On the Metric Coefficients

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Abstract

The article investigates the metric coefficients in the light of the Field Equations of General Relativity. We derive results connecting the metric coefficients(exclusively) that point to inconsistency in the formal [conventional] theory. They follow from the consistent application of mathematics on the existing theory of General Relativity and is quite starling in view of the fact that these results impose a an unacceptable constraint on the theory. These results do not apply where the energy density tensor has zero values.

Introduction

The metric coefficients are considered in the light of the Einstein's Field Equations. Results connecting the metric coefficients(exclusively) are derived. These results which follow by a consistent application of mathematics on existing theory is quite starling in view of the fact they impose a heavy constraint on the existing theory. The derived results ,incidentally, do not apply where the energy density tensor has zero values.

Reversed Cauchy Schwarz Inequality

We consider first a simple mathematical result and its derivation:

For arbitrary real numbers a_1, a_2, b_1 and b_2,

$$(a_1b_1 - a_2b_2)^2 \geq ({a_1}^2 - {a_2}^2)({b_1}^2 - {b_2}^2) \quad (1)$$

Proof: $(a_1b_1 - a_2b_2)^2 - ({a_1}^2 - {a_2}^2)({b_1}^2 - {b_2}^2)$

$$= {a_1}^2{b_1}^2 + {a_2}^2{b_2}^2 - 2a_1a_2b_1b_2 - \left({a_1}^2{b_1}^2 + {a_2}^2{b_2}^2 - {a_1}^2{b_2}^2 - {a_2}^2{b_1}^2\right)$$
$$= {a_1}^2{b_2}^2 + {a_2}^2{b_1}^2 - 2a_1a_2b_1b_2$$

$$= (a_1b_2 - a_2b_1)^2 \geq 0$$

Therefore$(a_1b_1 - a_2b_2)^2 - ({a_1}^2 - {a_2}^2)({b_1}^2 - {b_2}^2) \geq 0$

$$(a_1b_1 - a_2b_2)^2 \geq ({a_1}^2 - {a_2}^2)({b_1}^2 - {b_2}^2)$$

Now we proceed to drive the Reversed Cauchy Schwarz Inequality given by:

$$|a_1b_1 - a_2b_2 - a_3b_3 - a_4b_4 - \cdots - a_nb_n|^2 \geq \left({a_2}^2 - {a_2}^2 - {a_3}^2 - {a_4}^2 - \cdots - {a_n}^2\right)\left({b_1}^2 - {b_2}^2 - {b_3}^2 - {b_4}^2 - \cdots - {b_n}^2\right) \quad (2)$$

Proof:By the Cauchy Schwarz inequality[1] we have,

$$(a_2b_2 + a_3b_3 + a_4b_4 + \cdots + a_nb_n)^2 \leq ({a_2}^2 + {a_3}^2 + {a_4}^2 + \cdots + {a_n}^2)({b_2}^2 + {b_3}^2 + {b_4}^2 + \cdots + {b_n}^2)$$

$$\left[\frac{a_2b_2 + a_3b_3 + a_4b_4 + \cdots + a_nb_n}{\sqrt{a_2b_2 + a_3b_3 + a_4b_4 + \cdots + a_nb_n}\sqrt{{b_2}^2 + {b_3}^2 + {b_4}^2 + \cdots + {b_n}^2}}\right]^2 \leq 1 \quad (3)$$

$$-1 \leq \frac{a_2b_2 + a_3b_3 + a_4b_4 + \cdots + a_nb_n}{\sqrt{a_2b_2 + a_3b_3 + a_4b_4 + \cdots + a_nb_n}\sqrt{{b_2}^2 + {b_3}^2 + {b_4}^2 + \cdots + {b_n}^2}} \leq 1$$

Therefore we may write

$$\frac{a_2b_2 + a_3b_3 + a_4b_4 + \cdots + a_nb_n}{\sqrt{a_2b_2 + a_3b_3 + a_4b_4 + \cdots + a_nb_n}\sqrt{{b_2}^2 + {b_3}^2 + {b_4}^2 + \cdots + {b_n}^2}} = Cos\theta$$

$$a_2b_2 + a_3b_3 + a_4b_4 + \cdots + a_nb_n = \sqrt{a_2b_2 + a_3b_3 + a_4b_4 + \cdots + a_nb_n}\sqrt{{b_2}^2 + {b_3}^2 + {b_4}^2 + \cdots + {b_n}^2}Cos\theta \quad (4)$$

$$a_1b_1 - a_2b_2 - a_3b_3 - a_4b_4 - \cdots a_nb_n = a_1b_1 - \sqrt{{a_2}^2 + {a_3}^2 + {a_4}^2 + \cdots + {a_n}^2}\sqrt{{b_2}^2 + {b_3}^2 + {b_4}^2 + \cdots + {b_n}^2}Cos\theta$$

$$(a_1b_1 - a_2b_2 - a_3b_3 - a_4b_4 - \cdots a_nb_n)^2 = \left(a_1b_1 - \sqrt{{a_2}^2 + {a_3}^2 + {a_4}^2 + \cdots + {a_n}^2}\sqrt{{b_2}^2 + {b_3}^2 + {b_4}^2 + \cdots + {b_n}^2}Cos\theta\right)^2$$

$$(a_1b_1 - a_2b_2 - a_3b_3 - a_4b_4 - \cdots a_nb_n)^2 = \left(a_1b_1 - \sqrt{{a_2}^2 + {a_3}^2 + {a_4}^2 + \cdots + {a_n}^2}\sqrt{(b_2Cos\theta)^2 + (b_3Cos\theta)^2 + (b_4Cos\theta)^2 + \cdots + (b_nCos\theta)^2}\right)^2 \quad (4')$$

Applying (1) we have,

$$\left[a_1 b_1 - \sqrt{{a_2}^2 + {a_3}^2 + {a_4}^2 + \cdots + {a_n}^2}\sqrt{(b_2 Cos\theta)^2 + (b_3 Cos\theta)^2 + (b_4 Cos\theta)^2 + \cdots + (b_n Cos\theta)^2}\right]^2 \geq \left({a_2}^2 - {a_2}^2 - {a_3}^2 - {a_4}^2 - \cdots - {a_n}^2\right)\left({b_1}^2 - Cos^2\theta({b_2}^2 + {b_3}^2 + {b_4}^2 + \cdots + {b_n}^2)\right) \quad (5)$$

But the left side of (5) considering (4')is $(a_1 b_1 - a_2 b_2 - a_3 b_3 - a_4 b_4 - \cdots a_n b_n)^2$

$$\Longrightarrow (a_1 b_1 - a_2 b_2 - a_3 b_3 - a_4 b_4 - \cdots a_n b_n)^2 \geq \left({a_2}^2 - {a_2}^2 - {a_3}^2 - {a_4}^2 - \cdots - {a_n}^2\right)\left({b_1}^2 - {b_2}^2 - {b_3}^2 - {b_4}^2 - \cdots - {b_n}^2\right)$$

We have deduced the Reversed Cauchy Schwarz inequality

From the Field Equations

We start with the field equations[2]

$$R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta} = \frac{8\pi G}{c^4} T^{\alpha\beta} \quad (6)$$

The Christoffel symbols do not change when the metric signature reverses. Consequently the Riemann tensor and the Ricci tenors do not reverse sign when the metric signature changes. The Ricci scalar being defined by $R = g_{\mu\nu} R^{\mu\nu}$ reverses sign when the metric –signature reverses. The sign of $Rg^{\alpha\beta}$remains unchanged when the metric reverses it signature.

$$\Rightarrow R^{\alpha\beta} - \frac{8\pi G}{c^4} T^{\alpha\beta} = \frac{1}{2} R g^{\alpha\beta}$$

$$\Rightarrow g_{\alpha\beta}\left(R^{\alpha\beta} - \frac{8\pi G}{c^4} T^{\alpha\beta}\right) = \frac{1}{2} R g_{\alpha\beta} g^{\alpha\beta}$$

$$\Rightarrow g_{\alpha\beta}\left(R^{\alpha\beta} - \frac{8\pi G}{c^4} T^{\alpha\beta}\right) = 2R \quad (7)$$

In the orthogonal system (7) reduces to

$$\Rightarrow g_{\alpha\alpha}\left(R^{\alpha\alpha} - \frac{8\pi G}{c^4} T^{\alpha\alpha}\right) = 2R \quad (8)$$

We use the metric signature (+,-,-,-). $g_{tt} > 0, g_{ii} < 0; i \neq \alpha$. Henceforth we denote $|g_{ii}|$ by g_{ii}like

$$ds^2 = g_{\alpha\alpha} {dx^\alpha}^2 = g_{tt} dt^2 - g_{xx} dx - g_{yy} dy^2 - g_{zz} dz^2$$

We have from equation (8),

$$g_{00}\left(R^{00}-\frac{8\pi G}{c^4}T^{00}\right)-g_{11}\left(R^{11}-\frac{8\pi G}{c^4}T^{11}\right)-g_{22}\left(R^{22}-\frac{8\pi G}{c^4}T^{22}\right)-g_{33}\left(R^{33}-\frac{8\pi G}{c^4}T^{33}\right)$$
$$=2R\ \ ; g_{ii}>0 (9)$$

By the Reversed Cauchy Schwarz inequality

$$4R^2=\left[g_{00}\left(R^{00}-\frac{8\pi G}{c^4}T^{00}\right)-g_{11}\left(R^{11}-\frac{8\pi G}{c^4}T^{11}\right)-g_{22}\left(R^{22}-\frac{8\pi G}{c^4}T^{22}\right)\right.$$
$$\left.-g_{33}\left(R^{33}-\frac{8\pi G}{c^4}T^{33}\right)\right]^2\geq$$

$$({g_{00}}^2-{g_{11}}^2-{g_{22}}^2-{g_{33}}^2)\left[\left(R^{00}-\frac{8\pi G}{c^4}T^{00}\right)^2-\left(R^{11}-\frac{8\pi G}{c^4}T^{11}\right)^2-\left(R^{22}-\frac{8\pi G}{c^4}T^{22}\right)^2\right.$$
$$\left.-\left(R^{33}-\frac{8\pi G}{c^4}T^{33}\right)^2\right] (11)$$

$$\Rightarrow 4R^2\geq({g_{00}}^2-{g_{11}}^2-{g_{22}}^2-{g_{33}}^2)\left[\frac{1}{4}R^2(g^{00})^2-\frac{1}{4}R^2(g^{11})^2-\frac{1}{4}R^2(g^{22})^2-\frac{1}{4}R^2(g^{33})^2\right]$$

$$\Rightarrow 16\geq({g_{00}}^2-{g_{11}}^2-{g_{22}}^2-{g_{33}}^2)[(g^{00})^2-(g^{11})^2+(g^{22})^2+(g^{33})^2]$$

$$16\geq({g_{00}}^2-{g_{11}}^2-{g_{22}}^2-{g_{33}}^2)^2$$

$$-4\geq {g_{00}}^2-{g_{11}}^2-{g_{22}}^2-{g_{33}}^2\leq 4\ (12)$$

The inequality expressed by (12) is too restrictive :it is not a valid one.As an example it does not apply to Schwarzschild's geometry for suitable values of the coordinates. If $T^{\alpha\beta}=0$ [implying $R^{\alpha\beta}=0, R=0$]the issues considered in this article may be considered by a limiting process with $T^{\alpha\beta}\to 0$but $T^{\alpha\beta}\neq 0$.

The Issue of Dimensions

We may write equation (9) as

$$k_0 g_{00}\frac{1}{k_0}\left(R^{00}-\frac{8\pi G}{c^4}T^{00}\right)-k_1 g_{11}\frac{1}{k_1}\left(R^{11}-\frac{8\pi G}{c^4}T^{11}\right)-k_2 g_{22}\frac{1}{k_2}\left(R^{22}-\frac{8\pi G}{c^4}T^{22}\right)$$
$$-k_3 g_{33}\frac{1}{k_3}\left(R^{33}-\frac{8\pi G}{c^4}T^{33}\right)=2R\ (13)$$

The dimensions of k_i are decided in such a manner that

$$[k_0 g_{00}]=[k_1 g_{11}]=[k_2 g_{22}]=[k_3 g_{33}]\ \ (14)$$

Automatically we have

$$\left[\frac{1}{k_0}\left(R^{00}-\frac{8\pi G}{c^4}T^{00}\right)\right]=\left[\frac{1}{k_1}\left(R^{11}-\frac{8\pi G}{c^4}T^{11}\right)\right]=\left[\frac{1}{k_2}\left(R^{22}-\frac{8\pi G}{c^4}T^{22}\right)\right]$$
$$=\left[\frac{1}{k_3}\left(R^{33}-\frac{8\pi G}{c^4}T^{33}\right)\right]\ (15)$$

The reason behind this is, considering (9), and that addition can take place only with terms that have identical dimensions,

$$\left[g_{00}\left(R^{00}-\frac{8\pi G}{c^4}T^{00}\right)\right]=\left[g_{11}\left(R^{11}-\frac{8\pi G}{c^4}T^{11}\right)\right]=\left[g_{22}\left(R^{22}-\frac{8\pi G}{c^4}T^{22}\right)\right]$$
$$=\left[g_{33}\left(R^{33}-\frac{8\pi G}{c^4}T^{33}\right)\right]\ (16)$$

Lets take

$$\left[g_{ii}\left(R^{ii}-\frac{8\pi G}{c^4}T^{ii}\right)\right]=\left[g_{jj}\left(R^{jj}-\frac{8\pi G}{c^4}T^{jj}\right)\right]$$

$$\Rightarrow\left[k_i g_{ii}\frac{1}{k_i}\left(R^{ii}-\frac{8\pi G}{c^4}T^{ii}\right)\right]=\left[k_j g_{jj}\frac{1}{k_j}\left(R^{jj}-\frac{8\pi G}{c^4}T^{jj}\right)\right]\ (17)$$

Now,

$$[k_i g_{ii}]=[k_j g_{jj}]\Rightarrow\left[\frac{1}{k_i}\left(R^{ii}-\frac{8\pi G}{c^4}T^{ii}\right)\right]=\left[\frac{1}{k_j}\left(R^{jj}-\frac{8\pi G}{c^4}T^{jj}\right)\right]\ (18)$$

We may set the numerical value of k_i to be unity.

Conclusions

As claimed at the outset we have arrived at a surprising result connecting the metric coefficients . The constraint is quite unacceptable and calls for a revision of formal theory

References

1. Wikipedia, Cauchy Schwarz Inequality, link: https://en.wikipedia.org/wiki/Cauchy%E2%80%93Schwarz_inequality accessed on 8th December, 2019.
2. Hartle J B, Gravity, an Introduction to Einstein’s General Relativity, Pearson Education, Inc, Published by Dorling Kindrsley (India) Pvt Ltd licenses to Pearson Education , South Asia, First Impression, 2006,.page 506-507