Yes, P = NP=NP-Complete=NP-hard, says the NP-hard Traveling Salesman Problem

Data Ordering and Route Construction Approach

"5% of the people think; 10% of the people think that they think; and the other 85% would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

Abstract

The solution of the traveling salesman problem (TSP) in this paper makes this problem no longer an NP-hard problem, but rather, a P problem. The TSP was solved in polynomial time and its solution was also correctly checked in polynomial time. Also solved were an NP-Complete TSP, and six other NP-Complete problems. The TSP solution killed two (three) birds with one stone, because its solution made the NP-hard problems and NP-Complete problems become P problems. The shortest route as well as the longest route for the salesman to visit each of nine cities once and return to the base city was determined. In finding the shortest route, the first step was to arrange the data of the problem in increasing order, since one's interest is in the shortest distances; but in finding the longest route, the first step was to arrange the data of the problem in decreasing order, since one's interest is in the longest distances. For the shortest route, the main principle is that the shortest route is the sum of the shortest distances such that the salesman visits each city once and returns to the starting city; but for the longest route, the main principle is that the longest route is the sum of the longest distances such that the salesman visits each city once and returns to the starting city. Since ten cities are involved, ten distances would be needed for the salesman to visit each of nine cities once and return to the starting city. One started the construction of the shortest route using only the shortest ten distances; and if a needed distance was not among the set of the shortest ten distances, one would consider distances longer than those in the set of the shortest ten distances. For the longest route, the construction began using only the longest ten distances; and if a needed distance was not among the set of the longest ten distances, one would consider distances shorter than those in the set of the longest ten distances. It was found out that even though, the length of the shortest or the longest route is unique, the sequence of the cities involved is not unique. The approach used in this paper can be applied in work-force project management and hiring, as well as in a country's work-force needs and immigration quota determination. Since approaches that solve the TSP and NP-Complete problems can also solve other NP problems, and the TSP and NP-Complete problems have been solved, all NP problems can be solved. If all NP problems can be solved, then all NP problems are P problems, and therefore, P is equal to NP. The CMI Millennium Prize requirements have been satisfied.

Options

Preliminaries
Some Agreements
About P, NP, NP-Complete and NP-hard

P: P is the set of problems that can be solved and checked in polynomial time,

NP-NP is the set of problems whose solutions have not been found in polynomial time but whose solutions can be verified in polynomial time

NP-hard is the set of problems that have not been solved in polynomial time

NP-Complete are those problems that are NP-hard and are in NP

Example A: Find the sum of the least **10** numbers in the table below.

3	**21**	**10**	**12**	**1**	**9**	**8**	**4**	**14**	**13**	**25**	**27**
24	**17**	**6**	**5**	**15**	**35**	**18**	**32**	**38**	**2**	**19**	**29**
41	**26**	**40**	**23**	**20**	**34**	**42**	**38**	**31**	**44**	**37**	**33**
16	**45**	**43**	**22**	**30**	**29**	**36**	**7**	**11**			

Solution:
Arranging the numbers from the smallest to the largest

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
13	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
25	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
37	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**			

The least 10 numbers are 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10
Their sum = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = **55**

Example B: Find the sum of the least **14** numbers of in the table.
The least 14 numbers are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13. and 14
Their sum = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 = **105**

Example C: Find the sum of the largest 10 numbers
The largest 10 numbers are 45, 44, 43, 42, 41, 40, 39, 38, 37 and 36.
Their sum = 45 + 44 + 43 + 42 + 41 + 40 + 39 + 38 + 37 + 36 = 4**05**

Given: The distances between each pair of cities.

Required : Frm a starting city, find the shortest route to visit each of other cities once and return to the starting city. It is assumed that there is a direct route between each pair of cities.

Note: **1**. From a starting city, the number of distances needed to visit once each of other cities, and return to the starting city equals the number of cities involved in the problem.

2 The symbol $C_{1,2}$ can mean the distance from City 1 to City 2.
The distance $C_{1,2}$ = the distance $C_{2,1}$.
Used as a sentence, $C_{1,2}$ can mean, from City 1, one visits City 2.

3. C_1 is the home base (starting city) of the traveling salesman.

4. $C_{1,2}(3)$ shows that the numerical value of $C_{1,2}$ is 3.

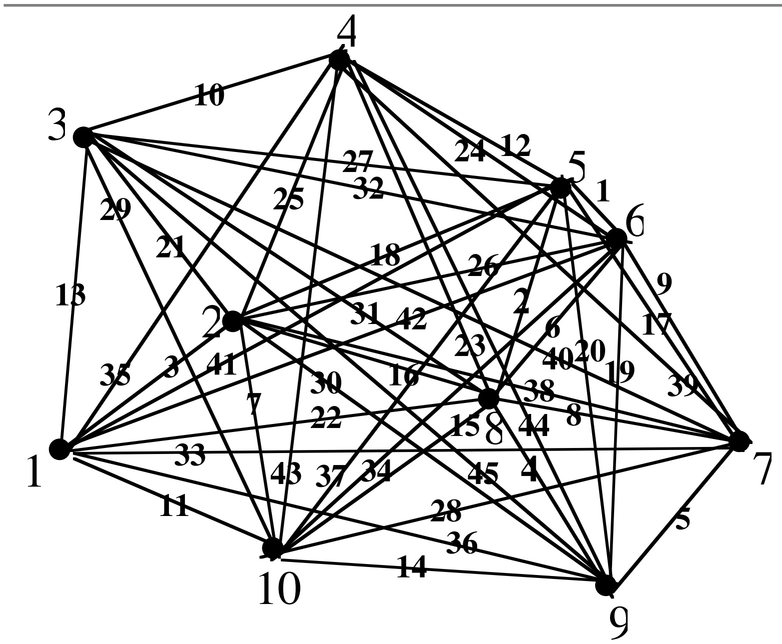
Traveling Salesman Problem: Shortest Route

Example 1a : From City 1, a traveling salesman would like to visit once each of **nine** other cities, namely, City 2, City 3, City 4, City 5, City 6, City 7, City 8, City 9, City 10; and return to City 1. Determine the shortest route.

Solution

At this step, this problem will be classified as an NP-hard problem. This classification will change after finding the solution and correctly checking the correctness of the solution.

The first step is to arrange the distances in this problem in increasing order. The main principle in finding the shortest route is that the shortest route is the sum of the shortest distances such that the salesman visits each city once and returns to the starting city.



Extra: For a ninth grade student
A ninth grade student is given 45 sticks of different lengths. Each stick is labeled as in Table 1, below (p.5).. Read also, **Note: 1**, 2, 3 and 4, above.. The student is asked to use 10 of the sticks to connect appropriately, end to end to 10 horizontal points on a table top so that the sum of the lengths of the sticks is as small as possible. Derermine the 10 sticks used as well as the sum of their lengths.

Distances Between Each Pair of Cities
(Distances are based on the relative lengths in the above diagram)

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
$C_{1,2}$ **3**	$C_{2,3}$ **21**	$C_{3,4}$ **10**	$C_{4,5}$ **12**	$C_{5,6}$ **1**	$C_{6,7}$ **9**	$C_{7,8}$ **8**	$C_{8,9}$ **4**	$C_{9,10}$ **14**
$C_{1,3}$ **13**	$C_{2,4}$ **25**	$C_{3,5}$ **27**	$C_{4,6}$ **24**	$C_{5,7}$ **17**	$C_{6,8}$ **6**	$C_{7,9}$ **5**	$C_{8,10}$ **15**	
$C_{1,4}$ **35**	$C_{2,5}$ **18**	$C_{3,6}$ **32**	$C_{4,7}$ **39**	$C_{5,8}$ **2**	$C_{6,9}$ **19**	$C_{7,10}$ **28**		
$C_{1,5}$ **41**	$C_{2,6}$ **26**	$C_{3,7}$ **40**	$C_{4,8}$ **23**	$C_{5,9}$ **20**	$C_{6,10}$ **34**			
$C_{1,6}$ **42**	$C_{2,7}$ **38**	$C_{3,8}$ **31**	$C_{4,9}$ **44**	$C_{5,10}$ **37**				
$C_{1,7}$ **33**	$C_{2,8}$ **16**	$C_{3,9}$ **45**	$C_{4,10}$ **43**					
$C_{1,8}$ **22**	$C_{2,9}$ **30**	$C_{3,10}$ **29**						
$C_{1,9}$ **36**	$C_{2,10}$ **7**							
$C_{1,10}$ **11**								

Step A: Arrange the numerical values of the distances in increasing order

$C_{5,6}$ **1**	$C_{5,8}$ **2**	$C_{1,2}$ **3**	$C_{8,9}$ **4**	$C_{7,9}$ **5**	$C_{6,8}$ **6**	$C_{2,10}$ **7**	$C_{7,8}$ **8**	$C_{6,7}$ **9**
$C_{3,4}$ **10**	$C_{1,10}$ **11**	$C_{4,5}$ **12**	$C_{1,3}$ **13**	$C_{9,10}$ **14**	$C_{8,10}$ **15**	$C_{2,8}$ **16**	$C_{5,7}$ **17**	$C_{2,5}$ **18**
$C_{6,9}$ **19**	$C_{5,9}$ **20**	$C_{2,3}$ **21**	$C_{1,8}$ **22**	$C_{4,8}$ **23**	$C_{4,6}$ **24**	$C_{2,4}$ **25**	$C_{2,6}$ **26**	$C_{3,5}$ **27**
$C_{7,10}$ **28**	$C_{3,10}$ **29**	$C_{2,9}$ **30**	$C_{3,8}$ **31**	$C_{3,6}$ **32**	$C_{1,7}$ **33**	$C_{6,10}$ **34**	$C_{1,4}$ **35**	$C_{1,9}$ **36**
$C_{5,10}$ **37**	$C_{2,7}$ **38**	$C_{4,7}$ **39**	$C_{3,7}$ **40**	$C_{1,5}$ **41**	$C_{1,6}$ **42**	$C_{4,10}$ **43**	$C_{4,9}$ **44**	$C_{3,9}$ **45**

Since there are ten cities, ten distances are needed for the salesman to visit each of nine cities once and return to City 1. For the departure from City 1, the first subscript of the distance from City 1 is 1, and for the return to City 1, the second subscript of the last distance is 1. One will select ten distances, one at a time, to obtain ten well-connected distances to allow the salesman to visit each city once and return to City 1. Ideally, if one were able to use only the shortest ten distances for the route construction, one would have surely, constructed the shortest route, since, numerically, one would have found the sum of the shortest ten distances.
One will always begin and concentrate on the distances in the box with thicker borders, and one will call this box, the Royal box. The Royal box contains the shortest 10 distances of the total 45 distances to choose from. One will start and the route construction by choosing from the set of the shortest ten distances; and if a needed distance is not among the set of the shortest ten distances, one would consider distances longer than those in the set of the shortest ten distances (that is, distances outside the Royal box)

Royal box Table 1

A $C_{5,6}(1)$ or $C_{6,5}(1)$	$C_{4,5}(12)$ or $C_{5,4}(12)$	$C_{4,8}(23)$ or $C_{8,4}(23)$	$C_{6,10}(34)$ or $C_{10,6}(34)$
B $C_{5,8}(2)$ or $C_{8,5}(2)$	$C_{1,3}(13)$ or $C_{3,1}(13)$	$C_{4,6}(24)$ or $C_{6,4}(24)$	$C_{1,4}(35)$ or $C_{4,1}(35)$
C $C_{1,2}(3)$ or $C_{2,1}(3)$	$C_{9,10}(14)$ or $C_{10,9}(14)$	$C_{2,4}(25)$ or $C_{4,2}(25)$	$C_{1,9}(36)$ or $C_{9,1}(36)$
D $C_{8,9}(4)$ or $C_{9,8}(4)$	$C_{8,10}(15)$ or $C_{10,8}(15)$	$C_{2,6}(26)$ or $C_{6,2}(26)$	$C_{5,10}(37)$ or $C_{10,5}(37)$
E $C_{7,9}(5)$ or $C_{9,7}(5)$	$C_{2,8}(16)$ or $C_{8,2}(16)$	$C_{3,5}(27)$ or $C_{5,3}(27)$	$C_{2,7}(38)$ or $C_{7,2}(38)$
F $C_{6,8}(6)$ or $C_{8,6}(6)$	$C_{5,7}(17)$ or $C_{7,5}(17)$	$C_{7,10}(28)$ or $C_{10,7}(28)$	$C_{4,7}(39)$ or $C_{7,4}(39)$
G $C_{2,10}(7)$ or $C_{10,2}(7)$	$C_{2,5}(18)$ or $C_{5,2}(18)$	$C_{3,10}(29)$ or $C_{10,3}(29)$	$C_{3,7}(40)$ or $C_{7,3}(40)$
H $C_{7,8}(8)$ or $C_{8,7}(8)$	$C_{6,9}(19)$ or $C_{9,6}(19)$	$C_{2,9}(30)$ or $C_{9,2}(30)$	$C_{1,5}(41)$ or $C_{5,1}(41)$
I $C_{6,7}(9)$ or $C_{7,6}(9)$	$C_{5,9}(20)$ or $C_{9,5}(20)$	$C_{3,8}(31)$ or $C_{8,3}(31)$	$C_{1,6}(42)$ or $C_{6,1}(42)$
J $C_{3,4}(10)$ or $C_{4,3}(10)$	$C_{2,3}(21)$ or $C_{3,2}(21)$	$C_{3,6}(32)$ or $C_{6,3}(32)$	$C_{4,10}(43)$ or $C_{10,4}(43)$
$C_{1,10}(11)$ or $C_{10,1}(11)$	$C_{1,8}(22)$ or $C_{8,1}(22)$	$C_{1,7}(33)$ or $C_{7,1}(33)$	$C_{4,9}(44)$ or $C_{9,4}(44)$
			$C_{3,9}(45)$ or $C_{9,3}(45)$

Important points in the route construction

1. Begin the route construction by choosing from the Royal box (the set of the shortest ten distances)

2. Branching begins only from distances in the Royal box, and each branch distance must go to a distance in the Royal box, if possible, but if a needed distance is not available in the Royal box, one will go outside the Royal box.).

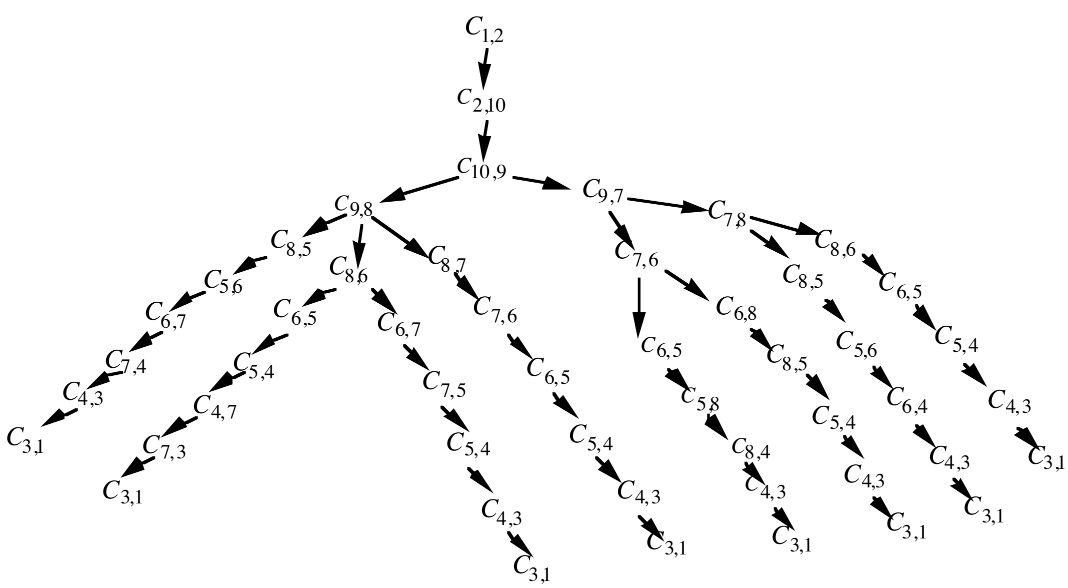
3. When choosing from outside the Royal box, do not skip the first (nearest) applicable distance. After choosing fom outside the Royal box, one should return immediately to the Royal box and continue. Note that one wishes that if it were possible, the selection of the ten distances would be done from the Royal box..

4. It is important that any possible branch is **not** missed, since such a branch may lead to the shortest route. By hand, draw the tree diagram and check by repeating the drawing

Analogy in the approach used in the route construction

If 45 workers are available to work on a project consisting of 10 consecutive tasks such that each worker is required to perform a single task out of the 10 tasks, an employer would hire the best 10 workers to begin the work. If at any step, any hired person cannot perform his or her task, the employer will replace this employee by the next qualified person among the remaining 35 possible hirees. Such a replacement may continue until all the 10 consecutive tasks have been completed. In this approach, the employer does not hire all the 45 available hirees. It may be possible that each of the first 10 hired is able to perform his or her task; and in this case, there would be no need to replace any of the first 10 people hired.

Tree Diagram for the Route Construction



Note that the tree diagram helps one to keep track of all the possible routes

Procedure: Use the above tree diagram to follow the solution steps below..
One will always begin the selection of the distances from the Royal box.
In the royal box ,$C_{1,2}$ (in box C) is the only distance with subscript 1, and it will be the starting (departure) distance.

Step 1: Begin with first city distance $C_{1,2}$ (from box C, above).

Note: $C_{1,2}$ means distance from City 1 to City 2. (From City 1, salesman visits City 2.)

Possible Routes

R1	R2	R3	R4	R5	R6	R7	R8
$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**
$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**
$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**
$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,7}$ **5**	$C_{9,7}$ **5**	$C_{9,7}$ **5**	$C_{9,7}$ **5**
$C_{8,5}$ **2**	$C_{8,6}$ **6**	$C_{8,6}$ **6**	$C_{8,7}$ **8**	$C_{7,8}$ **8**	$C_{7,8}$ **8**	$C_{7,6}$ **9**	$C_{7,6}$ **9**
$C_{5,6}$ **1**	$C_{6,5}$ **1**	$C_{6,7}$ **9**	$C_{7,6}$ **9**	$C_{8,5}$ **2**	$C_{8,6}$ **6**	$C_{6,5}$ **1**	$C_{6,8}$ **6**
$C_{6,7}$ **9**	$C_{5,4}$ **12**	$C_{7,5}$ **17**	$C_{6,5}$ **1**	$C_{5,6}$ **1**	$C_{6,5}$ **1**	$C_{5,8}$ **2**	$C_{8,5}$ **2**
$C_{7,4}$ **39**	$C_{4,7}$ **39**	$C_{5,4}$ **12**	$C_{5,4}$ **12**	$C_{6,4}$ **24**	$C_{5,4}$ **12**	$C_{8,4}$ **23**	$C_{5,4}$ **12**
$C_{4,3}$ **10**	$C_{7,3}$ **40**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**
$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**
102	**139**	**95**	**81**	**87**	**79**	**87**	**81**

Step: 2: Since the second subscript of $C_{1,2}$ is 2, the first subscript of the next distance will be 2. Inspect the boxes in the Royal box to pick a distance whose first subscript is 2. Box G contains a distance with 2 as a first subscript. We choose the distance $C_{2,10}$ in box G. Connect the chosen distance with the distance in Step 1 to obtain the connected distances $C_{1,2}-C_{2,10}$, shown vertically in the tree diagram and as the first two rows of column R1 of the possible routes.

Step 3: Since the second subscript of the last distance is 10, the first subscript of the next distance should be 10. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2), except that the first subscript of the next distance should be 10. Inspection of the entries in the Royal box indicates that there is no distance whose first subscript is 10. One will go outside the Royal box for the next applicable distance. One chooses $C_{10,9}$ (Note that $C_{10,1}$ is excluded). Never skip the nearest applicable distance. The excluded subscript numbers ,except 1, represent the cities already visited.

Step 4: Since the second subscript of the last distance is 9, the first subscript of the next distance should be 9. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2, 10), except that the first subscript of the next distance should be 9. Inspection of the Royal box shows that there are two distances, namely, $C_{9,8}$ and $C_{9,7}$ with 9 as first subscript. This situation implies that there are two branches from the last distance as in the tree diagram.

Step 5: One will work on distance $C_{9,8}$ followed by $C_{9,7}$.

For $C_{9,8}$	For $C_{9,7}$
Since the second subscript of this distance is 8, the first subscript of the next distance should be 8. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2,10, 9) except that the first subscript of the next distance should be 8, Inspection of the Royal box shows that there are three distances namely, $C_{8,5}$,$C_{8,6}$ and $C_{8,7}$, producing three branches from $C_{9,8}$.	Since the second subscript of this distance is 7, the first subscript of the next distance should be 7. Note that the next distance should not contain any of the subscripts already used (i.e., no 1, 2,10, 9) except that the first subscript of the next distance should be 7, Inspection of the Royal box shows that there are two applicable distances namely, $C_{7,6}$ and $C_{7,8}$,producing two tree branches from $C_{9,7}$

Step 6:

For $C_{9,8}$			For $C_{9,7}$	
$C_{8,5}$	$C_{8,6}$	$C_{8,7}$	$C_{7,6}$	$C_{7,8}$
Since the second subscript of this distance is 5, the first subscript of the next distance should be 5. Note that the next distance should not contain any of the subscripts already used, (No 1, 2,10, 9, 8) except that the first subscript of the next distance should be 5. Inspection of the Royal box shows that there is only one applicable distance namely, $C_{5,6}$ from box A.	Since the second subscript of this distance is 6, the first subscript of the next distance should be 6. Note that the next distance should not contain any of the subscripts already used, (No 1, 2,10, 9. 8) except that the first subscript of the next distance should be 6. Inspection of the Royal box shows that there are two applicable distances namely, $C_{6,5}$ and $C_{6,7}$ from boxes A and I respectively.	Since the second subscript of this distance is 7, the first subscript of the next distance should be 7. Note that the next distance should not contain any of the subscripts already used, (No 1, 2,10, 9. 8) except that the first subscript of the next distance should be 7. Inspection of the Royal box shows that there is only one applicable distance namely, $C_{7,6}$ from box **I**.	Since the second subscript of this distance is 6, the first subscript of the next distance should be 6. Note that the next distance should not contain any of the subscripts already used, (No 1, 2,10, 9. 7) except that the first subscript of the next distance should be 6. Inspection of the Royal box shows that there are two applicable distances namely, $C_{6,8}$ and $C_{6,5}$ from boxes F and A respectively.	Since the second subscript of this distance is 6, the first subscript of the next distance should be 8. Note that the next distance should not contain any of the subscripts already used, (No 1, 2,10, 9. 7) except that the first subscript of the next distance should be 8. Inspection of the Royal box shows that there are two applicable distances namely, $C_{8,5}$ and $C_{8,6}$ from boxes B and F respectively.

Step 7: One will next determine the next distances for some of the descendants of $C_{9,8}$.

That is, for $C_{5,6}$ $C_{6,5}$ $C_{6,7}$, $C_{7,6}$,

$C_{5,6}$	$C_{6,5}$	$C_{6,7}$	$C_{7,6}$
Since the second subscript of this distance is 6, the first subscript of the next distance should be 6. Note that the next distance should not contain any of the subscripts already used, (No 1, 2,10, 9. 8, 5) except that the first subscript of the next distance should be 6. Inspection of the Royal box shows that there is only one applicable distance namely, $C_{6,7}$ from box I.	Since the second subscript of this distance is 5, the first subscript of the next distance should be 5. Note that the next distance should not contain any of the subscripts already used. (No 1, 2,10, 9, 8,6) except that the first subscript of the next distance should be 5. Inspection of the entries in the Royal box indicates that there is no applicable distance whose first subscript is 5. Note that $C_{5,6}$ and $C_{5,8}$ are excluded here. One will go outside the Royal box for the next applicable distance. One chooses $C_{5,4}$ Never skip the nearest applicable distance.	Since the second subscript of this distance is 7, the first subscript of the next distance should be 7. Note that the next distance should not contain any of the subscripts already used. (No 1, 2,10, 9,8, 6) except that the first subscript of the next distance should be 7. Inspection of the entries in the Royal box indicates that there is no applicable distance whose first subscript is 7. Note that $C_{7,9}$, $C_{7,8}$ and $C_{7,6}$ are excluded here. One will go outside the Royal box for the next applicable distance. One chooses $C_{7,5}$.	Since the second subscript of this distance is 6, the first subscript of the next distance should be 6. Note that the next distance should not contain any of the subscripts already used, (No 1, 2,10, 9, 8 7) except that the first subscript of the next distance should be 6. Inspection of the Royal box shows hat there is only one applicable distance namely, $C_{6,5}$ from box A.

By imitating the above steps, one will obtain the following columns

$C_{6,7}$	$C_{5,4}$	$C_{7,5}$	$C_{6,5}$	$C_{5,6}$	$C_{6,5}$	$C_{5,8}$	$C_{8,5}$
$C_{7,4}$	$C_{4,7}$	$C_{5,4}$	$C_{5,4}$	$C_{6,4}$	$C_{5,4}$	$C_{8,4}$	$C_{5,4}$
$C_{4,3}$	$C_{7,3}$	$C_{4,3}$	$C_{4,3}$	$C_{4,3}$	$C_{4,3}$	$C_{4,3}$	$C_{4,3}$
$C_{3,1}$	$C_{3,1}$	$C_{3,1}$	$C_{3,1}$	$C_{3,1}$	$C_{3,1}$	$C_{3,1}$	$C_{3,1}$

Step 8: By combining steps 1-7, one obtains the possible routes, R1, R2, R3, R4, R5, R6, R7, R8

R1	**R2**	**R3**	**R4**	**R5**	**R6**	**R7**	**R8**
$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**
$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**
$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**
$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,7}$ **5**	$C_{9,7}$ **5**	$C_{9,7}$ **5**	$C_{9,7}$ **5**
$C_{8,5}$ **2**	$C_{8,6}$ **6**	$C_{8,6}$ **6**	$C_{8,7}$ **8**	$C_{7,8}$ **8**	$C_{7,8}$ **8**	$C_{7,6}$ **9**	$C_{7,6}$ **9**
$C_{5,6}$ **1**	$C_{6,5}$ **1**	$C_{6,7}$ **9**	$C_{7,6}$ **9**	$C_{8,5}$ **2**	$C_{8,6}$ **6**	$C_{6,5}$ **1**	$C_{6,8}$ **6**
$C_{6,7}$ **9**	$C_{5,4}$ **12**	$C_{7,5}$ **17**	$C_{6,5}$ **1**	$C_{5,6}$ **1**	$C_{6,5}$ **1**	$C_{5,8}$ **2**	$C_{8,5}$ **2**
$C_{7,4}$ **39**	$C_{4,7}$ **39**	$C_{5,4}$ **12**	$C_{5,4}$ **12**	$C_{6,4}$ **24**	$C_{5,4}$ **12**	$C_{8,4}$ **23**	$C_{5,4}$ **12**
$C_{4,3}$ **10**	$C_{7,3}$ **40**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**
$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**
102	**139**	**95**	**81**	**87**	**79**	**87**	**81**

Shortest Route

From the above table, the shortest route is **Route R6** of length 79 units.

$C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,7}(5)C_{7,8}(8)C_{8,6}(6)C_{6,5}(1)C_{5,4}(12)C_{4,3}(10)C_{3,1}(13)= 79$

Verification and Justification of the Shortest Route Determined

In the shortest route, **R6** (below), seven of the distances are from the Royal box (below) and the other three are the next three distances outside the Royal box (except 11 which is excluded here because of the subscript, 1), namely, 12, 13, and 14, are included in R6. Thus, no applicable relatively short distance was skipped or ignored. Note: all distances are in kilometers. Note that R4 and R8 are good competitors for the shortest route. Such a challenge makes the approach used in determining the shortest route very encouraging, since for road or weather conditions, the salesman can alternatively use routes R4 and R8.

R1	**R2**	**R3**	**R4**	**R5**	**R6**	**R7**	**R8**
$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**	$C_{1,2}$ **3**
$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**	$C_{2,10}$ **7**
$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**	$C_{10,9}$ **14**
$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,8}$ **4**	$C_{9,7}$ **5**	$C_{9,7}$ **5**	$C_{9,7}$ **5**	$C_{9,7}$ **5**
$C_{8,5}$ **2**	$C_{8,6}$ **6**	$C_{8,6}$ **6**	$C_{8,7}$ **8**	$C_{7,8}$ **8**	$C_{7,8}$ **8**	$C_{7,6}$ **9**	$C_{7,6}$ **9**
$C_{5,6}$ **1**	$C_{6,5}$ **1**	$C_{6,7}$ **9**	$C_{7,6}$ **9**	$C_{8,5}$ **2**	$C_{8,6}$ **6**	$C_{6,5}$ **1**	$C_{6,8}$ **6**
$C_{6,7}$ **9**	$C_{5,4}$ **12**	$C_{7,5}$ **17**	$C_{6,5}$ **1**	$C_{5,6}$ **1**	$C_{6,5}$ **1**	$C_{5,8}$ **2**	$C_{8,5}$ **2**
$C_{7,4}$ **39**	$C_{4,7}$ **39**	$C_{5,4}$ **12**	$C_{5,4}$ **12**	$C_{6,4}$ **24**	$C_{5,4}$ **12**	$C_{8,4}$ **23**	$C_{5,4}$ **12**
$C_{4,3}$ **10**	$C_{7,3}$ **40**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**	$C_{4,3}$ **10**
$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**	$C_{3,1}$ **13**
102	**139**	**95**	**81**	**87**	**79**	**87**	**81**

Royal box

A $C_{5,6}$ or $C_{6,5}$ **1**	$C_{4,5}$ or $C_{5,4}$ **12**	$C_{4,8}$ or $C_{8,4}$ **23**	$C_{6,10}$ or $C_{10,6}$ **34**
B $C_{5,8}$ or $C_{8,5}$ **2**	$C_{1,3}$ or $C_{3,1}$ **13**	$C_{4,6}$ or $C_{6,4}$ **24**	$C_{1,4}$ or $C_{4,1}$ **35**
C $C_{1,2}$ or $C_{2,1}$ **3**	$C_{9,10}$ or $C_{10,9}$ **14**	$C_{2,4}$ or $C_{4,2}$ **25**	$C_{1,9}$ or $C_{9,1}$ **36**
D $C_{8,9}$ or $C_{9,8}$ **4**	$C_{8,10}$ or $C_{10,8}$ **15**	$C_{2,6}$ or $C_{6,2}$ **26**	$C_{5,10}$ or $C_{10,5}$ **37**
E $C_{7,9}$ or $C_{9,7}$ **5**	$C_{2,8}$ or $C_{8,2}$ **16**	$C_{3,5}$ or $C_{5,3}$ **27**	$C_{2,7}$ or $C_{7,2}$ **38**
F $C_{6,8}$ or $C_{8,6}$ **6**	$C_{5,7}$ or $C_{7,5}$ **17**	$C_{7,10}$ or $C_{10,7}$ **28**	$C_{4,7}$ or $C_{7,4}$ **39**
G $C_{2,10}$ or $C_{10,2}$ **7**	$C_{2,5}$ or $C_{5,2}$ **18**	$C_{3,10}$ or $C_{10,3}$ **29**	$C_{3,7}$ or $C_{7,3}$ **40**
H $C_{7,8}$ or $C_{8,7}$ **8**	$C_{6,9}$ or $C_{9,6}$ **19**	$C_{2,9}$ or $C_{9,2}$ **30**	$C_{1,5}$ or $C_{5,1}$ **41**
I $C_{6,7}$ or $C_{7,6}$ **9**	$C_{5,9}$ or $C_{9,5}$ **20**	$C_{3,8}$ or $C_{8,3}$ **31**	$C_{1,6}$ or $C_{6,1}$ **42**
J $C_{3,4}$ or $C_{4,3}$ **10**	$C_{2,3}$ or $C_{3,2}$ **21**	$C_{3,6}$ or $C_{6,3}$ **32**	$C_{4,10}$ or $C_{10,4}$ **43**
$C_{1,10}$ or $C_{10,1}$ **11**	$C_{1,8}$ or $C_{8,1}$ **22**	$C_{1,7}$ or $C_{7,1}$ **33**	$C_{4,9}$ or $C_{9,4}$ **44**
			$C_{3,9}$ or $C_{9,3}$ **45**

From above, the traveling salesman problem has been solved in polynomial time, and the correctness of the solution has also been checked in polynomial time, Therefore, the above traveling salesman problem is no longer an **N**P-hard problem. It is a **P** problem.
In the next example, Example 1b, the problem will be classified as NP complete problem at the beginning of the solution, but after solving and checking the correctness of the solution, it will be reclassified as a **P** problem.

Example 1b : As in Example 1a, From City 1, a traveling salesman would like to visit once each of **nine** other cities, namely, City 2, City 3, City 4, City 5, City 6, City 7, City 8, City 9, City 10; and return to City 1. If the salesman has a limited supply of gasoline, to cover a total of 80 kilometers, can the salesman visit all nine cities and return to the home base city?

Solution & Checking

With the attached 80-kilometer condition, Example 1a, which was originally an NP-hard problem becomes an NP-Complete problem, since the 80 kilometer condition makes it easy to check the correctness of the answer
From Example 1a , since the shortest route determined was 79 kilometers, and 79 is less than 80, yes. the salesman can visit all nine cities and return to the home base city?

Sub-Discussion and Sub-Conclusion for the Shortest Route

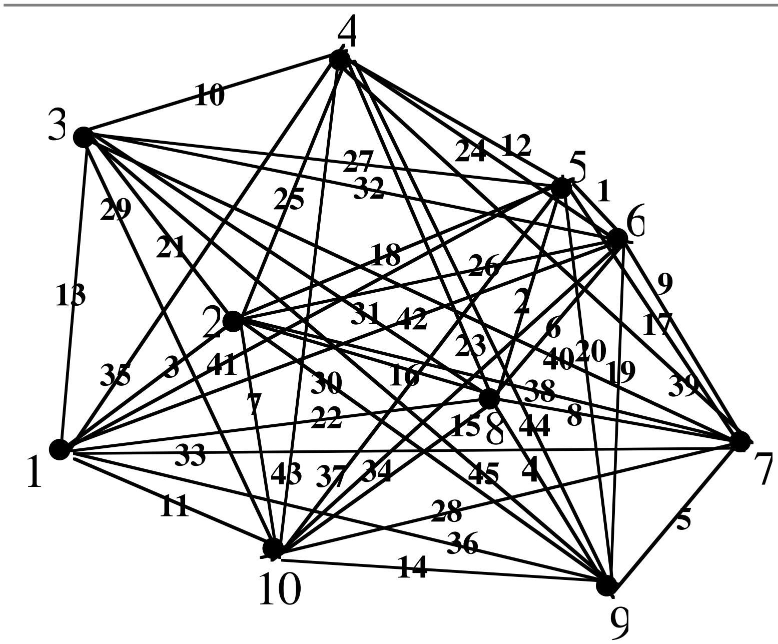
The length of the shortest route was found to be 79 kilometers; but the sequence of cities of the shortest route is not unique. One sequence of the cities of the shortest route is given by
$C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,7}(5)C_{7,8}(8)C_{8,6}(6)C_{6,5}(1)C_{5,4}(12)C_{4,3}(10)C_{3,1}(13) = 79$. If the direction of travel of this route is reversed, one obtains the route given by
$C_{1,3}(13)C_{3,4}(10)C_{4,5}(12)C_{5,6}(1)C_{6,8}(6)C_{8,7}(8)C_{7,9}(5)C_{9,10}(14)C_{10,2}(7)C_{2,1}(3) = 79$
The future in the approach for solving the traveling salesman problem lies in the approach (data ordering and route construction)) whereby one concentrates on the smallest distances, and by judicious selection, constructs the shortest route. Such an approach reduces the redundant use of brute force. For the nine cities visit, using brute-force, one would have to consider about 362,880 possibilities. Each possibility would be a column of nine distances. One of these 362,880 columns would be the shortest route to visit the nine cities without returning to City 1. In the approach used in this paper, only eight columns were constructed.

The error in the shortest route of length 79 units determined is zero or negligible.
From Example 1a and Example 1b, P = NP-hard; an P = NP Complete

Now, by moving the cursor (using the mouse), enjoy the following travel:

$C_{1,2}(3)C_{2,10}(7)C_{10,9}(14)C_{9,7}(5)C_{7,8}(8)C_{8,6}(6)C_{6,5}(1)C_{5,4}(12)C_{4,3}(10)C_{3,1}(13)= 79$ is equivalent to

$C_{1,2}(3) + C_{2,10}(7) + C_{10,9}(14) + C_{9,7}(5) + C_{7,8}(8) + C_{8,6}(6) + C_{6,5}(1) + C_{5,4}(12) + C_{4,3}(10) + C_{3,1}(13)= 79$



Shortest Travel Route

From City 1 to City 2;
from City 2 to City 10;
from City 10 to City 9;
from City 9 to City 7;
from City 7 to City 8;
from City 8 to City 6;
from City 6 to City 5;
from City 5 to City 4;
.from City 4 to City 3;
and finally, from City 3 to City 1.

Adonten

Traveling Salesman Problem: Longest Route

Example 1c: As in Example 1a, from City 1, a traveling salesman would like to visit once each of **nine** other cities, namely, City 2, City 3, City 4, City 5, City 6, City 7, City 8, City 9, City 10; and return to City 1. The salesman's family has a gasoline station in almost every city in the country. For the end of year bonus, the salesman's boss has informed him that to increase sales at his family's gas stations, he does not have to take the usual shortest route, but he can take the longest route. Determine the **longest route,** for the salesman so that he can maximize gasoline sales at the family gasoline stations.

Solution

The first step is to arrange the distances in this problem in **decreasing** order. The main principle in finding the longest route is finding the sum of the ten longest distances such that the salesman visits each city once and returns to the starting city.

At this step, this problem will be classified as an NP-hard problem. This classification will change after finding the solution and successfully checking the correctness of the solution.

Procedure: Imitate Example 1a, but note that one is interested in the longest distances. One will always begin the selection of the distances from the Royal box.

In the Royal box of example 1a, there was only one distance whose first subscript was 1, as in $C_{1,2}$, and therefore, there was only one departure distance.

In this example, there are three distances in the Royal box, with 1 as the first subscript, namely, D ($C_{1,6}$), E ($C_{1,5}$) and $C_{1,9}(36)$. Therefore, one will consider three possible departure distances for the route construction.

A $C_{3,9}(45)$ or $C_{9,3}(45)$	$C_{6,10}(34)$ or $C_{10,6}(34)$	$C_{4,8}(23)$ or $C_{8,4}(23)$	$C_{4,5}(12)$ or $C_{5,4}(12)$
B $C_{4,9}(44)$ or $C_{9,4}(44)$	$C_{1,7}(33)$ or $C_{7,1}(33)$	$C_{1,8}(22)$ or $C_{8,1}(22)$	$C_{1,10}(11)$ or $C_{10,1}(11)$
C $C_{4,10}(43)$ or $C_{10,4}(43)$	$C_{3,6}(32)$ or $C_{6,3}(32)$	$C_{2,3}(21)$ or $C_{3,2}(21)$	$C_{3,4}(10)$ or $C_{4,3}(10)$
D $C_{1,6}(42)$ or $C_{6,1}(42)$	$C_{3,8}(31)$ or $C_{8,3}(31)$	$C_{5,9}(20)$ or $C_{9,5}(20)$	$C_{6,7}(9)$ or $C_{7,6}(9)$
E $C_{1,5}(41)$ or $C_{5,1}(41)$	$C_{2,9}(30)$ or $C_{9,2}(30)$	$C_{6,9}(19)$ or $C_{9,6}(19)$	$C_{7,8}(8)$ or $C_{8,7}(8)$
F $C_{3,7}(40)$ or $C_{7,3}(40)$	$C_{3,10}(29)$ or $C_{10,3}(29)$	$C_{2,5}(18)$ or $C_{5,2}(18)$	$C_{2,10}(7)$ or $C_{10,2}(7)$
G $C_{4,7}(39)$ or $C_{7,4}(39)$	$C_{7,10}(28)$ or $C_{10,7}(28)$	$C_{5,7}(17)$ or $C_{7,5}(17)$	$C_{6,8}(6)$ or $C_{8,6}(6)$
H $C_{2,7}(38)$ or $C_{7,2}(38)$	$C_{3,5}(27)$ or $C_{5,3}(27)$	$C_{2,8}(16)$ or $C_{8,2}(16)$	$C_{7,9}(5)$ or $C_{9,7}(5)$
I $C_{5,10}(37)$ or $C_{10,5}(37)$	$C_{2,6}(26)$ or $C_{6,2}(26)$	$C_{8,10}(15)$ or $C_{10,8}(15)$	$C_{8,9}(4)$ or $C_{9,8}(4)$
J $C_{1,9}(36)$ or $C_{9,1}(36)$	$C_{2,4}(25)$ or $C_{4,2}(25)$	$C_{9,10}(14)$ or $C_{10,9}(14)$	$C_{1,2}(3)$ or $C_{2,1}(3)$
$C_{1,4}(35)$ or $C_{4,1}(35)$	$C_{4,6}(24)$ or $C_{6,4}(24)$	$C_{1,3}(13)$ or $C_{3,1}(13)$	$C_{5,8}(2)$ or $C_{8,5}(2)$
			$C_{5,6}(1)$ or $C_{6,5}(1)$

For the departure distance, $C_{1,6}$ (Beginning with C1,6 : (Six branches)

Step 1: Begin with first city distance $C_{1,6}$ (from box D, above).

Note: $C_{1,6}$ means distance from City 1 to City 6. (From City 1, salesman visits City 6.)

Imitate the procedure in Example 1a to obtain the following routes

Using a tree diagram (not shown) as in Example 1a, one obtains the following tables

Table 1

R1	**R2**	**R3**	**R4**	**R5**	**R6**
$C_{1,6}(42)$	$C_{1,6}(42)$	$C_{1,6}(42)$	$C_{1,6}(42)$	$C_{1,6}(42)$	$C_{1,6}(42)$
$C_{6,10}(34)$	$C_{6,10}(34)$	$C_{6,10}(34)$	$C_{6,10}(34)$	$C_{6,10}(34)$	$C_{6,10}(34)$
$C_{10,4}(43)$	$C_{10,4}(43)$	$C_{10,4}(43)$	$C_{10,5}(37)$	$C_{10,5}(37)$	$C_{10,5}(37)$
$C_{4,7}(39)$	$C_{4,7}(39)$	$C_{4,9}(44)$	$C_{5,3}(27)$	$C_{5,3}(27)$	$C_{5,3}(27)$
$C_{7,2}(38)$	$C_{7,3}(40)$	$C_{9,3}(45)$	$C_{3,9}(45)$	$C_{3,7}(40)$	$C_{3,7}(40)$
$C_{2,9}(30)$	$C_{3,9}(45)$	$C_{3,7}(40)$	$C_{9,4}(44)$	$C_{7,4}(39)$	$C_{7,2}(38)$
$C_{9,3}(45)$	$C_{9,2}(30)$	$C_{7,2}(38)$	$C_{4,7}(39)$	$C_{4,9}(44)$	$C_{2,9}(30)$
$C_{3,8}(31)$	$C_{2,5}(18)$	$C_{2,5}(18)$	$C_{7,2}(38)$	$C_{9,2}(30)$	$C_{9,4}(44)$
$C_{8,5}(2)$	$C_{5,8}(2)$	$C_{5,8}(2)$	$C_{2,8}(16)$	$C_{2,8}(16)$	$C_{4,8}(23)$
$C_{5,1}(41)$	$C_{8,1}(22)$	$C_{8,1}(22)$	$C_{8,1}(22)$	$C_{8,1}(22)$	$C_{8,1}(22)$
345	**315**	**328**	**344**	**331**	**337**

For the departure distance, $C_{1,5}$ (Beginning with **C1.5** (Three branches)

Table 2

R1 R2 R3 R4

$C_{1,5}(41)$	$C_{1,5}(41)$	$C_{1,5}(41)$
$C_{5,10}(37)$	$C_{5,10}(37)$	$C_{5,10}(37)$
$C_{10,4}(43)$	$C_{10,4}(43)$	$C_{10,4}(43)$
$C_{4,9}(44)$	$C_{4,7}(39)$	$C_{4,7}(39)$
$C_{9,3}(45)$	$C_{7,2}(38)$	$C_{7,3}(40)$
$C_{3,7}(40)$	$C_{2,9}(30)$	$C_{3,9}(45)$
$C_{7,2}(38)$	$C_{9,3}(45)$	$C_{9,2}(30)$
$C_{2,6}(26)$	$C_{3,6}(32)$	$C_{2,6}(26)$
$C_{6,8}(6)$	$C_{6,8}(6)$	$C_{6,8}(6)$
$C_{8,1}(22)$	$C_{8,1}(22)$	$C_{8,1}(22)$
342	**333**	**329**

For the departure distance, $C_{1,9}$ (Beginning with C1,9 : (Six branches)

Table 3

R1	**R2**	**R3**	**R4**	**R5**	**R6**
$C_{1,9}(36)$	$C_{1,9}(36)$	$C_{1,9}(36)$	$C_{1,9}(36)$	$C_{1,9}(36)$	$C_{1,9}(36)$
$C_{9,3}(45)$	$C_{9,3}(45)$	$C_{9,3}(45)$	$C_{9,4}(44)$	$C_{9,4}(44)$	$C_{9,4}(44)$
$C_{3,7}(40)$	$C_{3,7}(40)$	$C_{3,7}(40)$	$C_{4,10}(43)$	$C_{4,7}(39)$	$C_{4,7}(39)$
$C_{7,2}(38)$	$C_{7,2}(38)$	$C_{7,4}(39)$	$C_{10,5}(37)$	$C_{7,3}(40)$	$C_{7,2}(38)$
$C_{2,6}(26)$	$C_{2,6}(26)$	$C_{4,10}(43)$	$C_{5,3}(27)$	$C_{3,6}(32)$	$C_{2,6}(26)$
$C_{6,10}(34)$	$C_{6,10}(34)$	$C_{10,5}(37)$	$C_{3,7}(40)$	$C_{6,10}(34)$	$C_{6,10}(34)$
$C_{10,5}(37)$	$C_{10,4}(43)$	$C_{5,2}(18)$	$C_{7,2}(38)$	$C_{10,5}(37)$	$C_{10,5}(37)$
$C_{5,4}(12)$	$C_{4,8}(23)$	$C_{2,6}(26)$	$C_{2,6}(26)$	$C_{5,2}(18)$	$C_{5,3}(27)$
$C_{4,8}(23)$	$C_{8,5}(2)$	$C_{6,8}(6)$	$C_{6,8}(6)$	$C_{2,8}(16)$	$C_{3,8}(31)$
$C_{8,1}(22)$	$C_{5,1}(41)$	$C_{8,1}(22)$	$C_{8,1}(22)$	$C_{8,1}(22)$	$C_{8,1}(22)$
313	**328**	**312**	**319**	**318**	**334**

From the above Tales, 1,2 and 3, the longest rore is route 1 (**R1**) of Table 1

A $C_{3,9}(45)$or $C_{9,3}(45)$	$C_{6,10}(34)$or $C_{10,6}(34)$	$C_{4,8}(23)$ or $C_{8,4}(23)$	$C_{4,5}(12)$ or $C_{5,4}(12)$
B $C_{4,9}(44)$ or $C_{9,4}(44)$	$C_{1,7}(33)$ or $C_{7,1}(33)$	$C_{1,8}(22)$ or $C_{8,1}(22)$	$C_{1,10}(11)$ or $C_{10,1}(11)$
C $C_{4,10}(43)$ or $C_{10,4}(43)$	$C_{3,6}(32)$ or $C_{6,3}(32)$	$C_{2,3}(21)$ or $C_{3,2}(21)$	$C_{3,4}(10)$ or $C_{4,3}(10)$
D $C_{1,6}(42)$ or $C_{6,1}(42)$	$C_{3,8}(31)$ or $C_{8,3}(31)$	$C_{5,9}(20)$or $C_{9,5}(20)$	$C_{6,7}(9)$ or $C_{7,6}(9)$
E $C_{1,5}(41)$ or $C_{5,1}(41)$	$C_{2,9}(30)$ or $C_{9,2}(30)$	$C_{6,9}(19)$ or $C_{9,6}(19)$	$C_{7,8}(8)$ or $C_{8,7}(8)$
F $C_{3,7}(40)$ or $C_{7,3}(40)$	$C_{3,10}(29)$ or $C_{10,3}(29)$	$C_{2,5}(18)$ or $C_{5,2}(18)$	$C_{2,10}(7)$ or $C_{10,2}(7)$
G $C_{4,7}(39)$ or $C_{7,4}(39)$	$C_{7,10}(28)$ or $C_{10,7}(28)$	$C_{5,7}(17)$ or $C_{7,5}(17)$	$C_{6,8}(6)$ or $C_{8,6}(6)$
H $C_{2,7}(38)$ or $C_{7,2}(38)$	$C_{3,5}(27)$ or $C_{5,3}(27)$	$C_{2,8}(16)$ or $C_{8,2}(16)$	$C_{7,9}(5)$ or $C_{9,7}(5)$
I $C_{5,10}(37)$or $C_{10,5}(37)$	$C_{2,6}(26)$or $C_{6,2}(26)$	$C_{8,10}(15)$ or $C_{10,8}(15)$	$C_{8,9}(4)$or $C_{9,8}(4)$
J $C_{1,9}(36)$ or $C_{9,1}(36)$	$C_{2,4}(25)$or $C_{4,2}(25)$	$C_{9,10}(14)$ or $C_{10,9}(14)$	$C_{1,2}(3)$ or $C_{2,1}(3)$
$C_{1,4}(35)$ or $C_{4,1}(35)$	$C_{4,6}(24)$or $C_{6,4}(24)$	$C_{1,3}(13)$ or $C_{3,1}(13)$	$C_{5,8}(2)$ or $C_{8,5}(2)$
			$C_{5,6}(1)$ or $C_{6,5}(1)$

Checking and Justification of the Longest Route Determined

From Table 1

R1

$C_{1,6}(42)$
$C_{6,10}(34)$
$C_{10,4}(43)$
$C_{4,7}(39)$
$C_{7,2}(38)$
$C_{2,9}(30)$
$C_{9,3}(45)$
$C_{3,8}(31)$
$C_{8,5}(2)$
$C_{5,1}(41)$
345

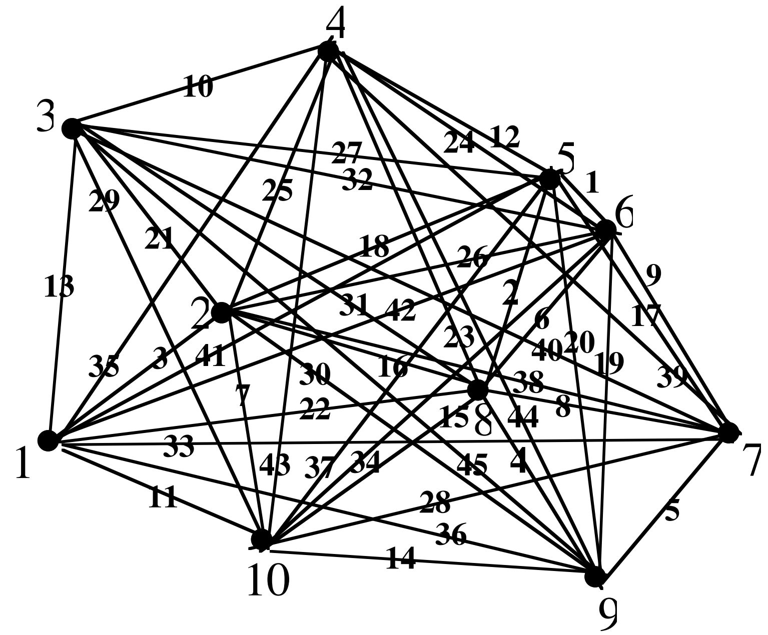
For the salesman to visit once each of nine cities and return to the home base city, ten distances are needed. Ideally, if all the distances were from the Royal box, one could immediately conclude that one has determined the longest route, since one would also have found the sum of the longest ten distances. However, in the longest route, **R1** of Table 1 (above) six of the distances namely, $C_{1,6}(42)$, $C_{10,4}(43)$, $C_{4,7}(39)$, $C_{7,2}(38)$, $C_{9,3}(45)$, $C_{5,1}(41)$ are from the Royal box. The first distance outside the Royal box, namely, $C_{1,4}(35)$ or $C_{4,1}(35)$ is not applicable because of the subscript 1, (which could either be a departure or return distance) the next three distances, (skipping $C_{3,6}(32)$) , namely $C_{6,10}(34)$, $C_{3,8}(31)$, and $C_{2,9}(30)$ are all part of the longest route. The only anomaly but necessary distnce is the distance, $C_{8,5}(2)$..

Longest Route

From Table 1. page 14, the longest **Route R1 is** of length 345 units.

$$C_{1,6}(42)C_{6,10}(34)C_{10,4}(43)C_{4,7}(39)C_{7,2}(38)C_{2,9}(30)C_{9,3}(45)C_{3,8}(31)C_{8,5}(2)C_{5,1}(41)$$

Now, by moving the cursor (using the mouse), enjoy the following travel:
$C_{1,6}(42)C_{6,10}(34)C_{10,4}(43)C_{4,7}(39)C_{7,2}(38)C_{2,9}(30)C_{9,3}(45)C_{3,8}(31)C_{8,5}(2)C_{5,1}(41)$=345 units is equivalent to
$C_{1,6}(42)+C_{6,10}(34)+C_{10,4}(43)+C_{4,7}(39)+C_{7,2}(38)+C_{2,9}(30)+C_{9,3}(45)+C_{3,8}(31)+C_{8,5}(2)+C_{5,1}(41)$



Longest Travel Route

From City 1 to City 6;
from City 6 to City 10;
from City 10 to City 4;
from City 4 to City 7;
from City 7 to City 2;
from City 2 to City 9;
from City 9 to City 3;
from City 3 to City 8;
.from City 8 to City 5;
and finally, from City 5 to City 1.

Comparison of the shortest route and the longest route

The shortest route was found to be of length 79 kilometers while the longest route was found to be of length 345 kilometers

The average of the above two lengths is $\frac{79+345}{2} = 212$ kilometers.

Perhaps, without finding the shortest route, the salesman's route would be 212 kilometers.

Sub-Conclusion

The longest route for the traveling salesman to visit once each of **nine** other cities, and return to the starting has been determined and confirmed; Therefore, the TSP is now a P problem, and P = NP-hard.

Next

Solutions of Six Other NP-Complete Problems

Solutions of NP-Complete Problems

Abstract

The simplest solution is usually the best solution---Albert Einstein

The NP-Complete problems covered include the division of items of different sizes, masses, or values into equal parts. The techniques and formulas developed for dividing these items into equal parts are based on an extended Ashanti fairness wisdom as exemplified below. If two people A and B are to divide items of different sizes which are arranged from the largest size to the smallest size, the procedure would be as follows. In the first round, A chooses the largest size, followed by B choosing the next largest size. In the second round, B chooses first, followed by A. In the third round, A chooses first, followed by B and the process continues up to the last item. To abbreviate the sequence in the above choices, one obtains the sequence "AB, BA AB". Let A and B divide the sum of the whole numbers, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1 as equally as possible, by merely always choosing the largest number. Then A chooses 10, B chooses 9 and 8, followed by A choosing 7 and 6; followed by B choosing 5 and 4; followed by A choosing 3 an 2; and finally, B chooses 1. The sum of A's choices is 10 +7+ 6 + 3 + 2 = 28; and the sum of B's choices is 9 + 8 + 5 + 4 + 1 = 27, with error, plus or minus 0.5 . Observe the sequence "AB, BA, AB, BA, AB". Observe also that the sequence is **not** "AB, AB, AB, AB, AB as one might think. The reason why the sequence is "AB, BA AB, BA, AB" is as follows. In the first round, when A chooses first, followed by B, A has the advantage of choosing the larger number and B has the disadvantage of choosing the smaller number. In the second round, if A were to choose first, A would have had two consecutive advantages, and therefore, in the second round, B will choose first to produce the sequence AB, BA. In the third round, A chooses first, because B chose first in the second round. After three rounds, the sequence would be AB, BA, AB. When this technique was applied to 100 items of different values or masses, by mere combinations, the total value or mass of A's items was equal to the total value or mass of B's items. Similar results were obtained for 1000 items. By hand, the techniques can be used to prepare final exam schedules for 100 or 1000 courses. Confirmed is the notion that an approach that solves one of these problems can also solve other similar NP problems. Since six problems from three different areas have been solved, all NP-Complete problems can be solved.

Solutions of NP-Complete Problems

The following sample problems will be solved and analyzed. They are based on the suggested sample problems from the Wikipedia (Simple English) website. Many Thanks to Wikipedia.

Basis of the method used in solving the NPComplete problems: **Ratios** To Page 21

Example 1 (Preliminaries)
By always choosing the largest number, A and B will divide the following set of numbers equally or nearly equally: 14,13,12,11,10,9,8,7, 6,5,4,3,2,1. To Page 22

Example 2a Consider the existence of dollar bills with denominations \$100, \$99, \$98,...\$2, down to \$1. Suppose the bills are on a table with the \$100 bill at the top, followed by the \$99, \$98, \$97 bills, and so on with the \$1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, divide the total value of these dollar bills equally between A and B. To Page 24

Example 2b Consider the existence of dollar bills with denominations \$100, \$99, \$98,...\$2, down to \$1. Suppose the bills are on a table with the \$100 bill at the top, followed by the \$99, \$98, \$97 bills, and so on with the \$1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, the total value of these dollar bills is to be divided equally between A and B.

Question: (a) If a computer costs \$2,000, can A afford to buy this computer?
(b) If a computer costs \$3,000, can A afford to buy this computer? To Page 26

Example 3 Let one randomly delete some of the bills in Example 2a, a previous example, and divide the remaining bills between A and B. After the deletion of some of the bills, there are 78 bills remaining. To Page 26

Example 4 A businessman wants to take 100 items of different masses to the market. These items are to be packed into boxes. Each box can only hold up to 560 units. The businessman would like to know if 10 boxes would be sufficient to carry all 100 items to the market. To Page 28

Example 5 A school offers 100 different courses, and each course requires one hour for the final exam. For each course, all students registered for that course must take the final exam at the same time. Since some students take more than one course, the final exam schedule must be such that students registered for two or more courses will be able to take the exams for all their registered courses. A teacher would like to know if it is possible to schedule all of the exams for the same day so that every student can take the exam for each course registered for. To Page 30

Example 6 A builder has 1000 concrete blocks of different masses arranged from 1000 units to one unit. The builder would like to divide the blocks into two piles A and B of equal masses. Prepare a list by masses of all the blocks in pile A, and all the blocks in pile B. To Page 32

Basis of the method used in solving the NP problems: Ratios

Method 2 below is the method used for the solutions of the **NP** problems.

Example 1: Divide \$12 between A and B in the ratio 1: 2

Method 1 (Usual arithmetic method)

Step 1: Fraction of the money A receives = $\frac{1}{1+2} = \frac{1}{3}$

Fraction of the money B receives = $\frac{2}{1+2} = \frac{2}{3}$

Step 2: Amount A receives = $\frac{1}{3} \times \frac{12}{1} = 4$

Amount B receives = $\frac{2}{3} \times \frac{12}{1} = 8$

Therefore, A receives \$4, and B receives \$8

(**Method 1** above is from the author's book entitled "Power of Ratios" by A. A. Frempong, and published by Yellowtextbooks.com.)

Method 2 (The process method)

The ratio 1:2 means whenever A receives \$1, B receives \$2.

Step 1: In the first round, A receives \$1, and B receives \$2.
After the first round, the amount of money remaining is \$12 - (\$1 + \$2) = \$9.

Step 2: In the second round, from this \$9, A receives \$1 and B receives \$2.
After the second round, the amount of money remaining = \$9 - (\$1+ \$2) = \$6

Step 3: In the third round, A receives \$1 and B receives \$2.
The amount remaining = \$6 - (\$1 + \$2) = \$3

Step 4: In the fourth and final round, A receives \$1 and B receives \$2.
The amount remaining = \$3 - (\$1 + \$2) = 0

Step 5: A's total = \$1 + \$1 + \$1 + \$1 = \$4
B's total = \$2 + \$2 + \$2 + \$2 = \$8

Example 2: Divide \$12 between A and B in the ratio 1: 1

Method 1 (Usual arithmetic method)

Step 1: Fraction of the money A receives = $\frac{1}{1+1} = \frac{1}{2}$

Fraction of the money B receives = $\frac{1}{1+1} = \frac{1}{2}$

Step 2: Amount A receives = $\frac{1}{2} \times \frac{12}{1} = 6$

Amount B receives = $\frac{1}{2} \times \frac{12}{1} = 6$

Therefore, A receives \$6, and B receives \$6.

Method 2 (The process method)

The ratio 1:1 means whenever A receives \$1, B receives \$1.

Step 1: In the first round, A receives \$1, and B receives \$1.
After the first round, the amount of money remaining is \$12 - (\$1 + \$1) = \$10

Step 2: In the second round, from this \$10, A receives \$1 and B receives \$1.
After the second round, the amount of money remaining = \$10 - (\$1+ \$1) = \$8

Step 3: In the third round, A receives \$1, and B receives \$1.
The amount remaining = \$8 - (\$1 + \$1) = \$6

Step 4: In the fourth round, A receives \$1 and B receives \$1
The amount remaining = \$6 - (\$1 + \$1) = 4

Step 5: In the fifth round, A receives \$1 and B receives \$1.
The amount remaining = \$4 - (\$1 + \$1) = 2

Step 6: In the sixth and final round, A receives \$1 and B receives \$1.
The amount remaining = \$2 - (\$1 + \$1) = 0

Step 7: A's total = \$1 + \$1 + \$1 + \$1 + \$1 + \$1 = \$6.
B's total = \$1 + \$1 + \$1 + \$1 + \$1 + \$1 = \$6.

Case 1: Only two devisors A and B

Example 1 (Preliminaries)
By always choosing the largest number, A and B will divide the following set of numbers equally or nearly equally.

14,13,12,11,10,9,8,7, 6,5,4,3,2,1.

Solution
For communication purposes, one will call the numbers to be divided the "dividends"; and one will call A and B the "divisors". Let the sum of A's choices be Q_A, and let the sum of B's choices be Q_B.

Step 1: Check to ensure that the numbers are arranged in decreasing order.
One will apply the wisdom method of the introduction.
That is, one applies "A, BB, AA, BB, AA, BB, AA, B"

Method 1 Using braces
Step 2: A chooses the first element, 14
Step 3: B chooses the next two elements, 13 and 12.,
Step 4: A chooses the next two elements 11, and 10, and the alternating consecutive choices continue to the end.

$$\underbrace{14}_{A},\ \underbrace{13,\ 12}_{B},\ \underbrace{11,\ 10}_{A},\ \underbrace{9,\ 8}_{B},\ \underbrace{7,\ 6}_{A},\ \underbrace{5,\ 4}_{B},\ \underbrace{3,\ 2}_{A},\ \underbrace{1}_{B} \qquad (1)$$

Step 5: Add the choices for A and add the choices for B.

$$Q_A = 14 + 11 + 10 + 7 + 6 + 3 + 2$$
$$= 53$$
$$Q_B = 13 + 12 + 9 + 8 + 5 + 4 + 1$$
$$= 52$$

The sum for A = 53; and the sum for B = 52.

Method 2 (Tabular form)
Step 1: List the dividends as shown below

14	13	12	11	10	9	8	7	6	5	4	3	2	1

Step 2: Write the divisors A, BB, AA, BB, AA, etc, above the numbers, This is the choosing step.

A	B	B	A	A	B	B	A	A	B	B	A	A	B
14	13	12	11	10	9	8	7	6	5	4	3	2	1

Note: A above [14] means A chooses 14. B above [13] means B chooses 13.

Step 3: Collect and add the corresponding (dividends) choices

Q_A	Q_B
14	13
11	12
10	9
7	8
6	5
3	4
2	1
Total: 53	52

Mathematical formulas for choosing the elements

Let $a_1 = 14,\ a_2 = 13,\ a_3 = 12,\ a_4 = 11,\ a_5 = 10,$
$a_6 = 9,\ a_7 = 8,\ a_8 = 7,\ a_9 = 6,\ a_{10} = 5,\ a_{11} = 4,\ a_{12} = 3,\ a_{13} = 2,\ a_{14} = 1$

By experimentation, one obtains the following formulas for A and B.

$$Q_A = a_1 + \sum_{n=2,4,6,}^{6} a_{2n} + a_{2n+1} \qquad (Q_A = a_1 + a_4 + a_5 + a_8 + a_9 + a_{12} + a_{13})$$

$$Q_B = \sum_{n=1,3,5.}^{5} a_{2n} + a_{2n+1} + a_{14} \qquad (Q_B = a_2 + a_3 + a_6 + a_7 + a_{10} + a_{11} + a_{14})$$

Apply the formulas to above (The above formulas are valid for only **two** divisors.)

$$Q_A = a_1 + \sum_{n=2,4,6}^{6} a_{2n} + a_{2n+1}$$
$$= 14 + 11 + 10 + 7 + 6 + 3 + 2$$
$$= 53$$

$$Q_B = \sum_{n=1,3,5.}^{5} a_{2n} + a_{2n+1} + a_{14}$$
$$= 13 + 12 + 9 + 8 + 5 + 4 + 1$$
$$= 52$$

Note that the above formulas using the sigma notation are valid for only two divisors, A and B. For three divisors A, B, and C, different formulas would have to be derived, based on the solutions of the problem.

Back to examples

Example 2a Consider the existence of dollar bills with denominations \$100, \$99, \$98,...\$2, down to \$1. Suppose the bills are on a table with the \$100 bill at the top, followed by the \$99, \$98, \$97 bills, and so on with the \$1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, the total value of these dollar bills is to be divided equally between A and B.

Method 2a: Using the numerical values and braces

Apply, "A, BB, AA, BB, AA, BB, AA,..." (as in Method 1 of Example 1)

Step 1: A chooses the first \$100 bill. (Only a single item is removed).

Step 2: B chooses the next two bills, the \$99 and \$98 bills, (two items removed consecutively)

Step 3: A chooses the next two bills, the \$97 and \$96 bills, and the alternating removal continues to the end.

$\underbrace{100}_{A}, \underbrace{99\ 98}_{B}, \underbrace{97,\ 96}_{A}, \underbrace{95,\ 94}_{B}, \underbrace{93,\ 92}_{A}\ \underbrace{91,\ 90}_{B}, \underbrace{89,\ 88}_{A}, \underbrace{87\ 86}_{B}, \underbrace{85,\ 84}_{A}, \underbrace{83,\ 82}_{B}, \underbrace{81,\ 80}_{A}\ \underbrace{79,\ 78}_{B},$

$\underbrace{77,\ 76}_{A},, \underbrace{75\ 74}_{B}, \underbrace{73,\ 72}_{A}, \underbrace{71,\ 70}_{B}, \underbrace{69,\ 68}_{A}\ \underbrace{67,\ 66}_{B}, \underbrace{65,\ 64}_{A}, \underbrace{63\ 62}_{B}, \underbrace{61,\ 60}_{A}, \underbrace{59,\ 58}_{B}, \underbrace{57,\ 56}_{A}\ \underbrace{55,\ 54}_{B},$

$\underbrace{53,\ 52}_{A}, ,\underbrace{51,\ 50}_{B}, \underbrace{49\ 48}_{A}, \underbrace{47,\ 46}_{B}, \underbrace{45,\ 44}_{A}, \underbrace{43,\ 42}_{B}\ \underbrace{41,\ 40}_{A}, \underbrace{39,\ 38}_{B}, \underbrace{37\ 36}_{A}, \underbrace{35,\ 34}_{B}, \underbrace{33,\ 32}_{A}, \underbrace{31,\ 30}_{B}$

$\underbrace{29,\ 28}_{A},, \underbrace{27,\ 26}_{B}, \underbrace{25\ 24}_{A}, \underbrace{23,\ 22}_{B}, \underbrace{21,\ 20}_{A}, \underbrace{19,\ 18}_{B}, \underbrace{17,\ 16}_{A}, \underbrace{15,\ 14}_{B}, \underbrace{13,\ 12}_{A}, \underbrace{11,\ 10}_{B}, \underbrace{9,\ 8}_{A}, \underbrace{7,\ 6}_{B}, \underbrace{5,\ 4}_{A}$

$\underbrace{3,\ 2}_{B}, \underbrace{1}_{A},$

Step 4: Collect and add the choices (dividends) for A and B

$Q_A = 100 + 97 + 96 + 93 + 92 + 89 + 88 + 85 + 84 + 81 + 80 + 77 + 76 + 73 + 72 + 69 + 68 + 65 + 64 + 61 + 60 + 57 + 56 + 53 + 52 + 49 + 48 + 45 + 44 + 41 + 40 + 37 + 36 + 33 + 32 + 29 + 28 + 25 + 24 + 21 + 20 + 17 + 16 + 13 + 12 + 9 + 8 + 5 + 4 + 1 = \mathbf{2525}$.

$Q_B = 99 + 98 + 95 + 94 + 91 + 90 + 87 + 86 + 83 + 82 + 79 + 78 + 75 + 74 + 71 + 70 + 67 + 66 + 63 + 62 + 59 + 58 + 55 + 54\ 51 + 50 + 47 + 46 + 43 + 42 + 39 + 38 + 35 + 34 + 31 + 30 + 27 + 26 + 23 + 22 + 19 + 18 + 15 + 14 + 11 + 10 + 7 + 6 + 3 + 2 = \mathbf{2525}$.

Conclusion: A receives \$2525 and B receives \$2525, Note **t**he zero error for A and B.

Method 2b: Using tabular form

Step 1: Write the divisors A and B above the numbers (as done in Method 2 of Example 1)

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
100	99	98	97	96	95	94	93	92	91	90	89	88	87	86	85	84	83	82	81
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	61
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Step 2: Collect and add the Choices (dividends)

$Q_A = 100 + 97 + 96 + 93 + 92 + 89 + 88 + 85 + 84 + 81 + 80 + 77 + 76 + 73 + 72 + 69 + 68 + 65 + 64 + 61 + 60 + 57 + 56 + 53 + 52 + 49 + 48 + 45 + 44 + 41 + 40 + 37 + 36 + 33 + 32 + 29 + 28 + 25 + 24 + 21 + 20 + 17 + 16 + 13 + 12 + 9 + 8 + 5 + 4 + 1 = \mathbf{2525.}$

$$Q_B = 99+98+95+94+91+90+87+86+83+82+79+78+75+74+71+70+67+66+ 63+62+59+58+55+54+51+50+47+46+43+42+39+38+35+34+31+30+27+ 26+23+22+19+18+15+14+11+10+7+6+3+2=\mathbf{2525}.$$

The above results are pleasantly astonishing. Of the 2^{100} possible ways to divide the above bills, the above technique and consequently the derived formulas divided the above mixture of bills into exactly two equal parts in value. Why has this technique been hiding for nearly 30 years? Note that the ratio $Q_A : Q_B$ is $1:1$.

Equations for above: $Q_A = a_1 + \sum_{n=2,4,6,...}^{48} a_{2n} + a_{2n+1} + a_{100}$ and $Q_B = \sum_{n=1,3,5,...}^{49} a_{2n} + a_{2n+1}$

Method 1b: Using term numbers and braces Apply, "A, BB, AA, BB, AA, BB, AA,..." .."

$\underbrace{a_1}_{A}, \underbrace{a_2\ a_3}_{B}, \underbrace{a_4, a_5}_{A}, \underbrace{a_6, a_7}_{B}, \underbrace{a_8, a_9}_{A} \underbrace{a_{10}, a_{11}}_{B}, \underbrace{a_{12}, a_{13}}_{A}, \underbrace{a_{14}\ a_{15}}_{B}, \underbrace{a_{16}, a_{17}}_{A}, \underbrace{a_{18}, a_{19}}_{B}, \underbrace{a_{20}, a_{21}}_{A} \underbrace{a_{22}, a_{23}}_{B},$

$\underbrace{a_{24}, a_{25}}_{A},, \underbrace{a_{26}, a_{27}}_{B} \underbrace{a_{28}, a_{29}}_{A} \underbrace{a_{30}, a_{31}}_{B}, \underbrace{a_{32}, a_{33}}_{A} \underbrace{a_{34}, a_{35}}_{B}, \underbrace{a_{36}, a_{37}}_{A} \underbrace{a_{38}\ a_{39}}_{B} \underbrace{a_{40}, a_{41}}_{A}, \underbrace{a_{42}, a_{43}}_{B}, \underbrace{a_{44}, a_{45}}_{A}$

$\underbrace{a_{46}, a_{47}}_{B}, \underbrace{a_{48}, a_{49}}_{A},, \underbrace{a_{50}, a_{51}}_{B}, \underbrace{a_{52}\ a_{53}}_{A}, \underbrace{a_{54}, a_{55}}_{B} \underbrace{a_{56}, a_{57}}_{A} \underbrace{a_{58}, a_{59}}_{B} \underbrace{a_{60}, a_{61}}_{A}, \underbrace{a_{62}, a_{63}}_{B} \underbrace{a_{64}\ a_{65}}_{A}, \underbrace{a_{66}, a_{67}}_{B}$

$\underbrace{a_{68}, a_{69}}_{A} \underbrace{a_{70}, a_{71}}_{B} \underbrace{a_{72}, a_{73}}_{A}, \underbrace{a_{74}, a_{75}}_{B}, \underbrace{a_{76}, a_{77}}_{A} \underbrace{a_{78}, a_{79}}_{B} \underbrace{a_{80}, a_{81}}_{A} \underbrace{a_{82}, a_{83}}_{B}, \underbrace{a_{84}, a_{85}}_{A}, \underbrace{a_{86}, a_{87}}_{B}, \underbrace{a_{88}, a_{89}}_{A},$

$\underbrace{a_{90}, a_{91}}_{B}, \underbrace{a_{92}, a_{93}}_{A}, \underbrace{a_{94}, a_{95}}_{B}, \underbrace{a_{96}, a_{97}}_{A} \underbrace{a_{98}, a_{99}}_{B}, \underbrace{a_{100}}_{A},$

Using the term numbers and tabular form

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}	a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}	a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}

Collect the terms for A and add them ; and similarly collect the terms for B and add them.

$Q_A = a_1 + a_4 + a_5 + a_8 + a_9 + a_{12} + a_{13} + a_{16} + a_{17} + a_{20} + a_{21} + a_{24} + a_{25} + a_{28} + a_{29} + a_{32} + a_{33}$
$+ a_{36} + a_{37} + a_{40} + a_{41} + a_{44} + a_{45} + a_{48} + a_{49} + a_{52} + a_{53} + a_{56} + a_{57} + a_{60} + a_{61} + a_{64} + a_{65}$
$+ a_{68} + a_{69} + a_{72} + a_{73} + a_{76} + a_{77} + a_{80} + a_{81} + a_{84} + a_{85} + a_{88} + a_{89} + a_{92} + a_{93} + a_{96} + a_{97} + a_{100}$

$Q_B = a_2 + a_3 + a_6 + a_7 + a_{10} + a_{11} + a_{14} + a_{15} + a_{18} + a_{19} + a_{22} + a_{23} + a_{26} + a_{27} + a_{30} + a_{31} + a_{34}$
$+ a_{35} + a_{38} + a_{39} + a_{42} + a_{43} + a_{46} + a_{47} + a_{50} + a_{51} + a_{54} + a_{55} + a_{58} + a_{59} + a_{62} + a_{63} + a_{66}$
$+ a_{67} + a_{70} + a_{71} + a_{74} + a_{75} + a_{78} + a_{79} + a_{82} + a_{83} + a_{86} + a_{87} + a_{90} + a_{91} + + a_{94} + a_{95} + a_{98} + a_{99}$

Example 2b Consider the existence of dollar bills with denominations \$100, \$99, \$98,...\$2, down to \$1. Suppose the bills are on a table with the \$100 bill at the top, followed by the \$99, \$98, \$97 bills, and so on with the \$1 bill at the bottom of the stack. Now, by mere grabbing in turns always from the top of the stack, the total value of these dollar bills is to be divided equally between A and B.

Question: (a) If a computer costs \$2,000, can A afford to buy this computer?
(b) If a computer costs \$3,000, can A afford to buy this computer?

Answers: (a) From the solution of Example 2a, A received \$2,525, and therefore can afford to buy this computer.
Yes. A can afford to buy this \$2,000 computer.

(b) Since from the solution of Example 2a, A received \$2,525, and the computer costs \$3,000, A cannot afford to buy \$3,000 computer
No. A cannot afford to buy this \$3,000 computer.

Example 3

Let one randomly delete some of the bills in Example 2a, a previous example, and divide as equally as possible the remaining bills between A and B. After the deletion of some of the bills, there are 78 bills remaining.

Solution: Using the numerical values and braces

$$\underbrace{98}_{A}.\underbrace{97,96}_{B},\underbrace{95,94}_{A},\underbrace{93,91}_{B},\underbrace{90,89}_{A},\underbrace{88,87}_{B},\underbrace{86.85}_{A},.\underbrace{81,80}_{B},\underbrace{79,78}_{A},\underbrace{77,76}_{B},$$

$$\underbrace{75,74}_{A},\underbrace{73,72}_{B},\underbrace{69.68}_{A},\underbrace{67,66}_{B},\underbrace{65,64}_{A},\underbrace{62,61}_{B},\underbrace{60,58}_{A},\underbrace{56,55}_{B},\underbrace{54,53}_{A}.\underbrace{51.50}_{B},$$

$$\underbrace{49,47}_{A},\underbrace{46,45}_{B},\underbrace{43.,41}_{A},\underbrace{40,39}_{B},\underbrace{37,36}_{A},\underbrace{35.34}_{B},\underbrace{33.32}_{A},\underbrace{31,30}_{B},\underbrace{29,26}_{A},\underbrace{25.24}_{B},$$

$$\underbrace{23,22}_{A},\underbrace{21,20}_{B},\underbrace{19,18}_{A},\underbrace{16,14}_{B},\underbrace{13,12}_{A},\underbrace{11,10}_{B},\underbrace{9,8}_{A},\underbrace{7,5}_{B},\underbrace{4,2}_{A},\underbrace{1}_{B}$$

$$\begin{aligned} Q_A &= 98+95+94+90+89+86+85+79+78+75+74+69+68+65+64+60+58+54+53 \\ &\quad +49+47+43+41+37+36+33+32+29+26+23+22+19+18+13+12+9+8+4+2 \\ &= \mathbf{1937} \end{aligned}$$

$$\begin{aligned} Q_B &= 97+96+93+91+88+87+81+80+77+76+73+72+67+66+62+61+56+55+51 \\ &\quad +50+46+45+40+39+35+34+31+30+25+24+21+20+16+14+11+10+7+5+1 \\ &= \mathbf{1933} \end{aligned}$$

Total of Q_A and $Q_B = 3870$. Division by 2 yields 1935.

For Q_A, relative error $= \dfrac{2}{1935} = 0.0010$ or about 0.1%

For Q_B, relative error $= \dfrac{-2}{1935} = -0.0010$

For equality, interchange the 47 bill in Q_A and the 45 bill in Q_B. Thus A gives \$2 to B, resulting in equality of **\$1,935** each. Other bills can be interchanged.

Using term numbers

$$\begin{aligned} Q_A &= a_1 + a_4 + a_5 + a_8 + a_9 + a_{12} + a_{13} + a_{16} + a_{17} + a_{20} + a_{21} + a_{24} + a_{25} + a_{28} + a_{29} + a_{32} + a_{33} \\ &\quad + a_{36} + a_{37} + a_{40} + a_{41} + a_{44} + a_{45} + a_{48} + a_{49} + a_{52} + a_{53} + a_{56} + a_{57} + a_{60} + a_{61} + a_{64} + a_{65} \\ &\quad + a_{68} + a_{69} + a_{72} + a_{73} + a_{76} + a_{77} \end{aligned}$$

$$\begin{aligned} Q_B &= a_2 + a_3 + a_6 + a_7 + a_{10} + a_{11} + a_{14} + a_{15} + a_{18} + a_{19} + a_{22} + a_{23} + a_{26} + a_{27} + a_{30} + a_{31} + a_{34} \\ &\quad + a_{35} + a_{38} + a_{39} + a_{42} + a_{43} + a_{46} + a_{47} + a_{50} + a_{51} + a_{54} + a_{55} + a_{58} + a_{59} + a_{62} + a_{63} + a_{66} \\ &\quad + a_{67} + a_{70} + a_{71} + a_{74} + a_{75} + a_{78} \end{aligned}$$

Observe above that the last term for Q_A is a_{77} and the last term for Q_B is a_{78} (there are 78 terms)

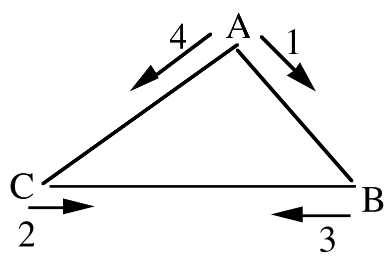
Back to examples

Case 2: Three or more divisors

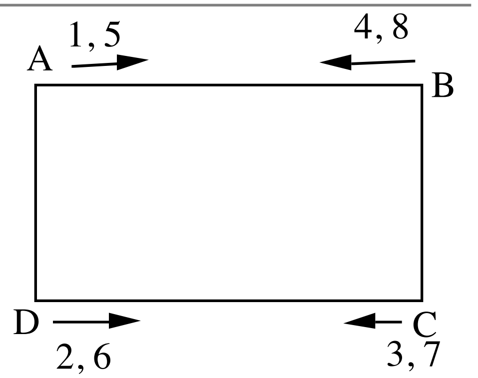
In the previous examples, for communication purposes, A and B were called the "divisors" and the numbers or terms to be divided were called "dividends". The concept of divisors A and B can be extended to three or more divisors such as A, B, C, or A, B, C, D, but in these cases, geometric figures will help keep track of the choices.

Geometric figures to keep track of the order and directions of the divisors
(For three or more divisors such as A, B, C; four divisors A, B, C, D)

The arrows are for directions



ABC / CBA / BCA / ACB



ABCD / DCBA / CDAB / BADC / ABCD

For ABC:

Step 1: Go Clockwise ABC (In the first round, A chooses first and C chooses last))
Step 2: Begin with C, reverse the direction in Step 1 and go CBA.
(Since C was at the largest disadvantage in the first round, by choosing last, C chooses first in the second round, followed by B)
Step 3: Begin with B, reverse previous direction (direction of C) and go clockwise BCA.
Step 4: Begin with A again, reverse previous direction (direction of B) and go counterclockwise ACB.

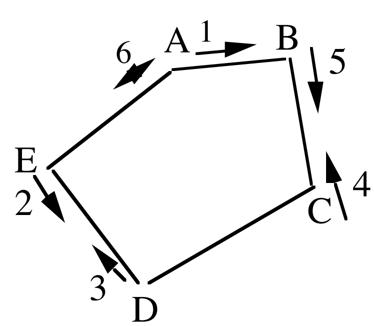
For ABCD

Step 1: Go Clockwise ABCD. (first round)
Step 2: Begin with D, reverse the direction in Step 1 (direction of A) and go DCBA.
Step 3: Begin with C, reverse previous direction (direction of D), and go clockwise CDAB.
Step 4: Begin with B, reverse previous direction (direction of C) and go counterclockwise BADC.
Step 5: Begin with A, reverse pevious direction, but by coincidence go clockwise ABCD. (5th round)

For five divisors A, B, C, D, E

ABCDE, *EDCBA*, *DEABC*, *CBAED* *BCDEA*

Step 1: Go Clockwise ABCDE
Step 2: Begin with E, reverse the direction in Step 1 and go EDCBA
Step 3: Begin with D, reverse previous direction and go clockwise DEABC
Step 4: Begin with C, reverse previous direction and go counterclockwise CBAED
Step 5: Begin with B, reverse previous direction and go clockwise BCDEA.
Step 6: Beginning again with A, reverse the direction (of B) and go counterclockwise AEDCB.



ABCDE, *EDCBA DEABC*
CBAED, *BCDEA*

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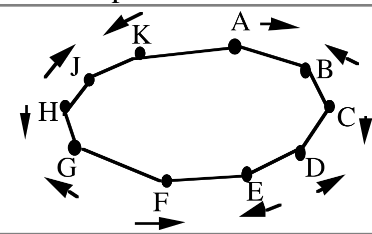
Example 4: A businessman wants to take 100 items of different masses to the market. These items are to be packed into boxes. Each box can only hold up to 560 units. The businessman would like to know if 10 boxes would be sufficient to carry all 100 items to the market.

Step 1: Arrange the items in decreasing order of their masses. Let the mass of the first item (largest) be 100 units, and let the masses of the rest of the items be respectively, 99, 98, 97, and so on down to smallest item of mass 1 unit. Let the 10 boxes be labeled A, B, C, D, E, F, G, H, J, and K The ten boxes are to divide the 100 items. **Imitate** Example 2 but with 10 divisors.

Guide1; *ABCDEFGHJK*
Guide 2; *KJHGFEDCBA*
Guide 3 *JKABCDEFGH*
Guide 4; *HGFEDCBAKJ*
Guide 5; *GHJKABCDEF*
Guide 6; *FEDCBAKJHG*
Guide 7: *EFGHJKABCD*
Guide 8; *DCBAKJHGFE*
Guide 9; *CDEFGHJKAB*
Guide10: *BAKJHGFEDC*



A	B	C	D	E	F	G	H	J	K	K	J	H	G	F	E	D	C	B	A
100	99	98	97	96	95	94	93	92	91	90	89	88	87	86	85	84	83	82	81
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}

J	K	A	B	C	D	E	F	G	H	H	G	F	E	D	C	B	A	K	J
80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	61
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}

G	H	J	K	A	B	C	D	E	F	F	E	D	C	B	A	K	J	H	G
60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}

E	F	G	H	J	K	A	B	C	D	D	C	B	A	K	J	H	G	F	E
40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}	a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}

C	D	E	F	G	H	J	K	A	B	B	A	K	J	H	G	F	E	D	C
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}	a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}

Step 2: **Collect the choices for A, B, C, D, E, F, G, H, J, K**

Q_A	Q_B	Q_C	Q_D	Q_E	Q_F	Q_G	Q_H	Q_J	Q_K
100 a_1	99, a_2	98 a_3	97 a_4	96 a_5	95 a_6	94 a_7	93 a_8	92 a_9	91 a_{10}
81 a_{20}	82, a_{19}	83 a_{18}	84 a_{17}	85 a_{16}	86 a_{15}	87 a_{14}	88 a_{13}	89 a_{12}	90 a_{11}
78 a_{23}	77, a_{24}	76 a_{25}	75 a_{26}	74 a_{27}	73 a_{28}	72 a_{29}	71 a_{30}	80 a_{21}	79 a_{22}
63 a_{38}	64, a_{37}	65 a_{36}	66 a_{35}	67 a_{34}	68 a_{33}	69 a_{32}	70 a_{31}	61 a_{40}	62 a_{39}
56 a_{45}	55, a_{46}	54 a_{47}	53 a_{48}	52 a_{49}	51 a_{50}	60 a_{41}	59 a_{42}	58 a_{43}	57 a_{44}
45 a_{56}	46 a_{55}	47 a_{54}	48 a_{53}	49 a_{52}	50 a_{51}	41 a_{60}	42 a_{59}	43 a_{58}	44 a_{57}
34 a_{67}	33 a_{68}	32 a_{69}	31 a_{70}	40 a_{61}	39 a_{62}	38 a_{63}	37 a_{64}	36 a_{65}	35 a_{66}
27 a_{74}	28 a_{73}	29 a_{72}	30 a_{71}	21 a_{80}	22 a_{79}	23 a_{78}	24 a_{77}	25 a_{76}	26 a_{75}
12 a_{89}	11 a_{90}	20 a_{81}	19 a_{82}	18 a_{83}	17 a_{84}	16 a_{85}	15 a_{86}	14 a_{87}	13 a_{88}
9 a_{92}	10 a_{91}	1 a_{100}	2 a_{99}	3 a_{98}	4 a_{97}	5 a_{96}	6 a_{95}	7 a_{94}	8 a_{93}
Total: 505	**505**	**505**	**505**	**505**	**505**	**505**	**505**	**505**	**505**

Condition for sufficiency:
The 10 boxes would be sufficient to carry all the 100 items to the market if the mass of the contents of each box is equal to or less than 560 units. Since the mass of the contents in each box is 505 units, which is less than 560 units, each box satisfies this sufficiency condition. Therefore, the 10 boxes would be sufficient to carry the 100 items to the market.
Note above that the ratio

$Q_A : Q_B : Q_C : Q_D : Q_E : Q_F : Q_G : Q_H : Q_{J:} : Q_K = 1:1:1:1:1:1:1:1:1:1$

4b Using the term numbers

A	B	C	D	E	F	G	H	J	K	K	J	H	G	F	E	D	C	B	A
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
J	K	A	B	C	D	E	F	G	H	H	G	F	E	D	C	B	A	K	J
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}
G	H	J	K	A	B	C	D	E	F	F	E	D	C	B	A	K	J	H	G
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}
E	F	G	H	J	K	A	B	C	D	D	C	B	A	K	J	H	G	F	E
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}	a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}
C	D	E	F	G	H	J	K	A	B	B	A	K	J	H	G	F	E	D	C
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}	a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}

Collect the terms for A, B, C, D, E, F, G, H, J, K

$Q_A = a_1 + a_{20} + a_{23} + a_{38} + a_{45} + a_{56} + a_{67} + a_{74} + a_{89} + a_{92}$

$Q_B = a_2 + a_{19} + a_{24} + a_{37} + a_{46} + a_{55} + a_{68} + a_{73} + a_{90} + a_{91}$

$Q_C = a_3 + a_{18} + a_{25} + a_{36} + a_{47} + a_{54} + a_{69} + a_{72} + a_{81} + a_{100}$

$Q_D = a_4 + a_{17} + a_{26} + a_{35} + a_{48} + a_{53} + a_{70} + a_{71} + a_{82} + a_{99}$

$Q_E = a_5 + a_{16} + a_{27} + a_{34} + a_{49} + a_{52} + a_{61} + a_{80} + a_{83} + a_{98}$

$Q_F = a_6 + a_{15} + a_{28} + a_{33} + a_{50} + a_{51} + a_{62} + a_{79} + a_{84} + a_{97}$

$Q_G = a_7 + a_{14} + a_{29} + a_{32} + a_{41} + a_{60} + a_{63} + a_{78} + a_{85} + a_{96}$

$Q_H = a_8 + a_{13} + a_{30} + a_{31} + a_{42} + a_{59} + a_{64} + a_{77} + a_{86} + a_{95}$

$Q_J = a_9 + a_{12} + a_{21} + a_{40} + a_{43} + a_{58} + a_{65} + a_{76} + a_{87} + a_{94}$

$Q_K = a_{10} + a_{11} + a_{22} + a_{39} + a_{44} + a_{57} + a_{66} + a_{75} + a_{88} + a_{93}$

Sub-Conclusion
The fairness wisdom method has performed perfectly.
Observe above in Step 2 that the totals for Q_A, Q_B, Q_C, Q_D, Q_E, Q_F, Q_G, Q_H, Q_J, Q_K are all the same. The technique applied picked combinations to produce these equal totals.
Note that in using the technique in this paper, the items involved must be arranged in decreasing order, preferably. Therefore, in programming, the first step should be to arrange the items in decreasing order, a task a computer performs very fast..

In the next example, Example 5, one will confirm the notion that a method that solves one of the NP problems can be used to solve other similar problems. One will use the results of the above example Example 4b to do the next problem.

Back to examples

Example 5: A school offers 100 different courses, and each course requires one hour for the final exam. For each course, all students registered for that course must take the final exam at the same time. Since some students take more than one course, the final exam schedule must be such that students registered for two or more courses will be able to take the exams for all their registered courses. A teacher would like to know if it is possible to schedule all of the exams for the same day so that every student can take the exam for each course registered for.

Step1: Final Exams 8AM – 6PM $\begin{cases} A=8-9;\ B=9-10;\ C=10-11;\ D=11-12; E=12-1; \\ F=1-2;\ G=2-3;\ H=3-4;\ \ J=4-5;\ K=5-6 \end{cases}$

Let the course numbers be $a_1,\ a_2,\ a_3,\ldots a_{100}$ **Using the result of Example 4b**

A	B	C	D	E	F	G	H	J	K	K	J	H	G	F	E	D	C	B	A
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}

J	K	A	B	C	D	E	F	G	H	H	G	F	E	D	C	B	A	K	J
a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{36}	a_{37}	a_{38}	a_{39}	a_{40}

G	H	J	K	A	B	C	D	E	F	F	E	D	C	B	A	K	J	H	G
a_{41}	a_{42}	a_{43}	a_{44}	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{51}	a_{52}	a_{53}	a_{54}	a_{55}	a_{56}	a_{57}	a_{58}	a_{59}	a_{60}

E	F	G	H	J	K	A	B	C	D	D	C	B	A	K	J	H	G	F	E
a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}	a_{67}	a_{68}	a_{69}	a_{70}	a_{71}	a_{72}	a_{73}	a_{74}	a_{75}	a_{76}	a_{77}	a_{78}	a_{79}	a_{80}

C	D	E	F	G	H	J	K	A	B	B	A	K	J	H	G	F	E	D	C
a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}	a_{89}	a_{90}	a_{91}	a_{92}	a_{93}	a_{94}	a_{95}	a_{96}	a_{97}	a_{98}	a_{99}	a_{100}

Step 2: Collect the choices for A, B, C, D, E, F, G, H, J, K

Final Exam Schedule: 8 AM-6 PM

$Q_A=8-9;\ Q_B=9-10;\ Q_C=10-11;\ Q_D=11-12;\ Q_E=12-1;$
$Q_F=1-2;\ Q_G=2-3;\ Q_H=3-4;\ Q_J=4-5;\ Q_K=5-6$

8- 9	9-10	10-11	11-12	12-1	1-2	2-3	3-4	4-5	5-6
Q_A	Q_B	Q_C	Q_D	Q_E	Q_F	Q_G	Q_H	Q_J	Q_K
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
a_{20}	a_{19}	a_{18}	a_{17}	a_{16}	a_{15}	a_{14}	a_{13}	a_{12}	a_{11}
a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{21}	a_{22}
a_{38}	a_{37}	a_{36}	a_{35}	a_{34}	a_{33}	a_{32}	a_{31}	a_{40}	a_{39}
a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{41}	a_{42}	a_{43}	a_{44}
a_{56}	a_{55}	a_{54}	a_{53}	a_{52}	a_{51}	a_{60}	a_{59}	a_{58}	a_{57}
a_{67}	a_{68}	a_{69}	a_{70}	a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}
a_{74}	a_{73}	a_{72}	a_{71}	a_{80}	a_{79}	a_{78}	a_{77}	a_{76}	a_{75}
a_{89}	a_{90}	a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}
a_{92}	a_{91}	a_{100}	a_{99}	a_{98}	a_{97}	a_{96}	a_{95}	a_{94}	a_{93}

The final exam for every course has been scheduled. However, if a student takes for example, Course a_1 and course a_{20}, because the duration for the final exams for these two courses is 8-9 AM, the student cannot take the final exams for these two courses simultaneously. Therefore, it is **not** possible to prepare a schedule to allow every student to take the final exams for all registered courses on the same day. However, below is what is possible.

In order for every student to take the final exam for all courses registered for, ten days would be needed as shown below, where the course numbers are $a_1, a_2, a_3, \ldots a_{100}$.

	8- 9	9-10	10-11	11-12	12-1	1-2	2-3	3-4	4-5	5-6
DAY	Q_A	Q_B	Q_C	Q_D	Q_E	Q_F	Q_G	Q_H	Q_J	Q_K
1	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
2	a_{20}	a_{19}	a_{18}	a_{17}	a_{16}	a_{15}	a_{14}	a_{13}	a_{12}	a_{11}
3	a_{23}	a_{24}	a_{25}	a_{26}	a_{27}	a_{28}	a_{29}	a_{30}	a_{21}	a_{22}
4	a_{38}	a_{37}	a_{36}	a_{35}	a_{34}	a_{33}	a_{32}	a_{31}	a_{40}	a_{39}
5	a_{45}	a_{46}	a_{47}	a_{48}	a_{49}	a_{50}	a_{41}	a_{42}	a_{43}	a_{44}
6	a_{56}	a_{55}	a_{54}	a_{53}	a_{52}	a_{51}	a_{60}	a_{59}	a_{58}	a_{57}
7	a_{67}	a_{68}	a_{69}	a_{70}	a_{61}	a_{62}	a_{63}	a_{64}	a_{65}	a_{66}
8	a_{74}	a_{73}	a_{72}	a_{71}	a_{80}	a_{79}	a_{78}	a_{77}	a_{76}	a_{75}
9	a_{89}	a_{90}	a_{81}	a_{82}	a_{83}	a_{84}	a_{85}	a_{86}	a_{87}	a_{88}
10	a_{92}	a_{91}	a_{100}	a_{99}	a_{98}	a_{97}	a_{96}	a_{95}	a_{94}	a_{93}

Observe how one used the results of the previous example (Example 4b) to solve the above problem, Example 5..
In the next example, one will cover an example involving 1000 items, which will be similar to Example 2a.

Example 6 A builder has 1000 concrete blocks of different masses arranged from 1000 units to one unit. The builder would like to divide the blocks into two piles A and B of equal masses. Prepare a list by masses of all the blocks in pile A, and all the blocks in pile B. Review Example 2a before proceeding

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
1000	999	998	997	996	995	994	993	992	991	990	989	988	987	986	985	984	983	982	981
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
980	979	978	977	976	975	974	973	972	971	970	969	968	967	966	965	964	963	962	961
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
960	959	958	957	956	955	954	953	952	951	950	949	948	947	946	945	944	943	942	941
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
940	939	938	937	936	935	934	933	932	931	930	929	928	927	926	925	924	923	922	921
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
920	919	918	917	916	915	914	913	912	911	910	909	908	907	906	905	904	903	902	901
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
900	899	898	897	896	895	894	893	892	891	890	889	888	887	886	885	884	883	882	881
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
880	879	878	877	876	875	874	873	872	871	870	869	868	867	866	865	864	863	862	861
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
860	859	858	857	856	855	854	853	852	851	850	849	848	847	846	845	844	843	842	841
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
840	839	838	837	836	835	834	833	832	831	830	829	828	827	826	825	824	823	822	821
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
820	819	818	817	816	815	814	813	812	811	810	809	808	807	806	805	804	803	802	801
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
800	799	798	797	796	795	794	793	792	791	790	789	788	787	786	785	784	783	782	781
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
780	779	778	777	776	775	774	773	772	771	770	769	768	767	766	765	764	763	762	761
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
760	759	758	757	756	755	754	753	752	751	750	749	748	747	746	745	744	743	742	741
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
740	739	738	737	736	735	734	733	732	731	730	729	728	727	726	725	724	723	722	721
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
720	719	718	717	716	715	714	713	712	711	710	709	708	707	706	705	704	703	702	701
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
700	699	698	697	696	695	694	693	692	691	690	689	688	687	686	685	684	683	682	681
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
680	679	678	677	676	675	674	673	672	671	670	669	668	667	666	665	664	663	662	661
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
660	659	658	657	656	655	654	653	652	651	650	649	648	647	646	645	644	643	642	641
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
640	639	638	637	636	635	634	633	632	631	630	629	628	627	626	625	624	623	622	621
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
620	619	618	617	616	615	614	613	612	611	610	609	608	607	606	605	604	603	602	601

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
600	599	598	597	596	595	594	593	592	591	590	589	588	587	586	585	584	583	582	581
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
580	579	578	577	576	575	574	573	572	571	570	569	568	567	566	565	564	563	562	561
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
560	559	558	557	556	555	554	553	552	551	550	549	548	547	546	545	544	543	542	541
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
540	539	538	537	536	535	534	533	532	531	530	529	528	527	526	525	524	523	522	521
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
520	519	518	517	516	515	514	513	512	511	510	509	508	507	506	505	504	503	502	501
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
500	499	498	497	496	495	494	493	492	491	490	489	488	487	486	485	484	483	482	481
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
480	479	478	477	476	475	474	473	472	471	470	469	468	467	466	465	464	463	462	461
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
460	459	458	457	456	455	454	453	452	451	450	449	448	447	446	445	444	443	442	441
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
440	439	438	437	436	435	434	433	432	431	430	429	428	427	426	425	424	423	422	421
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
420	419	418	417	416	415	414	413	412	411	410	409	408	407	406	405	404	403	402	401
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
400	399	398	397	396	395	394	393	392	391	390	389	388	387	386	385	384	383	382	381
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
380	379	378	377	376	375	374	373	372	371	370	369	368	367	366	365	364	363	362	361
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
360	359	358	357	356	355	354	353	352	351	350	349	348	347	346	345	344	343	342	341
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
340	339	338	337	336	335	334	333	332	331	330	329	328	327	326	325	324	323	322	321
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
320	319	318	317	316	315	314	313	312	311	310	309	308	307	306	305	304	303	302	301
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
300	299	298	297	296	295	294	293	292	291	290	289	288	287	286	285	284	283	282	281
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
280	279	278	277	276	275	274	273	272	271	270	269	268	267	266	265	264	263	262	261
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
260	259	258	257	256	255	254	253	252	251	250	249	248	247	246	245	244	243	242	241
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
240	239	238	237	236	235	234	233	232	231	230	229	228	227	226	225	224	223	222	221
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
220	219	218	217	216	215	214	213	212	211	210	209	208	207	206	205	204	203	202	201

A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
200	199	198	197	196	195	194	193	192	191	190	189	188	187	186	185	184	183	182	181
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
180	179	178	177	176	175	174	173	172	171	170	169	168	167	166	165	164	163	162	161
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
160	159	158	157	156	155	154	153	152	151	150	149	148	147	146	145	144	143	142	141
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
140	139	138	137	136	135	134	133	132	131	130	129	128	127	126	125	124	123	122	121
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
120	119	118	117	116	115	114	113	112	111	110	109	108	107	106	105	104	103	102	101
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
100	99	98	97	96	95	94	93	92	91	90	89	88	87	86	85	84	83	82	81
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	61
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21
A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A	A	B	B	A
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Concrete masses for Pile A

Step 2: Collect and add the Choices (dividends) :

$$Q_{A1} = 1000 + 997 + 996 + 993 + 992 + 989 + 988 + 985 + 984 + 981 + 980 + 977 + 976 + 973 + 972 + 969 + 968 + 965 + 964 + 961 + 960 + 957 + 956 + 953 + 952 + 949 + 948 + 945 + 944 + 941 + 940 + 937 + 936 + 933 + 932 + 929 + 928 + 925 + 924 + 921 + 920 + 917 + 916 + 913 + 912 + 909 + 908 + 905 + 904 + 901 \mathbf{= 47{,}525}$$

$$Q_A = 900 + 897 + 896 + 893 + 892 + 889 + 888 + 885 + 884 + 881 + 880 + 877 + 876 + 873 + 872 + 869 + 868 + 865 + 864 + 861 + 860 + 857 + 856 + 853 + 852 + 849 + 848 + 845 + 844 + 841 + 840 + 837 + 836 + 833 + 832 + 829 + 828 + 825 + 824 + 821 + 820 + 817 + 816 + 813 + 812 + 809 + 808 + 805 + 804 + 801 \mathbf{= 42{,}525}$$

$$Q_A = 800 + 797 + 796 + 793 + 792 + 789 + 788 + 785 + 784 + 781 + 780 + 777 + 776 + 773 + 772 + 769 + 768 + 765 + 764 + 761 + 760 + 757 + 756 + 753 + 752 + 749 + 748 + 745 + 744 + 741 + 740 + 737 + 736 + 733 + 732 + 729 + 728 + 725 + 724 + 721 + 720 + 717 + 716 + 713 + 712 + 709 + 708 + 705 + 704 + 701 \mathbf{= 37{,}525}$$

$$Q_A = 700 + 697 + 696 + 693 + 692 + 689 + 688 + 685 + 684 + 681 + 680 + 677 + 676 + 673 + 672 + 669 + 668 + 665 + 664 + 661 + 660 + 657 + 656 + 653 + 652 + 649 + 648 + 645 + 644 + 641 + 640 + 637 + 636 + 633 + 632 + 629 + 628 + 625 + 624 + 621 + 620 + 617 + 616 + 613 + 612 + 609 + 608 + 605 + 604 + 601 = \mathbf{32{,}525}$$

$$Q_A = 600 + 597 + 596 + 593 + 592 + 589 + 588 + 585 + 584 + 581 + 580 + 577 + 576 + 573 + 572 + 569 + 568 + 565 + 564 + 561 + 560 + 557 + 556 + 553 + 552 + 549 + 548 + 545 + 544 + 541 + 540 + 537 + 536 + 533 + 532 + 529 + 528 + 525 + 524 + 521 + 520 + 517 + 516 + 513 + 512 + 509 + 508 + 505 + 504 + 501 \mathbf{= 27{,}525}$$

$$Q_A = 500 + 497 + 496 + 493 + 492 + 489 + 488 + 485 + 484 + 481 + 480 + 477 + 476 + 473 + 472 + 469 + 468 + 465 + 464 + 461 + 460 + 457 + 456 + 453 + 452 + 449 + 448 + 445 + 444 + 441 + 440 + 437 + 436 + 433 + 432 + 429 + 428 + 425 + 424 + 421 + 420 + 417 + 416 + 413 + 412 \; 409 + 408 + 405 + 404 + 401 \mathbf{= 22{,}525}$$

$$Q_A = 400 + 397 + 396 + 393 + 392 + 389 + 388 + 385 + 384 + 381 + 380 + 377 + 376 + 373 + 372 + 369 + 368 + 365 + 364 + 361 + 360 + 357 + 356 + 353 + 352 + 349 + 348 + 345 + 344 + 341 + 340 + 337 + 336 + 333 + 332 + 329 + 328 + 325 + 324 + 321 + 320 + 317 + 316 + 313 + 312 \; 309 + 308 + 305 + 304 + 301 \mathbf{= 17{,}525}$$

$$Q_A = 300 + 297 + 296 + 293 + 292 + 289 + 288 + 285 + 284 + 281 + 280 + 277 + 276 + 273 + 272 + 269 + 268 + 265 + 264 + 261 + 260 + 257 + 256 + 253 + 252 + 249 + 248 + 245 + 244 + 241 + 240 + 237 + 236 + 233 + 232 + 229 + 228 + 225 + 224 + 221 + 220 + 217 + 216 + 213 + 212 \; 209 + 208 + 205 + 204 + 201 \mathbf{= 12{,}525}$$

$$Q_A = 200 + 197 + 196 + 193 + 192 + 189 + 188 + 185 + 184 + 181 + 180 + 177 + 176 + 173 + 172 + 169 + 168 + 165 + 164 + 161 + 160 + 157 + 156 + 153 + 152 + 149 + 148 + 145 + 144 + 141 + 140 + 137 + 136 + 133 + 132 + 129 + 128 + 125 + 124 + 121 + 120 + 117 + 116 + 113 + 112 \; 109 + 108 + 105 + 104 + 101 \mathbf{= 7{,}525}$$

$$Q_A = 100 + 97 + 96 + 93 + 92 + 89 + 88 + 85 + 84 + 81 + 80 + 77 + 76 + 73 + 72 + 69 + 68 + 65 + 64 + 61 + 60 + 57 + 56 + 53 + 52 + 49 + 48 + 45 + 44 + 41 + 40 + 37 + 36 + 33 + 32 + 29 + 28 + 25 + 24 + 21 + 20 + 17 + 16 + 13 + 12 \; 9 + 8 + 5 + 4 + 1 \mathbf{= 2{,}525}$$

Total for Pile A, Q_A = 250,250 units

Concrete masses for Pile B

$$Q_B = 999 + 998 + 995 + 994 + 991 + 990 + 987 + 986 + 983 + 982 + 979 + 978 + 975 + 974 + 971 + 970 + 967 + 966 + 963 + 962 + 959 + 958 + 955 + 954 + 951 + 950 + 947 + 946 + 943 + 942 + 939 + 938 + 935 + 934 + 931 + 930 + 927 + 926 + 923 + 922 + 919 + 918 + 915 + 914 + 911 + 910 + 907 + 906 + 903 + 902 = \mathbf{47,525}$$

$$Q_B = 899 + 898 + 895 + 894 + 891 + 890 + 887 + 886 + 883 + 882 + 879 + 878 + 875 + 874 + 871 + 870 + 867 + 866 + 863 + 862 + 859 + 858 + 855 + 854 + 851 + 850 + 847 + 846 + 843 + 842 + 839 + 838 + 835 + 834 + 831 + 830 + 827 + 826 + 823 + 822 + 819 + 818 + 815 + 814 + 811 + 810 + 807 + 806 + 803 + 802 = \mathbf{42,525}$$

$$Q_B = 799 + 798 + 795 + 794 + 791 + 790 + 787 + 786 + 783 + 782 + 779 + 778 + 775 + 774 + 771 + 770 + 767 + 766 + 763 + 762 + 759 + 758 + 755 + 754 + 751 + 750 + 747 + 746 + 743 + 742 + 739 + 738 + 735 + 734 + 731 + 730 + 727 + 726 + 723 + 722 + 719 + 718 + 715 + 714 + 711 + 710 + 707 + 706 + 703 + 702 = \mathbf{37,525}$$

$$Q_B = 699 + 698 + 695 + 694 + 691 + 690 + 687 + 686 + 683 + 682 + 679 + 678 + 675 + 674 + 671 + 670 + 667 + 666 + 663 + 662 + 659 + 658 + 655 + 654 + 651 + 650 + 647 + 646 + 643 + 642 + 639 + 638 + 635 + 634 + 631 + 630 + 627 + 626 + 623 + 622 + 619 + 618 + 615 + 614 + 611 + 610 + 607 + 606 + 603 + 602 = \mathbf{32,525}$$

$$Q_B = 599 + 598 + 595 + 594 + 591 + 590 + 587 + 586 + 583 + 582 + 579 + 578 + 575 + 574 + 571 + 570 + 567 + 566 + 563 + 562 + 559 + 558 + 555 + 554 + 551 + 550 + 547 + 546 + 543 + 542 + 539 + 538 + 535 + 534 + 531 + 530 + 527 + 526 + 523 + 522 + 519 + 518 + 515 + 514 + 511 + 510 + 507 + 506 + 503 + 502 = \mathbf{27,525}$$

$$Q_B = 499 + 498 + 495 + 494 + 491 + 490 + 487 + 486 + 483 + 482 + 479 + 478 + 475 + 474 + 471 + 470 + 467 + 466 + 463 + 462 + 459 + 458 + 455 + 454 + 451 + 450 + 447 + 446 + 443 + 442 + 439 + 438 + 435 + 434 + 431 + 430 + 427 + 426 + 423 + 422 + 419 + 418 + 415 + 414 + 411 + 410 + 407 + 406 + 403 + 402 = \mathbf{22,525}$$

$$Q_B = 399 + 398 + 395 + 394 + 391 + 390 + 387 + 386 + 383 + 382 + 379 + 378 + 375 + 374 + 371 + 370 + 367 + 366 + 363 + 362 + 359 + 358 + 355 + 354 + 351 + 350 + 347 + 346 + 343 + 342 + 339 + 338 + 335 + 334 + 331 + 330 + 327 + 326 + 323 + 322 + 319 + 318 + 315 + 314 + 311 + 310 + 307 + 306 + 303 + 302 = \mathbf{17,525}$$

$$Q_B = 299 + 298 + 295 + 294 + 291 + 290 + 287 + 286 + 283 + 282 + 279 + 278 + 275 + 274 + 271 + 270 + 267 + 266 + 263 + 262 + 259 + 258 + 255 + 254 + 251 + 250 + 247 + 246 + 243 + 242 + 239 + 238 + 235 + 234 + 231 + 230 + 227 + 226 + 223 + 222 + 219 + 218 + 215 + 214 + 211 + 210 + 207 + 206 + 203 + 202 = \mathbf{12,525}$$

$$Q_B = 199 + 198 + 195 + 194 + 191 + 190 + 187 + 186 + 183 + 182 + 179 + 178 + 175 + 174 + 171 + 170 + 167 + 166 + 163 + 162 + 159 + 158 + 155 + 154 + 151 + 150 + 147 + 146 + 143 + 142 + 139 + 138 + 135 + 134 + 131 + 130 + 127 + 126 + 123 + 122 + 119 + 118 + 115 + 114 + 111 + 110 + 107 + 106 + 103 + 102 = \mathbf{7,525}$$

$$Q_B = 99 + 98 + 95 + 94 + 91 + 90 + 87 + 86 + 83 + 82 + 79 + 78 + 75 + 74 + 71 + 70 + 67 + 66 + 63 + 62 + 59 + 58 + 55 + 54 + 51 + 50 + 47 + 46 + 43 + 42 + 39 + 38 + 35 + 34 + 31 + 30 + 27 + 26 + 23 + 22 + 19 + 18 + 15 + 14 + 11 + 10 + 7 + 6 + 3 + 2 = \mathbf{2,525}$$

Total for Pile B, Q_B = 250,250 units

Since the total mass for Pile A, Q_A = 250,250 units, and the total mass for Pile B, Q_B = 250,250 units, the 1000 concrete blocks of different masses have been divided into two piles of equal masses.

Sub-Conclusion for NP-Complete Problems

Two different types of NP-Complete problems were solved. The first type involved the division of items of different sizes, lengths, masses, volumes, or values into equal parts by combinations only. The second type covered possible final exam schedules for schools. The common approach for solving the different types of NP-Complete problems was to arrange the given data in either increasing or decreasing order. For the solutions of the first type of NP-Complete problems, an extended Ashanti fairness wisdom technique was applied to a set of 100 items of different values or masses. Two people A and B were able to divide items equally by merely choosing in turns from a set of ordered items. The total value or mass of A's items was found to be equal total value or mass of B's items, and these results are combinations of the items of different values or masses. It is very pleasing that such a simple technique can produce desired combinations. High school and middle school graduates could be taught the technique involved. From the solutions, formulas or simple equations were produced to help programmers apply the techniques. Note that in using the technique in this paper, the items involved must be arranged in decreasing order, preferably. Therefore, in programming, the first step should be to arrange the items in decreasing order. The technique was also applied to 1000 items; and the results were perfect, just like the results for the 100 items. Therefore, the technique covered does not care whether there are 2^{100} or 2^{1000} possibilities. There are social consequences of the method and principles used to divide the set of items into equal totals. The results can be applied by government agencies in the distribution of goods and services. Management personnel should be aware of the principles involved in the above technique. From the elementary school, through high school, and perhaps college, students should be taught the principles in the above wisdom technique, since throughout life, one is going to encounter situations in which two or more people are asked to choose in turns, from items of different values or sizes, and in this case, the sequence by which the choices are made matters; one may be either a participant or one may be in charge of the distribution process. By hand, the techniques can be used to prepare final exam schedules for 100 or 1000 courses. School secretaries and office assistants can learn and apply the techniques covered. Finally, if an approach can solve one NP problem, that approach can also solve other NP problems. Since three different types NP problems (seven problems) were solved using the same approach outlined above, all NP problems can be solved. The formerly NP problems are now P problems, and therefore, it is concluded that P is equal to NP.

Overall Conclusion

The traveling salesman problem (TSP) was solved in polynomial time and its solution was verified in polynomial time. This solution together with the verification no longer makes the TSP an NP-hard problem, but rather, a P problem. Also solved were an NP-Complete TSP, and six other NP-Complete problems. The TSP solution killed two (three) birds with one stone, because its solution made the NP-hard problems and NP-Complete problems become P problems. The shortest route as well as the longest route for the salesman to visit each of nine cities once and return to the base city was determined. The shortest route was found to be of length 79 kilometers, while the longest route was found to be of length 345 kilometers. In finding the shortest route, the first step was to arrange the data of the problem in increasing order, since one's interest was in the shortest distances; but in finding the longest route, the first step was to arrange the data of the problem in decreasing order, since one's interest was in the longest distances. For the shortest route, the main principle is that the shortest route is the sum of the shortest distances such that the salesman visits each city once and returns to the starting city; but for the longest route, the main principle is that the longest route is the sum of the longest distances such that the salesman visits each city once and returns to the starting city. One started the construction of the shortest route using only the shortest ten distances and if a needed distance was not among the set of the shortest; ten distances, one would consider distances greater than those in the set of the shortest ten distances. For the longest route, the construction began using only the longest ten distances; and if a needed distance was not among the set of the longest ten distances, one would consider distances shorter than those in the set of the longest ten distances. It was found out that even though, the length of each route is unique, the sequence of the cities involved is not unique, since the order of the sequence could be reversed. The approach used in this paper can be applied in work-force project management and hiring, as well as in a country's work-force needs and immigration quota determination. In the NP-complete problems, two people A and B were able to divide items equally by merely choosing in turns from a set of ordered items. Since approaches that solve the TSP and NP-Complete problems can also solve other NP problems, and TSP and NP-Complete problems have been solved, all NP problems can be solved. If all NP problems can be solved, then all NP problems are P problems, and therefore, P is equal to NP. The CMI Millennium Prize requirements have been satisfied.

Future and Next Task

Write computer codes to implement the solution processes in this paper.

Extra

Perhaps, one may make the following statements.
1. NP plus human ability equals P.
2. NP plus human inability is **not** equal to P.
3. NP minus human inability equals P.

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