EVIDENCE, THAT $x^2 + y^3 = 1$ AND OTHERS HAVE NO SOLUTION IN $Q > 0$

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Abstract. Due to the Incompleteness Theorems of Gödel one can say, that some true conjectures do not have valid proofs. One could think it also about my conjectures below, but I was lucky to find evidence for them.

1. Some my theorems

If number w is irrational, then $W = w^{1/n}$ can not be rational for any integer n: if W is rational, then $w = W^n$ is rational as well. We came to contradiction with the condition of the theorem, thus, the W is irrational.

Any irrational number β can be expressed as $\beta = A\,B$, with irrational A and rational B. Hereby the A can be arbitrary close to $A = 1$. Then, because $\beta^{1/n}$ is irrational for any integer $n \geq 1$, then the expression $A^{1/n}\,B^{1/n}$ can not be rational for arbitrary irrational A and rational B. Generally, the $A^{1/(n\,\kappa)}\,B^{m/n}$ can not be rational, where integers $\kappa, m \geq 1$.

2. On $x^2 + y^3 = 1$

The $x^m + y^3 = 1$ has solutions in positive rational numbers, if $m = 1$. As example, $y = 1/2$, $x = 1 - (1/2)^3$. But already $m = 3$ has no positive rational solution due to Fermat's Last theorem. Indeed, for positive rational $x = a/c$, $y = b/j$

$$(a/c)^3 + (b/j)^3 = (aj/(jc))^3 + (bc/(jc))^3\,.$$

Then the Fermat's Last Theorem stays, [3] that

$$x_2^n + y_2^n = 1$$

does not have positive rational solution for all integers $n \geq 3$.

It follows, that

$$x_3^{n\,k} + y_3^{n\,r} = 1$$

does not have positive rational solution for all integers $n \geq 3$, $k \geq 1$, $r \geq 1$. Because for rational x_3 the $(x_3)^k$ is rational. Same for rational y_3 the $(y_3)^r$ is rational.

So, the question arises: $m = 2$. Let us call this case Martila conjecture. If conjecture is right, then the $x_4^{2\,p} + y_4^{3\,h} = 1$ does not have positive rational solution for all integers $p \geq 1$, $h \geq 1$.

So, $2\,p = n\,k$; $3\,h = n\,r$, then $p = n$, $k = 2$, $h = n$, $r = 3$.

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Therefore, the Martila conjecture leads to infinite number of true conjectures, latter ones differ by the choice of $n = 3, 4, 5, 6, 7, 8, ...$ Therefore, this is evidence, that Martila Conjecture is true. You might argue, that if to forget about wrong-hood of the $m = 1$ case then this case also leads to infinite number of true conjectures. But, speaking about the probabilities, we are not speaking about our memories, but about the objective reality. Thus, we are not allowed to forget wrong-hood of $m = 1$ case.

3. Evidence, that $x^m + y^n = 1$ has no solution in positive rational numbers for all integers $n \geq 3,\ m \geq 2$

Let us call it the Martila Second Conjecture. From it follows that

$$x_5^{m\,k} + y_5^{n\,r} = 1$$

does not have solution in positive rational numbers x_5, y_5. From Euler's proof [1] for $n = 3$ case follows, that

$$x_6^{3\,p} + y_6^{3\,h} = 1$$

does not have solution.

Then $m\,k = 3\,p$, $n\,r = 3\,h$, so $k = 3$, $m = p$, $r = 3$, $n = h$.

Therefore, the Martila Second Conjecture leads to infinite number of true conjectures. Therefore, this is evidence, that Martila Second Conjecture is true.

From validity of Martila Second Conjecture follows the validity of Fermat's Last Theorem, when $m = n$. The humankind is not yet done with Fermat's Last Theorem: [2].

3.1. **Evidence for Martila's Third Conjecture.** Same way one finds evidence for

$$A^N + B^m = C^\kappa$$

having no solution in positive rational numbers A, B, C; if the integers $N, m, \kappa \geq 3$.

PROOF: Indeed, if for the six integers holds

$$(k_1/k_2)^n + (k_3/k_4)^n = (k_5/k_6)^n$$

then

$$(k_1\,k_4\,k_6/(k_2\,k_4\,k_6))^n + (k_3\,k_2\,k_6/(k_4\,k_2\,k_6))^n = (k_5\,k_4\,k_2/(k_6\,k_4\,k_2))^n\,,$$

$$(k_1\,k_4\,k_6)^n + (k_3\,k_2\,k_6)^n = (k_5\,k_4\,k_2)^n\,,$$

which due to Fermat's Last Theorem can not hold, if $n \geq 3$. Therefore, the

$$a^n + b^n = c^n$$

has no solution in positive rational numbers a, b, c; and then

$$x^{\alpha\,n} + y^{\beta\,n} = z^{\gamma\,n}$$

has no rational solution, if $\alpha, \beta, \gamma \geq 1$. Then equating $\alpha\,n = u\,N$, $\beta\,n = v\,m$, $\gamma\,n = d\,\kappa$, where $u, v, d \geq 1$, we have infinitely many true conjectures for each fixed three N, m, κ according to the values of $n = 3, 4, 5, ..., \infty$ in: $\alpha = N$, $n = u$, $n = v$, $\beta = m$, $n = d$, $\kappa = \gamma$.

4. Do numerical tests of Fermat's Conjecture and Riemann Hypothesis increase the probability, that they are true?

The distribution of counter-examples to theorem has a density: $\rho = n/N$, $0 \le \rho < 1$ where $N \gg 1$ is total number of tests (made or to be made), the n is the number of counter-examples. Because there is no preference between the different values of ρ, one uses unity as probability distribution function $p(\rho) = 1$. Because the $N < \infty$ is always finite, the $\rho = 0$ would mean $n = 0$. If $M \gg 1$ tests are made without found counter-example, then the average density of counter-examples $\rho \le 1/M$. Then probability to find counter-example is $P(\rho \le 1/M) = \int_0^{1/M} p(\rho) = 1/M$. Now because there are almost infinite number of tests made ($M \gg 1$), then probability is much better than 5σ (the 5σ is used in experimental physics), that Fermat's Theorem and Riemann Hypothesis are true.

References

[1] J. J. Mačys, "On Euler's hypothetical proof", Mathematical Notes **82**(3–4), 352–356 (2007).

[2] Colin McLarty, "What does it take to prove Fermat's last theorem? Grothendieck and the logic of number theory," Bulletin of Symbolic Logic **16**(3), 359–377 (2010).

[3] Andrew Wiles, "Modular elliptic curves and Fermat's last theorem", Annals of Mathematics. **141**(3), 443–551 (1995).

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