A Dynamical Theory of the Electromagnetic Four-Potential

James Conor O'Brien

29 Nov 2019 Brisbane

"Gin a body meet a body
Flyin' through the air,
Gin a body hit a body,
Will it fly? and where?" - J.C.M.

Keywords:

Wheeler–Feynman time-symmetric theory, Feynman–Stueckelberg interpretation, Cramer's transactional interpretation, Electromagnetic mass, quark model, Foot-Koide-Brennan rules for lepton masses, electromagnetic four-vector potential, Mach's principle.

Abstract

This paper applies the Wheeler-Feynman time-symmetric theory to the electromagnetic four-potential $A_\mu(\varphi, \mathbb{A})$ and derives a wave equation that puts Matter on the same foundation as Maxwell's electromagnetic equation for Light waves. Applying the Wheeler-Feynman summation it derives the various physical properties for matter of mass, charge, spin, momentum, phase, electromagnetic mass, Hamiltonian action, and gives an explicit derivation of Cramer's Quantum Handshake as a standing wave of the advanced and retarded electromagnetic four-potentials. The Wheeler-Feynman summation is then applied to the derive the lepton masses of the Standard Model, this leads to the suggestion that the masses, charges and spins of all charged leptons are based on different configurations of the electromagnetic four-vector potential. Mach principle appears as a natural consequence of the free potentials of the Wheeler-Feynman summation acting on the resultant particle's charge. Gaussian units are used throughout.

§1 Modelling Matter from the Wheeler-Feynman Summation

The following sections are an attempt to derive an electromagnetic wave equation for matter from the Wheeler–Feynman absorber theory. Before reading this section it is recommended if you knew of Wheeler-Feynman's paper [Wheeler-Feynman 1945]; John Cramer's paper [Cramer 1986] on the Transactional interpretation of quantum mechanics; and Carver Mead's paper on Collective Electrodynamics I [Mead 1997].

§1-1 An Electromagnetic Four-Vector Potential Wave Function

Wheeler-Feynman [Wheeler-Feynman 1945] discussed the problem of action at a distance through the radiative reaction on an absorber, and gave the following as one of the requirements: "... This field combines with the half-retarded, half-advanced field of the source to give for the total disturbance the full retarded field which accords with experience." This is labelled the Wheeler-Feynman time symmetric theory WFTST, and can be summarized for an electric field $\mathbb{E}$ as the total contribution of the advanced and retarded fields together

$$\mathbb{E}_{\text{total}} = \sum_n \frac{1}{2}\left(\mathbb{E}_n^{\text{ret}} + \mathbb{E}_n^{\text{adv}}\right) \tag{1}$$

while the contribution of the free advanced and retarded fields are,

$$\mathbb{E}_{\text{free}} = \sum_n \frac{1}{2}\left(\mathbb{E}_n^{\text{ret}} - \mathbb{E}_n^{\text{adv}}\right) = 0 \tag{2}$$

adding the total and the free fields together

$$\mathbb{E}^{\text{total}} = \mathbb{E}^{\text{total}} + \mathbb{E}^{\text{free}} = \sum_n \frac{1}{2}\left(\mathbb{E}_n^{\text{ret}} + \mathbb{E}_n^{\text{adv}}\right) + \sum_n \frac{1}{2}\left(\mathbb{E}_n^{\text{ret}} - \mathbb{E}_n^{\text{adv}}\right) = \sum_n \mathbb{E}_n^{\text{ret}} \tag{3}$$

If instead of an electric field $\mathbb{E}$ let's specify the Wheeler-Feynman time symmetric theory in terms of the advanced and retarded four-vector potentials A_μ,

$$A_\mu = (\varphi(t), \mathbb{A}(x, y, z)) \tag{4}$$

here the total contribution of the advanced and retarded potentials is,

$$A_\mu^{\text{total}} = \sum_n \frac{1}{2}\left(A_{\mu,n}^{\text{ret}} + A_{\mu,n}^{\text{adv}}\right) \tag{5}$$

and the contribution of the free advanced and retarded potentials is,

$$A_\mu^{\text{free}} = \sum_n \frac{1}{2}\left(A_{\mu,n}^{\text{ret}} - A_{\mu,n}^{\text{adv}}\right) = 0 \tag{6}$$

adding the two series together,

$$A_\mu^{\text{total}} = A_\mu^{\text{total}} + A_\mu^{\text{free}} = \sum_n \frac{1}{2}\left(A_{\mu,n}^{\text{ret}} + A_{\mu,n}^{\text{adv}}\right) + \sum_n \frac{1}{2}\left(A_{\mu,n}^{\text{ret}} - A_{\mu,n}^{\text{adv}}\right) = \sum_n A_\mu^{\text{ret}} \tag{7}$$

This will be referred to as the Wheeler–Feynman Electromagnetic Four-Vector Potential Time-Symmetric Theory or WFEMFPTST.

Now make the observation that since only the scalar potential φ component of the four-potential A_μ is in the temporal axis and the spatial magnetic potential $\mathbb{A}$ component is not, then *only* the temporal component of the four-vector potential translates across time,

$$(\varphi, \mathbb{A}) \rightarrow (\varphi, 0) \tag{8}$$

The WFTST can now be written purely in terms of the scalar potential components of electromagnetic four potential, (importantly the $\mathbb{A}$ will be added in later in accordance with Maxwell's equations),

$$\varphi_{\text{total}} = \sum_n \frac{1}{2}\left(\varphi_{\text{ret}} + \varphi_{\text{adv}}\right) + \sum_n \frac{1}{2}\left(\varphi_{\text{ret}} - \varphi_{\text{adv}}\right) = \varphi_{\text{ret}} \tag{9}$$

where on writing

$$\mathbb{E}_{\text{ret}} = -\nabla \varphi_{\text{ret}} \tag{10}$$

we can immediately recover Wheeler-Feynman use of a general field for the summation.

$$\mathbb{E}^{\text{total}} = \mathbb{E}^{\text{total}} + \mathbb{E}^{\text{free}} = \sum_n \frac{1}{2}\left(\mathbb{E}_n^{\text{ret}} + \mathbb{E}_n^{\text{adv}}\right) + \sum_n \frac{1}{2}\left(\mathbb{E}_n^{\text{ret}} - \mathbb{E}_n^{\text{adv}}\right) = \sum_n \mathbb{E}_n^{\text{ret}} \tag{11}$$

This may seem an unnecessary step to recover the thing we first started out with, however, the different behavior of φ compared to $\mathbb{E}$ over time requires a different approach to the problem of action at a distance. To quote David Griffiths - "*There is a very peculiar thing about the scalar potential in Coulomb gauge: it is determined by the distribution of charge right now. If I move an electron in my laboratory, the potential V on the moon immediately records this change*." [Griffiths 2017]. On using only φ and making the assumption that in flat space the resultant potential is the average of the advanced and retarded potentials and that it is located at the midpoint between them, this will be labelled as the intrinsic (or local) frame of reference. The advanced and retarded potentials are both extrinsic to the resultant potentials frame of reference, while the resultant summation is intrinsic to the resultant potentials frame of reference. Since the resultant potential appears midpoint in time between sources of the advanced and retarded potentials let's relabel this as the 'intrinsic potential' to avoid confusion,

$$\varphi_{\text{total}} = \sum_n \frac{1}{2}\left(\varphi_{\text{ret}} + \varphi_{\text{adv}}\right) + \sum_n \frac{1}{2}\left(\varphi_{\text{ret}} - \varphi_{\text{adv}}\right) = \varphi_{\text{int}} \tag{12}$$

obviously conservation of energy requires

$$|\varphi_{\text{ext}}| = |\varphi_{\text{int}}| \tag{13}$$

so we can equally write

$$\varphi_{\text{total}} = \sum_n \frac{1}{2}\left(\varphi_{\text{ret}} + \varphi_{\text{adv}}\right) + \sum_n \frac{1}{2}\left(\varphi_{\text{ret}} - \varphi_{\text{adv}}\right) = \varphi_{\text{ext}} \tag{14}$$

to extract the charges from this requires including the opposite polarity of the antiparticles,

$$\varphi_{\text{total}} = \varphi_{\text{total}} + \varphi_{\text{free}} = \sum_n \frac{q}{2}\left(\frac{1}{r_{\text{ret}}} - \frac{1}{r_{\text{adv}}}\right) + \sum_n \frac{q}{2}\left(\frac{1}{r_{\text{ret}}} + \frac{1}{r_{\text{adv}}}\right) = \frac{q_{\text{int}}}{r_{\text{int}}} \tag{15}$$

conservation of energy again requires

$$\left|\frac{q_{\text{ext}}}{r_{\text{ext}}}\right| = \left|\frac{q_{\text{int}}}{r_{\text{int}}}\right| \tag{16}$$

Therefore the charge of the resultant potential is identified with the original charges of the interaction, and is inversely proportional of the distance of those original charges.

Maxwell's equations must be applied to the intrinsic frame to prevent the system from being unbalanced, to do so first take the Coulomb gauge in the extrinsic frames, we can do this because in the intrinsic frame the $\mathbb{A}_{ext}$ vanishes,

$$\nabla \cdot \mathbb{A}_{ext} = 0 \tag{17}$$

However, since Maxwell's equations are Lorentz invariant, we can satisfy them by switching to the Lorenz gauge in the new frame, therefore in the intrinsic frame a new magnetic potential appears as due to the time varying extrinsic electric potentials, or

$$\nabla \cdot \mathbb{A}_{int} = -\frac{1}{c} \frac{\partial \varphi_{ext}}{\partial t} \tag{18}$$

summing over all the extrinsic potentials

$$\nabla \cdot \mathbb{A}_{int} = -\frac{1}{c} \sum_n \frac{1}{2}\left(\frac{\partial \varphi_{ret}}{\partial t} + \frac{\partial \varphi_{adv}}{\partial t}\right) - \frac{1}{c} \sum_n \frac{1}{2}\left(\frac{\partial \varphi_{ret}}{\partial t} - \frac{\partial \varphi_{adv}}{\partial t}\right) = -\frac{1}{c} \sum_n \frac{\partial \varphi_{ext}}{\partial t} \tag{19}$$

These results can be used to yield the magnetic field

$$\mathbb{B}_{int} = \nabla \times \mathbb{A}_{int} \tag{20}$$

and the electric field in the intrinsic frame,

$$\mathbb{E}_{int} = -\nabla \varphi_{int} \tag{21}$$

We have now recovered the original summation for the WFEMFPTST by writing

$$A_\mu^{int} = (\varphi_{int}, \mathbb{A}_{int}) \tag{22}$$

$$A_\mu^{tot} = \Sigma \frac{1}{2}\left(A_\mu^{ret} + A_\mu^{adv}\right) + \Sigma \frac{1}{2}\left(A_\mu^{ret} - A_\mu^{adv}\right) = A_\mu^{int} \tag{23}$$

now we can see the WHTST as a subspace of the WFEMFPTST.

Therefore there appears within the context of the Wheeler-Feynman model a cycle of electromagnetic potentials over the particles evolution,

$$(\mathbb{B}_{int}, \mathbb{E}_{int}) \to A_\mu^{total}(\varphi, \mathbb{A}) \to \sum_n A_\mu^{ret, adv}(\varphi_{ext}, 0) \to \sum_n \varphi_{ext} \to \mathbb{A}_{int} \to \varphi_{int} \to (\mathbb{B}_{int}, \mathbb{E}_{int}) \to A_\mu^{int} \tag{24}$$

this can be shown to be an electromagnetic wave by writing Ampere's Law

$$\nabla \times \mathbb{B} = \frac{4\pi}{c} \mathbb{J} + \frac{1}{c} \frac{\partial \mathbb{E}}{\partial t} \tag{25}$$

in the intrinsic frame we can use the vector triple product

$$\nabla \times (\nabla \times \mathbb{A}_{int}) = \nabla(\nabla \cdot \mathbb{A}_{int}) - \nabla^2 \mathbb{A}_{int} = \frac{4\pi}{c} \mathbb{j}^{int} - \frac{1}{c} \nabla \left(\frac{d\varphi_{int}}{dt}\right) - \frac{1}{c} \frac{d^2 \mathbb{A}_{int}}{dt^2} \tag{26}$$

and applying the Lorenz gauge transformation

$$A^{\mu} \rightarrow A^{\mu} + \partial^{\mu} f \tag{27}$$

yields the D'Alembertian for an electromagnetic wave

$$\Box^2 \mathbb{A}_{\text{int}} = -\frac{4\pi}{c} \mathbb{j}_{\text{int}} \tag{28}$$

Since the radius r must be greater than zero to avoid infinite energy then radius r can be derived from

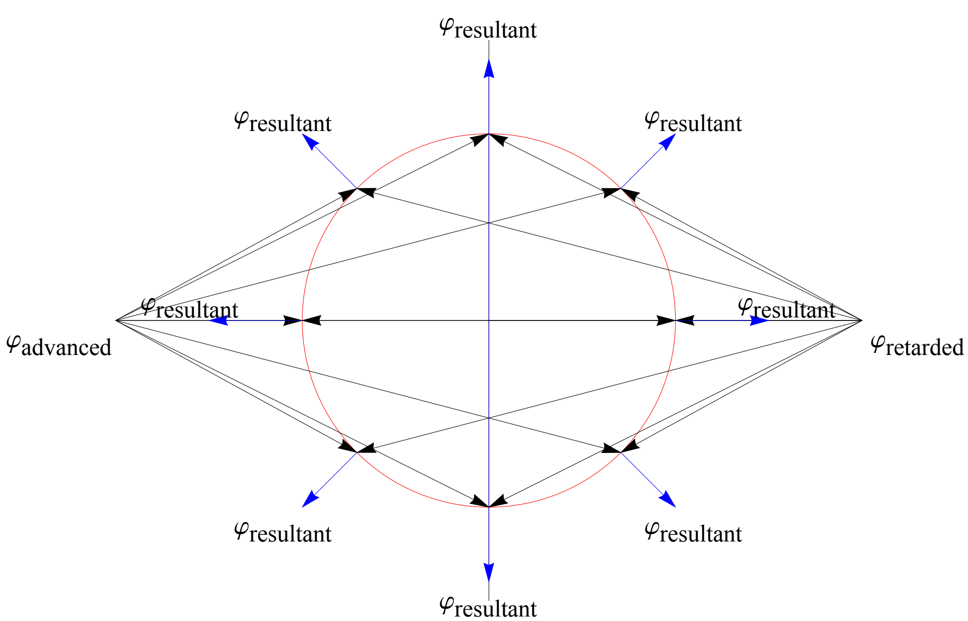
$$\mathbb{E}_{\text{int}} = -\nabla \frac{q_{\text{int}}}{r_{\text{int}}} \tag{29}$$

this implies this is the wave equation of a spherical particle travelling from the retarded source to the advanced source.

Finally by the mass-energy relationship there must be a mass associated with the energy of the charge, which must equal the rest mass of the particle. Therefore by applying the Wheeler–Feynman time-symmetric theory to the four-vector potential there appears in the inertial frame of the observed particle a four dimensional inhomogeneous electromagnetic wave function which has radius, mass and charge,

$$\partial_{\mu} \partial^{\mu} A_{\mu}^{\text{int}} = -\frac{4\pi}{c} j_{\mu}^{\text{int}} \tag{30}$$

Diagrammatically consider the following two dimensional diagram of the sum of the advanced and retarded potentials. Here the blue arrows are the resultant potential vectors, the black arrows are the source vectors. For symmetry let's assume that in four dimensions the distance from the advanced and retarded charges is equidistant to their loci on the circle, then we see the resultant potential is radially directed away from the center of the particle. In four dimensions each of the black arrows is equal in length. Importantly, none of the intrinsic potentials are within the radius of the particle, requiring the particle is hollow and the charge is entirely on the surface of the particle (it was Poincaré in 1906 who first pointed out the possibility of elementary particles as being cavities or holes in the dielectric of space [Poincaré in 1906]).



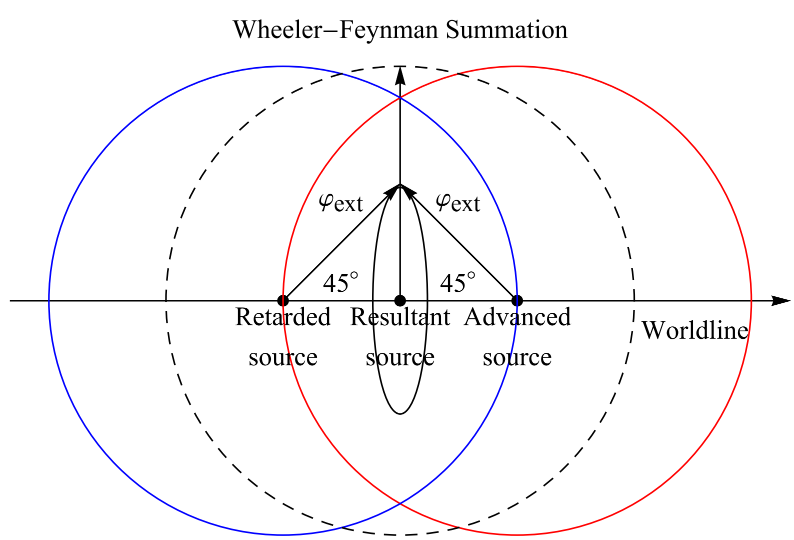
(31)

You will note the resultant sum is not in the frame of reference of either the advanced or retarded potentials, rather it is a loci midway between the advanced and retarded frames, so a better choice of language would be to distinguish the frames into past, present and future frames of reference. I think this was a problem with the Wheeler-Feynman formulation where it appeared the retarded field is the resultant field, while for this model the magnitude of the resultant potential is equal to the magnitudes of either advanced or retarded potentials, the result in this model is an average of those frames of reference, which is a much more reasonable idea (at least in flat space). The A_μ^{ret} and A_μ^{adv} are in a two-to-one relationship that forms a double-cover of the A_μ^{total}, and we can project the waveforms of the advanced and retarded potentials to form two four-dimensional vector potential spheres centered on the advanced and retarded charges. This resultant third four-dimensional sphere appears in physical space at the intersection of two four-dimensional hyperspheres as a charged three-dimensional sphere. Since this spherical potential is the result of extrinsic potentials and therefore does not interact with itself, it is in the language of Poincaré non self-interacting. Self-interaction was a major problem with the early electromagnetic program, this means we need to modify the above model to exclude self-interaction of the resultant potentials

$$A_\mu^{\text{total}} = \sum_{i \neq j} \frac{1}{2}\left(A_{\mu(i)}^{\text{ret}} + A_{\mu(j)}^{\text{adv}}\right) + \sum_{i,j} \frac{1}{2}\left(A_{\mu(i)}^{\text{ret}} - A_{\mu(j)}^{\text{adv}}\right) = \sum_i A_{\mu(i)}^{\text{int}} \tag{32}$$

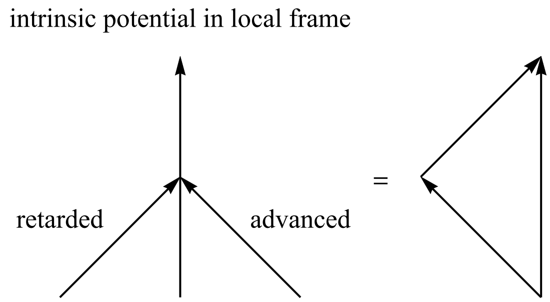
Mathematically this spherical three-dimensional charged sphere is referred to as a glome, and it should noted that the term ‘particle’ is no longer appropriate as it refers to an extremely small piece of matter, whereas in this model the potentials and by implication matter itself becomes a travelling spherical electromagnetic wave, and the name should be replaced with the mathematical term ‘glome’, or ‘glome-wave’ where the word ‘glome’ refers specifically to a spherical charged shell; but as the word glome is an obscure mathematical term to avoid confusion the term particle with it's historical usage shall be continued to be used when referring to electrons and quarks.

We can simplify even more by considering a two dimensional projection of the progression of the Wheeler-Feynman summation as a series of circles advancing along the time line. Here the vertical black arrow is sum of half retarded and half advanced sources; the circles are surfaces of the advanced(red) and retarded(blue) sources; the dashed circle is the surface of intermediate source; the vertical black arrow is the sum of time projected half retarded and half advanced sources, and importantly the radius of the resultant intrinsic potential is the same as the extrinsic potentials:

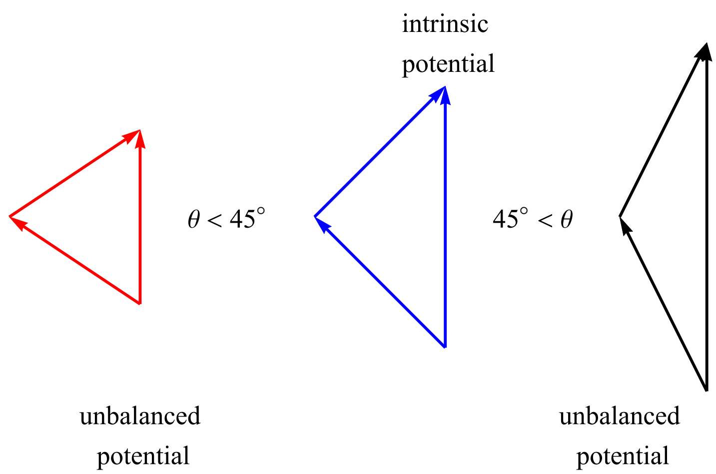


(33)

This next diagram examines the potentials in isolation as simple vectors, where we clearly see the resultant potential is sum of half the advanced potential and half the retarded potential,

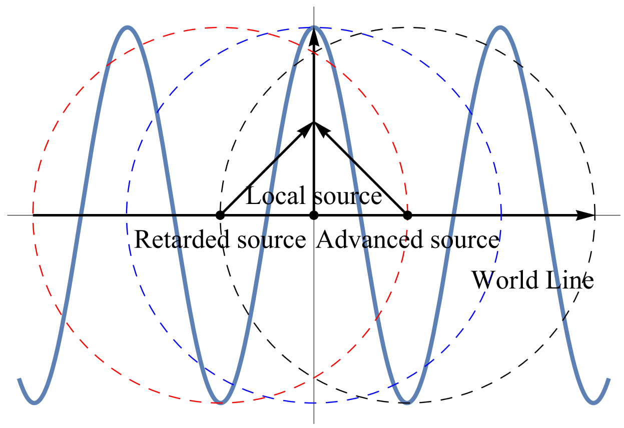


Note this intersection must take place at an oblique angle, for if on the one hand the two spheres intersect orthogonally from the axis of time then the resultant waveform is transverse and is identical to Maxwell’s derivation for the equation for light waves, on the other hand if the two spheres intersect on the axis of time then the sine of the angles of the potentials are zero, this leaves only an oblique angle for the intersection, and for reasons of symmetry and conservation of energy that will be latter demonstrated in this paper this angle must be 45°,



this 45° will be returned to in the later section on Robert Foot's formula for lepton masses.

For illustrative purposes it’s easy enough to superimpose a wavelength upon the diagram for spherical charge shells as the dashed lines for advanced-retarded-local particles; momentum moving along the world line; a wavelength beginning and ending on the advanced and retarded sources; and the amplitude is drawn as the radius of the spheres. While this is only for illustrative purposes as no attempt has been made to derived the wavelength and amplitude, it however will be demonstrated later in the paper there is indeed a standing wave function between the advanced and retarded sources that generates the local charge that is consistent with Cramer’s Quantum Handshake.



Comparing the above derivation for a matter wave to the equation for a photon, we note as Griffiths writes: "*In quantum electrodynamic A^μ become the wave function of the photon. The free photon satisfies this equation*

$$\Box^2 A^\mu = \frac{4\pi}{c} J^\mu \tag{36}$$

for $J^\mu = 0$

$$\Box^2 A^\mu = 0 \tag{37}$$

which we recognize in this context as the Klein-Gordon equation for a massless particle." - [Griffiths 1987 p227], and here we see an electromagnetic wave equation can be written for light as well as for matter. This model suggests that just like Maxwell's derivation of the electromagnetic wave equation for light there is an equivalent four dimensional electromagnetic wave equation for matter that corresponds to Maxwell's three dimensional electromagnetic field wave equation, (hence the title of this paper closely mirrors Maxwell's paper on light waves [Maxwell 1865]). For now, let's state this as a general principle that

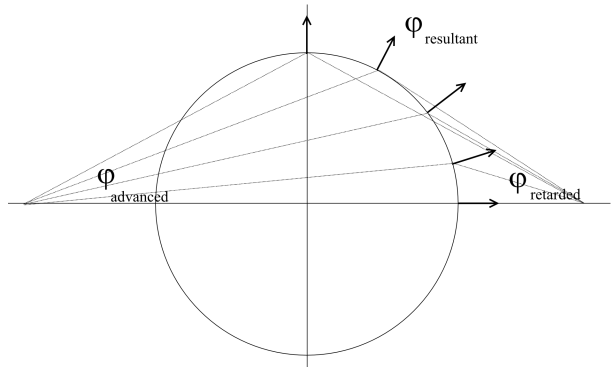
0: Light and Matter are both Electromagnetic Wave Equations.

§2 A Working Model for Elementary Particles

Now let's work with the idea that the charge of an elementary particle can be derived as part of the solution of a four dimensional inhomogeneous electromagnetic wave function in $\mathbb{R}^{1,3}$, where the charge exists on the surface of the three sphere in $\mathbb{R}^3$, not as a point particle, and importantly there is no interior to the particle (an idea first expressed by Poincaré) and try to develop this model to see how it might be applied to the particles of the Standard Model. If this spherical wave is to represent matter waves it needs to be able to define the charge, mass, momentum and spin of particles of the Standard Model and the following sections discuss these properties in light of the electromagnetic wave model for matter.

§2-1 Charge

As shown above the resultant extrinsic electric potential in the particle's frame of reference is radially directed from the surface of the intermediate particle.



(38)

We can then say that in any frame of reference there is a unique set of electrical field vectors that are radially directed from the origin of the particle and by Gauss' law this is equivalent to a source point charge q at the origin of the intermediate coordinate, and at every point in the evolution of the motion of the particle there exists a unique solution to the electromagnetic wave equation as a spherical charged shell. To establish this charge from the wave equation first write the four dimensional D'Alembertian from the A_μ

$$\partial_\mu \partial^\mu A_\mu = -\frac{4\pi}{c} j_\mu \tag{39}$$

from this we can extract the three dimensional charge from the current density

$$\nabla \cdot \mathbb{j} = -\frac{\partial \rho}{\partial t} \tag{40}$$

and applying Gauss' divergence theorem where the rate of change of charge in a fixed volume Ω equals the net current flowing through the boundary yields the charge

$$\frac{dq_\Omega}{dt} = \frac{d}{dt}\int\int\int_\Omega \rho\, dV = -\oiint_{\partial\Omega} \mathbb{j} \cdot d\mathbb{S} = -I_{\partial\Omega} \tag{41}$$

In other words we have transformed from a four dimensional space $\mathbb{R}^{1,3}$ to three dimensional space $\mathbb{R}^3$, we have gone from a four dimensional waveform to a three dimensional sphere, and if we now integrate over this unit sphere we obtain the charge which is identified with the sphere of the intrinsic electric field. Application of Gauss's divergence theorem immediately explains how these charged spheres can appear to be point particles, this electromagnetic sphere behaves as a point charge even where there is no point charge, and in so doing this resolves two important problems in particle physics, firstly there is no infinite energy associated with a point particle as the charge is no longer condensed into a point with an infinite energy density (a major sticking point in the history of particle physics), and secondly the origin of charge, charge itself now appears as a simple expression of Maxwell's equations, indeed we can even go so far as to say that four dimensionally there is no charge only the potentials, however since we can measure the potentials only the charges we are stuck with charge as a fundamental property of matter. It all comes down to a frame of reference, the potentials are dependent on extrinsic frames of reference, but the charges must be in the local intrinsic frame of reference, so depending on whether we are considering the abstract potentials or measuring particle charges depends on whether we are measuring the wave or the particle.

We can reasonably expect the potentials to be perfectly spherical about the extrinsic charges, this requires the resultant charge particles are perfect spherical, in fact if aspherical particles are detected that would invalidate the present model. As a proof of this concept it must be pointed out that a ten year study of the shape of the electron at the Imperial's Centre for Cold Matter have demonstrated that the electron is for all practical requirements perfectly spherical, the following is a verbatim of the paper's abstract:

"*The electron is predicted to be slightly aspheric, with a distortion characterized by the electric dipole moment (EDM), de. No experiment has ever detected this deviation. The standard model of particle physics predicts that de is far too small to detect, being some eleven orders of magnitude smaller than the current experimental sensitivity. However, many extensions to the standard model naturally predict much larger values of de that should be detectable. This makes the search for the electron EDM a powerful way to search for new physics and constrain the possible extensions. In particular, the popular idea that new supersymmetric particles may exist at masses of a few hundred GeV/c2 (where c is the speed of light) is difficult to reconcile with the absence of an electron EDM at the present limit of sensitivity. The size of the EDM is also intimately related to the question of why the Universe has so little antimatter. If the reason is that some undiscovered particle interaction breaks the symmetry between matter and antimatter, this should result in a measurable EDM in most models of particle physics. Here we use cold polar molecules to measure the electron EDM at the highest level of precision reported so far, providing a constraint on any possible new interactions. We obtain de = (−2.4 ±5.7stat ±1.5syst) ×10−28e cm, where e is the charge on the electron, which sets a new upper limit of |de| < 10.5 × 10−28e cm with 90 per cent confidence. This result, consistent with zero, indicates that the electron is spherical at this improved level of precision. Our measurement of atto-electronvolt energy shifts in a molecule probes new physics at the tera-electronvolt energy scale.*" [Hudson, Kara, Smallman, Sauer, Tarbutt, Hinds 2011]

This idea of the elementary particles being perfect spheres harks back to Poincaré's original conception of particles as being hollow charged spheres that led to his extraordinary statement in his paper "La fin de la matière" that matter no longer exists:

"One of the most surprising discoveries made by the physicists in recent years, amounts in the claim that matter doesn't exist. We simultaneously add, that this discovery is not definitely established. The essential property of matter is its mass and its inertia. Mass remains constant everywhere and always, it even remains when a chemical transformation changes all observable properties of matter, and has apparently created an entirely new body. Thus if one is able to show, that matter carries mass like a foreign jewelry, that this mass (always considered as constant) can also suffer variations: then one surely has the right to say that there is no matter. But this is precisely what is announced." [Poincaré 1906]

Contrary to Poincaré, I am not suggesting matter does not exist, rather I'm suggesting matter is a transitory state of the electromagnetic four dimensional waveform, the problem lies in choosing a frame of reference. Is it a potential? Is it a wave? Is it a particle? Is it a sphere? That all depends on what mathematical framework is used to describe the motion of matter, a better description would be that it is all contingent on where you stand in the universe.

For now let's state as a general rule:

1 : The intrinsic charge of the particle in $\mathbb{R}^3$ is derived from the sum of the extrinsic retarded and advanced potentials in $\mathbb{R}^{1,3}$.

§2-2 Momentum

Carver Mead in his paper Collective Electrodynamics I [Mead 1997] discussed and showed how momentum can be derived from the magnetic vector potential for a closed loop, his method is paraphrased in the following:

"*Our model system is a loop of superconducting wire—the two ends of the loop being collocated[sic] in space and either insulated or shorted, depending on the experimental situation. Experimentally, the voltage V between the two ends of the loop is related to the current I flowing through the loop by*" [Mead 1997]

$$L\,I = \int V\,dt \tag{42}$$

Mead then shows after energy considerations of the current flow and inductance

$$\Phi = \int V\, dt = n\,\Phi_o \tag{43}$$

for a closed loop... "*The rate at which phase accumulates with distance l is the component of the propagation vector k W in the direction 3dl along the path. Thus, the total phase accumulated around the loop is...*" [Mead 1997]

$$\varphi = \int (\omega_1 - \omega_2)\, dt \tag{44}$$

"*We can understand quantization as an expression of the single-valued nature of the phase of the wave function. When the two ends of the loop were connected to an external circuit, the two phases could evolve independently. When the ends are connected to each other, however, the two phases must match up. But the phase is a quantity that has a cyclic nature—matching up means being equal modulo 2p. Thus, for a wave that is confined to a closed loop, and has a single-valued,continuous phase, the integral of Eq.44 must be n2π, where n is an integer.*" [Mead 1997]

This leads to the observation

$$\varphi = \int \omega\, dt = \frac{q_o}{\hbar} \int V\, dt = \frac{q_o}{\hbar}\, n\,\Phi_o = n\, 2\,\pi \tag{45}$$

and this in turn leads to what Mead calls "*the first set of fundamental relations of collective electrodynamics.*" [Mead 1997]

$$\left.\begin{array}{r} \text{phase}\ \varphi = \int \omega\, dt = \oint \mathbb{k} \cdot d\mathbb{l} \\ \text{flux}\ \Phi = \int V\, dt = \oint \mathbb{A} \cdot dt \end{array}\right\} \Phi = \frac{\hbar}{q_o}\varphi \tag{46}$$

If $\mathbb{a}$ is area of the loop then the magnetic flux Φ_m is given by,

$$\Phi_m = \mathbb{B} \cdot \mathbb{a} \tag{47}$$

then the magnetic field is given by

$$\mathbb{B} = \frac{d\,\Phi_m}{d\mathbb{a}} \tag{48}$$

substituting $\mathbb{B} = \nabla \times \mathbb{A}$

$$\frac{d\,\Phi_m}{d\mathbb{a}} = \nabla \times \mathbb{A} \tag{49}$$

where dΩ is the volume differential for current density $\mathbb{J}$ we have the magnetic vector potential

$$\mathbb{A} = \int \frac{\mathbb{J}}{r}\, d\Omega \tag{50}$$

By writing the de Broglie relationship for the momentum vector in terms of planck's constant and the wave propagation vector $\mathbb{k}$

$$\mathbb{p} = \hbar\, \mathbb{k} \tag{51}$$

allows us to express Newton's law from the magnetic vector potential or as Mead put's it - "*From a classical point of view, Newton's law tells us that the force $q_o\mathbb{E}$ on a charge should be equal to the time rate of change of momentum.*" [Mead 1997]

$$q_o \mathbb{E} = \frac{\partial \mathbb{p}}{\partial t} = q_o \frac{\partial \mathbb{A}}{\partial t} \tag{52}$$

Mead expresses this novel idea as "*We found Newton's law masquerading as one of Maxwell's equations.*" [Mead 1997]

Let's now apply Mead's idea to the Wheeler-Feynman summation, first writing

$$A_\mu^{\text{tot}} = \Sigma \frac{1}{2}\left(A_\mu^{\text{ret}} + A_\mu^{\text{adv}}\right) + \Sigma \frac{1}{2}\left(A_\mu^{\text{ret}} - A_\mu^{\text{adv}}\right) = A_\mu^{\text{ext}} \tag{53}$$

again only the φ lies on the temporal axis, so drop the extrinsic $\mathbb{A}$ for now and use only the extrinsic φ, after the summation we have the extrinsic potential for the wave equation,

$$\varphi_{\text{total}} = \sum\nolimits_n \frac{1}{2}\left(\varphi_{\text{ret}} + \varphi_{\text{adv}}\right) + \sum\nolimits_n \frac{1}{2}\left(\varphi_{\text{ret}} - \varphi_{\text{adv}}\right) = \varphi_{\text{ext}} \tag{54}$$

Since Maxwell's equations require a changing extrinsic electric potential gives rise to an intrinsic magnetic potential, we need to add a $\mathbb{A}_{\text{int}}$ in local frame of reference,

$$-\frac{1}{c}\frac{\partial \varphi_{\text{ext}}}{\partial t} = \nabla \cdot \mathbb{A}_{\text{int}} \tag{55}$$

Thus we are lead to the intrinsic magnetic vector potential $\mathbb{A}_{\text{int}}$

$$-\frac{1}{c}\frac{d\varphi_\mu^{\text{tot}}}{dt} = -\Sigma \frac{1}{2c}\left(\frac{\partial \varphi_{\text{ret}}}{\partial t} + \frac{\partial \varphi_{\text{adv}}}{\partial t}\right) - \Sigma \frac{1}{2c}\left(\frac{\partial \varphi_{\text{ret}}}{\partial t} - \frac{\partial \varphi_{\text{adv}}}{\partial t}\right) = \nabla \cdot \mathbb{A}_{\text{int}} \tag{56}$$

where

$$\mathbb{A}_{\text{int}} = -\frac{1}{c}\int \frac{\partial \varphi_{\text{ext}}}{\partial t}\, d\Omega \tag{57}$$

again we recover the original Wheeler-Feynman summation with

$$A_\mu^{\text{int}} = \left(\varphi_{\text{int}}, \mathbb{A}_{\text{int}}\right) \tag{58}$$

$$A_\mu^{\text{tot}} = \Sigma \frac{1}{2}\left(A_\mu^{\text{ret}} + A_\mu^{\text{adv}}\right) + \Sigma \frac{1}{2}\left(A_\mu^{\text{ret}} - A_\mu^{\text{adv}}\right) = A_\mu^{\text{int}} \tag{59}$$

This suggests that not only $\mathbb{A}_{\text{int}}$ is distributed over the surface of the sphere, and it is uniformly distributed over the entire sphere without a distinct pole. It's as if the spin of the particle is simultaneously pointing in every direction, but you will note the sphere is derived from the two four-spheres of the A_μ^{ret} and A_μ^{adv}, this means the four-vector potential double-covers the sphere of the particle in $\mathbb{R}^3$. By applying Mead's conception of momentum as the result of the magnetic potential, and considering any great circle on the electromagnetic sphere as being a Mead Loop, then irrespective of the axis of the area of the loop the current density $\mathbb{J}$ flows through that loop, and the following can be applied to the intrinsic magnetic potential,

$$\frac{\partial \mathbb{p}}{\partial t} = -q \frac{\partial \mathbb{A}_{\text{int}}}{\partial t} \tag{60}$$

therefore the acceleration of a charged particle is dependent on the rate of change of the intrinsic magnetic vector potential.

If we now express the Lorentz Force Law in potential form [cf. Griffiths 2017 pp 443]

$$\mathbb{F} = d(\mathbb{p} + q\,\mathbb{A})\,/\,dt = -q\left(\mathbb{E} + \frac{\mathbb{v}}{c} \times \mathbb{B}\right) = -q\left[-\nabla\varphi - \frac{\partial\,\mathbb{A}}{\partial\,t} + \frac{\mathbb{v}}{c} \times (\nabla \times \mathbb{A})\right] \tag{61}$$

excluding the static terms we find this can be reduced to the effects of the time varying of $\mathbb{A}$

$$\mathbb{F} = -q \frac{\partial \mathbb{A}_{\text{int}}}{\partial t} \tag{62}$$

on rearranging to solve for the acceleration $\mathbb{a}$

$$\mathbb{a} = -\frac{q}{m} \frac{\partial \mathbb{A}_{\text{int}}}{\partial t} = -\frac{q}{m} \frac{\partial \mathbb{A}_{\text{int}}}{\partial t} \tag{63}$$

we find the acceleration is given by a combination of the charge per unit mass and the rate of change of the $\mathbb{A}$, implying the inertia of the particle is dependent on its own magnetic vector potential. It’s as if when the particle is accelerating there is an associated *back emf* from the particles own intrinsic magnetic vector potential acting on the charge of the particle, but in reality the back emf comes from the extrinsic electric potentials via the Wheeler-Feynman summation. This back emf is a . If q is a constant then m must also be a constant and therefore the ratio q/m is also a constant. To paraphrase David Griffiths in his book on electrodynamics where he discusses the Lorentz force law in potential form “*... the parallel between equation*

$$\frac{d}{dt}(\mathbb{p} + q\,\mathbb{A}) = -\nabla[q(\varphi - \mathbb{v} \cdot \mathbb{A})] \tag{64}$$

and equation

$$\frac{d}{dt}(T + q\,V) = \frac{\partial}{\partial\,t}[q(\varphi - \mathbb{v} \cdot \mathbb{A})] \tag{65}$$

... invites us to interpret $\mathbb{A}$ *as a kind of ‘potential momentum’ per unit charge, just as V is potential energy per unit charge*.” [cf. Griffiths 2017 p444].

One can’t help but be drawn to the idea that charge and mass are two sides of the same coin, that the ratio of mass to charge is a fundamental relationship that quantifies mechanics, in effect mass becomes a constant of proportionality necessary to satisfy Maxwell’s equations. This is equivalent to the statement there is an intrinsic inertial response that is associated with the extrinsic potentials and this prevents the particle from running away on it's own, more explicitly the inertia is derived from a back EMF of the $\mathbb{A}$, this strongly suggests that the inertia is solely dependent on the electromagnetic forces pushing back on a charged particle, or the momentum of the particle is determined by its intrinsic magnetic vector potential - it's as if there is no momentum - only electromagnetic inertia, obviously there is momentum and again this is a statement about the mathematical frame reference rather than a statement about a physical property like charge or momentum.

In the generalized case where the canonical momentum carries both the mechanical momentum and the electromagnetic momentum

$$\mathbb{p}_{\text{can}} = \mathbb{p}_{\text{mech}} + q\,\mathbb{A} \tag{66}$$

the mechanical momentum is in the momentum of the frame of reference that carries the particle, i.e., when we observe an electron that is on an atom in one frame of reference we need to include the momentum of the atom with the electron, however, since the atom itself is made up of particles, we can always add up the magnetic vector potentials of the individual particles to give the total momentum

$$p_{\text{can}} = \sum p_{\text{mech}} = \sum_{i,j} q_i\,\mathbb{A}_j \tag{67}$$

therefore mechanical momentum is replaced with the configuration of charges and the magnetic vector potentials, in effect there is no mechanical momentum only electromagnetic momentum. Since we have derived the intrinsic magnetic vector potential from the Wheeler-Feynman summation of the electromagnetic four potential wave function, this allows us to state as a general rule,

2: Momentum is an intrinsic property of the electromagnetic four potential wave function.

§2-3 Phase

Starting from the summation

$$\varphi_{\text{ext}} = \sum_{i\neq j} \frac{1}{2}\left(\varphi_{\text{ret}(i)} + \varphi_{\text{adv}(j)}\right) + \sum_{i,j} \frac{1}{2}\left(\varphi_{\text{ret}(i)} - \varphi_{\text{adv}(j)}\right) \tag{68}$$

we can determine the phase of the electromagnetic wave by first writing the energy E as

$$E = q\,(\varphi,\,0) = q\,\varphi_{\text{ext}} \tag{69}$$

if we consider the potentials that are greater or lesser than the intrinsic potential we find

$$q\,\varphi_3^{\text{adv}} < q\,\varphi_2^{\text{adv}} < q\,\varphi_1^{\text{adv}} < q\,\varphi_{\text{ext}} < q\,\varphi_1^{\text{ret}} < q\,\varphi_2^{\text{ret}} < q\,\varphi_3^{\text{ret}} \tag{70}$$

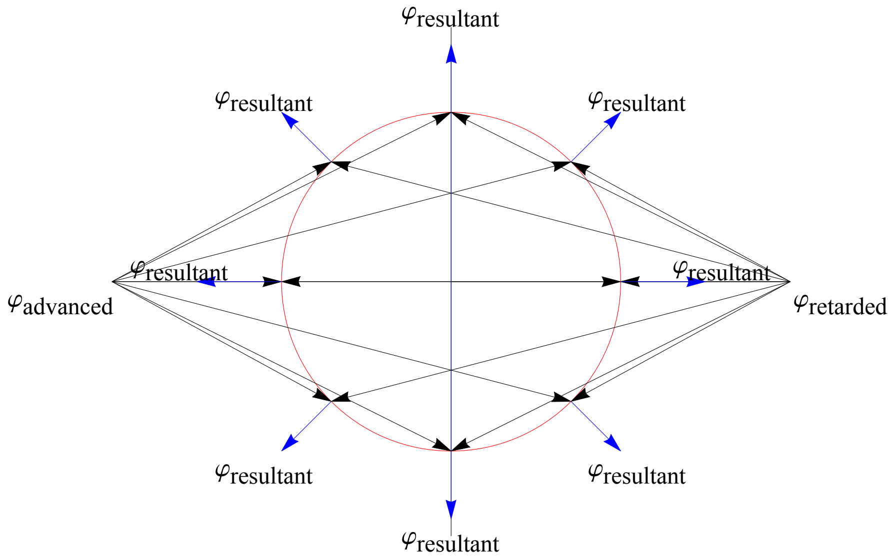
This implies there is one and only one set of extrinsic potentials whose sum equals the intrinsic potential, all others are either too large or too small to satisfy the equality, therefore there is a particular set of potentials that is identically symmetric about the intrinsic charge, suggesting only the following is applicable, where $q\,\varphi_i^{\text{ret}*}$ is the conjugate of $q\,\varphi_i^{\text{ret}}$,

$$\left|q\,\varphi_i^{\text{adv}}\right| = \left|q\,\varphi_i^{\text{ret}*}\right| \tag{71}$$

converting this to the potentials of the advanced and retarded potentials

$$\left|\varphi_i^{\text{adv}}\right| = \left|\varphi_i^{\text{ret}*}\right| \tag{72}$$

Since the charge is the intrinsic charge of the local particle, if the field lines are not radially symmetric then the charged particle is continuously accelerated by its own extrinsic potentials, and this in turn violates conservation of energy, therefore the resultant potential (the intrinsic potential) is radially directed from the loci of the equation of motion (again this following idealized diagram does not capture the intersection of two four dimensional hyperspheres), and the advanced and retarded potentials intercept obliquely on the surface of the intermediate particle in so doing forming that particle, to conserve energy this angle (as discussed above) must be at 45° between axis of movement of the particle and the extrinsic potentials.



(73)

Therefore we can say if the resultant potential is not radially directed from the origin then that would violate Conservation of Energy, so we can write in the particle's frame of reference

$$E_{adv} = E_{ret} \tag{74}$$

All of this implies there is a *unique phase for the world line of each particle that is dependent on the energy of the wave function*, we can use this to write,

$$\Psi_{temporal} = \exp\left[-\frac{i}{\hbar}\int E\,dt\right] = \exp\left[-\frac{i\,q}{\hbar}\int \varphi_{ext}\,\mathrm{dt}\right] \tag{75}$$

The model, however, is unbalanced as it does not consider the phase of the momentum, to quantize the momentum consider the definition of electromagnetic momentum as

$$q\,\mathbb{A} = \mathbb{p} \tag{76}$$

if r is the distance between potentials leads to

$$\int q\,\mathbb{A}\,\mathrm{dx} = \mathbb{p}\;r \tag{77}$$

in generalized coordinates

$$\int p_\mu\,d\,x^\mu = \int q\,A_\mu\,d\,x^\mu = q\int \varphi\,d\,t + q\int \mathbb{A}\,d\,x \tag{78}$$

to introduce the quantization of the wave function where the end points of the standing wave are the sources of the advanced and retarded potentials, write

$$2\,\pi\int q\,A_\mu\,d\,x^\mu = n\,h \tag{79}$$

$$\text{phase} = \frac{2\pi}{\hbar}\int A_\mu\, d x^\mu \tag{80}$$

note the similarity of this to the model of Bohr-Sommerfeld atom where there is an integral number of wavelengths along the path this will be returned to in the section Spin,

$$\oint p_\mu\, d x^\mu = n_i\, h \tag{81}$$

now we can write the spatial component of the wave function as

$$\Psi_{\text{spatial}} = \exp\left[-\frac{i\,q}{\hbar}\int \mathbb{A}_{\text{int}}\,\text{dx}\right] \tag{82}$$

combining this with the temporal definition of a wave function, yields

$$\Psi_{\text{temporal}}\,\Psi_{\text{spatial}} = \exp\left[-\frac{i\,q}{\hbar}\int \varphi_{\text{ext}}\,\text{dt}\right]\cdot\exp\left[-\frac{i\,q}{\hbar}\int \mathbb{A}_{\text{int}}\,\text{dx}\right] \tag{83}$$

leads to the quantization of the four vector potential wave function

$$\psi_{\text{tot}} = \Psi_{\text{temporal}}\,\Psi_{\text{spatial}} = -\frac{i\,q}{\hbar}\int A_\mu\, d x^\mu \tag{84}$$

(note the similarity to the total wave function of the Wheeler-Feynman model)

Thus we can say as a general rule,

3: The total phase of charged leptons is determined by the Wheeler-Feynman summation of their extrinsic potentials.

§2-4 Electromagnetic Mass

J.J. Thomson [Thomson 1881], Oliver Heaviside [Heaviside 1889], George Searle [Searle 1897], Wilhelm Wien [Wien 1900], Max Abraham [Abraham 1902], Hendrik Lorentz [Lorentz (1892,1904)], et al, developed the charged spherical shell model for the electron, and concluded the electromagnetic energy–mass relation for a static charge was

$$m_{\text{electrostatic}} = \frac{E_{\text{electrostatic}}}{c^2} \tag{85}$$

yet this contradicted when the mass is derived from the Lorentz-Abraham equations yielding

$$m_{\text{electromagnetic}} = \frac{4}{3}\left(E_{\text{electromagnetic}}\big/ c^2\right) \tag{86}$$

Clearly the 4/3 factor was a problem, to solve this Poincaré [Poincaré 1905] suggested the addition of internal stresses to give an extra $\frac{1}{3}$ Electromagnetic term.

$$\frac{E_{\text{total}}}{c^2} = \left(E_{\text{electrostatic}} + \frac{1}{3} E_{\text{Poincaré stresses}}\right) \Big/ c^2 = \frac{4}{3}\left(E_{\text{electromagnetic}} \big/ c^2\right) = \frac{4}{3} m_{\text{electrostatic}} = m_{\text{electromagnetic}} \tag{87}$$

I'm going to suggest these Poincaré stresses can be achieved by considering that a spinning electron also carries rotational field energy along with its electrical field energy and the source of this rotational energy lies in the magnetic vector potential. To demonstrate this, first write the magnetic vector potential in terms of the Wheeler-Feynman model, where in $\mathbb{R}^3$ the φ_{ext} is the intrinsic electric potential and $\mathbb{A}_{\text{int}}$ is the intrinsic magnetic vector potential

$$\nabla \cdot \mathbb{A}_{\text{int}} = -\frac{\partial}{\partial t}\varphi_{\text{ext}} = \frac{1}{2}\sum\left(-\frac{\partial}{\partial t}\varphi_{\text{ret}} - \frac{\partial}{\partial t}\varphi_{\text{adv}}\right) + \left(-\frac{\partial}{\partial t}\varphi_{\text{ret}} + \frac{\partial}{\partial t}\varphi_{\text{adv}}\right) \tag{88}$$

again the $\mathbb{A}_{\text{int}}$ and φ_{ext} can be used to write the action S of the wave function

$$S = q\int A_\mu \, dx^\mu = q\int \varphi_{\text{ext}} \, dt + q\int \mathbb{A}_{\text{int}} \, d\mathbb{x} \tag{89}$$

This allows us to express the total action in $\mathbb{R}^3$ in terms of energy and angular momentum

$$S = \int E \, dt + \int \mathbb{p} \cdot d\mathbb{x} \tag{90}$$

The first term can be used to derive the electrical field energy that gives rise to the $E_{\text{electrostatic}}$ field energy which in turn yields the electrostatic mass. The second term can be used to derive the rotational field energy from the angular momentum, this requires determining momentum from a mechanism involving $q\mathbb{A}$ as suggested by Mead’s work in the previous section on momentum.

If we now equate the mechanical angular momentum to the intrinsic electromagnetic spin we get,

$$I\,\omega = q\int_a^\infty \mathbb{A}_{\text{int}} \cdot d\mathbb{x} \tag{91}$$

this is the important step, for as the vector potential varies so does the angular momentum, by applying this to the momentum of inertia for a spherical shell with radius a

$$I_{\text{sphere}} = \frac{2}{3} m\, a^2 \tag{92}$$

we arrive at the rotational kinetic energy

$$E_{\text{rotational}} = \frac{1}{2} I_{\text{sphere}}\, \omega^2 \tag{93}$$

and if substitute the moment of inertia

$$E_{\text{rotational}} = \frac{1}{3} m\, a^2\, \omega^2 \tag{94}$$

then the angular velocity ω and the radius disappears

$$E_{\text{rotational}} = \frac{1}{3} m a^2 \frac{v^2}{a^2} \tag{95}$$

For an electromagnetic wave the velocity v is at c, yielding

$$E_{\text{rotational}} = \frac{1}{3} m_o c^2 \tag{96}$$

this equates the energy of the rotational component to the energy of the magnetic component

$$E_{\text{rotational}} \equiv E_{\text{magnetic}} \tag{97}$$

Now we can substitute result in place of the Poincaré stresses to obtain a balanced mass equation and show the masses are entirely electromagnetic in origin (at least for charged leptons),

$$\frac{E_{\text{total}}}{c^2} = \left(E_{\text{electric}} + \frac{1}{3} E_{\text{magnetic}}\right) \Big/ c^2 = \left(E_{\text{electrostatic}} + \frac{1}{3} E_{\text{Poincaré stresses}}\right) \Big/ c^2 = \frac{4}{3}\left(E_{\text{electromagnetic}} \big/ c^2\right) = m_{\text{electromagnetic}} \tag{98}$$

This equation balances if we assume the electron is an electromagnetic waves travelling at the speed of light, a straight out derivation of spin from an electromagnetic potential is given in the next section. For now we are lead to the conclusion that electromagnetic mass and spin are innately connected, and state as a general rule,

4: *The electromagnetic mass for charged leptons is implicit in the Wheeler-Feynman summation.*

(As a footnote it is a common misconception that the Higgs Mechanism generates all particle masses, in fact the Higgs Mechanism refers to the masses of the W^+ and $Z^\pm$ gauge bosons, the model presented here, however, does not touch on the structure of the gauge bosons so it does not contradict accepted particle physics.)

§2-5 Spin

We can now use the previous derivations to model quantum Spin as another property of the Wheeler-Feynman summation, and we can do this by following almost identical steps to Bohr's method for modelling the Rutherford–Bohr atom. First we will accept (for now) Cramer's premise of the Transaction Interpretation that there is a standing wave between the advanced and retarded sources of the Wheeler-Feynman summation. This standing wave can be used to show that in the same way Bohr noted there was a standing wave for an electron in orbit around the Rutherford atom with an integral number of wavelengths, there must also be a standing wave with an integral number of wavelengths between the sources of the advanced and retarded potentials.

If L is the distance between the advanced and retarded sources then a round trip of the particle (which is a necessary condition for a standing wave) is $2L$, and the distance across one wavelength is twice the radius a of the particle, or

$$2L = 4\pi a \tag{99}$$

this simplifies to Bohr's first condition

$$L = 2\pi a \tag{100}$$

the number of wavelengths is equal to *L,* Bohr’s second condition

$$n\lambda = 2\pi a \tag{101}$$

using de Broglie’s relationship λ p = h

$$n\frac{h}{p} = 2\pi a \tag{102}$$

$$p\,a = \hbar\,n \tag{103}$$

integrating to *a* gives angular momentum L

$$\int p\,dx = p\,a \tag{104}$$

For a round trip of a standing wave we have a closed curve integral and once again note the similarity of this to the model of Bohr-Sommerfeld atom,

$$\hbar\,n = \oint p\,dx \tag{105}$$

as was pointed out by Mead the momentum can be given in terms of the magnetic vector potential and the charge

$$p = q\,\mathbb{A}_{\text{int}} \tag{106}$$

this allows us to rewrite the integral as

$$\oint p\,dx = = q\oint \mathbb{A}_{\text{int}}\,d\mathbb{x} \tag{107}$$

finally allows us to recognize the spin *S*

$$S = \hbar\,n = q\oint \mathbb{A}_{\text{int}}\,d\mathbb{x} \tag{108}$$

Of course all this hinges on the assumption that not is that momentum can be given in terms of the magnetic vector potential but also that an electron spins at the speed of light as discussed in the previous section, and maybe this is where the model falls over, for an electron cannot spin with the speed of light without violating special relativity. To get around this we need to note the peculiar behavior of the four dimensional wave where the two hyperspheres of the advanced and retarded waves intersect in $\mathbb{R}^3$ to form a standing wave on the spherical shell. The magnetic field $\mathbb{A}$ appears the same at every point on the surface, in effect there is an identical magnetic pole appearing at every point on the surface, which in turn means the electron is spinning on an axis at every point, in effect every point is identical and appears to be moving with velocity c. This may seem a bizarre idea but then again we are talking about the double cover of two $\mathbb{R}^{1,3}$ spheres intersecting in $\mathbb{R}^3$. For now we can state as a general rule

5) the Spin of a charged elementary particle can be given by its charge and intrinsic magnetic vector potential.

§2-5 Action

From a consideration of Hamilton's Principle for action it will be shown there is generalized formulation of the Wheeler-Feynman model for wave functions. Starting with the definition Hamilton's Principle for least action, for action S where η(t) are the generalized coordinates

$$\frac{\delta S_{\text{tot}}}{\delta\eta(t)} = 0 \tag{109}$$

we can separate S into the advanced S_{adv} and retarded S_{ret} actions with an interference term ω(t)

$$S_{\text{tot}} = S_{\text{ret}} + S_{\text{adv}} + \omega(t) \tag{110}$$

Hamilton's Principle now becomes

$$\frac{\delta S_{\text{tot}}}{\delta\,\eta(t)} = \sum\left((\delta\,S_{\text{ret}})/\delta\eta(t) + \left(\delta\,S_{\text{adv}}\right)/\,\delta\eta(t) + (\delta\,\omega(t))/\delta\eta(t)\right) = 0 \tag{111}$$

Writing the half-wave function ψ in terms of the action S requires the introduction of the $\frac{1}{2}$ factor into the exponent

$$\psi = \exp\left[-\frac{i\,q}{2\,\hbar}\int A_{\mu}\,dx^{\mu}\right] = \exp\left[-\frac{i}{2\,\hbar}\int S\,dt + \omega(t)\right] \tag{112}$$

or

$$\psi_{(\text{tot})}(x,\,t) = \exp\left[-\frac{i}{2\,\hbar}\int\left(S_{\text{ret}} + S_{\text{adv}}\right)dt + \omega(t)\right] \tag{113}$$

Hamilton's Principle for the action S with generalized coordinates η(t) requires the ω(t) is zero

$$\frac{\delta S_{\text{tot}}}{\delta\eta(t)} = 0 \Rightarrow \omega(t) = 0 \tag{114}$$

for a plane wave make the assumption the interference term ω(t) is the difference of the free terms of advanced and retarded actions

$$\omega(t) = S_{\text{ret}} - S_{\text{adv}} = 0 \tag{115}$$

in effect the free terms set the phase difference for the total terms. Allowing ω(t) to be written as

$$\frac{\delta\,\omega(t)}{\delta\eta(t)} = \frac{\delta\,S_{\text{ret}}}{\delta\eta(t)} - \left(\delta\,S_{\text{adv}}\right)/\,\delta\eta(t) = 0 \tag{116}$$

now introduce this ω(t) to the Wheeler-Feynman summation

$$\frac{\delta\,S_{\text{tot}}}{\delta\eta(t)} = \frac{1}{2}\sum\left((\delta\,S_{\text{ret}})/\,\delta\eta(t) + \left(\delta\,S_{\text{adv}}\right)/\,\delta\eta(t)\right) + \left((\delta\,S_{\text{ret}})/\,\delta\eta(t) - \left(\delta\,S_{\text{adv}}\right)/\,\delta\eta(t)\right) = (\delta\,S_{\text{ret}})/\,\delta\eta(t) = 0 \tag{117}$$

extract the intrinsic total action

$$S_{\text{tot}} = \frac{1}{2}\sum\left(S_{\text{ret}} + S_{\text{adv}}\right) + \left(S_{\text{ret}} - S_{\text{adv}}\right) = S_{\text{int}} \tag{118}$$

or

$$\frac{i\,q}{\hbar} A_\mu^{\text{tot}} = \frac{i\,q}{2\,\hbar} \sum \left(A_\mu^{\text{ret}} + A_\mu^{\text{adv}}\right) + \left(A_\mu^{\text{ret}} - A_\mu^{\text{adv}}\right) = \frac{i\,q}{\hbar} A_\mu^{\text{int}} \tag{119}$$

and this allows us to transform to exponents and express the Wheeler-Feynman model as a wave functionally dependent on the four-vector potential

$$\psi_{\text{tot}}(x,\,t) = \exp\left[-\frac{i\,q}{2\,\hbar} \int \left(A_\mu^{\text{ret}} + A_\mu^{\text{adv}}\right) + \left(A_\mu^{\text{ret}} - A_\mu^{\text{adv}}\right) dt\right] = \Psi_{\text{int}}(x,\,t) \tag{120}$$

This result can be used to simplify the formulation for wave functions, first express the total wave function from the action of the half-wave four-vector potentials, half-wave functions are represented with the capital ψ, and full wave functions with the lower case Ψ

$$\psi_{\text{tot}}(x,\,t) = \exp\left[-\frac{i\,q}{2\,\hbar} \int \left(A_\mu^{\text{ret}} + A_\mu^{\text{adv}}\right) + \left(A_\mu^{\text{ret}} - A_\mu^{\text{adv}}\right) dt\right] = \Psi_{\text{int}}(x,\,t) \tag{121}$$

in this case the resultant intrinsic wave function is derived from the extrinsic action, again this reduces to the scalar potential projected over the time axis

$$\varphi^{\text{tot}} = \sum_n \frac{1}{2}\left(\varphi_n^{\text{ret}} + \varphi_n^{\text{adv}}\right) + \sum_n \frac{1}{2}\left(\varphi_n^{\text{ret}} - \varphi_n^{\text{adv}}\right) = \sum_n \varphi_n^{\text{ext}} \tag{122}$$

this allows us to construct the Wheeler-Feynman model as a summation of the half wave functions from the extrinsic scalar potentials

$$\psi_{\text{tot}}\,\psi_{\text{free}} = \exp\left[-\frac{i\,q}{2\,\hbar} \int \varphi^{\text{ret}} + \varphi^{\text{adv}} + \varphi^{\text{ret}} - \varphi^{\text{adv}}\, dt\right] = \exp\left[-\frac{i\,q}{\hbar} \int \varphi^{\text{ext}}\, dt\right] \tag{123}$$

In flat space where the sources charges are sufficiently close (on the order of the Compton wavelength) the magnitude of the advanced electric potentials are equal and opposite to the retarded electric potentials and because of temporal charge reversal between matter and antimatter we can write

$$\varphi^{\text{adv}} = -\varphi^{\text{ret}} \tag{124}$$

$$\left|\varphi^{\text{adv}}\right| = \left|\varphi^{\text{ret}}\right| \tag{125}$$

Remembering that according to the Feynman-Stueckelberg interpretation the positron is the negative energy mode of the electron field moving backward in time, we can write this simply as the advanced wave function being the complex conjugate of the retarded wave function, and identify the retarded wave function with matter and the advanced wave function with antimatter. This allows us to express the WFEMFPTST in terms of half wave functions $\psi_{\text{ret,adv}}$

$$\psi_{\text{ret}}(t) = \exp\left[\pm \frac{i\,q}{2\,\hbar} \int \varphi^{\text{ret}}\, dt\right] \tag{126}$$

with complex conjugate

$$\psi_{\text{adv}}^{*}(t) = \exp\left[\mp \frac{i\,q}{2\,\hbar} \int \varphi^{\text{adv}}\, dt\right] \tag{127}$$

This allows us to write the total wave function ψ_{tot} is

$$\psi_{tot} = \psi_{ret} \cdot \psi_{adv} \quad (128)$$

and the free wave function ψ_{free} is

$$\psi_{free} = \psi_{ret} \cdot \psi^*_{adv} \quad (129)$$

since the complex conjugates of the advanced and retarded wave functions take the convenient form of being conjugates of each other

$$\psi_{ret} = \psi^*_{adv} \quad (130)$$

$$\psi_{adv} = \psi^*_{ret} \quad (131)$$

and the magnitudes are equal

$$|\psi_{ret}| = |\psi^*_{adv}| \quad (132)$$

this allows us to write the free wave function as equal to unity

$$\psi_{free} = \psi_{ret} \cdot \psi^*_{adv} = I \quad (133)$$

combing the free terms with the total terms, the WFEMFPTST can now be written in the much simpler wave function form

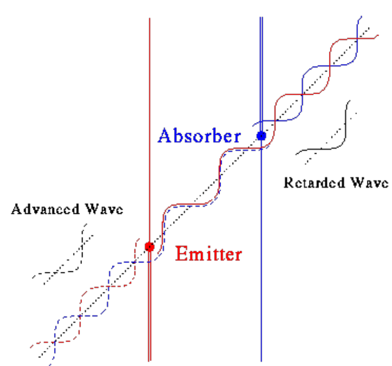
$$\psi_{total}\,\psi_{free} = \psi_{ret} \cdot \psi_{adv} \cdot \psi_{ret} \cdot \psi^*_{adv} = \psi_{ret} \cdot \psi^*_{ret} \cdot \psi_{ret} \cdot \psi^{**}_{ret} \quad (134)$$

on rearranging the advanced terms vanish, the conjugated phase cancel, and we are left with the Wheeler-Feynman model formulated as wave functions,

$$6)\ \psi_{total}\,\psi_{free} = \psi_{ret} \cdot \psi^*_{ret} \cdot \psi_{ret} \cdot \psi_{ret} = \psi_{ret} \cdot \psi_{ret} = \Psi_{total} \quad (135)$$

§2-7 The Quantum Handshake

In Cramer's work on the transactional interpretation of the Wheeler-Feynman model [Cramer 1986], a two dimensional representation of which he has kindly allowed me to reproduce here,



he posited the idea that there should exist a standing wave between the advanced and retarded charges which he labelled the Quantum Handshake, *"The basic element of the transactional interpretation is an emitter-absorber transaction through the* *exchange of advanced and retarded waves, as first described by Wheeler and Feynman (1945, 1949) [see also (Feynman, 1967b)]. Advanced waves are solutions of the electromagnetic wave equation and other similar wave equations which contain only the second time derivative. Advanced waves have characteristic eigenvalues of negative energy and frequency, and they propagate in the negative time direction."*

I want to now express this explicitly as a mathematical model in terms of the Wheeler–Feynman time-symmetric theory by including the initial phase or epoch angles ε of both the advanced and retarded half wave functions

$$\psi_{\text{ret}} = \exp\left[-\frac{i\,q}{2\,\hbar}\int A_{\mu}^{\text{ret}}\,dx^{\mu} + \varepsilon_{\text{ret}}\right] \tag{136}$$

$$\psi_{\text{adv}} = \exp\left[-\frac{i\,q}{2\,\hbar}\int A_{\mu}^{\text{adv}}\,dx^{\mu} + \varepsilon_{\text{adv}}\right] \tag{137}$$

$$\psi_{\text{ret}}\cdot\psi_{\text{adv}} = \exp\left[-\frac{i\,q}{2\,\hbar}\int A_{\mu}^{\text{ret}}\,dx^{\mu} + \varepsilon_{\text{ret}} - \frac{i\,q}{2\,\hbar}\int A_{\mu}^{\text{adv}}\,dx^{\mu} + \varepsilon_{\text{adv}}\right] \tag{138}$$

As shown above (§ 2-3) the conservation of energy and momentum requires the advanced wave function must be in phase with the retarded wave function, this means we require a zero phase difference

$$\varepsilon_{\text{ret}} + \varepsilon_{\text{adv}} = 0 \tag{139}$$

i.e.,

$$\varepsilon_{\text{ret}} = -\varepsilon_{\text{adv}} \tag{140}$$

since the epoch angles are directly dependent from the environment of the retarded and advanced potentials, and since the environment *is* the free terms, we can without loss of generality substitute the epoch angles with the retarded and advanced potentials of the free terms

$$\varepsilon_{\text{ret}} = -\frac{i\,q}{2\,\hbar}\int A_{\mu}^{\text{ret}}\,dx^{\mu} \tag{141}$$

$$-\varepsilon_{\text{adv}} = \frac{i\,q}{2\,\hbar}\int A_{\mu}^{\text{adv}}\,dx^{\mu} \tag{142}$$

writing in full we see this completely expresses the Wheeler-Feynman model in wave function formulation

$$\psi_{\text{total}}\,\psi_{\text{free}} = \exp\left[-\frac{i\,q}{2\,\hbar}\int A_{\mu}^{\text{ret}}\,dx^{\mu} - \frac{i\,q}{2\,\hbar}\int A_{\mu}^{\text{adv}}\,dx^{\mu} - \frac{i\,q}{2\,\hbar}\int A_{\mu}^{\text{ret}}\,dx^{\mu} + \frac{i\,q}{2\,\hbar}\int A_{\mu}^{\text{adv}}\,dx^{\mu}\right] \tag{143}$$

once again the advanced terms cancel out and we are left with the intrinsic four vector potential form of the wave function

$$\psi_{\text{total}}\,\psi_{\text{free}} = \exp\left[-\frac{i\,q}{2\,\hbar}\int A_{\mu}^{\text{ret}}\,dx^{\mu} - \frac{i\,q}{2\,\hbar}\int A_{\mu}^{\text{ret}}\,dx^{\mu}\right] = \exp\left[-\frac{i\,q}{\hbar}\int A_{\mu}^{\text{int}}\,dx^{\mu}\right] = \Psi_{\text{int}} \tag{144}$$

So simply by replacing the epoch angles with the free terms we can construct the Wheeler-Feynman model as a quantized wave function model from first principles, in other words the Wheeler-Feynman interaction is only possible if and only if the advanced and retarded potentials are identically in phase, and this is a direct mathematical statement of Cramer’s Quantum Handshake.

We can also express the above statement in the more convenient half-wave function formalism, by substituting the epoch angles with free terms

$$\psi_{\text{total}}\,\psi_{\text{free}} = \psi_r \cdot \psi_a \cdot e^{\varepsilon_{\text{ret}}} \cdot e^{\varepsilon_{\text{adv}}} = \psi_r \cdot \psi_a \cdot \psi_r \cdot \psi^*{}_a \tag{145}$$

for flat space and holding the potentials in close proximity, substitute the conjugates of the advanced half wave functions and cancelling, results in the most elegant expression of the Quantum Handshake,

$$\psi_{\text{total}}\,\psi_{\text{free}} = \psi_r \cdot \psi^*{}_r \cdot \psi_r \cdot \psi^{**}{}_r = \psi_r \cdot \psi_r = \Psi_{\text{int}} \tag{146}$$

The resulting final expression Ψ_{int} is a standing quantum wave in time and space that transfers energy and momentum between the advanced and retarded potentials, or more simply Ψ_{int} is the Quantum Handshake.

From this section we see there is a unique phase that determines the evolution of the four potential wave function, this follows from the observation that if the resultant φ is not radially directed away from the origin then energy is not conserved, for if the potential lines are not radially directed from the origin of the intrinsic particle then any interaction of the charged particle with the extrinsic potentials would be asymmetric and depending on the orientation of the particle would give a different solution to Gauss' law, and equally conservation of momentum is violated if the magnetic vector potential is not radially symmetric about the origin. It is shown that the free terms of the Wheeler-Feynman model are equivalent to the initial angles of the total wave functions of the advanced and retarded wave forms.

Mathematically this leads to a peculiar idea, it is not that a wave function collapses rather it is the collapse of the potentials in free spacetime as they sum to zero that allows the Quantum Handshake to take place, which is like saying it is not the particle that collapses it is spacetime that collapses around the particle, and this is a very peculiar idea. If, however, we can accept this peculiarity as a mathematical necessity, then the method presented here gives an explicit statement of what Cramer's Quantum Handshake, which was a problem that Cramer noted in his book [Cramer 2016]: "Some critics of the Transaction Interpretation have asked why it does not provide a detailed mathematical description of transaction formation." Clearly the Quantum Handshake is expressed directly from a formulation of conservation of energy and momentum in the wave function phases, formally,

$$\text{7) The Quantum Handshake is the Wheeler} - \text{Feynman Summation.} \tag{147}$$

§3 How to make a Quark

The following sections are an attempt to derive the masses of the leptons in a manner that is consistent with the Wheeler-Feynman summation. Before reading this section it is recommended if you knew of Robert Foot's paper [Foot 1994] and Yoshio Koide's paper [Koide 1983] on lepton masses.

§3-1 Foot's Footnote

To derive Foot's formula for the masses of charged leptons it will be necessary to first derive a vector space over momentum and kinetic energy, using an inner product between the scalar φ_{ext} and the vector $\mathbb{A}_{\text{int}}$. To do this first write the Lagrangian

$$\mathcal{L} = T - V \tag{148}$$

where $\mathcal{L}$ equals zero in the inertial frame of the particle,

$$T = V \tag{149}$$

Equate the extrinsic potential to the kinetic energy of the particle,

$$q\,\varphi_{\text{ext}} = \frac{1}{2} m_o\, v^2 \tag{150}$$

or

$$E_{\text{ke}} = \frac{1}{2} m_o\, v^2 \tag{151}$$

this can be expressed as a dot product of the velocity

$$E_{\text{ke}(i)} = \sum_i \frac{1}{2} m_i\, \mathbb{v} \cdot \mathbb{v} \tag{152}$$

We can now redefine the kinetic energy as a vector space by transforming it to its square root,

$$E_{\text{ke}_i}{}^{1/2} = \sum_i \sqrt{\left(\frac{1}{2} m_i\, \mathbb{v} \cdot \mathbb{v}\right)} = \sum_i \sqrt{\left(\frac{m_i}{2}\right)}\, \mathbb{v}_i \tag{153}$$

where the sum of the $\mathbb{v}_i$ yields a vector, or

$$E_{\text{ke}_i}{}^{1/2} = \sqrt{\left(\frac{m_x}{2}\right)}\, \mathbb{v}_x + \sqrt{\left(\frac{m_y}{2}\right)}\, \mathbb{v}_y + \sqrt{\left(\frac{m_z}{2}\right)}\, \mathbb{v}_z = \sum_i \sqrt{\left(\frac{m_i}{2}\right)}\, \mathbb{v}_i \tag{154}$$

this allows us to form the inner product of an energy-momentum vector space of the $E_{\text{ke}}{}^{1/2}$ vector with the momentum $\mathbb{p}$ vector to give the cosine,

$$\text{Cos}\,\theta = \frac{E_{\text{ke}}{}^{1/2}}{\left|E_{\text{ke}}{}^{1/2}\right|} \cdot \frac{\mathbb{p}}{|\mathbb{p}|} \tag{155}$$

$$\text{Cos}\,\theta = \left(\sum_i \sqrt{(2\, m_i)}\, \mathbb{v}_i\right) \Big/ \left(\sum_i \left|\sqrt{(2\, m_i)}\, \mathbb{v}_i\right|\right) \cdot \frac{m_o\, \mathbb{v}}{|m_o\, \mathbb{v}|} \tag{156}$$

where

$$\frac{m_o\, \mathbb{v}}{|m_o\, \mathbb{v}|} = (m_o\, \mathbb{v},\, m_o\, \mathbb{v},\, m_o\, \mathbb{v}) / |(m_o\, \mathbb{v},\, m_o\, \mathbb{v},\, m_o\, \mathbb{v})| \tag{157}$$

this is equivalent to taking the inner product of momentum with the time projected electric potential, and if the solutions for the particles have a common velocity, such that

$$\mathbb{v}_i = \mathbb{v} \tag{158}$$

then the velocities in the numerator and the denominator cancel out, along with the $\sqrt{2}$, leaving

$$\text{Cos}\,\theta = \left(\sum_i \sqrt{m_i}\right) / \left(\sum_i \left|\sqrt{m_i}\right|\right) \cdot m_{o,i} / \left|m_{o,i}\right| \tag{159}$$

the m_O also cancels out leaving

$$\text{Cos}\,\theta = \left(\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3}\right) / \sqrt{(m_1 + m_2 + m_3)} \cdot (1, 1, 1)/|(1, 1, 1)| \tag{160}$$

if we take for the masses the masses of the electron, muon or tauon

$$m_i = m_e \text{ or } m_\mu \text{ or } m_\tau \tag{161}$$

and since

$$(1, 1, 1)/|(1, 1, 1)| = \frac{1}{\sqrt{3}} \tag{162}$$

we have Foot’s general equation for the masses of the leptons

$$\text{Cos}\,\theta = \left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right) / \sqrt{(m_e + m_\mu + m_\tau)} \cdot 1 / \left(\sqrt{3}\right) \tag{163}$$

Therefore we can say a general equation for the lepton masses can be derived from the Wheeler-Feynman summation of the four vector potential.

§3-2 Koide’s formula

Foot makes the comment: “*Theoretically the only constraint on θ (assuming that at least one lepton has a non-zero mass) is that* cos $\theta > \frac{1}{\sqrt{3}}$ *(which means that θ < 54.7 degrees). Putting in the measured lepton mass values, then*

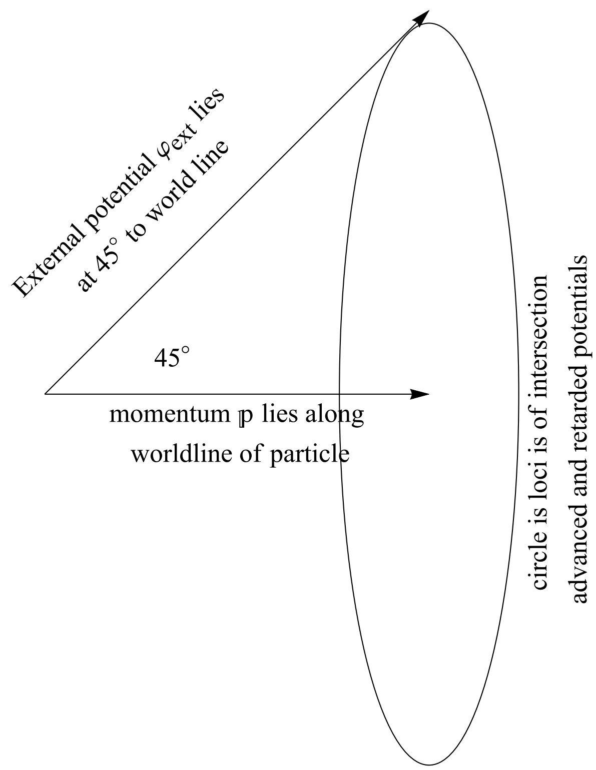
$$\theta = 45^\circ \pm 0.001 \tag{164}$$

Note that if we replaced the 2/3 factor in Koide’s formula with another factor the θ is nothing special (e.g. if we replaced the 2/3 with 3/4 or 1/2 then this corresponds with θ = 48.189...and 35.264...respectively). If Koide’s formula has something to do with the real world, then maybe this “geometrical formulation” of the mass relation may be useful in order to derive the formula from some underlying theory.” [Foot 1994]

In the most general expression of the four-vector potential A_μ assume the intrinsic momentum is derived from the magnetic vector potential $q\,\mathbb{A}$, and the energy (read rest energy) is derived from the extrinsic electric potential $q\,\varphi$, where the φ_{ext} and $\mathbb{A}_{\text{int}}$ are at arbitrary angles to each other.

$$p^\mu = q\,A^\mu = q\,(\varphi, \mathbb{A}) = \left(\frac{E}{c}, \mathbb{p}\right) \tag{165}$$

In this paper it has been shown this angle between the axis of momentum and electric potential is determined by reasons of conservation of energy to be 45°, therefore taking



taking Foot’s equation

$$\text{Cos}\,45^\circ = \left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)\Big/\sqrt{\left(m_e + m_\mu + m_\tau\right)} \cdot 1\Big/\left(\sqrt{3}\right) = 1\Big/\left(\sqrt{2}\right) \tag{167}$$

we can rearrange this to express Koide’s equation for lepton masses[Koide 1983],

$$\left(m_e + m_\mu + m_\tau\right)\Big/\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2 = \frac{2}{3} \tag{168}$$

§3-3 Brennan's neutrino rule

As was pointed out by Carl Brennan[Brennan 2006] Foot’s model can also be applied to neutrinos masses if negative velocities are included and letting the magnitude of the individual neutrino velocities (η) equal the total velocity, we can do this by allowing retrograde or antiparticles into the formula, i.e., by the Stueckelberg interpretation that antiparticles are negative energy particle running backwards in time, and introducing a negative velocity to the kinetic energy and momentum vector space relation

$$\text{Cos}\,\theta = \left(-\sqrt{m_{\nu_e}}\ \eta_e + \sqrt{m_{\nu_\mu}}\ \eta_\mu + \sqrt{m_{\nu_\tau}}\ \eta_\tau\right)\Big/\left(\sqrt{\left(m_{\nu_e} + m_{\nu_\mu} + m_{\nu_\tau}\right)\eta}\right) \cdot (1,\ 1,\ 1)/|(1,\ 1,\ 1)| \tag{169}$$

under neutrino oscillation we can assume the velocities of the particles are the same magnitude,

$$|\eta| = |\eta_e| = \left|\eta_\mu\right| = |\eta_\tau| \tag{170}$$

this leads to

$$\text{Cos}\,\theta = \left(-\sqrt{m_{\nu_e}} + \sqrt{m_{\nu_\mu}} + \sqrt{m_{\nu_\tau}}\right) \Big/ \sqrt{\left(m_{\nu_e} + m_{\nu_\mu} + m_{\nu_\tau}\right)} \cdot (1,\,1,\,1)/|(1,\,1,\,1)| \tag{171}$$

inserting the neutrino masses results in Brennan's neutrino rule

$$\frac{3}{2} = \left(-\sqrt{m_{\nu_e}} + \sqrt{m_{\nu_\mu}} + \sqrt{m_{\nu_\tau}}\right) \Big/ \sqrt{\left(m_{\nu_e} + m_{\nu_\mu} + m_{\nu_\tau}\right)} \tag{172}$$

§3-4 de Broglie relation

We can express the wavelength and frequency of the resultant particles in terms of the above energy-momentum vector space using the de Broglie relations

$$\frac{h}{p} = \lambda \tag{173}$$

$$E = \frac{h}{\omega} \tag{174}$$

equating Planck's constant result in

$$E\,\omega = \lambda\,p \tag{175}$$

the ratio of the energy-momentum vector space now becomes

$$\frac{E}{p} = \frac{\lambda}{\omega} \tag{176}$$

if we use φ_{int} as the summation of the advanced and retarded potentials of the Wheeler-Feynman summation, and the derived $\mathbb{A}_{\text{int}}$ we can show

$$\frac{E}{p} = \frac{q\,|\varphi_{\text{int}}|}{q\,|\mathbb{A}_{\text{int}}|} = \frac{\lambda}{\omega} \tag{177}$$

therefore the wavelength and frequency result from the relationship between the energy and momentum

$$\frac{|\varphi|}{|\mathbb{A}|} = \frac{\lambda}{\omega} \tag{178}$$

§3-5 Quarks as Electromagnetic Potentials

This section attempts to derive the charges from the partial fields of the Wheeler-Feynman summation, to do so first assume the D'Alembertian to describe the equation of inhomogeneous electromagnetic wave equation is derived from the WFEMFPTST

$$\partial_\mu \partial^\mu A_\mu = -\frac{4\pi}{c} j_\mu \tag{179}$$

where once more over the temporal axis the $\mathbb{A}_{\text{ext}}$ drops out

$$\psi = \exp\left[-\frac{i\,q}{\hbar}\int A_\mu \, dx^\mu\right] \to \exp\left[-\frac{i\,q}{\hbar}\int \varphi_{\text{total}}\, dt\right] \tag{180}$$

this has a potential φ that can be decomposed into three partial potentials on the three spatial axes

$$\varphi = \frac{\varphi x}{3} + \frac{\varphi y}{3} + \frac{\varphi z}{3} \tag{181}$$

substitute this into the WFEMFPTST

$$\varphi^{\text{total}} = \sum_{\text{total}} \left(\left(\varphi_x^{\text{ret}} + \varphi_y^{\text{ret}} + \varphi_z^{\text{ret}}\right)/3! + \left(\varphi_x^{\text{adv}} + \varphi_y^{\text{adv}} + \varphi_z^{\text{adv}}\right)\Big/3!\right) + \sum_{\text{free}}\left(\left(\varphi_x^{\text{ret}} + \varphi_y^{\text{ret}} + \varphi_z^{\text{ret}}\right)/3! - \left(\varphi_x^{\text{adv}} + \varphi_y^{\text{adv}} + \varphi_z^{\text{adv}}\right)\Big/3!\right) \tag{182}$$

we can rearrange the fractions and “do some voodoo”

$$3!\,\varphi^{\text{total}} = \sum\left(\varphi_x^{\text{ret}} + \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} + \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} + \varphi_z^{\text{adv}}\right) + \left(\varphi_x^{\text{ret}} - \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} - \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} - \varphi_z^{\text{adv}}\right) \tag{183}$$

$$3!\,\varphi^{\text{total}} = \sum\left(\varphi_x^{\text{ret}} + \varphi_y^{\text{ret}}\right) + \left(\varphi_x^{\text{ret}} + \varphi_y^{\text{ret}}\right) + \left(\varphi_z^{\text{ret}} + \varphi_z^{\text{ret}}\right) + \left(\varphi_x^{\text{adv}} - \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{adv}} - \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{adv}} - \varphi_z^{\text{adv}}\right) \tag{184}$$

the result is identical with the original decomposition into the three axes, thus,

$$\varphi^{\text{total}} = \sum \frac{\varphi_x^{\text{ret}}}{3} + \varphi_y^{\text{ret}}\,/3 + \frac{\varphi_z^{\text{ret}}}{3} = \varphi \tag{185}$$

Importantly since the total potential derived from the D’Alembertian moves with velocity $\mathbb{v}$ then we expect these fractional charges to also move with $\mathbb{v}$

$$\mathbb{v}^{\text{total}} = \mathbb{v}_x^{\text{ret}} = \mathbb{v}_y^{\text{ret}} = \mathbb{v}_z^{\text{ret}} \tag{186}$$

However, given the common 1/3 charge this suggests quarks, and given the common velocity this suggests hadrons, let’s assume this provides a foundation for the quark bag model and see where it goes, what we are searching for is a total charge equal to one, we are looking for something along the lines:

$$\varphi^{\text{total}} = \sum -\frac{\varphi_x^{\text{ret}}}{3} + \left(2\,\varphi_y^{\text{ret}}\right)/3 + \left(2\,\varphi_z^{\text{ret}}\right)/3 = \varphi \tag{187}$$

First ansatz: If we add a free term for every spatial axis of spacetime.

$$3!\,\varphi^{\text{total}} = \sum\left[\left(\varphi_x^{\text{ret}} + \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} + \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} + \varphi_z^{\text{adv}}\right)\right] + \left[\left(\varphi_x^{\text{ret}} - \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} - \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} - \varphi_z^{\text{adv}}\right)\right] + \left[\left(\varphi_x^{\text{ret}} - \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} - \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} - \varphi_z^{\text{adv}}\right)\right] + \left[\left(\varphi_x^{\text{ret}} - \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} - \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} - \varphi_z^{\text{adv}}\right)\right] \tag{188}$$

$$3!\,\varphi^{\text{total}} = \sum \varphi_x^{\text{ret}} + \varphi_x^{\text{adv}} + \varphi_y^{\text{ret}} + \varphi_y^{\text{adv}} + \varphi_z^{\text{ret}} + \varphi_z^{\text{adv}} + \varphi_x^{\text{ret}} - \varphi_x^{\text{adv}} + \varphi_y^{\text{ret}} - \varphi_y^{\text{adv}} + \varphi_z^{\text{ret}} - \varphi_z^{a} + \varphi_x^{\text{ret}} - \varphi_x^{\text{adv}} + \varphi_y^{\text{ret}} - \varphi_y^{\text{adv}} + \varphi_z^{\text{ret}} - \varphi_z^{\text{adv}} + \varphi_x^{\text{ret}} - \varphi_x^{\text{adv}} + \varphi_y^{\text{ret}} - \varphi_y^{\text{adv}} + \varphi_z^{\text{ret}} - \varphi_z^{\text{adv}} \tag{189}$$

$$3!\,\varphi^{\text{total}} = \sum \varphi_x^{\text{ret}} + \varphi_y^{\text{ret}} + \varphi_z^{\text{ret}} + \varphi_x^{\text{ret}} + \varphi_y^{\text{ret}} + \varphi_z^{\text{ret}} + \varphi_x^{\text{ret}} - \varphi_x^{\text{adv}} + \varphi_y^{\text{ret}} - \varphi_y^{\text{adv}} + \varphi_z^{\text{ret}} - \varphi_z^{\text{adv}} + \varphi_x^{\text{ret}} - \varphi_x^{\text{adv}} + \varphi_y^{\text{ret}} - \varphi_y^{\text{adv}} + \varphi_z^{\text{ret}} - \varphi_z^{\text{adv}} \tag{190}$$

$$3!\,\varphi^{\text{total}} = \sum 4\left(\varphi_x^{\text{ret}} + \varphi_y^{\text{ret}} + \varphi_z^{\text{ret}}\right) + 2\left(-\varphi_x^{\text{adv}} - \varphi_y^{\text{adv}} - \varphi_z^{\text{adv}}\right) \tag{191}$$

$$\varphi^{\text{total}} = \sum \left(4\left(\varphi_x^{\text{ret}} + \varphi_y^{\text{ret}} + \varphi_z^{\text{ret}}\right) + 2\left(-\varphi_x^{\text{adv}} - \varphi_y^{\text{adv}} - \varphi_z^{\text{adv}}\right)\right) \Big/ 3! \tag{192}$$

$$\varphi^{\text{total}} = \sum \left(2\left(\varphi_x^{\text{ret}} + \varphi_y^{\text{ret}} + \varphi_z^{\text{ret}}\right) + \left(-\varphi_x^{\text{adv}} - \varphi_y^{\text{adv}} - \varphi_z^{\text{adv}}\right)\right) \Big/ 3 \tag{193}$$

$$\varphi^{\text{total}} = \sum \left(2\left(\varphi_x^{\text{ret}} + \varphi_y^{\text{ret}} + \varphi_z^{\text{ret}}\right)\right) / 3 - \left(\varphi_x^{\text{adv}} + \varphi_y^{\text{adv}} + \varphi_z^{\text{adv}}\right) \Big/ 3 \tag{194}$$

but this doesn’t look like the charges of a hadron

$$\varphi^{\text{total}} = \sum \left(2\,\varphi^{\text{ret}}\right) / 3 - \varphi^{\text{adv}} \Big/ 3 \tag{195}$$

so this is not a solution for quark charges.

Second ansatz: On the other hand, since we know two particles are involved let’s add the Wheeler-Feynman terms *twice*, effectively colliding two particles together

$$3!\;2\,\varphi^{\text{total}} = \sum \left[\left(\varphi_x^{\text{ret}} + \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} + \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} + \varphi_z^{\text{adv}}\right)\right] + \left[\left(\varphi_x^{\text{ret}} - \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} - \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} - \varphi_z^{\text{adv}}\right)\right] + \left[\left(\varphi_x^{\text{ret}} - \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} - \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} - \varphi_z^{\text{adv}}\right)\right] + \left[\left(\varphi_x^{\text{ret}} - \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} - \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} - \varphi_z^{\text{adv}}\right)\right] + \left[\left(\varphi_x^{\text{ret}} + \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} + \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} + \varphi_z^{\text{adv}}\right)\right] + \left[\left(\varphi_x^{\text{ret}} - \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} - \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} - \varphi_z^{\text{adv}}\right)\right] + \left[\left(\varphi_x^{\text{ret}} - \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} - \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} - \varphi_z^{\text{adv}}\right)\right] + \left[\left(\varphi_x^{\text{ret}} - \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} - \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} - \varphi_z^{\text{adv}}\right)\right] \tag{196}$$

rearrange to yield

$$2\,\varphi^{\text{total}} = \left(-\varphi^{\text{ret}} - \varphi^{\text{ret}} + 2\,\varphi^{\text{adv}} + 2\,\varphi^{\text{adv}} + 2\,\varphi^{\text{adv}} + 2\,\varphi^{\text{adv}}\right) \Big/ 3 \tag{197}$$

then picking out one term for the total, and we have the necessary quark charges for a proton and this time it does look like the charges of a hadron

$$\varphi^{\text{total}} = -\frac{1}{3}\varphi^{\text{ret}} + \frac{2}{3}\varphi^{\text{adv}} + \frac{2}{3}\varphi^{\text{adv}} \tag{198}$$

This may seem to violate conservation of particles, in that only one particle arises out of the interaction, however, if we reverse temporal frame of reference in the WFEMFPTST the result is an anti-proton

$$\varphi^{\text{total}} = +\frac{1}{3}\varphi^{\text{ret}} - \frac{2}{3}\varphi^{\text{adv}} - \frac{2}{3}\varphi^{\text{adv}} \tag{199}$$

in other words if we do WFEMFPTST backwards we get the necessary second hadron, and the underlying principle this whole paper is based on Stueckelberg’s original idea that antimatter is matter going backwards in time, since the antiproton and the proton can’t simultaneously be in the same space as they are moving away from each other in time then we can treat the particle and antiparticle as separate solutions.

Third ansatz: Also if we collide a particle and an anti-particle we obtain no particles, which is inline with observation

$$2\,\varphi^{\text{total}} = \left(-\,\varphi^{\text{ret}} + 4\,\varphi^{\text{adv}} + \varphi^{\text{ret}} - 4\,\varphi^{\text{adv}}\right)\big/3 = 0 \tag{200}$$

Fourth ansatz: Finally we can substitute without favor the quarks for the potentials by assuming the radius of the potentials are equals

$$\frac{q^{\text{total}}}{r} = -\frac{1}{3}\frac{q^{\text{ret}}}{r} + \frac{2}{3}\frac{1}{r}q^{\text{adv}} + \frac{2}{3}\frac{1}{r}q^{\text{adv}} \tag{201}$$

and we have the total charge of a proton derived from the extrinsic potentials of the four vector potential model as

$$p^{+} = d + u + u \tag{202}$$

in other words the quark charges can be expressed as being the result of a decomposition of the total potential of the extrinsic potentials of the Wheeler-Feynman summation.

§3-6 Gluons and how to make one

(The following section is a speculative discussion as to the structure of gluons and should not be taken too seriously.)

David Griffiths noted [p227, Griffiths 1987] "*In quantum electrodynamic A^{μ} become the wave function of the photon. The free photon satisfies this equation*

$$\Box^2 A^{\mu} = \frac{4\pi}{c} J^{\mu} \tag{203}$$

for $J^{\mu} = 0$

$$\Box^2 A^{\mu} = 0 \tag{204}$$

which we recognize in this context as the Klein-Gordon equation a massless particle."

Let's see if we can apply this idea to gluons,

Returning to the potential formulation and examining the charges of this model, where the potentials are multiplied by arbitrary structure constants f^{abc} which determine the structure of the fields

$$F^{\text{abc}} A_{\mu}^{\text{total}} = \Sigma \frac{1}{2}\left(f^{\text{abc}} A_{\mu}^{\text{ret}} + f^{\text{def}} A_{\mu}^{\text{adv}}\right) + \Sigma \frac{1}{2}\left(f^{\text{hij}} A_{\mu}^{\text{ret}} - f^{\text{klm}} A_{\mu}^{\text{adv}}\right) \tag{205}$$

making the observation the wave function is in phase if and only if

$$f^{\text{abc}} = f^{\text{def}} \tag{206}$$

and

$$f^{\text{hij}} = f^{\text{klm}} \tag{207}$$

$$f^{\text{abc}} = f^{\text{hij}} \quad \text{and} \quad f^{\text{def}} = f^{\text{klm}} \tag{208}$$

then we can write

$$f^{\text{abc}} = f^{\text{hij}} = f^{\text{def}} = f^{\text{klm}} \tag{209}$$

$$F^{\text{abc}} A_{\mu}^{\text{total}} = \Sigma f^{\text{abc}} A_{\mu}^{\text{ret}} \tag{210}$$

To each of these axes we can write an electromagnetic wave in three-space where i ∈ {x,y,z}

$$\partial_{\mu} \partial^{\mu} f^{\text{abc}} A_{\mu} = -\frac{4\pi}{c} f^{\text{abc}} J_{\mu} \tag{211}$$

and to each of these axes we associate a charge, we don't know that the charges look like or how they behave, we do know they obey Dirac's equation and are subject to the Pauli matrices; we know they must also satisfy Maxwell's equations; and when divided up into the three spatial coordinates they are still subject to Wheeler-Feynman's model. So let's assume the structure of charges of the axes are determined by the Pauli matrices,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{212}$$

this allows us to associate a unique EM field to each of the axes, we can do this because the matrices are unique and this in turn allows us to write the charges as giving rise to unique fields.

Comparing the Lagrangians of QED and QCD we see

$$\mathcal{L}_{\text{QED}} = \overline{\psi}(i\gamma^{\mu} D_{\mu} - m)\psi_j - \frac{1}{4} F_{\mu\nu}^{a} F_{a}^{\mu\nu} \tag{213}$$

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \tag{214}$$

$$\mathcal{L}_{\text{QED}} = \overline{\psi}(i(\gamma^{\mu} D_{\mu})_{\text{ij}} - m\,\delta_{\text{ij}})\psi_j - \frac{1}{4} G_{\mu\nu}^{a} G_{a}^{\mu\nu} \tag{215}$$

$$G_{\mu\nu}^{a} = \partial_{\mu} A_{\nu}^{a} - \partial_{\nu} A_{\mu}^{a} + g f^{\text{abc}} A_{\mu}^{b} A_{\nu}^{c} \tag{216}$$

Where the f^{abc} are the structure constants and g the coupling, in fact the only difference between the two Lagrangians is the final term, so according to this model the Pauli matrices give structure to the electromagnetic field, importantly these partial fields exist only during the expansion of the Wheeler-Feynman terms, that is, the potentials expand into the three axes, quarks arise and interact due to their unique EM potentials. In effect, suggesting

gluons might be photons with structure.

Clearly this is a speculative suggestion rather than a definitive derivation from first principles and it is to be taken with a quark of salt.

§3-7 Gell-Mann–Nishijima formula

All this leads to a generalization of the Wheeler-Feynman time symmetric theory electromagnetic four-potential WFTSTEMFPG (maybe at this point I should start using smaller acronyms), first write

$$\varphi^{\text{total}} = \sum \frac{2}{3}\left[\left(\varphi_x^{\text{ret}} - \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} - \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} - \varphi_z^{\text{adv}}\right)\right] - \frac{1}{3}\left[\left(\varphi_x^{\text{ret}} - \varphi_x^{\text{adv}}\right) + \left(\varphi_y^{\text{ret}} - \varphi_y^{\text{adv}}\right) + \left(\varphi_z^{\text{ret}} - \varphi_z^{\text{adv}}\right)\right] \tag{217}$$

and noting that

$$\varphi^{\text{total}} = \frac{q}{R} \tag{218}$$

$$\varphi_x^{\text{ret}} = \frac{q^{\text{ret}}}{x};\ \varphi_x^{\text{ret}} = \frac{q^{\text{ret}}}{y};\ \varphi_x^{\text{ret}} = \frac{q^{\text{ret}}}{z}; \tag{219}$$

$$\varphi_x^{\text{adv}} = \frac{q^{\text{adv}}}{x};\ \varphi_x^{\text{adv}} = \frac{q^{\text{adv}}}{y};\ \varphi_x^{\text{adv}} = \frac{q^{\text{adv}}}{z} \tag{220}$$

then

$$Q = q \sum n(\text{charges}) \tag{221}$$

and assuming that all the radii of the potentials are equal (which we can do for the quark bag model), then we can remove the radius and rewrite the total charge purely in terms of the number of partial charges which simplifies as

$$Q = q \sum \frac{2}{3}\left[\left(n_x^{\text{ret}} - n_x^{\text{adv}}\right) + \left(n_y^{\text{ret}} - n_y^{\text{adv}}\right) + \left(n_z^{\text{ret}} - n_z^{\text{adv}}\right)\right] - \frac{1}{3}\left[\left(n_x^{\text{ret}} - n_x^{\text{adv}}\right) + \left(n_y^{\text{ret}} - n_y^{\text{adv}}\right) + \left(n_z^{\text{ret}} - n_z^{\text{adv}}\right)\right] \tag{222}$$

let's make an additional assumption that each of the axes holds the charge of a different particle, and *provisionally* identify these particles with the up, charm, top, down, strange, bottom, top and their antiquarks

$$Q = q \sum \frac{2}{3}\left[(n_u - n_{\overline{u}}) + (n_c - n_{\overline{c}}) + \left(n_t - n_{\overline{t}}\right)\right] - \frac{1}{3}\left[\left(n_d - n_{\overline{d}}\right) + (n_s - n_{\overline{s}}) + \left(n_b - n_{\overline{b}}\right)\right] \tag{223}$$

then write where S, C, B', and T are the strangeness, charm, bottomness and topness flavor quantum numbers respectively,

$$S = -(n_s - n_{\overline{s}}) \tag{224}$$

$$C = +(n_c - n_{\overline{c}}) \tag{225}$$

$$B' = -\left(n_b - n_{\overline{b}}\right) \tag{226}$$

$$T = +\left(n_t - n_{\overline{t}}\right) \tag{227}$$

then we can equate the WFTSTEMFPG to the Gell-Mann–Nishijima formula,

$$Q = I_3 + \frac{1}{2}\left[B + S + C + B' + T\right]$$

(On a historical note, Feynman referred to partially charged particles like quarks and gluons as Partons, that nomenclature in no way justifies the above reasoning from the Wheeler-Feynman summation, but like Poincaré's conception of matter as hollow charged spheres one can't help but wonder if the intuition of Feynman or Poincaré was correct.)

§4 Short Notes and Observations

§4-1 Wave-Particle Duality

In his book The Quantum Handshake, Cramer [p181, 2016] made the observation that the Transactional Interpretation "*...pictures a transaction as emerging from an offer-confirmation handshake as a four-vector standing wave normal in three-dimensional space with endpoints at the emission and absorption vertices*..."

As suggested above this model is equivalent to the Wheeler-Feynman summation over potentials is the sphere in $\mathbb{R}^3$, and an electromagnetic wave in $\mathbb{R}^{1,3}$, where

$$\partial_\mu \partial^\mu A_\mu = -\frac{4\pi}{c} j_\mu \tag{228}$$

is an electromagnetic wave distributed over time, and we find in the particles frame of reference it is a static charged spherical shell,

$$\nabla^2 \varphi = -\frac{1}{\epsilon_o} \rho \tag{229}$$

To counterpoint Cramer's picture, the model presented here as a four-potential vector potential wave gives rises to a four-dimensional system where the signal travels along the electric potential on the time axis, this signal moves in a zig-zag (read wave motion) pattern, up to the apex of the charged sphere and back down, repeating this process from the beginning of the particle movement to its termination, about the axis of movement. On transforming from $\mathbb{R}^{1,3}$ to $\mathbb{R}^3$ we have transformed from a $\mathbb{R}^{1,3}$ frame of reference to a $\mathbb{R}^3$ *frame of reference,* here the resultant is static within the frame of reference of the particle, and appears as a three-dimensional object that has mass, charge, and spin of what we call a particle. In this manner we obtain the contradictory notion of a wave that has mass, charge and spin, where by a choice of dimensional frames of reference we obtain a explanation to the problem of wave–particle duality. The beauty of this four-dimensional wave argument is that it puts elementary particles like the electron and quark in the same category as the photon, we could even go so far as to suggest the electron is a kind of slowed down photon with kinks, internally it still travels at the speed of light but the resultant moves with the velocity of a subluminal particle. Stating this as a general principle: the transformation from a four-dimensional vector potential wave to a three-dimension object manifests the mass, charge and spin as the intrinsic properties of a three-dimensional sphere, or more simply

9) *Elementary particles are electromagnetic spheres (or glomes) in* $\mathbb{R}^3$ *and electromagnetic Waves in* $\mathbb{R}^{1,3}$.

Thus the problem of whether the particle is a point or a extended wave is neatly reduced to a problem of frames of reference, in effect there are no point particles and in their place there are charged spheres.

§4-2 Infinite Energy

In the same way the problem of infinite energy of a point particle is obviated as the envelope of the sphere never equals the zero point, and thus the radius of the sphere never approaches zero except relativistically, infinite energy from the potential of a point particle was -and still is- a tremendous problem for Standard Model. It simply isn't a problem for this model.

§4-3 Self-Interaction

There is no particle self-interaction as the retarded and advanced potentials are extrinsic to the loci of the particle, the potentials of the particles do not operate upon themselves but on their retarded and advanced counterparts, the only interaction is between the extrinsic scalar potential and the accompanying intrinsic vector potential.

§4-4 Motion

The equation of motion for matter is the derived directly from the Wheeler–Feynman time-symmetric theory, where the four-vector potential there appears in the inertial frame a four dimensional inhomogeneous electromagnetic wave function

$$\partial_\mu \partial^\mu A_\mu = -\frac{4\pi}{c} j_\mu \tag{230}$$

comparing this to the homogeneous electromagnetic wave function for light

$$\partial_\mu \partial^\mu A_\mu = 0 \tag{231}$$

we find the only difference is the orientation of the potentials describing either a transverse or oblique wave functions. In other this model gives an equation of motion that places matter in same category as light.

§4-5 Historical note on Electromagnetic Mass

In 1881 J.J.Thomson working from Maxwell's equations showed that a charged sphere would undergo a resistive force in a dielectric media and thereby the electrostatic energy of charged sphere possessed some form of electromagnetic momentum:

"*The first case we shall consider is that of a charged sphere moving through an unlimited space filled with a medium of specific inductive capacity K. The charged sphere will produce an electric displacement throughout the field; and as the sphere moves the magnitude of this displacement at any point will vary. Now, according to Maxwell's theory, a variation in the electric displacement produces the same effect as an electric current; and a field in which electric currents exist is a seat of energy; hence the motion of the charged sphere has developed energy, and consequently the charged sphere must experience a resistance as it moves through the dielectric. But as the theory of the variation of the electric displacement does not take into account any thing corresponding to resistance in conductors, there can be no dissipation of energy through the medium; hence the resistance cannot be analogous to an ordinary frictional" resistance, but must correspond to the resistance theoretically experienced by a solid in moving through a perfect fluid. In other words, it must be equivalent to an increase in the mass of the charged moving sphere ...*" [Thomson 1881]

Henri Poincaré in his paper 'The End of Matter' then introduced a form of pressure (what is now called the Poincaré stresses) to counteract the electron's electrostatic energy from tearing itself apart:

"*It is thus necessary to return from here to the theory of Lorentz; but if one wants to preserve it and avoid intolerable contradictions, it is necessary to suppose a special force which explains at the same time the contraction and the constancy of two of* *the axes. I sought to determine this force, I found that it can be compared to a constant external pressure, acting on the deformable and compressible electron, and whose work is proportional to the variations of the volume of the electron. So if the inertia of matter is exclusively of electromagnetic origin, as it is generally admitted since the experiment of Kaufmann, and except that constant pressure from which I come to speak, all forces are of electromagnetic origin...*" [Poincaré 1906]

Poincaré even went so far as to suggest that matter doesn't exist at all and electrons are spherical holes in the aether:

"This question was further investigated by Lorentz; he found that all atoms as well as all positive and negative electrons have an inertia, which (for all of them) varies with velocity by the same laws. Any material atom would therefore be composed by small and heavy positive electrons, and when the observable matter doesn't appear to us as electric, then this is caused by the fact that both kinds of electrons are present in approximately the same amount. They all have no measures, and their inertia is borrowed from the aether. In this system there is no actual matter, there are only holes in the aether." [Poincaré 1906]

You will note how closely Thomson and Poincaré ideas matches the four vector potential model I have present here, this does not justify model, but it does give food for thought.

§4-6 Einstein's Bubble

In 1927 Einstein attempted to break quantum non-locality with the Bubble paradox, quoting Cramer's [Cramer, 2016, p78]:

"*A source emits a single photon isotropically, so that there is no preferred emission direction. According to the Copenhagen view of quantum formalism, this should produce a spherical wave function ψ that expands like an inflating bubble centered on the source. At some time later, the photon is detected, and, since the photon does not propagate further, its wave function bubble should 'pop', disappearing instantaneously from all locations except the position of the detector. Einstein asked how the parts of the wave function away from the detector could 'know' that they should disappear, and how it could arrange that only a single photon was always detected when only one was emitted*?"

You will observe the four-potential model allows an escape mechanism from the Bubble paradox, for the scalar component four-potential is not constrained by relativity and here I will again quote from Griffiths' book: [Griffiths, 2017, p441]:

"*There is a very peculiar thing about the scalar potential in Coulomb gauge: it is determined by the distribution of charge right now. If I move an electron in my laboratory, the potential V on the moon immediately records this change.*"

All of this makes sense if once more we look at the four-vector potential

$$A_\mu = (\varphi_t, \mathbb{A}_x, \mathbb{A}_y, \mathbb{A}_z) \tag{232}$$

we clearly see that only the electric potential is time dependent in A_μ, yes, the $\mathbb{A}$ can be written as $\partial_t \mathbb{A}$, but not within the context of the external A_μ it is only when we apply Maxwell's equations does this become significant. In other words a change in the electric potential can happen simultaneously over the surface of the four-sphere without violating causality, as Cause and Effect only takes place with the $\mathbb{E}$ and $\mathbb{B}$ fields which are located in three-space and dependent upon the four-potential traveling along a path of least action, this allows the Quantum Handshake to be the signal, either as a photon *or* an electromagnetic massive particle from the center of the potential to a resonant particle on the shell. In effect:

Cramer's Quantum Handshake pops Einstein's Bubble.

§4-7 Mach's Principle

To derive Mach's principle from the Wheeler-Feynman summation, we return to the observation there is a source of back emf acting on the magnetic vector potential of particle from the extrinsic potential

$$\mathbb{F} = -\frac{\partial q_{\text{int}} \mathbb{A}_{\text{int}}}{\partial t} \tag{233}$$

where $\mathbb{A}$ is derived from

$$\nabla \cdot \mathbb{A}_{\text{int}} = -\frac{1}{c} \frac{\partial \varphi_{\text{ext}}}{\partial t} \tag{234}$$

we need only apply this principle to the sum of the free potentials acts on the intrinsic charge of the particle

$$q_{\text{int}} \varphi_{\text{free}} = \sum_n \frac{1}{2} \left(q_{\text{int}} \varphi_{\text{ret}} - q_{\text{int}} \varphi_{\text{adv}} \right) = 0 \tag{235}$$

to find the free back emf acting on the particle to be zero

$$\nabla \cdot q\, \mathbb{A}_{\text{int}} = -\frac{q_{\text{int}}}{c} \frac{\partial \varphi_{\text{free}}}{\partial t} = \sum_n \frac{q_{\text{int}}}{2\,c} \left(\frac{\partial \varphi_{\text{adv}}}{\partial t} - \frac{\partial \varphi_{\text{ret}}}{\partial t} \right) = 0 \tag{236}$$

it can be seen the resultant force must be zero on the particle otherwise it will accelerate. If we now generalize the WFTSTEFP to include the total extrinsic potentials plus all the free potentials A_μ^{free} across the entire universe,

$$A_\mu^{\text{universe}} = \Sigma^{\text{total}} \frac{1}{2} \left(A_\mu^{\text{ret}} + A_\mu^{\text{adv}} \right) + \Sigma^{\text{infinite free potentials}} \frac{1}{2} \left(A_\mu^{\text{ret}} - A_\mu^{\text{adv}} \right) = \Sigma A_\mu \tag{237}$$

and assume that nearly all the non-local extrinsic potential cancel each other out. Then since A_μ is independent of c the principle of superposition of waves of the wave equation for A_μ this allows for an instantaneous quantum handshake across the entire universe, and therefore the intrinsic back emf allows the momentum of the particle to react against all the potentials of the universe without breaking any conservation laws, this allows us to use set a set of extrinsic reference frames to determine the local reference frame, effectively

Mach’s Principle is implicit in the Wheeler-Feynman absorber model .

Therefore I’m going to suggest that Mach's principle can be derived as a natural consequence of the Wheeler-Feynman model, the problem of inertial frames is reduced to a simple infinite sum, and in turn Newton's Bucket remains as still as a teacup floating in the center of an raging hurricane.

§5 Predictions

§5-1 This model predicts for charged leptons there can be no charge without a generalized electromagnetic field, as a result there is a necessary condition that lepton charge and mass are tied inextricably to each other, effectively there can be no charge without mass, therefore finding a subluminal lepton with a massless charge would break this model, on the other hand mass without charge is still possible provided it is a relativistic mass.

§5-2 All elementary massive charged particles are at their basis electromagnetic waves and therefore all particle fields and interactions must be derived ultimately from Maxwell’s equations, in effect gluons and leptons, the strong force, nuclear strong force, and the weak interaction, are all ultimately electromagnetic in origin. (I will however, leave out gravity from this list, as this model barely touches upon the structure of spacetime.) If it can be shown that leptons and quarks are not derived from Maxwell’s equations that would break this model.

§5-3 Ultimately this model predicts that all magnetic fields have their sources as electrically charged electromagnetic waves, in fact we can always transform to a frame of reference where ***B*** can be substituted by ***E***, which in Gaussian units.

$$|\mathbb{B}| = |\mathbb{E}| \tag{238}$$

so in a very real sense the magnetic field is an electric field, and by seeing elementary particles as electromagnetic waves just like light are electromagnetic waves, this in turn forces us to definitively write Maxwell's equations with

$$\nabla \cdot \mathbb{B} = 0 \tag{239}$$

We can generalize this even further via the Wheeler-Feynman model where the electric charge is absorbed into four-vector potential, in other words, by looking at this four-dimensionally the charge disappears and only the electromagnetic wave remains. This allows us to throw away the possibility of magnetic charge, it allows us to make the prediction that no magnetic monopoles exist, and no magnetic monopoles will be detected, and from this we can definitively write Maxwell’s equations without magnetic monopoles, and therefore the observation of magnetic monopoles breaks this model.

§5-4 The universe is not made of bits of string, clearly this model does not resort to higher dimensions than four to explain fundamental physics in the same way as String Theory is used to explain away point particles, should a definitive experimental proof of String theory ever be found that would break this model.

§5-5 Schrödinger [Schrödinger 1930] showed that the interference of positive and negative charged energy states gives rise to to the Zitterbewegung but it has never been observed. Since the retarded and advanced potentials are positive and negative charged energy states, and in this model the free term of the Wheeler-Feynman model corresponds to a zero phase difference between the negative and positive energy states, then the zero phase takes away the need for Zitterbewegung. Therefore if Zitterbewegung is observed for free relativistic particles that would break this model.

§5-6 It is suggested that Mach’s Principle is solely derived from the four-potential, if there exists an exclusively non-electromagnetic mechanism for Mach’s principle, that would break this model.

§5-7 This model predicts that electrons, muon and tauons are the charged shells of perfect spheres, failure to do so disproves this model, if you ever find an electron in the shape of a duck then this model is in serious trouble.

§6 Conclusions

It has been shown:

1: The Wheeler–Feynman time-symmetric theory can be expressed and simplified in terms of the electromagnetic four potential (WFTSTEFP), this is possible by projecting the extrinsic electric scalar potentials over time, then to satisfy Maxwell's equation the magnetic vector potential is reintroduced in the intrinsic frame, in the process an electromagnetic wave equation for matter was found as a solution of the Wheeler-Feynman summation that closely matches Maxwell's equation for light, and as a result it is suggested that light and matter are both forms of electromagnetism.

2: The WFTSTEFP is shown to be consistent with the laws of conservation of momentum, energy, action, spin, and phase, this allows the model to be expressed entirely as wave functions, thus providing a useful and elegant means of expressing the Wheeler-Feynman summation in the formalism of quantum mechanics.

3: Cramer's Transactional Interpretation is derived from the free potentials of the Wheeler-Feynman summation, confirming Cramer's suggestion that matter is a standing wave between advanced and retarded sources, and it is suggested the Quantum Handshake is in fact a standing electromagnetic four vector potential wave between the past and the future, this standing wave constitutes what we call the observed particle, and that the observed particle is a particle is both distinct and derived from the extrinsic advanced and retarded source particles.

4: The Gell-Mann–Nishijima formula is derived from first principles through manipulation of the partial extrinsic potentials over an infinite space of the free potentials leading to the suggestion that the charges of quarks are in fact solutions of the WFTSTEFP, leading to the suggestion that quarks are in fact the partial fields of the four vector potential subject to the Wheeler-Feynman summation.

5: A proof of the Brennan-Foot-Koide formulas for leptons masses appears through an inner product of the extrinsic scalar potential to the intrinsic vector potential, leading to the suggestion that mass is intrinsic property of the electromagnetic four potential.

6: Mach's principle is given in terms of the Wheeler-Feynman summation through a projection of the free scalar potentials over infinite space and time, the method give does not violate causality, nor conservation of energy, the effect appears locally and the process does not violate Special Relativity.

7: In this model it is suggested matter is a kind of slowed down light, where the potentials are rotated to an oblique angle of 45° out of the axis of motion to form matter and charge, the fields still move at the speed of light but it moves up and down out of the axis of translation, with this observation we can begin to see the entire universe as being formed from different modes of electromagnetic radiation.

But there remains unanswered problems with this model, obviously this is not the Standard Model as it lacks gravity, it lacks the electroweak gauge bosons, nor does it explicitly explain the structure of neutrinos only their masses. Yet despite these limits a considerable tract of physics can be derived through a simple manipulation of the Wheeler-Feynman summation. Personally I'm suspect the electroweak gauge bosons might be a type of electromagnetic evanescent waveforms, some kind of asymmetric electromagnetic potential that rapidly decays into stable potentials, or some form of evanescent Bessel equation that transits between states. As for neutrinos, they are distinct from the charged leptons, yet one of the predictions of this model is that electromagnetic mass and charge are intrinsically related, to overcome this problem it is suggested their neutrino masses must be relativistic in origin. To explain this you will note the electromagnetic fields of a photon are transverse (read 90 degrees) to the axis of movement, whereas for the charged leptons of this model the electromagnetic fields of the charged lepton are rotated at 45 degrees to the axis of movement, which suggests the neutrinos might be parallel (read longitudinal) to the axis of movement. If we return to the original observation that φ exists along the axis of time independently of space, then we can rotate the φ field in any direction, in this case the axis of time would be on a direct path between two points. So how would that behave, how would it look? Obviously Maxwell's equations must still apply, so we can write

$$\nabla \cdot \mathbb{A} = -\frac{1}{c}\frac{\partial \varphi}{\partial t} \tag{240}$$

if φ is orientated along a straight line on the axis of movement, then the magnetic potential $\mathbb{A}$ appears as a *ring* around the axis of movement, then applying

$$\nabla \times \mathbb{A} = \mathbb{B} \tag{241}$$

$$-\nabla \cdot \varphi \tag{242}$$

that suggests the neutrinos electric potential appears along the axis of movement and the magnetic potential appears as radial field around the axis of movement. If this is the case then neutrinos like the Aharonov–Bohm effect should be subject to displacement by magnetic fields, of course, given the velocity of neutrinos and their trivial masses, then any magnetic field required to test this would be astronomical in magnitude - which begs the question what's a black hole and where I can buy one? Of course this is mere speculation and a more robust model must be found for neutrinos.

From the point of nomenclature, the elementary particle is no longer a point particle but rather a wave in $\mathbb{R}^{1,3}$ and a sphere in $\mathbb{R}^3$, however, continuing on in the tradition of calling it a particle rather than a sphere, sphere-particle or a spherical particle or a glome would simplify the discussion as long as it is understood in the context that we are referring to a sphere particle rather than a point particle - personally I prefer bubbles of light.

So what does this all mean? Well, I think it allows us to return to the electromagnetic program began by Maxwell in his 1865 paper on Light "A Dynamical Theory of the Electromagnetic Field"; furthered by J. J. Thomson in his 1881 paper "On the Electric and Magnetic Effects produced by the Motion of Electrified Bodies"; and elaborated by Henri Poincaré in his 1906 paper "La fin de la matière". Thomson made the simple calculation that a charged sphere moving in a space (Maxwell's electromagnetic aether) with a specific inductive capacity is harder to set in motion than an uncharged body and used this as a step to explain the problem of inertia. Later Poincaré suggested that electromagnetic effects alone would account for momentum, and that matter could exist on the surface of a charged spherical shell, and he made the most extreme proposal that mass distinct from electromagnetism would no longer exist. Poincaré' conception, however, brought with it the problem of 4/3 electromagnetic masses and the need for Poincaré's stresses to balance those masses. All of this was dropped when mass–energy equivalence removed the necessity for electromagnetic masses by reducing everything to a simple equation that held everything together without actually explaining the underlying mechanisms. The four vector potential wave model presented in this paper not only gives a much clearer understanding of what actually constitutes matter, it also allows Maxwell's, Thomson's and Poincaré's insights to be expressed in a manner that not only answers many of the problems it also charts a whole realm of the physics. Indeed, if we compare the four vector potential wave model to Maxwell's model for Light, we can see most of physics is reduced to just two equations, for Light

$$\partial_\mu \partial^\mu A_\mu = 0 \tag{243}$$

and for Matter

$$\partial_\mu \partial^\mu A_\mu = -\frac{4\pi}{c} j_\mu \tag{244}$$

Clearly it can be seen the left hand side holds the orientation of the potentials so for Light the fields are transverse to the axis of movement, and for Matter the fields appear along an oblique angle to the axis of move, while the right hand sides holds the mass, momentum, spin and charges of the charge lepton, a more wonderfully elegant and concise model of Maxwell's electromagnetic program is hard to imagine.

§7 Acknowledgments and final words

Thanks to Mitch Porter for initial discussions about the Feynman-Stueckelberg interpretation of antimatter.

Thanks to John Cramer for his correspondence and his work on the Quantum Handshake.

Thanks to David Griffiths and his erudite textbooks in elementary particles, electrodynamics and quantum mechanics, now if only he would write one for gravity...

Thanks to Sean Court,

To paraphrase Maxwell's 1865 quote on the discovery of the electromagnetic equations of light

I too *have a paper afloat,*
with an electromagnetic theory of matter*, which,*
till I am convinced to the contrary,
I hold to be great guns.

and naturally

"The changing of bodies into light,
and light into bodies,

is very conformable to the course of Nature."
- I.Newton, Opticks 1704

References

Wheeler, J. A.; Feynman, R. P. ,“Interaction with the Absorber as the Mechanism of Radiation”, Reviews of Modern Physics. 17 (2–3): 157–181, (April 1945).

Wheeler, J. A.; Feynman, R. P. , “Classical Electrodynamics in Terms of Direct Interparticle Action”. Reviews of Modern Physics. 21 (3): 425–433. (July 1949).

Griffiths D. J., “Introduction to Electrodynamics 4th Edition”, Cambridge University Press, (2017).

Griffiths D. J., “Introduction to Elementary Particles”, John Wiley and Sons, (1987).

Poincaré H., “La fin de la matière”, [The End of Matter], Athenæum, (1906).

Hudson J. J. , Kara D.J., Smallman I.J., Sauer B.E., Tarbutt M.R., Hinds E.A., “Improved measurement of the shape of the electron” Nature volume 473, pages493–496(2011).

Maxwell J.C., “A Dynamical Theory of the Electromagnetic Field”, Philosophical Transactions of the Royal Society of London. 155: 459–512., (1865).

Mead C.A. "Collective electrodynamics I", Proceedings National Academy Science U.S.. Vol. 94, pp. 6013-6018, (June 1997).

Thomson J.J., “On the Electric and Magnetic Effects produced by the Motion of Electrified Bodies”, Philosophical Magazine, 1881, 5 11 (68): 229-249, (1881).

Heaviside O., “On the Electromagnetic Effects due to the Motion of Electrification through a Dielectric”, Philosophical Magazine, 5, 27 (167): 324–339 (1889).

Searle G.F.C., “On the Steady Motion of an Electrified Ellipsoid”, Philosophical Magazine, 5, 44 (269): 329–341, (1897).

Lorentz H.A.,”La Théorie electromagnétique de Maxwell et son application aux corps mouvants”, Néerlandaises des Sciences Exactes et Naturelles, 25: 363–552, (1892).

Wien W., “Über die Möglichkeit einer elektromagnetischen Begründung der Mechanik” [On the Possibility of an Electromagnetic Foundation of Mechanics], Annalen der Physik, 310 (7): 501–513 (1900).

Abraham M., “Prinzipien der Dynamik des Elektrons” [Principles of the dynamics of the electron], Annalen der Physik, 315 (1): 105–179, (1903).

Poincaré H., “Sur la dynamique de l’électron”, [On the Dynamics of the Electron], Comptes Rendus de l’Académie des Sciences, t. 140, p. 1504–1508
Session from June 5, (1905).

Cramer J. G., “Transactional Interpretation of quantum mechanic”, Reviews of Modern Physics 58, 647-687 (1986).

Cramer J. G., “The Quantum Handshake”, Springer International Publishing, (2016).

Foot R., “A note on Koide’s lepton mass relation”, arXiv:hep-ph/9402242v1, (7 Feb 1994).

Koide, Y., "New view of quark and lepton mass hierarchy". Physical Review D. 28 (1): 252–254. (1983).

Brannen, C. A., "Koide Mass Formula for Neutrinos". https://ui.adsabs.harvard.edu/abs/2006APS..NWS.B3014B/abstract, (May, 2006).

Schrödinger E., "Über die kräftefreie Bewegung in der relativistischen Quantenmechanik" [On the free movement in relativistic quantum mechanics], (1930).