# Intrinsic vector potential and electromagnetic mass

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**Abstract**

Electric charges may have mass in part or in full because they are charged. The explanation here avoids charge distribution models by associating the charge's mass with intrinsic quantum mechanical quantities, similar to the way spin angular momentum dispenses with mechanical models. Inhomogeneous Lorentz, *i.e.* 'Poincaré', dual fermion, 8-spinor fields are needed. Poincaré fields have a probability current that acts as an intrinsic vector potential. The potential obeys a Maxwell-like equation which identifies the charged source. Intrinsic gauge freedom allows the chosen intrinsic gauge to provide the charged source with mass, which is, therefore, 'electromagnetic mass'. One of the two fermions obeys the Dirac equation for a massless, chargeless particle while the other is charged and massive. These conventional equations describe neutrinos and electrons and similar lepton pairs with well-known accuracy.

Keywords: Quantum field theory, Translations, Inhomogeneous Lorentz group, Poincaré group

## 1 Introduction

As the only long-range, fairly strong force that can vanish when positive and negative charges balance, electromagnetism is perhaps the best studied of the four fundamental forces. Yet a classical problem persists. If the interaction of electric charges is mediated by an electromagnetic field and if the electromagnetic field has energy, then carrying the electromagnetic field should act as a drag on a charge's motion, an inertia.

The problem is called "electromagnetic mass" and its explanations in terms of charge distributions are controversial. There is a vast literature. Some charge distributions are solid structures, often described with special relativity. There are gravity-bound hydrodynamic models and many others. See, for example, Ref. [1, 6, 7, 10, 17]. Electromagnetic mass is frequently presented as an unsolved problem in introductory textbooks. [2, 5]

The proposed solution here avoids charge distributions. We show how a charge may have mass due to its being charged, but in a way that is related to intrinsic quantities. Electromagnetic field energy does not enter into the process.

Consider angular momentum. Given enough grease and quality bearings, a large flywheel rotating about a stationary axis can maintain a constant angular momentum. To keep the flywheel intact, forces are needed. Failing flywheels can do impressive damage.

Now consider quantum effects. An electron whether at rest or in motion has 'intrinsic' or 'spin' angular momentum. It is widely accepted that, with spin, "a consistent mechanical model doesn't exist". See, for

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example, page 374 of [12]. The physical description of intrinsic electron spin does not have a place for the forces that prevent rotating matter flying apart like a crumbling flywheel.

If electromagnetic mass could be explained as an intrinsic quantum effect, then one would not need to introduce non-electromagnetic forces to counter the repulsion of like-charges in order to hold an extended charge distribution model together. There simply would not be an extended charge distribution model.

To begin, note that physics is locally invariant under spacetime rotations, i.e. spacial rotations and boosts, as well as translations. Also consider that the quantum fields of the Standard Model, 'Lorentz fields', transform with the (homogeneous) Lorentz group of spacetime rotations, but not with translations. [11, 16]

It may, therefore, be interesting to investigate fields, "Poincaré fields", that transform non-trivially under both spacetime rotations and translations, so-named because the Poincaré group is the Lorentz group plus translations.

As one might expect, the probability current density $J^{\mu}(x,y)$ for a Poincaré field is more complicated than the Lorentz field probability density $j^{\mu}(x)$, where $\mu \in \{1,2,3,4\}$ with $4 = t$ the time index. For example, arbitrary parameters associated with the translations introduce an intrinsic spacetime's worth of new parameters, $y^{\mu}$. We take the intrinsic coordinates $y^{\mu}$ to be independent of spacetime coordinates $x^{\mu}$, for simplicity.

Assuming $J^{\mu}(x,y)$ is sufficiently smooth, there are derivatives of $J^{\mu}(x,y)$ with respect to $y^{\nu}$. A surprising identity is found: The Poincaré probability density $J^{\mu}(x,y)$ satisfies a Maxwell-like equation in intrinsic coordinates $y^{\nu}$. Thus, the probability density $J^{\mu}(x,y)$ becomes an intrinsic vector potential, labeled "$A_i^{\mu}$" with $i$ for 'intrinsic'. And there is intrinsic gauge invariance of the Maxwell-like equation. One can freely choose an intrinsic gauge $\chi(y)$, a matrix function of intrinsic coordinates $y^{\mu}$.

Intrinsic gauge freedom allows one to purpose the intrinsic gauge $\chi(y)$ to introduce a mass term into a lagrangian for otherwise massless fields. But the intrinsic vector potential sits in the interaction lagrangian with a charge current density. And that current density is also determined by the Maxwell-like equation for the probability density $J^{\mu}(x,y)$. The mass is thus associated with that charge current density. These facts support the claim that the particle has acquired mass because it is charged.

Intrinsic matrix translations have previously been used to obtain a universal fermion mass in the context of the Poincaré gauge theory of gravitation. [4, 15] The intrinsic momenta there mix components of a 4-spinor fermion, while we mix components of one 4-spinor with those of another. Thus, while somewhat similar in ingredients, the 8-spinor scheme here and the 4-spinor formalism there are distinct approaches with distinct outcomes.

The Higgs mechanism [13], explains how massless particles in gauge theory acquire mass. The properties of a candidate Higgs boson are being studied in on-going experiments, for a recent example of such an experiment see Ref. [14]. Alternate explanations may be sought if, as tentatively considered in Ref. [13], there is some "unresolved paradox" or if the recently discovered particle fails to have the properties of a Higgs boson. The nascent scheme in this article would need significant development to become a serious alternative to the Higgs mechanism.

Section 2 briefly covers Lorentz and Poincaré quantum fields and their probability current densities. The probability current density of the Poincaré field is shown to satisfy a Maxwell-like equation in intrinsic coordinates. That equation is the foundation of the article. The Maxwell-like equation distinguishes the charged from the uncharged 4-spinor fermion. Gauge invariance and parity are also discussed in Sec. 2.

An Appendix describes the 8-spinor Poincaré representation (rep) and includes notation and conventions such as the spacetime metric.

Section 3 sets up the lagrangian using the choice of intrinsic gauge to produce traditional field equations. Among the results is the standard Maxwell equation with the charged fermion as the source. The equations for the 8-spinor field describe a charged massive Dirac fermion and, independently, a massless 4-spinor fermion. Since there is no deviation from standard equations, verification is provided the well-documented agreement with experiment of predictions based on these standard equations. The essential difference with standard equations is the non-phenomenological explanation of the charged fermion's mass.

Concluding remarks are collected in Sec. 4.

## 2 Currents and the intrinsic E-M vector potential

'Poincaré fields' transform with translations as well as spacetime rotations. Otherwise, they are like the 'Lorentz fields' of the Standard Model. In this section, the probability current density of a Poincaré field is shown to be the intrinsic vector potential.

Let $\psi_0$ be an 8-spinor Lorentz quantum field that is constructed from the annihilation and creation operators for a massless spin 1/2 particle. By the properties of quantum fields, when spacetime undergoes the Poincaré transformation $(\Lambda, b)$, there is a unitary transformation $U(\Lambda, b)$, for the operators. The Lorentz field $\psi_0$ transforms by the in general non-unitary matrix $D^{-1}(\Lambda, 0)$. This requirement largely determines the coefficients of the operators in the sum that makes up the field $\psi_0$. See, for example, Ref. [16].

Define the associated Poincaré field $\Phi_0$ by

$$\Phi_0 \equiv D(1, y)\psi_0(x) \quad , \tag{1}$$

where the coordinate-like set of parameters $y^\mu$ are arbitrary, assumed, for simplicity, to be independent of spacetime coordinates $x^\mu$. Proper behavior of $D(1, y)$ requires that the $y^\mu$ transform just like coordinates, $y \to \Lambda y + b$, with the spacetime Poincaré transformation $(\Lambda, b)$.

Applying the unitary transformation $U(\Lambda, b)$ of operators to the Lorentz field $\psi_0$ yields, by assumption, [16]

$$U(\Lambda, b)\psi_0(x)U^{-1}(\Lambda, b) = D^{-1}(\Lambda, 0)\psi_0(\Lambda x + b) \quad . \tag{2}$$

For the Poincaré field, one finds that

$$U(\Lambda, b)\Phi_0(x, y)U^{-1}(\Lambda, b) = D^{-1}(\Lambda, b)\Phi_0(\Lambda x + b, \Lambda y + b) \quad . \tag{3}$$

To show that (3) follows from (1) and (2), note that the field transformation $D(1, y)$ commutes with the operator transformation $U(\Lambda, b)$. One uses the well-known rule for successive Poincaré transformation, $A$ followed by $B$, $(\Lambda_B, b_B)(\Lambda_A, b_A) = (\Lambda_B\Lambda_A, \Lambda_B b_A + b_B)$, to show that

$$D^{-1}(\Lambda, b)D(1, \Lambda y + b) = D(1, y)D^{-1}(\Lambda, 0) \quad , \tag{4}$$

which gives (3) from (2).

In this article, the Lorentz field $\psi_0$ is the direct sum of two free massless 4-spinor fields $\psi_{01}$ and $\psi_{02}$, and it transforms with $8 \times 8$ matrix $D(\Lambda, b)$ of the Poincaré rep in the Appendix. Thus its probability current density $j^\mu$ is the sum of the current densities of the two 4-spinor fields. One has

$$j^\mu = \bar{\psi}_0\gamma^\mu\psi_0 = \bar{\psi}_{01}\gamma^\mu_{11}\psi_{01} + \bar{\psi}_{02}\gamma^\mu_{22}\psi_{02} = j^\mu_1 + j^\mu_2 \quad , \tag{5}$$

where normalization constants are dropped for brevity. Here $j^\mu_i \equiv \bar{\psi}_{0i}\gamma^\mu\psi_{0i}$ are the currents for the two 4-spinors $\psi_{01}$ and $\psi_{02}$ in the 8-spinor field.

By the definition of $\Phi_0$, (1), the current $J^\mu$ of the Poincaré field can be written in terms of the Lorentz field $\psi_0$ as

$$J^\mu \equiv \bar{\Phi}_0\gamma^\mu\Phi_0 = -\bar{\psi}_0(x)\gamma^t D^\dagger(1, y)\gamma^t\gamma^\mu D(1, y)\psi_0(x) = a^{-1}\bar{\psi}_0 \, \alpha^\mu \, \psi_0 \quad , \tag{6}$$

where the constant $a$ is introduced now to be determined later. Comparing (5) and (6) shows that the $\alpha^{\mu}$ in $J^{\mu}$ replaces the $\gamma^{\mu}$ in the current $j^{\mu}$.

One sees from (6) that $\alpha^{\mu}(y)$ is the combination

$$\alpha^{\mu}(y) = -a\gamma^{t} D^{\dagger}(1,y)\gamma^{t}\gamma^{\mu} D(1,y) \quad . \tag{7}$$

We can work with this expression using the details of the 8-spinor Poincaré rep in the Appendix. By (A.7), the products of the off-diagonal linear momentum generators vanish, $\pi^{\mu}\pi^{\nu} = 0$. Thus the translation matrix $D(1,y) = \exp\left(-iy_{\sigma}\pi^{\sigma}\right) = 1 - iy_{\sigma}\pi^{\sigma}$. And $D^{\dagger}(1,y)$ is also linear in $y$. By various gamma identities, one finds that the matrix $\alpha^{\mu}$ in (7) is

$$\alpha^{\mu}(y) = a\left[\gamma^{\mu} - iky_{\rho}\left(\gamma^{\rho}_{12}\gamma^{\mu}_{22} + \gamma^{\mu}_{22}\gamma^{\rho}_{21}\right) + k^{2}y^{2}\gamma^{\mu}_{11} - 2k^{2}y^{\mu}y_{\rho}\gamma^{\rho}_{11}\right] \quad , \tag{8}$$

where, see the Appendix, $\gamma^{\rho}_{12}$ has the 4-spinor Dirac matrix $\gamma^{\rho}_{D}$ in the 12-block and vanishes elsewhere. In (8), $y^{2} = y_{\rho}y^{\rho}$ and one sees that $\alpha^{\mu}(y)$ is a matrix quadratic in $y$.

Since $\alpha^{\mu}(y)$ is quadratic in $y$, second order partial derivatives of $\alpha^{\mu}(y)$ with respect to $y$ are constant. One finds an identity,

$$\partial^{\lambda\prime}\partial^{\prime}_{\lambda}\alpha^{\mu} - \partial^{\mu\prime}\partial^{\prime}_{\kappa}\alpha^{\kappa} = 12ak^{2}\gamma^{\mu}_{11} \quad , \tag{9}$$

where the primed partial derivatives are with respect to the $y$s, $\partial^{\prime}_{\lambda}\alpha^{\mu} \equiv \partial\alpha^{\mu}/\partial y^{\lambda}$. Unprimed partials, $\partial_{\rho}f(x) \equiv \partial f/\partial x^{\rho}$, are saved for later with $x^{\mu}$.

The current $J^{\mu}(x,y) = a^{-1}\bar{\psi}_{0}\alpha^{\mu}\psi_{0}$ has its $y$-dependence confined to $\alpha^{\mu}(y)$ and the identity (9) implies that

$$\partial^{\lambda\prime}\partial^{\prime}_{\lambda}(aJ^{\mu}) - \partial^{\mu\prime}\partial^{\prime}_{\kappa}(aJ^{\kappa}) = 12ak^{2}\bar{\psi}_{0}\gamma^{\mu}_{11}\psi_{0} = 12ak^{2}j^{\mu}_{1} \quad , \tag{10}$$

since spacetime and intrinsic coordinates are independent, $\partial x^{\mu}/\partial y^{\nu} = 0$.

Compare (10) with one of Maxwell's equations,

$$\partial^{\lambda}\partial_{\lambda}A^{\mu}_{q} - \partial^{\mu}\partial_{\kappa}A^{\kappa}_{q} = \rho^{\mu} \quad , \tag{11}$$

where $\partial_{\lambda}A^{\mu}_{q} \equiv \partial A^{\mu}_{q}/\partial x^{\lambda}$ and $A^{\mu}_{q}$ is the vector potential due to a charged current density $\rho^{\mu}$. Clearly, the two equations (10) and (11) have the same form. We say that (9) and (10) are 'Maxwell-like' with 'current density' proportional to $j^{\mu}_{1}$.

Based on this comparison, we identify $A^{\mu}_{i} = aJ^{\mu}$ as the "intrinsic vector potential" and $\alpha^{\mu}$ as the "intrinsic vector potential matrix" or "matrix vector potential." Thus (10) becomes the Maxwell-like equation

$$\partial^{\lambda\prime}\partial^{\prime}_{\lambda}A^{\mu}_{i} - \partial^{\mu\prime}\partial^{\prime}_{\kappa}A^{\kappa}_{i} = qj^{\mu}_{1} \quad , \tag{12}$$

where the constant $a$ has been chosen to be

$$a = \frac{q}{12k^{2}} \tag{13}$$

Note that the intrinsic vector potential $A^{\mu}_{i}$ does not satisfy Maxwell's equation (11) because the parameters $y^{\mu}$ introduced with the Poincaré field $\Phi$ in (1) are not spacetime coordinates $x$. Unlike the matrix quantity $\alpha^{\mu}(y)$, the vector potential $A^{\mu}_{q}(x)$ is a function of the same coordinates $x$ as the current density $\rho^{\mu} = qj^{\mu}_{1}(x)$.

However, that is all right because no intrinsic quantity can be a function of $x$. The intrinsic quantity would then have "mechanical" properties, which is like proposing a rotating mass with a density function of $x$ as the source of the electron's spin. No mechanical model accounts for electron spin and no mechanical model should exist for the intrinsic vector potential. Hence, such considerations suggest that the intrinsic vector potential $A^{\mu}_{i}$ should be a function of intrinsic coordinates $y$, not spacetime coordinates $x$.

The Maxwell-like equation (12) is our justification for characterizing the first 4-spinor as charged and the second 4-spinor as uncharged.

Intrinsic gauge invariance is a property of the Maxwell-like equation (12), meaning that the equation is unchanged when the intrinsic vector potential $\alpha^{\mu}$ undergoes an 'intrinsic gauge transformation',

$$\tilde{\alpha}^{\mu} = \alpha^{\mu} + \partial^{\mu}\chi \quad , \tag{14}$$

where the gauge function $\chi(y)$ is a matrix whose 64 components must have symmetric second partials, $\partial^{\nu\prime}\partial^{\mu\prime}\chi = \partial^{\mu\prime}\partial^{\nu\prime}\chi$.

Intrinsic gauge freedom could be used to make the intrinsic vector potential divergence-free. Instead we find a gauge that makes a mass term and simplifies a lagrangian.

# 3 Gauge, mass term, field equations

In this section, field equations are determined by a lagrangian built traditionally, but with the intrinsic momentum and intrinsic vector potential from Sec. 2.

To start with, combine the free-field and interaction lagrangians $L_{\psi} = \bar{\psi} p_{\lambda} \gamma^{\lambda} \psi$, $L_{\mathrm{A}} = -F^2/4$, and $L_{\mathrm{int}} = -qA_{\mu}j_1^{\mu}$ for the particles, electromagnetism and the electromagnetic interaction. Then one includes the intrinsic momentum $\pi^{\mu}$ with the momentum $p^{\mu}$ and the intrinsic vector potential $\alpha_{\mu}$ with the vector potential $A_{\mu}$. Putting all this together yields the initial lagrangian $L_1$,

$$L_1 \equiv \bar{\psi}\left[\left(i\partial_{\lambda} + \pi_{\lambda}\right)\gamma^{\lambda} - q\left(A_{\lambda} + \alpha_{\lambda}\right)\gamma_{11}^{\lambda}\right]\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad , \tag{15}$$

where $p_{\lambda} = i\partial_{\lambda}$ is the momentum, $F_{\mu\nu}$ is the electromagnetic field, $F_{\mu\nu} \equiv \partial_{\nu}A_{\mu} - \partial_{\mu}A_{\nu}$, with vector potential $A^{\mu}$. The charge current is $qj_1^{\mu} = q\bar{\psi}\gamma_{11}^{\lambda}\psi$,

The interaction lagrangian, *i.e.* $L_{\mathrm{int}} = -qA_{\mu}j_1^{\mu}$, does not include $j_2^{\mu}$ because we interpret the Maxwell-like equation (12) as showing that $j_1^{\mu}$, not $j_2^{\mu}$, carries electromagnetic current. The functions of spacetime coordinates $x$ are $\psi$, $A^{\mu}$, and $F^{\mu\nu}$. The intrinsic vector potential $\alpha^{\mu}$ is the only function of the intrinsic coordinates $y$.

The field equations depend on the choice of intrinsic gauge. The intrinsic gauge $\chi$ is a matrix function of the $y$s constrained only by having symmetric second partial derivatives. The gauge transformation replaces the intrinsic potential $\alpha^{\mu}$ by $\alpha^{\mu} \rightarrow \tilde{\alpha}^{\mu} = \alpha^{\mu} + \partial^{\mu\prime}\chi$. Let the intrinsic gauge $\chi$ be

$$\chi = \frac{m}{4q}y_{\lambda}\gamma^{\lambda} - a\left[y_{\lambda}\gamma^{\lambda} + iky^2\left(\mathbf{1}_{12} + \mathbf{1}_{21}\right) + \frac{k^2}{3}y^2 y_{\lambda}\gamma_{11}^{\lambda}\right] + \frac{1}{q}y_{\lambda}\pi^{\lambda} \quad , \tag{16}$$

where the constant $a$ is given by (13), $a = q/(12k^2)$, and $\mathbf{1}_{ij}$ is the $4 \times 4$ identity matrix in the $ij$-block. The intrinsic gauge $\chi$ simplifies the lagrangian and introduces the mass term.

With the expressions for the intrinsic gauge $\chi$ in (16) and the intrinsic vector potential $\alpha^{\mu}$ in (8), the lagrangian $L_1$ in (15) becomes the lagrangian $L$ that gives field equations. One finds

$$L(\phi, \partial\phi) = \bar{\psi}\left(i\partial_{\lambda}\gamma^{\lambda} - m\mathbf{1}_{11} - qA_{\lambda}\gamma_{11}^{\lambda}\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad , \tag{17}$$

where the placeholder $\phi$ indicates the fields $\bar{\psi}, \psi, A^{\mu}$. All these functions depend on spacetime coordinates $x$.

The intrinsic gauge $\chi$ removed the functions of intrinsic coordinates $y$ in $\alpha_\lambda$ from the Lagrangian $L_1$. By (15), this means that the $y$-dependence of the expression $\alpha_\lambda\gamma_{11}^\lambda$ is in the form $\partial_\lambda^\prime\chi\,\gamma_{11}^\lambda$. In one sense, the scalar product 'vanishes', $\alpha_\lambda\gamma_{11}^\lambda \approx 0$, because the $y$-dependence can be gauged away.

The choice of $\chi$ in (16) also made the intrinsic momentum term $\pi^\mu$ disappear. This is possible because $\pi^\mu$ is the gradient $\pi^\mu = \partial^{\mu\,\prime}\left(y_\lambda\pi^\lambda\right)$ and, therefore, can be absorbed by the gauge. By (A.8), including $\pi^\mu$ would have mixed the components of the two four spinors, which are not mixed without it.

The field equations are found by the Euler-Lagrange equations,

$$\frac{\partial L}{\partial\phi} - \partial_\lambda\left[\frac{\partial L}{\partial(\partial_\lambda\phi)}\right] = 0 \quad , \tag{18}$$

with suitable boundary conditions, *i.e.* the fields vanish properly at infinity.

The field equations for the vector potential, $\phi \rightarrow A^\mu(x)$, are Maxwell's equations for a current source $qj_1^\mu$,

$$\partial_\lambda F^{\mu\lambda} = \partial^\lambda\partial_\lambda A^\mu - \partial^\mu\partial_\lambda A^\lambda = qj_1^\mu \quad . \tag{19}$$

As discussed previously, one can trace the assignment of $qj_1^\mu$ as the source to the identity (9), a Maxwell-like equation, obeyed by the intrinsic vector potential matrix $\alpha^\mu$.

Varying $L$ with $\phi \rightarrow \bar{\psi}(x)$ gives a field equation for the 8-spinor $\psi$,

$$\left(i\partial_\lambda\gamma^\lambda - qA_\lambda\gamma_{11}^\lambda\right)\psi = m\mathbf{1}_{11}\psi \quad , \tag{20}$$

where $m\mathbf{1}_{11}$ is written in the block notation defined in the Appendix,

$$m\mathbf{1}_{11} = \begin{pmatrix} m & 0 \\ 0 & 0 \end{pmatrix} \quad . \tag{21}$$

With the parity matrix $\beta$ in the Appendix and applying spacial inversion to the partial derivative and vector potential, the field equation (20) can be shown to be parity-invariant.

Finally, varying $\phi \rightarrow \psi(x)$ in $L$ gives equations for the 8-spinor field $\bar{\psi}(x)$,

$$-i\partial_\lambda\bar{\psi}\gamma^\lambda - qA_\lambda\bar{\psi}\gamma_{11}^\lambda = m\bar{\psi}\mathbf{1}_{11} \quad . \tag{22}$$

While this looks nothing like the equation for $\psi$, (20), there is a standard process to manipulate it and show that the two equations are almost identical.

By work shown in the Appendix, with the charge conjugation matrix $\mathcal{C}$, *i.e.* $\mathcal{C} = i\gamma^2\gamma^t$, we have $\gamma^{\mu\,\mathrm{T}} = -\mathcal{C}\gamma^\mu\mathcal{C}^{-1}$ and since $\bar{\psi} = \psi^\dagger\gamma^t = \psi^{*\mathrm{T}}\gamma^t$, we find that $(\bar{\psi}\gamma^\lambda)^{\mathrm{T}} = \mathcal{C}\gamma^\lambda\gamma^t\mathcal{C}^{-1}\psi^* = \mathcal{C}\gamma^\lambda\psi^{\mathrm{c}}$, where the "charge conjugate" 8-spinor field $\psi^{\mathrm{c}}$ is the combination

$$\psi^{\mathrm{c}} = \gamma^t\mathcal{C}^{-1}\psi^* \quad . \tag{23}$$

Thus the transpose of (22) can be restated to look like (20),

$$\left(i\partial_\lambda\gamma^\lambda + qA_\lambda\gamma_{11}^\lambda\right)\psi^{\mathrm{c}} = m\mathbf{1}_{11}\psi^{\mathrm{c}} \quad . \tag{24}$$

Comparing (20) and (24), one sees that $\psi$ and $\psi^{\mathrm{c}}$ satisfy the same equation aside from the sign of the charge $q$.

By (20) and (A.2), the first 4-spinor $\psi_1$ obeys the Dirac equation for a charged, massive particle,

$$\left(i\partial_\lambda - qA_\lambda\right)\gamma_D^\lambda\psi_1 = m_1\psi_1 \quad . \tag{25}$$

See, for example, Ref. [8], Chapter XX.9 for a discussion. And, the second 4-spinor $\psi_2$ obeys the Dirac equation

$$\gamma^{\lambda}_{D}\partial_{\lambda}\psi_2 = 0 \quad . \tag{26}$$

The second fermion $\psi_2$ has neither electromagnetic charge nor mass and obeys free-particle field equations.

Since we recognize the field equation (25) as the traditional 4-spinor Dirac equation, it known to be invariant under the continuous rep Poincaré group with Dirac formalism rep of the Lorentz group. [8] Likewise, the field equation (26) has the well-known symmetries of a massless free spin 1/2 particle. For the symmetries of these traditional equations, see Ref. [3] .

# 4 Concluding remarks

The field equations do not reflect the underlying 8-spinor translation rep. Just as a displacement shifts all points in spacetime equally, the 8-spinor translation, *i.e.* $D(1,y)$ in (A.8), adds portions of one 4-spinor to the other. Since the field equations (20,22) that are written with the Lorentz field $\psi$, where $\psi = \{\psi_1,\psi_2\}$, keep $\psi_1$ and $\psi_2$ well separated and independent, the field equations can not be written easily in terms of the Poincaré field $\Phi$ which mixes components when translated.

Indeed, the arbitrary parameter space introduced with the Poincaré field $\Phi$ in (1), *i.e.* the vast intrinsic spacetime $y^{\mu}$, vanishes from the lagrangian by the choice of intrinsic gauge in (16). All trace of the 8-spinor translation rep is lost in the lagrangian $L$ in (17), aside from the mass term which does not depend on $y^{\mu}$. Supplying that mass term for the charged fermion was the goal of the exercise.

# A The 8-spinor Rep of the Poincaré group

The 8-spinor matrix rep of the Poincaré group of spacetime rotations and translations is used throughout the text. Terms like 'the first 4-spinor' are keyed to this Appendix. Functions of spacetime also occur and these transform by the usual "continuous rep" of the Poincaré group with momentum proportional to the divergence.

Label the points of 3 + 1 Minkowski spacetime with coordinates $x^{\mu}$, where $\mu \in \{1,2,3,4\}$. The time component is $x^4 = x^t$. The Kronecker delta $\delta_{ab}$ is unity for equal indices $a = b$ and vanishes otherwise, a unit matrix. The spacetime metric $\eta_{\mu\nu}$ is diagonal with $\eta_{11} = \eta_{22} = \eta_{33} = +1$ and $\eta_{44} = -1$. Repeated indices are summed, as in $v_{\nu} \equiv \eta_{\mu\nu}v^{\mu}$.

The $8 \times 8$ matrices in the rep are conveniently split into four $4 \times 4$ blocks. Write $M_{ij}$ for a matrix that has nonzero components only in the $ij$ block, $i,j \in \{1,2\}$. It follows that $M_{ij}M_{kl} = \delta_{jk}N_{il}$, where matrix $N_{il}$ has nonzero components confined to the $il$ block.

The nonzero $4 \times 4$ blocks of each generator are multiples of standard $4 \times 4$ Dirac gamma matrices $\gamma^{\mu}_{D}$,

$$\gamma^{\mu}_{D} = i\begin{pmatrix} 0 & -\tau^{\mu} \\ \tau_{\mu} & 0 \end{pmatrix} \; ; \; \tau^{\mu} = \{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix},\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\} \; , \tag{A.1}$$

with $2 \times 2$ Pauli matrices $\tau^{\mu}$.

Define $\gamma^{\mu}_{ij}$ to be the $8 \times 8$ matrix with $\gamma^{\mu}_{D}$ in the $ij$-block and which vanishes elsewhere. These matrices satisfy the defining requirement of Dirac gammas [8] in the form $\gamma^{\mu}_{ij}\gamma^{\nu}_{kl} + \gamma^{\nu}_{ij}\gamma^{\mu}_{kl} = 2\delta_{jk}\eta^{\mu\nu}\mathbf{1}_{il}$, where $\mathbf{1}_{il}$ is null everywhere except for a $4 \times 4$ identity matrix in the $il$ block.

Write $(\Lambda, b)$ for any Poincaré transformation reorganized, if needed, to be the spacetime rotation $\Lambda$ followed by the translation along the displacement $b$. A spacetime rotation matrix $D(\Lambda, 0)$ is generated by angular momentum matrices $\sigma^{\mu\nu}$ and linear momentum matrices $\pi^{\mu}$ generate the translation matrix $D(1, b)$.

The '8-spinor gamma matrices' $\gamma^{\mu}$ are the block diagonal matrices

$$\gamma^{\mu} \equiv \gamma^{\mu}_{11} + \gamma^{\mu}_{22} \quad . \tag{A.2}$$

For the algebra's generators, angular momentum matrices $\sigma^{\mu\nu}$ and the linear momentum matrices $\pi^{\mu}$, choose

$$\sigma^{\mu\nu} = -\frac{i}{4}\left(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu}\right) \quad ; \quad \pi^{\mu} = k\gamma^{\mu}_{21} \quad . \tag{A.3}$$

The generators $\sigma^{\mu\nu}$ and $\pi^{\mu}$ obey the commutation relations,

$$i\left[\sigma^{\mu\nu}, \sigma^{\rho\lambda}\right] = \eta^{\nu\rho}\sigma^{\mu\lambda} - \eta^{\mu\rho}\sigma^{\nu\lambda} - \eta^{\nu\lambda}\sigma^{\mu\rho} + \eta^{\mu\lambda}\sigma^{\nu\rho} \, , \tag{A.4}$$

$$i\left[\sigma^{\mu\nu}, \pi^{\rho}\right] = \eta^{\nu\rho}\pi^{\mu} - \eta^{\mu\rho}\pi^{\nu} \quad \text{and} \quad i\left[\pi^{\mu}, \pi^{\nu}\right] = 0 \, , \tag{A.5}$$

which is the Poincaré algebra.

Space inversion, *i.e.* parity, changes the sign of spacial vector components, $x^i \rightarrow -x^i$, where $i \in \{1, 2, 3\}$ now indicates 3D space coordinates. Thus, we can say that $x^{\mu} \rightarrow -x_{\mu}$ since $-\eta_{ii} = -1$ for our choice of metric. Also, left and right-handed quantities are switched. [9, 16] In the matrix rep, applying the parity matrix $\beta$, with $\beta = i\gamma^t$, to the gammas has the form $\beta\gamma^{\mu}\beta^{-1} = -\gamma_{\mu}$. For the generators, Parity lowers the indices of $\sigma^{\mu\nu}$, i.e. $\beta\sigma^{\mu\nu}\beta^{-1} = \sigma_{\mu\nu}$, and the $\pi^{\mu}$ transform like the gammas.

The so-called 'charge-conjugation' matrix $\mathcal{C}$, with $\mathcal{C} = i\gamma^2\gamma^t$, is needed for the discussion of the adjoint Dirac equation (24). The matrix $\mathcal{C}$ produces block-wise transposes. These are true transposes for block-diagonal matrices like the gammas $\gamma^{\mu}$, $\mathcal{C}\gamma^{\mu}\mathcal{C}^{-1} = -\gamma^{\mu\,\mathrm{T}}$. But, for example with the momentum, charge conjugation gives an in-block-transpose. The result $\left(\mathcal{C}\pi^{\mu}\mathcal{C}^{-1}\right)_{(21)} = -\left(\pi^{\mu}\right)^{\mathrm{T}}_{(21)}$ remains in the 21-block. Compare the preceding 8-spinor Dirac formalism with, for example, the 4-spinor Dirac formalism in [16, 18].

Generators combine with real-valued parameters to make transformations. For a spacetime rotation with parameters $\omega_{\mu\nu}$, $= -\omega_{\nu\mu}$, let $\Lambda$ be the transformation of spacetime tensors in the continuous rep and let $D(\Lambda, 0)$ denote the $8 \times 8$ matrix transformation for 8-spinors. A translation along a displacement $b^{\mu}$ transforms spacetime coordinates by $x^{\mu} \rightarrow x^{\mu} + b^{\mu}$, in the continuous rep. In the matrix rep, denote the translation as $D(1, b)$. Let $D(\Lambda, b)$ be the matrix for the Poincaré transformation $(\Lambda, b)$. One has

$$D(\Lambda, b) = D(1, b)D(\Lambda, 0) = e^{-ib_{\mu}\pi^{\mu}} e^{i\omega_{\mu\nu}\sigma^{\mu\nu}/2} \quad , \tag{A.6}$$

with matrix exponentiation understood.

By definition in (A.3), the nonzero components of the momentum matrices $\pi^{\mu}$ are confined to the 21-block. It follows that their products vanish, simplifying many expressions,

$$\pi^{\mu}\pi^{\nu} = 0 \quad \text{and} \quad D(1, b) = e^{-ib_{\mu}\pi^{\mu}} = \mathbf{1} - ib_{\mu}\pi^{\mu} \quad , \tag{A.7}$$

so the translation matrix $D(1, b)$ is linear in $b^{\mu}$. Applied to an 8-spinor $\psi$, a linear combination of the first 4-spinor's components is added to the second 4-spinor. We have

$$D(1, b)\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} - ib_{\mu} \begin{pmatrix} 0 & 0 \\ k\gamma^{\mu}_D & 0 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 - ikb_{\mu}\gamma^{\mu}_D\psi_1 \end{pmatrix} \quad . \tag{A.8}$$

The first 4-spinor $\psi_1$ is the "donor" and the second 4-spinor $\psi_2$ is the "receiver".

This additive behavior is in keeping with the inhomogeneous nature of translations. No eigenspinors nor any eigenvalues of intrinsic translations are possible because there is no 8-spinor $\psi$ with $D(1,b)\psi$ proportional to $\psi$. No translation eigenvalues for translations means no contributions to linear momentum. Unlike spin, which does contribute to the total angular momentum, intrinsic translations do not contribute to the observed linear momentum of a quantum system.

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