The Riemann flow (II)

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Abstract

The Riemann flow is defined with help of the riemannian curvature.

1 The Ricci flow

The Ricci flow is a flow for the metrics g of a riemannian manifold M [GHL]. It is defined with help of the Ricci curvature $Ric(g)$ which is a contraction of the riemannian curvature R_g [T].

$$r_g(x,y,z,t) = g(R_g(x,y)z,t)$$

$$Ric(g)(x,y) = \sum_i r_g(x,e_i,y,e_i)$$

$$\frac{\partial g}{\partial t} = -2Ric(g)$$

2 The Riemann flow

The Riemann flow is defined for the metrics g with help of the riemannian curvature [GHL] R_g. The operator d^*_∇ is defined by:

$$d^*_\nabla = * \circ d_\nabla \circ *$$

with $*$ the Hodge operator and d_∇ the differential.

$$d^*_\nabla R_g \in \Lambda(TM) \otimes End(TM)$$

The Riemann flow is:

$$\frac{\partial g}{\partial t}(X,Y) = -tr((d^*_\nabla R_g(X)) \circ (d^*_\nabla R_g(Y)))$$

3 The Einstein-Riemann metrics

The Einstein-Riemann metrics [B] are such that:

$$\lambda g(X,Y) = tr((d^*_\nabla R_g(X)) \circ (d^*_\nabla R_g(Y)))$$

with λ a scalar.

References

[B] A.Besse, "Einstein Manifolds", Springer Verlag, Berlin, 1987.

[GHL] S.Gallot, D.Hulin, J.Lafontaine, "Riemannian geometry", 3ed., Springer, Berlin, 2004.

[T] P.Topping, "Lectures on the Ricci flow", Cambridge University Press, Cambridge, 2006.