A SECOND NOTE ON A POSSIBLE ANOMALY IN THE COMPLEX NUMBERS

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Abstract. The paper gives an additional reason why, initially, there are two different solutions associated to a quadratic equation that indicates an anomaly in complex numbers. It is demonstrated that one of the solutions is impossible but plausible & necessary.

1. Introduction

In a previous note [1] it was already indicated that an anomaly in complex numbers is possible. This anomaly is based on Euler's identity and the DeMoivre rule [2]. In this present short note we will look at the problem from a different angle and find support for the basic requirement of two solutions. Nevertheless, one of the solutions cannot exist.

2. Complex number anomaly looked at differently

In elementary complex number theory [2] there are two basic principles that will be employed here. The first is Euler's identity. This is $\forall_{t\in\mathbb{R}}\ e^{it} = \cos(t) + i\sin(t)$. The second is the power rule of DeMoivre. This is, $\forall_{n\in\mathbb{N}}\left(\cos(x) + i\sin(x)\right)^n = \cos(nx) + i\sin(nx)$. Here we will use the easy to be verified form for $n = 2$.

Now let us look at the following expression for $\varphi \in \mathbb{R}$ and $\psi \in \mathbb{R}$.

$$z = \exp\left[i(\varphi + \psi)^2\right] \tag{2.1}$$

Hence, [2, p 68], for any $u \in \mathbb{C}$ and $w \in \mathbb{C}$, $\exp[(u + w)] = \exp(u)\exp(w)$.

$$z = \exp\left[i(\varphi^2 + \psi^2)\right]\exp\left[2i\chi\right] \tag{2.2}$$

and $\chi = \varphi\psi$. Let us, subsequently, look at $\varphi + \psi = \sqrt{\pi}$. According to (2.1) $z = e^{i\pi} = -1$. Moreover, if $\alpha = \varphi\left(\varphi - \sqrt{\pi}\right)$ then, via $\psi = \sqrt{\pi} - \varphi$

$$\begin{aligned}\chi &= -\alpha \\ \varphi^2 + \psi^2 &= \pi + 2\alpha\end{aligned} \tag{2.3}$$

Note we may take $\varphi \not\equiv 0$.

The previous gives rise to the following equation

$$\exp\left[2i\alpha\right] = -\exp\left[i(\varphi^2 + \psi^2)\right] \tag{2.4}$$

hence,

$$\exp\left[2i\alpha\right] = -\exp\left[i(\pi + 2\alpha)\right] = -\exp(i\pi) \times \exp\left[i2\alpha\right] \tag{2.5}$$

Because $\exp(i\pi) = -1$ and using Euler and DeMoivre, we find the obvious "triviality"

$$\left(\cos(\alpha) + i\sin(\alpha)\right)^2 = \left(\cos(\alpha) + i\sin(\alpha)\right)^2 \tag{2.6}$$

Obviously (2.6) is equivalent to $\cos(2\alpha) + i\sin(2\alpha) = \cos(2\alpha) + i\sin(2\alpha)$ but there is no compelling reason to *exclusively* look at this expression of the equation. The DeMoivre rule allows to continue from (2.6). Now let us take $\eta_1 = \pm 1$, and note, $\forall_{\eta_1\in\{-1,1\}}\eta_1^2 = 1$, then

$$(\eta_1)^2\left(\cos(\alpha) + i\sin(\alpha)\right)^2 = 1 \times \left(\cos(\alpha) + i\sin(\alpha)\right)^2 \tag{2.7}$$

Identically

$$\{\eta_1 (\cos(\alpha) + i \sin(\alpha))\}^2 = (\cos(\alpha) + i \sin(\alpha))^2 \tag{2.8}$$

So, with $\eta_2 = \pm 1$, and $\forall_{\eta_2 \in \{-1,1\}} \eta_2^2 = 1$,

$$1 \times \{\eta_1 (\cos(\alpha) + i \sin(\alpha))\}^2 = (\eta_2)^2 (\cos(\alpha) + i \sin(\alpha))^2 \tag{2.9}$$

or, equivalently

$$\{\eta_1 (\cos(\alpha) + i \sin(\alpha))\}^2 = \{\eta_2 (\cos(\alpha) + i \sin(\alpha))\}^2 \tag{2.10}$$

Obviously, we may have $(\eta_1, \eta_2) \in \{(-1,-1), (-1,1), (1,-1), (1,1)\}$.

Then we may note that from (2.10) it follows, introducing yet another $\eta_3 = \pm 1$

$$(\cos(\alpha) + i \sin(\alpha)) = \eta_1 \eta_2 \eta_3 (\cos(\alpha) + i \sin(\alpha)) \tag{2.11}$$

Then we may have

$$\begin{aligned}(\eta_1, \eta_2, \eta_3) \in \{&(-1,-1,-1), (-1,1,-1), (1,-1,-1), (1,1,-1),\\ &(-1,-1,1), (-1,1,1), (1,-1,1), (1,1,1)\} \equiv \Xi\end{aligned} \tag{2.12}$$

Hence, there is a subset of (η_1, η_2, η_3) is

$$\begin{aligned}\Xi_{neg} &= \{(\eta_1, \eta_2, \eta_3) \in \Xi \mid \eta_1 \eta_2 \eta_3 = -1\}\\ \Xi_{pos} &= \{(\eta_1, \eta_2, \eta_3) \in \Xi \mid \eta_1 \eta_2 \eta_3 = +1\}\end{aligned} \tag{2.13}$$

and $\Xi = \Xi_{neg} \cup \Xi_{pos}$. There is no compelling reason whatsoever to *only* have $(\eta_1, \eta_2, \eta_3) \in \Xi_{pos}$. Therefore we may use $(\eta_1, \eta_2, \eta_3) \in \Xi_{neg}$. With those η values we see an anomaly because we have $\nexists_{\alpha \in \mathbb{R}} \cos(\alpha) = \sin(\alpha) = 0$. In the derivation of the anomaly it is claimed that *valid* steps were taken. If a reader disagrees then the error in the sequence of derivation *must* be indicated.

3. Conclusion & discussion

In the paper it is demonstrated that the to be expected two solutions of a particular "trivial" equation

$$\cos(\alpha) + i \sin(\alpha) = \eta (\cos(\alpha) + i \sin(\alpha))$$

has only *one* solution, with $\eta = 1$ and not the expected two, $\eta = \pm 1$. The $\eta = -1$ is as plausible as the $\eta = 1$, because $\forall_{\eta \in \{-1,1\}} \eta^2 = 1$. We recall, $\nexists_{\alpha \in \mathbb{R}} \cos(\alpha) = \sin(\alpha) = 0$, which makes the $\eta = -1$ impossible. However, because of the quadrate, there is a solution which is based on the breakdown with $\eta = -1$. We showed that this is solution is contradictory, therefore, is impossible. Do please observe that there is no compelling reason, save arbitrary elimination, to reject the $\eta = -1$ breakdown. The $\eta = -1$ is derived with the same valid means as the $\eta = 1$, because for both $\eta^2 = 1$. If the reader thinks there is an error in the reasoning then the reader should *indicate* the error and *not* come with his or her own version of the story. The latter is absolutely not a valid indication of an error.

Finally, we note that the ± 1 manipulation given here has its application in e.g. Bell's theorem for spin measurement [3]. It is claimed that the consequences of ± 1 manipulation in Bell's formula, in case of ± 1 spin functions, is false in a different number of ways viz also [4]. The presented anomaly shows one of the falsehoods of that theorem in that the use of $\forall_{\eta \in \{-1,1\}} \eta^2 = 1$ may give unexpected results.

References

[1] Han Geurdes, A note on a possible anomaly in the complex numbers, vixra 1911.0311v3, (2019).
[2] J.V. Deshpande, Complex Analysis, 69 - 77 McGraw Hill Int., New Dehli (1986).
[3] J.S. Bell, On the Einstein Podolsky Rosen Paradox, Physics 1, 195-200, (1964).
[4] Han Geurdes & Koji Nagata, Bell's correlation formula and anomalous spin, vixra 1907.0448 (2019).

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