

# Some Problems about the Massive Neutrino Oscillation and New Explanation for the Neutrino Observations

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**Abstract** – We review the massive neutrino oscillation and find some problems about it. The original theory predicts the mass differences existing on three kinds of neutrinos. However, if no external energy or mass participates in the transition process, it will experience the non-conservation of mass when one neutrino transits to others and then transits back to itself again. It also violates one of the conservation of laws of energy and momentum. Furthermore, according to the conservation of momentum in the special relativity, the speeds of neutrinos before and after transition must be different because the mass is non-conserved. It results in the special physical phenomena of self-acceleration and self-deceleration. **Especially, it clearly reveals the unhappened neutrino transition at some positions varying with kinetic conditions which means the massive neutrino oscillation is controlled by its kinetic value! On the other words, when the measurements are proceeded at these zero transition places, then the results show no neutrino oscillation phenomenon.** Even the violation of the Lorentz invariance is proposed in the standard model extension to discuss the neutrino oscillation without the existence of the mass difference, the all other original elementary particles predicting by the standard model will lose their criteria because they seriously obey the Lorentz invariance. After reviewing the results of Super-Kamiokande Collaboration and Sudbury Neutrino Observatory, both results strongly imply the ratio of number between three kinds of neutrinos is approximately  $\nu_e:\nu_\mu:\nu_\tau=1:1:1$ . According to this, we propose a new explanation for the observation data. The detection of neutrinos in the supernova SN 1987A event earlier than light may tell us the truth that the mass of neutrino is very possibly zero. Otherwise, the non-zero mass neutrino must be dragged by gravity to slow down its average velocity whole the traveling period.

**Keywords:** neutrino, neutrino oscillation, lepton, weak interaction, standard model

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## I. Introduction

The neutrino was first proposed by W. Pauli in 1930 for explanation of missing energy and momentum in  $\beta$  decays [1,2]. **Since neutrinos interact very weakly with other known particles, they are much difficult to detect. The investigation of the**

neutrino is based on the conservations of momentum and energy which must be satisfied for each detecting case. Due to the conservations of energy and momentum, the energy spectrum of  $\beta$  decay does have a clear upper limit, and therefore it was concluded that neutrinos do exist in 1933. The experimental group led by Cowan and Reines detected the electron neutrinos  $\nu_e$  firstly produced by nuclear reactors in 1956 [1-4]. Next, the conclusion that the chiral characteristics of neutrinos produced by weak-interaction decay are all left-handed was obtained in 1958 [1,2,5]. The  $\nu_\mu$  neutrinos associated with the  $\mu$  charged lepton were confirmed by experiments in 1962 [6]. Until 2000, the neutrino of the third-generation lepton,  $\nu_\tau$ , was detected in Fermilab [7]. Nowadays in the standard model, all the neutrinos only appear in the left-handed form and it is zero mass in this model.

Neutrino is thought to have a special characteristic, the neutrino oscillation, referring to the phenomenon that different types of neutrinos can transit from one to others which was first proposed in 1957 [8]. The mixing and oscillation of the different generation neutrinos was first discussed in 1962 [9]. The neutrino oscillation further indicates neutrinos having non-zero mass and it exists the mixing between the different flavors of neutrinos. However, the difficulty to detect neutrinos is that they really interact with matter very weakly. Averagely speaking, one neutrino has to pass through water more than several hundred light-years to take place one interaction. Therefore, the detection of neutrinos is a challenge that requires a lot of detectors and the cutting-edge technology.

The first experiment in the gold mine of Homestake about the detection of the solar neutrinos was announced by David in 1968 [9]. Recently, neutrino detections in Super-Kamiokande (SK) Collaboration [11] and Sudbury Neutrino Observatory (SNO) [12,13] revealed the observation data which could prove the massive neutrino oscillation. But an immediate problem is that the existence of the mass difference between neutrinos causes neutrinos before and after transition exhibit different speeds. If there were no other mass or energy involving this transition, these neutrinos with different mass will result in the violation of some physical conservation laws. This problem has been tried to solve by the standard model extension which discusses the neutrino oscillation without the existence of the mass difference. Therefore, we propose some serious problems about the neutrino oscillation and offer new explanation for the neutrino observations in SK and SNO as well as the supernova SN 1987A event.

## II. The Problems About The Massive Neutrino Oscillation

In 1932, the electron neutrino  $\nu_e$  was first investigated by Sir James Chadwick in the neutron beta-decay [2]

$$n^0 \rightarrow p^+ + e^- + \bar{\nu}_e. \quad (1)$$

The Feynman diagram describing this reaction is shown in Fig. 1(a) which involves the weak interaction with the charged boson  $W^-$ . In 1956, Cowan and Reines measured this following reaction near the nuclear reactor [2]

$$\bar{\nu}_e + p^+ \rightarrow n^0 + e^+. \quad (2)$$

The electron antineutrino was first found in the experiments. In 1962, the muon antineutrino was also investigated from the similar reaction at Brookhaven by replacing  $(\bar{\nu}_e, e^+)$  with  $(\bar{\nu}_\mu, \mu^+)$  [2]

$$\bar{\nu}_\mu + p^+ \rightarrow n^0 + \mu^+. \quad (3)$$

In 1956, the neutrino flux was recorded as high as  $5 \times 10^{13} \text{ particles/cm}^2 \cdot \text{sec}$  [2]. However, total  $10^{14}$  antineutrinos from  $\pi^-$  decays were used in the experiments but only 29 instances were identified in 1962 [2]. Seriously speaking, the neutrino physics is like a field to study the highly invisible particles.

In quantum field theory (QFT), neutrinos are produced by the weak interactions. Each neutrino and its corresponding charged lepton are generated by the  $W^-$  or  $W^+$  decays, where  $W^-$  and  $W^+$  are the charged, spin-1, and massive bosons [1,2,14,15]. In the pion decay for the first generation in the lepton section, the reaction is [1,2,14,15]

$$\pi^- \rightarrow e^- + \bar{\nu}_e, \quad (4)$$

and it takes place through the weak charged current [1,2,14,15] as shown in Fig. 1(b). The interaction is the  $\Phi^3$  structure and its Lagrangian is [1,2,14,15]

$$L_{W\pi} = -g_{W\pi}(J_\mu^{W^-} W^{+\mu} + J_\mu^{W^+} W^{-\mu}), \quad (5)$$

where  $J_\mu^{W^-}$  and  $J_\mu^{W^+}$  are weak charged currents and  $g_{W\pi}$  is the coupling constant. It is similar to the one that can describe the pair-production process by a photon [14,15]

$$L_{EM} = eJ_{EM}^\mu A_\mu, \quad (6)$$

where  $e$  is the unit charge,  $J_{EM}^\mu$  is the electromagnetic current, and  $A_\mu$  is the photon field. The weak charged current has two parts which are shown as

$$J_\mu^{W^-} = J_{e\mu}^{W^-} + J_{H\mu}^{W^-}, \quad (7)$$

where

$$J_{e\mu}^{W^-} = \bar{\Psi}_e \gamma^\mu (1 - \gamma^5) \Psi_{\nu_e}, \quad (8)$$

and

$$J_{H\mu}^{W^-} = f_\pi \partial_\mu \Phi_\pi, \quad (9)$$

where  $\gamma^\mu$  ( $\mu = 0,1,2,3$ ) are the Dirac's 4 matrices,  $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ ,  $\Phi_\pi$  is the positively charged pion field, and  $f_\pi$  is the pion decay constant. The general Lagrangian of the charged current for three generations of leptons is [14,15]

$$L = -\frac{g}{\sqrt{2}}\bar{\Psi}_L\gamma^\mu(1 - \gamma^5)\Psi_{\nu_L}W^- + H.C., \quad (10)$$

where all the first elements of the left-handed doublets for the three-generation leptons are  $\Psi_L = (\Psi_e, \Psi_\mu, \Psi_\tau)^T$ , and all the second elements of the left-handed doublets for the three-generation leptons are  $\Psi_{\nu_L} = (\Psi_{\nu_e}, \Psi_{\nu_\mu}, \Psi_{\nu_\tau})^T$ . If the neutrinos have mass, their quantity would be very small. Direct measurements, such as the electron energy spectrum of the Tritium's beta decay, give the electron neutrino mass less than 1 eV [16]. Recently, the research in cosmology even revealed that the upper limit of three generations of lepton neutron mass is reported as 0.3 eV [17]. But, in the standard model, the mass of each neutrino is supposed to be zero [14,15].

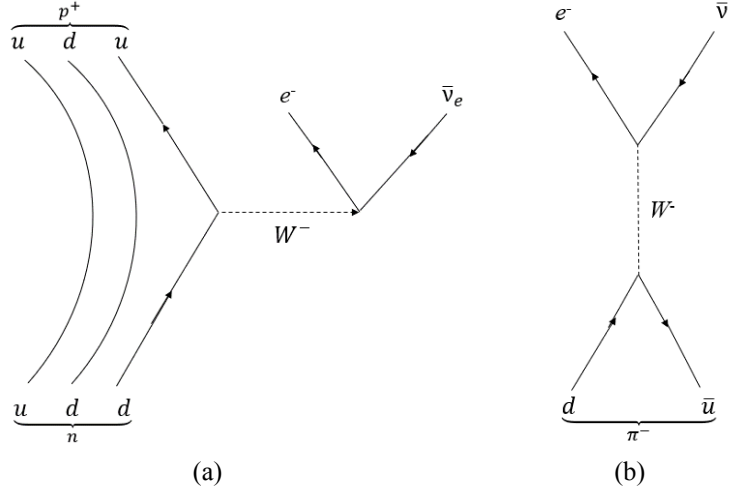


Figure 1. (a) The neutron decay through the weak interaction [1,2]. (b) The negative pion decay in the weak interaction [1,2].

However, SK only tells us that the oscillation from  $\nu_e$  to  $\nu_\mu$  or  $\nu_\tau$  can explain the flow problem of the solar neutrinos, but it does not prove that the missing part of  $\nu_e$  just transits to  $\nu_\mu$  and  $\nu_\tau$ . Fortunately, the experiments in SNO can give us more information about it. The SNO's experiments use heavy water  $D_2O$  as a target to detect neutrinos. They mainly measure three reaction processes [12]:

$$\nu_e + d^+ \rightarrow p^+ + p^+ + e^- \quad (11)$$

$$\nu_x + d^+ \rightarrow p^+ + n^0 + \nu_x \quad (12)$$

$$\nu_x + e^- \rightarrow \nu_x + e^- \quad (13)$$

The first reaction is the charged-current (CC) process only for  $\nu_e$ , the second one is the

neutral-current (NC) process for all three kinds of neutrinos, and the third one is the elastic process (ES) also for all three kinds of Neutrinos. The first and third statistical data are respectively [12]

$$\Phi^{CC} = 1.75 \pm 0.07(stat.)_{-0.11}^{+0.12}(sys.) \pm 0.05(theor.) \times 10^6 \text{ cm}^{-2}\text{s}^{-1} \quad (14)$$

and

$$\Phi^{ES} = 2.39 \pm 0.34(stat.)_{-0.14}^{+0.16} \times 10^6 \text{ cm}^{-2}\text{s}^{-1}. \quad (15)$$

In general, the neutrinos of the weakly acting eigenstates are not the eigenstates of mass  $\nu_m$ . But quantum mechanics tells us that due to the completeness of the eigenstates, the eigenstates of the weak interactions and mass eigenstates can represent each other in this formula  $N_L = V_{PMNS}N_L^m$ , that is,  $\nu_e$  is a linear combination of different  $\nu_m$  eigenstates. For three generations of neutrinos,  $V_{PMNS}$  is a  $3 \times 3$  positive matrix, which describes the mixing characteristics of three generations of neutrinos. The matrix  $V_{PMNS}$  is often written as [14,15]

$$V_{PMNS} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu1} & V_{\mu2} & V_{\mu3} \\ V_{\tau1} & V_{\tau2} & V_{\tau3} \end{pmatrix}. \quad (16)$$

$V_{PMNS}$  can be theoretically described by four independent parameters, three mixing angles  $\theta_{12}$ ,  $\theta_{23}$ ,  $\theta_{13}$ , and one phase term  $\delta$ . The commonly standard form for use is [18]

$$V_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \text{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2}), \quad (17)$$

where  $c_{ij} = \cos\theta_{ij}$  and  $s_{ij} = \sin\theta_{ij}$ . The oscillation probability from  $\nu_\alpha$  to  $\nu_\beta$  is given by the following formula [18]

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i \neq j} \text{Re}(V_{\alpha i}^* V_{\alpha j} V_{\beta i} V_{\beta j}^*) \sin^2(\Delta m_{ij}^2 x / 4E) + 2 \sum_{i \neq j} \text{Im}(V_{\alpha i}^* V_{\alpha j} V_{\beta i} V_{\beta j}^*) \sin(\Delta m_{ij}^2 x / 2E), \quad (18)$$

where  $x$  is the propagation distance from the origin at time  $t=0$  and  $E$  is the total energy of neutrino.  $E$  is assumed to be a constant before and after the transition.

Next, consider the case of only two-kind neutrinos for the easily understanding purpose, i.e.  $\nu_e$  and  $\nu_\mu$ , and  $\theta$  is the mixing angle. Since neutrinos have very low activity interacting with other substances, the energy of the propagation process can be treated as a conserved quantity. The individual energy of the  $\nu_m$  eigenstate is  $E_i = (m_i^2 c^4 + c^2 p_i^2)^{1/2}$  where  $m_i$  is the mass and  $p_i$  is the momentum for the  $i$ th eigenstate

with  $i=1,2$ . In the quantum-mechanical language,  $\nu_e$  and  $\nu_\mu$  states can be related to the mass eigenstates of  $\nu_{m1}$  and  $\nu_{m2}$  by using the mixing matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_{m1} \\ \nu_{m2} \end{pmatrix}. \quad (19)$$

This equation gives two  $\nu_e$ - and  $\nu_\mu$ - neutrino states in terms of the mass eigenstates which are respectively

$$|\nu_e\rangle = \cos \theta |\nu_{m1}\rangle + \sin \theta |\nu_{m2}\rangle, \quad (20)$$

and

$$|\nu_\mu\rangle = -\sin \theta |\nu_{m1}\rangle + \cos \theta |\nu_{m2}\rangle. \quad (21)$$

However, both momenta of the mass eigenstates are  $p_1$  and  $p_2$ , respectively. It means that two parts of  $\nu_e$  move inconsistently in space if  $v_1 \neq v_2$ . That will cause  $\nu_e$  separated as shown in Fig. 2, so its two parts must have the same speed at  $t=0$ . It means

$$\frac{p_1 c^2}{E_1} = v_1 = v_2 = \frac{p_2 c^2}{E_2}. \quad (22)$$

In reality, the experiments tell us that the detection of each coming particle shows the characteristic definitely belong to one of the three neutrinos, not the one part of one neutrino and the other part of the other neutrinos. Based on this, the neutrino doesn't move in separation. Otherwise, different parts of the original neutrino will move in different speed and we have to explain where the second part goes if only the early arrival part is detected?

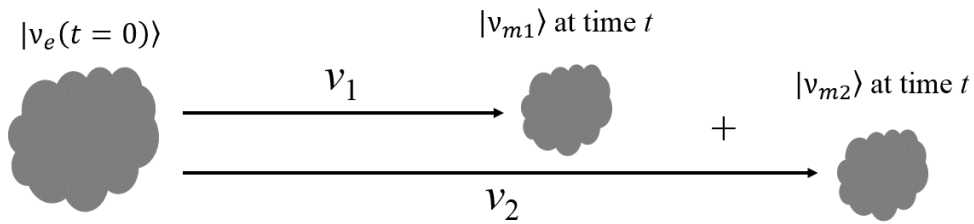


Figure 2. According to the *PMNS* matrix, if two parts of the neutrino  $\nu_e$  moving in different speeds in space, they will separate from each other in some period.

Next, using Dirac's notation, the  $\nu_e$ -neutrino state generated at  $x=0$  and  $t=0$  is

$$|\nu_e(0)\rangle = |\nu_e\rangle = \cos \theta |\nu_{m1}\rangle + \sin \theta |\nu_{m2}\rangle, \quad (23)$$

where the  $\nu_\mu$ -neutrino state is

$$|\nu_\mu(0)\rangle = 0 \neq |\nu_\mu\rangle. \quad (24)$$

After the generation of neutrino, it will propagate in terms of the mass eigenstates in

the free space. At time  $t$  and the propagation distance  $x$ , the  $\nu_e$ -neutrino state becomes

$$|\nu_e(t)\rangle = \cos\theta e^{i(p_1x - E_1t)/\hbar} |\nu_{m1}\rangle + \sin\theta e^{i(p_2x - E_2t)/\hbar} |\nu_{m2}\rangle. \quad (25)$$

Hence, the probability amplitudes of  $\nu_e$  and  $\nu_\mu$  measured at time  $t$  are

$$\langle \nu_e | \nu_e(t) \rangle = \cos^2\theta e^{i(p_1x - E_1t)/\hbar} + \sin^2\theta e^{i(p_2x - E_2t)/\hbar} \quad (26)$$

and

$$\langle \nu_\mu | \nu_e(t) \rangle = -\sin\theta \cos\theta e^{i(p_1x - E_1t)/\hbar} + \sin\theta \cos\theta e^{i(p_2x - E_2t)/\hbar}. \quad (27)$$

Therefore, the probability of the transition from  $\nu_e$  to  $\nu_\mu$  is

$$P(\nu_e \rightarrow \nu_\mu) = |\langle \nu_\mu | \nu_e(t) \rangle|^2 = \sin^2(2\theta) \cdot \sin^2\left(\frac{\Delta px - \Delta Et}{2\hbar}\right), \quad (28)$$

where the speed  $x/t = v = v_1 = v_2$ ,  $\Delta p = (p_2 - p_1)$ , and  $\Delta E = (E_2 - E_1)$ . Similarly, the probability to hold on the  $\nu_e$ -neutrino state is

$$P(\nu_e \rightarrow \nu_e) = |\langle \nu_e | \nu_e(t) \rangle|^2 = 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta px - \Delta Et}{2\hbar}\right). \quad (29)$$

The above two equations show oscillation in time dependent on the initial condition  $(v\Delta p - \Delta E)$  so the period  $T$  between two maxima that the part of  $\nu_e$  transits mostly to  $\nu_\mu$  is

$$T = \frac{\pi\hbar}{v\Delta p - \Delta E}. \quad (30)$$

From Eqs. (28) to (30), it clearly reveals that the massive neutrino oscillation does not happen when  $v = \Delta E / \Delta p$ . Considering the average momentum in the  $x$  direction at time  $t$ , it is

$$\begin{aligned} \overline{p(t)} &= \sum_{i=e,\mu} \langle \nu_e(t) | \nu_i \rangle \langle \nu_i | \hat{p} | \nu_i \rangle \langle \nu_i | \nu_e(t) \rangle \\ &= (p_1 \cos^2\theta + p_2 \sin^2\theta) + (p_2 - p_1) \cos 2\theta \sin^2(2\theta) \sin^2\left(\frac{\Delta px - \Delta Et}{2\hbar}\right), \end{aligned} \quad (31)$$

where  $\hat{p}$  is the momentum operator. This result tells us the variation of momentum in time for the initially free neutrino  $\nu_e$ . Furthermore, the average total mass  $M$  at time  $t$  is

$$\begin{aligned} \overline{M} &= \sum_{i=e,\mu} \langle \nu_e(t) | \nu_i \rangle \langle \nu_i | \hat{m} | \nu_i \rangle \langle \nu_i | \nu_e(t) \rangle \\ &= (m_1 \cos^2\theta + m_2 \sin^2\theta) + (m_2 - m_1) \cos 2\theta \sin^2(2\theta) \sin^2\left(\frac{\Delta px - \Delta Et}{2\hbar}\right), \end{aligned} \quad (32)$$

where  $\hat{m}$  is the mass operator and  $m_1$  and  $m_2$  are the aforementioned eigenvalues of the two-mass eigenstates. It obviously that the average total mass  $M$  is not a constant and dependent on time. If the **massive** neutrino oscillation happens, mass will exist some difference in time after one neutrino transits to another because the average total mass is non-conserved. According to the transition period  $T$ , a part of  $\nu_e$  transits to  $\nu_\mu$  and then transits back to itself totally after another period time  $T$  as shown in Fig.3. The mass will change from  $m_{\nu_e}$  to  $m_{\nu_\mu}$  and back to  $m_{\nu_e}$  again. If there were no additional mass or energy participating in this transition and  $m_{\nu_\mu} \neq m_{\nu_e}$ , then the conservation of mass is directly broken. If this transition exists, the mass difference also causes one serious problem. Supposing the initial speed of the neutrino  $\nu_e$  is  $u_e$  and the final speed of the neutrino is  $u_\mu$ . According to the conservation of energy in the special relativity, it gives the equivalence

$$\gamma_e m_{\nu_e} c^2 = \gamma_\mu m_{\nu_\mu} c^2 \quad (33)$$

where  $\gamma$  is the Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}. \quad (34)$$

Substituting Eq. (34) into Eq. (33) and using the condition  $m_{\nu_\mu} \neq m_{\nu_e}$ , then we have

$$u_\mu \neq u_e. \quad (35)$$

It clearly shows that the speed before and after transition must be different because two neutrinos have different mass if no other particles participate in the transition. Then neutrino will perform self-acceleration or self-deceleration without any external force after time  $2T$ ! This directly violate the conservation of momentum especially in the elementary particle physics. It means that the **massive** neutrino oscillation cannot happen due to the violations of the conservations of mass and momentum. No matter what kind of neutrinos is detected, the equipment must detect one complete neutrino belong to one of the three neutrinos,  $\nu_e$ ,  $\nu_\nu$ , and  $\nu_\mu$ , in each detection like the reaction in Eq. (11). Therefore, we can check the mass before and after the transition by the conservations of momentum and energy in the special relativity. The conservation of momentum is

$$\gamma_e m_{\nu_e} u_e = \gamma_\mu m_{\nu_\mu} u_\mu. \quad (36)$$

Combing Eq. (36) with Eq. (33), it gives

$$u_\mu = u_e, \quad (37)$$

and

$$m_{\nu_\mu} = m_{\nu_e}. \quad (38)$$



The laws of conservations of momentum and energy clearly tell us that even the transition exists, the mass as well as velocity must be constant before and after the transition!

Even some research points out the neutrino oscillation possibly existing at Lorentz and  $CPT$  violation [19], the direct violation of Lorentz invariance still makes some serious problem. The Lorentz violation of neutrino means the existence of this violation from the beginning since the neutrino's birth. Such unique spacetime for the neutrino makes it inconsistent with other elementary particles describing by the standard model based on the Lorentz invariance so as to result in neutrino oscillation more questionable and doubtful. Then without the Lorentz invariance, how to describe and calculate the following reaction because it obeys the Lorentz invariance [2]?

$$\nu + n \rightarrow p^+ + e^- . \quad (39)$$

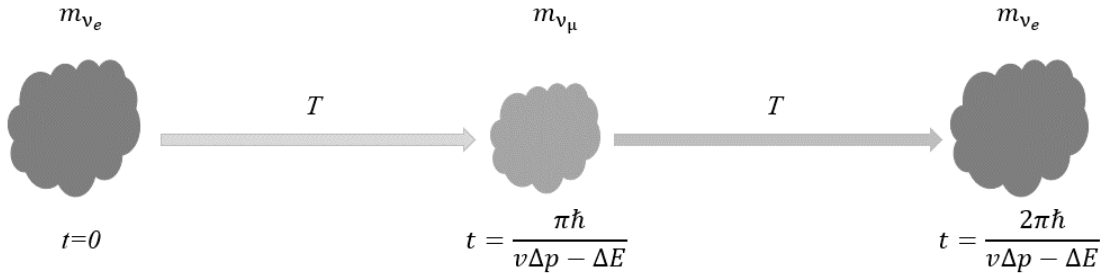


Figure 3.  $\nu_e$  transits to  $\nu_\mu$  maximally after time  $T$  and transits itself totally after  $2T$ . If there were no additional mass or energy and the mass of both neutrinos  $m_{\nu_\mu} \neq m_{\nu_e}$ , then the conservations of mass as well as momentum are directly broken in this case which are shown in Eqs. (31) and (32).

### III. Dose The Heisenberg Uncertainty Principle Allow The Violation Of The Conservations Of Energy And Momentum Significantly?

Since we deal with the quantum problem, then we have to face the Heisenberg uncertainty Principle. Dose the Heisenberg uncertainty principle allow the significant violation of the conservations of momentum and energy? Someone might say that the Heisenberg uncertainty principle allows for coherence of two superposition mass eigenstates, without violating the conservations of energy and momentum. We might ask whether it is true or not?

We can explain what problems the massive neutrino oscillation phenomenon will encounter when applying the Heisenberg uncertainty principle. The upper limit of three generations of lepton neutron mass is reported as 0.3 eV [17], so the average maximum mass of each neutrino is 0.1 eV. Considering that solar neutrinos have an initial energy of 1 MeV, it belongs to an extremely relativistic case, and the corresponding Lorentz factor  $\gamma$  is about  $10^7$ . Using Eq. (28) for the two massive neutrino oscillation probabilities, we suppose that the maximum probability of an electron neutrino becoming a muon neutrino is 0.5, and the mass difference between the two neutrinos is about  $\Delta m \sim 0.0086$  eV [18]. Then the average energy measured at the maximum

transition probability on the Earth is

$$E \sim \gamma [0.5 \times (0.1) + 0.5 \times (0.1 + 0.086)] = 1.043 \text{ MeV}. \quad (40)$$

That is to say, when this neutrino travels from the sun to the Earth, the average energy at the maximal transition probability of neutrons will increase by 43,000 eV. Where does so much energy come from? When calculating the energy uncertainty according to the uncertainty principle, in order to produce such an energy uncertainty effect, it is necessary to be able to measure the neutrino position to an accuracy of 1Å! For neutrinos, such accuracy is a fantasy. If we think that the Lorentz factor using in Eq. (40) is not a constant, then we must use the conservations of energy and momentum to calculate the Lorentz factor. Therefore, we cannot avoid to use the conservation of energy and momentum here. In the field of astronomy, neutrino energy can be as high as 1 GeV or even 1 TeV. For a 1 TeV neutrino, if the aforementioned two massive neutrino oscillations exist, the energy gap will reach 43 GeV! Such energy gap means that we have measure the position of neutrino at much higher accuracy than the solar neutrino. Thus, it even leads to such conclusion that energy comes from nothing, and it cannot satisfy the conservation of momentum! Actually, it has been stated that two or more virtual processes must be combined so that energy conservation is only violated in a short time  $\Delta\tau$  compatible with the principle of energy-time uncertainty, that is,  $\Delta\tau\Delta E \sim \hbar$  [1]. Therefore, the energy uncertainty  $\Delta E$  is totally not a way to explain the energy non-conservation in the massive neutrino oscillation.

Neutrino mixing is a quantum phenomenon and can be described in Quantum Mechanics. Neutrino states are represented as a wave function describing probability of measurements as shown in Eqs. (18), (28), and (29). According to the Quantum-Mechanics rules, only one of eigenstates can be observed with the probability described by the wave function in each measurement. The neutrino wave function evolves periodically in time, which is described by Eq. (25) in a neutrino superposition state. Actually, we do not know which state it will be before measurement. In a measurement, when one of actual eigenstates is observed, it means that this neutrino interacts with the experimental setup and we will know which kind of neutrinos is detected. All the detections can give a statistical distribution and, therefore, the probability of each neutrino at certain measuring place can be measured. As mentioned in Introduction, the neutrino was first proposed by W. Pauli in 1930 for explanation of missing energy and momentum in  $\beta$  decays [1,2]. Since neutrinos interact very weakly with other known particles, they are much difficult to detect. The investigation of the neutrino is based on the conservations of momentum and energy which must be satisfied for each detecting case. Due to the conservations of energy and momentum, the energy spectrum of  $\beta$  decay does have a clear upper limit, and therefore it was concluded that neutrinos do

exist in 1933. The other thing that we have to notice is the value in Eq. (40), where the detecting place is at the maximum transition probability in Eq. (28). If we adopt to detect muon neutrino at its minimum transition probability, then we will not detect anyone of them. It means that the total detecting neutrino is electron neutrino and thus, the conservations of energy and momentum are automatically preserved. This place is not like the one describing in Eq. (40) and on the contrary, the Heisenberg uncertainty principle is not necessary to be considered. Therefore, the detecting places at the maximum transition probability and the minimum transition probability give a totally different result. The former needs to consider the energy uncertainty and the later does not need to think about it. These two places give the violation results for the same detecting thing.

Except for the viewpoint of the Heisenberg uncertainty principle, let us look at how to calculate some decay rates and scattering involving the weak interaction as well as neutrinos. The Golden rule gives the decay rate of the muon decay in the muon rest frame [2]

$$d\Gamma = \frac{\langle |M|^2 \rangle}{2\hbar m_\mu} \left[ \frac{cd^3\mathbf{p}_2}{(2\pi^3)2E_2} \right] \left[ \frac{cd^3\mathbf{p}_3}{(2\pi^3)2E_3} \right] \left[ \frac{cd^3\mathbf{p}_4}{(2\pi^3)2E_4} \right] (2\pi^4) \delta^4(p_1 - p_2 - p_3 - p_4), \quad (41)$$

where  $p_1, p_2, p_3$ , and  $p_4$  are the four momenta of muon, electron anti-neutrino, muon neutrino, and electron, respectively,  $m_\mu$  is the muon mass, and the average amplitude square with the weak coupling constant  $g_w$  and the mass of the  $W$  boson  $M_W$  is [2]

$$\begin{aligned} \langle |M|^2 \rangle &= 2 \left( \frac{g_w}{M_W c} \right)^4 (p_1 \cdot p_2)(p_3 \cdot p_4) \\ &= \left( \frac{g_w}{M_W c} \right)^4 m_\mu^2 E_2 \left[ \left( m_\mu - \frac{m_e^2}{m_\mu} \right) c^2 - 2E_2 \right]. \end{aligned} \quad (42)$$

Similarly, in the neutron decay as the reaction in Eq. (1), the Golden rule gives the decay rate in the rest frame of the neutron [2]

$$d\Gamma = \frac{\langle |M|^2 \rangle}{2\hbar m_n} \left[ \frac{cd^3\mathbf{p}_2}{(2\pi^3)2E_2} \right] \left[ \frac{cd^3\mathbf{p}_3}{(2\pi^3)2E_3} \right] \left[ \frac{cd^3\mathbf{p}_4}{(2\pi^3)2E_4} \right] (2\pi^4) \delta^4(p_1 - p_2 - p_3 - p_4), \quad (43)$$

where  $p_1, p_2, p_3$ , and  $p_4$  are the four momenta of neutron, electron antineutrino, proton, and electron, respectively,  $m_n$  is the neutron mass, and the expression of the average amplitude square is the same as Eq. (42). Another case is about the elastic muon neutrino-electron scattering in the CM frame, in which the Golden rule gives the differential cross-section [2]

$$d\sigma = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{\langle |M|^2 \rangle_c}{(E_1 + E_2)|\mathbf{p}_1|} \frac{d^3\mathbf{p}_3 d^3\mathbf{p}_4}{E_3 E_4} \delta^4(p_1 + p_2 - p_3 - p_4), \quad (44)$$

where  $p_1, p_2, p_3$ , and  $p_4$  are the four momenta of the incoming muon neutrino, incoming electron, outgoing muon neutrino, and outgoing electron, respectively, and the average amplitude square with the weak coupling constant  $g_z$ , the mass of the  $Z$  boson  $M_Z$ , and the electron mass  $m_{e \rightarrow 0}$  is [2]

$$\langle |M|^2 \rangle = 2 \left(\frac{g_z E}{M_Z c^2}\right)^4 \left[ (c_V + c_A)^2 + (c_V - c_A)^2 \cos^4 \frac{\theta}{2} \right]. \quad (45)$$

In the above equation  $c_V$  and  $c_A$  are the neutral and axial vector couplings. The same significance in Eqs. (41), (44), and (45) is the conservations of momentum and energy. In the quantum field theory or high-energy physics, when we calculate the decay rate and scattering cross-section by the Feynman rules, it is necessary to consider the conservations of momentum and energy at each vortex. At most parts of the high-energy physics, we have to consider the conservations of momentum and energy to explain a lot of phenomena. Even in the Noether's theorem, it shows a conserved quantity associated with every continuous symmetry [14]. For example, the conservation of energy is associated with invariance under time translation, and the conservation of momentum is associated with invariance under space translation [14]. Therefore, the conservations of momentum and energy are indeed the bases in the quantum field theory. In addition, it has been clearly claimed that energy and momentum are always conserved in a relativistic collision [2].

Even in the quantum mechanics, the Heisenberg uncertainty principle is the characteristic of the particle-wave duality. The Golden rule to describe the transition from the initial state  $i$  to the final state  $f$  is [20]

$$R_{i \rightarrow f} = \frac{2\pi}{\hbar} \int \prod_k \frac{V d^3\mathbf{p}_k}{(2\pi\hbar)^3} |M_{fi}|^2 \delta\left(E_i^0 - E_f^0 - \sum_K E_k\right) \delta\left(\mathbf{p}_i - \mathbf{p}_f - \sum_k \mathbf{p}_k\right), \quad (46)$$

where  $E_i^0$ ,  $E_f^0$ , and  $E_k$  are the energy of the initial state, final state, and the joining particles due to decay, respectively,  $\mathbf{p}_i$ ,  $\mathbf{p}_f$ , and  $\mathbf{p}_k$  are the momenta of the initial state, final state, and the joining particles due to decay,  $V$  is the occupied space, and  $M_{fi}$  is the matrix element of the perturbation between the initial and final states. Two delta functions express the conservations of energy and momentum, and the energy and momentum carried off by the free particle are equal to the change in the system [19 20]. It also clearly reveals the conservations of energy and momentum in the quantum system.

#### IV. The Estimation Of The Upper Rest Mass Limit For Neutrino From The Supernova SN 1987A Event

If neutrino has mass, then it must be affected by gravity so the Lorentz violation would be questionable and unbelievable based on the instantaneously inertial coordinate frame of General Relativity. According to the supernova SN1987A event [21-26], neutrinos were detected three hours earlier than photons that causes a problem: if neutrinos have mass, why they came to the Earth faster than photons? The role of this supernova, SN 1987A, is 168,000 light years far away from the Earth [27]. It means that the early arrival neutrinos move averagely faster than photons even their speeds should be very slightly slower than the speed of light in the free space. The time difference of three hours means the average speed difference between neutrinos and photons from SN 1987A is  $2 \times 10^{-9}$  or  $0.6 \text{ m/sec}$ . Maybe someone might say that photons were detected later than neutrinos just because the core collapse process could emit neutrino way earlier than the photon. However, this traveling time is 168,000 years, the core collapse process is only a very tiny period comparing to this traveling time. Therefore, we still can identify that the neutrinos arrived Earth 3 hours than the photons. Even the core collapse last for three hours, it still means that the average speed of the detected neutrinos is almost the same as that of the detected photons. If the rest mass of one neutrino  $m_\nu$  were non-zero, then the relativistic effect and gravitational affection must be considered. Supposing the speed of neutrino very close to the speed of light, the difference is as small as  $10^{-8}$  in speed. According to the Lorentz factor, this difference in speed gives  $\gamma \approx 7,000$ . If the supernova event exhausted one solar mass to transform energy to all neutrinos and produce approximate  $10^{60}$  neutrinos, then the upper rest mass limit of each neutrino would be

$$m_\nu c^2 \approx (1.99 \times 10^{30}) \times (3 \times 10^8)^2 / 10^{60} / \gamma \approx 150 \text{ eV}. \quad (47)$$

However, the energy of one neutrino occupies the nuclear reaction very small, the upper mass limit should be at least  $10^3$  times smaller than the above value. Roughly speaking, the upper mass limit of one neutrino is

$$m_\nu c^2 \leq 0.15 \text{ eV}. \quad (48)$$

This rough estimation is from the SN 1987A event. Theoretically speaking, the neutrinos must be affected by the gravitation if they have non-zero mass. However, the early arrival neutrinos revealed that their average speeds were larger than the photons based on the three-hour difference. As we know, the escaping velocity to get rid of the solar gravitation, the third cosmic velocity, is  $1.67 \times 10^4 \text{ m/sec} \approx 5.57 \times 10^{-5} c$ , so each neutrino escaping the gravitation of the supernova would also lose its speed at the same order in the whole time. If the photons interact with the interstellar materials very few, their average speeds should be still very close to  $c$ . Only when photons pass

through the interstellar materials and interact with them, photons will slow down. On the other hand, in the empty space far away from gravitation, they will propagate in the velocity of light. But it is not the same thing for the neutrinos with non-zero mass. Once the neutrinos leave the initial gravitation, their velocities are affected and decrease until they arrive the detectors on the Earth, about 168,000 light years [27]. Their average speeds must be slower than the speed of light in the free space during the whole traveling time. Hence, an easy way to explain the phenomenon of the arrival neutrinos three hours earlier than photons is the zero-mass neutrino. Photons take part in the electromagnetic interaction so they slow down when passing through the interstellar materials but it is not the truth for the zero-mass neutrinos. They are very hard to interact with matters so not easily decelerated by matters. According to the supernova SN 1987A event, the average speed of neutrinos is  $2 \times 10^{-9}$  faster than the average speeds of photons, therefore it is a reasonable factor that neutrinos have zero mass resulting in them not slowing down by gravitation!

## V. New Explanation For The Neutrino Observations

According to the SNO's observations in 2001, the occupation of  $\nu_e$  from the sun is about 0.32, close to 1/3 [12]. If no neutrino oscillation takes place,  $\nu_\mu$  and  $\nu_\tau$  will occupy the rest about 2/3 neutrino flux from the sun. One thing is possible that the solar model needs to be corrected, and the other thing is to boldly predict that only one kind of neutrino exists which can be a linear combination of three different neutrino states. Because neutrino is hard to detect and the present recorded data cannot completely avoid such possibility. For example, the decays of  $Z^0$  bosons can produce three kinds of neutrino. The atmospheric observations from SK in 1998 also revealed the close 1:1 ratio between  $\nu_\mu$  and  $\nu_e$ , and the missing part of  $\nu_\mu$  was very possibly to be  $\nu_\tau$  which is roughly equal to  $\nu_\mu$  [10]. Both results of SK and SNO imply three equal neutrinos in number and lead to a unified neutrino state as a linear combination of three neutrino states

$$|\nu_{unified}\rangle = \frac{1}{\sqrt{3}}|\nu_e\rangle + \frac{1}{\sqrt{3}}|\nu_\mu\rangle + \frac{1}{\sqrt{3}}|\nu_\tau\rangle. \quad (49)$$

The laws of conservations of energy and momentum in Eqs. (33) and (36) clearly tell us the result in Eq. (37) that the mass of neutrino is fixed no matter which kind of neutrino is detected. Therefore, all three kinds of neutrinos have the same rest mass zero or non-zero and the Eq. (49) is a reasonable expression. Besides, we also have to consider the possibility of the cross terms at the same time that all neutrinos may interact with all leptons so the correction of Lagrangian is multiplied by a matrix  $U$

$$U = \begin{pmatrix} 1 - \delta_{e2} - \delta_{e3} & \delta_{e2} & \delta_{e3} \\ \delta_{\mu1} & 1 - \delta_{\mu2} - \delta_{\mu3} & \delta_{\mu3} \\ \delta_{\tau1} & \delta_{\tau2} & 1 - \delta_{\tau3} - \delta_{\tau3} \end{pmatrix}, \quad (50)$$

where  $\delta_{e2}$ ,  $\delta_{e3}$ ,  $\delta_{\mu1}$ ,  $\delta_{\mu3}$ ,  $\delta_{\tau1}$ , and  $\delta_{\tau2}$  are possibly non-zero values, and each element should be not negative. Therefore, the matrix will appear in the Lagrangian which becomes

$$L = -\frac{g}{\sqrt{2}} \bar{\psi}_L U \gamma^\mu (1 - \gamma^5) (\sqrt{3} \Psi_{L\nu}) W^- + H.C. \quad (51)$$

In fact, the inverse muon decay [28] reveals the possibility of the form in Eq. (50). The reaction is

$$\nu_\mu + e^- \rightarrow \mu^- + \nu_e. \quad (52)$$

The role of the muon neutrino is changed to the electron neutrino after this reaction so it makes the unified neutrino model become much reasonable.

## VI. Conclusions

In conclusion, the **massive** neutrino oscillation violates several conservations if the mass difference exists. The original theory predicts the mass differences existing on three kinds of neutrino. However, one neutrino transits to another and then transits back to itself again that causes the mass non-conservation if no external energy or additional mass participates in the transition process. It also violates one of the conservations of energy and momentum and the speeds of neutrinos before and after transition must be different that results in self-acceleration and deceleration. Even the Lorentz violation is proposed in the standard model extension, the all other originally elementary particles predicting by the standard model will lose their criteria because all of them obey the Lorentz invariance. Based on the assumption, once neutrino is generated, its spacetime will violate the Lorentz symmetry everywhere. Neutrino shall not have so special spacetime different from all other elementary particles. Thus, the Lorentz violation is still not reasonable to explain the neutrino oscillation even the mass differences between three kinds of neutrinos are not existing. The non-zero mass of neutrinos might cause a problem because they must be affected by gravity. If so, the fact that the neutrinos arrived Earth three hours earlier than photons in the supernova SN 1987A event would be not easy to reasonably explain because the traveling distance is 16,800 light years! An easy way to explain the observations from the supernova SN 1987A is the zero mass of neutrinos.

After reviewing the results of SK and SNO, both results strongly imply the ratio of number between three kinds of neutrinos is approximate  $\nu_e:\nu_\mu:\nu_\tau=1:1:1$ . According to this, we propose a new explanation for the observation data. Only one unified neutrino



exists in nature which is a linear combination of three neutrino states. This is a reasonable expression because the laws of conservations of energy and momentum tell us that the three kinds of neutrinos have the same mass zero or non-zero. Each lepton not only interacts with the corresponding neutrino state, but also interacts with other lepton neutrino states and the correction of the Lagrangian described by a new matrix is revealed here.

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## Additional Information

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