ideas introduced by Bernhard Riemann (in particular, the Riemann zeta function). The first such distribution found is $\pi(N) \sim \frac{N}{\log N}$, where $\pi(N)$ is the prime-counting function and $\log N$ is the natural logarithm of N. This means that for large enough N, the probability that a random integer not greater than N is prime is very close to $\frac{1}{\log N}$. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ can also be written as $\lim_{n\to\infty}\left(2+\frac{2}{n}\right)^n$. For this same reason prime number theorem works nicely and primes appear through zeta zeros on critical half line in analytic continuation of zeta function.

6 Primes product = 2.Sum of numbers

We know :

$\zeta(-1) = \zeta(1) + \zeta(0)$

$$or\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}...\right)+\left(1+1+1+1+...\right)=\frac{1}{2}$$

$$or\left(1+1\right)+\left(1+\frac{1}{2}\right)+\left(1+\frac{1}{3}\right)+\left(1+\frac{1}{4}\right)+...=\frac{1}{2}$$

$$or\left(\frac{2}{1}+\frac{3}{2}+\frac{4}{3}+\frac{5}{4}+\frac{6}{5}...\right)=\frac{1}{2}$$

LCM of the denominators can be shown to equal the square root of primes product.

Reversing the numerator sequence can shown to equal the sum of integers.

$$or\left(\frac{1+2+3+4+5+6+7...*}{2.3.5.7.11...**}\right)=\frac{1}{2}$$

$$or 2.\sum_{N=1}^{\infty} N = \prod_{i=1}^{\infty} P_i$$

*Series of terms written in reverse order.

**Product of All numbers can be written as 2 series of infinite product of all prime powers

**One arises from individual numbers and another from the number series.Then

$$LCM = \prod_{i=1}^{\infty} P_i^1.P_i^2.P_i^3.P_i^4.P_i^5.P_i^6...P_i^1.P_i^2.P_i^3.P_i^4.P_i^5.P_i^6...$$

$$LCM = \prod_{i=1}^{\infty} P_i^{(1+2+3+4+5+6+7...)+(1+2+3+4+5+6+7...)}...$$

$$LCM = \prod_{i=1}^{\infty} P_i^{\frac{1}{2}+\frac{1}{2}}...$$

$$LCM = 2.3.5.7.11...$$

Intuitively the above relation between sum of numbers and product of primes including the sole even prime must be universally true as it re-proves the fundamental theorem of arithmetic.We can use this to prove Goldbach conjecture and Twin prime conjecture.

7 Negative Zeta values redefined in light of new evidences

Having found that zeta function can take two equally likely values for negative arguments we get the chance of redefining negative zeta values as follows.

7.1 Negative even zeta values redefined removing trivial zeros

We can apply Euler's reflection formula for Gamma function $\Gamma(1-s)\Gamma(s) = \frac{\pi}{\sin(\pi s)}$, $s \notin \mathbb{Z}$ in Riemann's functional equation $\zeta(s) = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s)$ to get another representation of zeta function as follows:

$$\zeta(s) = 2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\frac{\pi}{\Gamma(s)\sin(\pi s)}\zeta(1-s)$$

$$\implies \zeta(s) = 2^s\pi^{(s)}\sin\left(\frac{\pi s}{2}\right)\frac{1}{\Gamma(s)2\sin(\frac{\pi s}{2})\cos(\frac{\pi s}{2})}\zeta(1-s)$$

$$\implies \zeta(s) = 2^{s-1}\pi^{(s)}\frac{1}{\Gamma(s)\cos(\frac{\pi s}{2})}\zeta(1-s)$$

When x=-2, $\zeta(-2) = 2^{-2-1}\pi^{(-2)}\frac{1}{\Gamma(-2)\cos(\frac{-2\pi}{2})}\zeta(1+2) = \frac{\zeta(3)}{4\pi^2} \approx 0.030448282$

When x=-4, $\zeta(-4) = 2^{-4-1}\pi^{(-4)}\frac{1}{\Gamma(-4)\cos(\frac{-4\pi}{2})}\zeta(1+4) = \frac{3\zeta(5)}{8\pi^4} \approx 0.003991799$

When x=-6, $\zeta(-6) = 2^{-6-1}\pi^{(-6)}\frac{1}{\Gamma(-6)\cos(\frac{-6\pi}{2})}\zeta(1+6) = \frac{15\zeta(7)}{8\pi^6} \approx 0.001966568$

When x=-8, $\zeta(-8) = 2^{-8-1}\pi^{(-8)}\frac{1}{\Gamma(-8)\cos(\frac{-8\pi}{2})}\zeta(1+8) = \frac{315\zeta(9)}{16\pi^8} \approx 0.00207904$

⋮

And the pattern continues for all negative even numbers upto negative infinity.

7.2 Negative odd zeta values defined following zeta harmonic conjugate function

Earlier we found that following harmonic conjugate theorem Riemann's functional equation which is an extension of real valued zeta function can also be represented as its harmonic conjugate function which mimic the extended function.

$$\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$$

We can get the harmonic conjugates of negative zeta values as follows:

When s=-1 $\zeta(-1) = -2^{-1}\pi^{(-1-1)}\sin\left(\frac{-1\pi}{2}\right)\Gamma(3+1)\zeta(1+1) = \frac{1}{2}$

When s=-3 $\zeta(-3) = -2^{-3}\pi^{(-3-1)}\sin\left(\frac{-3\pi}{2}\right)\Gamma(3+3)\zeta(1+3) = \frac{-1}{6}$

When s=-5 $\zeta(-5) = -2^{-5}\pi^{(-5-1)}\sin\left(\frac{-5\pi}{2}\right)\Gamma(3+5)\zeta(1+5) = \frac{1}{6}$

When s=-7 $\zeta(-7) = -2^{-7}\pi^{(-7-1)}\sin\left(\frac{-7\pi}{2}\right)\Gamma(3+7)\zeta(1+7) = \frac{-3}{10}$

⋮

And the pattern continues for all negative odd numbers upto negative infinity.

7.3 Negative even zeta values following zeta harmonic conjugate function

We can apply Euler's reflection formula for Gamma function $\Gamma(2-s)\Gamma(s-1) = \frac{\pi}{\sin(\pi s - \pi)}, s \notin \mathbb{Z}$ in Riemann's functional equation $\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(3-s)\zeta(1-s)$ to get another representation of zeta function as follows:

$$\zeta(s) = -2^s\pi^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\frac{\pi(2-s)}{\Gamma(s-1)\sin(\pi s - \pi)}\zeta(1-s)$$

$$\implies \zeta(s) = -2^s \pi^{(s-1)} \sin\left(\frac{\pi s}{2}\right) \frac{\pi(2-s)}{\Gamma(s-1)\sin(\pi s)} \zeta(1-s)$$

$$\implies \zeta(s) = -2^s \pi^{(s)} \sin\left(\frac{\pi s}{2}\right) \frac{2-s}{\Gamma(s-1) 2 \sin(\frac{\pi s}{2}) \cos(\frac{\pi s}{2})} \zeta(1-s)$$

$$\implies \zeta(s) = -2^{s-1} \pi^{(s)} \frac{2-s}{\Gamma(s-1)\cos(\frac{\pi s}{2})} \zeta(1-s)$$

When x=-2, $\zeta(-2) = 2^{-2-1}\pi^{(-2)} \frac{2+2}{\Gamma(-3)\cos(\frac{-2\pi}{2})} \zeta(1+2) = \frac{\zeta(3)}{\pi^2} \approx 0.121793129$

When x=-4, $\zeta(-4) = 2^{-4-1}\pi^{(-4)} \frac{2+4}{\Gamma(-5)\cos(\frac{-4\pi}{2})} \zeta(1+4) = \frac{9\zeta(5)}{2\pi^4} \approx 0.04790251$

When x=-6, $\zeta(-6) = 2^{-6-1}\pi^{(-6)} \frac{2+6}{\Gamma(-7)\cos(\frac{-6\pi}{2})} \zeta(1+6) = \frac{45\zeta(7)}{\pi^6} \approx 0.047197639$

When x=-8, $\zeta(-8) = 2^{-8-1}\pi^{(-8)} \frac{2+8}{\Gamma(-9)\cos(\frac{-8\pi}{2})} \zeta(1+8) = \frac{45\zeta(7)}{\pi^6} \approx 0.047197639$

$\vdots$

And the pattern continues for all negative even numbers upto negative infinity.

8 Proof of Goldbach's Conjecture and Twin Prime Conjecture

8.1 Twin Prime Conjecture Problem statement

A twin prime is a prime number that is either 2 less or 2 more than another prime numberfor example, either member of the twin prime pair (41, 43). In other words, a twin prime is a prime that has a prime gap of two.The question of whether there exist infinitely many twin primes has been one of the great open questions in number theory for many years. This is the content of the twin prime conjecture, which states that there are infinitely many primes p such that p + 2 is also prime. In 1849, de Polignac made the more general conjecture that for every natural number k, there are infinitely many primes p such that p + 2k is also prime.The case k = 1 of de Polignac's conjecture is the twin prime conjecture.

8.2 Goldbach's Conjecture Problem statement

Goldbach's conjecture is one of the oldest unsolved problems in number theory and all of mathematics. It states:

Every even integer greater than 2 can be expressed as the sum of two primes.

The conjecture has been shown to hold for all integers less than 4×10^{18} but remains unproven to date.

8.3 Proof of Twin Prime Conjecture

Let N be a arbitrarily large number. Sum of all the natural numbers upto N shall be $\frac{N(1+N)}{2}$ which includes sum of all the primes upto N too. Double of the sum shall be $N(1+N)$ which shall include double of sum of all the primes upto N too. According to PNT we know that there shall be $\frac{N}{\ln(N)}$ number of primes with an average prime gap of $\ln(N)$. Sum of all the natural numbers upto N being an ever growing number any theorem proved in the interval N and $N(1+N)$ shall apply upto infinity. We can visualise $\frac{N}{\ln(N)}$ as a prime number itself we can allow the prime gaps to change equivalently and complete the number in between. Now if we take logarithm of $N(1+N)$ with respect to the base of $\frac{N}{\ln(N)}$ the result shall give us the lower bound of prime powers that can comfortably be applied on that prime less than N to reach double of the sum of all the natural numbers upto N i.e. $N(1+N)$. In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of $P < \frac{N}{\ln(N)}$ then that will lead us also to lower limit of prime gaps which will satisfy the equation $P+R = P^{\log_{\frac{N}{\ln(N)}} N(1+N)} = N(1+N)$ where $R \geq$ lowest bound of prime gap. As we are comparing double of the sum of all the natural numbers we can always half it and do the same test again and again to descend along the even number line from any arbitrarily large height. If our resultant exponent is greater than 2 then that would imply that there shall be a lower bound of prime gaps and that bound will lie near to very initial gaps along the number line whereas due to continuity there shall not be any upper bound on the prime gaps, it may grow as the number sequence grows. Clearly the result $\log_{\frac{N}{\ln(N)}} N(1+N) = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N)$ shall be greater than 2 meaning that the lower bound of prime gaps would be the gap between sole even prime 2 and its immediate successor even number i.e. 4. Thus the lower bound of prime gaps equals 2. As a prime gap of 2 is lesser than the above highest possible exponent, there shall be infinitely many twin primes satisfying the equation $p_1 + 2 = N(1+N) - 1 = p_2$.Hence Twin prime conjecture stands proved and it can be called as Twin prime theorem.

8.4 Proof of Goldbach's Conjecture

Similarly we can proof Goldbach conjecture too. Before we proceed to proof Goldbach conjecture let us have an understanding how it works. We take the identity $(p+q)^2 = p^2 + q^2 + 2pq$. Now let us set p equals an odd prime p_1 and q equals the sole even prime 2.As a result $(p_1+2)^2$ gives a confirmed odd number as follows:$(p_1+2)^2 = p_1^2 + 4 + 4p_1$.This can be rewritten as sum of one even and one one odd prime as $(p_1+2)^2 = (2)+(p_1^2+4p_1+2)$ as $p_1^2+4p_1+2$ cannot be factorized in a real way.We know that there are infinite number of primes out of which 2 is the sole even prime which essentially means there are infinite number of odd primes.For all this odd primes there will be infinite number of odd numbers which differs an odd prime by 2.Ensuring that atleast one odd prime is there in the right hand side by way of adding such an odd number r to both side of $(p_1+2)^2 = 2+p_1^2+4p_1+2$ we will turn both side into an even number capable of being expressed as sum of two odd primes as follows:$(p_1+2)^2+r = (2+r)+(p_1^2+4p_1+2) = p_2+p_3$.$\mathbf{(p_1+2)^2+r = (2+r)+(p_1^2+4p_1+2) = p_2+p_3}$ can be regarded as standard prime sum form. Standard prime sum form can also be written in vertex form $y = \frac{1}{2}(p_1+2)^2 + (\frac{r}{2} - 1)$.On which, due to infinitude of prime, there shall be infinite number of points satisfying the equation. Now to prove that above equation goes through all the even numbers we go back to our earlier approach of using arithmetic sum.

Let N be a arbitrarily large number. Sum of all the natural numbers upto N shall be $\frac{N(1+N)}{2}$ which includes sum of all the primes upto N too. Double of the sum shall be $N(1+N)$ which shall include double of sum of all the primes upto N too. According to PNT we know that there shall be $\frac{N}{\ln(N)}$ number of primes with an average prime gap of $\ln(N)$. Sum of all the natural numbers upto N being an ever growing number any theorem proved in the interval N and $N(1+N)$ shall apply upto infinity. We can visualise $\frac{N}{\ln(N)}$ as a prime number itself we can allow the prime gaps to change equivalently and complete the number in between. Now if we take logarithm of $N(1+N)$ with respect to the base of $\frac{N}{\ln(N)}$ the result shall give us the lower bound of prime powers that can comfortably be applied on that prime less than N to reach double of the sum of all the natural numbers upto N i.e. $N(1+N)$. In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of $P < \frac{N}{\ln(N)}$ then that will lead us also to lower limit of number of primes sum of which will satisfy the equation $\sum p_i = P^{\log_{\frac{N}{\ln(N)}} N(1+N)} = N(1+N)$ where $i =$ number of Goldbach partitions. As we are comparing double of the sum of all the natural numbers we can always half it and do the same test again and again to descend along the even number line from any arbitrarily large height. If our resultant exponent is greater than 2 then that would imply that there shall be a lower bound of number of primes sum of which can express all the even numbers and that bound will lie near to very initial primes along the number line whereas due to continuity there shall not be any upper bound on the same, it may grow as the number sequence grows. Clearly the result $\log_{\frac{N}{\ln(N)}} N(1+N) = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N)$ shall be greater than 2 meaning that the lower bound of such Goldbach partition would be the same of number 4 the very first non-prime even number. 4 can be written 4=2+2 i.e 4 has got 2 Goldbach partitions. As 2 Goldbach partition is always lesser than the general value of the exponent as calculated above, all the even numbers greater than 2 can be expressed as $p_1+p_2 = N(1+N)-1$.Hence Goldbach conjecture stands proved and it can be called as Goldbach theorem. The weaker version of Goldbach conjecture (ternary Goldbach conjecture) immediately follows from the stronger version (binary Goldbach conjecture) proved above.

9 Proof of other unsolved problems

In the light of identities derived most of the unsolved prime conjectures turns obvious as follows:

9.1 Legendre's prime conjecture

Conjecture. (Adrien-Marie Legendre) There is always a prime number between n^2 and $(n+1)^2$ provided that $n \neq -1$ or 0. In terms of the prime counting function, this would mean that $\pi((n+1)^2) - \pi(n^2) > 0$ for all $n > 0$. Jing Run Chen proved in 1975 that there is always a prime or a semiprime between n^2 and $(n+1)^2$ for large enough n. A natural question to ask is: Why doesn't Bertrand's postulate prove Legendre's conjecture? The reason is that actually $(n+1)^2 < 2n^2$ when $n > 2$. For example, for $n = 3$, Bertrand's postulate guarantees that there is at least one prime between 9 and 18, but for Legendre's conjecture to be true we need a prime between 9 and 16. Suppose, just for the sake of argument, that 17 is prime but 11 and 13 are composite. Bertrand's postulate would still be true but Legendre's conjecture would be false. Of course the gap between $(n+1)^2$ and $2n^2$ gets larger as n gets larger, Legendre's conjecture holds true for $n = 3$, and indeed it has been checked up to $n = 10^{10}$.

Let N be a arbitrarily large number. Sum of square of all the natural numbers upto N shall be $\frac{N(N+1)(2N+1)}{6}$. Double of the sum shall be $\frac{N(N+1)(2N+1)}{3}$. According to PNT we know that there shall be $\frac{N}{\ln(N)}$ number of primes with an average prime gap of $\ln(N)$. Sum of all the natural numbers upto N being an ever growing number any theorem proved in the interval N and $N(1+N)$ shall apply upto infinity. We can visualise $\frac{N}{\ln(N)}$ as a prime number itself we can allow the prime gaps to change equivalently and complete the number in between.Now if we take logarithm of $\frac{N(N+1)(2N+1)}{3}$ with respect to the base of $\frac{N}{\ln(N)}$ the result shall give us the lower bound of prime powers that can comfortably be applied on that prime less than N to reach double of the sum of squares of all the natural numbers upto N i.e. $\frac{N(N+1)(2N+1)}{3}$. In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of $P < \frac{N(N+1)(2N+1)}{3}$ then that will lead us also to lower bound of primes which will satisfy the equation $P + R = P^{\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3}} = \frac{N(N+1)(2N+1)}{3}$ where $R \geq$ lowest bound of prime gap. Similarly replacing sum of N^2 by sum of $(N+1)^2$ we get $P + R = P^{\log_{\frac{N}{\ln(N)}} \frac{(N+1)(N+2)(2N+3)}{3}} = P^{\log_{\frac{N}{\ln(N)}} \frac{(N+1)(N+2)(2N+3)}{3}}$. As we are comparing double of the sum of squares of all the natural numbers or its successors we can always half it and do the same test again and again to descend along the even number line from any arbitrarily large height. If our resultant exponent is greater than 2 then that would imply that there shall be a lower bound of prime gaps in the interval and that bound will lie near to very initial gaps along the number line whereas due to continuity there shall not be any upper bound on the prime gaps, it may grow as the number sequence grows. Clearly the result $\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3} = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (N+1) + \log_{\frac{N}{\ln(N)}} (2N+1)$ shall be significantly greater than $\log_{\frac{N}{\ln(N)}} \frac{(N+1)(N+2)(2N+3)}{3} = \log_{\frac{N}{\ln(N)}} (N+1)((N+1)+1)(\frac{2N}{3}+1)$ (due to complete pattern of extra little quantity of +1) such that another prime can occur in the interval meaning that the lower bound of number of primes in the interval between $\frac{N(N+1)(2N+1)}{3}$) and $N(1+N)$ would be greater than 1. Thus there shall be atleast one prime between n^2 and $(n+1)^2$ as Legendre conjectured.Hence Legendre's prime conjecture stands proved and it can be called as Legendre's theorem.

9.2 Sophie Germain prime conjecture

In number theory, a prime number p is a Sophie Germain prime if 2p + 1 is also prime. The number 2p + 1 associated with a Sophie Germain prime is called a safe prime. For example, 11 is a Sophie Germain prime and 2 11 + 1 = 23 is its associated safe prime. Sophie Germain primes are named after French mathematician Sophie Germain, who used them in her investigations of Fermat's Last Theorem.

The conjecture states that there are infinitely many prime numbers of the form $2P + 1$.

Sum of all the natural numbers upto N shall be $\frac{N(1+N)}{2}$ which includes sum of all the primes upto N too. Double of the sum shall be $N(1 + N)$ which shall include double of sum of all the primes upto N too. According to PNT we know that there shall be $\frac{N}{\ln(N)}$ number of primes with an average prime gap of $\ln(N)$. Sum of all the natural numbers upto N being an ever growing number any theorem proved in the interval N and $N(1 + N)$ shall apply upto infinity. We can visualise $\frac{N}{\ln(N)}$ as a prime number itself we can allow the prime gaps to change equivalently and complete the number in between.Now if we take logarithm of $N(1 + N)$ with respect to the base of $\frac{N}{\ln(N)}$ the result shall give us the lower bound of prime powers that can comfortably be applied on that prime less than N to reach double of the sum of all the natural numbers upto N i.e. $N(1 + N)$. In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of $P < \frac{N}{\ln(N)}$ then that will lead us also to lower limit of prime gaps which will satisfy the equation $P + R = P^{\log_{\frac{N}{\ln(N)}} N(1+N)} = N(1 + N)$ where $R \geq$ lowest bound of prime gap. As we are comparing double of the sum of all the natural numbers we can always half it and do the same test again and again to descend along the even number line from any arbitrarily large height. If our resultant exponent is greater than 2 which is the lower bound of prime gaps then due to continuity infinitude of prime of the underlying pattern is guaranteed otherwise not. Clearly the result $\log_{\frac{N}{\ln(N)}} N(1 + N) = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1 + N)$ shall be greater than 2 meaning that there shall be infinitely many primes of the form $2P + 1$.Hence Sophie Germain conjecture stands proved and it can be called as Sophie Germain's prime theorem.

9.3 Landau's prime conjecture

The conjecture states that there are infinitely many prime numbers of the form $N^2 + 1$.

Let N be a arbitrarily large number. Sum of square of all the natural numbers upto N shall be $\frac{N(N+1)(2N+1)}{6}$. Double of the sum shall be $\frac{N(N+1)(2N+1)}{3}$. According to PNT we know that there shall be $\frac{N}{\ln(N)}$ number of primes with an average prime gap of $\ln(N)$. Sum of all the natural numbers upto N being an ever growing number any theorem proved in the interval N and $N(1+N)$ shall apply upto infinity. We can visualise $\frac{N}{\ln(N)}$ as a prime number itself we can allow the prime gaps to change equivalently and complete the number in between. Now if we take logarithm of $\frac{N(N+1)(2N+1)}{3}$ with respect to the base of $\frac{N}{\ln(N)}$ the result shall give us the lower bound of prime powers that can comfortably be applied on that prime less than N to reach double of the sum of squares of all the natural numbers upto N i.e. $\frac{N(N+1)(2N+1)}{3}$. In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of $P < \frac{N(N+1)(2N+1)}{3}$ then that will lead us also to lower bound of primes which will satisfy the equation $P + R = P^{\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3}}$ where $R \geq$ lowest bound of prime gap. As we are comparing double of the sum of square of all the natural numbers we can always half it and do the same test again and again to descend along the even number line from any arbitrarily large height. If our resultant exponent is greater than 2 which is the lower bound of prime gaps then due to continuity infinitude of prime of the underlying pattern is guaranteed otherwise not. Clearly the result $\log_{\frac{N}{\ln(N)}} N(1 + N) = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1 + N)$ shall be greater than 2 meaning that there shall be infinitely many primes of the form $N^2 + 1$.Hence Landau's prime conjecture stands proved and it can be called as Landau's prime theorem.

9.4 Brocard's prime conjecture

Brocard's conjecture pertains to the squares of prime numbers. Here we denote the nth prime as p_n. With the exception of 4, there are always at least four primes between the square of a prime and the square of the next prime. In terms of the prime counting function, this would mean that $\pi({p_{n+1}}^2) - \pi({p_n}^2) > 3$ for all $n > 1$.

Let N be a arbitrarily large number. Sum of square of all the natural numbers upto N shall be $\frac{N(N+1)(2N+1)}{6}$. Double of the sum shall be $\frac{N(N+1)(2N+1)}{3}$. According to PNT we know that there shall be $\frac{N}{\ln(N)}$ number of primes with an average prime gap of $\ln(N)$. Sum of all the natural numbers upto N being an ever growing number any theorem proved in the interval N and $N(1+N)$ shall apply upto infinity. We can visualise $\frac{N}{\ln(N)}$ as a prime number itself we can allow the prime gaps to change equivalently and complete the number in between. Now if we take logarithm of $\frac{N(N+1)(2N+1)}{3}$ with respect to the base of $\frac{N}{\ln(N)}$ the result shall give us the lower bound of prime powers that can comfortably be applied on that prime less than N to reach double of the sum of squares of all the natural numbers upto N i.e. $\frac{N(N+1)(2N+1)}{3}$. In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of $P < \frac{N(N+1)(2N+1)}{3}$ then that will lead us also to lower bound of primes which will satisfy the equation $P+R = P^{\log_{\frac{N}{\ln(N)}} \frac{N(N+1)(2N+1)}{3}}$ where $R \geq$ lowest bound of prime gap. As we are comparing double of the sum of square of all the natural numbers we can always half it and do the same test again and again to descend along the even number line from any arbitrarily large height. If our resultant exponent is greater than 2 which is the lower bound of prime gaps then due to continuity there shall be at least two primes in the underlying interval . Clearly the result $\log_{\frac{N}{\ln(N)}} N(1+N) = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N)$ shall be greater than 2. In case of interval between two consecutive primes the above limit get raised to the power of its own value meaning that there shall be at least 4 primes the square of a prime and the square of the next prime. Hence Brocard's prime conjecture stands proved and it can be called as Landau's prime theorem.

9.5 Opperman's prime conjecture

Oppermann's conjecture is an unsolved problem in mathematics on the distribution of prime numbers. It is closely related to but stronger than Legendre's conjecture, Andrica's conjecture, and Brocard's conjecture. It is named after Danish mathematician Ludvig Oppermann, who announced it in an unpublished lecture in March 1877.The conjecture states that, for every integer $x > 1$, there is at least one prime number between $x(x-1)$ and x^2,and at least another prime between x^2 and $x(x+1)$.It can also be phrased equivalently as stating that the prime-counting function must take unequal values at the endpoints of each range.That is: $\pi(x^2 - x) < \pi(x^2) < \pi(x^2 + x)$ for $x > 1$ with $\pi(x)$ being the number of prime numbers less than or equal to x. The end points of these two ranges are a square between two pronic numbers, with each of the pronic numbers being twice a pair triangular number. The sum of the pair of triangular numbers is the square.

Let N be a arbitrarily large number. Sum of square of all the natural numbers upto N shall be $\frac{N(N+1)(2N+1)}{6}$. Double of the sum shall be $\frac{N(N+1)(2N+1)}{3}$. Sum of all the natural numbers upto N shall be $\frac{N(1+N)}{2}$ which includes sum of all the primes upto N too. Now subtracting the latter from the former we get $\frac{N(N+1)(2N+1)}{3} - \frac{N(1+N)}{2} = N(N+1).\frac{2(2N+1)-3}{6} = N(N+1).\frac{4N-1}{6}$. Double of such difference shall be $N(N+1).\frac{4N-1}{3}$. According to PNT we know that there shall be $\frac{N}{\ln(N)}$ number of primes with an average prime gap of $\ln(N)$. Sum of all the natural numbers upto N being an ever growing number any theorem proved in the interval N and $N(1+N)$ shall apply upto infinity. We can visualise $\frac{N}{\ln(N)}$ as a prime number itself we can allow the prime gaps to change equivalently and complete the number in between. Now if we take logarithm of $N(N+1).\frac{4N-1}{3}$ with respect to the base of $\frac{N}{\ln(N)}$ the result shall give us the lower bound of prime powers that can comfortably be applied on that prime less than N to reach double of the sum of squares of all the natural numbers upto N less double of the sum of all the natural numbers upto Ni.e. $N(N+1).\frac{4N-1}{3}$. In other words if we consolidate the average prime gaps into a relatively large prime having approximate value of $P < N(N+1).\frac{4N-1}{3}$ then that will lead us also to lower bound of primes which will satisfy the equation $P+R = P^{\log_{\frac{N}{\ln(N)}} N(N+1).\frac{4N-1}{3}}$ where $R \geq$ lowest bound of prime gap. Clearly the result $\log_{\frac{N}{\ln(N)}} N(N+1).\frac{4N-1}{3} = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N) + \log_{\frac{N}{\ln(N)}} \frac{4N-1}{3}$ shall be greater than 2 meaning that there shall be atleast one prime between $x(x-1)$ and x^2. Again adding $\frac{N(1+N)}{2}$ with $\frac{N(N+1)(2N+1)}{3}$ we get $\frac{N(N+1)(2N+1)}{3} + \frac{N(1+N)}{2} = N(N+1).\frac{2(2N+1)+3}{6} = N(N+1).\frac{4N+5}{6}$. Double of such difference shall be $N(N+1).\frac{4N+5}{3}$. Clearly the result $\log_{\frac{N}{\ln(N)}} N(N+1).\frac{4N+5}{3} = \log_{\frac{N}{\ln(N)}} N + \log_{\frac{N}{\ln(N)}} (1+N) + \log_{\frac{N}{\ln(N)}} \frac{4N+5}{3}$ shall be greater than 2 meaning that there shall be atleast one prime between x^2 and $x(x+1)$. Altogether Opperman's conjecture stands proved and it can be called as Opperman's theorem.

9.6 Collatz conjecture

The Collatz conjecture is a conjecture in mathematics that concerns a sequence defined as follows: start with any positive integer n. Then each term is obtained from the previous term as follows: if the previous term is even, the next term is one half the previous term. If the previous term is odd, the next term is 3 times the previous term plus 1. The conjecture is that no matter what value of n, the sequence will always reach 1.

As fundamental formula of numbers is proved to be continuous, Collatz conjectured operations on any number (i.e. halving the even numbers or simultaneously tripling and adding 1 to odd numbers) may either blow up to infinity or come down to singularity. We have seen that among the odd numbers odd primes are descendants of sole even prime 2. This small bias turns the game of equal probability into one sided game i.e Collatz conjecture cannot blow upto infinity, it ends with 2 and one last step before the final whistle bring it down to singularity 1 as Collatz conjectured. Hence Collatz conjecture is proved to be trivial.

10 Complex logarithm simplified

Thanks to Roger cots who first time used i in complex logarithm. Thanks to euler who extended it to exponential function and tied i, pi and exponential function to unity in his famous formula. Now taking lead from both of their work and applying results of Zeta function which are simultaneously continuous logarithmic function and continuous exponential function we can redefine Complex number and complex logarithm as follows.

10.1 Difficulties faced in present concept of Complex logarithm

The complex exponential function is not injective, because $ew + 2\pi i = ew$ for any w, since adding $i\theta$ to w has the effect of rotating ew counterclockwise θ radians. So the points equally spaced along a vertical line, are all mapped to the same number by the exponential function. That is why the exponential function does not have an inverse (Complex logarithm) function in true sense.One is to restrict the domain of the exponential function to a region that does not contain any two numbers differing by an integer multiple of $2\pi i$: this leads naturally to the definition of branches of log z, which are certain functions that single out one logarithm of each number in their domains. Another way to resolve the indeterminacy is to view the logarithm as a function whose domain is not a region in the complex plane, but a Riemann surface that covers the punctured complex plane in an infinite-to-1 way. Branches have the advantage that they can be evaluated at complex numbers. On the other hand, the function on the Riemann surface is elegant in that it packages together all branches of the logarithm and does not require an arbitrary choice as part of its definition. The function Log z is discontinuous at each negative real number, but continuous everywhere else in $\mathbb{C}^{\times}$. To explain the discontinuity, consider what happens to Arg z as z approaches a negative real number a. If z approaches a from above, then Arg z approaches π, which is also the value of Arg a itself. But if z approaches a from below, then Arg z approaches $-\pi$. So Arg z "jumps" by 2 as z crosses the negative real axis, and similarly Log z jumps by $2\pi i$. All logarithmic identities are satisfied by complex numbers. It is true that $e^{\ln z} = z$ for all $z \neq 0$ (this is what it means for Log z to be a logarithm of z), but the identity Log $e^z = z$ fails for z outside the strip S. For this reason, one cannot always apply Log to both sides of an identity $e^z = e^w$ to deduce z = w. Also, the identity $\ln z_1 z_2 = \ln z_1 + \ln z_2$ can fail: the two sides can differ by an integer multiple of $2\pi i$: for instance,

$$\mathrm{Log}((-1)i) = \mathrm{Log}(-i) = \ln(1) - \frac{\pi i}{2} = -\frac{\pi i}{2}$$

but

$$\mathrm{Log}(-1) + \mathrm{Log}(i) = (\ln(1) + \pi i) + \left(\ln(1) + \frac{\pi i}{2}\right) = \frac{3\pi i}{2} \neq -\frac{\pi i}{2}$$

10.2 Eulers formula, the unit circle, the unit sphere

$z = r(\cos x + i \sin x)$ is the trigonometric form of complex numbers. Using Eulers formula $e^{ix} = \cos x + i \sin x$ we can write $z = re^{ix}$. Putting $x = \pi$ in Eulers formula we get , $e^{i\pi} = -1$.Putting $x = \frac{\pi}{2}$ we get $e^{\frac{i\pi}{2}} = i$. So the general equation of the points lying on unit circle $|z| = |e^{ix}| = 1$. But that's not all. If $x = \frac{\pi}{3}$ in trigonometric form then

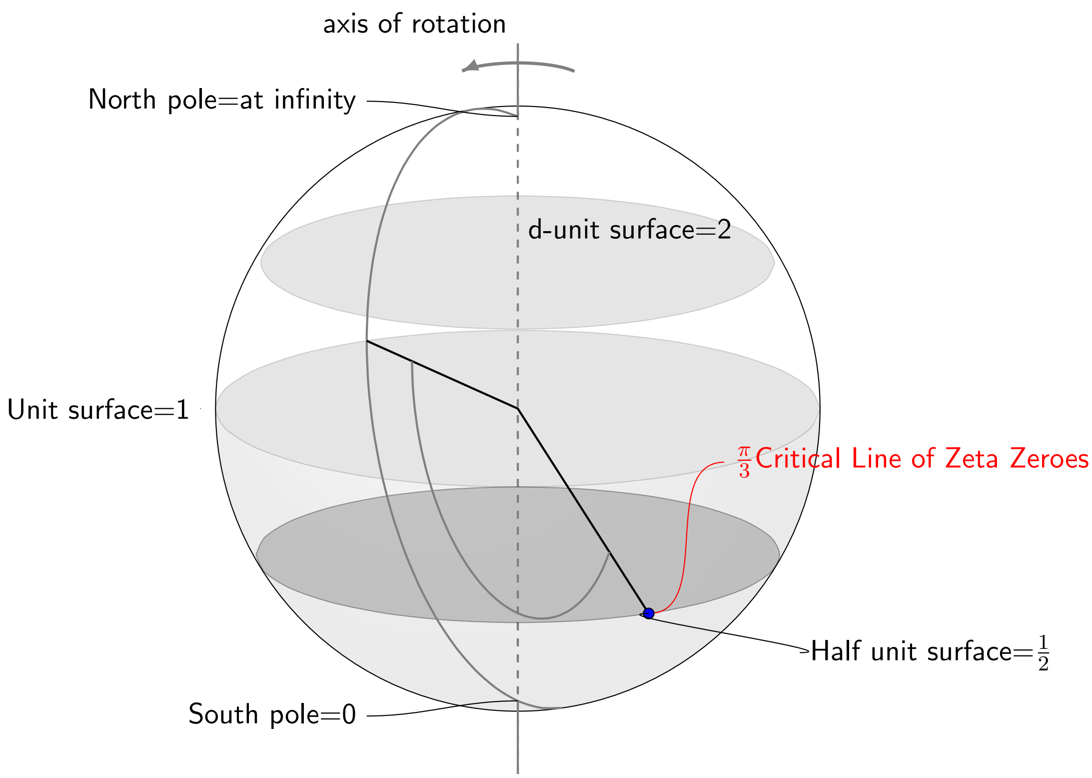
$z = cos(\frac{\pi}{3})+i.sin(\frac{\pi}{3}) = \frac{1}{2}(\sqrt{3}+i)$.So $|z| = r = \sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2} = \frac{1}{2}.\sqrt{4} = \frac{1}{2}.2 = 1$.So another equation of the points lying on unit circle $|\mathbf{z}| = |\frac{\mathbf{1}}{\mathbf{2}}\mathbf{e^{ix}}| = \mathbf{1}$. Although both the equation are of unit circle, usefulness of $|\mathbf{z}| = |\frac{\mathbf{1}}{\mathbf{2}}\mathbf{e^{ix}}| = \mathbf{1}$ is greater than $|z| = |e^{ix}| = 1$ as $|\mathbf{z}| = |\frac{\mathbf{1}}{\mathbf{2}}\mathbf{e^{ix}}| = \mathbf{1}$ bifurcates mathematical singularity and introduces unavoidable mathematical duality particularly in studies of primes and Zeta function. $|\mathbf{z}| = |\frac{\mathbf{1}}{\mathbf{2}}\mathbf{e^{ix}}| = \mathbf{1}$ can be regarded as d-unit circle. When Unit circle in complex plane is stereo-graphically projected to unit sphere the points within the area of unit circle gets mapped to southern hemisphere, the points on the unit circle gets mapped to equatorial plane, the points outside the unit circle gets mapped to northern hemisphere. d-unit circle can also be easily projected to Riemann sphere. Projection of d-unit circle to d-unit sphere will have three parallel disc (like three dimensions hidden in one single dimension of numbers) for three (equivalent unit values in three different sense) magnitude of $\frac{1}{2}, 1, 2$ in the southern hemisphere, on the equator, in the northern hemisphere respectively as shown in the following diagram.

Explanation 8 *One may attempt to show that* $|\mathbf{z}| = |\frac{\mathbf{1}}{\mathbf{2}}\mathbf{e^{ix}}| = \mathbf{1}$ *will mean 1= 2. This may not be right interpretation. Correct way to interpret is given here under.*

We know: $e^{ix} = r(\cos\theta + i\sin\theta)$. *Taking derivative both side we get*

$$ie^{ix} = (\cos\theta + i\sin\theta)\frac{dr}{dx} + r(-\sin\theta + i\cos\theta)\frac{d\theta}{dx}.$$

Now Substituting $r(\cos\theta + i\sin\theta)$ *for* e^{ix} *and equating real and imaginary parts in this formula gives* $\frac{dr}{dx} = 0$ *and* $\frac{d\theta}{dx} = 1$. *Thus, r is a constant, and θ is x + C for some constant C. Now if we assign* $r = \frac{1}{2}$ *and* $ix = \ln 2$ *then* $re^{ix} = \frac{1}{2}.e^{\ln 2} = 1$ *The initial value x=1 then gives* $i = \ln 2$.*This proves the formula* $|\mathbf{z}| = |\frac{\mathbf{1}}{\mathbf{2}}\mathbf{e^{ix}}| = \mathbf{1}$.*Thus we see* $ix = \ln(\cos\theta + i\sin\theta)$ *is a multivalued function not only because of infinite rotation around the unit circle but also due to different real solutions to i. Square root of minus 1 is a general concept of complex numbers which can have different real values.*



10.3 Introduction of Quaternions for closure of logarithmic operation

Hamilton knew that the complex numbers could be interpreted as points in a plane, and he was looking for a way to do the same for points in three-dimensional space. Points in space can be represented by their coordinates, which are triples of numbers, and for many years he had known how to add and subtract triples of numbers. However, Hamilton had been stuck on the problem of multiplication and division for a long time. He could not figure out how to calculate the quotient of the coordinates of two points in space. The great breakthrough in quaternions finally came on Monday 16 October 1843 in Dublin, when Hamilton was on his way to the Royal Irish Academy where he was going to preside at a council meeting. Hamilton could not resist the urge to carve the formula for the quaternions, $\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$ into the stone of Brougham Bridge as he paused on it. A quaternion is an expression of the form : $a + b\ \mathbf{i} + c\ \mathbf{j} + d\ \mathbf{k}$ where a, b, c, d, are real numbers, and i, j, k, are symbols that can be interpreted as 'imaginary operators' which define how the scalar values combine. The set of quaternions is made a 4 dimensional vector space over the real numbers, with $\{1, \mathbf{i}, \mathbf{j}, \mathbf{k}\}$ as a basis, by the componentwise addition

$$(a_1 + b_1\ \mathbf{i} + c_1\ \mathbf{j} + d_1\ \mathbf{k}) + (a_2 + b_2\ \mathbf{i} + c_2\ \mathbf{j} + d_2\ \mathbf{k}) = (a_1 + a_2) + (b_1 + b_2)\ \mathbf{i} + (c_1 + c_2)\ \mathbf{j} + (d_1 + d_2)\ \mathbf{k}$$

and the componentwise scalar multiplication

$$\lambda(a + b\ \mathbf{i} + c\ \mathbf{j} + d\ \mathbf{k}) = \lambda a + (\lambda b)\ \mathbf{i} + (\lambda c)\ \mathbf{j} + (\lambda d)\ \mathbf{k}\ .$$

A multiplicative group structure, called the Hamilton product, can be defined on the quaternions. The real quaternion 1 is the identity element.The real quaternions commute with all other quaternions, that is aq = qa for every quaternion q and every real quaternion a. In algebraic terminology this is to say that the field of real quaternions are the center of this quaternion algebra. The product is first given for the basis elements, and then extended to all quaternions by using the distributive property and the center property of the real quaternions. The Hamilton product is not commutative, but associative, thus the quaternions form an associative algebra over the reals.
For two elements $a_1 + b_1 i + c_1 j + d_1 k$ and $a_2 + b_2 i + c_2 j + d_2 k$, their product, called the Hamilton product $(a_1 + b_1 i + c_1 j + d_1 k)(a_2 + b_2 i + c_2 j + d_2 k)$, is determined by the products of the basis elements and the distributive law.The distributive law makes it possible to expand the product so that it is a sum of products of basis elements. This gives the following expression:

$$a_1a_2 + a_1b_2i + a_1c_2j + a_1d_2k + b_1a_2i + b_1b_2i^2 + b_1c_2ij + b_1d_2ik$$

$$+c_1a_2j + c_1b_2ji + c_1c_2j^2 + c_1d_2jk + d_1a_2k + d_1b_2ki + d_1c_2kj + d_1d_2k^2$$

Now the basis elements can be multiplied using the rules given above to get:

$$a_1a_2 - b_1b_2 - c_1c_2 - d_1d_2 + (a_1b_2 + b_1a_2 + c_1d_2 - d_1c_2)i$$

$$+(a_1c_2 - b_1d_2 + c_1a_2 + d_1b_2)j + (a_1d_2 + b_1c_2 - c_1b_2 + d_1a_2)k$$

If some ask what quaternion has to do with complex logarithm then I wont say "shut up and calculate" (quantum mechanics instructors famous instruction). Let us recall that a third dimension is hidden inside 2nd dimension, a d-unit circle is hidden inside the unit circle, a d-unit sphere is hidden inside the unit sphere.In spite of his great success in conceptualising Riemannian geometry, Riemannian manifold, Stereo graphic projection, Riemann mapping to R3 Riemann sphere etc.. Riemann missed a vital fact which Hamilton realised that to go 3D we need 4D. Had this idea come to Riemannian's mind he could have figured it out himself why zeta zeros falls on half line, he would not have leave us in dark for more than 150 years searching light for proving his unfinished hypothesis. More than 150 years of world's best brain's run time! huge loss of talent. Let us fix the problem we faced in complex logarithm defining the principal value by way of introducing quaternions in the picture. If we visualise natural logarithm of product of two pairs of -1 as natural logarithm of two pairs of quaternion then we can arive zero at part with the definition of logarithm and solve the issue of indeterminacy of the principal value i.e. $\ln 1 = 0 = \ln -1. -1 = \ln i^2.j^2.k^2.i.j.k = 3(\ln i + \ln j + \ln k)$. This way numbers are very complexly 3 dimensional hidden in other dimensions of quaternions although we do not need it to be so in our everyday use of numbers. Now the question may arise can't we make our life simpler restricting ourselves to complex numbers. The answer is yes we can do so provided we add correction to our end results. Suppose Alice do not know addition she knows only multiplication and she is a member of team A involved in project estimating the percentage of dark matter and dark energy. Bob who does not know multiplication is a member of Team B for the same project. Alice found 96% dark matter and dark energy where as bobs result was 0%. Using quaternion find who is right. In the world of

Alice everything is real, time is a One Way Street where entropy rules, fastest method of mathematical operation she knows is scalar multiplication and she applied that, she did not account for CPT symmetry, she overlooked very complex conjugation, rotational matrix and the unit quaternions. And the end result was she have over estimated 69% dark energy which might have got squared off if she would have used natural logarithm of 2 as real replacement for imaginary number i and if she would have given due weight to noncommutative multiplication of quaternions, 25% dark matter could have squared off if she would not have completely missed hidden dimensions in complex 4 dimensional calculation. On the other hand Bob was right because just adding numbers meticulously he did not committed the mistakes Alice committed.
Now let see how quaternion helps in product of exponentials or simplifying the complex logarithm. Let us recall the power addition identity, which is,

$$e^{(a+b)} = e^a * e^b$$

However this only applies when 'a' and 'b' commute, so it applies when a or b is a scalar for instance. The more general case where 'a' and 'b' don't necessarily commute is given by:

$$e^c = e^a * e^b$$

where:

$$c = c = a + b + ab + 1/3(a(ab) + b(ba)) + \text{...series known as the Baker-Campbell-Hausdorff formula}$$

where:x = vector cross product. This shows that if when a_1 and a_2 become close to becoming parallel then ab approaches zero and c approaches a + b so the rotation algebra approaches vector algebra. As we have seen all the three unit discs appear parallel to each other our life gets easier and we can do complex exponentiation and logarithm as we do natural logarithm in real life. This becomes real and simple logarithm.

10.4 Properties of Real and simple (RS) Logarithm

If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ then RS logarithm has the following property.

Theorem 1

$$\ln(z_1.z_2) = \ln(\mathrm{Re}(z_1)) + \ln(\mathrm{Re}(z_2)) + i(\ln(\mathrm{Im}(z_1) + \ln(\mathrm{Im}(z_2))$$

Proof:

$$\begin{aligned}
&\ln(z_1.z_2.z_3.z_4.z_5.z_6.z_7....)\\
&= \ln\Big(p_1.p_2.p_3.p_4.p_5.p_6.p_7...\Big) + i\ln\Big(p_1.p_2.p_3.p_4.p_5.p_6.p_7...\Big)\\
&= \ln(1) + \ln(2) + \ln(3) + \ln(4) + \ln(5) + ... + i\ln\Big(\ln(1) + \ln(2) + \ln(3) + \ln(4) + \ln(5) + ...\Big)\\
&= \ln(\mathrm{Re}(z_1)) + \ln(\mathrm{Re}(z_2)) + \ln(\mathrm{Re}(z_3)) + ... + i\Big(\ln(\mathrm{Im}(z_1)) + \ln(\mathrm{Im}(z_2)) + \ln(\mathrm{Im}(z_3)) + ...\Big)
\end{aligned}$$

Following Zeta functions analytic continuation or bijective holomorphic property, we can write:

$$\ln(z_1.z_2) = \ln(\mathrm{Re}(z_1)) + \ln(\mathrm{Re}(z_2)) + i\Big(\ln(\mathrm{Im}(z_1)) + \ln(\mathrm{Im}(z_2))\Big)$$

Corrolary 1

$$\ln(\frac{z_1}{z_2}) = \ln(\mathrm{Re}(z_1)) - \ln(\mathrm{Re}(z_2)) + i\Big(\ln(\mathrm{Im}(z_1)) - \ln(\mathrm{Im}(z_2))\Big)$$

Corrolary 2

$$\exp(z_1 + z_2) = \exp(\mathrm{Re}(z_1)).\exp(\mathrm{Re}(z_2)) + i\Big(\exp(\mathrm{Im}(z_1)).\exp(\mathrm{Im}(z_2))\Big)$$

Corrolary 3

$$\exp(z_1 - z_2) = \frac{\exp(\mathrm{Re}(z_1))}{\exp(\mathrm{Re}(z_2))} + i\left(\frac{\exp(\mathrm{Im}(z_1))}{\exp(\mathrm{Im}(z_2))}\right)$$

Corrolary 4

$$\ln(z_1 + z_2) = \ln(\mathrm{Re}(z_1 + z_2)) + i\left(\ln(\mathrm{Im}(z_1 + z_2))\right)$$

Corrolary 5

$$\ln(z_1 - z_2) = \ln(\mathrm{Re}(z_1 - z_2)) + i\left(\ln(\mathrm{Im}(z_1 - z_2))\right)$$

10.5 First real quaternion root of i

In d-unit circle we have seen $|\mathbf{z}| = |\frac{\mathbf{1}}{\mathbf{2}}\mathbf{e}^{\mathbf{ix}}| = \mathbf{1}$ is another form of unit circle. We can rewrite :

$z = \frac{1}{2}e^{ix} = 1 = \frac{1}{2}e^{\ln 2}$

we can say :

$e^{ix} = e^{ln2}$

taking logarithm both side :

$ix = ln(2)$

setting x=1 :

$\boxed{ln(2) = e^{ln(ln(2))} = e^{ln(i)} = i \approx e^{-\frac{1}{e}} \approx 2^{-\frac{1}{2}} \approx e - 2*}$

or

$\boxed{ln(2)^{\frac{1}{ln(ln(2))}} = i^{\frac{1}{ln(i)}} = e \approx -\frac{1}{ln(i)} \approx 2 + i*}$

we get two more identity like $e^{i\pi} + 1 = 0$:

$\boxed{\frac{1}{e} + ln(i) = 0 = e + \frac{1}{ln(i)}}$

again we know $i^2 = -1$, taking log both side

$\boxed{\ln(-1) = 2\ln i = 2ln(ln(2))}$

* Not an exhaustively computed value (even wolfram alpha can't be that match accurate as the nature, there may be slight difference based on the devices capabilities). MS-Excel based calculation done on dual core PC approximately matches our definition.

Example 1 *Find natural logarithm of -5 using first root of i*

$$\ln(-5) = \ln(-1) + \ln(5) = 2ln(ln(2)) + \ln(5) = 0.876412071(approx)$$

Example 2 *Find natural logarithm of -5i using first root of i*

$$\ln(-5i) = \ln(-1) + \ln(5) + \ln(i) = 2ln(ln(2)) + \ln(5) + ln(ln(2)) = 0.509899151(approx)$$

Example 3 *Find natural logarithm of 5-5i using first root of i*

$$\ln(5 - 5i) = \ln(5) + \ln(-1) + \ln(5) + \ln(i) = \ln(5) + 2ln(ln(2)) + \ln(5) + ln(ln(2)) = 2.119337063(approx)$$

Example 4 *Transform the complex number 2+9i using first root of i.*

$$e^{2+9i} = e^{2+9\mathrm{X}0.693147181} = e^{8.238324625} = 3783.196723(approx)$$

10.6 Middle scale constants from 1st root of i and its 6 unit partitions

Puting the value of i in Eulers identity we get constants of the middle scale.

Constant 1

$$\boxed{e^{i\pi} = e^{ln(2).\pi} = 8.824977827 = e^{2.17758609}...(approx)}$$

Constant 2

$$\boxed{e^{i\frac{\pi}{2}} = e^{\frac{ln(2).\pi}{2}} = 2.970686424 = e^{1.088793045}...(approx)}$$

Constant 3

$$\boxed{e^{i\frac{\pi}{3}} = e^{\frac{ln(2).\pi}{3}} = 2.066511728 = e^{0.72586203}...(approx)}$$

Constant 4

$$\boxed{e^{i\frac{\pi}{4}} = e^{\frac{ln(2).\pi}{4}} = 1.723567934 = e^{0.544396523}...(approx)}$$

Constant 5

$$\boxed{e^{i\frac{\pi}{5}} = e^{\frac{ln(2).\pi}{5}} = 1.545762348 = e^{0.435517218}...(approx)}$$

Constant 6

$$\boxed{e^{i\frac{\pi}{6}} = e^{\frac{ln(2).\pi}{6}} = 1.437536687 = e^{0.362931015}...(approx)}$$

10.7 Second real quaternion root of i

From $i^2 = -1$ we know that i shall have at least two roots or values, one we have already defined, another we need to find out. We know that at $\frac{\pi}{3}$ Zeta function (which is bijectively holomorphic and deals with both complex exponential and its inverse i.e. complex logarithm) attains zero. Let us use Eulers formula to define another possible value of i as Eulers formula deals with unity which comes from the product of exponential and its inverse i.e. logarithm.

Lets assume:

$e^{i\frac{\pi}{3}} = z$

taking natural log both side :

$\frac{i\pi}{3} = ln(z)$

Lets set:$ln(z) = i + \frac{1}{3}$

$i\pi = 1 + 3i$

$i(\pi - 3) = 1$

$$\boxed{i = \frac{1}{\pi - 3}}$$

$$\boxed{\pi * = 3 + \frac{1}{i}}$$

we get two more identity like $e^{i\pi} + 1 = 0$:

$$\boxed{ln(i) - 2 = 0 = \frac{1}{ln(i)} - \frac{1}{2}*}$$

again we know $i^2 = -1$, taking log both side

$$\boxed{\ln(-1) = 2\ln i = 2ln\left(\frac{1}{\pi - 3}\right)*}$$

* Not an exhaustively computed value (even wolfram alpha can't be that match accurate as the nature, there may be slight difference based on the devices capabilities). MS-Excel based calculation done on dual core PC approximately matches our definition.

Example 5 *Find natural logarithm of -5 using second root of i*

$$\ln(-5) = \ln(-1) + \ln(5) = 2ln\left(\frac{1}{\pi - 3}\right) + \ln(5) = 5.519039873(approx)$$

Example 6 *Find natural logarithm of -5i using second root of i*

$$\ln(-5i) = \ln(-1) + \ln(5) + \ln(i) = 2ln\left(\frac{1}{\pi - 3}\right) + \ln(5) + ln\left(\frac{1}{\pi - 3}\right) = 7.473840854(approx)$$

Example 7 *Find natural logarithm of 5-5i using second root of i*

$$\ln(5 - 5i) = \ln(5) + \ln(-1) + \ln(5) + \ln(i) = \ln(5) + 2ln\left(\frac{1}{\pi - 3}\right) + \ln(5) + ln\left(\frac{1}{\pi - 3}\right) = 9.083278766(approx)$$

Example 8 *Transform the complex number 3+i using second root of i.*

$$e^{3+i} = e^{3+1X7.06251330593105} = e^{10.0625133059311} = 23447.3627750323(approx)$$

10.8 Large scale constants from 2nd root of i and its 6 unit partitions

Putting the value of i in Eulers identity we get large constants applicable for cosmic scale and their reciprocals are useful constants to deal with quantum world.

Constant 7

$$\boxed{e^{i\pi} = e^{\frac{\pi}{\pi-3}} = 4,324,402,934 = e^{22.18753992}...(approx)}$$

Constant 8

$$\boxed{e^{i\frac{\pi}{2}} = e^{\frac{\pi}{2(\pi-3)}} = 65,760 = e^{11.09376703}...(approx)}$$

Constant 9

$$\boxed{e^{i\frac{\pi}{3}} = e^{\frac{\pi}{3(\pi-3)}} = 1,629 = e^{7.395721609}...(approx)}$$

Constant 10

$$\boxed{e^{i\frac{\pi}{4}} = e^{\frac{\pi}{4(\pi-3)}} = 256.4375 = e^{5.54688497}...(approx)}$$

Constant 11

$$\boxed{e^{i\frac{\pi}{5}} = e^{\frac{\pi}{5(\pi-3)}} = 84.5639441 = e^{4.43750798}...(approx)}$$

Constant 12

$$\boxed{e^{i\frac{\pi}{6}} = e^{\frac{\pi}{6(\pi-3)}} = 40.36339539 = e^{3.69792332}...(approx)}$$

10.9 Third but not the final real quaternion root of i

Can there be a third root of i, why not? Three sides of a triangle can enclose a circle, value of pi is just little more than 3, we see 3 generation of stars in the universe, there are 3 generation of matter in the standard model, spatially we cannot imagine more than three dimensions. Let us use Eulers formula to define another possible value of i as Eulers formula deals with unity which comes from the product of exponential and its inverse i.e. logarithm.

Lets assume:

$e^{i\frac{\pi}{3}} = z$

taking natural log both side :

$\frac{i\pi}{3} = ln(z)$

Lets set:$ln(z) = \frac{4\pi - 3}{6(\pi - 3)}$

$i\pi = \frac{4\pi - 3}{2(\pi - 3)}$

$\boxed{i = \frac{4\pi - 3}{2\pi(\pi - 3)}}$

$\boxed{\pi * = 3 + \frac{75.9426752}{i}}$

we get two more identity like $e^{i\pi} + 1 = 0$:

$\boxed{ln(i) - \frac{19}{8} = 0 = \frac{1}{ln(i)} - \frac{8}{19} *}$

again we know $i^2 = -1$, taking log both side

$\boxed{\ln(-1) = 2\ln i = 2ln\left(\frac{4\pi - 3}{2\pi(\pi - 3)}\right) *}$

* Not an exhaustively computed value (even wolfram alpha can't be that match accurate as the nature, there may be slight difference based on the devices capabilities). MS-Excel based calculation done on dual core PC approximately matches our definition.

Example 9 *Find natural logarithm of -5 using third root of i*

$$\ln(-5) = \ln(-1) + \ln(5) = 2ln\left(\frac{4\pi - 3}{2\pi(\pi - 3)}\right) + \ln(5) = 6.359793515(approx)$$

Example 10 *Find natural logarithm of -5i using third root of i*

$$\ln(-5i) = \ln(-1) + \ln(5) + \ln(i) = 2ln\left(\frac{4\pi - 3}{2\pi(\pi - 3)}\right) + \ln(5) + ln\left(\frac{4\pi - 3}{2\pi(\pi - 3)}\right) = 8.734971317(approx)$$

Example 11 *Find natural logarithm of 5-5i using third root of i*

$$\ln(5-5i) = \ln(5)+\ln(-1)+\ln(5)+\ln(i) = \ln(5)+2ln\left(\frac{4\pi - 3}{2\pi(\pi - 3)}\right)+\ln(5)+ln\left(\frac{4\pi - 3}{2\pi(\pi - 3)}\right) = 10.34440923(approx)$$

Example 12 *Transform the complex number 3+i using third root of i.*

$$e^{3+i} = e^{3+1X10.7529249} = e^{13.7529249} = 939332.598(approx)$$

10.10 Super Large scale constants from 3rd root of i and its 6 unit partitions

Putting the value of i in Eulers identity we get large constants applicable for cosmic scale and their reciprocals are useful constants to deal with quantum world.

Constant 13

$$\boxed{e^{i\pi} = e^{\frac{4\pi-3}{2(\pi-3)}} = 4.68853E + 14 = e^{33.7813104}...(approx)}$$

Constant 14

$$\boxed{e^{i\frac{\pi}{2}} = e^{\frac{4\pi-3}{4(\pi-3)}} = 21653007.96 = e^{16.89065494}...(approx)}$$

Constant 15

$$\boxed{e^{i\frac{\pi}{3}} = e^{\frac{4\pi-3}{6(\pi-3)}} = 77686.488314 = e^{11.26043663}...(approx)}$$

Constant 16

$$\boxed{e^{i\frac{\pi}{4}} = e^{\frac{4\pi-3}{8(\pi-3)}} = 4653.279269 = e^{8.445327469}...(approx)}$$

Constant 17

$$\boxed{e^{i\frac{\pi}{5}} = e^{\frac{4\pi-3}{10(\pi-3)}} = 859.4236373 = e^{6.756261975}...(approx)}$$

Constant 18

$$\boxed{e^{i\frac{\pi}{6}} = e^{\frac{4\pi-3}{12(\pi-3)}} = 278.7229598 = e^{5.630218313}...(approx)}$$

11 Pi based logarithm

One thing to notice is that pi is intricately associated with e. We view pi mostly associated to circles, what it has to do with logarithm? Can it also be a base to complex logarithm? Although base pi logarithm are not common but this can be handy in complex logarithm. We know:

$$\begin{aligned}
&ln(2).\frac{\pi}{4}\\
&= \left(\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots\right)\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{11} - \frac{1}{13} + \cdots\right)\\
&= \left(1 + \frac{1}{\lfloor 3} - \frac{1}{\lfloor 5} + \frac{1}{\lfloor 7} - \cdots\right) + \left(1 + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 4} + \frac{1}{\lfloor 6} + \cdots\right) - \left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \cdots\right)\\
&= \left(1 - \frac{i^3}{\lfloor 3} + \frac{i^5}{\lfloor 5} - \frac{i^7}{\lfloor 7} - \cdots\right) + \left(1 - \frac{i^2}{\lfloor 2} + \frac{i^4}{\lfloor 4} - \frac{i^6}{\lfloor 6} + \cdots\right) - \frac{1}{1 - \frac{1}{2}}\\
&= \sin(i) + \cos(i) - 2
\end{aligned}$$

Lets set:$\pi = \sin(i) + \cos(i)$ and replacing $\pi - 2 = \ln(\pi)$ we can write

$$\frac{\ln\left(e^{\frac{ln(2)}{4}}\right)}{\ln(\pi)} = \frac{1}{\pi} = \pi^{-1}\text{Lets set:}e^{\frac{ln(2)}{4}} = \pi^{\pi^{je}} \text{ we can write } \boxed{\pi^{je} = -1}$$

11.1 Constants from 3 roots of j and its 6 unit partitions

Similar to 3 roots of i, there can be 3 roots of j which will give another complex logarithmic scales. I will not make the readers crazy anymore with this nasty mind-boggling staff. I am writing directly the scales because it feels so boring writing the same staff again and again. Do not ask me how I got it? Same head spinning and mind twisting algorithms. Better I will ask my readers to undergo these processes themselves little bit to have a better feel of my work on complex logarithm. Remember that you will get always hints from the transcendental parts of pi to set i similar to transcendental parts of e. My inspiration for trying this was recursive nature of Zeta function, Mandelbrot fractal, Rogers Ramanujan continued fraction, Ramanujan's infinite radicals, Ramanujan's Sum etc.

Constant 19

$$\boxed{\pi^{je} = \pi^{\frac{1}{\ln(2)}.e} = 89.05301963 = e^{4.489231919}...(approx)}$$

Constant 20

$$\boxed{\pi^{\frac{je}{2}} = \pi^{\frac{\frac{1}{\ln(2)}.e}{2}} = 9.436790748 = e^{2.244615959}...(approx)}$$

Constant 21

$$\boxed{\pi^{\frac{je}{3}} = \pi^{\frac{\frac{1}{\ln(2)}.e}{3}} = 4.465631508 = e^{1.49641064}...(approx)}$$

Constant 22

$$\boxed{\pi^{\frac{je}{4}} = \pi^{\frac{\frac{1}{\ln(2)}.e}{4}} = 3.071935994 = e^{1.12230798}...(approx)}$$

Constant 23

$$\boxed{\pi^{\frac{je}{5}} = \pi^{\frac{\frac{1}{\ln(2)}.e}{5}} = 2.4543118 = e^{0.897846384}...(approx)}$$

Constant 24

$$\boxed{\pi^{\frac{je}{6}} = \pi^{\frac{\frac{1}{\ln(2)}.e}{6}} = 2.1132041 = e^{0.74820532}...(approx)}$$

Constant 25

$$\boxed{\pi^{je} = \pi^{\frac{e}{e-2}.e} = 130089.9289 = e^{11.77598125}...(approx)}$$

Constant 26

$$\boxed{\pi^{\frac{je}{2}} = \pi^{\frac{\frac{e}{e-2}.e}{2}} = 360.6798149 = e^{5.887990625}...(approx)}$$

Constant 27

$$\boxed{\pi^{\frac{je}{3}} = \pi^{\frac{\frac{e}{e-2}.e}{3}} = 50.66964856 = e^{3.925327084}...(approx)}$$

Constant 28

$$\boxed{\pi^{\frac{je}{4}} = \pi^{\frac{\frac{e}{e-2}.e}{4}} = 18.99157221 = e^{2.943995313}...(approx)}$$

Constant 29

$$\boxed{\pi^{\frac{je}{5}} = \pi^{\frac{\frac{e}{e-2}.e}{5}} = 10.54019717 = e^{2.35519625}...(approx)}$$

Constant 30

$$\boxed{\pi^{\frac{je}{6}} = \pi^{\frac{\frac{e}{e-2}.e}{6}} = 7.118261625 = e^{1.962663542}...(approx)}$$

Constant 31

$$\boxed{\pi^{je} = \pi^{e^2.e} = 9672129983 = e^{22.99251439}...(approx)}$$

Constant 32

$$\boxed{\pi^{\frac{je}{2}} = \pi^{\frac{e^2.e}{2}} = 98346.98767 = e^{11.49625719}...(approx)}$$

Constant 33

$$\boxed{\pi^{\frac{je}{3}} = \pi^{\frac{e^2.e}{3}} = 2130.626748 = e^{7.664171463}...(approx)}$$

Constant 34

$$\boxed{\pi^{\frac{je}{4}} = \pi^{\frac{e^2.e}{4}} = 313.6032329 = e^{5.748128597}...(approx)}$$

Constant 35

$$\boxed{\pi^{\frac{je}{5}} = \pi^{\frac{e^2.e}{5}} = 99.33548691 = e^{4.598502878}...(approx)}$$

Constant 36

$$\boxed{\pi^{\frac{je}{6}} = \pi^{\frac{e^2.e}{6}} = 46.15871259 = e^{3.832085732}...(approx)}$$

12 Grand unified scale

In nature around us we see things grow or decay exponentially. In calculus e is the magic number whose derivative and integration is itself. Thats why we took e as the base of natural logarithm and we analyze very big data related to nature in natural logarithmic scale. How immensely big numbers can be scaled down to that small number e without having smoothing problem just like horizon problem faced in modern cosmology. Wherever infinitely big as well as infinitesimally small numbers are involved nature do not follow natural logarithmic scale i.e. $e^1, e^2, e^3, e^4, e^5, e^6, e^7...$ or inversely $\frac{1}{e^1}, \frac{1}{e^2}, \frac{1}{e^3}, \frac{1}{e^4}, \frac{1}{e^5}, \frac{1}{e^6}, \frac{1}{e^7}...$ will not give us 5 sigma answer,rather we will be off by 4 sigma(jokes apart, we may not be that much wrong but of course frequently we will be that much wrong e.g calculation of the age of universe or time since big bang) . Howsoever strange it may sound it is real and it is logical too. Here nature plays number theory. Believe it or not a truly invariant scale will be as given in the following table. I propose that, we shall call it Grand unified scale as it integrates infinity to unity. Natural logarithmic scale is just the linear trend line to Grand unified scale. In middle scale this is not felt. In very large or very small scale the smooth exponential/natural do not fit the trend line, step up or step down in the ladder becomes inevitable.

SL	Formula	i/j	g_1	g_2	g_3	g_4	g_5	g_6
1	$e^{\frac{i\pi}{g_p}}$	$\ln(2)$	$e^{2.18}$	$e^{1.09}$	$e^{0.73}$	$e^{0.54}$	$e^{0.44}$	$e^{0.36}$
2	$\pi^{\frac{je}{g_p}}$	$\frac{1}{\ln(2)}$	$e^{4.49}$	$e^{2.24}$	$e^{1.5}$	$e^{1.12}$	$e^{0.9}$	$e^{0.75}$
3	$\pi^{\frac{je}{g_p}}$	$\frac{e}{e-2}$	$e^{11.78}$	$e^{5.89}$	$e^{3.93}$	$e^{2.94}$	$e^{2.36}$	$e^{1.96}$
4	$e^{\frac{i\pi}{g_p}}$	$\frac{1}{\pi-3}$	$e^{22.19}$	$e^{11.09}$	$e^{7.4}$	$e^{5.55}$	$e^{4.44}$	$e^{3.7}$
5	$\pi^{\frac{je}{g_p}}$	e^2	$e^{22.99}$	$e^{11.50}$	$e^{7.66}$	$e^{5.75}$	$e^{4.60}$	$e^{3.83}$
6	$e^{\frac{i\pi}{g_p}}$	$\frac{4\pi-3}{2\pi(\pi-3)}$	$e^{33.78}$	$e^{16.89}$	$e^{11.26}$	$e^{8.45}$	$e^{6.76}$	$e^{5.63}$

Table 1: Tabulated value of Grand unified scale in ascending order

13 Factorial functions revisited

The factorial function is defined by the product

$$n! = 1 \cdot 2 \cdot 3 \cdots (n-2) \cdot (n-1) \cdot n,$$

for integer $n \geq 1$ This may be written in the Pi product notation as

$$n! = \prod_{i=1}^{n} i.$$

$$n! = n \cdot (n-1)!.$$

Euler in the year 1730 proved that the following indefinite integral gives the factorial of x for all real positive numbers,

$$x! = \Pi(x) = \int_0^\infty t^x e^{-t} dt, x > 1$$

Eulers Pi function satisfies the following recurrence relation for all positive real numbers.

$$\Pi(x+1) = (x+1)\Pi(x), x > 0$$

In 1768, Euler defined Gamma function, $\Gamma(x)$, and extended the concept of factorials to all real negative numbers, except zero and negative integers. $\Gamma(x)$, is an extension of the Pi function, with its argument shifted down by 1 unit.

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$

Eulers Gamma function is related to Pi function and factorial function as follows:

$$\Gamma(x+1) = \Pi(x) = x!$$

Ibrahim [1] defined the factorial of negative integer n as the product of first n negative integers.

$$-n! = \prod_{k=1}^{n} (-1)^k, -n \leq -1$$

The relation $n! = n \cdot (n-1)!$ allows one to compute the factorial for an integer given the factorial for a smaller integer. The relation can be inverted so that one can compute the factorial for an integer given the factorial for a larger integer:

$$(n-1)! = \frac{n!}{n}$$

13.1 Factorial of rational numbers

For positive half-integers, factorials are given exactly by

$$\Gamma\left(\tfrac{n}{2}\right) = \left(\tfrac{n}{2} - 1\right)! = \sqrt{\pi}\frac{(n-2)!!}{2^{\frac{n-1}{2}}}$$

or equivalently, for non-negative integer values of n:

$$\Gamma\left(\tfrac{1}{2}+n\right) = \left(n-\tfrac{1}{2}\right)! = \frac{(2n-1)!!}{2^n}\sqrt{\pi} = \frac{(2n)!}{4^n n!}\sqrt{\pi}$$
$$\Gamma\left(\tfrac{1}{2}-n\right) = \left(-n-\tfrac{1}{2}\right)! = \frac{(-2)^n}{(2n-1)!!}\sqrt{\pi} = \frac{(-4)^n n!}{(2n)!}\sqrt{\pi}$$

similarly based on gamma function factorials can be calculated for other rational numbers as follows,

$$\Gamma\left(n+\tfrac{1}{3}\right) = \left(n-\tfrac{2}{3}\right)! = \Gamma\left(\tfrac{1}{3}\right)\frac{(3n-2)!!!}{3^n}$$
$$\Gamma\left(n+\tfrac{1}{4}\right) = \left(n-\tfrac{3}{4}\right)! = \Gamma\left(\tfrac{1}{4}\right)\frac{(4n-3)!!!!}{4^n}$$
$$\Gamma\left(n+\tfrac{1}{p}\right) = \left(n-1+\tfrac{1}{p}\right)! = \Gamma\left(\tfrac{1}{p}\right)\frac{\big(pn-(p-1)\big)!^{(p)}}{p^n}$$

13.2 Limitation of factorial functions

However, this recursion does not permit us to compute the factorial of a negative integer; use of the formula to compute $(-1)!$ would require a division by zero, and thus blocks us from computing a factorial value for every negative integer. Similarly, the gamma function is not defined for zero or negative integers, though it is defined for all other complex numbers.Representation through the gamma function also allows evaluation of factorial of complex argument.

$$z! = (x+iy)! = \Gamma(x+iy+1), z = \mathbb{C} \setminus \{0,-1,-2,\dots\}$$

For example the gamma function with real and complex unit arguments returns

$$\Gamma(1+i) = i! = i\Gamma(i) \approx 0.498 - 0.155i$$

$$\Gamma(1-i) = -i! = -i\Gamma(-i) \approx 0.498 + 0.155i$$

13.3 Extended factorials using Delta function

Now let us extend factorials of negative integers by way of shifting the argument of Gamma function further down by 1 unit.Let us define Delta function as follows:

$$\Delta(x) = \int_0^\infty t^{x-2}e^{-t}dt$$

The extended Delta function shall have the following recurrence relation.

$$\Delta(x+2) = (x+2)\Delta(x+1) = (x+2)(x+1)\Delta(x) = x!$$

Newly defined Delta function is related to Eulers Gamma function and Pi function as follows:

$$\Delta(x+2) = \Gamma(x+1) = \Pi(x)$$

Plugging into $x = 2$ above

$$\Delta(4) = \Gamma(3) = \Pi(2) = 2$$

Putting $x = 1$ above

$$\Delta(3) = \Gamma(2) = \Pi(1) = 1$$

Putting $x = 0$ above

$$\Delta(2) = \Gamma(1) = \Pi(0) = 1$$

Putting $x = -1$ above we can remove poles of Gamma and Pi function as follows:

$$\Delta(1) = \Gamma(0) = \Pi(-1) = 1.\Delta(0) = -1.\Delta(-1) = \int_0^\infty t^{1-1}e^{-t}dt = \Big[-e^{-x}\Big]_0^\infty = \lim_{x\to\infty} -e^{-x} - e^{-0} = 0+1 = 1$$

Therefore we can say $\Delta(-1) = -1$. Similarly Putting $x = -2$ above

$$\Delta(0) = \Gamma(-1) = \Pi(-2) = -1.\Delta(-1) = -2.\Delta(-2) = \int_0^\infty t^0 e^{-t}dt = \Big[-e^{-x}\Big]_0^\infty = \lim_{x\to\infty} -e^{-x} - e^{-0} = 0+1 = 1$$

Therefore we can say $\Delta(-2) = -\frac{1}{2}$. Continuing further we can remove poles of Gamma and Pi function:

Putting $x = -3$ above and equating with result found above

$$\Delta(-1) = \Gamma(-2) = \Pi(-3) = -2. - 1.\Delta(-3) = -1 \implies \Delta(-3) = -\frac{1}{2}$$

Putting $x = -4$ above and equating with result found above

$$\Delta(-2) = \Gamma(-3) = \Pi(-4) = -3. - 2.\Delta(-4) = -\frac{1}{2} \implies \Delta(-4) = -\frac{1}{12}$$

Putting $x = -5$ above and equating with result found above

$$\Delta(-3) = \Gamma(-4) = \Pi(-5) = -4. - 3.\Delta(-5) = -\frac{1}{2} \implies \Delta(-5) = -\frac{1}{24}$$

Putting $x = -6$ above and equating with result found above

$$\Delta(-4) = \Gamma(-5) = \Pi(-6) = -5. - 4.\Delta(-6) = -\frac{1}{12} \implies \Delta(-6) = -\frac{1}{240}$$

$\vdots$

And the pattern continues upto negative infinity.
We can extend concept of factorials as follows:

1. We can define $(-1)! = \Delta(-1) = \Gamma(-2) = \Pi(-3) = -1$.

2. We can use Delta function to formulate factorial of negative integer $-n < -1$ as follows:
For even negative integers factorial can be obtained using the following formula:

$$(-n-1)! = \frac{-1}{\Delta(-n-2)} = \frac{-1}{\Gamma(-n-3)} = \frac{-1}{\pi(-n-4)}$$

For odd negative integers factorial can be obtained using the following formula:

$$-n! = \frac{-1}{(-n+1)\Delta(-n-1)} = \frac{-1}{(-n+1)\Gamma(-n-2)} = \frac{-1}{(-n+1)\Pi(-n-3)}$$

3. Through the extended Delta, Gamma, Pi function trio we can evaluate factorial of all complex argument.

$$z! = (x+iy)! = \Delta(x+iy+2) = \Gamma(x+iy+1) = \Pi(x+iy)$$

For example:

$$\Gamma(-1+i) = (-2+i)! = (-1+i)!.(-2+i) \approx (-2+i)(-0.155-0.498i)$$

4. Hence factorials satisfy the closure property and $\mathbb{C}$ is closed under the factorial operation.

14 A narrative description of my journey to numbers world*

Purpose of writing this narrative description is scaling the gap between quantum scale and middle scale or cosmic scale and middle scale with the help of number theory. We know that there is a huge scale gap between classical mechanics and Quantum Mechanics and also between general relativity and Quantum mechanics. Instead of doing something to fix this gap we rely upon our existing theories and math which we know are incomplete. We interpret math presuming that nature is scale invariant although the same can be interpreted the other way. At the grandest scale spacetime maybe scale invariant over time in long run just like number of primes are guaranteed vide PNT but the proven fact is nature is quantized or spacetime is discrete in short run just like the uncertainty about the exact sequence when a prime will appear on the number line. Both general relativity and quantum mechanics have got gravitational constant and planck constant respectively working as it's scale factor. But is that sufficient? I mean can a single constant fit into all the underlying dimensions. Why I am asking so? I would not have asked this type of questions if I would not have solved most of the number theory problems and see that numbers collectively do not fit into one particular scale rather they have got hierarchy of RSL scale factors. Numbers are said to be the foundation of mathematics together with mathematical logic. Although Russell Paradox put a question mark on the logic of mathematics, my answer to Russell Paradox is the barber will train a man from all other mans who does not shave themselves to become a barber and the barber 1 will get himself shaved by the barber 2. This way logic gives birth to numbers and mathematics cannot be pure logic without numbers. Both numbers and logic are inseparable parts of mathematics. I can safely declare every causal event and its effects are scripted in the language of numbers even before they happen and that's normal because numbers were already casted immediately after the absolute zero.Physics also require numbers to describe the physical phenomena around us. In general relativity equation we see number 3 pops up to take care of the three spatial dimensions pressure on energy density. In Planck's law we see an integer is required to save us from the ultraviolet catastrophe. Are this numbers safe to use such a way. I mean to say when this numbers are not properly scaled itself how can they fit into the given equations scale accurately. Numbers are not so innocent we think of them. And the kingpin of all the mischievous numbers is number 2. It is behind all the quantum weirdness observed in wave particle duality, measurement problem, quantum entanglement and what not. From Dark energy to Cosmological constant problem or Vacuum catastrophe wherever we face a problem at the deepest root we will see that number 2 is somehow involved. So is the situation of pure mathematics too. Riemann hypothesis remained unsolved for more than 150 years just because we don't understand the number 2 yet. I feel not so excited about my proof of Riemann Hypothesis, than the excitement I feel about connecting imaginary number i to Dark energy, connecting zeta function to cosmological constant problem or vacuum catastrophe, biggest challenges faced by the contemporary theoretical physicists. 96% of the universe is made of dark things is just the darkest side of science. Even if no one bothers (I know lot of research is happening but still the urgency is not felt somehow), I bother a lot as it takes away my sleep. I want my son to read science which always enlightened us with knowledge and wisdom required to explain how things work. It's high time for correcting the misconceptions build over time. I wanted to give my readers a full disclosure of my total thought process, so I took the narrative approach. Language used is kept simple that of day to day use so that it can reach more audience and they can relate it to something of their use. All my readers can freely pick up relevant portion according to their area of interest and use it with an one liner credit note. Elegance and elementary is always preferred over the rigor. Here I take my readers through the detour of my journey to numbers world.

14.1 I am the imaginary number i, and I am every where

The percentage of dark energy always hinted me that it could be a mathematical constant in the form of natural logarithm of 2 because numerically they are same and negative sign of dark energy resembles infinite rotation in the Eulers unit circle via Eulers formula. Natural logarithm of the redshift expansion scale factor of (1000-1100 time) is also approximately close to 10 times of natural logarithm of 2. String theorists treat this as extra dimensions but deeper I went stronger I felt that nature is scale variant in short run so that time itself remain eternally open in long run. This was not enough. I cross checked double natural logarithm of 2 and found that the value is arbitrarily close to a thousandth part of a years time in days. This way Natural logarithm of 2 is also bridging the scale of the solar system and the universal cosmic scale. These two natural signatures prompted me that I have correctly cracked the imaginary number i. Good news! is'nt it. The second root of i is a product of physical constants (dimensionless) as follows:

$$\frac{\text{2.mass of electron.speed of light squared.charles ideal gas constant}}{\text{boltzman constant}} \approx e^{\frac{\pi}{(\pi-3)}}$$

This constant time period entropy correction may take place and cosmological changes happen,in the last such event our planet earth was formed. Einstein should be happy now knowing that his initial idea of eternal universe is true. For those who may feel it is against the second law of thermodynamics I would ask them to study the distribution of primes, how the most disordered thing called primes lines up with military precision in descending order of prime density meaning constancy of prime number theorem. Similarly at grandest scale universe has no entropy or its entropy stands still with endless time. Why the arrow of time points towards the future i.e. Why Yesterday had low entropy than today and why tomorrow will have higher entropy than today? My reply is because numbers also do the same thing, all the numbers upto infinity have a continuous connection to the number 2 as composite numbers are made of primes and primes are all descendants of the sole even prime number 2. The seamless strange connection is reflected through the arrow of time. There may be Big bounces when we plug the infinite series of natural logarithm of 2 in his cosmological constant and the universe become ultimate perpetual machine. With these constants we can solve Cosmological Constant Problem or Vacuum Catastrophe because numerically it is near the same orders of magnitude that QM utterly worstly predicted for zero pint energy resulting scale difference of the order of $10^{120} = 10^{\frac{1}{\zeta(3)}}$. We should extensively use this grand unified scale to fix the scale gap in general relativity and quantum mechanics. I have a thought experiment for wave function collapse or quantum decoherence in double slit experiment. In a regular double slit experiment with slit detectors on if we simultaneously measure the spin of the passing by particles then we will see that the spin of the particles passed through one slit is just opposite of the spin of the particles passed through the other slit restoring the wave pattern. What does that prove? Quantum uncertainty can be eliminated by way of setting the apparatus and deterministic measurement can be made. $\Delta p \Delta x \geq \frac{1}{2}\hbar$ can be transformed to $\Delta p \Delta x = 1$ using the techniques of Fourier transformation provided duality is not break opened into singularity situation. To prove that Quantum entanglement is local and do not violate special relativity I have another thought experiment. Let's form a triangle selecting 3 cities randomly from the ATLAS. Labs in city (A,B), (B,C), (C,A) will entangle a pair of particles each among themselves and they will hold the entanglement to ensure that they are synced among themselves. With this 3 pair of particles in entanglement and synced in time if now any of the Labs try to entangle another pair of particle with another Lab located in city D they wont succeed and they may end up breaking the entanglement of all the particles. This shall prove that entanglement is local and do not violate Faster than light principle.

Theoretical physicists will benefit the most out this new mathematics as they will get a better insight to rewrite the physics written so far whether in the form of quantum mechanics or cosmology. Apart from solving many of the unsolved physics as hinted above, RSL scale will give us the data points to search for interesting events that happens in nature directly for example in astronomy if we plot available astronomical data in this scale we may see that supernova trend line coincides the RSL scale. Surely it can be applied to today's technologies to further optimise it.RSL scale will open up immense computing power challenging P versus NP problem. With this increase computing power and knowledge of riemann hypothesis intelligent hackers will try their hands on RSA algorithm. Honestly speaking RSA algorithm is not Invincible. Internet security can be strengthened by way of strengthening RSA algorithm. Quantum computing can be boosted further so that it overtakes digital computing.First thing I search in the internet after solving riemann hypothesis was 60th degree parallel North and South to find a natural signature to zeros of Zeta function inside our planet which is also a riemann sphere. Geo-physicists may consider the idea of exploring 60 degree Parallel South where there is no land mass for new discoveries as that latitude is the critical line following zeta zeros. Who can say where the road goes, may be with the understanding of RH and RSL we invent new lean technology tomorrow to optimise usage of prime natural resources which is depleting day by day. We can take one step forward towards becoming type 1 civilisation in Kardashev scale and gradually move along the scale. Weather control, climate control shall become reality. I may be sounding too much like sci-fi movies. Lets stop it here. Anything further realistic comes to my mind, surely I will bring it in my next paper if I succeed in publishing the current paper. If I do not succeed then I wont blame any body as I understand that's part of life. Boys don't cry, they are supposed to stand up absorbing the pains of failure. So many star falls everyday nobody keeps the account. I believe that my ideas have enough spark to en-light another beautiful mind on this earth. I will continue to search for that wise man. If I find him out I will consider that my job is done, atleast enough for this life. I being stardust (collection of particles or elements that form in nuclear fusion reaction in a star) and being a subject matter of causality I shall beat entropy rules and reincarnate into Boltzmann brain again and again to see whether mankind have adopted my work or they are still struggling and going round and round the problems of today's physics and mathematics. Until then my wishes for a good luck to all the haunters trying to haunt Riemann hypothesis, Dark energy etc.

14.2 Searching for triangularity into the Duality

I remember the day I came to know about Euler's formula first time. Initially I was not getting fully convinced with Eulers unit circle concept as it does not give us concentric circles representing every natural numbers. Euler's formula do not jumps like the numbers instead it rotates the numbers around the same unit circle. An Idea came to my mind, what if I find a way which will give me a jump to another number and come back again to Unit circle. I took the help of trigonometric form of complex number. I looked into the table of sine and cosine and was searching for the argument which will give me a modulus of 2 on half unit circle. I found the angle pi upon 3 give a modulus of 2 on half unit circle. Then I thought that using the same logic I'll be able to get a modulus of 3 on one-third unit circle. But I could not find a modulus of 3 on one-third unit circle. I was not aware of Fermat's last theorem. Later when I came to know about Fermat's last theorem, I understood the reason why it is not possible to get a modulus of 3 on one-third unit circle. It is because before we reach a modulus of 3 we will have a 2 pi rotation on the unit circle and as such we will never reach a modulus of 3 on one-third unit circle. Triangularity is hidden inside another complex dimension perpendicular to the edge of duality, not easily detectable just like one might have missed the fact that number 3 has already appeared when i said pi upon 3. Proving Fermat's last theorem involves downsizing that extra dimension by 1 (i.e. 4D to 3D) and completing the cube is impossible. When Fermat was writing in the margin that he had the proof of his own last theorem I guess he was talking something similar to my approach of proving his last theorem by mathematical induction. I wondered if a modulus of 3 is not there then why we don't face any problem in getting a fractional modulus like one third, one fifth and so on. I found answer to that question later when I came to know about Cantors theorem. Cantor has given a nice proof why there are much more ordinal numbers than cardinal numbers. I was able to find the value of Zeta 1 (Sum of unit fractions) which is just double of Zeta (-1) (Sum of natural numbers). This proves another version of cantor's theorem numerically that there shall be more rational numbers than natural numbers.

14.3 On the proofs of Riemann Hypothesis

Immediately after discovery of mathematical duality I started trying to solve Riemann hypothesis. Here also Euler's initial work on Zeta function helped me a lot. I started with Euler's original product form. Although Euler product form does not involve imaginary numbers I called it into the product form based on the fact that Zeta function has got analytical continuity in the complex domain. Now Eulers product form of Zeta function in exponential form of complex numbers can be zero if and only if, any of the factors can be shown equals to zero. Manipulating this way each term of Zeta function can be equated to Eulers formula in unit circle. One step ahead I have shown the sum of all the arguments in one of such factors equals Pi and the sum of the entire radius equals 1. Apparently this may sound illegal but that's the logic of infinite sum to unity. This way it was possible to solve the argument and radius which will be responsible for non trivial zeros of Zeta function. It was also possible to prove Riemann hypothesis using the alternate product form. The only new thing I had to apply here was when multiplying a positive complex number with a negative complex number instead of adding we can subtract the lower argument from the higher one. I believed that Zeta pole could be removed using Euler's induction method too. I started from there where Euler left. I took infinite product of positive Zeta values both from the side of sum of numbers and the side of product of primes. This gave me the value of Zeta 1 = 1. Apart from this I got a nice relation between the sum of fractions and the product of primes reciprocals. Similar concept I used to calculate Zeta(-1). I got second root of Zeta(-1) which equals half apart from the known one. Also I got a nice relationship between sum of numbers and the product of primes which I used to formulate fundamental formula of numbers. All this manipulation may not be permissible in conventional mathematics but it does make complete sense when we apply deeper logic applied by Euler, Cantor, Ramanujan while dealing with infinity. But I knew that such a easy proof may not be well accepted although it involved almost an years effort to figure it out. I thought I will proof Riemann hypothesis using Riemanns own functional equation. Here it took less than another years time but at the end I succeeded. And the success came using newly defined Delta function for factorial and shifting gamma functions argument by 2 units. The proof came after removal of pole at Zeta 1.

14.4 On the complex logarithm simplified an pi based logarithm

Even after solving these conjectures I was having a feel that I was missing something. Mathematical duality is ok, specialty of number 2 is understood, prime numbers take birth at Zeta zeros, Zeta zeros fall on the half line in complex plane all this are ok but someone said to solve Riemann hypothesis one has to introduce new mathematics. So far my work does not give anything new. Intuitively I was not clear even with my own proof. Almost a months time elapsed I emptied all my thoughts. When I came back to revisit my work, the first thing struck my mind that I have not applied the mysterious Euler's formula yet on complex logarithm(ultimately RH was a complex logarithm problem) , although it had still more potential. Imaginary number i remained still mysterious to me. I thought I will do something with imaginary number i as it cannot remain undefined eternally. I needed to understand how can I define imaginary number i such a way that it vanishes or it becomes real like i squared. I had realised that Zeta function has simultaneous and continuous properties of exponential as well as logarithmic function. Just like natural logarithm of 1 give us zero we get zeros of Zeta function on the half line which is the base of all bases. Can we extend the concept of Zeta function to complex logarithm just like Riemann extended Euler's Zeta function to the whole Complex plane which will unify complex numbers, complex logarithm and number theory. Why not, in fact Roger Cotes started that way and showed that complex logarithm will always involve a complex number later Euler used the concept in exponential form. I thought I will be doing the opposite. I will use Euler's formula to do complex logarithm. But I failed perhaps because I was getting lost in Cantors paradise. I took u turn and concentrated on how to find out i. I knew that Zeta function have a close relation with eta function which is again nothing but alternate Zeta function. Eta of 1 results natural logarithm of 2. After falling many times on the slippery road I stood up with the conclusion that natural logarithm 2 is the first solution to i (Although that time I didn't knew that I will get another 2 solution to i). Now connecting zeta results found earlier I could set the properties of real and simple (RS) logarithm which unlike complex logarithm do not need branch cuts. Although complex logarithm still remain a multivalued function but it's number of solutions drastically come down from infinity to number of ways we can define imaginary number i. While working on this I was getting a feel that pi was equally mysterious from the perspective of complex logarithm. I solved the mystery of pi based Complex logarithm too with my crooked manipulating algorithm. When wondering about the possible number of solution to i/j a crazy idea came to my mind connecting the problem to number of dimensions we see in nature. As every number upto infinity can be traced back on the d-unit circle then some property of the individual numbers progression upto infinity should also reflect unity in some sense. In other words the idea was if all the numbers can be plotted in maximum 3 dimension then can all the numbers have three constants which then can explain some kind of cyclic behavior of numbers globally. The painful part was arranging all the jigsaw puzzles to figure out those exact three constants both for natural logarithmic scale and pi based logarithmic scale. Gradually I found the second and third root of i using same methods. Plugging the different values of i/j into Euler's formula I discovered the scale natural exponential scale proceeds which give birth to prime numbers at higher frequencies, I am sure.

* This part of the document do neither form part of the proof nor the authors personal views to be considered seriously(although the author is quite sure that considering the propositions seriously may prove to be beneficial). Its just an addendum to the document.

References

[1] *Extension of factorial concept to negative numbers*,Ibrahim AM.,2013.

Surajit Ghosh, 11/6 T.M.G Road, Kolkata-700 041 Or 593 Jawpur Road, Supari Bagan, Kolkata-700074
E-mail address, Surajit Ghosh: surajit.ghosh@yahoo.com *Mobile*, Surajit Ghosh: +918777677384