In the following I talk about functions in particolar I study the behavior of this near infity

assuming the notion of function, limit and derivate is know.

Let a function $f : \mathbb{R} \to \mathbb{R}$ such that $\lim_{x\to+\infty} f(x) = +\infty$ if $lim_{x\to+\infty}\frac{f(x)}{x} > 1$ than exist a(∞)$\in$ $\mathbb{R}$ such that $\forall x_0 \geqslant a(\infty)$ $lim_{x\to x_0} f(x) = +\infty$ so the function f is not bijective.

Example 0.1. Let function $f : \mathbb{R} \to \mathbb{R}$ defined by $x \mapsto x^2$. $\lim_{x\to+\infty} \frac{x^2}{x} = +\infty > 1$ so the funcion is bijective for x$\in$ [0,a(∞)[$\subset$ $\mathbb{R}\, and\, f(x) \in [0, \infty[$ so $f^{-1}(x) : [0, \infty[\to [0, a(\infty)[$ with $\lim_{x\to+\infty} f^{-1}(x) = a(\infty)$.

An important result is the following

Theorem 0.2. *Let f a function, f have an oblique asymptote if and only if f' have an orizontal asymptote.*

Proof. we prove the Theorem for x tends $+\infty$ (idem for x tends $-\infty$)
$\Rightarrow$) f have asymptote y=mx+q for x tends $+\infty$ so q= $\lim_{x\to+\infty}(f(x) - mx)$. $\frac{d}{dx}(q) = 0 = lim_{x\to+\infty}\frac{d}{dx}f - m \Rightarrow lim_{x\to+\infty}f' = m$.
$\Leftarrow$) f' have an asymptote y=m that is $lim_{x\to+\infty}f' - m = 0 \Rightarrow \int 0dx = c_1 = lim_{x\to+\infty}\int(f' - m)dx = lim_{x\to+\infty}(f - mx + c_2)$ if $q = c_1 - c_2$ so q=$lim_{x\to+\infty}(f - mx)$. □

Example 0.3. Let the function y=ln(x) y'=$\frac{1}{x}$ $\lim_{x\to+\infty} \frac{1}{x} = 0$ so y' have an orizzontal asymptote y=0 so the function have asymptote y=q where q is the real a(∞).