[... *Title and contents of Section 1 omitted for succinctness*]

Abstract

With the vector form of ζ, RH's validity is direct.

1 Introduction

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2 Proof

Let:

$V(c) = (v_1(c), v_2(c), v_3(c), ...)$ where $v_n(c) = n^{-c}$ for $c \in \mathbb{C}$
$\sigma = a + it$
$\circ$ be Hadamard product and $\bullet$ be dot product
$COS = (\cos(t\ln(1)), \cos(t\ln(2)), \cos(t\ln(3)), ...)$
δ be any real value

and take note of the identity:

$$n^{\sigma} = n^{a}\cos(t\ln(n)) + i\sin(t\ln(n))$$

If $\sigma + \delta^{\dagger}$ and σ are both ζ roots, then $\zeta(\sigma) = \Sigma_{n=1}^{\infty} n^a(\cos(t\ln(n)) + i\sin(t\ln(n))) = (V(a) \circ COS) \bullet V(0) + i((V(a) \circ ...) \bullet V(0)) = 0$. But for a complex number to be zero, both the real and imaginary components will have to be simultaneously zero. Therefore:

$$\Sigma_{n=1}^{\infty} n^a(\cos(t\ln(n)) = 0 \qquad (\text{and } \Sigma_{n=1}^{\infty} n^a \sin(t\ln(n)) = 0)$$

$$\Sigma_{n=1}^{\infty} n^a n^{\delta}(\cos(t\ln(n)) = 0 \qquad (\text{and } \Sigma_{n=1}^{\infty} n^a n^{\delta} \sin(t\ln(n))) = 0)$$

It can be observed that the following vectors

$$V(a) \circ COS \tag{1}$$
$$V(a) \circ V(\delta) \circ COS \tag{2}$$
$$V(\beta)^{\ddagger} \tag{3}$$

are linearly independent, so not coplanar†† (unless $\delta = 0$). (*Q. E. D.*)

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†We ignore the symmetry and prove, equivalently, that there can not be more than one root among complex numbers with the same non-zero imaginary part it.

‡$V(\beta) \bullet V(0) = (V(\alpha + \delta) - V(\alpha)) \bullet V(0)$ for some β, observing that $V(x - y) \neq V(x) - V(y)$.

††If the assumption (of two distinct symmetric roots) holds, the dot products of (1), (2) and (3) with $V(0)$ will have to all result in 0, so (1), (2) and (3) will have to be all orthogonal to $V(0)$, implying they need be coplanar.