

The coupled Einstein equations (II)

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Abstract

The coupled Einstein equations are defined for a manifold with two riemannian metrics. We make use of the mixed Riemann curvature.

1 The Einstein equations

Let (M, g) be a riemannian manifold with riemannian curvature R .

$$r_g(x, y, z, t) = g(R(x, y)z, t)$$

$$Ric_g(x, y) = \sum_i r_g(x, e_i, y, e_i)$$

The Einstein equations are then [Be]:

$$Ric_g = \lambda g$$

2 The coupled Einstein equations

Let (g, g') be two metrics over the manifold M , the two Levi-Civita connections are (∇, ∇') . Then the mixed Riemann curvature is:

$$R_{g, g'}(X, Y) = \nabla_X \nabla'_Y + \nabla'_X \nabla_Y - \nabla_Y \nabla'_X - \nabla'_Y \nabla_X - \\ - \nabla_{[X, Y]} - \nabla'_{[X, Y]}$$

The mixed Ricci curvature is:

$$Ric_{g, g'}(X, Y) = tr(Z \rightarrow R_{g, g'}(X, Z)Y)$$

The coupled Einstein equations are:

$$Ric_{g, g'} = \mu(g + g')$$

$$Ric_g = \lambda g$$

$$Ric_{g'} = \lambda' g'$$

References

[Be] A.Besse, "Einstein Manifolds", Springer Verlag, Berlin, 1987.