Numbers: Part 6 , “three integrals”

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ABSTRACT. We give three definite integrals.

keywords: definite integrals, number Pi.

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I. Introduction: the number Pi

Recall that

$$\pi = 4\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-...\right) = 3.1415926535\ ... \tag{1}$$

this note presents three definite integrals.

II. Integrals

$$\frac{\pi}{4} = \int_0^{\infty} \frac{\sqrt{2\sqrt{32+8x+x^2}-8-2x}}{\sqrt{x}\left(x+\sqrt{32+8x+x^2}+\sqrt{x}\sqrt{2\sqrt{32+8x+x^2}+8+2x}\right)}dx \tag{2}$$

$$\frac{\pi}{4} = \int_0^{\infty} \frac{\sqrt{2\sqrt{32x^2+8x+1}-2-8x}}{x\left(1+\sqrt{32x^2+8x+1}+\sqrt{2\sqrt{32x^2+8x+1}+2+8x}\right)}dx \tag{3}$$

$$\frac{\pi}{4} = \int_{\ln\left(1+\sqrt{2}\right)}^{\infty} \frac{\left(1+e^{-2x}\right)e^{-x}}{1-2e^{-x}-e^{-2x}+\left(1-e^{-x}\right)\sqrt{1-2e^{-x}-e^{-2x}}}dx \tag{4}$$

III. References

1. Gradshteyn, I.S., and Ryzhik, I.M.: Table of Integrals, Series and Products. 7th ed.,edited by Alan Jeffrey and Daniel Zwillinger, Academic Press, 2007.