SunQM-3s11: Using {N,n} QM's probability density 3D map to build a complete Solar system with time-dependent orbital movement (semi-QM deduction)

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Abstract

Combining the results from the previous SunQM series papers, a (high resolution) 3D probability density map has been constructed and it is able to describe the whole Solar system with time-dependent orbital movement. It is the Eigen description of our Solar system using Schrodinger equation's solution. In it, the Eigen n' values of all planet have been calculated. These Eigen n' values give both the orbital r and the surface r information for each planet. The result revealed that for all planets, their Eigen n' values in all three dimensions are equal ($n'_r = n'_\theta = n'_\varphi = n'$). For example, for a planet at orbit {1,5//6} (in the Solar {N,n//6} QM structure), if it has Eigen n' = n*q^w = 5*6^11 = 1.81E+9 in each of rθφ-3D dimension, then it will have an orbital r = 1.57E+11 m, and surface r = 7.89E+6 m. This is very close to Earth's orbital r = 1.49E+11 m and surface r = 6.38E+6 m. For Asteroid belt and the cold-KBO, their Eigen n'(s) in the r- and θ-dimension are equal ($n'_r = n'_\theta$). For example, Asteroid belt's Eigen n' = 48 in both r- and θ-dimension. We found that a planet's wave function in the φ-dimension is composed by a group of wave functions that further forms a (group) wave packet out of the phase wave. Because Schrodinger equation/solution can accurately describe a Solar system as well as a hydrogen atom, it implies that either the whole universe or a single quark can also be described by Schrodinger equation and solution (simultaneously at the resolution levels of either quark, or proton, or star, or galaxy, or even our universe). Several lower resolution 3D probability density maps (also based on Schrodinger equation's solution) for the whole Solar system have also been successfully built. A summary of the major results from the phase-1 study of the SunQM series has been listed. This work indisputably proved that Solar system is a QM system.

Introduction

The SunQM series papers [1] ~ [15] have shown that the formation of Solar system (as well as each planet) was governed by its {N,n} QM. In papers SunQM-3s6, SunQM-3s7, and SunQM-3s8, it has been shown that the formation of planet's and star's (radial) internal structure is governed by the planet's or star's radial QM. In papers SunQM-3s3 and -3s9, it has been shown that the surface mass (atmosphere) movement of Sun, Jupiter, Saturn, and Earth, etc., is governed by Star's (or planet's) θφ-2D dimension QM. In paper SunQM-3s4 and -3s10, it has been shown that the formation of either ring structures of a planet, or the belt structures in Solar system, is also governed by the {N,n} QM (the nLL effect). In current paper, we want to use {N,n} QM and Schrodinger equation's solution to build a 3D probability density map for a complete Solar system with time-dependent orbital movement. Note: for {N,n} QM nomenclature as well as the general notes for {N,n} QM model, please see SunQM-1 section VII. Note: Microsoft Excel's number format is often used in this paper, for example: x^2 = x^2, 3.4E+12 = $3.4*10^{12}$, 5.6E-9 = $5.6*10^{-9}$. Note: The reading sequence for SunQM series papers is: SunQM-1, 1s1, 1s2, 1s3, 2, 3, 3s1, 3s2, 3s6, 3s7, 3s8, 3s3, 3s9, 3s4, 3s10, and 3s11. Note: for all SunQM series papers, reader should check "SunQM-4s5: Updates and Q/A for SunQM series papers" for the most recent updates and corrections.

I. To build a (time-independent) 3D probability density r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 for the current Sun from the center to the surface at {0,2//6}

In the Solar {N,n} QM theory, the whole Solar system is primarily governed by a single super large three dimensional r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 QM probability density structure which covers the Sun, all 8 planets, Asteroid and Kuiper belts, 4 undiscovered planets, and Oort cloud. Here we name this super large r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 probability density function as Sun's primary (or master) r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2. For this primary r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2, we use Sun core{0,1//6} as n=1, so it matches the Sun {N,n//q} QM structure naturally. Another reason for using {0,1//6} as n=1 is that in paper SunQM-3s10, Asteroid belt at {1,8} = {0,48//6} can be perfectly described by |48,47,47> Eigen QM state alone, and also Kuiper belt at {2,6} = {0,192//6} can be perfectly described by |192,191,m> Eigen QM state alone. This strongly suggests that using {0,1//6} as n=1 is the right choice for Sun's primary r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2. As we defined before, |nLL> means |n,*l*,m> with *l* = n-1, and m = n-1 (see SunQM-3s1). Under Sun core's total n=1, planets/belts' n(s) are calculated according to the {N,n//6} QM model and listed in column 10 of Table 1 (see also in SunQM-1 Table 3 column 7).

Table 1. Calculation of a planet's Eigen n' (in both r- and θ-dimension), orbital angular velocity ω (group-ω and phase-ω), φ position (day-0 and day-60).

NASA's data of planets						assigned N, n, period factor			set total n=1 at Sun core calc model n, rn, vn			Determine planet r-dimensional n' & w			
	mass	Sun's body-r or planets' orbit-r_n	vn	orbit period	planet's body-r, b=	N	n	period factor	total n from Sun core	rn= r1*n^2	v_n = sqrt(G M/r_n)	n' = ln(0.1) / [ln(1 + b /r_n) - (b /r_n)]	w= log(n' / n) / log(6)	round up w	n'_r = n*q^w
unit	kg	m	m/s	days	m					m	m/s				
Sun core		1.74E+08				0	1	6	1	1.74E+08					
SUN	1.99E+30	6.96E+08				0	2	6	2	6.96E+08					
{0,3} corona						0	3	6	3	1.57E+09					
{0,4} corona						0	4	6	4	2.78E+09					
{0,5} corona						0	5	6	5	4.35E+09					
{0,6} corona end						0	6	6	6	6.26E+09					
{1,2}						1	2	6	12	2.50E+10					
Mercury	3.3E+23	5.79E+10	47400	88	2.44E+06	1	3	6	18	5.64E+10	48533	2.46E+09	11.45	11	1.09E+09
Venus	4.87E+24	1.08E+11	35000	224.7	6.05E+06	1	4	6	24	1.00E+11	36400	1.26E+09	10.92	11	1.45E+09
Earth	5.97E+24	1.49E+11	29800	365.2	6.38E+06	1	5	6	30	1.57E+11	29120	2.77E+09	11.24	11	1.81E+09
Mars	6.42E+23	2.28E+11	24100	687	3.40E+06	1	6	6	36	2.25E+11	24266	2.03E+10	12.25	12	1.31E+10
Asteroid belt	2.92E+21	4.02E+11				1	8	6	48	4.01E+11	18200				
Jupiter	1.90E+27	7.78E+11	13100	4331	7.15E+07	2	2	5.33	64.0	7.12E+11	13658	4.56E+08	10.67	11	7.26E+08
Saturn	5.68E+26	1.43E+12	9700	10747	6.03E+07	2	3	5.33	95.9	1.60E+12	9106	3.25E+09	11.54	12	6.53E+09
Uranus	8.68E+25	2.97E+12	6800	30589	2.56E+07	2	4	5.33	127.9	2.85E+12	6829	5.71E+10	12.98	13	5.22E+10
Neptune	1.02E+26	4.51E+12	5400	59800	2.48E+07	2	5	5.33	159.9	4.45E+12	5463	1.48E+11	13.39	13	6.53E+10
Kuiper belt	1.46E+22	5.91E+12				2	6	5.33	191.9	6.40E+12	4553				
{3,2}	7.12E+25				2.18E+07	3	2	6	383.8	2.56E+13	2276	6.35E+12	16.00	16	5.64E+12
{3,3}	3.99E+25				1.80E+07	3	3	6	575.6	5.76E+13	1518	4.73E+13	16.89	17	5.08E+13
{3,4}	2.75E+25				1.59E+07	3	4	6	767.5	1.02E+14	1138	1.92E+14	17.51	18	4.06E+14
{3,5}	1.98E+25				1.42E+07	3	5	6	959.4	1.60E+14	911	5.67E+14	18.00	18	5.08E+14

				Determine planet θ-dimensional n' & w Note: w(ϕ) = w(θ), and n'_ϕ = n'_θ					{N,n} calculated ω			initial ϕ= day-0	day-0	day-60	day-60
	±b = at n'=n*q^w & Porb=0.01	r_1 = $r_{n'}$ / (n*q^w)^2	r_1 at {N,1//6}	w(θ) =	n'_θ = n*q^w	0.01^(1/(2n'))	θ'= acos[0.01^(1/(2n'))]	b= r*sin(θ')	phase $\omega_{n,ph}$ = v_n / r_n/2	group ω = ω_n = v_n / r_n	period T= 2π/(2ω)/(3600*24)	day-0, 8/14/2019	day-0	60	60
unit	m	m					arc	m	arc/s	arc/s	day	degree	arc	degree	Δdegree
Sun core															
SUN															
{0,3} corona															
{0,4} corona															
{0,5} corona															
{0,6} corona end															
{1,2}															
Mercury	2.59E+06	4.76E-08	{-10,1//6}	11	1.09E+09	0.999999997884414	6.50E-05	3.67E+06	4.31E-07	8.61E-07	84	62	1.08	318	256
Venus	6.91E+06	4.76E-08	{-10,1//6}	11	1.45E+09	0.999999998413311	5.63E-05	5.64E+06	1.82E-07	3.63E-07	200	180	3.14	288	108
Earth	7.89E+06	4.76E-08	{-10,1//6}	11	1.81E+09	0.999999998730648	5.04E-05	7.89E+06	9.30E-08	1.86E-07	391	0	0.00	55	55
Mars	5.18E+06	1.32E-09	{-11,1//6}	12	1.31E+10	0.999999999823701	1.88E-05	4.23E+06	5.38E-08	1.08E-07	676	191	3.33	223	32
Asteroid belt									2.27E-08	4.54E-08	1601				
Jupiter	6.94E+07	1.35E-06	{-9,1//6}	11	7.26E+08	0.999999996826621	7.97E-05	5.67E+07	9.60E-09	1.92E-08	3788	304	5.31	310	6
Saturn	5.21E+07	3.75E-08	{-10,1//6}	12	6.53E+09	0.999999999647402	2.66E-05	4.25E+07	2.84E-09	5.69E-09	1.28E+04	329	5.74	331	2
Uranus	3.27E+07	1.04E-09	{-11,1//6}	13	5.22E+10	0.999999999955925	9.39E-06	2.67E+07	1.20E-09	2.40E-09	3.03E+04	73	1.27	74	1
Neptune	3.73E+07	1.04E-09	{-11,1//6}	13	6.53E+10	0.999999999964740	8.40E-06	3.73E+07	6.14E-10	1.23E-09	5.92E+04	30	0.52	30	0
Kuiper belt									3.55E-10	7.11E-10	1.02E+05				
{3,2}	2.31E+07	8.05E-13	{-13,1//6}	16	5.64E+12	0.999999999999592	9.03E-07	2.31E+07	4.44E-11	8.89E-11	8.18E+05				
{3,3}	2.12E+07	2.24E-14	{-14,1//6}	17	5.08E+13	0.999999999999955	3.01E-07	1.73E+07	1.32E-11	2.63E-11	2.76E+06				
{3,4}	1.33E+07	6.21E-16	{-15,1//6}	18	4.06E+14	0.999999999999994	1.06E-07	1.09E+07	5.55E-12	1.11E-11	6.55E+06				
{3,5}	1.53E+07	6.21E-16	{-15,1//6}	18	5.08E+14	0.999999999999995	9.54E-08	1.53E+07	2.84E-12	5.69E-12	1.28E+07				

Note: all planets data is obtained from NASA's Planetary Fact Sheet at: http://nssdc.gsfc.nasa.gov/planetary/factsheet/, Sun's data is from: https://en.wikipedia.org/wiki/Sun. According to wiki "Asteroid belt", Asteroid belt's mass = 4% of Moon mass = 0.04 * 7.3E+22 = 2.92E+21 kg. Note: Kuiper belt's mass was assumed to be 5x of Earth's mass here, in comparison with wiki "Kuiper belt" 's ~0.1x of measured, or ~30x of modeled (of Earth's mass). Note: 1.99E+30 kg is for the whole Sun (including Sun core). Note: In column 12, the modeled orbit v_n is calculated by using classical physics $F = ma = mv_n^2 / r_n$, $F = GMm / r_n^2$, $mv^2 / r_n = GMm / r_n^2$, $r_nv_n^2 = GM$, $v_n = sqrt(GM / r_n)$. Note: for the undiscovered {3,n=2..5}planets, the estimated mass in column 2 is copied from Table 3b in SunQM-1s1, and the estimated surface-r in column 6 is copied from Table 2 in SunQM-3s6. Note: in column 15, the integer w is the round-up (or round-down) according to column 17's calculated b value (which should close to planet's surface-r at column 6). Note: From column 18's r_1 value, check SunQM-1s2 Table 1 to find the corresponding {N,1}, and fill to column 19. Notice that {1,n=3..6}'s r_1 fits to the Hot-G r track in SunQM-1s2 Table 1, and {2,n=3..6} 's and {3,n=3..6}'s r_1 fits to the Cold-G r track in SunQM-1s2 Table 1.

I-a. To build a (time-independent) 3D probability density r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 for the current Sun from the center to the surface at {0,2//6}

For the current Sun ball, a detailed r-dimension description is given in SunQM-3s8 section-I. Because from Sun surface to Sun core, it belongs to a {0,1//6}o orbit shell space, therefore we also can described it with n=1, *l* = n-1 =0, m = -*l* … +*l* = 0, or |1,0,0>, or by

$$r^2 \, |R(1,0)|^2 |Y(0,0)|^2 \qquad \text{eq-1}$$

We know that either r^2 * |R(1,0)|^2 function or |Y(0,0)|^2 function is a perfect sphere, so the production of these two is still a perfect sphere. Because the current Sun has 100% mass occupancy up to surface-r at {0,2}, therefore the shape of our Sun follows the primary r^2 * |R(1,0)|^2 * |Y(0,0)|^2 and it is a perfect sphere.

In the Solar {N,n} QM, the space inside Sun core with r less than the r of {0,1//6} is described by {-1,n=1..5//6}o orbit shells, and the space inside the r of {-1,1//6} is described by {-2,n=1..5//6}o orbit shells, and the space inside the r of {-2,1//6} is described by {-3,n=1..5//6}o orbit shells, and so on so forth (see SunQM-3 Figure 3a). To use (Sun's) primary r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2, we have to set N=0 for Sun's whole {N,n//6} QM structure. In the {N,n} QM's nomenclature, we have {-1,n//6} = {0, n/(6^1)//6}, {-2,n//6} = {0, n/(6^2)//6}, and {-3,n//6} = {0, n/(6^3)//6} (see SunQM-1 section-VII). So {-1,n=1..5//6}o orbit shells is re-written as {0,n=(1/6, 2/6, 3/6, 4/6, 5/6)//6}o orbit shells, with the fractional quantum number as n = 1/6, 2/6, 3/6, 4/6 and 5/6. And {-2,n=1..5//6}o orbit shells is re-written as {0,n=(1/6^2, 2/6^2, 3/6^2, 4/6^2, 5/6^2)//6}o orbit shells, with the fractional quantum number as n = 1/6^2, 2/6^2, 3/6^2, 4/6^2 and 5/6^2, and so on so forth.

So, the original probability function r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 is only suitable for each specific N's {N,n//q} QM where n is always the base n (i.e., N=0, or N=2, etc.). For a {N,n//q} QM, **let's define that {N,n//q} is a general form (where n is base n)**, and **let's define {0,n*q^N//q} is the primary form**. Obviously, in the primary form {0,n*q^N//q}, the quantum number is no longer base n. It is either high frequency n (if N>0), or sub-base n (if N <0). For general form {N,n//q}, we write it as |n,l,m>. For primary form {0,n*q^N//q}, we write the QM state as |n*q^N,*l*,m> (in which we do NOT calculate out the value of n*q^N). According to the rule of "all mass between r_n and r_{n+1} belongs to orbit n (see paper SunQM-3s2)", the uncalculated and calculated n*q^N have different n and covers different r ranges. For example, for {1,3//6}, it is |3,*l*,m>. For {0,3*6^1//6}, it is |3*6^1,*l*,m>, we do NOT write it as |18,*l*,m> because they have different meaning: |18,*l*,m> QM state covers r-dimension from r of n=18 to n=19 with r_1 at {0,1//6}, while |3*6^1,*l*,m> QM state covers r-dimension from r of n=3 to n=4 with r_1 at {1,1//6}, which equivalent to r of n = 3*6^1 =18 to n = 4*6^1 =24 with r_1 at {0,1//6}. Considering the rule of "all mass between r_n and r_{n+1} belongs to orbit n", so |3*6^1,*l*,m> QM state is always a linear combination of |18,*l*,m>, |19,*l*,m>, |20,*l*,m>, |21,*l*,m>, |22,*l*,m>, and |23,*l*,m> QM states. This is always true when |3*6^1,*l*,m> QM state is at ~100% mass occupancy. Only when the mass occupancy << 1%, then |3*6^1,*l*,m> QM state may equal to |18,*l*,m> QM state.

Accordingly, we write the probability function for a primary form {0,n*q^N//q} QM state as

$$r^2 \, |R(n\,q^N, l)|^2 |Y(l,m)|^2 \qquad \text{eq-2}$$

However, due to that if N < 0, then *l* = n*q^N - 1 < 0, and we don't know how to handle it. So to calculate eq-2, we have to use the general formed {N,n//q} to move r_1 inward if N < 0 or outward if N > 0. For example, the {-3,4//6}o orbit shell's probability function will be calculated as r^2 * |R(4,*l*)|^2 * |Y(*l*,m)|^2, with r_1 at {-3,1//6}, and with *l* = 0, 1, 2, 3, m = -*l*, … +*l*, even it can be written as r^2 * |R(4/6^3,*l*)|^2 * |Y(*l*,m)|^2, or its |n,*l*,m> can be written as |4/6^3,*l*,m> . Similarly, the {1,3//6}o orbit shell's probability function will be calculated as r^2 * |R(3,*l*)|^2 * |Y(*l*,m)|^2, with r_1 at {1,1//6}, and with *l* = 0, 1, 2, m = -*l*, … +*l*, even it can be written as r^2 * |R(3*6^1,*l*)|^2 * |Y(*l*,m)|^2, or its |n,*l*,m> can be written as |3*6,*l*,m> .

Then, according to SunQM-3s8, Sun's {0,1//6}o orbit space shell can be accurately described by eq-1. Note: the small contribution from r^2 * |R(2,1)|^2 in the outer edge of {0,1}o orbit shell is ignored here (see SunQM-3s8). Sun's {-1,n=1..5//6}o orbit shells can be accurately described by

$$r^2 \left[a_1 \left| R\left(\tfrac{1}{6},0\right) \, Y(0,0) \right|^2 + a_2 \left| R\left(\tfrac{2}{6}, l=0..1\right) \, Y(l,m) \right|^2 + a_3 \left| R\left(\tfrac{3}{6}, l=0..2\right) \, Y(l,m) \right|^2 + a_4 \left| R\left(\tfrac{4}{6}, l=0..3\right) \, Y(l,m) \right|^2 + a_5 \left| R\left(\tfrac{5}{6}, l=0..4\right) \, Y(l,m) \right|^2 \right] \qquad \text{eq-3}$$

where, l = 0, … n-1, and m = -l, …, +l, and a_1 …a_5 are the linear combination coefficients. Note: the small contribution from r^2 * |R(6/6,5)|^2 * |Y(5,5)|^2 in the outer edge of {-1,n=1..5//6}o orbit shell is ignored here too (see SunQM-3s8). Similarly, Sun's {-2,n=1..5//6}o orbit shells can be accurately described by

$$r^2 \left[a_1 \left|R\left(\frac{1}{6^2}, 0\right)\ Y(0,0)\right|^2 + a_2 \left|R\left(\frac{2}{6^2}, l = 0..1\right)\ Y(l, m)\right|^2 + a_3 \left|R\left(\frac{3}{6^2}, l = 0..2\right)\ Y(l, m)\right|^2 + a_4 \left|R\left(\frac{4}{6^2}, l = 0..3\right)\ Y(l, m)\right|^2 + a_5 \left|R\left(\frac{5}{6^2}, l = 0..4\right)\ Y(l, m)\right|^2\right]$$

eq-4

where l = 0, …, n-1, and m = -l, …, +l, and a_1 …a_5 are the linear combination coefficients which have values different than those in eq-3. And, so on so forth for probability functions of N = -2, -3, etc.

Now let's determine how many negative valued N super-shells are needed for Sun's probability function at the minimum acceptable accuracy, is it {0,1//6}o orbit shell space only, or {0,1//6}o plus {-1,n=1..5//6}o orbit shell spaces, or {0,1//6}o plus {-1,n=1..5//6}o plus {-2,n=1..5//6}o orbit shell spaces, or even more? If we only count the {0,1//6}o orbit shell space and not include Sun core {0,1//6}, then the volume ratio of total Sun (with radius R) and Sun core (with radius r) is R^3 / r^3 = 4^3 / 1^3 = 64.However, from wiki "Solar core", "*The core inside 0.20 of the solar radius, contains 34% of the Sun's mass, but only 0.8% of the Sun's volume*". So it is obvious that only count Sun's {0,1//6}o orbit shell space and ignore the Sun core is not accurate enough for the Sun's probability function. Now if we count the total Sun as {0,1//6}o plus {-1,n=1..5//6}o orbit shell spaces, and ignore Sun's center ball part within {-1,1//6} (and define its radius as r'). Then the volume ratio of total Sun (with radius R= 6^2 * 2^2 * r' =144 r') vs. the ignored center ball (with radius r') is R^3 / r^3 = 2985984. Then the mass in {0,1//6}o plus {-1,n=1..5//6}o orbit shell spaces can easily pass 99% of the Sun (even though the center part of Sun has much higher mass density). Therefore, we believe that counting {0,1//6}o plus {-1,n=1..5//6}o orbit shell spaces is enough to give the minimum acceptable accuracy for the Sun's probability function.

Accordingly, we can obtain a Sun's probability density function by adding eq-3 to eq-1, as shown in below:

$$\mathbf{r^2\ |\Psi(r, \theta, \phi)_{Sun}|^2 \propto r^2 \left[a_1 \left|R\left(\frac{1}{6}, 0\right)\ Y(0,0)\right|^2 + a_2 \left|R\left(\frac{2}{6}, \mathit{l} = 0..1\right)\ Y(\mathit{l}, m)\right|^2 + a_3 \left|R\left(\frac{3}{6}, \mathit{l} = 0..2\right)\ Y(\mathit{l}, m)\right|^2 + a_4 \left|R\left(\frac{4}{6}, \mathit{l} = 0..3\right)\ Y(\mathit{l}, m)\right|^2 + a_5 \left|R\left(\frac{5}{6}, \mathit{l} = 0..4\right)\ Y(\mathit{l}, m)\right|^2 + a_6 |R(1, 0)\ Y(0, 0)|^2\right]}$$

eq-5

where, l = 0, …, n-1, and m = -l, …, +l, and a_1 …a_6 are the linear combination coefficients. By integrating eq-5, we can get the integration form of a Sun's probability function:

$$\mathrm{Mass}(r, \theta, \varphi) = \int_0^{6.96\times10^8} \int_0^{\pi} \int_0^{2\pi} r^2 \left[a_1 \left|R\left(\frac{1}{6}, 0\right)\ Y(0,0)\right|^2 + a_2 \left|R\left(\frac{2}{6}, l\right)\ Y(l, m)\right|^2 + a_3 \left|R\left(\frac{3}{6}, l\right)\ Y(l, m)\right|^2 + a_4 \left|R\left(\frac{4}{6}, l\right)\ Y(l, m)\right|^2 + a_5 \left|R\left(\frac{5}{6}, l\right)\ Y(l, m)\right|^2 + a_6 |R(1,0)\ Y(0,0)|^2\right] \sin(\theta)\ dr\ d\theta\ d\varphi$$

eq-6

where, l = 0, …, n-1, m = -l, …, +l, and a_1 …a_6 are the normalized linear combination coefficients that makes the integrated eq-6's value equals to Sun's total mass. Notice that in eq-6, the normalization coefficient of each radial wave function R(n,l) is no longer the same as the original R(n,l) (which is normalized for H-atom's $r_1 = a_0$ = 5.29E-11 m, also see section I-b for detailed explanation). The normalization coefficient of each R(n,l) in eq-6 (and eq-7) includes Sun's radial mass density distribution information. In eq-6, the integration of each θφ probability ∬ |Y(l,m)|^2 *sin(θ) dθ dφ, [θ=0, π; φ=0, 2π] with l = 0, …, n-1, and m = -l, …, +l is always independent of r-dimension's integration, and due to the ~ 100% mass occupancy inside Sun, it always give a constant value. So we can put this constant value into the coefficient a_j (where j = 1, 2, …). Therefore, eq-6 can also be simplified as

$$1.99 \times 10^{30}(\text{kg}) = \int_0^{6.98\times10^8} r^2 \left[a_1 \left|\mathrm{R}\left(\frac{1}{6},0\right)\right|^2 + a_2 \left|\mathrm{R}\left(\frac{2}{6}, l = 0..1\right)\right|^2 + a_3 \left|\mathrm{R}\left(\frac{3}{6}, l = 0..2\right)\right|^2 + a_4 \left|\mathrm{R}\left(\frac{4}{6}, l = 0..3\right)\right|^2 + a_5 \left|\mathrm{R}\left(\frac{5}{6}, l = 0..4\right)\right|^2 + a_6 |\mathrm{R}(1,0)|^2\right] \mathrm{dr}$$

eq-7

where R(n,*l*) is normalized for Sun's r_1 = 1.74E+8 m, and also contains Sun's radial mass density distribution information, and a_j = 1 ... 6 are the r-independent values, and they are the normalized linear combination constants that makes the integrated eq-7's value equals to Sun's total mass.

Now if we use H-atom's R(n,*l*), and it should not contain Sun's radial mass density distribution information, then we have an integration formula that is similar to that in SunQM-3s8 section-I:

$$1.99 \times 10^{30}(\text{kg}) = \int_0^{6.98\times10^8} r^2 \left[\left|\mathrm{R}\left(\frac{1}{6},0\right)\right|^2 + \left|\mathrm{R}\left(\frac{2}{6}, l = 0..1\right)\right|^2 + \left|\mathrm{R}\left(\frac{3}{6}, l = 0..2\right)\right|^2 + \left|\mathrm{R}\left(\frac{4}{6}, l = 0..3\right)\right|^2 + \left|\mathrm{R}\left(\frac{5}{6}, l = 0..4\right)\right|^2 + |\mathrm{R}(1,0)|^2\right] \times \mathrm{W} \times \mathrm{D}\ \mathrm{dr}$$

eq-8

where R(n,*l*) is normalized for H-tom's $r_1 = a_0$ = 5.29E-11 m, and does not contains Sun's radial mass density distribution information, and D(r) is the Sun's radial mass density r-distribution function (in SunQM-3 section I-f, D(r) was determined to be D ≈ 1.26E+23 / r^2.33 kg/m^3), and W is a normalization constant to make the integrated eq-8's value equals to Sun's total mass.

I-b. A major correction for inside Sun's (or inside planet's, or inside pre-Sun ball's) radial wave function R(r)

Solving Schrodinger equation for a single (nonrelativistic) particle doing orbital movement under a central attractive force (valid for both H-atom and pre-Sun ball models) gives the eigenstates of this equation

$$\Psi(\mathrm{r}, \theta, \varphi, \mathrm{t}) = \mathrm{R}(\mathrm{r})\ \Theta(\theta)\ \Phi(\varphi)\ \mathrm{T}(\mathrm{t}) \qquad \text{eq-9}$$

The two functions of Ψ are usually grouped together as spherical harmonics (because of the RF, or rotation diffusion, or RotaFusion, see SunQM-2 for details)

$$\mathrm{Y}(l, \mathrm{m}) = \Theta(\theta)\ \Phi(\varphi) \qquad \text{eq-10}$$

The solution of Y(*l*,m) is valid for both outside the current Sun, as well as inside the current Sun and pre-Sun ball. The radial wave function R(r) for H-atom (ignoring the normalization coefficient) is valid for outside the current Sun, but for inside the current Sun or a pre-Sun ball, there is significant deviation (due to that the center mass of the gravity potential V(r) inside Sun is also dependent on r). For example, if inside the Sun, the center Mass can be simplified to have a M ∝ r^b (where 0 > b < 1) relationship, then the G-potential V(r) = ∫ F dr ∝ ∫ G * M(r^b) * m / r^2 dr = G * M * m ∫ 1 / r^(2 - b) dr = -G * M * m / r^(1 - b) / (1 - b) + constant. If b = 0, then it goes back to V(r) = -GMm / r. For inside the Sun, if we guess b ≈ 0.5, then V(r) ∝ -2 * G * M * m / r^(0.5) + constant. So we need to solve the Schrodinger equation for inside the current Sun with this equation (rather than the H-atom equivalent with gravity's V(r) = -GMm / r), and to get an accurate R(r). As a citizen scientist level QM physicist, I don't have the ability to solve it. However, I do believe that the curve shape of the inside Sun's true R(r) is very similar as that of outside Sun's R(r), or the H-atom's R(r). For this reason, in the SunQM series papers, I use

H-atom's R(r) to mimic inside Sun's (or inside planet's, or inside pre-Sun ball's) radial wave function with two minor modifications:
1) In H-atom's R(r) formula, Bohr radius a_0 is replaced by Sun or planet's r_1;
2) When plotting the R(r) curve vs. r for Sun (or for planet), at the outside of Sun (or planet) surface, the r is log compressed with the formula r (r > b) = b + log(r -b), where b= $r_{surface}$.
We need to emphasis that the more accurate R(r) function (which we don't have it now) is only needed for inside Sun's (or planet's) R(r), it is not needed for the outside Sun's region (where the H-atom's R(r) formula works fine after the r_1 re-normalization).

II. To build a (time-independent) 3D probability density r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 for Solar system's region from {0,2} to {5,1} at median or low resolutions

Note: for most readers, you can skip this section (with median or low resolution description), and directly read the section III (with high resolution description).

II-a. At median resolution (good resolution for belts, poor resolution for planets)

To simplify the primary r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 for Sun and all planets/belts, those n orbits with (practically) zero mass should be omitted. We only need to show those n orbit regions with mass. According to Table 1, these n regions with mass are n < 1, and n = 1, 18, 24, 30, 36, 48, 64, 96, 128, 160, 192, 384, 576, 768, and 959 (the Oort cloud is not included for the moment). For the current Sun's {0,1//6}o orbit shell space, it is in |1,0,0> QM state. For the rest n(s) that > 1, due to they all have < 1% mass occupancy, the spinning Sun's nLL QM effect causes all mass stay in |n, *l*=n-1, m> QM state (where m ≤*l*). The result in SunQM-3s10 confirmed that all Asteroid belt's mass is in the |48,47,47> QM state, and Kuiper belt's cold-KBO mass is in the |192,191,191> QM state. SunQM-3s10 also defined that if all mass of an object is perfectly in a single |nLL> state, then this |nLL> is the Eigen QM state of this object. So |48,47,47> is the Eigen QM state of Asteroid belt, and |192,191,191> is the Eigen QM state of the cold-KBO. We can confidently to say that the mass in QM states of n=18, 24, 30, 36, 48, 64, 96, 128, 160, 192, 384, 576, 768, and 959 can be described by |nLL> as |18,17,17>, |24,23,23>, |30,29,29>, |36,35,35>, |48,47,47>, |64,63,63>, |96,95,95>, |128,127,127>, |160,159,159>, |192,191,191>, |384,383,383>, |576,575,575>, |768,767,767>, and |959,958,958>. Because |48,47,47> and |192,191,191> are the Eigen QM state of Asteroid belt and the cold-KBO, |384,383,383>, |576,575,575>, |768,767,767>, and |959,958,958> are also expected to be the Eigen QM states for all four undiscovered belts at {3,n=2..5//6} (if they have not accreted into planets by now). For eight planets, |18,17,17>, |24,23,23>, |30,29,29>, |36,35,35>, |64,63,63>, |96,95,95>, |128,127,127>, |160,159,159> are not the Eigen QM states, these are the low-resolution QM description, not perfect but still OK. This is because as shown in Figure 3 of SunQM-3s10, an n state in r-dimension can be described from high resolution to low resolution by using high-frequency n' or sub-base frequency n. Let's use Earth as the example, section-III of this paper will show that although Earth is better to be described by a high-frequency n'=5*6^11=1.81E+9, (because it describes both Earth's orbit-r and surface-r), the relative low frequency n=30 also describes Earth's orbit-r, but not the surface-r. Both two descriptions give the same Earth's orbit-r at r = r_1 * n^2 = 1.74E+8 * 30^2 = 4.76E-8 * 1.81E+9^2 ≈ 1.57E+11 m. So before the mass in {1,5//6}o orbit shell (or {0,30//6}o orbit shell) starts to accrete, |30,29,29> would be a Eigen description for a belt (made of pre-Earth's mass).

For a set of QM description that gives good resolution for belts but poor resolution for planets, we call it a median resolution. Therefore, for eight planets, |18,17,17>, |24,23,23>, |30,29,29>, |36,35,35>, |64,63,63>, |96,95,95>, |128,127,127>, |160,159,159> are the medium resolution description of their QM state, Now we can have a median resolution |n,*l*,m> QM state description for the whole solar system as

$$|\mathrm{n},l,\mathrm{m}>_{\mathrm{SolarSystem}} = \mathrm{a}\,|\mathrm{n},l,\mathrm{m}>_{\mathrm{SunCore}} + \mathrm{b}\,|1{,}0{,}0> + c_1\,|18{,}17{,}17> + \mathrm{c}_2\,|24{,}23{,}23> + \mathrm{c}_3\,|30{,}29{,}29>$$
$$+ \mathrm{c}_4\,|36{,}35{,}35> + \mathrm{c}_5\,|48{,}47{,}47> + \mathrm{c}_6\,|64{,}63{,}63> + \mathrm{c}_7\,|96{,}95{,}95> + \mathrm{c}_8\,|128{,}127{,}127> + \mathrm{c}_9\,|160{,}159{,}159>$$

$+c_{10}\,|192,191,191> +c_{11}\,|384,383,383> +c_{12}\,|576,575,575> +c_{13}\,|768,767,767> +c_{14}\,|959,958,958>$

eq-11

where a, b, c_1, … c_{14} are the coefficients of the linear combination. also where $|n,l,m>_{SunCore}$ = [a_{11} * |1/6,*l*,m> + a_{12} * |2/6,*l*,m> + a_{13} * |3/6,*l*,m> + a_{14} * |4/6,*l*,m> + a_{15} * |5/6,*l*,m> + …], and a_{11}, a_{12}, … are the coefficients of the linear combination. Eq-11 described a Sun's RF ball up to {0,2}, plus 14 belts (two for Asteroid and Kuiper, 8 for planets, and 4 for undiscovered {3,n=2..5//6} planer/belt) with a good accuracy. Then the corresponding primary 3D probability density r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 for Solar system up to {3,5//6} is the linear combination of:

$$r^2\left|\Psi(r,\theta,\phi)_{\mathrm{SolarSystemMedianResolution}}\right|^2 \propto r^2\,\{[eq-3] + |b\times R(1,0)\times Y(0,0)|^2 + |c_1\times R(18,17)\times Y(17,17)|^2 + |c_2\times R(24,23)\times Y(23,23)|^2 + |c_3\times R(30,29)\times Y(29,29)|^2 + |c_4\times R(36,35)\times Y(35,35)|^2 + |c_5\times R(48,47)\times Y(47,47)|^2 + |c_6\times R(64,63)\times Y(63,63)|^2 + |c_7\times R(96,95)\times Y(95,95)|^2 + |c_8\times R(128,127)\times Y(127,127)|^2 + |c_9\times R(160,159)\times Y(159,159)|^2 + |c_{10}\times R(192,191)\times Y(191,191)|^2 + |c_{11}\times R(384,383)\times Y(383,383)|^2 + |c_{12}\times R(576,575)\times Y(575,575)|^2 + |c_{13}\times R(768,767)\times Y(767,767)|^2 + |c_{14}\times R(959,958)\times Y(958,958)|^2\}$$

eq-12

where a, b, c_1, … c_{14} are the coefficients of the linear combination, and [eq-3] is the formula of eq-3 (without r^2). So, after combining eq-5 with eq-12, we have built a (time-independent) 3D probability density r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 for the whole Solar system in a median resolution level (not including Oort cloud).

We also can describe the primary r^2 *|R(n,*l*)|^2 *|Y(*l*,m)|^2 for solar system up to {3,5//6} using a integration form:

$$\mathrm{Mass}(r,\theta,\varphi) = 1.99\times 10^{30}(\mathrm{kg}) = \int_0^{1500\times 1.49\times 10^{11}}\int_0^{\pi}\int_0^{2\pi}[eq-12]\,\times\sin(\theta)\,dr\,d\theta\,d\varphi \qquad \text{eq-13}$$

where the radial integration from 0 m, to 1550 AU = 1550* 1.49E+11 m (where {3,5//6} orbit ends), and 1.99E+30 kg is the total mass of our solar system. Notice that the integration of each item in eq-13 will generate the mass for each planet, belt, and Sun as listed in column 2 in Table 1.

Although eq-12 and eq-13 give good description for our solar system, they do not have the enough resolution to describe the eight planets (for their size). Therefore in section-III, we will build more accurate description for eight planets using planet's Eigen n' based r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 probability function so that both planets' orbit-r and surface-r can be described with the acceptable accuracy.

II-b. At low resolution (good resolution for each N super shells, poor resolution for each belts and planets)

Similarly, according to SunQM-3s10's Figure 4a (or Figure 5a), we can have the eq-11 equivalent equation with lower resolution as

$$r^2\left|\Psi(r,\theta,\phi)_{\mathrm{SolarSystemLowResolution}}\right|^2 \propto r^2\left[[eq-3] + |b\times R(1,0)\times Y(0,0)|^2 + \left|c_1\times R\left(\frac{4}{6^1},3\right)\times Y(3,3)\right|^2 + \left|c_2\times R\left(\frac{2}{6^2},1\right)\times Y(1,1)\right|^2 + \left|c_3\times R\left(\frac{2}{6^3},1\right)\times Y(1,1)\right|^2\right]$$

eq-14

where a, b, c_1, … c_3 are the coefficients of the linear combination. After combining eq-5 with eq-14, we can built a (time-independent) 3D probability density r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 for the whole Solar system in a low resolution level (not including Oort cloud).

II-c. At very low resolution (good resolution for the whole {N=1..3,n//6} region as a unit, poor resolution for belts and planets, even for each N super shell)

Also according to SunQM-3s10's Figure 4a (or Figure 5a), we can have the eq-11 equivalent equation with very low resolution as

$$r^2 \left|\Psi(r,\theta,\phi)_{\mathrm{SolarSystemVeryLowResolution}}\right|^2 \propto r^2 \left[[eq-3] + |b \times R(1,0) \times Y(0,0)|^2 + \left|c \times R\left(\frac{4}{6^2}, 3\right) \times Y(3,3)\right|^2\right]$$

eq-15

where a, b, c are the coefficients of the linear combination. After combining eq-5 with eq-15, we can built a (time-independent) 3D probability density r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 for the whole Solar system in a very low resolution level (not including Oort cloud).

II-d. To build a Oort cloud at {N=4,n=1..5}o orbit space using |n,*l*,m> QM state with *l* = 0.., n-1, m = -*l*, ... +*l*

From wiki "Oort cloud", "*Oort cloud is a theoretical cloud of predominantly icy planetesimals believed to surround the Sun. The region can be subdivided into a spherical outer Oort cloud of 20,000–50,000 AU, and a torus-shaped inner Oort cloud of 2,000–20,000 AU*". Therefore in Figure 2 of SunQM-1, the inner Oort cloud is assigned as in {4,n=1..3//6}o orbit shells with Δθ' ≈ ± 30°, and the outer Oort cloud is assigned as in {4,n=4..5//6}o orbit shells with Δθ' ≈ ± 90°. So Oort cloud is not in a |nLL> QM state. It is in a general |n,*l*,m> form with *l* = 0 .. n-1, m = -*l* .. +*l*, although for the inner Oort, both the low valued *l*(s) and the low valued |±m| (s) are missing. Then we can write Oort cloud's primary form |n,*l*,m> as a combination of

$$|n, l, m>_{\mathrm{Oort}} = d_1\ |1 \times 5.33 \times 6^3, l, m> + d_2\ |2 \times 5.33 \times 6^3, l, m> + d_3\ |3 \times 5.33 \times 6^3, l, m> + d_4\ |4 \times 5.33 \times 6^3, l, m> + d_5\ |5 \times 5.33 \times 6^3, l, m>$$

eq-16

where d_1 … d_5 are the linear combination coefficients, and *l* = 0 ... n-1, m = -*l* ... +*l*, and both the low valued *l*(s) and |±m| (s) are missing, and the missing weight is heavy at the small n, *l*, m side, and light at large n, *l*, m side. Note: for accuracy, the original n * 6^4 is now become n * 5.33 * 6^3 due to the compression in N=2 super shell. Note: here we need to use the primary form |n*q^N,*l*,m>. Then Oort cloud's primary 3D probability density r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 is:

$$\mathbf{r^2\ |\Psi(r,\theta,\phi)_{OortCloud}|^2 \propto r^2\ [|d_1 \times R(1 \times 5.33 \times 6^3, \boldsymbol{l}) \times Y(\boldsymbol{l}, m)|^2 + |d_2 \times R(2 \times 5.33 \times 6^3, \boldsymbol{l}) \times Y(\boldsymbol{l}, m)|^2 + |d_3 \times R(3 \times 5.33 \times 6^3, \boldsymbol{l}) \times Y(\boldsymbol{l}, m)|^2 + |d_4 \times R(4 \times 5.33 \times 6^3, \boldsymbol{l}) \times Y(\boldsymbol{l}, m)|^2 + |d_5 \times R(5 \times 5.33 \times 6^3, \boldsymbol{l}) \times Y(\boldsymbol{l}, m)|^2]}$$

eq-17

and its integration form is:

$$\mathrm{Mass}(r,\theta,\varphi) = 1.99 \times 10^{30}(\mathrm{kg}) = \int_0^{8000 \times 1.49 \times 10^{11}} \int_0^{\pi} \int_0^{2\pi} [eq-17] \times \sin(\theta)\, dr\, d\theta\, d\varphi \qquad \text{eq-18}$$

So, after combining eq-5, eq-17, and eq-12 (or eq-14, or eq-15), we have built a time-independent 3D probability density r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 for the whole Solar system in a median (or low, or very) resolution level including Oort cloud.

III. To build a (time-dependent) 3D probability density r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 for a planet in region from {1,3//6} to {2,5//6} at high resolution

In eq-11 and eq-12, Earth is described as |30,29,29> QM state or a 3D probability density of r^2 * |R(30,29)|^2 * |Y(29,29)|^2. It gives the good description for Earth's orbit-r, the poor description for Earth's surface-r, because the probability curve is too broad. So we need an r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 function that can describe both Earth's orbit-r and Earth's surface-r with good accuracy. From the knowledge we learned from Figure 3 of SunQM-3s10, we know that we can achieve this goal by using the high frequency multiplier n'. Since the mass ratio of Moon to Earth is 7.3E+22 kg / 5.97E+24 kg ≈ 1.2% , so we can set our goal as: to looking for n' to make r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 curve's width (with 99% of Earth mass included) equals to 2 times of Earth's surface-r in all r, θ, φ, 3D-dimensions. In SunQM-3s10, an Eigen quantum n' is defined as the maximum n' that can describe one orbit space's > 90% mass in a single |nLL> = |n',n'-1,n'-1> QM state. Therefore, the n' we are looking for is the Eigen n' of this planet.

III-a. Determination of Solar {N,n} QM's r-dimensional probability density for Earth using multiplier $n'_r = n * q\text{^}w_r$

Here we try to figure out n'_r for the r^2 * |R(n',*l*=n'-1)|^2 curve, so that the width at 1% of its peak covers Earth's diameter d = 2r = 1.28E+7 m. According to SunQM-3s10's eq-1, |nLL> QM state's R(n,*l*=n-1) formula is

$$\mathrm{R}(\mathrm{n}, l=\mathrm{n}-1) \propto r_1{}^{-3/2}\left(\frac{r}{r_1}\right)^{n-1} e^{-\frac{r}{r_1 n}} \qquad \text{eq-19}$$

After normalize to the maximum value, it becomes (see SunQM-3s10, eq-5)

$$\mathrm{r}^2\,|\mathrm{R}(\mathrm{n}, l=\mathrm{n}-1)|^2 \propto \left(\frac{\mathrm{r}}{\mathrm{r_n}}\mathrm{e}^{1-\frac{\mathrm{r}}{\mathrm{r_n}}}\right)^{2\mathrm{n}} \qquad \text{eq-20}$$

If we plot eq-20, we can see that this is an exponential rising curve times an exponential declining curve, with maximum always at r_n, and the higher the n value, the narrower the peak. Eq-19 and eq-20 are valid for both base-frequency n and high-frequency n', with the limitation that R(n,*l*=n-1).

Now the task becomes to adjust n' value to make eq-20 curve peak's half-width (at 1% of its peak value) Δr roughly equals a planet's $r_{surface}$. Since this n' is specifically in r-dimension, we name it as n'_r. For a planet, we know its orbital r_n and its $r_{surface}$ (here we use b = $r_{surface}$ in the formula), so we need to find an n'_r so that at r_n ±b, its probability of eq-20 equals to 0.01. Using eq-20, we have

$$\left(\frac{r_n \pm b}{r_n} e^{1-\frac{r_n \pm b}{r_n}}\right)^{2n'_r} = 0.01 \qquad \text{eq-21}$$

or,

$$n'_r = \frac{ln(0.1)}{ln(1 \pm b/r_n) - (\pm b/r_n)} \qquad \text{eq-22}$$

For Earth, r_n = 1.57E+11 m (or more accurately, 1.565E+11 m), b = $r_{surface}$ = 6.38E+6 m, so we calculate n'_r at r_n ±b (either r = 1.565E+11 + 6.38E+6 m, or r = 1.565E+11 - 6.38E+6 m) as n'_r = LN(0.1) / [LN(1 + 6.38E+6 /1.565E+11) - (6.38E+6

/1.565E+11)] = 2.77E+9 (see Table 1 column 13), and r_1' = 1.565E+11 / n'^2 = 2.01E-8 m. Then, for {N,n//q} QM, an n' need to satisfy n'= n *q^w (where w is integer). Therefore, we have 2.77E+9 = 5 * 6^w, w = log(2.79E+9 / 5) / log(6) = 11.24, or round w down to 11. So, at w = 11, n' = n * q^w, we have the final n'_r = 5* 6^11 ≈ 1.81E+9 (see Table 1 column 16). This is also the Eigen n' of Earth in r-dimension.

We know that at r equals to r_n = 1.57E+11 m, the probability is maximum, or = 100%. Then we can ask at what probability % eq-20 gives (a peak's half-width) Δr exactly equals to Earth's $r_{surface}$? The answer is, it equals to [(1 ± 6.38E+6 / 1.57E+11) * exp((1-(1 ± 6.38E+6 / 1.57E+11)))] ^(2 * 1.81E+9) ≈ 5% of maximum. If we choose w=12 (rather than w=11), then n'= n * q^w, n' = 5* 6^12 ≈ 1.09E+10, or at r = 1.57E+11 + 6.38E+6 m, probability = [(1+6.38E+6 / 1.57E+11) * exp((1-(1+6.38E+6 / 1.57E+11)))] ^(2*1.09E+10) = 1.52E-8 of maximum probability. It obvious that 1.52E-8 is too strict. So here we choose w=11 for Earth's n'. For Earth's w=11, r_1' = r_n' / (n * q^w)^2 = 1.565E+11 / (5 * 6^11)^2 = 4.76E-8 m (see Table 1 column 18). Check SunQM-1s2 Table 1, we find that 4.76E-8 m equals to the Hot-G r track at {-10,1}, and then it is recorded in Table 1 column 19. In Table 1 columns 13 through 19, all other planets' n'_r(s) were determined by using eq-22. In column 14's calculation, for N=1 super-shell {1,n//6} planets,

$$n' = n\,6^{w}, \qquad w = \frac{\log(n'/n)}{\log(6)} \qquad \text{eq-23}$$

and for N=2, N=3 super-shell (or n at {2,2//6} and above),

$$n'\left(\frac{5.33}{6}\right) = n\,6^{w}, \quad w = \frac{\log[n'(5.33/6)/n]}{\log(6)} \qquad \text{eq-24}$$

III-b. Determination of Solar {N,n} QM's θ-dimensional probability density for a planet using multiplier n'_θ = n * q^w_θ

QM text books (e.g., from Davis J Griffiths' book "Introduction to Quantum mechanics", 2nd ed. 2005. pp135, combining eq-2.7, eq-4.15 and eq-4.19) tell us that the wave function for the Schrodinger equation can be written as eq-9 and eq-10. For |nLL> = |n,*l*=n-1,m=n-1>, we know that (see John S. Townsed, A Modern Approach to Quantum Mechanics, 2nd ed., 2012, pp334, eq-9.146, or from wiki "Table of spherical harmonics"),

$$Y(l = n-1, m = n-1) \propto e^{im\varphi} \sin(\theta)^{l} = e^{i(n-1)\varphi} \sin(\theta)^{(n-1)} \qquad \text{eq-25}$$

Or

$$|\Theta(\theta)|^2 \propto \sin(\theta)^{[2(n-1)]} \qquad \text{eq-26}$$

where n can be either base frequency n or high-frequency n'_θ. Note: both eq-25 and eq-26 are only valid for nLL QM state which means *l* = n-1, m = n-1. Similar as what we have done in section III-a, let's define that 1% of the probability density peak is the acceptable (probability density peak's) size for a planet, and let's define θ = π/2 - θ', and define n'_θ is the n' in θ-dimension, then eq-26 under n'_θ >> 1 can be rewritten as

$$\sin(\theta)^{[2(n'_\theta-1)]} = \sin\left(\frac{\pi}{2} - \theta'\right)^{[2(n'_\theta-1)]} = \cos(\theta')^{[2(n'_\theta-1)]} \approx \cos(\theta')^{2n'_\theta} = 0.01 \qquad \text{eq-27}$$

or (see Table 1 column 23)

$$\theta' = \mathrm{acrcos}\left[0.01^{1/(2n'_\theta)}\right] \qquad \text{eq-28}$$

Then, we can calculate planet's surface-r as $\sin(\theta') * r_n$, where r_n is planet's orbit r (see Table 1 column 11). In Table 2, using n' = n * q^w = 5 * 6^w for Earth, we searched w value to fit Earth's surface-r = 6.38E+6 m. The result shows that at w=11, it gives the best fitting. So, w(θ) = w(r) = 11, and $n'_\theta = n'_r$ = 1.81E+9. This is also the Eigen n' of Earth in θ-dimension.

In the same way, in columns 20 through 24 of Table 1, values of w(θ) and n'_θ were determined for all other planets. As expected, all planets have w(θ) = w(r), and $n'_\theta = n'_r$.

Table 2. Searching the right w(r) for Earth

n=	5	5	5
q=	6	6	6
w=	10	11	12
n'=n*q^w	3.02E+08	1.81E+09	1.09E+10
0.01^[1/(2n'$_\theta$)]	0.999999992	0.999999999	1.000000000
arccos	0.000123419	0.000050386	0.000020570
r=	1.57E+11	1.57E+11	1.57E+11
b=	1.94E+07	7.91E+06	3.23E+06

III-c. Determination of Solar {N,n} QM's φ-dimensional probability density for Earth with time-dependent orbital movement

QM text books tell us that for the |n,*l*,m> = |n,*l*=n-1,m=n-1> state in eq-25,

$$\Phi(\varphi) \propto e^{im\varphi} \qquad \text{eq-29}$$

And

$$T(t) \propto e^{-i\omega t} \qquad \text{eq-30}$$

where ω = E/ℏ. In the traditional QM probability calculation, |exp(imφ)|^2 = exp(imφ) * exp(-imφ) = 1, and |exp(-iωt)|^2 = exp(-iωt) * exp(+iωt) =1 (see David J. Griffiths, "Introduction to Quantum Mechanics", 2nd ed., 2015, pp345). Therefore, both the probability peak and its time-dependent movement in φ-dimension cancelled out (for any situation, even for an orbital moving planet). We know that Earth on orbit {1,5} should correspond to a probability density peak, and this peak should have a time-dependent orbital movement in φ-dimension. Therefore, the traditional QM probability calculation method is no longer correct for this situation. A new method to calculate the Solar {N,n} QM's φ-dimensional probability density (and its time-dependency) has to be established.

Let's first try to remove the complex number in the φ-dimensional wave function (because a probability value has to be a positive (≥ 0) real value). According to wiki "Table of spherical harmonics" (see "https://en.wikipedia.org/wiki/Table of_spherical_harmonics"), we see

1) For a -m, Y(*l*=n-1,-m=-(n-1)) ∝ exp(-imφ) * [sin(θ)]^m;
2) For a +m equals to an even number, Y(*l*=n-1,+m=n-1) ∝ exp(imφ) * [sin(θ)]^m;
3) For a +m equals to an odd number, Y(*l*=n-1,+m=n-1) ∝ -exp(imφ) * [sin(θ)]^m.

Let's construct a function as

$$Y(l,\pm m) = \begin{cases} \dfrac{[Y(l,m)+Y(l,-m)]}{2} \propto \dfrac{[e^{im\varphi}(\sin\theta)^m + e^{-im\varphi}(\sin\theta)^m]}{2} = \cos(m\varphi)(\sin\theta)^m \ldots\ldots\ldots \text{when } m = \text{even} \\ \dfrac{[-Y(l,m)+Y(l,-m)]}{2} \propto \dfrac{[e^{im\varphi}(\sin\theta)^m + e^{-im\varphi}(\sin\theta)^m]}{2} = \cos(m\varphi)(\sin\theta)^m \ldots\ldots\ldots \text{when } m = \text{odd} \end{cases}$$

eq-31

where *l* = m = n-1, and n can either base n or multiplier n'. Or simply,

$$\mathrm{Y}(l, \pm\mathrm{m}) = \cos(\mathrm{m}\varphi)\,[\sin(\theta)]^{\mathrm{m}} \qquad \text{eq-32}$$

where l = m = n-1. (Notice that eq-31 and eq-32 is only valid for nLL QM state, or l = m = n-1). Here we used a real value wave function Y(l,±m) (notice that this is not the Eigen function but a linear combination of the Eigen functions of Schrodinger equation) to replace the original complex value wave function Y(l,m) (notice that this is the Eigen function of Schrodinger equation). Eq-32 is still a solution of Schrodinger equation. This is because that the linear partial differential equation (which Schrodinger equation belongs to) has an important property: a linear combination of its solutions is still a valid solution of this equation [16].

The probability of eq-32 is

$$|\mathrm{Y}(l, \pm\mathrm{m})|^2 = [\cos(\mathrm{m}\varphi)]^2\,[\sin(\theta)]^{2\mathrm{m}} \qquad \text{eq-33}$$

where l = m = n-1. In eq-33, we know that [sin(θ)]^(2m) curve produces a single narrow peak in θ-dimension at θ = π/2, and [cos(mφ)]^2 curve produces 2*m of (equal amplitude) peaks evenly distributed in the whole range of φ-dimension from -π to +π. Because a spherical planet's projection in the Solar {N,n} QM structure's θφ-2D dimension is a perfect circle, we expect that its probability density function should be something like

$$|\mathrm{Y}(l, \pm\mathrm{m})|^2 = [\cos(\varphi)]^{2\mathrm{m}}\,[\sin(\theta)]^{2\mathrm{m}} \qquad \text{eq-34}$$

because eq-34 produces a circular contour line probability density in Solar system's θφ-2D dimension. However, normally we can't use eq-34 directly because: 1) it has an extra (or the 2nd) peak at φ = ±π besides the right peak at φ = 0: 2) its corresponding wave function Y(l,±m)| = [cos(φ)]^m * [sin(θ)]^m is not a solution of Schrodinger equation. Using above analysis, the problem can be simplified as: if we can construct eq-34 -like probability density curve (but without the second peak) from eq-34, then we are able to describe a planet's probability density in the θφ-2D dimension of Solar {N,n} QM structure. Luckily, we found a way to do that.

Inspired by Fourier transformation [16], let's construct a (normalized) linear combination of a group of cos[(m+ δ)φ)] with the integer number m (= n-1) is deviated by a small integer δ (with δ << m):

$$\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(\mathrm{m}+\delta)\varphi] \qquad \text{eq-35}$$

where 1/(1+2* δ) is the normalization factor. When we plot eq-35 at m = 1024 and δ = 36 (shown in Figure 1), it shows that a wave packet is formed beyond the single cos[(m+ δ)φ)] wave, and the envelop of this wave packet (almost) perfectly fits to [cos(φ)]^m curve's one peak at φ=0. More interestingly, after we squared eq-35 (see eq-36 and Figure 2), the plot of squared eq-35 produced a wave packet that (almost) perfectly fits to [cos(φ)]^(2m) curve's one peak at φ=0 (shown in Figure 2), and it does not have the second peak at φ = ±π!

$$\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(\mathrm{m}+\delta)\varphi]\right\}^2 \qquad \text{eq-36}$$

Furthermore, the modeling shows that as δ value increasing, the curve of eq-36 is approaching infinitely to the curve of [cos(φ)]^(2m) that without 2nd peak. Therefore, as shown in eq-37, when δ is large enough (but still << m), we can replace eq-36 by [cos(φ)]^(2m) in a practical calculation (if we can ignore the 2nd peak of [cos(φ)]^(2m)).

$$\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(\mathrm{m}+\delta)\varphi]\right\}^2 \to \cos(\varphi)^{2m}$$

eq-37

where δ (<< m) value is high enough. Therefore, our result revealed that a planet's wave function in the φ-dimension is composed by a group of wave functions (which are the Schrodinger equation's solution) that further forms a (group) wave packet out of the phase wave. Because eq-35 shows that a planet's matter wave in φ-dimension is composed of a group of cosine waves, and the cosine wave is a typical plane wave function, then we can use (a group of) plane wave to describe a planet's matter wave in φ-dimension (see SunQM-4 section I-c).

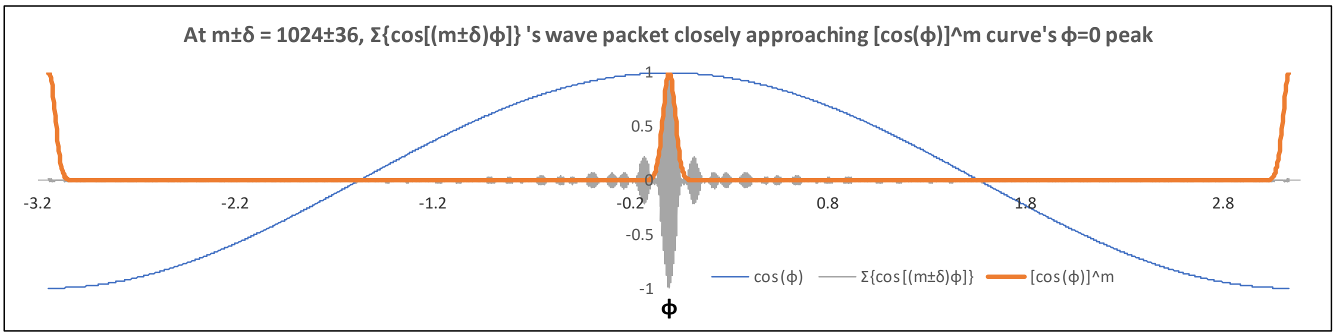

Figure 1. A wave function plot of the true calculation of eq-35 at m = 1024 and δ = 36 shows that the envelop of its wave packet fits to [cos(φ)]^m curve's one peak at φ=0. Notice that the rest small wave packets (at φ≠0) can be further suppressed if we increase δ to a higher value. Also notice that eq-35 does not produce any wave packets at φ=±π.

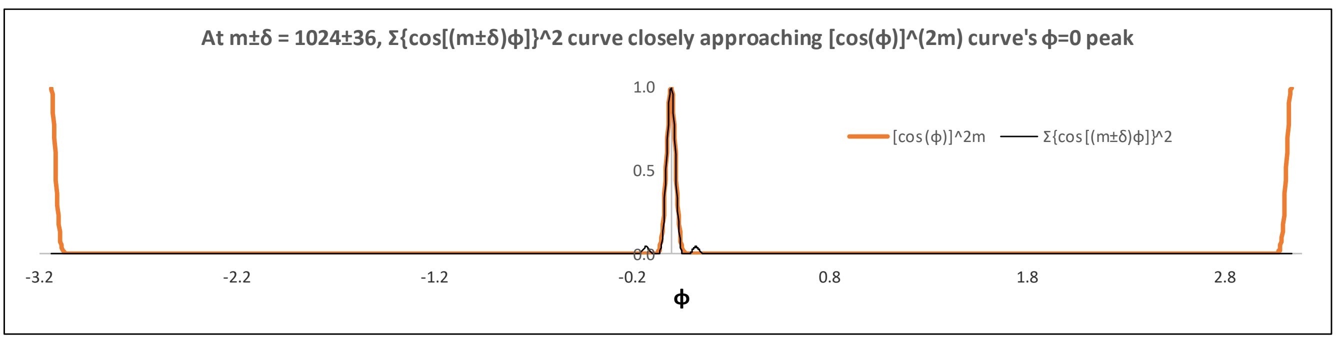

Figure 2. A probability density plot of the true calculation of eq-36 at m = 1024 and δ = 36 shows that the envelop of its wave packet fits to [cos(φ)]^(2m) curve's one peak at φ=0. Notice that the rest small wave packets (at φ≠0) can be further suppressed if we increase δ to a higher value. Also notice that eq-36 does not produce any peak at φ=±π.

With this knowledge, we start to deduce a planet (that in Solar {N,n} QM structure)'s probability density function. According to eq-20, its r-dimensional (normalized) wave function is

$$\mathrm{r\,R}(\mathrm{n}, l=\mathrm{n}-1) = \left(\frac{\mathrm{r}}{\mathrm{r_n}}\mathrm{e}^{\left(1-\frac{\mathrm{r}}{\mathrm{r_n}}\right)}\right)^{\mathrm{n}} \qquad \text{eq-38}$$

Its θφ-2D dimensional wave function is eq-32. Combining these two functions, we have a planet's 3D (normalized) wave function as

$$r\,R(n, l = n-1)\,Y(l, \pm m) = \left(\frac{\mathrm{r}}{\mathrm{r_n}}\mathrm{e}^{\left(1-\frac{\mathrm{r}}{\mathrm{r_n}}\right)}\right)^{\mathrm{n}} \cos(m\varphi)\,[\sin(\theta)]^m \qquad \text{eq-39}$$

The corresponding probability function is

$$r^2|R(n,l)|^2|Y(l,\pm m)|^2 = \left\{\left(\frac{\mathrm{r}}{\mathrm{r_n}}\mathrm{e}^{\left(1-\frac{\mathrm{r}}{\mathrm{r_n}}\right)}\right)^{\mathrm{n}} \sin(\theta)^{\mathrm{m}} \cos(m\varphi)\right\}^2 \qquad \text{eq-40}$$

where l = m = n-1 (or n'-1). To transform φ-dimension's flat probability into a single peak probability, we need to linearly combine a group of m±δ so that eq-40 becomes eq-41 (notice that all m (=n-1), or n related integer numbers need to be deviated to become m+δ, or n-1+ δ)

$$r^2|R(n,l)|^2|Y(l,\pm m)|^2 = \left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\left(\frac{\mathrm{r}}{\mathrm{r_{n+\delta}}}\mathrm{e}^{\left(1-\frac{\mathrm{r}}{\mathrm{r_{n+\delta}}}\right)}\right)^{\mathrm{n+\delta}} \sin(\theta)^{\mathrm{n-1+\delta}} \cos[(\mathrm{n}-1+\delta)\varphi]\right\}^2$$

eq-41

where l = m = n-1=n'-1.

Inspired by the math treatment of partial derivatization, we can use the δ-dependent perturbation method to treat r-, θ-, φ-dimension's wave function one-by-one separately. First, for a group of δ-dependent r-dimensional (normalized) wave function r * R(n,l), each one has a single (positive and narrow) peak, so the shape of this peak should not be that sensitive to the variable δ if δ << m=n-1. So under the condition of δ << m=n-1 (remember that in Table 1 all planets have n' > 1.0E+9, so we can choose δ < 1E+6), eq-41 can be approximated as one δ-independent r *R(n,l) (which equivalent to the zero-order perturbation item) plus the high order perturbations such as 1 + O(1) + O(2) ..., where O(1) represents the first order perturbation item, and O(2) represents the second order perturbation item. We can discard all high order perturbations and only keep the zero-order item:

$$\begin{aligned} r^2|R(n,l)|^2|Y(l,\pm m)|^2 &= \left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\left(\frac{\mathrm{r}}{\mathrm{r_n}}\mathrm{e}^{\left(1-\frac{\mathrm{r}}{\mathrm{r_n}}\right)}\right)^{\mathrm{n}}[1+O(1)+\cdots]\sin(\theta)^{\mathrm{n-1+\delta}}\cos[(\mathrm{n}-1+\delta)\varphi]\right\}^2 \\ &\approx \left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\left(\frac{\mathrm{r}}{\mathrm{r_n}}\mathrm{e}^{\left(1-\frac{\mathrm{r}}{\mathrm{r_n}}\right)}\right)^{\mathrm{n}}\sin(\theta)^{\mathrm{n-1+\delta}}\cos[(\mathrm{n}-1+\delta)\varphi]\right\}^2 \\ &= \left(\frac{\mathrm{r}}{\mathrm{r_n}}\mathrm{e}^{\left(1-\frac{\mathrm{r}}{\mathrm{r_n}}\right)}\right)^{2\mathrm{n}}\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\sin(\theta)^{\mathrm{n-1+\delta}}\cos[(\mathrm{n}-1+\delta)\varphi]\right\}^2 \end{aligned}$$

eq-42

where n >> 1, and 0 ≤ δ << m=n-1=n'-1.

Similarly, for a group of δ-dependent θ-dimensional wave function [sin(θ)]^m, each one has a single (positive and narrow) peak, its shape should not be that sensitive to the variable δ if δ << m=n-1. So under the condition of δ << m=n-1, eq-42 can be approximated as one δ-independent [sin(θ)]^m (which equivalent to the zero-order perturbation item) plus the high order perturbations such as 1 + O(1) + …, and we can discard all high order perturbations items

$$r^2\,|\Psi(r,\theta,\phi-\omega t)_{Planet}|^2 = r^2|R(n,l)|^2|Y(l,\pm m)|^2$$
$$\approx \left(\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\right)^{2n}\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\sin(\theta)^{n-1}\,[1+O(1)+\cdots]\cos[(n-1+\delta)\varphi]\right\}^2$$
$$\approx \left(\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\right)^{2n}\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\sin(\theta)^{n-1}\cos[(n-1+\delta)\varphi]\right\}^2$$
$$= \left(\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\right)^{2n}\sin(\theta)^{2(n-1)}\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(n-1+\delta)\varphi]\right\}^2$$

eq-43

where n >> 1, and $0 \le \delta << m=n-1=n'-1$. Eq-43 is the (time-independent) 3D probability density function for a planet in Solar {N,n} QM structure. After replacing φ by φ-ωt, we have **the final (time-dependent) 3D probability density function for a planet in Solar {N,n} QM structure** as shown in eq-44:

$$r^2\,|\Psi(r,\theta,\phi-\omega t)_{Planet}|^2 \approx \left(\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\right)^{2n}\sin(\theta)^{2(n-1)}\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(n-1+\delta)(\varphi-\omega t)]\right\}^2$$

eq-44

where n >> 1, and $0 \le \delta << m=n-1=n'-1$, and δ value is high enough, and ω is the planet's orbital angular frequency. Eq-44 is valid for both base-frequency n and multiplier n'. Notice that in φ-dimension's range from – π to + π, eq-44 produces only a single peak at φ=0, and the shape of this peak is infinitely approaching to the shape of [cos(φ)]^(2m) curve's one peak at φ=0. This is exactly what we wanted. (Note: although the wave function needs to be the solution of Schrodinger equation, the probability function does not need to be the solution of Schrodinger equation, so that eq-44 is a valid probability density function for a planet in Solar {N,n} QM structure).

At n >> 1 (e.g., Earth's n' = 5 * 6^11 = 1813985280 >> 1), n-1 ≈ n. So we can simplify eq-44 to be:

$$r^2\,|\Psi(r,\theta,\phi-\omega t)_{Planet}|^2 \approx \left[\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\sin(\theta)\right]^{2n}\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(n+\delta)(\varphi-\omega t)]\right\}^2$$

eq-45

Notice that eq-44 and eq-45 only valid for the nLL QM state (like a planet in Solar {N,n} QM structure).

Using eq-37, we can re-write eq-45 as

$$r^2\,|\Psi(r,\theta,\phi-\omega t)_{Planet}|^2 \approx \left[\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\sin(\theta)\,\cos(\varphi-\omega t)\right]^{2n}$$

eq-46

where n >>1 , in nLL QM state, and if we can manually ignore the 2nd peak at φ = ±π.

When looking into the formula of eq-44 or eq-45 (which can be represented by eq-46), we are amazed on how simple the formula is and how straightforward meaning it is: r^2 * |R(r)|^2 ∝ [r / r_n * exp(1 – r / r_n)]^(2 * n) produces an exponential rising curve times an exponential declining curve, with the peak always at r = r_n, and the higher the n, the narrower the peak. |Θ(θ)|^2 ∝ [sin(θ)]^(2 * n) produces a peak at θ = π/2, and the higher the n, the narrower the peak. |Φ(φ-ωt)|^2 ∝ [cos(φ-ωt)]^(2 * n') produces a peak at φ-ωt = 0, or φ = ωt, and the higher the n', the narrower the peak (Note: need to ignore the φ-ωt = π peak).

We try to plot eq-44 (or eq-45) in θφ-2D-dimension but no 3D plotting software can handle eq-44. Instead, we have to use eq-46 to represent eq-44 to make the plot. Figure 3 shows that eq-45 with $2n = 64$ gives a narrow peak in θφ-2D-dimension and moving in +φ direction with $\varphi = \omega t = 0$, 0.5, and 1. Of cause, we need to manually ignore the second peak at $\varphi\text{-}\omega t = \pm\pi$.

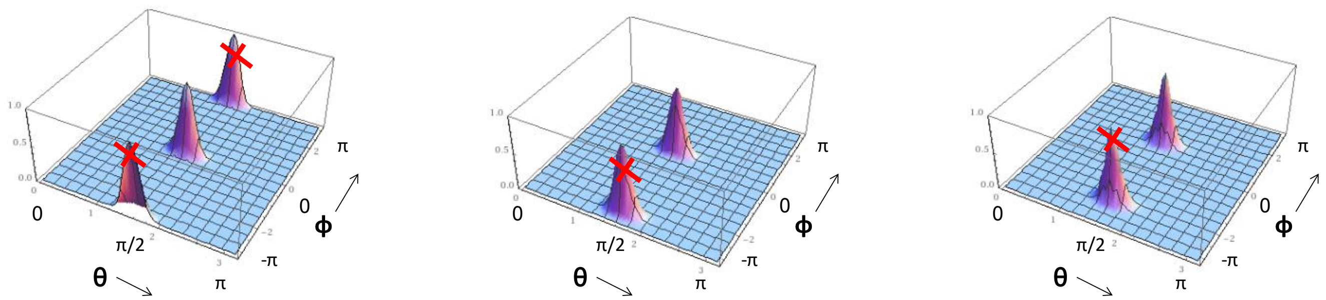

Figure 3. Using WolframAlpha plot to show eq-46's [sin(θ) * cos(φ-ωt)]^(2 * n) at 2 * n = 64 gives a narrow peak in θφ-2D dimension and moving in +φ direction. Figure 3a (left), ωt = 0; Figure 3b (middle), ωt = 0.5; Figure 3c (right), ωt = 1.

IV. To build a complete 3D (high resolution) probability density map to describe the whole Solar system with time-dependent orbital movement

Using an online information (https://www.theplanetstoday.com/astrology.html), the initial φ values (at time of 8/14/2019) for all eight known planets were listed in Table 1 column 29 (Note: Earth's initial φ is set to be 0, and rest planets' initial φ values are relative to Earth's φ = 0). Eq-45 gives the general form of a planet's time-dependent 3D probability density formula (with n or n' >> 1), and values of r_n, φ, ω, n' for each planet can be obtained from columns of 11, 29, 26, 16 in Table 1. Therefore, we are able to construct the time-dependent 3D probability density peak for each planet (with t=0 on Aug. 14, 2019) as shown below (Note: due to all planets have n' > 1E+9, here we choose δ =1.0E+6 for all planets):

$$r^2 \left|\Psi(r,\theta,\phi-\omega t)_{Mercury}\right|^2 \approx \left[\frac{r}{5.64\times10^{10}} e^{\left(1-\frac{r}{5.64\times10^{10}}\right)} \times \sin(\theta)\right]^{2\times1.09\times10^{9}} \times \left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(1.09\times10^{9}+\delta)\times(1.08-8.61\times10^{-7}t)]\right\}^2 \quad \text{eq-47}$$

$$r^2 \left|\Psi(r,\theta,\phi-\omega t)_{Venus}\right|^2 \approx \left[\frac{r}{1.00\times10^{11}} e^{\left(1-\frac{r}{1.00\times10^{11}}\right)} \times \sin(\theta)\right]^{2\times1.45\times10^{9}} \times \left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(1.45\times10^{9}+\delta)\times(3.14-3.63\times10^{-7}t)]\right\}^2 \quad \text{eq-48}$$

$$r^2 \left|\Psi(r,\theta,\phi-\omega t)_{Earth}\right|^2 \approx \left[\frac{r}{1.57\times10^{11}} e^{\left(1-\frac{r}{1.57\times10^{11}}\right)} \times \sin(\theta)\right]^{2\times1.81\times10^{9}} \times \left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(1.81\times10^{9}+\delta)\times(0-1.86\times10^{-7}t)]\right\}^2 \quad \text{eq-49}$$

$$r^2 \left|\Psi(r,\theta,\phi-\omega t)_{Mars}\right|^2 \approx \left[\frac{r}{2.25\times10^{11}} e^{\left(1-\frac{r}{2.25\times10^{11}}\right)} \times \sin(\theta)\right]^{2\times1.31\times10^{10}} \times \left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(1.31\times10^{10}+\delta)\times(3.33-1.08\times10^{-7}t)]\right\}^2 \quad \text{eq-50}$$

$$r^2\left|\Psi(r,\theta,\phi-\omega t)_{Jupiter}\right|^2 \approx \left[\frac{r}{7.12\times10^{11}}e^{\left(1-\frac{r}{7.12\times10^{11}}\right)}\times\sin(\theta)\right]^{2\times7.26\times10^{8}}\times\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(7.26\times10^{8}+\delta)\times(5.31-1.92\times10^{-8}t)]\right\}^2 \quad \text{eq-51}$$

$$r^2\left|\Psi(r,\theta,\phi-\omega t)_{Saturn}\right|^2 \approx \left[\frac{r}{1.60\times10^{12}}e^{\left(1-\frac{r}{1.60\times10^{12}}\right)}\times\sin(\theta)\right]^{2\times6.53\times10^{9}}\times\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(6.53\times10^{9}+\delta)\times(5.74-5.69\times10^{-9}t)]\right\}^2 \quad \text{eq-52}$$

$$r^2\left|\Psi(r,\theta,\phi-\omega t)_{Uranus}\right|^2 \approx \left[\frac{r}{2.85\times10^{12}}e^{\left(1-\frac{r}{2.85\times10^{12}}\right)}\times\sin(\theta)\right]^{2\times5.22\times10^{10}}\times\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(5.22\times10^{10}+\delta)\times(1.27-2.40\times10^{-9}t)]\right\}^2 \quad \text{eq-53}$$

$$r^2\left|\Psi(r,\theta,\phi-\omega t)_{Neptune}\right|^2 \approx \left[\frac{r}{4.45\times10^{12}}e^{\left(1-\frac{r}{4.45\times10^{12}}\right)}\times\sin(\theta)\right]^{2\times6.53\times10^{10}}\times\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(6.53\times10^{10}+\delta)\times(0.52-1.23\times10^{-9}t)]\right\}^2 \quad \text{eq-54}$$

For a solar {N,n} QM structure's belt, we can directly use eq-40, and replacing φ by φ-ωt:

$$r^2\left|\Psi(r,\theta,\phi-\omega t)_{Belt}\right|^2 = r^2|R(n,l)|^2|Y(l,\pm m)|^2 = \left\{\left(\frac{r}{r_n}e^{\left(1-\frac{r}{r_n}\right)}\right)^n \sin(\theta)^{n-1}\cos[(n-1)\times(\phi-\omega t)]\right\}^2 \quad \text{eq-55}$$

For Asteroid belt, according to SunQM-3s10 eq-6, its Eigen quantum number n' = 48, so we have

$$r^2\left|\Psi(r,\theta,\phi-\omega t)_{AsteroidBelt}\right|^2 = \left\{\left(\frac{r}{4.01\times10^{11}}e^{\left(1-\frac{r}{4.01\times10^{11}}\right)}\right)^{48}\sin(\theta)^{47}\cos[47\times(\phi-4.54\times10^{-8}t)]\right\}^2 \quad \text{eq-56}$$

where $\omega = \omega_n$ = 4.54E-8 arc/s is the (averaged) angular frequency/velocity of the rotating Asteroid belt at orbit {0,48//6} in φ-dimension, and φ can be any position inside φ-dimension. For the cold-KBO in Kuiper belt, according to SunQM-3s10 eq-13, its Eigen quantum number n' = 192, so we have

$$r^2\left|\Psi(r,\theta,\phi-\omega t)_{Cold-KBO}\right|^2 = \left\{\left(\frac{r}{6.40\times10^{12}}e^{\left(1-\frac{r}{6.40\times10^{12}}\right)}\right)^{192}\sin(\theta)^{191}\cos[191\times(\phi-7.11\times10^{-10}t)]\right\}^2 \quad \text{eq-57}$$

where $\omega = \omega_n$ = 7.11E-10 arc/s is the (averaged) angular frequency of the rotating Kuiper belt at orbit {0,192//6}, and φ can be any position inside φ-dimension.

For the undiscovered {3,n=2..5//6} planets/belts, we assumed that they have formed planets, so they have the forms of those eight known planets, except the initial φ positions are unknown.

$$r^2\left|\Psi(r,\theta,\phi-\omega t)_{\{3,2\}Planet}\right|^2 \approx \left[\frac{r}{2.56\times10^{13}}e^{\left(1-\frac{r}{2.56\times10^{13}}\right)}\times\sin(\theta)\right]^{2\times5.64\times10^{12}}\times\left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos\left[(5.64\times10^{12}+\delta)\times\left(\varphi_{\{3,2\}}-8.89\times10^{-11}t\right)\right]\right\}^2 \quad \text{eq-58}$$

$$r^2 \left|\Psi(r,\theta,\phi-\omega t)_{\{3,3\}\text{Planet}}\right|^2 \approx \left[\frac{r}{5.76\times10^{13}} e^{\left(1-\frac{r}{5.76\times10^{13}}\right)} \times \sin(\theta)\right]^{2\times5.08\times10^{13}} \times \left\{\frac{1}{1+2\delta} \sum_{-\delta}^{+\delta} \cos\left[(5.08\times10^{13}+\delta)\times\left(\varphi_{\{3,3\}} - 2.63\times10^{-11}t\right)\right]\right\}^2 \quad \text{eq-59}$$

$$r^2 \left|\Psi(r,\theta,\phi-\omega t)_{\{3,4\}\text{Planet}}\right|^2 \approx \left[\frac{r}{1.02\times10^{14}} e^{\left(1-\frac{r}{1.02\times10^{14}}\right)} \times \sin(\theta)\right]^{2\times4.06\times10^{14}} \times \left\{\frac{1}{1+2\delta} \sum_{-\delta}^{+\delta} \cos\left[(4.06\times10^{14}+\delta)\times\left(\varphi_{\{3,4\}} - 1.11\times10^{-11}t\right)\right]\right\}^2 \quad \text{eq-60}$$

$$r^2 \left|\Psi(r,\theta,\phi-\omega t)_{\{3,5\}\text{Planet}}\right|^2 \approx \left[\frac{r}{1.60\times10^{14}} e^{\left(1-\frac{r}{1.60\times10^{14}}\right)} \times \sin(\theta)\right]^{2\times5.08\times10^{14}} \times \left\{\frac{1}{1+2\delta} \sum_{-\delta}^{+\delta} \cos\left[(5.08\times10^{14}+\delta)\times\left(\varphi_{\{3,5\}} - 5.69\times10^{-12}t\right)\right]\right\}^2 \quad \text{eq-61}$$

Then we can use a matrix production to constitute a complete probability density function for Solar {N,n} QM structure (as shown in eq-62). Eq-62 will produce a high-resolution 3D map of probability density peaks for a complete Solar system, including a Sun (in standing), eight known planets (doing orbital movement), two known belts (doing orbital movement), four undiscovered (supposed) planets (doing orbital movement), and Oort cloud (in standing). Notice that in eq-62, the coefficient matrix is a diagonal only matrix, all non-diagonal cells have values of zero. The (most right) vector space column is composed by the probability density functions (in bold font) with the equation numbers of eq-5, eq-47 through eq-54, eq-56 through eq-61, and eq-17.

$$r^2 \left|\Psi(r,\theta,\phi-\omega t)_{\text{SolarSystem}}\right|^2 = \begin{bmatrix} a^2 &&&&&&&&&&&&&&& \\ & c_1^2 &&&&&&&&&&&&&& \\ && c_2^2 &&&&&&&&&&&&& \\ &&& c_3^2 &&&&&&&&&&&& \\ &&&& c_4^2 &&&&&&&&&&& \\ &&&&& c_5^2 &&&&&&&&&& \\ &&&&&& c_6^2 &&&&&&&&& \\ &&&&&&& c_7^2 &&&&&&&& \\ &&&&&&&& c_8^2 &&&&&&& \\ &&&&&&&&& c_9^2 &&&&&& \\ &&&&&&&&&& c_{10}^2 &&&&& \\ &&&&&&&&&&& c_{11}^2 &&&& \\ &&&&&&&&&&&& c_{12}^2 &&& \\ &&&&&&&&&&&&& c_{13}^2 && \\ &&&&&&&&&&&&&& c_{14}^2 & \\ &&&&&&&&&&&&&&& c_{15}^2 \end{bmatrix} \begin{bmatrix} \text{Sun, eq-5} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Mercury}}|^2\text{, eq-47} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Venus}}|^2\text{, eq-48} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Earth}}|^2\text{, eq-49} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Mars}}|^2\text{, eq-50} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Jupiter}}|^2\text{, eq-51} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Saturn}}|^2\text{, eq-52} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Uranus}}|^2\text{, eq-53} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{Neptune}}|^2\text{, eq-54} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{AsteoridBelt}}|^2\text{, eq-56} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\text{KuiperBelt}}|^2\text{, eq-57} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\{3,2\}\text{Planet}}|^2\text{, eq-58} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\{3,3\}\text{Planet}}|^2\text{, eq-59} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\{3,4\}\text{Planet}}|^2\text{, eq-60} \\ r^2 |\Psi(r,\theta,\phi-\omega t)_{\{3,5\}\text{Planet}}|^2\text{, eq-61} \\ \text{Oort cloud, eq-17} \end{bmatrix}$$

eq-62

Here we name eq-62 as the "Solar r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 master matrix formula". Eq-62 is the Eigen description of our Solar system using Schrodinger equation's solution. It can also be written as the integration form:

$$\text{Mass}(r,\theta,\varphi) = 1.99\times10^{30}(\text{kg}) = \int_0^{1500\times1.49\times10^{11}} \int_0^{\pi} \int_0^{2\pi} [\text{eq}-62] \times \sin(\theta)\, dr\, d\theta\, d\varphi \quad \text{eq-63}$$

Therefore each coefficient in eq-62's diagonal matrix can be obtained because each item (Sun, planet, belt, cloud)'s integration should equal to this item's mass.

V. More discussion on the Solar r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 master matrix formula eq-62

1) The r-dimensional probability density distribution of eq-62 is illustrated in Figure 4 (and Table 3). Notice that the previous low-resolution diagram of probability density r-distribution (in SunQM-3s1 Figure 4 where all eight planets' probability density peak widths were very broad) is now updated to a high-resolution diagram (where all eight planets' probability density peak widths are close to planets' true diameters).

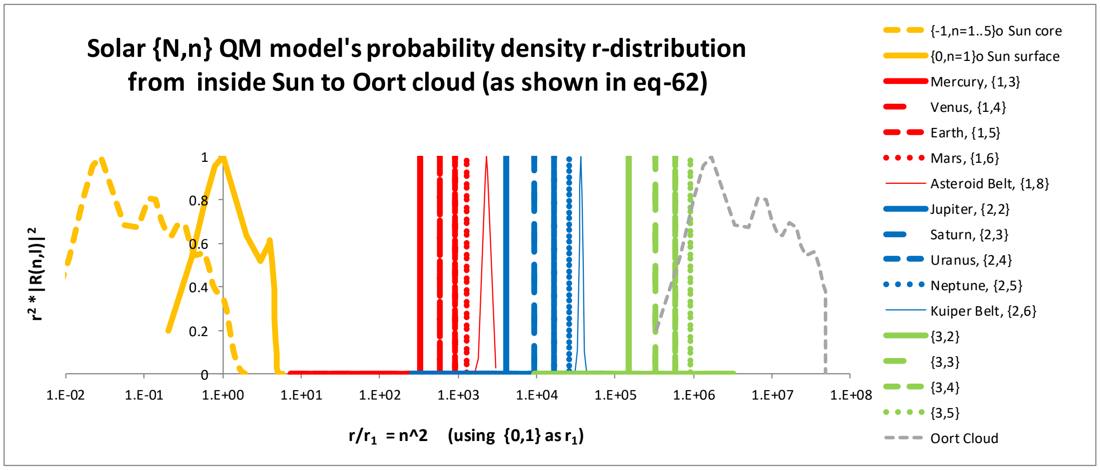

Figure 4. Probability density distribution in r-dimension for eq-62 (where all eight planets' probability density peak widths are close to planets' diameters).

Table 3. Solar {N,n} QM model's probability density r-distribution from inside Sun to Oort cloud (as shown in eq-62).

r cut-off = 4	{0,n=1}o Sun surface	r_1=	{-1,n=1..5}o Sun core			Mercury, {1,3}	Venus, {1,4}	Earth, {1,5}	Mars, {1,6}	Asteroid Belt, {1,8}		Jupiter, {2,2}	Saturn, {2,3}	Uranus, {2,4}	Neptune, {2,5}	Kuiper Belt, {2,6}		{3,2}	{3,3}	{3,4}	{3,5}		Oort Cloud
r/r_1= 4 + log(4-r/r1)	Σn=1,2	r/r_1=	r/r_1 /36=	Σn=1..5	r/r_1 *36 =	[r/r_n* exp(1-r/r_n)]^(2*n')	[r/r_n* exp(1-r/r_n)]^(2*n')	[r/r_n* exp(1-r/r_n)]^(2*n')	[r/r_n* exp(1-r/r_n)]^(2*n')	[r/r_n* exp(1-r/r_n)]^(2*n')	r/r_1 *36^2=	[r/r_n* exp(1-r/r_n)]^(2*n')	[r/r_n* exp(1-r/r_n)]^(2*n')	[r/r_n* exp(1-r/r_n)]^(2*n')	[r/r_n* exp(1-r/r_n)]^(2*n')	[r/r_n* exp(1-r/r_n)]^(2*n')	r/r_1 *36^3=	[r/r_n* exp(1-r/r_n)]^(2*n')	[r/r_n* exp(1-r/r_n)]^(2*n')	[r/r_n* exp(1-r/r_n)]^(2*n')	[r/r_n* exp(1-r/r_n)]^(2*n')	r/r_1 *36^4=	r/r_1 *36^4=, log(r/r1)
0.2	0.200064	0.2	0.005556	0.20103	7.2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.17E-199	259	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9331.2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.36E+05	3.36E+05
0.4	0.535308	0.4	0.011111	0.53731	14.4	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.86E-171	518	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	18662.4	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.72E+05	6.72E+05
0.6	0.80492	0.6	0.016667	0.8067	21.6	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.08E-154	778	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	27993.6	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.01E+06	1.01E+06
0.8	0.956247	0.8	0.022222	0.9569	28.8	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.98E-142	1037	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	37324.8	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.34E+06	1.34E+06
1	1	1	0.027778	1	36	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.44E-133	1296	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	46656	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.68E+06	1.68E+06
2	0.635976	2	0.055556	0.68264	72	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.85E-105	4090	2.26E-15	0.00E+00	0.00E+00	0.00E+00	8.21E-219	147271	3.62E-62	0.00E+00	0.00E+00	0.00E+00	3.36E+06	3.36E+06
3	0.519828	3	0.083333	0.67274	108	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.41E-88	4091	1.00E+00	0.00E+00	0.00E+00	0.00E+00	8.84E-219	147271.74	1.00E+00	0.00E+00	0.00E+00	0.00E+00	5.04E+06	5.04E+06
4	0.602935	4	0.111111	0.80469	144	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.10E-77	4092	2.74E-24	0.00E+00	0.00E+00	0.00E+00	9.70E-219	147274	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.72E+06	6.72E+06
4.00	0.613141	5	0.138889	0.79999	180	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.39E-68	6480	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.18E-153	233280	0.00E+00	0.00E+00	0.00E+00	0.00E+00	8.40E+06	8.40E+06
4.30	0.519713	6	0.166667	0.70777	216	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.24E-61	9203	0.00E+00	2.07E-74	0.00E+00	0.00E+00	8.55E-107	331360	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.01E+07	1.01E+07
4.48	0.383339	7	0.194444	0.64013	252	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.38E-56	9204	0.00E+00	1.00E+00	0.00E+00	0.00E+00	8.96E-107	331361.41	0.00E+00	1.00E+00	0.00E+00	0.00E+00	1.18E+07	1.18E+07
4.60	0.255361	8	0.222222	0.63545	323.9	7.88E-46	0.00E+00	0.00E+00	0.00E+00	1.08E-46	9205	0.00E+00	1.19E-09	0.00E+00	0.00E+00	9.10E-107	331363	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.34E+07	1.34E+07
4.70	0.157584	9	0.25	0.66801	324	1.00E+00	0.00E+00	0.00E+00	0.00E+00	1.10E-46	11664	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.94E-78	419904	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.51E+07	1.51E+07
4.78	0.091657	10	0.277778	0.69698	324.1	8.22E-46	0.00E+00	0.00E+00	0.00E+00	1.13E-46	16363	0.00E+00	0.00E+00	3.46E-24	0.00E+00	2.72E-43	589086	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.68E+07	1.68E+07
4.90	0.02716	12	0.333333	0.67102	432	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.21E-36	16364	0.00E+00	0.00E+00	1.00E+00	0.00E+00	2.74E-43	589086.95	0.00E+00	0.00E+00	1.00E+00	0.00E+00	2.02E+07	2.02E+07
5.00	0.007076	14	0.388889	0.58702	575.9	0.00E+00	1.04E-19	0.00E+00	0.00E+00	2.92E-27	16364	0.00E+00	0.00E+00	1.02E-19	0.00E+00	2.76E-43	589089	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.35E+07	2.35E+07
5.08	0.001681	16	0.444444	0.54516	576	0.00E+00	1.00E+00	0.00E+00	0.00E+00	2.96E-27	20736	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.30E-23	746496	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.69E+07	2.69E+07
5.15	0.000373	18	0.5	0.55268	576.1	0.00E+00	1.05E-19	0.00E+00	0.00E+00	3.00E-27	25567	0.00E+00	0.00E+00	0.00E+00	5.56E-45	1.41E-10	920446	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.02E+07	3.02E+07
5.20	7.82E-05	20	0.555556	0.56076	720	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.48E-20	25568	0.00E+00	0.00E+00	0.00E+00	1.00E+00	1.42E-10	920448.36	0.00E+00	0.00E+00	0.00E+00	1.00E+00	3.36E+07	3.36E+07
5.26	1.57E-05	22	0.611111	0.53753	899.9	0.00E+00	0.00E+00	1.97E-10	0.00E+00	1.63E-14	25569	0.00E+00	0.00E+00	0.00E+00	3.04E-43	1.43E-10	920450	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.70E+07	3.70E+07
5.30	3.05E-06	24	0.666667	0.48804	900	0.00E+00	0.00E+00	1.00E+00	0.00E+00	1.64E-14	28000	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.94E-06	1119744	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.03E+07	4.03E+07
5.36	2.47E-07	27	0.75	0.4135	900.1	0.00E+00	0.00E+00	1.98E-10	0.00E+00	1.65E-14	29000	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.06E-05	1259712	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.53E+07	4.53E+07
5.41	1.89E-08	30	0.833333	0.37731	1080	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.63E-10	31000	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.68E-03	1399680	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.04E+07	4.77E+07
5.46	1.39E-09	33	0.916667	0.36774	1295.9	0.00E+00	0.00E+00	0.00E+00	1.34E-34	1.78E-06	33000	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.09E-01	1539648	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.54E+07	4.77E+07
5.51	9.9E-11	36	1	0.35241	1296	0.00E+00	0.00E+00	0.00E+00	1.00E+00	1.79E-06	35000	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.16E-01	1679616	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.05E+07	4.77E+07
5.56	2.79E-12	40	1.111111	0.29678	1296.1	0.00E+00	0.00E+00	0.00E+00	1.35E-34	1.79E-06	35800	0.00E+00	0.00E+00	0.00E+00	0.00E+00	8.61E-01	1866240	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.72E+07	4.77E+07
5.61	3.03E-14	45	1.25	0.19314	1620	0.00E+00	0.00E+00	0.00E+00	0.00E+00	4.93E-03	36818	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E+00	2099520	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.56E+07	4.77E+07
5.66	3.14E-16	50	1.388889	0.10212	1800	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.73E-02	38000	0.00E+00	0.00E+00	0.00E+00	0.00E+00	8.24E-01	2332800	0.00E+00	0.00E+00	0.00E+00	0.00E+00	8.40E+07	4.77E+07
5.71	3.11E-18	55	1.527778	0.04579	2200	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.04E-01	39000	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.23E-01	2566080	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.24E+07	4.77E+07
5.75	2.98E-20	60	1.666667	0.01801	2304	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E+00	41000	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E-01	2799360	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.01E+08	4.77E+07
5.79	2.78E-22	65	1.805556	0.00638	2400	0.00E+00	0.00E+00	0.00E+00	0.00E+00	9.22E-01	43000	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.66E-03	3032640	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.09E+08	4.77E+07
5.82	2.53E-24	70	1.944444	0.00207	3000	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.58E-02	90720	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.98E-94	3265920	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.18E+08	

Note: In column-2 and -5, probability curve of [Σn=1,2] and [Σn=1..5] are copied from SunQM-3's Table 2 column-23 and -22 and then normalized to one. Note: In column-1, r/r_1 value is cut off at ≥ 4, with the formula: [4 + log(r/r_1 -4)]. Note: In column-24, r/r_1 value is cut off at ≥ 36^4 * 5.33^2, with the formula: [36^4 * 5.33^2 + log(r/r_1 - 36^4 * 5.33^2)] where r/r_1 is shown in column-23.

2) The real time-dependency of eight planets in $\theta = \pi/2$ plane (or x-y plane) is illustrated in Figure 5 with the initial φ-dimensional positions set on Aug. 14, 2019. The new φ position of each planet (after 60 days) is shown in the same figure. Three values associated with each planet (in Figure 5) are planet's orbital-r, body-r, and the orbital angular frequency/velocity obtained from the eq-62 (or from Table 1's Solar {N,n} QM structure model, not from the NASA's data). Notice that SunQM-3s10 eq-6 showed that the width of Asteroid belt is from 1.95 AU to 3.32 AU, so its Eigen $\Delta r \approx 2.0E+11$ m. Also SunQM-3s10 eq-13 showed that the width of Kuiper belt is from 38.5 AU to 47.8 AU, so its Eigen $\Delta r \approx 1.39E+12$ m.

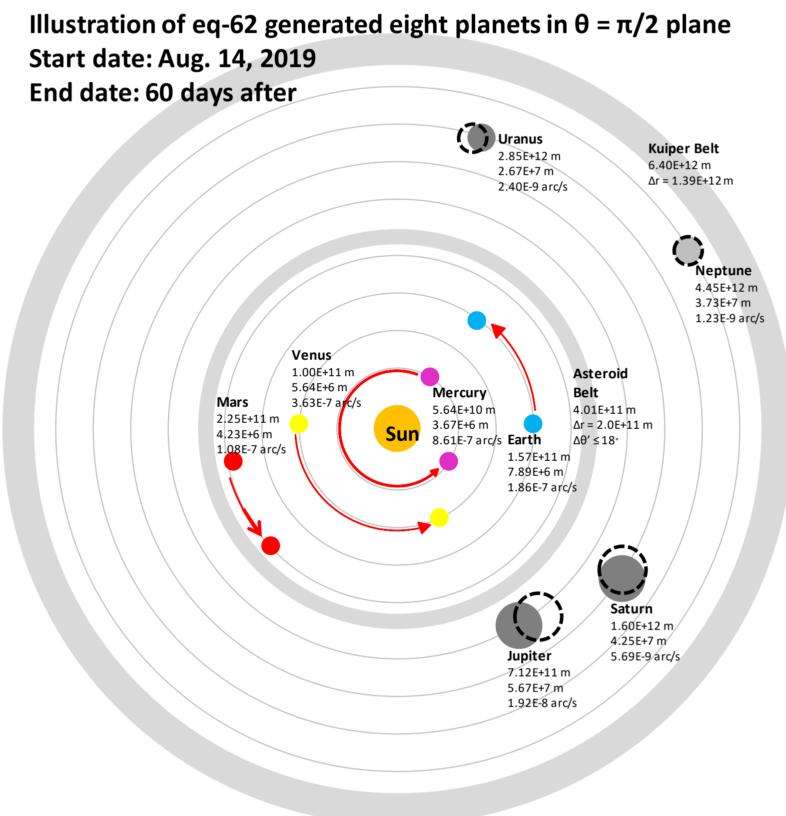

Figure 5. Illustration of eq-62 generated eight planets in $\theta = \pi/2$ plane with initial φ positions on Aug. 14, 2019, and the φ-ωt positions after 60 days of orbital movement.

3) In eq-62, only planets and belts have the time-dependent description (as orbital movement), the rest objects (Sun and cloud) are described in static. For Sun and Oort cloud, since they are (mostly) not in nLL QM state, their descriptions are much more complicated.

4) In eq-62, we can set all bases of probability density functions (the most right-side matrix in eq-62) to be time-independent, and then to describe the time-dependent circular movement of Solar system (including planet's orbital movement and Sun's self-spin) by using a spatial rotation matrix. Notice that it is not a regular rotation matrix (that provide only one rotation velocity at one time for all r-distance), this spatial rotation matrix must have $\omega_{n\text{-spin}} = \omega_{1\text{-spin}}$ / n^x function (see SunQM-3s1 Table 1), and also must be able to cover both planet's orbital movement and Sun's self-spin. However, this is beyond my citizen scientist's math level.

5) Here is a summary of what we have achieved in eq-62: We used a single gigantic r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 to describe the whole Solar system. To do this, we picked 6 of base-n, {-1,n=1..5//6} and {0,1//6}, to describe the Sun, we picked 8 base-

n ({1,n=3..6//6} and {2,n=2..5//6}) to describe 8 known planets, we picked 2 base-n ({1,8//6} and {2,6//6}) to describe Asteroid belt and Kuiper belt, we picked 4 base-n ({3,n=2..5//6}) to describe the four undiscovered planet/belt, and we picked 5 base-n ({4,n=1..5//6}) to describe the Oort cloud. Together, with many Eigen n'(s) or high-frequency n'(s), we can use a relatively simple QM probability density formula to describe the whole Solar system with time-dependent orbital movement at high accuracy. This is a big achievement for the {N,n} QM.

VI. To build a complete (full-QM probability density) 3D map to describe the whole Solar system with time-dependent orbital movement

A full-QM deduced time-dependent |Φ(φ)|^2 * |T(t)|^2 probability density function has been made. It has been moved to SunQM-4 series because that in SunQM-3 series, we study Solar {N,n} QM within the frame of the traditional QM, i.e., the traditional Schrodinger equation, the Born rule, etc. However, in SunQM-4 series, the traditional Schrodinger equation, the Born rule, etc. is no longer the boundary for us to study the Solar {N,n} QM.

VII. Matter waves of galaxies, universe, protons, quarks are all (superposition) running simultaneously in Solar system's matter wave resonance chamber

In SunQM-2 section IV-c, we introduced the concept of MWRC (matter wave resonance chamber) and MWP (matter wave packet): a mass entity (like Solar system, or a proton, or our universe, etc.) can be treated as a matter wave packet (MWP) runs inside a matter wave resonance chamber (MWRC), For example, our Solar system (as a mass entity) itself is a MWRC, and our Solar system produces MWP running in its own MWRC (with high level of RF). Meanwhile, other MWPs produced by other mass entities (e.g., our universe, or galaxies, or protons, or quarks, etc.) also run inside our Solar system's MWRC (with different levels of RF) simultaneously. At the same time, our Solar system produced MWP also runs inside Milky way galaxy's MWRC (with much lower RF), or in our universe's MWRC (with very low RF), or in many proton's MWRCs (with extremely high RF), or even in many quark's MWRCs (with even higher RF).

Now let us use SunQM-3s10's Figure 3 to illustrate this idea in a more intuitive description. In that figure, the cold KBO is a small part of Solar system's MWP running inside Solar system's MWRC. Its orbit at {3,1//6} can be Eigen described as {0,n=6*6*5.33//6} = {0,192//6}, or n=192, or in |192,191,192> QM state. For simplicity, let's ignore the pFactor of 5.33 and still use 6, so {3,1//6} can be Eigen described as {0,n=6^3//6} = {0,216//6}, or n=216, or in |216,215,215> QM state. Then its radial wave function is r^2 * |R(n=216,*l*=215)|^2. Remember that this Eigen n = 6^3 = 216 is based on r_1 at Sun's {0,1//6}. The same cold-KBO can also be described by n = 6^j where j is a (either positive or negative) integer number, or n = … 6^(-5), ... 6^(-1), 6^0, 6^1, 6^2, 6^3, 6^4, … 6^18, etc. Recall that the Milky way galaxy at {8,1//6}, or a proton at {-15,1//6}, have ΔN = +8, or ΔN = -15 relative to {0,1//6} (see SunQM-1s2 Table 1). So if we choose r_1 at {8,1//6}, then comparing to cold-KBO's n = 6^3, the n shifted from n = 6^3 to n = 6^(3-8) = 6^(-5), or the cold-KBO can also be described by r^2 * |R(n=6^(-5),*l*)|^2. Then the cold-KBO's r^2 * |R(n=6^(-5),*l*)|^2 description is expected to have a strong contribution from Milky way galaxy's MWP (because its r_1 is equivalent to the size of Milky way galaxy). Similarly, if we choose r_1 at {-15,1//6}, then comparing to cold-KBO's n = 6^3, the n shifted from n = 6^3 to n = 6^(3-(-15)) = 6^18, or the cold-KBO's can also be described by r^2 * |R(n=6^18,*l*)|^2. Then the cold-KBO's r^2 * |R(n=6^18,*l*)|^2 description is expected to have a strong contribution from proton's MWP (because its r_1 is equivalent to the size of a proton). Intuitively, we can easily understand that at r_1 at {-15,1//6}, the cold-KBO's r^2 * |R(n=6^18,*l*)|^2 description will have a strong contribution from protons' MWPs because the cold-KBO is made of proton, neutron, electron, etc. However, without {N,n} QM's help, we can hardly imagine that the cold-KBO's r^2 * |R(n,*l*)|^2 description at n = 6^(-5) is expected to have a strong contribution from Milky way galaxy's MWPs, simply because this specific n = 6^(-5) correlates to a r_1 at {8,1//6}. Using the terminology of the traditional QM, this phenomenon is called superposition of QM states.

Based on the {N,n//6} QM structure model, eq-62 describes a Solar system not only at the Eigen description (of a planet or a belt) level, but can be (simultaneously) at any possible levels of resolution (down to atom level, proton level, or up to galaxy level, even to the whole universe level)! In SunQM-4, we named this phenomenon as the "Simultaneous-Multi-

Eigen-Description", or "SMED". Also in SunQM-4, we pointed out that the SMED character is the nature attribute of both wave mechanics and matrix mechanics, therefore "Simultaneous-Multi-Eigen-Description (SMED)" is one of many nature attributes of quantum mechanics! Some of the other known nature attributes of QM are: the particle-wave duality, uncertainty principle, RF (or RotaFusion, or rotation diffusion), etc.

VIII. Can we use 3D probability density r^2 * |R(n,*l*)|^2 * |Y(*l*,m)|^2 map to calculate out the ϕ-positions of the four undiscovered planets at {3,n=2..5//6} orbits?

Suppose that all mass in the {3,n=2..5//6}o orbit spaces have accreted into planets. Based on the discussion in section VII, we believe that all twelve planets' orbital movements are entangled not only through r-dimension's wave function r^2 * |R(n,*l*)|^2 (as shown in SunQM-3s10's Figure 3), but also through the whole 3D wave function r^2 * |R(n,*l*)| * |Y(*l*,m)|, which means that their φ-positions are entangled also. When without the entanglement, the Solar system would have formed exactly as the Solar {N,n//6} QM model with the accurate $r_n = r_1$ * n^2 orbit relationship between each circular orbits in x-y plane, and each planet's ϕ–position is unrelated to other planets' ϕ–positions. With the entanglement, these orbits have their own eccentricities, inclinations, and their averaged r(s) are deviated from the $r_n = r_1$ * n^2, and the mass is part of the entanglement parameters, and φ-positions are also become part of the entangled parameters. A "global fitting" for all parameters of all twelve planets at same time is needed. Although some studies using the classical physics have shown some interesting results (see wiki "Planet Nine", and see [17] ~ [18]), we do believe that the correct solution can only be obtained through the {N,n} QM kind of "global fitting" modeling. A true quantum computer may be needed to solve this true QM problem.

IX. A prediction that all mass entities (from the whole universe to a single quark) can be described by Schrodinger equation and solution

Since {N,n//6} QM structure covers from quark {-17,1//6} to the Virgo super cluster {10,1//6} with good consistency (see SunQM-1s2 Table 1), and Schrodinger equation/solution has accurately described the Solar system from {-2,1//6} to {5,1//6} (see SunQM-3s11, and SunQM-4) as well as the atom system from {-15,1//6} to {-11,1//6} (see SunQM-1s2 Table 1), we believe that the whole universe can be described by Schrodinger equation and solution, and a single quark can also be described by Schrodinger equation and solution. After searching wiki, we found that there are some other scientists have also pointed that "*Solutions to Schrodinger's equation describe not only molecular, atomic, and subatomic systems, but also macroscopic systems, possibly even the whole universe*" (see wiki "Schrodinger equation", and also see [19]).

X. The wrap-up discussion on the phase-1 study of Solar {N,n} QM modeling

This paper marks (almost) the end of the phase-1 study on the {N,n} QM modeling to our Solar system. In the phase-1 study, we expanded Bohr's equation ($r_n = r_1$ * n^2), Einstein's QM equation (E = h * f), and Schrodinger equation from micro-world to Solar system and established a brand new {N,n//q} QM. The final result of this study revealed that the whole Solar system can be described by a single solution of Schrodinger equation as shown in eq-62.

Just like that from Einstein field equations of general relativity, Karl Schwarzschild discovered the possible existence of black hole (see wiki "black hole"), and Georges Lemaitre discovered that the recession of nearby galaxies can be explained by an expanding universe, and this expanding universe can be further traced back to time zero as a single point (which leads to the big bang theory, see wiki "Big Bang"), here from Bohr equation and Schrodinger equation and {N,n} QM, we have also made many astonishing new discoveries. Here we list some major ones:

1) Our observable universe from Virgo super cluster at {10,1//6}, down to Milky way galaxy at {8,1//6}, Solar system at {5,1//6}, Sun at {0,2//6}, black hole at {-3,1//6}, H-atom at {-12,1//6}, proton at {-15,1//6}, and quark at {-17,1//6}, are all mysteriously follow {N,n//6} QM structure in size (see SunQM-1s2 table 1).

2) The whole current Solar system can be accurately described by Schrodinger equation and a single solution as shown in eq-62. From this QM description, we predict that there are four undiscovered planets/belts in our Solar system at orbit of {3,n=2..5}, each with ~ 12x, 7x, 5x, and 3x of Earth's mass. If they had already formed planets, then their radius are predicted to be 2.18E+7 m, 1.80E+7 m, 1.59E+7 m, and 1.42E+7 m respectively with the interior {N,n} QM structure similar as that of Neptune (see SunQM-3s6 Table 2). If they are still in belt form, then these four belts' r ± Δr and Δθ' ranges are predicted to be 173 ±13 AU, 390 ±25 AU, 693 ±38 AU, and 1081 ±53 AU, and ±6.3 degree, ±5.1 degree, ±4.4 degree, and ±4.0 degree, respectively (see SunQM-3s10 section V).

3) The pre-Sun ball nebula collapsed quantumly, might first at size around of {6,1} and then down to {5,1}, {4,1}, {3,1} {2,1}, {1,1}, {0,2} one by one, and this process can be described by Schrodinger equation and solution (see SunQM-1s1, SunQM-3s2).

4) A Sun-massed white dwarf, neutron star, and black hole also follows {N,n//6} QM structure with size of {-1,1}, {-3,2}, and {-3,1}. The {N,n} QM also predicted that a Sun-massed black hole (with the Schwarzschild radius = 2.95E+3 meters), instead of having size of "singularity", it may have a stable {N,n} QM structure at size of {-5,1} with r ≈ 2 meters (see SunQM-1s2 table 1).

5) The quantum expansion of Sun's H-fusion ball, He-fusion ball, C-fusion ball, etc. and red giant, may also follow {N,n//6} QM dynamics (see SunQM-1s1 table 7b). {N,n,Cold-G} vs. {N,n,Hot-G} recorded the history that the pre-Sun ball started the massive H-fusion after the {2,1} pre-Sun ball was formed (during a series of quantum collapses).

6) Inside the current Sun, the inward expansion (in-pansion?) of the convective zoom may also follow {N,n//6} QM structure (see SunQM-3s8 section II). The previous quantum in-pansion of convective zoom to {-1,11}o orbit shell (~2400 Mya) and to {-1,10}o orbit shell (~650 Mya) might have caused two "Snowball Earth" periods in the geological history of Earth. The Solar {N,n} QM model predicts that in the next 5 billion years, there are four more quantum in-pansion of convective zoom from the current {-1,10}o to {-1,n=9..6}o orbit shells. The next quantum in-pansion of convective zoom to {-1,9}o orbit shell is estimated to happen in ~ 650 million years.

7) It is predicted that there is an expanding rock-evap-line which has passed the {1,2}, it might have burned off all mass of an ancient planet at orbit {1,2}, and has been burning off most mass of Mercury at orbit {1,3}, and start to burn off the (light element) mass of Venus at orbit {1,4} (see SunQM-3s6). It is predicted that there is an expanding ice-evap-line which has passed the {1,8}, it had burned off all of (~ 20,000 km thick) original atmosphere on each of four rock planet, and most evaporated H/He/H_2O molecules were captured by Jupiter, and made Jupiter 10x more massive than the original one (see SunQM-1s1). It also produced Asteroid belt as the (dried) "ring stain" of this expanding ice-evap-line. It is predicted that there is an expanding methane-evap-line which has just arrived {2,6}, and it produced the "cold KBO" as the (wet) "ring stain" of this expanding methane-evap-line (see SunQM-3s10).

8) It has been shown that the formation of planet's and star's (radial) internal structure is governed by the planet's or star's radial (gravity-forced) QM (see SunQM-3s6, SunQM-3s7, and SunQM-3s8).

9) It has been shown that the surface mass (atmosphere, or rock, or liquid iron) movement of Sun, Jupiter, Saturn, and Earth, etc., is governed by Star's (or planet's) θφ-2D dimension QM (see SunQM-3s3). The famous Jupiter surface cloud bands pattern is explained as the |5,4,m> zonal bands embedded in the background |400> belt bands. The Earth's atmospheric circulation is caused by the same QM peaking/depleting effect of |211> state on top (not embedded) of |100> state.

10) The sunspot drift, the continental drift, and Sun's and Earth's magnetic dynamo has been explained by a (single) Yml cycle model (see SunQM-3s9). Under this model, the apparent random drift of post-Pangaea continents can be nicely depicted as an expected hydrodynamic result of a broken dam through a mouth located near the south end of South America continent.

11) All planets in Solar system are formed in {N,1//2} QM structure (see SunQM-3s6 Table 2), and Sun itself can be as a {N,1//2} QM structure (see SunQM-3s7 section VIII), so that {N,1//2} seems to be the basic building block of {N,n//q} QM structure for the formation of celestial body in the macro world. Will this also be true in the micro world?

As the result, the success of {N,n//q} QM expanded the traditional QM, and make it more self-consistent, and more complete.

Conclusion

A number of high, medium, low resolution 3D probability density maps (based on Schrodinger equation's solution) have been constructed and they are able to describe the whole Solar system with time-dependent orbital movement. In the study, the Eigen n' of a planet has been calculated. This Eigen n' gives the planet's information not only on the orbital r, but also for the surface r. The analysis revealed that for all planets, their Eigen n(s) in all three dimensions are equal ($n'_r = n'_\theta = n'_\varphi = n'$). A group of linearly combined wave functions produced a perfect probability density peak in the φ-dimension. The successful of construction of this 3D probability density map for the Solar {N,n//6} QM structure in the range from {-2,1//6} to {5,1//6} implies that both the whole universe and a single quark may also be described by Schrodinger equation and solution.

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SunQM-3s11: Using {N,n} QM's probability density 3D map to build a complete Solar system with time-dependent orbital movement.
SunQM-4: (Full-QM) deduction of using {N,n} QM's probability density 3D map to build a complete Solar system with time-dependent orbital movement.
SunQM-4s1: Schrodinger equation and Solar {N,n} QM.
SunQM-4s2: Using {N,n} QM to analyze Earth's atmosphere pattern and its effect on the weather
SunQM-4s3: Using {N,n} QM to explain how planets are formed through accretion.
SunQM-4s5: Addendums, Updates and Q/A for SunQM series papers
SunQM-5: A new version of QM based on interior {N,n}, multiplier n', |R(n,l)|^2 * |Y(l,m)|^2 guided mass occupancy, and RF, and its application from string to universe (drafted in April 2018).
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Special thanks to: All scientific writers who have contributed to the wiki writing. Your contribution truly democratized the scientific knowledge to anyone who interested to know.