Analyzing some parts of Ramanujan’s Manuscripts. Mathematical connections between several Ramanujan’s equations, the Rogers-Ramanujan continued fractions and the Dilaton value.

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Abstract

In this research thesis, we have analyzed some parts of Ramanujan’s Manuscripts and obtained new mathematical connections between several Ramanujan’s equations, the Rogers-Ramanujan continued fractions and the Dilaton value.

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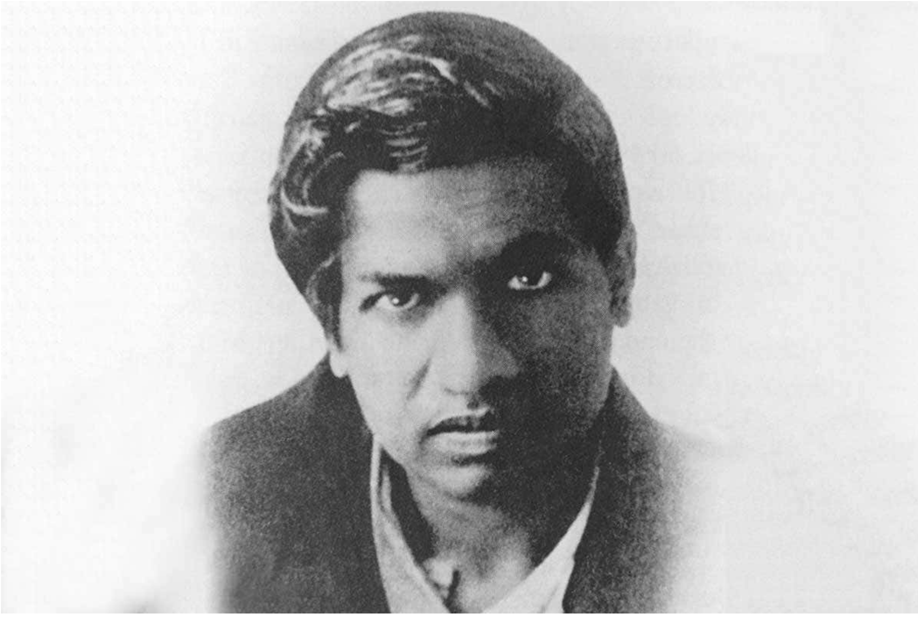


https://www.newscientist.com/article/2209213-computer-attempts-to-replicate-the-dream-like-maths-of-ramanujan/

From the Ramanujan Manuscript Book III

9. No. of the form p^2+q^2. 3

$\frac{1}{1^k}+\frac{1}{2^k}+\frac{1}{4^k}+\frac{1}{5^k}+\frac{1}{8^k}+\cdots$

$=\frac{1}{1-2^{-k}}\cdot\frac{1}{1-5^{-k}}\cdot\frac{1}{1-13^{-k}}\cdot\frac{1}{1-17^{-k}}\cdots$

$\times\frac{1}{1-3^{-2k}}\cdot\frac{1}{1-7^{-2k}}\cdot\frac{1}{1-11^{-2k}}\cdots$

$=\frac{\sqrt{s_k\cdot s'_k}}{1-2^{-k}}\sqrt{\frac{1}{1-3^{-2k}}\cdot\frac{1}{1-7^{-2k}}\cdot\frac{1}{1-11^{-2k}}\cdots}$

where $s_k=\frac{1}{1^k}+\frac{1}{3^k}+\frac{1}{5^k}+\cdots$

and $s'_k=\frac{1}{1^k}-\frac{1}{3^k}+\frac{1}{5^k}-\cdots$

$\frac{s_k}{s'_k}=\frac{1+3^{-k}}{1-3^{-k}}\cdot\frac{1+7^{-k}}{1-7^{-k}}\cdot\frac{1+11^{-k}}{1-11^{-k}}\cdots$

Hence the series =

$\frac{s'_k}{1-2^{-k}}\sqrt{\frac{s_k}{s'_k}}\cdot\sqrt[4]{\frac{s_{2k}}{s'_{2k}}}\sqrt[8]{\frac{s_{4k}}{s'_{4k}}}\sqrt[16]{\frac{s_{8k}}{s'_{8k}}}\cdots$

$=\frac{A}{\sqrt{k-1}}+\frac{B}{\sqrt[4]{2k-1}}+\frac{C}{\sqrt[8]{4k-1}}+\frac{D}{\sqrt[16]{8k-1}}+\cdots$

where $A=\sqrt{\frac{\pi}{2(1-\frac{1}{3^2})(1-\frac{1}{7^2})(1-\frac{1}{11^2})\cdots}}$ and

B, C, D are depending upon A.

Hence the reqd nos between m and n

is $C\int_n^m\frac{dx}{\sqrt{\log x}}+\theta(x)$ where $C=\frac{1}{\sqrt{2(1-\frac{1}{3^2})(1-\frac{1}{7^2})\cdots}}$

and $\theta(x)$ is of the order $\frac{\sqrt{x}}{(\log x)^{\frac{3}{4}}}$.

Obs. $\sqrt{2(1-\frac{1}{3^2})(1-\frac{1}{7^2})(1-\frac{1}{11^2})(1-\frac{1}{19^2})\cdots}=(1+\frac{1}{7})(1+\frac{1}{11})(1+\frac{1}{19})\cdots$

At the bottom, on the last line of the page, we find the following expression that we are going to analyze

sqrt((((2(1-1/9)(1-1/49)(1-1/121)(1-1/361)))) = (1+1/7)(1+1/11)(1+1/19)

Input:

$$\sqrt{2\left(1-\frac{1}{9}\right)\left(1-\frac{1}{49}\right)\left(1-\frac{1}{121}\right)\left(1-\frac{1}{361}\right)} = \left(1+\frac{1}{7}\right)\left(1+\frac{1}{11}\right)\left(1+\frac{1}{19}\right)$$

Result:

True

Left hand side:

$$\sqrt{2\left(1-\frac{1}{9}\right)\left(1-\frac{1}{49}\right)\left(1-\frac{1}{121}\right)\left(1-\frac{1}{361}\right)} = \frac{1920}{1463}$$

Right hand side:

$$\left(1+\frac{1}{7}\right)\left(1+\frac{1}{11}\right)\left(1+\frac{1}{19}\right) = \frac{1920}{1463}$$

We have that:

sqrt((((2(1-1/9)(1-1/49)(1-1/121)(1-1/361))))

Input:

$$\sqrt{2\left(1-\frac{1}{9}\right)\left(1-\frac{1}{49}\right)\left(1-\frac{1}{121}\right)\left(1-\frac{1}{361}\right)}$$

Exact result:

$$\frac{1920}{1463}$$

Decimal approximation:

1.312371838687628161312371838687628161312371838687628161312...

1.312371838687….

Repeating decimal:

$1.\overline{312371838687628161}$ (period 18)

All 2nd roots of 3686400/2140369:

$\frac{1920\, e^0}{1463} \approx 1.3124$ (real, principal root)

$\frac{1920\, e^{i\pi}}{1463} \approx -1.3124$ (real root)

(1+1/7)(1+1/11)(1+1/19)

Input:

$$\left(1+\frac{1}{7}\right)\left(1+\frac{1}{11}\right)\left(1+\frac{1}{19}\right)$$

Exact result:

$$\frac{1920}{1463}$$

Decimal approximation:

1.312371838687628161312371838687628161312371838687628161312...

Repeating decimal:

$1.\overline{312371838687628161}$ (period 18)

1.312371838687….

We observe that:

[sqrt(((2(1-1/9)(1-1/49)(1-1/121)(1-1/361))))]^16

Input:

$$\sqrt{2\left(1-\frac{1}{9}\right)\left(1-\frac{1}{49}\right)\left(1-\frac{1}{121}\right)\left(1-\frac{1}{361}\right)}^{16}$$

Exact result:

$$\frac{34\,105\,126\,070\,941\,954\,606\,390\,978\,313\,516\,482\,560\,000\,000\,000\,000\,000}{440\,462\,782\,507\,829\,638\,853\,407\,196\,959\,489\,747\,504\,132\,268\,442\,241}$$

Decimal approximation:

77.43021073599039896176690209685563130875920683708724925579...

77.4302107359.... result that is very near to 76 that is the value of a(n) for n = 96 of a 5^{th} order mock theta function.

The formula of mock theta function is:

a(n) ~ exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(phi*n))

and for n = 96.554, we obtain:

exp(Pi*sqrt(96.554/15)) / (2*5^(1/4)*sqrt(golden ratio*96.554))

Input interpretation:

$$\frac{\exp\left(\pi\sqrt{\frac{96.554}{15}}\right)}{2\sqrt[4]{5}\sqrt{\phi\times 96.554}}$$

ϕ is the golden ratio

Result:

77.4325...

77.4325…

Series representations:

$$\frac{\exp\left(\pi\sqrt{\frac{96.554}{15}}\right)}{2\sqrt[4]{5}\sqrt{\phi\, 96.554}} = \frac{\exp\left(\pi\sqrt{5.43693}\sum_{k=0}^{\infty} e^{-1.69322\,k}\binom{\frac{1}{2}}{k}\right)}{2\sqrt[4]{5}\sqrt{-1+96.554\,\phi}\sum_{k=0}^{\infty}(-1+96.554\,\phi)^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{\exp\left(\pi\sqrt{\frac{96.554}{15}}\right)}{2\sqrt[4]{5}\sqrt{\phi\, 96.554}} = \frac{\exp\left(\pi\sqrt{5.43693}\sum_{k=0}^{\infty}\frac{(-0.183927)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}{2\sqrt[4]{5}\sqrt{-1+96.554\,\phi}\sum_{k=0}^{\infty}\frac{(-1)^k(-1+96.554\,\phi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\exp\left(\pi\sqrt{\frac{96.554}{15}}\right)}{2\sqrt[4]{5}\sqrt{\phi\, 96.554}} = \frac{\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6.43693-z_0)^k z_0^{-k}}{k!}\right)}{2\sqrt[4]{5}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(96.554\,\phi-z_0)^k z_0^{-k}}{k!}}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

Thence, we obtain the following mathematical connection:

$$\left(\sqrt{2\left(1-\frac{1}{9}\right)\left(1-\frac{1}{49}\right)\left(1-\frac{1}{121}\right)\left(1-\frac{1}{361}\right)}^{16}\right) = 77.43021 \Rightarrow$$

$$\Rightarrow \left(\frac{\exp\left(\pi\sqrt{\frac{96.554}{15}}\right)}{2\sqrt[4]{5}\sqrt{\phi \times 96.554}}\right) = 77.4325$$

Now, we have:

((((1/ sqrt((((2(1-1/9)(1-1/49)(1-1/121)(1-1/361)))))))))^1/8

Input:

$$\sqrt[8]{\frac{1}{\sqrt{2\left(1-\frac{1}{9}\right)\left(1-\frac{1}{49}\right)\left(1-\frac{1}{121}\right)\left(1-\frac{1}{361}\right)}}}$$

Exact result:

$$\frac{\sqrt[8]{\frac{1463}{15}}}{2^{7/8}}$$

Decimal approximation:

0.966591311823517666029439786015911710690223079081958938941...

0.966591311823517666.... result very near to the spectral index n_s and to the mesonic Regge slope (see Appendix)

And:

1/10^27 (((((47+4)/10^3+((((2sqrt((((2(1-1/9)(1-1/49)(1-1/121)(1-1/361))))))))^1/2)))))

Input:

$$\frac{1}{10^{27}}\left(\frac{47+4}{10^3}+\sqrt{2\sqrt{2\left(1-\frac{1}{9}\right)\left(1-\frac{1}{49}\right)\left(1-\frac{1}{121}\right)\left(1-\frac{1}{361}\right)}}\right)$$

Result:

$$\frac{\frac{51}{1000}+16\sqrt{\frac{15}{1463}}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

Decimal approximation:

$$1.671106069791498658659204600114344811307021850050794 7\ldots\times 10^{-27}$$

$1.671106069791498\ldots*10^{-27}$

We note that $1.6711060697*10^{-27}$ kg is a result practically equal to the value of the formula:

$$m_{p\prime} = 2\times\frac{\eta}{R}m_P = 1.6714213\times 10^{-24}\text{ gm}$$

that is the holographic proton mass (N. Haramein)

Alternate forms:

$$\frac{74\,613+16\,000\sqrt{21\,945}}{1\,463\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{51}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}+\frac{\sqrt{\frac{3}{7315}}}{12\,500\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{51+16\,000\sqrt{\frac{15}{1463}}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

Minimal polynomial:

$$1463000000000000000000000000000000000000000000000000000000000000000\,x^2 - 149226000000000000000000000000000000000\,x - 3836194737$$

We note also:

1+ 1/2 sqrt((((2(1-1/9)(1-1/49)(1-1/121)(1-1/361))))

Input:

$$1+\frac{1}{2}\sqrt{2\left(1-\frac{1}{9}\right)\left(1-\frac{1}{49}\right)\left(1-\frac{1}{121}\right)\left(1-\frac{1}{361}\right)}$$

Exact result:

$$\frac{2423}{1463}$$

Decimal approximation:

1.656185919343814080656185919343814080656185919343814080656...

1.656185919.... is very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164,2696 i.e. 1,65578...

24/10^3 + sqrt((((2sqrt((((2(1-1/9)(1-1/49)(1-1/121)(1-1/361))))))))

Input:

$$\frac{24}{10^3}+\sqrt{2\sqrt{2\left(1-\frac{1}{9}\right)\left(1-\frac{1}{49}\right)\left(1-\frac{1}{121}\right)\left(1-\frac{1}{361}\right)}}$$

Result:

$$\frac{3}{125}+16\sqrt{\frac{15}{1463}}$$

Decimal approximation:

1.644106069791498658659204600114344811307021850050794739300...

1.64410606979149…. ≈ ζ(2) = $\frac{\pi^2}{6} = 1.644934$ ...

Alternate forms:

$$\frac{4389+2000\sqrt{21945}}{182875}$$

$$\frac{16\sqrt{21945}}{1463}+\frac{3}{125}$$

$$\frac{1}{125}\left(3+2000\sqrt{\frac{15}{1463}}\right)$$

Minimal polynomial:

$$22859375x^2-1097250x-59986833$$

From which, we obtain:

sqrt[6(((((24/10^3 + sqrt((((2sqrt((((2(1-1/9)(1-1/49)(1-1/121)(1-1/361))))))))))))))]

Input:

$$\sqrt{6\left(\frac{24}{10^3}+\sqrt{2\sqrt{2\left(1-\frac{1}{9}\right)\left(1-\frac{1}{49}\right)\left(1-\frac{1}{121}\right)\left(1-\frac{1}{361}\right)}}\right)}$$

Result:

$$\sqrt{6\left(\frac{3}{125}+16\sqrt{\frac{15}{1463}}\right)}$$

Decimal approximation:

3.140801875118676114669146964339020007199446557636698755753...

3.1408018751186….. ≈ π

Alternate forms:

$$\frac{\sqrt{43890\left(4389+2000\sqrt{21945}\right)}}{36575}$$

$$\sqrt{\frac{18}{125}+96\sqrt{\frac{15}{1463}}}$$

$$\frac{1}{5}\sqrt{\frac{6\left(4389+2000\sqrt{21945}\right)}{7315}}$$

Minimal polynomial:

$$22859375x^4-6583500x^2-2159525988$$

And:

sqrt((((2sqrt((((2(1-1/9)(1-1/49)(1-1/121)(1-1/361))))))))-(2/10^3)

where 2 is a Fibonacci number, a Lucas number and a prime number

Input:

$$\sqrt{2\sqrt{2\left(1-\frac{1}{9}\right)\left(1-\frac{1}{49}\right)\left(1-\frac{1}{121}\right)\left(1-\frac{1}{361}\right)}} - \frac{2}{10^3}$$

Result:

$$16\sqrt{\frac{15}{1463}} - \frac{1}{500}$$

Decimal approximation:

1.618106069791498658659204600114344811307021850050794739300...

1.6181060697914….

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Alternate forms:

$$\frac{8000\sqrt{21945} - 1463}{731500}$$

$$\frac{16\sqrt{21945}}{1463} - \frac{1}{500}$$

$$\frac{1}{500}\left(8000\sqrt{\frac{15}{1463}} - 1\right)$$

Minimal polynomial:

$$365750000x^2 + 1463000x - 959998537$$

And:

((((sqrt((((2(1-1/9)(1-1/49)(1-1/121)(1-1/361))))))))^41+4096+144+13

Input:

$$\sqrt{2\left(1-\frac{1}{9}\right)\left(1-\frac{1}{49}\right)\left(1-\frac{1}{121}\right)\left(1-\frac{1}{361}\right)}^{41}+4096+144+13$$

Exact result:

437 764 646 480 702 599 243 514 267 218 143 063 788 891 226 400 828 897 567 168 \
330 439 988 736 338 669 550 890 191 393 901 543 031 559 712 082 212 766 561 499 \
882 619 915 692 539 /
5 956 859 696 113 511 164 673 312 709 859 971 238 074 122 046 659 128 253 758 \
395 900 607 517 684 577 369 826 629 973 487 501 300 625 373 167 696 394 677 \
051 465 464 358 263

Decimal approximation:

73489.16523354031981668912966891476626892221502921417475764...

73489.1652335403

Thence, we have the following mathematical connections:

$$\left[\sqrt{2\left(1-\frac{1}{9}\right)\left(1-\frac{1}{49}\right)\left(1-\frac{1}{121}\right)\left(1-\frac{1}{361}\right)}^{41}+4096+144+13\right] = 73489.16523\ldots \Rightarrow$$

$$\Rightarrow -3927+2\left(\sqrt[13]{\begin{array}{c} N\exp\left[\int d\hat{\sigma}\left(-\frac{1}{4u^2}\mathbf{P}_i D\mathbf{P}_i\right)\right]|Bp\rangle_{\mathrm{NS}} + \\ \int[d\mathbf{X}^\mu]\exp\left\{\int d\hat{\sigma}\left(-\frac{1}{4v^2}D\mathbf{X}^\mu D^2\mathbf{X}^\mu\right)\right\}|\mathbf{X}^\mu,\mathbf{X}^i=0\rangle_{\mathrm{NS}}\end{array}}\right) =$$

$$-3927+2\sqrt[13]{2.2983717437\times10^{59}+2.0823329825883\times10^{59}}$$

$$= 73490.8437525\ldots. \Rightarrow$$

$$\Rightarrow\left(A(r)\times\frac{1}{B(r)}\left(-\frac{1}{\phi(r)}\right)\times\frac{1}{e^{\Lambda(r)}}\right)\Rightarrow$$

$$\Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621} \left(-\frac{1}{0.0005946833} \right) \times \frac{1}{0.00183393} \right) =$$

$$= 73491.788325481187105491595720422205480251957265634133987 00\ldots$$

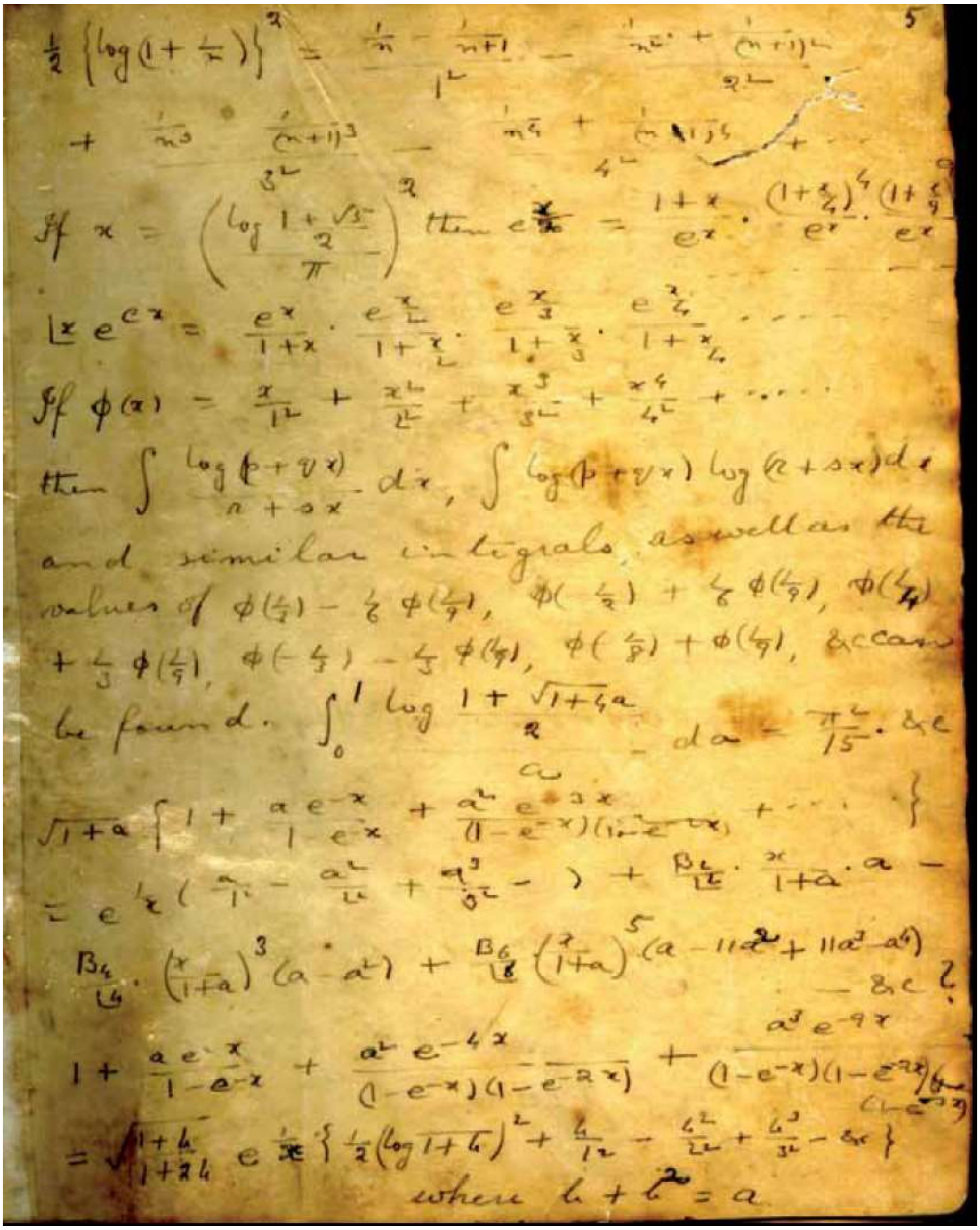
$$= 73491.7883254\ldots \Rightarrow$$

$$\left(\begin{array}{c} I_{21} \ll \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leq P^{1-\varepsilon_1}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \\ \ll H \left\{ \left(\frac{4}{\varepsilon_2 \log T} \right)^{2r} (\log T)(\log X)^{-2\beta} + (\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r}) T^{-\varepsilon_1} \right\} \end{array} \right) /$$

$$/(26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = 73493.30662\ldots$$

Mathematical connections with the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of u → ∞, with the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and with the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

Now:



5

$$\frac{1}{2}\left\{\log\left(1+\frac{1}{n}\right)\right\}^2 = \frac{\frac{1}{n}-\frac{1}{n+1}}{1^2} - \frac{\frac{1}{n^2}+\frac{1}{(n+1)^2}}{2^2} + \frac{\frac{1}{n^3}-\frac{1}{(n+1)^3}}{3^2} - \frac{\frac{1}{n^4}+\frac{1}{(n+1)^4}}{4^2} + \cdots$$

If $x = \left(\frac{\log\frac{1+\sqrt{5}}{2}}{\pi}\right)^2$ then $e^{\frac{x}{x}} = \frac{1+x}{e^x}\cdot\frac{(1+\frac{x}{4})^4}{e^x}\cdot\frac{(1+\frac{x}{9})^9}{e^x}$

$$\lfloor x\, e^{Cx} = \frac{e^x}{1+x}\cdot\frac{e^{\frac{x}{2}}}{1+\frac{x}{2}}\cdot\frac{e^{\frac{x}{3}}}{1+\frac{x}{3}}\cdot\frac{e^{\frac{x}{4}}}{1+\frac{x}{4}}\cdots$$

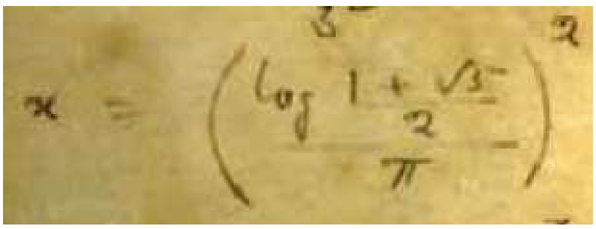
If $\phi(x) = \frac{x}{1^2}+\frac{x^2}{2^2}+\frac{x^3}{3^2}+\frac{x^4}{4^2}+\cdots$ then $\int\frac{\log(p+qx)}{r+sx}dx$, $\int\log(p+qx)\log(r+sx)dx$ and similar integrals as well as the values of $\phi(\frac{1}{3})-\frac{1}{6}\phi(\frac{1}{9})$, $\phi(-\frac{1}{2})+\frac{1}{6}\phi(\frac{1}{9})$, $\phi(\frac{1}{4})+\frac{1}{3}\phi(\frac{1}{9})$, $\phi(-\frac{1}{3})-\frac{1}{3}\phi(\frac{1}{9})$, $\phi(\frac{1}{8})+\phi(\frac{1}{9})$, &c can be found. $\int_0^1\frac{\log\frac{1+\sqrt{1+4a}}{2}}{a}da = \frac{\pi^2}{15}$. &c

$$\sqrt{1+a}\left\{1+\frac{ae^{-x}}{1-e^{-x}}+\frac{a^2e^{-3x}}{(1-e^{-x})(1-e^{-2x})}+\cdots\right\} = e^{\frac{1}{x}\left(\frac{a}{1^2}-\frac{a^2}{2^2}+\frac{a^3}{3^2}-\cdots\right)+\frac{B_2}{\lfloor 2}\cdot\frac{x}{1+a}\cdot a-\frac{B_4}{\lfloor 4}\cdot\left(\frac{x}{1+a}\right)^3(a-a^2)+\frac{B_6}{\lfloor 6}\left(\frac{x}{1+a}\right)^5(a-11a^2+11a^3-a^4)-\&c}\,?$$

$$1+\frac{ae^{-x}}{1-e^{-x}}+\frac{a^2e^{-4x}}{(1-e^{-x})(1-e^{-2x})}+\frac{a^3e^{-9x}}{(1-e^{-x})(1-e^{-2x})(1-e^{-3x})} = \sqrt{\frac{1+b}{1+2b}}\,e^{\frac{1}{x}\left\{\frac{1}{2}(\log 1+b)^2+\frac{b}{1^2}-\frac{b^2}{2^2}+\frac{b^3}{3^2}-\&c\right\}}$$

where $b+b^2=a$

We analyze this formula:



$$x = \left(\frac{\log\frac{1+\sqrt{5}}{2}}{\pi}\right)^2$$

Now, we consider the following variant of the above formula, performing the logarithm of the result of the whole fraction (numerator and denominator)

$$\left(\log\frac{1+\sqrt{5}}{2}\cdot\frac{1}{\pi}\right)^2 = \left(\log 1.6180339887498\cdot\frac{1}{\pi}\right)^2 = \left(\log\frac{1.6180339887498}{\pi}\right)^2 =$$

$$(\log 0.5150362148)^2 = -0.6635180607907362^2 = 0.440256216995499$$

Indeed, we have:

(((((ln(((1+sqrt(5))/2) / Pi))))^2

Input:

$$\log^2\left(\frac{\frac{1}{2}\left(1+\sqrt{5}\right)}{\pi}\right)$$

log(x) is the natural logarithm

Exact result:

$$\log^2\left(\frac{1+\sqrt{5}}{2\pi}\right)$$

Decimal approximation:

0.4402562169942523843403473280955627102809070168463269553 34...

0.4402562169….

Alternate forms:

$$\left(\text{csch}^{-1}(2)-\log(\pi)\right)^2$$

$$\log^2\left(\frac{2\pi}{1+\sqrt{5}}\right)$$

$$\left(-\log(2)+\log\left(1+\sqrt{5}\right)-\log(\pi)\right)^2$$

Alternative representations:

$$\log^2\left(\frac{1+\sqrt{5}}{\pi 2}\right)=\log_e^2\left(\frac{1+\sqrt{5}}{2\pi}\right)$$

$$\log^2\left(\frac{1+\sqrt{5}}{\pi 2}\right)=\left(\log(a)\log_a\left(\frac{1+\sqrt{5}}{2\pi}\right)\right)^2$$

$$\log^2\left(\frac{1+\sqrt{5}}{\pi 2}\right)=\left(-\text{Li}_1\left(1-\frac{1+\sqrt{5}}{2\pi}\right)\right)^2$$

Series representations:

$$\log^2\left(\frac{1+\sqrt{5}}{\pi\,2}\right) = \left(\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1+\frac{1+\sqrt{5}}{2\pi}\right)^k}{k}\right)^2$$

$$\log^2\left(\frac{1+\sqrt{5}}{\pi\,2}\right) = \left(2\,i\,\pi \left\lfloor \frac{\arg(1+\sqrt{5}-2\pi x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2\pi}\right)^k x^{-k} \left(1+\sqrt{5}-2\pi x\right)^k}{k}\right)^2$$

for $x < 0$

$$\log^2\left(\frac{1+\sqrt{5}}{\pi\,2}\right) = \left(2\,i\,\pi \left\lfloor \frac{\arg\left(\frac{1+\sqrt{5}}{2\pi}-x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2\pi}\right)^k x^{-k} \left(1+\sqrt{5}-2\pi x\right)^k}{k}\right)^2$$

for $x < 0$

Integral representation:

$$\log^2\left(\frac{1+\sqrt{5}}{\pi\,2}\right) = \left(\int_1^{\frac{1+\sqrt{5}}{2\pi}} \frac{1}{t}\,dt\right)^2$$

Now, we have:

(((((ln(((1+sqrt(5))/2) / Pi))))^2)))^1/64

Input:

$$\sqrt[64]{\log^2\left(\frac{\frac{1}{2}\left(1+\sqrt{5}\right)}{\pi}\right)}$$

log(x) is the natural logarithm

Exact result:

$$\sqrt[32]{-\log\left(\frac{1+\sqrt{5}}{2\pi}\right)}$$

Decimal approximation:

0.987263084758650033899699895808408258403170137670263112520...

0.98726308475…. result very near to the dilaton value $\mathbf{0.989117352243 = \phi}$ (see Appendix)

Alternate forms:

$$\sqrt[32]{\log(\pi) - \operatorname{csch}^{-1}(2)}$$

$$\sqrt[32]{\log\left(\frac{2\pi}{1+\sqrt{5}}\right)}$$

$$\sqrt[32]{-1}\ e^{-(i\pi)/16}\ \sqrt[32]{\operatorname{csch}^{-1}(2) - \log(\pi)}$$

$\operatorname{csch}^{-1}(x)$ is the inverse hyperbolic cosecant function

Alternative representations:

$$\sqrt[64]{\log^2\left(\frac{1+\sqrt{5}}{\pi\, 2}\right)} = \sqrt[64]{\log_e^2\left(\frac{1+\sqrt{5}}{2\pi}\right)}$$

$$\sqrt[64]{\log^2\left(\frac{1+\sqrt{5}}{\pi\, 2}\right)} = \sqrt[64]{\left(\log(a)\log_a\left(\frac{1+\sqrt{5}}{2\pi}\right)\right)^2}$$

$$\sqrt[64]{\log^2\left(\frac{1+\sqrt{5}}{\pi\, 2}\right)} = \sqrt[64]{\left(-\operatorname{Li}_1\left(1-\frac{1+\sqrt{5}}{2\pi}\right)\right)^2}$$

Series representations:

$$\sqrt[64]{\log^2\left(\frac{1+\sqrt{5}}{\pi\, 2}\right)} = \sqrt[32]{\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{1+\sqrt{5}}{2\pi}\right)^k}{k}}$$

$$\sqrt[64]{\log^2\left(\frac{1+\sqrt{5}}{\pi\, 2}\right)} =$$

$$\sqrt[32]{-2 i\pi\left\lfloor\frac{\arg(1+\sqrt{5}-2\pi x)}{2\pi}\right\rfloor - \log(x) + \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2\pi}\right)^k x^{-k}\left(1+\sqrt{5}-2\pi x\right)^k}{k}} \quad \text{for } x<0$$

$$\sqrt[64]{\log^2\left(\frac{1+\sqrt{5}}{\pi\,2}\right)} = \sqrt[32]{-2\,i\,\pi\left\lfloor\frac{\arg\left(\frac{1+\sqrt{5}}{2\pi}-x\right)}{2\pi}\right\rfloor - \log(x) + \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2\pi}\right)^k x^{-k}\left(1+\sqrt{5}-2\pi x\right)^k}{k}} \quad \text{for } x<0$$

Integral representation:

$$\sqrt[64]{\log^2\left(\frac{1+\sqrt{5}}{\pi\,2}\right)} = \sqrt[32]{-\int_1^{\frac{1+\sqrt{5}}{2\pi}}\frac{1}{t}\,dt}$$

From which:

log base 0.98726308475865 (((((ln(((1+sqrt(5))/2) / Pi))))^2))

Input interpretation:

$$\log_{0.98726308475865}\left(\log^2\left(\frac{\frac{1}{2}\left(1+\sqrt{5}\right)}{\pi}\right)\right)$$

$\log(x)$ is the natural logarithm

$\log_b(x)$ is the base- b logarithm

Result:

64.0000000000...

64 (see Appendix)

Alternative representations:

$$\log_{0.987263084758650000}\left(\log^2\left(\frac{1+\sqrt{5}}{\pi\,2}\right)\right) = \frac{\log\left(\log^2\left(\frac{1+\sqrt{5}}{2\pi}\right)\right)}{\log(0.987263084758650000)}$$

$$\log_{0.987263084758650000}\left(\log^2\left(\frac{1+\sqrt{5}}{\pi\,2}\right)\right) = \log_{0.987263084758650000}\left(\log_e^2\left(\frac{1+\sqrt{5}}{2\pi}\right)\right)$$

$$\log_{0.987263084758650000}\left(\log^2\left(\frac{1+\sqrt{5}}{\pi\, 2}\right)\right) = \log_{0.987263084758650000}\left(\left(\log(a)\log_a\left(\frac{1+\sqrt{5}}{2\pi}\right)\right)^2\right)$$

Series representations:

$$\log_{0.987263084758650000}\left(\log^2\left(\frac{1+\sqrt{5}}{\pi\, 2}\right)\right) = -\frac{\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\log^2\left(\frac{1+\sqrt{5}}{2\pi}\right)\right)^k}{k}}{\log(0.987263084758650000)}$$

$$\log_{0.987263084758650000}\left(\log^2\left(\frac{1+\sqrt{5}}{\pi\, 2}\right)\right) = \log_{0.987263084758650000}\left(\left(\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{1+\sqrt{5}}{2\pi}\right)^k}{k}\right)^2\right)$$

Integral representation:

$$\log_{0.987263084758650000}\left(\log^2\left(\frac{1+\sqrt{5}}{\pi\, 2}\right)\right) = \log_{0.987263084758650000}\left(\left(\int_1^{\frac{1+\sqrt{5}}{2\pi}}\frac{1}{t}\,dt\right)^2\right)$$

We note that the inverse of this formula, elevated to the power of eight, where 8 is a Fibonacci number, provides

[1 / (((((((ln(((1+sqrt(5))/2) / Pi))))^2))))]^8

Input:

$$\left(\frac{1}{\log^2\left(\frac{\frac{1}{2}\left(1+\sqrt{5}\right)}{\pi}\right)}\right)^8$$

log(x) is the natural logarithm

Exact result:

$$\frac{1}{\log^{16}\left(\frac{1+\sqrt{5}}{2\pi}\right)}$$

Decimal approximation:

708.5263725917167947661152245609603448069820652407384271951...

708.52637259…

Alternate forms:

$$\frac{1}{(\operatorname{csch}^{-1}(2) - \log(\pi))^{16}}$$

$$\frac{1}{\log^{16}\left(\frac{2\pi}{1+\sqrt{5}}\right)}$$

$$\frac{1}{\left(-\log(2) + \log\left(1+\sqrt{5}\right) - \log(\pi)\right)^{16}}$$

Alternative representations:

$$\left(\frac{1}{\log^2\left(\frac{1+\sqrt{5}}{\pi 2}\right)}\right)^8 = \left(\frac{1}{\log_e^2\left(\frac{1+\sqrt{5}}{2\pi}\right)}\right)^8$$

$$\left(\frac{1}{\log^2\left(\frac{1+\sqrt{5}}{\pi 2}\right)}\right)^8 = \left(\frac{1}{\left(\log(a)\log_a\left(\frac{1+\sqrt{5}}{2\pi}\right)\right)^2}\right)^8$$

$$\left(\frac{1}{\log^2\left(\frac{1+\sqrt{5}}{\pi 2}\right)}\right)^8 = \left(\frac{1}{\left(-\operatorname{Li}_1\left(1-\frac{1+\sqrt{5}}{2\pi}\right)\right)^2}\right)^8$$

Series representations:

$$\left(\frac{1}{\log^2\left(\frac{1+\sqrt{5}}{\pi 2}\right)}\right)^8 = \frac{1}{\left(\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1+\frac{1+\sqrt{5}}{2\pi}\right)^k}{k}\right)^{16}}$$

$$\left(\frac{1}{\log^2\left(\frac{1+\sqrt{5}}{\pi 2}\right)}\right)^8 = \frac{1}{\left(2\pi\left\lfloor\frac{\arg\left(1+\sqrt{5}-2\pi x\right)}{2\pi}\right\rfloor - i\left(\log(x) - \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2\pi}\right)^k x^{-k}\left(1+\sqrt{5}-2\pi x\right)^k}{k}\right)\right)^{16}} \quad \text{for } x < 0$$

$$\left(\frac{1}{\log^2\left(\frac{1+\sqrt{5}}{\pi 2}\right)}\right)^8 = \frac{1}{\left(2 i\pi\left\lfloor\frac{\arg\left(\frac{1+\sqrt{5}}{2\pi}-x\right)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2\pi}\right)^k x^{-k}\left(1+\sqrt{5}-2\pi x\right)^k}{k}\right)^{16}} \quad \text{for } x < 0$$

Integral representation:

$$\left(\frac{1}{\log^2\left(\frac{1+\sqrt{5}}{\pi 2}\right)}\right)^8 = \frac{1}{\left(\int_1^{\frac{1+\sqrt{5}}{2\pi}}\frac{1}{t}\,dt\right)^{16}}$$

The result 708.52637259… is very near to 706 that is the value of a(n) for n = 166 of a 5th order mock theta function and adding 21, that is a Fibonacci number, we obtain 729.52637

The formula of mock theta function is:

a(n) ~ exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(phi*n))

and for n = 166.15, we obtain:

exp(Pi*sqrt(166.15/15)) / (2*5^(1/4)*sqrt(golden ratio*166.15))

Input interpretation:

$$\frac{\exp\left(\pi\sqrt{\frac{166.15}{15}}\right)}{2\sqrt[4]{5}\sqrt{\phi\times 166.15}}$$

ϕ is the golden ratio

Result:

708.516...

708.516…

Series representations:

$$\frac{\exp\left(\pi\sqrt{\frac{166.15}{15}}\right)}{2\sqrt[4]{5}\sqrt{\phi\,166.15}} = \frac{\exp\left(\pi\sqrt{10.0767}\sum_{k=0}^{\infty} e^{-2.31022\,k}\binom{\frac{1}{2}}{k}\right)}{2\sqrt[4]{5}\sqrt{-1+166.15\,\phi}\sum_{k=0}^{\infty}(-1+166.15\,\phi)^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{\exp\left(\pi\sqrt{\frac{166.15}{15}}\right)}{2\sqrt[4]{5}\sqrt{\phi\,166.15}} = \frac{\exp\left(\pi\sqrt{10.0767}\sum_{k=0}^{\infty}\frac{(-0.0992392)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}{2\sqrt[4]{5}\sqrt{-1+166.15\,\phi}\sum_{k=0}^{\infty}\frac{(-1)^k(-1+166.15\,\phi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\exp\left(\pi\sqrt{\frac{166.15}{15}}\right)}{2\sqrt[4]{5}\sqrt{\phi\,166.15}} = \frac{\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(11.0767-z_0)^k z_0^{-k}}{k!}\right)}{2\sqrt[4]{5}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(166.15\,\phi-z_0)^k z_0^{-k}}{k!}}$$

for not $((z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0))$

Thence, we have the following mathematical connection:

$$\left(\frac{1}{\log^{16}\left(\frac{1+\sqrt{5}}{2\pi}\right)}\right) = 708.52637259 \Rightarrow$$

$$\Rightarrow \left(\frac{\exp\left(\pi\sqrt{\frac{166.15}{15}}\right)}{2\sqrt[4]{5}\sqrt{\phi\times 166.15}}\right) = 708.516$$

We observe that from the two results of the connections 77.43021 and 708.52637, and the continued fraction constant:

(1/6)π^2/(log(2)log(10))

Input:

$$\frac{1}{6} \times \frac{\pi^2}{\log(2)\log(10)}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\pi^2}{6\log(2)\log(10)}$$

Decimal approximation:

1.030640834100712935881776094116936840925920311120726281770...

1.0306408341007…..

Alternate forms:

$$\frac{\pi^2}{\log(10)\log(64)}$$

$$\frac{\pi^2}{6\log(2)(\log(2)+\log(5))}$$

Alternative representations:

$$\frac{\pi^2}{(\log(2)\log(10))6} = \frac{\pi^2}{6(\log_e(2)\log_e(10))}$$

$$\frac{\pi^2}{(\log(2)\log(10))6} = \frac{\pi^2}{6(\log^2(a)\log_a(2)\log_a(10))}$$

$$\frac{\pi^2}{(\log(2)\log(10))6} = \frac{\pi^2}{6(\text{Li}_1(-9)\text{Li}_1(-1))}$$

Series representations:

$$\frac{\pi^2}{(\log(2)\log(10))\,6} = -\left(\pi^2 \Big/ \left(6\left(2\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor - i\log(x) + i\sum_{k=1}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}}{k}\right)\left(2\pi\left\lfloor\frac{\arg(10-x)}{2\pi}\right\rfloor - i\log(x) + i\sum_{k=1}^{\infty}\frac{(-1)^k (10-x)^k x^{-k}}{k}\right)\right)\right) \text{ for } x<0$$

$$\frac{\pi^2}{(\log(2)\log(10))\,6} = -\left(\pi^2 \Big/ \left(6\left(2\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor - i\log(z_0) + i\sum_{k=1}^{\infty}\frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)\left(2\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor - i\log(z_0) + i\sum_{k=1}^{\infty}\frac{(-1)^k (10-z_0)^k z_0^{-k}}{k}\right)\right)\right)$$

$$\frac{\pi^2}{(\log(2)\log(10))\,6} = \pi^2 \Big/ \left(6\left(\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0) - \sum_{k=1}^{\infty}\frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)\left(\left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor\frac{\arg(10-z_0)}{2\pi}\right\rfloor\log(z_0) - \sum_{k=1}^{\infty}\frac{(-1)^k (10-z_0)^k z_0^{-k}}{k}\right)\right)$$

Integral representations:

$$\frac{\pi^2}{(\log(2)\log(10))\,6} = \frac{\pi^2}{6\left(\int_1^2 \frac{1}{t}\,dt\right)\int_1^{10}\frac{1}{t}\,dt}$$

$$\frac{\pi^2}{(\log(2)\log(10))\,6} = -\frac{2\pi^4}{3\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{9^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \text{ for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

We obtain:

((((708.52637 * 77.43021)^1.0306408341)))-(2048+1024+64+16)

Input interpretation:

$(708.52637 \times 77.43021)^{1.0306408341} - (2048 + 1024 + 64 + 16)$

-

Result:

73492.59...

73492.59...

Or:

((((708.52637 * 77.43021)^((1/6)π^2/(log(2)log(10))))))-(2048+1024+64+16)

Input interpretation:

$(708.52637 \times 77.43021)^{1/6 \times \pi^2/(\log(2)\log(10))} - (2048 + 1024 + 64 + 16)$

log(x) is the natural logarithm

Result:

- More digits

73492.59...

73492.59…

Alternative representations:

$$(708.526 \times 77.4302)^{\pi^2/((\log(2)\log(10))\,6)} - (2048 + 1024 + 64 + 16) = -3152 + 54\,861.3^{\pi^2/(6(\log_e(2)\log_e(10)))}$$

$$(708.526 \times 77.4302)^{\pi^2/((\log(2)\log(10))\,6)} - (2048 + 1024 + 64 + 16) = -3152 + 54\,861.3^{\pi^2/\left(6\left(\log^2(a)\log_a(2)\log_a(10)\right)\right)}$$

$$(708.526 \times 77.4302)^{\pi^2/((\log(2)\log(10))\,6)} - (2048 + 1024 + 64 + 16) = -3152 + 54\,861.3^{\pi^2/(6(\mathrm{Li}_1(-9)\,\mathrm{Li}_1(-1)))}$$

$\log_b(x)$ is the base- b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations:

$$(708.526\times 77.4302)^{\pi^2/((\log(2)\log(10))6)} - (2048+1024+64+16) = -3152 + 54861.\therefore 3^{\pi^2/\left(6\left(2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}\right)\left(2 i \pi \left\lfloor \frac{\arg(10-x)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (10-x)^k x^{-k}}{k}\right)\right)}$$

for $x < 0$

$$(708.526\times 77.4302)^{\pi^2/((\log(2)\log(10))6)} - (2048+1024+64+16) = -3152 + 54861.\therefore 3^{\pi^2/\left(6\left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi}\right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)\left(\log(z_0) + \left\lfloor \frac{\arg(10-z_0)}{2\pi}\right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (10-z_0)^k z_0^{-k}}{k}\right)\right)}$$

$$(708.526\times 77.4302)^{\pi^2/((\log(2)\log(10))6)} - (2048+1024+64+16) = -3152 + 54861.\therefore 3^{\pi^2/\left(6\left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{2}{z_0}\right) - \arg(z_0)}{2\pi}\right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)\left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{10}{z_0}\right) - \arg(z_0)}{2\pi}\right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (10-z_0)^k z_0^{-k}}{k}\right)\right)}$$

Integral representations:

$$(708.526\times 77.4302)^{\pi^2/((\log(2)\log(10))6)} - (2048+1024+64+16) = -3152 + e^{\frac{1.81876\,\pi^2}{\left(\int_1^2 \frac{1}{t}\,dt\right)\int_1^{10} \frac{1}{t}\,dt}}$$

$$(708.526\times 77.4302)^{\pi^2/((\log(2)\log(10))6)} - (2048+1024+64+16) = -3152 + \exp\left(\frac{7.27504\, i^2 \pi^4}{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{9^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right) \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

Thence, we obtain the following mathematical connections:

$$\left[(708.52637\times 77.43021)^{1/6\times\pi^2/(\log(2)\log(10))} - (2048+1024+64+16)\right] = 73492.59 \Rightarrow$$

$$\Rightarrow -3927 + 2\left(\sqrt[13]{\frac{N\exp\left[\int d\hat{\sigma}\left(-\frac{1}{4u^2}\mathrm{P}_i D\mathrm{P}_i\right)\right]|Bp\rangle_{\mathrm{NS}}}{\int [d\mathrm{X}^\mu]\exp\left\{\int d\hat{\sigma}\left(-\frac{1}{4v^2}D\mathrm{X}^\mu D^2\mathrm{X}^\mu\right)\right\}|\mathrm{X}^\mu,\mathrm{X}^i=0\rangle_{\mathrm{NS}}} +}\right) =$$

$$-3927 + 2\sqrt[13]{2.2983717437\times 10^{59} + 2.0823329825883\times 10^{59}}$$

$= 73490.8437525.... \Rightarrow$

$$\Rightarrow\left(A(r)\times\frac{1}{B(r)}\left(-\frac{1}{\phi(r)}\right)\times\frac{1}{e^{\Lambda(r)}}\right)\Rightarrow$$

$$\Rightarrow\left(-0.000029211892\times\frac{1}{0.0003644621}\left(-\frac{1}{0.0005946833}\right)\times\frac{1}{0.00183393}\right) =$$

$= 73491.78832548118710549159572042220548025195726563413398700...$

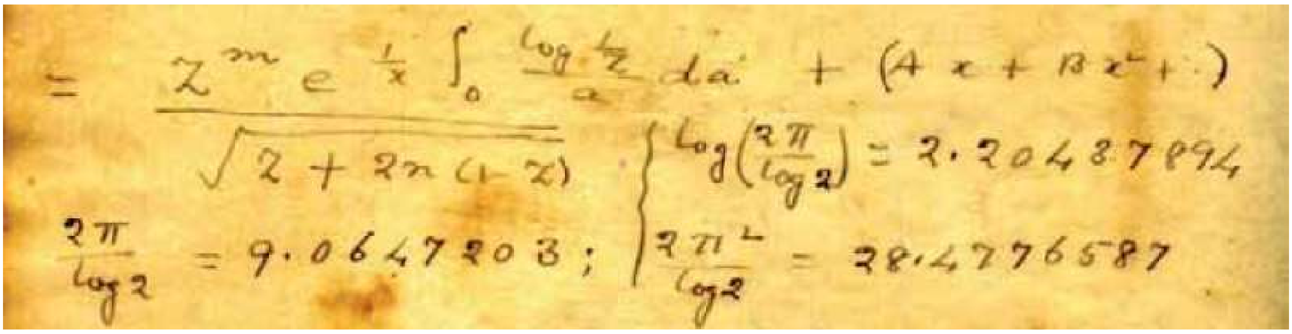
$= 73491.7883254... \Rightarrow$

$$\left(\begin{array}{c} I_{21} \ll \int_{-\infty}^{+\infty}\exp\left(-\left(\frac{t}{H}\right)^2\right)\left|\sum_{\lambda\leq P^{1-\varepsilon_2}}\frac{a(\lambda)}{\sqrt{\lambda}}B(\lambda)\lambda^{-i(T+t)}\right|^2 dt \ll \\ \ll H\left\{\left(\frac{4}{\varepsilon_2\log T}\right)^{2r}(\log T)(\log X)^{-2\beta} + (\varepsilon_2^{-2r}(\log T)^{-2r} + \varepsilon_2^{-r}h_1^r(\log T)^{-r})T^{-\varepsilon_1}\right\}\end{array}\right)/$$

$$/(26\times 4)^2 - 24 = \left(\frac{7.9313976505275\times 10^8}{(26\times 4)^2 - 24}\right) = 73493.30662\ldots$$

Mathematical connections with the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of u → ∞, with the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and with the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

Now, we have that:



$\log\left(\frac{2\pi}{\log 2}\right) = 2.20487894$

$\frac{2\pi}{\log 2} = 9.0647203$; $\frac{2\pi^2}{\log 2} = 28.4776587$

ln(2Pi/ln2)

Input:

$$\log\left(2\times\frac{\pi}{\log(2)}\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\log\left(\frac{2\pi}{\log(2)}\right)$$

Decimal approximation:

2.204389986991009810573098631043904749177058395112672088687...

2.2043899869910….

Alternate form:

$$\log(2)+\log(\pi)-\log(\log(2))$$

Alternative representations:

$$\log\left(\frac{2\pi}{\log(2)}\right)=\log_e\left(\frac{2\pi}{\log(2)}\right)$$

$$\log\left(\frac{2\pi}{\log(2)}\right)=\log(a)\log_a\left(\frac{2\pi}{\log(2)}\right)$$

$$\log\left(\frac{2\pi}{\log(2)}\right)=-\text{Li}_1\left(1-\frac{2\pi}{\log(2)}\right)$$

Series representations:

$$\log\left(\frac{2\pi}{\log(2)}\right)=\log\left(-1+\frac{2\pi}{\log(2)}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{\log(2)}{2\pi-\log(2)}\right)^k}{k}$$

$$\log\left(\frac{2\pi}{\log(2)}\right) = 2 i \pi \left\lfloor \frac{\arg\left(-x + \frac{2\pi}{\log(2)}\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} \left(-x + \frac{2\pi}{\log(2)}\right)^k}{k} \quad \text{for } x < 0$$

$$\log\left(\frac{2\pi}{\log(2)}\right) = 2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{2\pi}{\log(2)} - z_0\right)^k z_0^{-k}}{k}$$

Integral representations:

$$\log\left(\frac{2\pi}{\log(2)}\right) = \int_1^{\frac{2\pi}{\log(2)}} \frac{1}{t}\, dt$$

$$\log\left(\frac{2\pi}{\log(2)}\right) = -\frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) \left(-1 + \frac{2\pi}{\log(2)}\right)^{-s}}{\Gamma(1-s)}\, ds \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

(2Pi^2/ln2)

Input:

$$2 \times \frac{\pi^2}{\log(2)}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{2\pi^2}{\log(2)}$$

Decimal approximation:

28.477658649975010867721351422733690893640556875329304062 90...

28.477658649…

Alternative representations:

$$\frac{2\pi^2}{\log(2)} = \frac{2\pi^2}{\log_e(2)}$$

$$\frac{2\pi^2}{\log(2)} = \frac{2\pi^2}{\log(a)\log_a(2)}$$

$$\frac{2\pi^2}{\log(2)} = \frac{2\pi^2}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi^2}{\log(2)} = \frac{2\pi^2}{2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi^2}{\log(2)} = \frac{2\pi^2}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi^2}{\log(2)} = \frac{2\pi^2}{2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi^2}{\log(2)} = \frac{2\pi^2}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi^2}{\log(2)} = \frac{4 i \pi^3}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

(2Pi/ln2)

Input:

$$2 \times \frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.064720283654387619255365891433333620343722935447591168372...

9.06472028…

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor + \log(z_0) - \sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\,dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4i\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1 < \gamma < 0$$

Now, we have that:

ln(2Pi/ln2) * (2Pi^2/ln2) * (2Pi/ln2)

Input:

$$\log\left(2\times\frac{\pi}{\log(2)}\right)\left(2\times\frac{\pi^2}{\log(2)}\right)\left(2\times\frac{\pi}{\log(2)}\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{4\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)}$$

Decimal approximation:

569.0456620556244658364918972442354124629248429568863086987...

569.045662…

Alternate forms:

$$\frac{4\pi^3(\log(2)+\log(\pi)-\log(\log(2)))}{\log^2(2)}$$

$$\frac{4\pi^3\log(\pi)}{\log^2(2)}-\frac{4\pi^3\log(\log(2))}{\log^2(2)}+\frac{4\pi^3}{\log(2)}$$

Alternative representations:

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)2\pi^2}{\log(2)\log(2)}=4\pi\log_e\left(\frac{2\pi}{\log(2)}\right)\pi^2\left(\frac{1}{\log_e(2)}\right)^2$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)2\pi^2}{\log(2)\log(2)}=4\pi\log(a)\log_a\left(\frac{2\pi}{\log(2)}\right)\pi^2\left(\frac{1}{\log(a)\log_a(2)}\right)^2$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)2\pi^2}{\log(2)\log(2)}=-4\pi\,\mathrm{Li}_1\left(1-\frac{2\pi}{\log(2)}\right)\pi^2\left(-\frac{1}{\mathrm{Li}_1(-1)}\right)^2$$

Series representations:

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)2\pi^2}{\log(2)\log(2)}=\frac{4\pi^3\left(-2i\pi\left\lfloor\frac{\arg\left(-x+\frac{2\pi}{\log(2)}\right)}{2\pi}\right\rfloor-\log(x)+\sum_{k=1}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\frac{2\pi}{\log(2)}\right)^k}{k}\right)}{\left(2\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor-i\log(x)+i\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\right)^2}$$

for $x<0$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)2\pi^2}{\log(2)\log(2)} = \frac{4\pi^3\left(-2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-\log(z_0)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2\pi}{\log(2)}-z_0\right)^k z_0^{-k}}{k}\right)}{\left(2\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-i\log(z_0)+i\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^2}$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)2\pi^2}{\log(2)\log(2)} = \frac{4\pi^3\left(\left\lfloor\frac{\arg\left(\frac{2\pi}{\log(2)}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg\left(\frac{2\pi}{\log(2)}-z_0\right)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2\pi}{\log(2)}-z_0\right)^k z_0^{-k}}{k}\right)}{\left(\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^2}$$

Integral representations:

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)2\pi^2}{\log(2)\log(2)} = \frac{4\pi^3\int_1^{\frac{2\pi}{\log(2)}}\frac{1}{t}\,dt}{\left(\int_1^2\frac{1}{t}\,dt\right)^2}$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)2\pi^2}{\log(2)\log(2)} = \frac{8i\pi^4\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\Gamma(1+s)\left(-1+\frac{2\pi}{\log(2)}\right)^{-s}}{\Gamma(1-s)}\,ds}{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2} \quad \text{for } -1<\gamma<0$$

(((2*(((ln(2Pi/ln2) * (2Pi^2/ln2) * (2Pi/ln2)))))))^1/14

Input:

$$\sqrt[14]{2\left(\log\left(2\times\frac{\pi}{\log(2)}\right)\right)\left(2\times\frac{\pi^2}{\log(2)}\right)\left(2\times\frac{\pi}{\log(2)}\right)}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{(2\pi)^{3/14}\sqrt[14]{\log\left(\frac{2\pi}{\log(2)}\right)}}{\sqrt[7]{\log(2)}}$$

Decimal approximation:

1.653097104485619556424528909360107223893861476019894811244...

1.6530971044…. is very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164,2696 i.e. 1,65578...

Alternate form:

$$\frac{(2\pi)^{3/14}\sqrt[14]{\log(2)+\log(\pi)-\log(\log(2))}}{\sqrt[7]{\log(2)}}$$

All 14th roots of (8 π^3 log((2 π)/log(2)))/(log^2(2)):

$$\frac{(2\pi)^{3/14}\, e^{0}\sqrt[14]{\log\left(\frac{2\pi}{\log(2)}\right)}}{\sqrt[7]{\log(2)}} \approx 1.6531 \text{ (real, principal root)}$$

$$\frac{(2\pi)^{3/14}\, e^{(i\pi)/7}\sqrt[14]{\log\left(\frac{2\pi}{\log(2)}\right)}}{\sqrt[7]{\log(2)}} \approx 1.4894 + 0.7173\, i$$

$$\frac{(2\pi)^{3/14}\, e^{(2i\pi)/7}\sqrt[14]{\log\left(\frac{2\pi}{\log(2)}\right)}}{\sqrt[7]{\log(2)}} \approx 1.0307 + 1.2924\, i$$

$$\frac{(2\pi)^{3/14}\, e^{(3i\pi)/7}\sqrt[14]{\log\left(\frac{2\pi}{\log(2)}\right)}}{\sqrt[7]{\log(2)}} \approx 0.36785 + 1.6117\, i$$

$$\frac{(2\pi)^{3/14}\, e^{(4i\pi)/7}\sqrt[14]{\log\left(\frac{2\pi}{\log(2)}\right)}}{\sqrt[7]{\log(2)}} \approx -0.3678 + 1.6117\, i$$

Alternative representations:

$$\sqrt[14]{\frac{2\log\left(\frac{2\pi}{\log(2)}\right)\left((2\pi^2)(2\pi)\right)}{\log(2)\log(2)}} = \sqrt[14]{8\pi\log_e\left(\frac{2\pi}{\log(2)}\right)\pi^2\left(\frac{1}{\log_e(2)}\right)^2}$$

$$\sqrt[14]{\frac{2\log\left(\frac{2\pi}{\log(2)}\right)\left((2\pi^2)(2\pi)\right)}{\log(2)\log(2)}} = \sqrt[14]{8\pi\log(a)\log_a\left(\frac{2\pi}{\log(2)}\right)\pi^2\left(\frac{1}{\log(a)\log_a(2)}\right)^2}$$

$$\sqrt[14]{\frac{2\log\left(\frac{2\pi}{\log(2)}\right)\left((2\pi^2)(2\pi)\right)}{\log(2)\log(2)}} = \sqrt[14]{-8\pi\,\text{Li}_1\left(1-\frac{2\pi}{\log(2)}\right)\pi^2\left(-\frac{1}{\text{Li}_1(-1)}\right)^2}$$

Series representations:

$$\sqrt[14]{\frac{2\log\left(\frac{2\pi}{\log(2)}\right)\left((2\pi^2)(2\pi)\right)}{\log(2)\log(2)}} = \frac{(2\pi)^{3/14}\sqrt[14]{2i\pi\left\lfloor\frac{\arg\left(-x+\frac{2\pi}{\log(2)}\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\frac{2\pi}{\log(2)}\right)^k}{k}}}{\sqrt[7]{2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}}} \quad \text{for } x<0$$

$$\sqrt[14]{\frac{2\log\left(\frac{2\pi}{\log(2)}\right)\left((2\pi^2)(2\pi)\right)}{\log(2)\log(2)}} = \frac{(2\pi)^{3/14}\sqrt[14]{2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2\pi}{\log(2)}-z_0\right)^k z_0^{-k}}{k}}}{\sqrt[7]{2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}}}$$

$$\sqrt[14]{\frac{2\log\left(\frac{2\pi}{\log(2)}\right)\left((2\pi^2)(2\pi)\right)}{\log(2)\log(2)}} = \left((2\pi)^{3/14}\left(\left\lfloor\frac{\arg\left(\frac{2\pi}{\log(2)}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg\left(\frac{2\pi}{\log(2)}-z_0\right)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2\pi}{\log(2)}-z_0\right)^k z_0^{-k}}{k}\right)^{\wedge}(1/14)\right)\Big/$$
$$\left(\sqrt[7]{\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}}\right)$$

Integral representations:

$$\sqrt[14]{\frac{2\log\left(\frac{2\pi}{\log(2)}\right)\left(\left(2\pi^2\right)(2\pi)\right)}{\log(2)\log(2)}} = \frac{(2\pi)^{3/14}\sqrt[14]{\int_1^{\frac{2\pi}{\log(2)}}\frac{1}{t}\,dt}}{\sqrt[7]{\int_1^2\frac{1}{t}\,dt}}$$

$$\sqrt[14]{\frac{2\log\left(\frac{2\pi}{\log(2)}\right)\left(\left(2\pi^2\right)(2\pi)\right)}{\log(2)\log(2)}} = \frac{i(2\pi)^{2/7}\left(-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^{6/7}\sqrt[14]{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\Gamma(1+s)\left(-1+\frac{2\pi}{\log(2)}\right)^{-s}}{\Gamma(1-s)}\,ds}}{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\Gamma(1+s)}{\Gamma(1-s)}\,ds} \text{ for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

1/10^27 * (((((18/10^3+(((2*(((ln(2Pi/ln2) * (2Pi^2/ln2) * (2Pi/ln2)))))))^1/14)))))

Input:

$$\frac{1}{10^{27}}\left(\frac{18}{10^3}+\sqrt[14]{2\left(\log\left(2\times\frac{\pi}{\log(2)}\right)\left(2\times\frac{\pi^2}{\log(2)}\right)\left(2\times\frac{\pi}{\log(2)}\right)\right)}\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\frac{9}{500}+\frac{(2\pi)^{3/14}\sqrt[14]{\log\left(\frac{2\pi}{\log(2)}\right)}}{\sqrt[7]{\log(2)}}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

Decimal approximation:

$1.671097104485619556424528909360107223893861476019894 8\ldots\times10^{-27}$

$1.6710971044\ldots*10^{-27}$

We note that 1.6710971044... is a result practically equal to the value of the formula:

$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-24}$ gm

that is the holographic proton mass (N. Haramein)

Alternate forms:

$$\frac{9}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{\pi^{3/14} \sqrt[14]{\log\left(\frac{2\pi}{\log(2)}\right)}}{500\,000\,000\,000\,000\,000\,000\,000\,000 \times 2^{11/14} \sqrt[7]{\log(2)}}$$

$$\frac{\frac{9}{500} + \frac{(2\pi)^{3/14} \sqrt[14]{\log(2)+\log(\pi)-\log(\log(2))}}{\sqrt[7]{\log(2)}}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{9 \sqrt[7]{\log(2)} + 500\,(2\pi)^{3/14} \sqrt[14]{\log\left(\frac{2\pi}{\log(2)}\right)}}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000 \sqrt[7]{\log(2)}}$$

Alternative representations:

$$\frac{\frac{18}{10^3} + \sqrt[14]{\frac{2\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)}{\log(2)\log(2)}}}{10^{27}} = \frac{\frac{18}{10^3} + \sqrt[14]{8\pi \log_e\left(\frac{2\pi}{\log(2)}\right)\pi^2 \left(\frac{1}{\log_e(2)}\right)^2}}{10^{27}}$$

$$\frac{\frac{18}{10^3} + \sqrt[14]{\frac{2\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)}{\log(2)\log(2)}}}{10^{27}} = \frac{\frac{18}{10^3} + \sqrt[14]{8\pi \log(a) \log_a\left(\frac{2\pi}{\log(2)}\right)\pi^2 \left(\frac{1}{\log(a)\log_a(2)}\right)^2}}{10^{27}}$$

$$\frac{\frac{18}{10^3} + \sqrt[14]{\frac{2\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)}{\log(2)\log(2)}}}{10^{27}} = \frac{\frac{18}{10^3} + \sqrt[14]{-8\pi \operatorname{Li}_1\left(1-\frac{2\pi}{\log(2)}\right)\pi^2 \left(-\frac{1}{\operatorname{Li}_1(-1)}\right)^2}}{10^{27}}$$

Series representations:

$$\frac{\frac{18}{10^3}+\sqrt[14]{\frac{2\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)}{\log(2)\log(2)}}}{10^{27}} =$$

$$\left(9\sqrt[7]{2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} + 500 (2\pi)^{3/14} \sqrt[14]{2 i \pi \left\lfloor \frac{\arg\left(-x+\frac{2\pi}{\log(2)}\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} \left(-x+\frac{2\pi}{\log(2)}\right)^k}{k}}\right) \Big/ \left(500\,000\,000\,000\,000\,000\,000\,000\,000\,000 \sqrt[7]{2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}}\right) \text{ for } x < 0$$

$$\frac{\frac{18}{10^3}+\sqrt[14]{\frac{2\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)}{\log(2)\log(2)}}}{10^{27}} =$$

$$\left(9\sqrt[7]{2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}} + 500 (2\pi)^{3/14} \sqrt[14]{2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{2\pi}{\log(2)} - z_0\right)^k z_0^{-k}}{k}}\right) \Big/ \left(500\,000\,000\,000\,000\,000\,000\,000\,000\,000 \sqrt[7]{2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}\right)$$

$$\frac{\frac{18}{10^3} + \sqrt[14]{\frac{2\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)}{\log(2)\log(2)}}}{10^{27}} =$$

$$\left(9\left(\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)\wedge\right.$$
$$(1/7)+500(2\pi)^{3/14}\left(\left\lfloor\frac{\arg\left(\frac{2\pi}{\log(2)}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\right.$$
$$\left.\left.\left\lfloor\frac{\arg\left(\frac{2\pi}{\log(2)}-z_0\right)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2\pi}{\log(2)}-z_0\right)^k z_0^{-k}}{k}\right)\wedge(1/14)\right)\Big/$$
$$\left(500\,000\,000\,000\,000\,000\,000\,000\,000\,000\sqrt[7]{\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}}\right)$$

Integral representations:

$$\frac{\frac{18}{10^3} + \sqrt[14]{\frac{2\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)}{\log(2)\log(2)}}}{10^{27}} = \frac{9\sqrt[7]{\int_1^2\frac{1}{t}\,dt}+500(2\pi)^{3/14}\sqrt[14]{\int_1^{\frac{2\pi}{\log(2)}}\frac{1}{t}\,dt}}{500\,000\,000\,000\,000\,000\,000\,000\,000\,000\sqrt[7]{\int_1^2\frac{1}{t}\,dt}}$$

$$\frac{\frac{18}{10^3} + \sqrt[14]{\frac{2\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)}{\log(2)\log(2)}}}{10^{27}} =$$

$$\left(9\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds+500\,i\,(2\pi)^{2/7}\left(-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^{6/7}\right.$$
$$\left.\sqrt[14]{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\frac{2\pi}{\log(2)}\right)^{-s}}{\Gamma(1-s)}\,ds}\right)\Big/$$
$$\left(500\,000\,000\,000\,000\,000\,000\,000\,000\,000\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\quad\text{for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

1/9*[(((ln(2Pi/ln2))))^4 + ((((2Pi/ln2))))^4 + (2Pi^2/ln2)^4] - (21*8*2)

Input:

$$\frac{1}{9}\left(\log^4\left(2\times\frac{\pi}{\log(2)}\right)+\left(2\times\frac{\pi}{\log(2)}\right)^4+\left(2\times\frac{\pi^2}{\log(2)}\right)^4\right)-21\times 8\times 2$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{1}{9}\left(\frac{16\,\pi^4}{\log^4(2)}+\frac{16\,\pi^8}{\log^4(2)}+\log^4\left(\frac{2\,\pi}{\log(2)}\right)\right)-336$$

Decimal approximation:

73492.79399207621478061723189938204922675911283263122564600...

73492.793992…

Alternate forms:

$$\frac{1}{9}\left(-3024+\frac{16\,(\pi^4+\pi^8)}{\log^4(2)}+\log^4\left(\frac{2\,\pi}{\log(2)}\right)\right)$$

$$-336+\frac{16\,\pi^4}{9\log^4(2)}+\frac{16\,\pi^8}{9\log^4(2)}+\frac{1}{9}\log^4\left(\frac{2\,\pi}{\log(2)}\right)$$

$$\frac{16\,\pi^4+16\,\pi^8+\log^4(2)\log^4\left(\frac{2\,\pi}{\log(2)}\right)}{9\log^4(2)}-336$$

Alternative representations:

$$\frac{1}{9}\left(\log^4\left(\frac{2\,\pi}{\log(2)}\right)+\left(\frac{2\,\pi}{\log(2)}\right)^4+\left(\frac{2\,\pi^2}{\log(2)}\right)^4\right)-21\,(8\times 2)=$$
$$-336+\frac{1}{9}\left(\log_e^4\left(\frac{2\,\pi}{\log(2)}\right)+\left(\frac{2\,\pi}{\log_e(2)}\right)^4+\left(\frac{2\,\pi^2}{\log_e(2)}\right)^4\right)$$

$$\frac{1}{9}\left(\log^4\left(\frac{2\,\pi}{\log(2)}\right)+\left(\frac{2\,\pi}{\log(2)}\right)^4+\left(\frac{2\,\pi^2}{\log(2)}\right)^4\right)-21\,(8\times 2)=$$
$$-336+\frac{1}{9}\left(\left(\log(a)\log_a\left(\frac{2\,\pi}{\log(2)}\right)\right)^4+\left(\frac{2\,\pi}{\log(a)\log_a(2)}\right)^4+\left(\frac{2\,\pi^2}{\log(a)\log_a(2)}\right)^4\right)$$

$$\frac{1}{9}\left(\log^4\left(\frac{2\,\pi}{\log(2)}\right)+\left(\frac{2\,\pi}{\log(2)}\right)^4+\left(\frac{2\,\pi^2}{\log(2)}\right)^4\right)-21\,(8\times 2)=$$
$$-336+\frac{1}{9}\left(\left(-\text{Li}_1\left(1-\frac{2\,\pi}{\log(2)}\right)\right)^4+\left(-\frac{2\,\pi}{\text{Li}_1(-1)}\right)^4+\left(-\frac{2\,\pi^2}{\text{Li}_1(-1)}\right)^4\right)$$

Series representations:

$$\frac{1}{9}\left(\log^4\left(\frac{2\pi}{\log(2)}\right)+\left(\frac{2\pi}{\log(2)}\right)^4+\left(\frac{2\pi^2}{\log(2)}\right)^4\right)-21\,(8\times 2)=$$

$$-336+\frac{1}{9}\left(\frac{16\pi^4}{\left(2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\right)^4}+\frac{16\pi^8}{\left(2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\right)^4}+\left(2i\pi\left\lfloor\frac{\arg\left(-x+\frac{2\pi}{\log(2)}\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\frac{2\pi}{\log(2)}\right)^k}{k}\right)^4\right)\quad\text{for } x<0$$

$$\frac{1}{9}\left(\log^4\left(\frac{2\pi}{\log(2)}\right)+\left(\frac{2\pi}{\log(2)}\right)^4+\left(\frac{2\pi^2}{\log(2)}\right)^4\right)-21\,(8\times 2)=$$

$$-336+\frac{1}{9}\left(\frac{16\pi^4}{\left(\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^4}+\frac{16\pi^8}{\left(\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^4}+\left(\log(z_0)+\left\lfloor\frac{\arg\left(\frac{2\pi}{\log(2)}-z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2\pi}{\log(2)}-z_0\right)^k z_0^{-k}}{k}\right)^4\right)$$

$$\frac{1}{9}\left(\log^4\left(\frac{2\pi}{\log(2)}\right)+\left(\frac{2\pi}{\log(2)}\right)^4+\left(\frac{2\pi^2}{\log(2)}\right)^4\right)-21\,(8\times 2)=$$

$$-336+\frac{1}{9}\left(\frac{16\pi^4}{\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^4}+\frac{16\pi^8}{\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^4}+\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2\pi}{\log(2)}-z_0\right)^k z_0^{-k}}{k}\right)^4\right)$$

We have the following mathematical connection:

$$\left(\frac{1}{9}\left(\frac{16\pi^4}{\log^4(2)}+\frac{16\pi^8}{\log^4(2)}+\log^4\left(\frac{2\pi}{\log(2)}\right)\right)-336\right) = 73492.793\ldots \Rightarrow$$

$$\Rightarrow -3927+2\left(\sqrt[13]{\frac{N\exp\left[\int d\hat{\sigma}\left(-\frac{1}{4u^2}\mathbf{P}_i D\mathbf{P}_i\right)\right]|Bp\rangle_{NS}}{\int[d\mathbf{X}^\mu]\exp\left\{\int d\hat{\sigma}\left(-\frac{1}{4v^2}D\mathbf{X}^\mu D^2\mathbf{X}^\mu\right)\right\}|\mathbf{X}^\mu,\mathbf{X}^i=0\rangle_{NS}}+}\right)=$$

$$-3927+2\sqrt[13]{2.2983717437\times10^{59}+2.0823329825883\times10^{59}}$$

$= 73490.8437525.... \Rightarrow$

$$\Rightarrow\left(A(r)\times\frac{1}{B(r)}\left(-\frac{1}{\phi(r)}\right)\times\frac{1}{e^{\Lambda(r)}}\right)\Rightarrow$$

$$\Rightarrow\left(-0.000029211892\times\frac{1}{0.0003644621}\left(-\frac{1}{0.0005946833}\right)\times\frac{1}{0.00183393}\right)=$$

$= 73491.788325481187105491595720422205480251957265634133987 00...$

$= 73491.7883254... \Rightarrow$

$$\left(\begin{array}{c} I_{21}\ll\int_{-\infty}^{+\infty}\exp\left(-\left(\frac{t}{H}\right)^2\right)\left|\sum_{\lambda\leqslant P^{1-\varepsilon_2}}\frac{a(\lambda)}{\sqrt{\lambda}}B(\lambda)\lambda^{-i(T+t)}\right|^2 dt\ll \\ \ll H\left\{\left(\frac{4}{\varepsilon_2\log T}\right)^{2r}(\log T)(\log X)^{-2\beta}+(\varepsilon_2^{-2r}(\log T)^{-2r}+\varepsilon_2^{-r}h_1^r(\log T)^{-r})T^{-\varepsilon_1}\right\}\end{array}\right)/$$

$$/(26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = 73493.30662\ldots$$

Mathematical connections with the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of u → ∞, with the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and with the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

The result 569.0456… is very near to 566 that is the value of a(n) for n = 142 of a 5th order mock theta function.

The formula of mock theta function is:

a(n) ≈ sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n))

sqrt(golden ratio) * exp(Pi*sqrt(142.36/15)) / (2*5^(1/4)*sqrt(142.36))

Input interpretation:

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{142.36}{15}}\right)}{2 \sqrt[4]{5} \sqrt{142.36}}$$

ϕ is the golden ratio

Result:

569.18234407420948635569472150857601093490468713356929833 89…

569.182344074…

Series representations:

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{142.36}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{142.36}} = \frac{\exp\left(\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(9.49067-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}}{2\sqrt[4]{5}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(142.36-z_0)^k z_0^{-k}}{k!}}$$

for not $((z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0))$

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{142.36}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{142.36}} = \left(\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\right.$$
$$\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg(9.49067-x)}{2\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k(9.49067-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$
$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Bigg/$$
$$\left(2\sqrt[4]{5}\,\exp\left(i\pi\left\lfloor\frac{\arg(142.36-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(142.36-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{142.36}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{142.36}} = \left(\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(9.49067-z_0)/(2\pi)\rfloor}\right.\right.$$
$$\left.z_0^{1/2(1+\lfloor\arg(9.49067-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(9.49067-z_0)^k z_0^{-k}}{k!}\right)$$
$$\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(142.36-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}$$
$$\left.z_0^{-1/2\lfloor\arg(142.36-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\Bigg/$$
$$\left(2\sqrt[4]{5}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(142.36-z_0)^k z_0^{-k}}{k!}\right)$$

We have the following mathematical connection:

$$\left[\frac{4\pi^3 \log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)}\right] = 569.0456\ldots \Rightarrow$$

$$\Rightarrow \left[\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{142.36}{15}}\right)}{2\sqrt[4]{5}\sqrt{142.36}}\right] = 569.18234\ldots$$

From the two following results: 1.6710971044…*10^{-27} that represent the proton mass, thence a like-particle solution and 73492.793992…, that is the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane, we obtain a solution very near to the dilaton value:

[((((((1/10^27 * (((((18/10^3+(((2*(((ln(2Pi/ln2) * (2Pi^2/ln2) * (2Pi/ln2)))))))^1/14)))))))))) * 73492.793992]^1/4096

Input interpretation:

$$\sqrt[4096]{\left(\frac{1}{10^{27}}\left(\frac{18}{10^3}+\sqrt[14]{2\left(\log\left(2\times\frac{\pi}{\log(2)}\right)\left(2\times\frac{\pi^2}{\log(2)}\right)\left(2\times\frac{\pi}{\log(2)}\right)\right)}\right)\right)\times 73492.793992}$$

log(x) is the natural logarithm

Result:

0.987758316480298...

0.9877583… result very near to the dilaton value $\mathbf{0.989117352243} = \boldsymbol{\phi}$ (see Appendix)

And:

sqrt((((log base 0.98775831648 [((((((1/10^27*(((((18/10^3+(((2*(((ln(2Pi/ln2)*(2Pi^2/ln2)*(2Pi/ln2)))))))^1/14)))))))))))*73492.793992]))))

Input interpretation:

$$\sqrt{\log_{0.98775831648}\left(\left(\frac{1}{10^{27}}\left(\frac{18}{10^3}+\sqrt[14]{2\left(\log\left(2\times\frac{\pi}{\log(2)}\right)\left(2\times\frac{\pi^2}{\log(2)}\right)\left(2\times\frac{\pi}{\log(2)}\right)\right)}\right)\right)\times 73492.793992\right)}$$

$\log(x)$ is the natural logarithm

$\log_b(x)$ is the base- b logarithm

Result:

64.0000000...

64 (see Appendix)

All 2nd roots of 4096.00000:

$64.0000000\, e^0 \approx 64.000$ (real, principal root)

$64.0000000\, e^{i\pi} \approx -64.000$ (real root)

Alternative representations:

$$\sqrt{\log_{0.987758316480000}\left(\frac{73492.7939920000\left(\frac{18}{10^3}+\sqrt[14]{\frac{2\left(\log\left(\frac{2\pi}{\log(2)}\right)\left(2\pi^2\right)(2\pi)\right)}{\log(2)\log(2)}}\right)}{10^{27}}\right)} = \sqrt{\frac{\log\left(\frac{73492.7939920000\left(\frac{18}{10^3}+\sqrt[14]{8\pi\log\left(\frac{2\pi}{\log(2)}\right)\pi^2\left(\frac{1}{\log(2)}\right)^2}\right)}{10^{27}}\right)}{\log(0.987758316480000)}}$$

$$\sqrt{\log_{0.987758316480000}\left(\frac{73492.7939920000\left(\frac{18}{10^3}+\sqrt[14]{\frac{2\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)}{\log(2)\log(2)}}\right)}{10^{27}}\right)} = \sqrt{\log_{0.987758316480000}\left(\frac{73492.7939920000\left(\frac{18}{10^3}+\sqrt[14]{8\pi\log_e\left(\frac{2\pi}{\log(2)}\right)\pi^2\left(\frac{1}{\log_e(2)}\right)^2}\right)}{10^{27}}\right)}$$

$$\sqrt{\log_{0.987758316480000}\left(\frac{73492.7939920000\left(\frac{18}{10^3}+\sqrt[14]{\frac{2\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)}{\log(2)\log(2)}}\right)}{10^{27}}\right)} = \sqrt{\log_{0.987758316480000}\left(\frac{73492.7939920000\left(\frac{18}{10^3}+\sqrt[14]{8\pi\log(a)\log_a\left(\frac{2\pi}{\log(2)}\right)\pi^2\left(\frac{1}{\log(a)\log_a(2)}\right)^2}\right)}{10^{27}}\right)}$$

Series representations:

$$\sqrt{\log_{0.987758316480000}\left(\frac{73492.7939920000\left(\frac{18}{10^3}+\sqrt[14]{\frac{2\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)}{\log(2)\log(2)}}\right)}{10^{27}}\right)} =$$

$$\exp\left(i\pi\left\lfloor\frac{1}{2\pi}\arg\left(-x+\log_{0.987758316480000}\left(1.32287029185600\times10^{-24}+8.52611499811343\times10^{-23}\sqrt[14]{\frac{\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)}}\right)\right)\right\rfloor\right)\sqrt{x}$$

$$\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^k x^{-k}\left(-x+\log_{0.987758316480000}\left(1.32287029185600\times10^{-24}+8.52611499811343\times10^{-23}\sqrt[14]{\frac{\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)}}\right)\right)^k$$

$$\left(-\frac{1}{2}\right)_k \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\sqrt{\log_{0.987758316480000}\left(\frac{73492.7939920000\left(\frac{18}{10^3}+\sqrt[14]{\frac{2\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)}{\log(2)\log(2)}}\right)}{10^{27}}\right)} =$$

$$\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\log_{0.987758316480000}\left(1.32287029185600\times10^{-24}+8.52611499811343\times10^{-23}\sqrt[14]{\frac{\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)}}\right)-z_0\right)/(2\pi)\right\rfloor}$$

$$z_0^{1/2\left(1+\left\lfloor\arg\left(\log_{0.987758316480000}\left(1.32287029185600\times10^{-24}+8.52611499811343\times10^{-23}\sqrt[14]{\frac{\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)}}\right)-z_0\right)/(2\pi)\right\rfloor\right)}$$

$$\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^k\left(-\frac{1}{2}\right)_k\left(\log_{0.987758316480000}\left(1.32287029185600\times10^{-24}+8.52611499811343\times10^{-23}\sqrt[14]{\frac{\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)}}\right)-z_0\right)^k z_0^{-k}$$

Integral representations:

$$\sqrt{\log_{0.987758316480000}\left(\frac{73492.7939920000\left(\frac{18}{10^3}+\sqrt[14]{\frac{2\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)}{\log(2)\log(2)}}\right)}{10^{27}}\right)} =$$

$$\sqrt{\log_{0.987758316480000}\left(1.32287029185600\times10^{-24}+8.52611499811343\times10^{-23}\sqrt[14]{\frac{\pi^3\int_1^{\frac{2\pi}{\log(2)}}\frac{1}{t}\,dt}{\left(\int_1^2\frac{1}{t}\,dt\right)^2}}\right)}$$

$$\sqrt{\log_{0.987758316480000}\left(\frac{73492.7939920000\left(\frac{18}{10^3}+\sqrt[14]{\frac{2\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)}{\log(2)\log(2)}}\right)}{10^{27}}\right)} =$$

$$\sqrt{\log_{0.987758316480000}\left(1.32287029185600\times10^{-24}+8.95887193618846\times10^{-23}\sqrt[14]{\frac{i\,\pi^4\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\frac{2\pi}{\log(2)}\right)^{-s}}{\Gamma(1-s)}\,ds}{\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2}}\right)} \quad \text{for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

Now, we have that:

$$\log 2\left\{e^{-x}+2e^{-2x}+4e^{-4x}+8e^{-8x}+\dots+1-\frac{x}{3\lfloor 1}+\frac{x^2}{7\lfloor 2}-\frac{x^3}{15\lfloor 3}+\frac{x^4}{31\lfloor 4}-\&c\right\} = \frac{1+\cdot 0000098844\cos\left(2\pi\frac{\log x}{\log 2}+\cdot 872811\right)}{x}$$

For x equal to the below formula:

$$x=\left(\frac{\log\frac{1+\sqrt{5}}{2}}{\pi}\right)^2$$

where we take this other version of it:

$$\left(\frac{\log\frac{1+\sqrt{5}}{2}}{\pi}\right)^2 = \left(\frac{\log 1.6180339887498}{\pi}\right)^2 = \left(\frac{0.4812118250596544828}{\pi}\right)^2$$
$$= (0.1531744812649979)^2 = 0.023462421710$$

We have from the inverse of result:

1/0.023462421710

Input interpretation:

$$\frac{1}{0.023462421710}$$

Result:

42.62134626852208258258264849847846332994753762781974989912...

42.621346268522....

Thence, we obtain:

(((((1+0.0000098844 cos((2Pi*ln0.023462422/(log2)+0.872811))))/0.023462422))))

Input interpretation:

$$\frac{1 + 9.8844\times 10^{-6} \cos\left(2\,\pi\times \frac{\log(0.023462422)}{\log(2)} + 0.872811\right)}{0.023462422}$$

log(x) is the natural logarithm

Result:

42.621281...

42.621281…

Addition formulas:

$$\frac{1+9.8844\times 10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624} = 42.6213 + 0.000421286\cos(0.872811)\cos\left(-\frac{2\pi\log(0.0234624)}{\log(2)}\right) + 0.000421286\sin(0.872811)\sin\left(-\frac{2\pi\log(0.0234624)}{\log(2)}\right)$$

$$\frac{1+9.8844\times 10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624} = 42.6213 + 0.000421286\cos(0.872811)\cos\left(\frac{2\pi\log(0.0234624)}{\log(2)}\right) - 0.000421286\sin(0.872811)\sin\left(\frac{2\pi\log(0.0234624)}{\log(2)}\right)$$

$$\frac{1+9.8844\times 10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624} = 42.6213 + 0.000421286\cosh\left(-\frac{2i\pi\log(0.0234624)}{\log(2)}\right)\cos(0.872811) - 0.000421286\, i\sinh\left(-\frac{2i\pi\log(0.0234624)}{\log(2)}\right)\sin(0.872811)$$

$$\frac{1+9.8844\times 10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624} = 42.6213 + 0.000421286\cosh\left(\frac{2i\pi\log(0.0234624)}{\log(2)}\right)\cos(0.872811) + 0.000421286\, i\sinh\left(\frac{2i\pi\log(0.0234624)}{\log(2)}\right)\sin(0.872811)$$

Alternative representations:

$$\frac{1+9.8844\times 10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624} = \frac{1+9.8844\times 10^{-6}\cosh\left(i\left(0.872811+\frac{2\pi\log(0.0234624)}{\log(2)}\right)\right)}{0.0234624}$$

$$\frac{1+9.8844\times 10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624} = \frac{1+9.8844\times 10^{-6}\cosh\left(-i\left(0.872811+\frac{2\pi\log(0.0234624)}{\log(2)}\right)\right)}{0.0234624}$$

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}=\frac{1}{0.0234624}\left(1+4.9422\times10^{-6}\left(e^{-i(0.872811+(2\pi\log(0.0234624))/\log(2))}+e^{i(0.872811+(2\pi\log(0.0234624))/\log(2))}\right)\right)$$

Series representations:

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}=$$
$$42.6213+0.000421286\sum_{k=0}^{\infty}\frac{(-1)^k\left(0.872811+\frac{2\pi\log(0.0234624)}{\log(2)}\right)^{2k}}{(2k)!}$$

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}=$$
$$42.6213-0.000421286\sum_{k=0}^{\infty}\frac{(-1)^k\left(0.872811+\pi\left(-\frac{1}{2}+\frac{2\log(0.0234624)}{\log(2)}\right)\right)^{1+2k}}{(1+2k)!}$$

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}=$$
$$42.6213+0.000421286\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left(0.872811+\frac{2\pi\log(0.0234624)}{\log(2)}-z_0\right)^k}{k!}$$

$n!$ is the factorial function

Integral representations:

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}=$$
$$42.6213-0.000421286\int_{\frac{\pi}{2}}^{0.872811+\frac{2\pi\log(0.0234624)}{\log(2)}}\sin(t)\,dt$$

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}=$$
$$42.6218+\int_0^1\frac{1}{\log(2)}(-0.000842573\,\pi\log(0.0234624)-0.000367703\log(2))\sin\left(t\left(0.872811+\frac{2\pi\log(0.0234624)}{\log(2)}\right)\right)dt$$

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}=$$

$$42.6213+\frac{0.000210643\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-\frac{(\pi\log(0.0234624)+0.436406\log(2))^2}{s\log^2(2)}}}{\sqrt{s}}\,ds \quad \text{for } \gamma>0$$

Performing the following calculations, we obtain:

(((((1+0.0000098844 cos((2Pi*ln0.023462422/(log2)+0.872811))))/0.023462422))))^3 - (64^2 - 64*3 + 64/2)

Input interpretation:

$$\left(\frac{1+9.8844\times10^{-6}\cos\left(2\pi\times\frac{\log(0.023462422)}{\log(2)}+0.872811\right)}{0.023462422}\right)^3-\left(64^2-64\times3+\frac{64}{2}\right)$$

log(x) is the natural logarithm

Result:

73488.69...

73488.69…

Addition formulas:

$$\left(\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}\right)^3-\left(64^2-64\times3+\frac{64}{2}\right)=$$
$$-3936+77425.\left(1+9.8844\times10^{-6}\cos(0.872811)\cos\left(-\frac{2\pi\log(0.0234624)}{\log(2)}\right)+9.8844\times10^{-6}\sin(0.872811)\sin\left(-\frac{2\pi\log(0.0234624)}{\log(2)}\right)\right)^3$$

$$\left(\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}\right)^3-\left(64^2-64\times3+\frac{64}{2}\right)=$$
$$-3936+7.47709\times10^{-11}\left(101170.+\cos(0.872811)\cos\left(\frac{2\pi\log(0.0234624)}{\log(2)}\right)-\sin(0.872811)\sin\left(\frac{2\pi\log(0.0234624)}{\log(2)}\right)\right)^3$$

$$\left(\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}\right)^3-\left(64^2-64\times3+\frac{64}{2}\right)=$$
$$-3936+77425.\left(1+9.8844\times10^{-6}\cosh\left(\frac{2i\pi\log(0.0234624)}{\log(2)}\right)\cos(0.872811)+9.8844\times10^{-6}\,i\sinh\left(\frac{2i\pi\log(0.0234624)}{\log(2)}\right)\sin(0.872811)\right)^3$$

$$\left(\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}\right)^3-\left(64^2-64\times3+\frac{64}{2}\right)=$$
$$-3936+7.47709\times10^{-11}\left(101170.+\cosh\left(-\frac{2i\pi\log(0.0234624)}{\log(2)}\right)\cos(0.872811)-i\left(\sinh\left(-\frac{2i\pi\log(0.0234624)}{\log(2)}\right)\sin(0.872811)\right)\right)^3$$

Alternative representations:

$$\left(\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}\right)^3-\left(64^2-64\times3+\frac{64}{2}\right)=$$
$$160-64^2+\left(\frac{1+9.8844\times10^{-6}\cosh\left(i\left(0.872811+\frac{2\pi\log(0.0234624)}{\log(2)}\right)\right)}{0.0234624}\right)^3$$

$$\left(\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}\right)^3-\left(64^2-64\times3+\frac{64}{2}\right)=$$
$$160-64^2+\left(\frac{1+9.8844\times10^{-6}\cosh\left(-i\left(0.872811+\frac{2\pi\log(0.0234624)}{\log(2)}\right)\right)}{0.0234624}\right)^3$$

$$\left(\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}\right)^3-\left(64^2-64\times3+\frac{64}{2}\right)=$$
$$160-64^2+\left(\frac{1}{0.0234624}\left(1+4.9422\times10^{-6}\left(e^{-i(0.872811+(2\pi\log(0.0234624))/\log(2))}+e^{i(0.872811+(2\pi\log(0.0234624))/\log(2))}\right)\right)\right)^3$$

Series representations:

$$\left(\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}\right)^3-\left(64^2-64\times3+\frac{64}{2}\right)=$$

$$-3936+77425.\left(1+9.8844\times10^{-6}\sum_{k=0}^{\infty}\frac{(-1)^k\left(0.872811+\frac{2\pi\log(0.0234624)}{\log(2)}\right)^{2k}}{(2k)!}\right)^3$$

$$\left(\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}\right)^3-\left(64^2-64\times3+\frac{64}{2}\right)=-3936+$$

$$77425.\left(1-9.8844\times10^{-6}\sum_{k=0}^{\infty}\frac{(-1)^k\left(0.872811+\pi\left(-\frac{1}{2}+\frac{2\log(0.0234624)}{\log(2)}\right)\right)^{1+2k}}{(1+2k)!}\right)^3$$

$$\left(\frac{1+9.8844\times10^{-6}\cos\left(\frac{(2\pi)\log(0.0234624)}{\log(2)}+0.872811\right)}{0.0234624}\right)^3-\left(64^2-64\times3+\frac{64}{2}\right)=-3936+$$

$$77425.\left(1+9.8844\times10^{-6}\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left(0.872811+\frac{2\pi\log(0.0234624)}{\log(2)}-z_0\right)^k}{k!}\right)^3$$

$n!$ is the factorial function

Thence, we have the following mathematical connections:

$$\left(\left(\frac{1+9.8844\times10^{-6}\cos\left(2\pi\times\frac{\log(0.023462422)}{\log(2)}+0.872811\right)}{0.023462422}\right)^3-\left(64^2-64\times3+\frac{64}{2}\right)\right)=73488.69\Rightarrow$$

$$\Rightarrow -3927+2\left(\sqrt[13]{\begin{array}{c}N\exp\left[\int d\hat{\sigma}\left(-\frac{1}{4u^2}\mathbf{P}_iD\mathbf{P}_i\right)\right]|Bp\rangle_{\rm NS}+\\ \int[d\mathbf{X}^\mu]\exp\left\{\int d\hat{\sigma}\left(-\frac{1}{4v^2}D\mathbf{X}^\mu D^2\mathbf{X}^\mu\right)\right\}|\mathbf{X}^\mu,\mathbf{X}^i=0\rangle_{\rm NS}\end{array}}\right)=$$

$$-3927+2\sqrt[13]{2.2983717437\times10^{59}+2.0823329825883\times10^{59}}$$

$= 73490.8437525.... \Rightarrow$

$$\Rightarrow \left(A(r) \times \frac{1}{B(r)}\left(-\frac{1}{\phi(r)}\right) \times \frac{1}{e^{\Lambda(r)}}\right) \Rightarrow$$

$$\Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621} \left(-\frac{1}{0.0005946833}\right) \times \frac{1}{0.00183393} \right) =$$

$= 73491.788325481187105491595720422205480251957265634133987 00...$

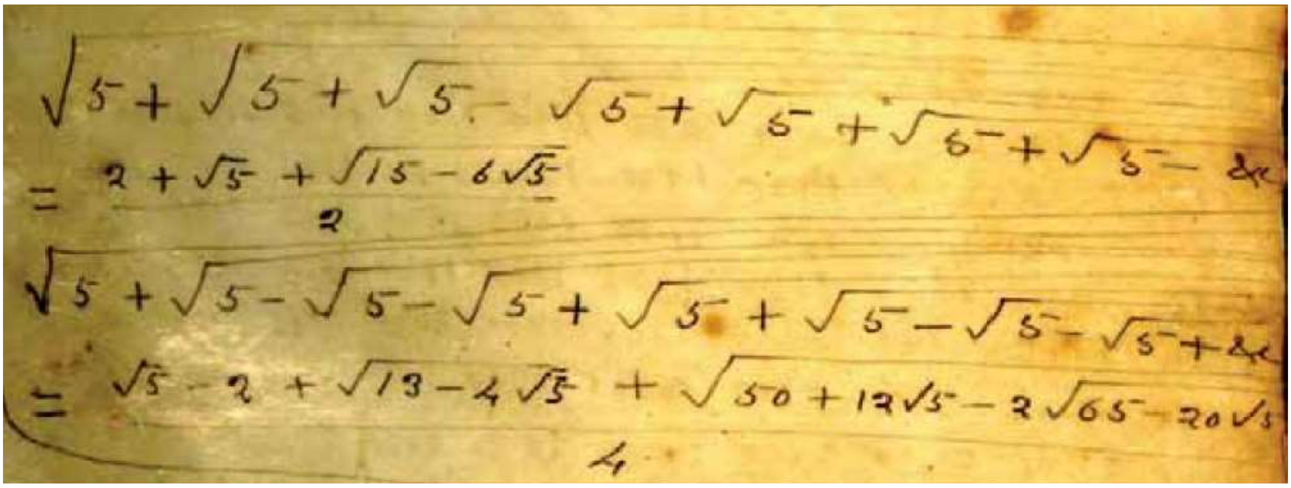
$= 73491.7883254... \Rightarrow$

$$\left(I_{21} \ll \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leq P^{1-\varepsilon_1}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \right.$$
$$\left. \ll H\left\{\left(\frac{4}{\varepsilon_2 \log T}\right)^{2r} (\log T)(\log X)^{-2\beta} + \left(\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r}\right) T^{-\varepsilon_1}\right\} \right) /$$

$$/(26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = 73493.30662\ldots$$

Mathematical connections with the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of u → ∞, with the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and with the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

Now, we have that:



$$\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5-\dots}}}}}}} = \frac{2+\sqrt{5}+\sqrt{15-6\sqrt{5}}}{2}$$

$$\sqrt{5+\sqrt{5-\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5-\sqrt{5+\dots}}}}}}}} = \frac{\sqrt{5}-2+\sqrt{13-4\sqrt{5}}+\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}}{4}$$

From the first formula, we obtain:

(((2+sqrt(5)+sqrt((15-6*sqrt(5))))))/2

Input:

$$\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)$$

Decimal approximation:

2.7472382749323043330574651861342028267581638787761679877 83...

2.7472382749323….

Alternate forms:

$$\frac{1}{2}\left(\sqrt{15-6\sqrt{5}}+\sqrt{5}\right)+1$$

$$\frac{1}{2}\left(2+\sqrt{5}+\sqrt{3\left(5-2\sqrt{5}\right)}\right)$$

$$1+\frac{\sqrt{5}}{2}+\frac{1}{2}\sqrt{15-6\sqrt{5}}$$

Minimal polynomial:

$$x^4-4x^3-4x^2+31x-29$$

We observe that from the square root of this expression, we obtain:

sqrt[(((2+sqrt(5)+sqrt((15-6*sqrt(5))))))/2]

Input:

$$\sqrt{\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)}$$

Decimal approximation:

1.65747949457370492474048304740677519034762309401832220 5669...

1.6574794945737…. is very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164,2696 i.e. 1,65578...

Alternate forms:

$$\frac{1}{2}\sqrt{\left(\sqrt{15-6\sqrt{5}}+\sqrt{5}+2\right)2}$$

$$\sqrt{\frac{1}{2}\left(2+\sqrt{5}+\sqrt{3\left(5-2\sqrt{5}\right)}\right)}$$

Minimal polynomial:

$$x^8-4x^6-4x^4+31x^2-29$$

Note that, from the 64th root of the inverse of this last result, we obtain:

((((1/(((sqrt[(((2+sqrt(5)+sqrt((15-6*sqrt(5))))))/2])))))))^1/64

Input:

$$\sqrt[64]{\frac{1}{\sqrt{\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)}}}$$

Result:

$$\sqrt[128]{\frac{2}{2+\sqrt{5}+\sqrt{15-6\sqrt{5}}}}$$

Decimal approximation:

0.992135803507096101450414990761542045073653305428180160362...

0.9921358035…. result very near to the dilaton value **0.989117352243** = ϕ (see Appendix)

Alternate form:

$$\sqrt[128]{\frac{2}{2+\sqrt{5}+\sqrt{3(5-2\sqrt{5})}}}$$

Minimal polynomial:

$$29x^{512}-31x^{384}+4x^{256}+4x^{128}-1$$

Now, performing the following calculations, we obtain:

24*[(((2+sqrt(5)+sqrt((15-6*sqrt(5))))))/2]^8-((64^2+(24*11+12)+8))

Input:

$$24\left(\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)\right)^8-(64^2+(24\times 11+12)+8)$$

Result:

$$\frac{3}{32}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)^8-4380$$

Decimal approximation:

73492.09699195285555876457006030735768486335147173325118542...

73492.09699…

Alternate forms:

$$\frac{1}{32}\left(478080\sqrt{3\left(85-38\sqrt{5}\right)}-180864\sqrt{5}+213696\sqrt{15\left(85-38\sqrt{5}\right)}+198528\sqrt{5\left(15-6\sqrt{5}\right)}+313152\sqrt{3\left(5-2\sqrt{5}\right)}+1519488\right)$$

$$12\left(3957-471\sqrt{5}+\sqrt{3\left(2286505+523582\sqrt{5}\right)}\right)$$

$$12\sqrt{3\left(5-2\sqrt{5}\right)}\left(1108+649\sqrt{5}\right)-36\left(157\sqrt{5}-1319\right)$$

Minimal polynomial:

$x^4 - 189\,936\,x^3 + 11\,233\,390\,176\,x^2 - 184\,735\,480\,018\,176\,x -$
$875\,005\,420\,177\,868\,544$

Thence, the following mathematical connections:

$$\left(\frac{3}{32}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)^8-4380\right)=73492.0969\ldots\Rightarrow$$

$$\Rightarrow -3927+2\left(\sqrt[13]{\begin{array}{c}N\exp\left[\int d\hat{\sigma}\left(-\frac{1}{4u^2}\mathbf{P}_i D\mathbf{P}_i\right)\right]|Bp\rangle_{\rm NS}+\\ \int[d\mathbf{X}^\mu]\exp\left\{\int d\hat{\sigma}\left(-\frac{1}{4v^2}D\mathbf{X}^\mu D^2\mathbf{X}^\mu\right)\right\}|\mathbf{X}^\mu,\mathbf{X}^i=0\rangle_{\rm NS}\end{array}}\right)=$$

$$-3927+2\sqrt[13]{2.2983717437\times10^{59}+2.0823329825883\times10^{59}}$$

$$=73490.8437525\ldots.\Rightarrow$$

$$\Rightarrow\left(A(r)\times\frac{1}{B(r)}\left(-\frac{1}{\phi(r)}\right)\times\frac{1}{e^{\Lambda(r)}}\right)\Rightarrow$$

$$\Rightarrow\left(-0.000029211892\times\frac{1}{0.0003644621}\left(-\frac{1}{0.0005946833}\right)\times\frac{1}{0.00183393}\right)=$$

$$=73491.788325481187105491595720422205480251957265634133987\ldots$$

$$=73491.7883254\ldots\Rightarrow$$

$$\left(\begin{array}{c} I_{21} \ll \int\limits_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leq P^{1-\varepsilon_1}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \\ \ll H \left\{ \left(\frac{4}{\varepsilon_2 \log T} \right)^{2r} (\log T)(\log X)^{-2\beta} + (\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r}) T^{-\varepsilon_1} \right\} \end{array} \right) /$$

$$/(26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = 73493.30662\ldots$$

Mathematical connections with the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of u → ∞, with the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and with the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

With regard 24, 8 and 11, they are numbers concerning the string theory/ M-theory.

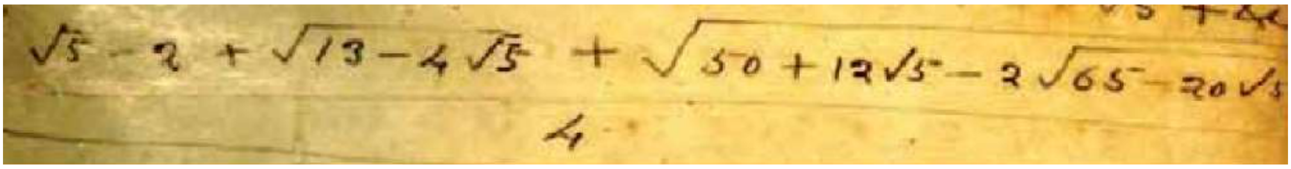
1968 "Veneziano model" Euler beta function describes the strong nuclear force. When a string moves in space-time by splitting and recombining (see worldsheet diagram at right), a large number of mathematical identities must be satisfied. These are the identities of Ramanujan's modular function. The KSV loop diagrams of interacting strings can be described using modular functions. **The "Ramanujan function"** (an elliptic modular function satisfies the need for "conformal symmetry") has **24** "modes" that correspond to the physical vibrations of a bosonic string. When the Ramanujan function is generalized, 24 is replaced by **8** (8 + 2 = 10) for fermion strings.

The **Ramanujan tau function**, studied by Ramanujan (1916), is the function:

$$\sum_{n\geq 1} \tau(n) q^n = q \prod_{n \geq 1} (1 - q^n)^{24} = \eta(z)^{24} = \Delta(z)$$

One notable feature of string theories is that these theories require extra dimensions of spacetime for their mathematical consistency. In bosonic string theory, spacetime is 26-dimensional (**24** + 2 = 26), while in superstring theory it is 10-dimensional (**8** + 2 = 10), and in M-theory it is 11-dimensional (8 + 2 + 1 = **11**)

From the second formula, we obtain:



((((sqrt(5)-2+((sqrt((13-4*sqrt(5))))+sqrt(((50+12*sqrt(5)-2*sqrt((65-20*sqrt(5))))))))))/4

Input:

$$\frac{1}{4}\left(\sqrt{5}-2+\left(\sqrt{13-4\sqrt{5}}+\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}\right)\right)$$

Result:

$$\frac{1}{4}\left(-2+\sqrt{5}+\sqrt{13-4\sqrt{5}}+\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}\right)$$

Decimal approximation:

2.621408383075861505698495280612243127797970614721167679664...

2.621408383…

Alternate forms:

$$\frac{1}{4}\left(\sqrt{13-4\sqrt{5}}+\sqrt{5}+\sqrt{-2\sqrt{5\left(13-4\sqrt{5}\right)}+12\sqrt{5}+50}-2\right)$$

$$\frac{1}{4}\left(-2+\sqrt{5}+\sqrt{13-4\sqrt{5}}+\sqrt{2\left(25+6\sqrt{5}-\sqrt{5\left(13-4\sqrt{5}\right)}\right)}\right)$$

root of $x^8+4x^7-10x^6-54x^5+9x^4+226x^3+125x^2-301x-269$ near $x=2.62141$

Minimal polynomial:

$$x^8+4x^7-10x^6-54x^5+9x^4+226x^3+125x^2-301x-269$$

sqrt[((((sqrt(5)-2+((sqrt((13-4*sqrt(5))))+sqrt(((50+12*sqrt(5)-2*sqrt((65-20*sqrt(5)))))))))))/4]

Input:

$$\sqrt{\frac{1}{4}\left(\sqrt{5}-2+\left(\sqrt{13-4\sqrt{5}}+\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}\right)\right)}$$

Result:

$$\frac{1}{2}\sqrt{-2+\sqrt{5}+\sqrt{13-4\sqrt{5}}+\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}}$$

Decimal approximation:

1.619076398159105247383508829602269202039776657295266862292...

1.61907639….

This result is a good approximation to the value of the golden ratio 1,618033988749...

Alternate forms:

$$\frac{1}{2}\sqrt{\sqrt{13-4\sqrt{5}}+\sqrt{5}+\sqrt{-2\sqrt{5\left(13-4\sqrt{5}\right)}+12\sqrt{5}+50}-2}$$

$$\frac{1}{2}\sqrt{-2+\sqrt{5}+\sqrt{13-4\sqrt{5}}+\sqrt{2\left(25+6\sqrt{5}-\sqrt{5\left(13-4\sqrt{5}\right)}\right)}}$$

$$\sqrt{\text{root of } x^8+4x^7-10x^6-54x^5+9x^4+226x^3+125x^2-301x-269 \text{ near } x=2.62141}$$

Minimal polynomial:

$$x^{16}+4x^{14}-10x^{12}-54x^{10}+9x^8+226x^6+125x^4-301x^2-269$$

Note that, from the 64th root of the inverse result, we obtain:

(1/1.6190763981591052473835)^1/64

Input interpretation:

$$\sqrt[64]{\frac{1}{1.6190763981591052473835}}$$

Result:

0.99249927406191969005673 83...

0.99249927... result very near to the dilaton value **0.989117352243** = $\boldsymbol{\phi}$ (see Appendix)

The two results obtained 0.9921358035… and 0.99249927..., are very similar. This means that the two values 1.6574794945737…. and 1.61907639… belong to the same interval, which could be 1.6-1.675 (so-called "golden numbers". M. Nardelli)

Performing the 64^{th} root of the difference between the results of the two expressions, we obtain:

[(((2+sqrt(5)+sqrt((15-6*sqrt(5))))))/2 - ((((sqrt(5)-2+((sqrt((13-4*sqrt(5))))+sqrt(((50+12*sqrt(5)-2*sqrt((65-20*sqrt(5))))))))))))/4]^1/64

Input:

$$\left(\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)-\frac{1}{4}\left(\sqrt{5}-2+\left(\sqrt{13-4\sqrt{5}}+\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}\right)\right)\right)^{\wedge}(1/64)$$

Result:

$$\left(\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)+\frac{1}{4}\left(2-\sqrt{5}-\sqrt{13-4\sqrt{5}}-\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}\right)\right)^{\wedge}(1/64)$$

Decimal approximation:

0.968130990157095087429750492357828586803931884018623914873...

0.96813099… result that is equal to the spectral index n_s

Alternate forms:

$$\frac{1}{2}\left(-\sqrt{13-4\sqrt{5}}+\sqrt{5}+2\sqrt{3\left(5-2\sqrt{5}\right)}-\sqrt{-2\sqrt{5\left(13-4\sqrt{5}\right)}+12\sqrt{5}+50}+6\right)\text{^}(1/64)\,2^{31/32}$$

$$\frac{\sqrt[64]{6+\sqrt{5}+2\sqrt{15-6\sqrt{5}}-\sqrt{13-4\sqrt{5}}-\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}}}{\sqrt[32]{2}}$$

$$\frac{\sqrt[64]{6+\sqrt{5}-\sqrt{13-4\sqrt{5}}+2\sqrt{3(5-2\sqrt{5})}-\sqrt{2\left(25+6\sqrt{5}-\sqrt{5(13-4\sqrt{5})}\right)}}}{\sqrt[32]{2}}$$

And:

[(((2+sqrt(5)+sqrt((15-6*sqrt(5))))))/2 - ((((sqrt(5)-2+((sqrt((13-4*sqrt(5))))+sqrt(((50+12*sqrt(5)-2*sqrt((65-20*sqrt(5)))))))))))/4]^1/(89+55+21)

Where 89, 55 and 21 are Fibonacci numbers

Input:

$$\left(\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)-\frac{1}{4}\left(\sqrt{5}-2+\left(\sqrt{13-4\sqrt{5}}+\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}\right)\right)\right)\text{^}\left(\frac{1}{89+55+21}\right)$$

Result:

$$\left(\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)+\frac{1}{4}\left(2-\sqrt{5}-\sqrt{13-4\sqrt{5}}-\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}\right)\right)\text{^}(1/165)$$

Decimal approximation:

0.987516007895626396616021042094032985575752359030610763431...

0.987516007…. result very near to the dilaton value $\mathbf{0.989117352243} = \boldsymbol{\phi}$ (see Appendix)

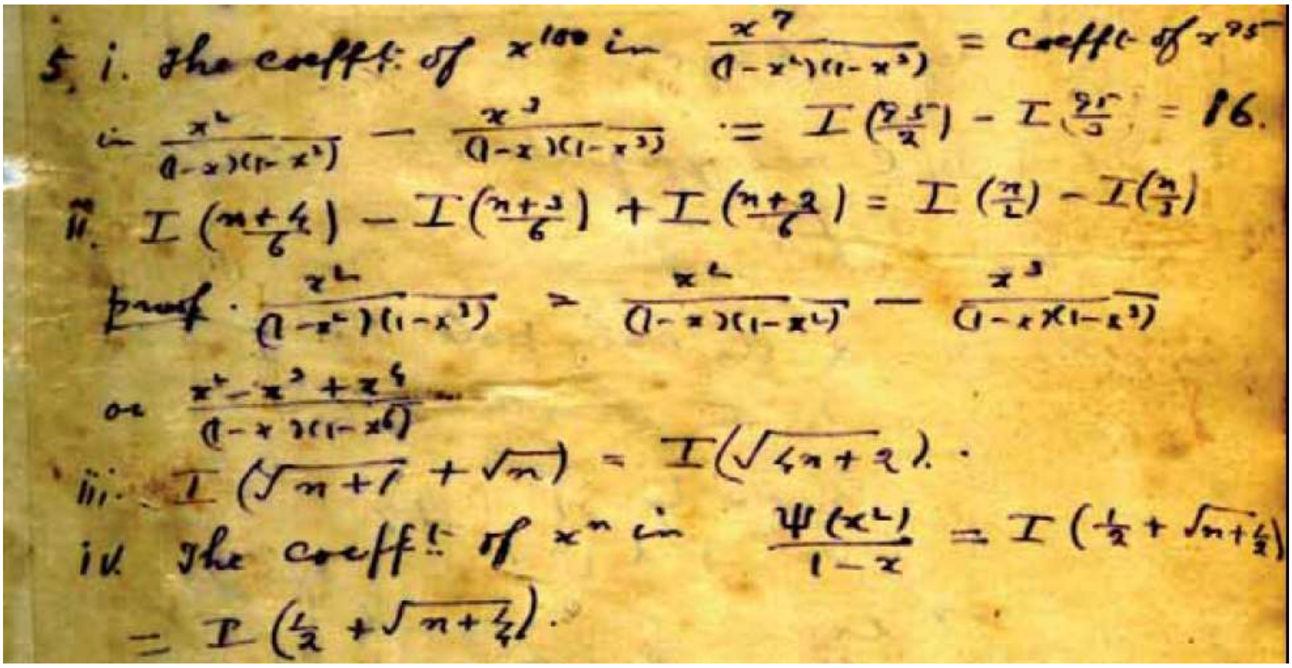
Alternate forms:

$$\frac{1}{2}\left(-\sqrt{13-4\sqrt{5}}+\sqrt{5}+2\sqrt{3\left(5-2\sqrt{5}\right)}-\sqrt{-2\sqrt{5\left(13-4\sqrt{5}\right)}+12\sqrt{5}+50}+6\right)\text{^}(1/165)\,2^{163/165}$$

$$\frac{\sqrt[165]{6+\sqrt{5}+2\sqrt{15-6\sqrt{5}}-\sqrt{13-4\sqrt{5}}-\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}}}{2^{2/165}}$$

$$\frac{\sqrt[165]{6+\sqrt{5}-\sqrt{13-4\sqrt{5}}+2\sqrt{3(5-2\sqrt{5})}-\sqrt{2\left(25+6\sqrt{5}-\sqrt{5(13-4\sqrt{5})}\right)}}}{2^{2/165}}$$

We have that:


5. i. The coeff.t of x^{100} in $\frac{x^7}{(1-x^2)(1-x^3)}$ = coeff.t of x^{95} in $\frac{x^2}{(1-x)(1-x^3)} - \frac{x^3}{(1-x)(1-x^3)} = I(\frac{95}{2}) - I(\frac{95}{3}) = 16.$

ii. $I(\frac{n+4}{6}) - I(\frac{n+3}{6}) + I(\frac{n+2}{6}) = I(\frac{n}{2}) - I(\frac{n}{3})$

proof. $\frac{x^2}{(1-x^2)(1-x^3)} = \frac{x^2}{(1-x)(1-x^2)} - \frac{x^3}{(1-x)(1-x^3)}$

or $\frac{x^2 - x^3 + x^4}{(1-x)(1-x^6)}$

iii. $I(\sqrt{n+1} + \sqrt{n}) = I(\sqrt{4n+2}).$

iv. The coeff.t of x^n in $\frac{\psi(x^2)}{1-x} = I(\frac{1}{2} + \sqrt{n+\frac{1}{2}})$

$= I(\frac{1}{2} + \sqrt{n+\frac{1}{4}}).$

x(95/2)-x(95/3) = 16

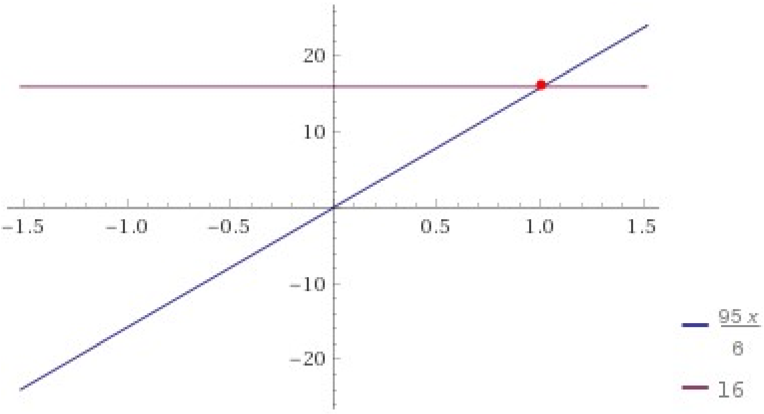
Input:

$$x \times \frac{95}{2} - x \times \frac{95}{3} = 16$$

Result:

$$\frac{95\,x}{6} = 16$$

Plot:



Alternate form:

$$\frac{95\,x}{6} - 16 = 0$$

Solution:

$$x \approx 1.0105$$

1.0105

1.0105((n+4)/6)- 1.0105((n+3)/6) + 1.0105((n+2)/6) - 1.0105(n/2) + 1.0105(n/3)

Input interpretation:

$$1.0105 \times \frac{n+4}{6} + \frac{n+3}{6} \times (-1.0105) + 1.0105 \times \frac{n+2}{6} + \frac{n}{2} \times (-1.0105) + 1.0105 \times \frac{n}{3}$$

Result:

$$-0.168417\,n + 0.168417\,(n+2) - 0.168417\,(n+3) + 0.168417\,(n+4)$$

Values:

n	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$-0.168417\, n + 0.168417\,(n+2) - 0.168417\,(n+3) + 0.168417\,(n+4)$	0.50525	0.50525	0.50525	0.50525	0.50525

Alternate form:

0.50525

0.50525

Property as a function:

Parity

even

Indefinite integral:

$$\int \left(\frac{1}{6} \times 1.0105\,(n+4) - \frac{1}{6} \times 1.0105\,(n+3) + \frac{1}{6} \times 1.0105\,(n+2) - \frac{1.0105\, n}{2} + \frac{1.0105\, n}{3}\right) dn = 0.50525\, n + \text{constant}$$

Global maximum:

$$\max\{-0.168417\, n + 0.168417\,(n+2) - 0.168417\,(n+3) + 0.168417\,(n+4)\} = \frac{2021}{4000} \text{ at } n = \frac{33}{10}$$

Global minimum:

$$\min\{-0.168417\, n + 0.168417\,(n+2) - 0.168417\,(n+3) + 0.168417\,(n+4)\} = \frac{2021}{4000} \text{ at } n = \frac{33}{10}$$

Limit:

$$\lim_{n \to \pm\infty} (-0.168417\, n + 0.168417\,(2+n) - 0.168417\,(3+n) + 0.168417\,(4+n)) = 0.50525$$

Definite integral after subtraction of diverging parts:

$$\int_0^\infty ((-0.168417\, n + 0.168417\,(2+n) - 0.168417\,(3+n) + 0.168417\,(4+n)) - 0.50525)\, dn = 0$$

1.0105((0.50525+4)/6)- 1.0105((0.50525+3)/6) + 1.0105((0.50525+2)/6) - 1.0105(0.50525/2) + 1.0105(0.50525/3)

Input interpretation:

$$1.0105\left(\frac{1}{6}(0.50525+4)\right)+\left(\frac{1}{6}(0.50525+3)\right)\times(-1.0105)+1.0105\left(\frac{1}{6}(0.50525+2)\right)+\frac{0.50525}{2}\times(-1.0105)+1.0105\times\frac{0.50525}{3}$$

Result:

0.50525

0.50525

(0.50525)^1/64

Input:

$$\sqrt[64]{0.50525}$$

Result:

0.98938948...

0.98938948…. result very near to the dilaton value **0.989117352243** = ϕ (see Appendix)

1.0105*((sqrt(0.50525+1)+sqrt(0.50525)))

Input interpretation:

$$1.0105\left(\sqrt{0.50525+1}+\sqrt{0.50525}\right)$$

Result:

1.95804...

1.95804…

1.0105*sqrt(4*0.50525+2)

Input interpretation:

$$1.0105\sqrt{4\times 0.50525+2}$$

Result:

2.02630...

2.02630…

Note that: 1.95804 ≈ 2.02630… where 1.95804 is a result practically near to the mean value 1.962 * 10^{19} of DM particle

1.0105*(1/2+sqrt(0.50525+2/4)) = 1.0105*(((1/2+sqrt(0.50525+1/2))))

Input interpretation:

$$1.0105\left(\frac{1}{2}+\sqrt{0.50525+\frac{2}{4}}\right)=1.0105\left(\frac{1}{2}+\sqrt{0.50525+\frac{1}{2}}\right)$$

Result:

True

1.0105*(((1/2+sqrt(0.50525+1/2))))

Input interpretation:

$$1.0105\left(\frac{1}{2}+\sqrt{0.50525+\frac{1}{2}}\right)$$

Result:

1.518399090120748220770459294979364005796997850491968076736...

1.51839909…

1/((((1.0105*(((1/2+sqrt(0.50525+1/2)))))))

Input interpretation:

$$\frac{1}{1.0105\left(\frac{1}{2}+\sqrt{0.50525+\frac{1}{2}}\right)}$$

Result:

0.658588382004678772991651006593101421040284856877452090717...

0.65858838…

1/((((1.0105*(((1/2+sqrt(0.50525+1/2)))))))^1/32

Input interpretation:

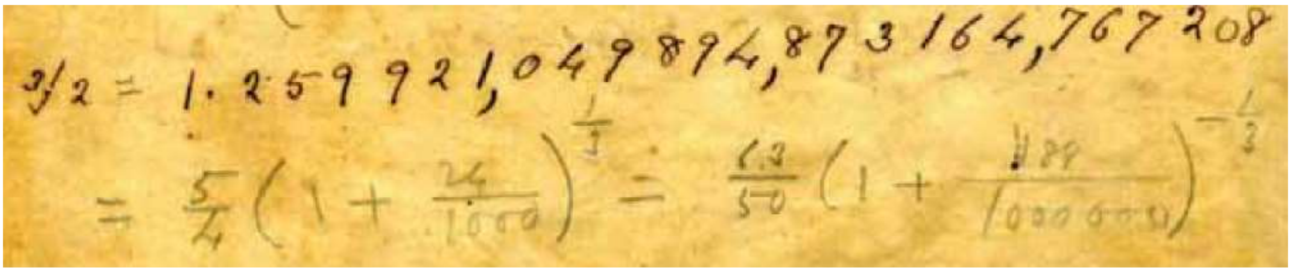
$$\frac{1}{\sqrt[32]{1.0105\left(\frac{1}{2}+\sqrt{0.50525+\frac{1}{2}}\right)}}$$

Result:

0.987033037784555433254102906818403726263980419370770744357...

0.987033037…. result very near to the dilaton value $\mathbf{0.989117352243} = \boldsymbol{\phi}$ (see Appendix)

We have that:



(2)^1/3 = 5/4(1+24/1000)^1/3

Input:

$$\sqrt[3]{2} = \frac{5}{4}\sqrt[3]{1+\frac{24}{1000}}$$

Result:

1.259921049894873164767210607278228350570251464701507980081...

1.259921049…

True

5/4(1+24/1000)^1/3

Input:

$$\frac{5}{4}\sqrt[3]{1+\frac{24}{1000}}$$

Result:

$$\sqrt[3]{2}$$

Decimal approximation:

1.259921049894873164767210607278228350570251464701507980081...

1.259921049…

63/50(1+189/1000000)^-(1/3)

Input:

$$\frac{63}{50}\left(1+\frac{189}{1000000}\right)^{-1/3}$$

Result:

$$\frac{126}{\sqrt[3]{1000189}}$$

Decimal approximation:

1.259920630000409955170602146632394291220340764820090733770...

1.25992063

Alternate form:

$$\frac{126 \times 1\,000\,189^{2/3}}{1\,000\,189}$$

Note that: 1.259921049… ≈ 1.25992063…

(((5/4(1+24/1000)^1/3)))/2

Input:

$$\frac{1}{2}\left(\frac{5}{4}\sqrt[3]{1+\frac{24}{1000}}\right)$$

Result:

$$\frac{1}{2^{2/3}}$$

Decimal approximation:

0.629960524947436582383605303639114175285125732350753990040…

0.629960524…

Alternate form:

$$\frac{\sqrt[3]{2}}{2}$$

((((((5/4(1+24/1000)^1/3)))/2)))^1/32

Input:

$$\sqrt[32]{\frac{1}{2}\left(\frac{5}{4}\sqrt[3]{1+\frac{24}{1000}}\right)}$$

Result:

$$\frac{1}{\sqrt[48]{2}}$$

Decimal approximation:

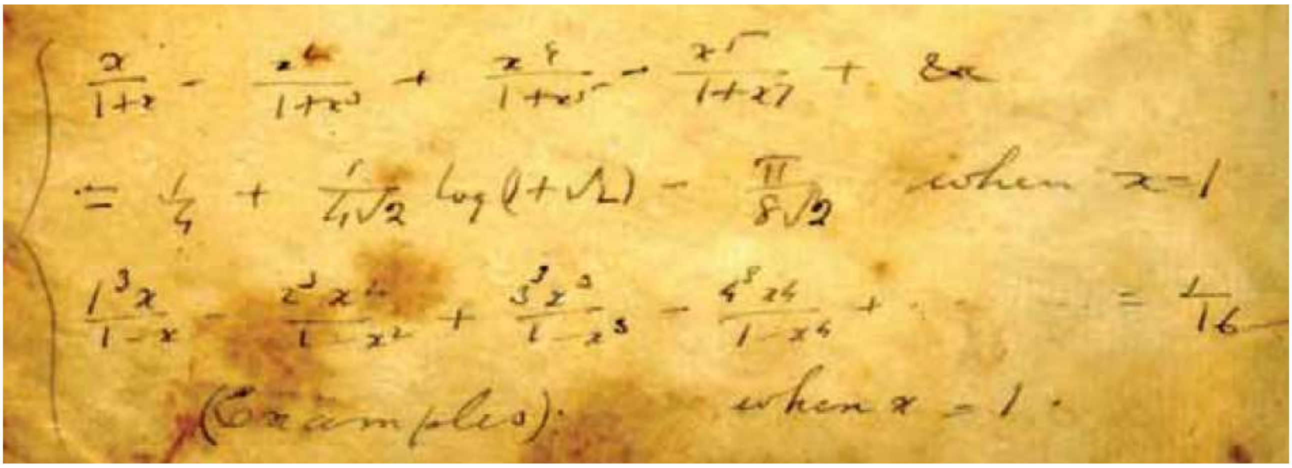
0.98566319864018757466759415575870742147534151843498039585 5...

0.9856631986… result very near to the dilaton value $\mathbf{0.989117352243} = \boldsymbol{\phi}$ (see Appendix)

Alternate form:

$$\frac{2^{47/48}}{2}$$

From:



We obtain:

1/4+1/(4sqrt(2)) ln (1+sqrt(2)) - Pi/(8sqrt(2))

Input:

$$\frac{1}{4}+\frac{1}{4\sqrt{2}}\log\left(1+\sqrt{2}\right)-\frac{\pi}{8\sqrt{2}}$$

log(x) is the natural logarithm

Exact result:

$$\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}$$

Decimal approximation:

0.128126126400159737910012458183848644499259972500661174281...

0.128126126…

Alternate forms:

$$\frac{1}{16}\left(4-\sqrt{2}\,\pi+2\sqrt{2}\,\sinh^{-1}(1)\right)$$

$$-\frac{\pi-2(\sqrt{2}+\log(1+\sqrt{2}))}{8\sqrt{2}}$$

$$\frac{1}{16}\left(4-\sqrt{2}\,\pi+2\sqrt{2}\,\log\left(1+\sqrt{2}\right)\right)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}=\frac{1}{4}+\frac{\log_e(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}$$

$$\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}=\frac{1}{4}+\frac{\log(a)\log_a(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}$$

$$\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}=\frac{1}{4}-\frac{\text{Li}_1(-\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}$$

Series representations:

$$\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}=\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log(2)}{8\sqrt{2}}-\frac{\sum_{k=1}^{\infty}\frac{(-1)^k 2^{-k/2}}{k}}{4\sqrt{2}}$$

$$\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}=$$
$$\frac{1}{16}\left(4-\sqrt{2}\,\pi+2\sqrt{2}\left(\log(z_0)+\left\lfloor\frac{\arg(1+\sqrt{2}-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(1+\sqrt{2}-z_0)^k z_0^{-k}}{k}\right)\right)$$

$$\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}=$$
$$\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{i\pi\left\lfloor\frac{\arg(1+\sqrt{2}-x)}{2\pi}\right\rfloor}{2\sqrt{2}}+\frac{\log(x)}{4\sqrt{2}}-\frac{\sum_{k=1}^{\infty}\frac{(-1)^k(1+\sqrt{2}-x)^k x^{-k}}{k}}{4\sqrt{2}} \quad \text{for } x<0$$

Integral representations:

$$\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}=\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{1}{4\sqrt{2}}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt$$

$$\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}=\frac{1}{4}-\frac{\pi}{8\sqrt{2}}-\frac{i}{8\sqrt{2}\,\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

((((1/4+1/(4sqrt(2)) ln (1+sqrt(2)) - Pi/(8sqrt(2))))))^1/64

Input:

$$\sqrt[64]{\frac{1}{4}+\frac{1}{4\sqrt{2}}\log\left(1+\sqrt{2}\right)-\frac{\pi}{8\sqrt{2}}}$$

log(x) is the natural logarithm

Exact result:

$$\sqrt[64]{\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}}$$

Decimal approximation:

0.968404589516534760779003269981247785539409185680817017342...

0.968404589…. result very near to the spectral index n_s and to the mesonic Regge slope (see Appendix)

Alternate forms:

$$\frac{\sqrt[64]{4-\sqrt{2}\,\pi+2\sqrt{2}\,\sinh^{-1}(1)}}{\sqrt[16]{2}}$$

$$\frac{\sqrt[64]{2(\sqrt{2}+\log(1+\sqrt{2}))-\pi}}{2^{7/128}}$$

$$\frac{\sqrt[64]{4-\sqrt{2}\,\pi+2\sqrt{2}\,\log(1+\sqrt{2})}}{\sqrt[16]{2}}$$

All 64th roots of 1/4 - π/(8 sqrt(2)) + log(1 + sqrt(2))/(4 sqrt(2)):

$$e^{0}\sqrt[64]{\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}}\approx 0.96840 \text{ (real, principal root)}$$

$$e^{(i\pi)/32}\sqrt[64]{\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}}\approx 0.96374+0.09492\,i$$

$$e^{(i\pi)/16}\sqrt[64]{\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}}\approx 0.94980+0.18893\,i$$

$$e^{(3i\pi)/32}\sqrt[64]{\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}}\approx 0.92671+0.28111\,i$$

$$e^{(i\pi)/8}\sqrt[64]{\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}}\approx 0.89469+0.37059\,i$$

Alternative representations:

$$\sqrt[64]{\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}}=\sqrt[64]{\frac{1}{4}+\frac{\log_e(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}}$$

$$\sqrt[64]{\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}}=\sqrt[64]{\frac{1}{4}+\frac{\log(a)\log_a(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}}$$

$$\sqrt[64]{\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}}=\sqrt[64]{\frac{1}{4}-\frac{\mathrm{Li}_1(-\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}}$$

Series representations:

$$\sqrt[64]{\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}}=\frac{\sqrt[64]{4-\sqrt{2}\,\pi+\sqrt{2}\,\log(2)-2\sqrt{2}\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}}}{\sqrt[16]{2}}$$

$$\sqrt[64]{\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}}=\sqrt[64]{\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\frac{\log(2)}{2}-\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}}{4\sqrt{2}}}$$

$$\sqrt[64]{\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}} = \frac{\sqrt[64]{4-\sqrt{2}\,\pi+2\sqrt{2}\left(2\,i\,\pi\left\lfloor\frac{\arg(1+\sqrt{2}-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(1+\sqrt{2}-x\right)^k x^{-k}}{k}\right)}}{\sqrt[16]{2}} \text{ for } x<0$$

Integral representations:

$$\sqrt[64]{\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}} = \frac{\sqrt[64]{4-\sqrt{2}\,\pi+2\sqrt{2}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt}}{\sqrt[16]{2}}$$

$$\sqrt[64]{\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}} = \frac{\sqrt[64]{4-\sqrt{2}\,\pi-\frac{i\sqrt{2}}{\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}}{\sqrt[16]{2}} \text{ for } -1<\gamma<0$$

log base 0.96840458951653476 ((((1/4+1/(4sqrt(2)) ln (1+sqrt(2)) - Pi/(8sqrt(2))))))

Input interpretation:

$$\log_{0.96840458951653476}\left(\frac{1}{4}+\frac{1}{4\sqrt{2}}\log\left(1+\sqrt{2}\right)-\frac{\pi}{8\sqrt{2}}\right)$$

Result:

64.00000000000000...

64 (see Appendix)

Alternative representations:

$$\log_{0.968404589516534760000}\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right) = \log_{0.968404589516534760000}\left(\frac{1}{4}+\frac{\log_e(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)$$

$$\log_{0.96840458951653476000}\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=\frac{\log\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}{\log(0.96840458951653476000)}$$

$$\log_{0.96840458951653476000}\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=\log_{0.96840458951653476000}\left(\frac{1}{4}+\frac{\log(a)\log_a(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)$$

Series representations:

$$\log_{0.96840458951653476000}\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=-\frac{\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{8}\right)^k\left(-\frac{\pi-2\log(1+\sqrt{2})+6\sqrt{2}}{\sqrt{2}}\right)^k}{k}}{\log(0.96840458951653476000)}$$

$$\log_{0.96840458951653476000}\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=\log_{0.96840458951653476000}\left(\left(-\pi+2\log(\sqrt{2})+2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}-2\sum_{k=1}^{\infty}\frac{(-1)^k\sqrt{2}^{-k}}{k}\right)\Big/\left(8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\quad \text{for } (x\in\mathbb{R}\text{ and }x<0)$$

$$\log_{0.96840458951653476000}\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=\log_{0.96840458951653476000}\left(\frac{1}{4}-\frac{\pi\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor)}}{8\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}+\frac{\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\left(\log(\sqrt{2})-\sum_{k=1}^{\infty}\frac{(-1)^k\sqrt{2}^{-k}}{k}\right)}{4\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}\right)$$

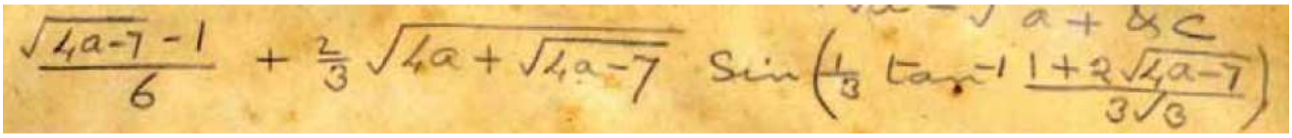
Integral representations:

$$\log_{0.96840458951653476000}\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=$$
$$\log_{0.96840458951653476000}\left(-\frac{\pi-2\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt-2\sqrt{2}}{8\sqrt{2}}\right)$$

$$\log_{0.96840458951653476000}\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=\log_{0.96840458951653476000}\left(\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\,\sqrt{2}^{-s}}{\Gamma(1-s)}\,ds-i\left(\pi\left(\pi-2\sqrt{2}\right)\right)}{8\,i\,\pi\sqrt{2}}\right)\quad\text{for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

Now, from the “**Manuscript Book 2 of Srinivasa Ramanujan**”, we have that


$$\frac{\sqrt{4a-7}-1}{6}+\frac{2}{3}\sqrt{4a+\sqrt{4a-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\frac{1+2\sqrt{4a-7}}{3\sqrt{3}}\right)$$

For a = 3

(sqrt(12-7)-1)/6+2/3*sqrt(12+sqrt(12-7))*sin(1/3*tan^-1(((1+2*sqrt(12-7))/(3*sqrt3)))

Input:

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{2}{3}\sqrt{12+\sqrt{12-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{1}{6}\left(\sqrt{5}-1\right)+\frac{2}{3}\sqrt{12+\sqrt{5}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)$$

(result in radians)

Decimal approximation:

0.8779621799993875532527799194946725106197613150374609036 32...

(result in radians)

0.8779621799…

Alternate forms:

$$\frac{1}{6}\left(-1+\sqrt{5}+4\sqrt{12+\sqrt{5}}\sin\left(\frac{1}{3}\tan^{-1}\left(\sqrt{\frac{7}{9}+\frac{4\sqrt{5}}{27}}\right)\right)\right)$$

$$\frac{1}{6}\left(-1+\sqrt{5}+4\sqrt{12+\sqrt{5}}\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)$$

$$-\frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{2}{3}\sqrt{12+\sqrt{5}}\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)$$

Alternative representations:

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=\frac{1}{6}\left(-1+\sqrt{5}\right)+\frac{2}{3}\cos\left(\frac{\pi}{2}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\sqrt{12+\sqrt{5}}$$

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=\frac{1}{6}\left(-1+\sqrt{5}\right)-\frac{2}{3}\cos\left(\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\sqrt{12+\sqrt{5}}$$

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=\frac{1}{6}\left(-1+\sqrt{5}\right)+\frac{2\left(-e^{-1/3\,i\tan^{-1}\left(\left(1+2\sqrt{5}\right)/\left(3\sqrt{3}\right)\right)}+e^{1/3\,i\tan^{-1}\left(\left(1+2\sqrt{5}\right)/\left(3\sqrt{3}\right)\right)}\right)\sqrt{12+\sqrt{5}}}{3(2i)}$$

Series representations:

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$
$$-\frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{2}{3}\sqrt{12+\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)^{1+2k}}{(1+2k)!}$$

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$
$$-\frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{2}{3}\sqrt{12+\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)^{2k}}{(2k)!}$$

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$
$$-\frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{1}{9}\sqrt{\left(12+\sqrt{5}\right)\pi}\ \tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{36^s\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}$$

Integral representations:

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$
$$-\frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{2}{9}\sqrt{12+\sqrt{5}}\ \tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\int_0^1\cos\left(\frac{1}{3}\,t\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)dt$$

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=-\frac{1}{6}+\frac{\sqrt{5}}{6}-$$
$$\frac{1}{18}\,i\sqrt{\frac{12+\sqrt{5}}{\pi}}\ \tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)^2/(36s)}}{s^{3/2}}\,ds\quad\text{for }\gamma>0$$

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$
$$-\frac{1}{6}+\frac{\sqrt{5}}{6}-\frac{1}{3}\,i\sqrt{\frac{12+\sqrt{5}}{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(\frac{1}{6}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)^{1-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\quad\text{for }0<\gamma<1$$

$$\frac{\sqrt{4a-7}-1}{6} + \frac{2}{3}\sqrt{4a+\sqrt{4a-7}}\ \sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\frac{1+2\sqrt{4a-7}}{3\sqrt{3}}\right)$$

(sqrt(12-7)-1)/6+2/3*sqrt(12+sqrt(12-7))*sin(Pi/3-1/3*tan^-1(((1+2*sqrt(12-7))/(3*sqrt3)))

Input:

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{2}{3}\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{1}{6}\left(\sqrt{5}-1\right)+\frac{2}{3}\sqrt{12+\sqrt{5}}\ \cos\left(\frac{\pi}{6}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)$$

(result in radians)

Decimal approximation:

1.969254219241230305114453041420413075023762093998880011607...

(result in radians)

1.96925421924…. result practically near to the mean value $1.962 * 10^{19}$ of DM particle

Alternate forms:

$$\frac{1}{6}\left(-1+\sqrt{5}+4\sqrt{12+\sqrt{5}}\ \cos\left(\frac{1}{6}\left(\pi+2\tan^{-1}\left(\sqrt{\frac{7}{9}+\frac{4\sqrt{5}}{27}}\right)\right)\right)\right)$$

$$\frac{1}{6}\left(-1+\sqrt{5}+4\sqrt{12+\sqrt{5}}\ \cos\left(\frac{1}{6}\left(\pi+2\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)\right)$$

$$\frac{1}{6}\left(-1+\sqrt{5}+4\sqrt{12+\sqrt{5}}\ \cos\left(\frac{\pi}{6}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)$$

Addition formulas:

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$
$$\frac{1}{6}\left(-1+\sqrt{5}\right)+\frac{2}{3}\sqrt{12+\sqrt{5}}\left(\frac{1}{2}\sqrt{3}\cos\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)-\frac{1}{2}\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)$$

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$
$$\frac{1}{6}\left(-1+\sqrt{5}+2\sqrt{3\left(12+\sqrt{5}\right)}\cos\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)-2\sqrt{12+\sqrt{5}}\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)$$

Alternative representations:

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$
$$\frac{1}{6}\left(-1+\sqrt{5}\right)+\frac{2}{3}\cos\left(\frac{\pi}{6}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\sqrt{12+\sqrt{5}}$$

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$
$$\frac{1}{6}\left(-1+\sqrt{5}\right)-\frac{2}{3}\cos\left(\frac{5\pi}{6}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\sqrt{12+\sqrt{5}}$$

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$
$$\frac{1}{6}\left(-1+\sqrt{5}\right)+\frac{2\left(-e^{-i\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)}+e^{i\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)}\right)\sqrt{12+\sqrt{5}}}{3(2i)}$$

Series representations:

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$

$$-\frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{2}{3}\sqrt{12+\sqrt{5}}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{36}\right)^k\left(\pi+2\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)^{2k}}{(2k)!}$$

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$

$$-\frac{1}{6}+\frac{\sqrt{5}}{6}-\frac{2}{3}\sqrt{12+\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)^{1+2k}}{(1+2k)!}$$

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$

$$-\frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{2}{3}\sqrt{\left(12+\sqrt{5}\right)\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{144^s\left(\pi+2\tan^{-1}\left(\frac{1}{9}\left(\sqrt{3}+2\sqrt{15}\right)\right)\right)^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}$$

Integral representations:

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$

$$-\frac{1}{6}+\frac{\sqrt{5}}{6}-\frac{2\sqrt{12+\sqrt{5}}}{3}\int_{\frac{\pi}{2}}^{\frac{1}{6}\left(\pi+2\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)}\sin(t)\,dt$$

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$

$$\frac{1}{6}\left(-1+\sqrt{5}\right)+\frac{2}{3}\sqrt{12+\sqrt{5}}\left(1-\frac{\pi}{6}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\int_0^1\sin\left(t\left(\frac{\pi}{6}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)dt\right)$$

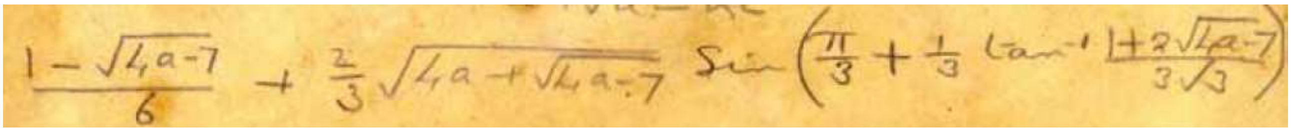
$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\,\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2 =$$

$$-\frac{1}{6}+\frac{\sqrt{5}}{6}-\frac{1}{3}\,i\sqrt{\frac{12+\sqrt{5}}{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\exp\left(s-\frac{\left(\pi+2\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)^2}{144\,s}\right)}{\sqrt{s}}\,ds \quad \text{for } \gamma>0$$

$$\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\,\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2 =$$

$$-\frac{1}{6}+\frac{\sqrt{5}}{6}-\frac{1}{3}\,i\sqrt{\frac{12+\sqrt{5}}{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(\frac{12}{\pi+2\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)}\right)^{2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

And, from this formula, we obtain:



(1-sqrt(12-7))/6)+2/3*sqrt(12+sqrt(12-7))*sin(Pi/3+1/3*tan^-1(((1+2*sqrt(12-7))/(3*sqrt3)))

Input:

$$\frac{1}{6}\left(1-\sqrt{12-7}\right)+\frac{2}{3}\sqrt{12+\sqrt{12-7}}\,\sin\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{1}{6}\left(1-\sqrt{5}\right)+\frac{2}{3}\sqrt{12+\sqrt{5}}\,\cos\left(\frac{\pi}{6}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)$$

(result in radians)

Decimal approximation:

2.229182410490723010162646126549447467923214229230578053104...

(result in radians)

2.2291824104…

Alternate forms:

$$\frac{1}{6}\left(1-\sqrt{5}+4\sqrt{12+\sqrt{5}}\ \sin\left(\frac{1}{3}\left(\pi+\tan^{-1}\left(\sqrt{\frac{7}{9}+\frac{4\sqrt{5}}{27}}\right)\right)\right)\right)$$

$$\frac{1}{6}\left(1-\sqrt{5}+4\sqrt{12+\sqrt{5}}\ \sin\left(\frac{1}{3}\left(\pi+\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)\right)$$

$$\frac{1}{6}\left(1-\sqrt{5}+4\sqrt{12+\sqrt{5}}\ \cos\left(\frac{\pi}{6}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)$$

Addition formulas:

$$\frac{1}{6}\left(1-\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$
$$\frac{1}{6}\left(1-\sqrt{5}\right)+$$
$$\frac{2}{3}\sqrt{12+\sqrt{5}}\ \left(\frac{1}{2}\sqrt{3}\ \cos\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)+\frac{1}{2}\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)$$

$$\frac{1}{6}\left(1-\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$
$$\frac{1}{6}\left(1-\sqrt{5}+2\sqrt{3\left(12+\sqrt{5}\right)}\ \cos\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)+\right.$$
$$\left.2\sqrt{12+\sqrt{5}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)$$

Alternative representations:

$$\frac{1}{6}\left(1-\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$
$$\frac{1}{6}\left(1-\sqrt{5}\right)+\frac{2}{3}\cos\left(\frac{\pi}{6}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\sqrt{12+\sqrt{5}}$$

$$\frac{1}{6}\left(1-\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$
$$\frac{1}{6}\left(1-\sqrt{5}\right)-\frac{2}{3}\cos\left(\frac{5\pi}{6}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\sqrt{12+\sqrt{5}}$$

$$\frac{1}{6}\left(1-\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$

$$\frac{1}{6}\left(1-\sqrt{5}\right)+\frac{2\left(-e^{-i\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)}+e^{i\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)}\right)\sqrt{12+\sqrt{5}}}{3(2i)}$$

Series representations:

$$\frac{1}{6}\left(1-\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$

$$\frac{1}{6}-\frac{\sqrt{5}}{6}+\frac{2}{3}\sqrt{12+\sqrt{5}}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{36}\right)^k\left(\pi-2\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)^{2k}}{(2k)!}$$

$$\frac{1}{6}\left(1-\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$

$$\frac{1}{6}-\frac{\sqrt{5}}{6}-\frac{2}{3}\sqrt{12+\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)^{1+2k}}{(1+2k)!}$$

$$\frac{1}{6}\left(1-\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$

$$\frac{1}{6}-\frac{\sqrt{5}}{6}+\frac{2}{3}\sqrt{12+\sqrt{5}}\sum_{k=0}^{\infty}\frac{e^{3ik\pi}\left(\frac{3}{\pi+\tan^{-1}\left(\frac{1}{9}\left(\sqrt{3}+2\sqrt{15}\right)\right)}\right)^{-1-2k}}{(1+2k)!}$$

$$\frac{1}{6}\left(1-\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\ \sin\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2=$$

$$\frac{1}{6}-\frac{\sqrt{5}}{6}+\frac{2}{3}\sqrt{\left(12+\sqrt{5}\right)\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{144^s\left(\pi-2\tan^{-1}\left(\frac{1}{9}\left(\sqrt{3}+2\sqrt{15}\right)\right)\right)^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}$$

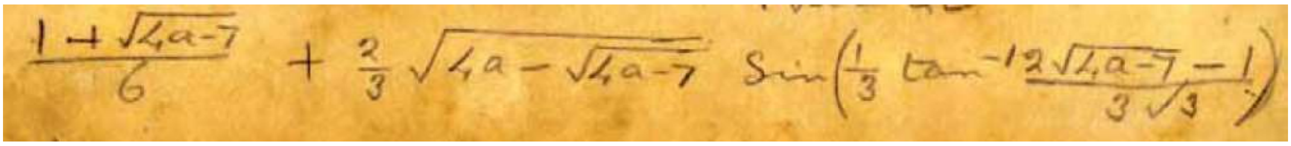
Integral representations:

$$\frac{1}{6}\left(1-\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\,\sin\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2 = \frac{1}{6}-\frac{\sqrt{5}}{6}-\frac{2\sqrt{12+\sqrt{5}}}{3}\int_{\frac{\pi}{2}}^{\frac{1}{6}\left(\pi-2\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)}\sin(t)\,dt$$

$$\frac{1}{6}\left(1-\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\,\sin\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2 = \frac{1}{6}\left(1-\sqrt{5}-\frac{2}{3}\sqrt{12+\sqrt{5}}\left(-6+\pi-2\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\int_0^1\sin\left(\frac{1}{6}\,t\left(\pi-2\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)dt\right)$$

$$\frac{1}{6}\left(1-\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12+\sqrt{12-7}}\,\sin\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)\right)2 = \frac{1}{6}\left(1-\sqrt{5}\right)+\frac{2}{3}\sqrt{12+\sqrt{5}}\left(1-\frac{\pi}{6}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\int_0^1\sin\left(t\left(\frac{\pi}{6}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)dt$$

Now, we have:



(1+sqrt(12-7))/6)+2/3*sqrt(12-sqrt(12-7))*sin(1/3*tan^-1(((2*sqrt(12-7)-1))/(3*sqrt3)))

Input:

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{2}{3}\sqrt{12-\sqrt{12-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{1}{6}\left(1+\sqrt{5}\right)+\frac{2}{3}\sqrt{12-\sqrt{5}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{5}-1}{3\sqrt{3}}\right)\right)$$

(result in radians)

Decimal approximation:

0.9457637221963984461555361224558654408175115229371060979 26...

(result in radians)

0.945763722…

Alternate forms:

$$\frac{1}{6}\left(1+\sqrt{5}+4\sqrt{12-\sqrt{5}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{5}-1}{3\sqrt{3}}\right)\right)\right)$$

$$\frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{2}{3}\sqrt{12-\sqrt{5}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{5}-1}{3\sqrt{3}}\right)\right)$$

$$\frac{2}{3}\sqrt{12-\sqrt{5}}\;\left(\text{root of } 569344x^{12}-1708032x^{10}+1921536x^8-990464x^6+224688x^4-16704x^2+361 \text{ near } x=0.195098\right)+\frac{1}{6}\left(1+\sqrt{5}\right)$$

Alternative representations:

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2=\frac{1}{6}\left(1+\sqrt{5}\right)+\frac{2}{3}\cos\left(\frac{\pi}{2}-\frac{1}{3}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\sqrt{12-\sqrt{5}}$$

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2=\frac{1}{6}\left(1+\sqrt{5}\right)-\frac{2}{3}\cos\left(\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\sqrt{12-\sqrt{5}}$$

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2 =$$

$$\frac{1}{6}\left(1+\sqrt{5}\right)+\frac{2\left(-e^{-\frac{1}{3}i\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)}+e^{1/3\,i\tan^{-1}\left(\left(-1+2\sqrt{5}\right)/\left(3\sqrt{3}\right)\right)}\right)\sqrt{12-\sqrt{5}}}{3(2i)}$$

Series representations:

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2 =$$

$$\frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{2}{3}\sqrt{12-\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)^{1+2k}}{(1+2k)!}$$

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2 =$$

$$\frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{2}{3}\sqrt{12-\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)^{2k}}{(2k)!}$$

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2 = \frac{1}{6}+\frac{\sqrt{5}}{6}+$$

$$\frac{1}{9}\sqrt{\left(12-\sqrt{5}\right)\pi}\ \tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\ \frac{36^s\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}$$

Integral representations:

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2 =$$

$$\frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{2}{9}\sqrt{12-\sqrt{5}}\ \tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\int_0^1\cos\left(\frac{1}{3}\,t\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)dt$$

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2=\frac{1}{6}+\frac{\sqrt{5}}{6}-\frac{1}{18}i\sqrt{\frac{12-\sqrt{5}}{\pi}}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)^2/(36s)}}{s^{3/2}}\,ds \quad \text{for } \gamma>0$$

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2=\frac{1}{6}+\frac{\sqrt{5}}{6}-\frac{1}{3}i\sqrt{\frac{12-\sqrt{5}}{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(\frac{1}{6}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)^{1-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds \quad \text{for } 0<\gamma<1$$

Dividing the four results obtained, and multiplying by 6, we obtain:

6(1/0.8779621799993875 *1/ 1.969254219241230305 *1/ 2.2291824104907230 *1/ 0.9457637221963984)

Input interpretation:

$$6\left(\frac{1}{0.8779621799993875}\times\frac{1}{1.969254219241230305}\times\frac{1}{2.2291824104907230}\times\frac{1}{0.9457637221963984}\right)$$

Result:

1.6460590041096573824233586903454769755430036340930024605 72...

1.6460590041.... ≈ ζ(2) = $\frac{\pi^2}{6} = 1.644934$ ...

And:

2Pi*0.97004937(1/0.8779621799993875 *1/ 1.969254219241230305 *1/ 2.2291824104907230 *1/ 0.9457637221963984)

Where 0.97004937 * (1/0.8779621799993875 *1/ 1.969254219241230305 *1/ 2.2291824104907230 *1/ 0.9457637221963984) = 0.26612641665... is the radius of a circumference

Input interpretation:

$$2\pi \times 0.97004937\left(\frac{1}{0.8779621799993875} \times \frac{1}{1.969254219241230305} \times \frac{1}{2.2291824104907230} \times \frac{1}{0.9457637221963984}\right)$$

Result:

1.6721216...

1.6721216… result very near to the proton mass

Alternative representations:

$$(2\pi\, 0.970049)/((1.9692542192412303050000 \times 2.2291824104907230000 \times 0.94576372219639840000)\, 0.87796217999938750000) = (349.218\,°)/(0.87796217999938750000 \times 0.94576372219639840000 \times 1.9692542192412303050000 \times 2.2291824104907230000)$$

$$(2\pi\, 0.970049)/((1.9692542192412303050000 \times 2.2291824104907230000 \times 0.94576372219639840000)\, 0.87796217999938750000) = -((1.9401\, i \log(-1))/(0.87796217999938750000 \times 0.94576372219639840000 \times 1.9692542192412303050000 \times 2.2291824104907230000))$$

$$(2\pi\, 0.970049)/((1.9692542192412303050000 \times 2.2291824104907230000 \times 0.94576372219639840000)\, 0.87796217999938750000) = \left(1.9401 \cos^{-1}(-1)\right)/(0.87796217999938750000 \times 0.94576372219639840000 \times 1.9692542192412303050000 \times 2.2291824104907230000)$$

Series representations:

$$(2\pi\, 0.970049)/((1.9692542192412303050000 \times 2.2291824104907230000 \times 0.94576372219639840000)\, 0.87796217999938750000) = 2.12901 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$(2\pi\, 0.970049)/((1.9692542192412303050000 \times 2.2291824104907230000 \times 0.94576372219639840000)\, 0.87796217999938750000) = -1.06451 + 1.06451 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$(2 \pi 0.970049) / ((1.9692542192412303050000 \times 2.2291824104907230000 \times 0.94576372219639840000) 0.87796217999938750000) = 0.532253 \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50 k)}{\binom{3k}{k}}$$

Integral representations:

$$(2 \pi 0.970049) / ((1.9692542192412303050000 \times 2.2291824104907230000 \times 0.94576372219639840000) 0.87796217999938750000) = 1.06451 \int_0^{\infty} \frac{1}{1+t^2} \, dt$$

$$(2 \pi 0.970049) / ((1.9692542192412303050000 \times 2.2291824104907230000 \times 0.94576372219639840000) 0.87796217999938750000) = 2.12901 \int_0^{1} \sqrt{1-t^2} \, dt$$

$$(2 \pi 0.970049) / ((1.9692542192412303050000 \times 2.2291824104907230000 \times 0.94576372219639840000) 0.87796217999938750000) = 1.06451 \int_0^{\infty} \frac{\sin(t)}{t} \, dt$$

From the following algebraic sums, we obtain:

1/(0.8779621799993875 + 1.969254219241230305 - 2.2291824104907230 + 0.9457637221963984)

Input interpretation:

$$1 / (0.8779621799993875 + 1.969254219241230305 - 2.2291824104907230 + 0.9457637221963984)$$

Result:

0.639468898694623952580442123580603914914227777913978462046...

0.63946889...

And:

(-0.8779621799993875 + 1.969254219241230305 + 2.2291824104907230 - 0.9457637221963984)

Input interpretation:

−0.8779621799993875 + 1.969254219241230305 + 2.2291824104907230 − 0.9457637221963984

Result:

2.374710727536167405

2.3747107...

(0.8779621799993875 + 1.969254219241230305 - 2.2291824104907230 + 0.9457637221963984)

Input interpretation:

0.8779621799993875 + 1.969254219241230305 − 2.2291824104907230 + 0.9457637221963984

Result:

1.563797710946293205

1.5637977109...

From the following difference between 2.3747107... and 1.5637977109..., multiplied by 2, we obtain:

2(2.374710727536167405 - 1.563797710946293205)

Input interpretation:

2 (2.374710727536167405 − 1.563797710946293205)

Result:

1.6218260331797484

1.621826033...

And from the mean of the above results, we obtain:

1/2(2.374710727536167405 + 1.563797710946293205)

Input interpretation:

$$\frac{1}{2}(2.374710727536167405 + 1.563797710946293205)$$

Result:

1.969254219241230305

1.96925421... result equal to the solution of a previous formula and practically near to the mean value 1.962 * 10^{19} of DM particle

(Pi*1/0.98593794)*1/1.969254219241230305

Input interpretation:

$$\left(\pi \times \frac{1}{0.98593794}\right) \times \frac{1}{1.969254219241230305}$$

Result:

1.6180745...

1.6180745…

This result is a very good approximation to the value of the golden ratio 1,618033988749...

Alternative representations:

$$\frac{\pi}{1.96925421924123030500000 \times 0.985938} = \frac{180°}{0.985938 \times 1.96925421924123030500000}$$

$$\frac{\pi}{1.96925421924123030500000 \times 0.985938} = -\frac{i \log(-1)}{0.985938 \times 1.96925421924123030500000}$$

$$\frac{\pi}{1.96925421924123030500000 \times 0.985938} = \frac{\cos^{-1}(-1)}{0.985938 \times 1.96925421924123030500000}$$

Series representations:

$$\frac{\pi}{1.9692542192412303050000 \times 0.985938} = 2.0602 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{\pi}{1.9692542192412303050000 \times 0.985938} = -1.0301 + 1.0301 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$\frac{\pi}{1.9692542192412303050000 \times 0.985938} = 0.515049 \sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}$$

Integral representations:

$$\frac{\pi}{1.9692542192412303050000 \times 0.985938} = 1.0301 \int_0^{\infty} \frac{1}{1+t^2}\,dt$$

$$\frac{\pi}{1.9692542192412303050000 \times 0.985938} = 2.0602 \int_0^{1} \sqrt{1-t^2}\,dt$$

$$\frac{\pi}{1.9692542192412303050000 \times 0.985938} = 1.0301 \int_0^{\infty} \frac{\sin(t)}{t}\,dt$$

In conclusion, we can to obtain a result very near to the dilaton value from the following equation, containing 1.96925421... and the golden ratio:

(Pi*1/x)*1/1.969254219241230305 = 1.61803398

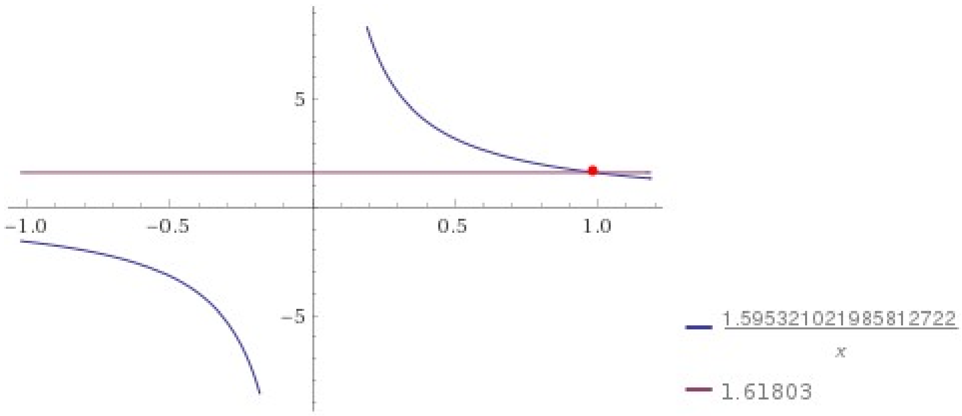
Input interpretation:

$$\left(\pi \times \frac{1}{x}\right) \times \frac{1}{1.969254219241230305} = 1.61803398$$

Result:

$$\frac{1.595321021985812722}{x} = 1.61803$$

Plot:



Alternate form assuming x is real:

$$\frac{0.985963}{x} = 1$$

Alternate form assuming x is positive:

$x = 0.985963$ (for $x \neq 0$)

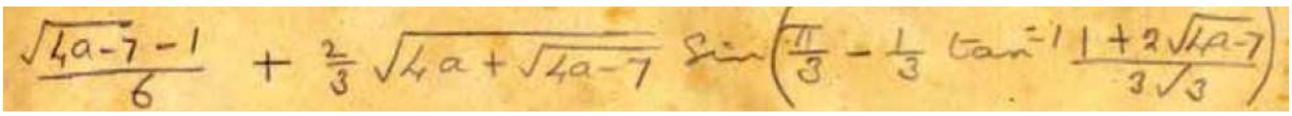
Solution:

$x \approx 0.985963$

0.985963 result that is an excellent approximation to the dilaton value **0.989117352243** = $\boldsymbol{\phi}$ very near also to the result of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

Thus, utilizing the previous formula,



with π and ϕ, we obtain:

(Pi*1/x)*1/(((((sqrt(12-7)-1)/6+2/3*sqrt(12+sqrt(12-7))*sin(Pi/3-1/3*tan^-1(((1+2*sqrt(12-7))/(3*sqrt3)))))))) = 1.61803398

Input interpretation:

$$\left(\pi \times \frac{1}{x}\right) \times \frac{1}{\frac{1}{6}\left(\sqrt{12-7}-1\right)+\frac{2}{3}\sqrt{12+\sqrt{12-7}}\,\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{12-7}}{3\sqrt{3}}\right)\right)} =$$

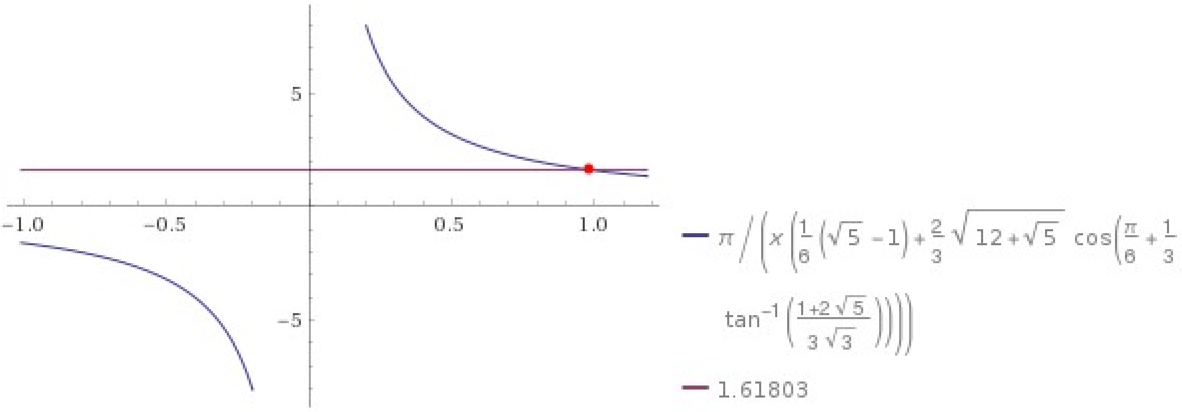
1.61803398

$\tan^{-1}(x)$ is the inverse tangent function

Result:

$$\frac{\pi}{x\left(\frac{1}{6}\left(\sqrt{5}-1\right)+\frac{2}{3}\sqrt{12+\sqrt{5}}\,\cos\left(\frac{\pi}{6}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)} = 1.61803$$

Plot:



Alternate forms:

$$\frac{6\pi}{x\left(-1+\sqrt{5}+4\sqrt{12+\sqrt{5}}\,\cos\left(\frac{1}{6}\left(\pi+2\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)\right)} = 1.61803$$

$$\frac{6\pi}{x\left(-1+\sqrt{5}+4\sqrt{12+\sqrt{5}}\,\cos\left(\frac{\pi}{6}+\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)} = 1.61803$$

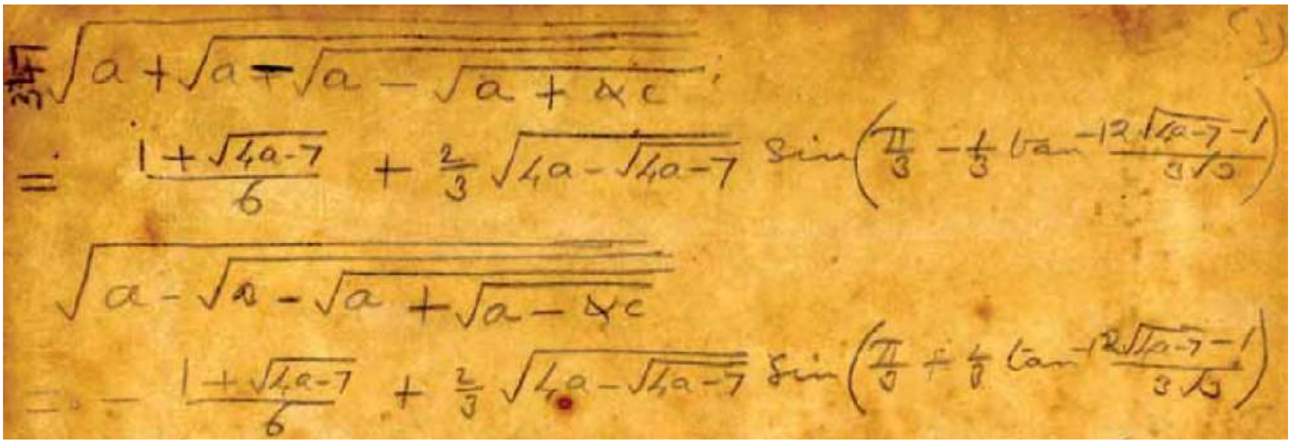
$$\pi \Bigg/ \left(\frac{\sqrt{5}\,x}{6}-\frac{x}{6}-\frac{1}{3}\sqrt{12+\sqrt{5}}\;x\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)+\sqrt{\frac{1}{3}\left(12+\sqrt{5}\right)}\;x\cos\left(\frac{1}{3}\tan^{-1}\left(\frac{1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right) = 1.61803$$

Solution:

$x \approx 0.985963$

0.985963 as above

We have also:


$$\sqrt{a+\sqrt{a+\sqrt{a-\sqrt{a+\&c}}}} = \frac{1+\sqrt{4a-7}}{6} + \frac{2}{3}\sqrt{4a-\sqrt{4a-7}}\,\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\frac{2\sqrt{4a-7}-1}{3\sqrt{3}}\right)$$
$$\sqrt{a-\sqrt{a-\sqrt{a+\sqrt{a-\&c}}}} = -\frac{1+\sqrt{4a-7}}{6} + \frac{2}{3}\sqrt{4a-\sqrt{4a-7}}\,\sin\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\frac{2\sqrt{4a-7}-1}{3\sqrt{3}}\right)$$

((1+sqrt(12-7))/6)+2/3*sqrt(12-sqrt(12-7))*sin((((Pi/3-1/3*tan^-1(((2*sqrt(12-7)-1))/(3*sqrt3))))))

Input:

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{2}{3}\sqrt{12-\sqrt{12-7}}\,\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{1}{6}\left(1+\sqrt{5}\right)+\frac{2}{3}\sqrt{12-\sqrt{5}}\,\cos\left(\frac{\pi}{6}+\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{5}-1}{3\sqrt{3}}\right)\right)$$

(result in radians)

Decimal approximation:

2.105530981777213665099274782189076818403932320866293894603...

(result in radians)

2.10553098177….

Alternate forms:

$$\frac{1}{6}\left(1+\sqrt{5}+4\sqrt{12-\sqrt{5}}\,\cos\left(\frac{1}{6}\left(\pi+2\cot^{-1}\left(\frac{3}{\sqrt{7-\frac{4\sqrt{5}}{3}}}\right)\right)\right)\right)$$

$$\frac{1}{6}\left(1+\sqrt{5}+4\sqrt{12-\sqrt{5}}\,\cos\left(\frac{1}{6}\left(\pi+2\tan^{-1}\left(\frac{2\sqrt{5}-1}{3\sqrt{3}}\right)\right)\right)\right)$$

$$\frac{1}{6}\left(1+\sqrt{5}+4\sqrt{12-\sqrt{5}}\cos\left(\frac{\pi}{6}+\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{5}-1}{3\sqrt{3}}\right)\right)\right)$$

$\cot^{-1}(x)$ is the inverse cotangent function

Addition formula:

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2 = \frac{1}{6}\left(1+\sqrt{5}\right)+ \frac{2}{3}\sqrt{12-\sqrt{5}}\left(\frac{1}{2}\sqrt{3}\cos\left(\frac{1}{3}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)-\frac{1}{2}\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)$$

And:

(-(1+sqrt(12-7))/6)+2/3*sqrt(12-sqrt(12-7))*sin((((Pi/3-1/3*tan^-1(((2*sqrt(12-7)-1))/(3*sqrt3))))))

Input:

$$-\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{2}{3}\sqrt{12-\sqrt{12-7}}\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{1}{6}\left(-1-\sqrt{5}\right)+\frac{2}{3}\sqrt{12-\sqrt{5}}\cos\left(\frac{\pi}{6}+\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{5}-1}{3\sqrt{3}}\right)\right)$$

(result in radians)

Decimal approximation:

1.026841655943950432962883559278651406590392867662451986513...

(result in radians)

1.02684165594395…..

Alternate forms:

$$\frac{1}{6}\left(-1-\sqrt{5}+4\sqrt{12-\sqrt{5}}\cos\left(\frac{1}{6}\left(\pi+2\cot^{-1}\left(\frac{3}{\sqrt{7-\frac{4\sqrt{5}}{3}}}\right)\right)\right)\right)$$

$$\frac{1}{6}\left(-1-\sqrt{5}+4\sqrt{12-\sqrt{5}}\cos\left(\frac{1}{6}\left(\pi+2\tan^{-1}\left(\frac{2\sqrt{5}-1}{3\sqrt{3}}\right)\right)\right)\right)$$

$$\frac{1}{6}\left(-1-\sqrt{5}+4\sqrt{12-\sqrt{5}}\cos\left(\frac{\pi}{6}+\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{5}-1}{3\sqrt{3}}\right)\right)\right)$$

$\cot^{-1}(x)$ is the inverse cotangent function

Addition formula:

$$-\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2 = \frac{1}{6}\left(-1-\sqrt{5}\right)+ \frac{2}{3}\sqrt{12-\sqrt{5}}\left(\frac{1}{2}\sqrt{3}\cos\left(\frac{1}{3}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)-\frac{1}{2}\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\right)$$

From the two results, we obtain:

1/(((Pi *1/(2.105530981777213 *1/ 1.02684165594395)))

Input interpretation:

$$\frac{1}{\pi\times\frac{1}{2.105530981777213\times\frac{1}{1.02684165594395}}}$$

Result:

0.652691993245873...

0.6526919932….

Alternative representations:

$$\frac{1}{\frac{\frac{\pi}{2.10553098177721 30000}}{1.02684165594395 0000}} = \frac{1}{\frac{\frac{180\,°}{2.10553098177721 30000}}{1.02684165594395 0000}}$$

$$\frac{1}{\frac{\frac{\pi}{2.10553098177721 30000}}{1.02684165594395 0000}} = -\frac{1}{\frac{\frac{i\log(-1)}{2.10553098177721 30000}}{1.02684165594395 0000}}$$

$$\frac{1}{\frac{\frac{\pi}{2.10553098177721 30000}}{1.02684165594395 0000}} = \frac{1}{\frac{\frac{\cos^{-1}(-1)}{2.10553098177721 30000}}{1.02684165594395 0000}}$$

Series representations:

$$\frac{1}{\frac{\frac{\pi}{2.10553098177721 30000}}{1.02684165594395 0000}} = \frac{0.5126230927595284217}{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$\frac{1}{\frac{\frac{\pi}{2.10553098177721 30000}}{1.02684165594395 0000}} = \frac{1.025246185519056843}{-1.000000000000000000 + \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}}$$

$$\frac{1}{\frac{\frac{\pi}{2.10553098177721 30000}}{1.02684165594395 0000}} = \frac{2.050492371038113687}{\sum_{k=0}^{\infty} \frac{2^{-k}(-6+50k)}{\binom{3k}{k}}}$$

Integral representations:

$$\frac{1}{\frac{\frac{\pi}{2.10553098177721 30000}}{1.02684165594395 0000}} = \frac{1.025246185519056843}{\int_0^{\infty} \frac{1}{1+t^2}\,dt}$$

$$\frac{1}{\frac{\frac{\pi}{2.10553098177721 30000}}{1.02684165594395 0000}} = \frac{0.5126230927595284217}{\int_0^{1} \sqrt{1-t^2}\,dt}$$

$$\frac{1}{\frac{\frac{\pi}{2.10553098177721 30000}}{1.02684165594395 0000}} = \frac{1.025246185519056843}{\int_0^{\infty} \frac{\sin(t)}{t}\,dt}$$

From the all six results, from the sign of the following formula

$$\sqrt{a-\sqrt{a+\sqrt{a+\sqrt{a-\sqrt{a+\sqrt{a+\sqrt{a-\sqrt{a+\&c}}}}}}}}$$

we obtain:

(2.105530981777213 -1.02684165594395 + 0.8779621799993875 + 1.969254219241230305 - 2.2291824104907230 + 0.9457637221963984)

Input interpretation:

2.105530981777213 − 1.02684165594395 + 0.8779621799993875 + 1.969254219241230305 − 2.2291824104907230 + 0.9457637221963984

Result:

2.642487036779556205

Repeating decimal:

2.642487036779556205

2.6424870....

Performing the square root, we obtain:

sqrt(2.10553098 -1.02684165 + 0.87796217 + 1.96925421 - 2.22918241 + 0.94576372)

Input interpretation:

$\sqrt{2.10553098 - 1.02684165 + 0.87796217 + 1.96925421 - 2.22918241 + 0.94576372}$

Result:

1.625572828267008158901357645933814583208432649369004303817...

1.625572828267.....

And performing the 64th root of the inverse the above expression, we obtain:

1/(2.10553098 -1.02684165 + 0.87796217 + 1.96925421 - 2.22918241 + 0.94576372)^1/64

Input interpretation:

1/((2.10553098 − 1.02684165 + 0.87796217 + 1.96925421 − 2.22918241 + 0.94576372)^(1/64))

Result:

0.9849315494...

0.9849315494 result that is an excellent approximation to the dilaton value **0.989117352243** = $\boldsymbol{\phi}$ very near also to the result of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

Furthermore, we have also the following result:

(2.10553098 +1.02684165 + 0.87796217 + 1.96925421 + 2.22918241 + 0.94576372)^5+(144*64)-16

Input interpretation:

$$(2.10553098 + 1.02684165 + 0.87796217 + 1.96925421 + 2.22918241 + 0.94576372)^5 + 144 \times 64 - 16$$

Result:

73495.6335890676502034282686403408072448817824

73495.633589...

Thence, we have the following mathematical connections:

$$\left((2.10553098 + 1.02684165 + 0.87796217 + 1.96925421 + 2.22918241 + 0.94576372)^5 + 144 \times 64 - 16\right) = 73495.633589 \Rightarrow$$

$$\Rightarrow -3927 + 2\left(\sqrt[13]{\frac{N \exp\left[\int d\hat{\sigma}\left(-\frac{1}{4u^2}\mathrm{P}_i D\mathrm{P}_i\right)\right] |Bp\rangle_{\mathrm{NS}}}{\int [d\mathrm{X}^\mu] \exp\left\{\int d\hat{\sigma}\left(-\frac{1}{4v^2} D\mathrm{X}^\mu D^2 \mathrm{X}^\mu\right)\right\} |\mathrm{X}^\mu, \mathrm{X}^i = 0\rangle_{\mathrm{NS}}} +}\right) =$$

$$-3927 + 2\sqrt[13]{2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59}}$$

$$= 73490.8437525.... \Rightarrow$$

$$\Rightarrow \left(A(r) \times \frac{1}{B(r)}\left(-\frac{1}{\phi(r)}\right) \times \frac{1}{e^{\Lambda(r)}}\right) \Rightarrow$$

$$\Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621}\left(-\frac{1}{0.0005946833}\right) \times \frac{1}{0.00183393}\right) =$$

$$= 73491.788325481187105491595720422205480251957265634133987 00...$$

$$= 73491.7883254... \Rightarrow$$

$$\left(\begin{array}{c} I_{21} \ll \int\limits_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum\limits_{\lambda \leq P^{1-\varepsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \\ \ll H\left\{\left(\frac{4}{\varepsilon_2 \log T}\right)^{2r} (\log T)(\log X)^{-2\beta} + (\varepsilon_2^{-2r}(\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r}) T^{-\varepsilon_1}\right\} \end{array}\right) /$$

$$/(26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24}\right) = 73493.30662\ldots$$

Mathematical connections with the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of u → ∞, with the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and with the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

Appendix

Table of connection between the physical and mathematical constants and the very closed approximations to the dilaton value.

Table 1

Elementary charge = 1.602176	**$1 / (1{,}602176)^{1/64} = 0{,}992662013$**
Golden ratio = 1.61803398	**$1 / (1{,}61803398)^{1/64} = 0{,}992509261$**
ζ(2) = 1.644934	**$1 / (1{,}644934)^{1/64} = 0{,}992253592$**
$\sqrt[14]{Q = (G_{505}/G_{101/5})^3} = 1.65578$	**$1 / (1{,}65578)^{1/64} = 0{,}992151706$**
Proton mass = 1.672621	**$1 / (1{,}672621)^{1/64} = 0{,}991994840$**
Neutron mass = 1.674927	**$1 / (1{,}674927)^{1/64} = 0{,}991973486$**

From:

Rotating strings confronting PDG mesons
Jacob Sonnenschein and Dorin Weissman - arXiv:1402.5603v1 [hep-ph] 23 Feb 2014

$c\bar{c}$. **The Ψ trajectory:** The left side of figure (15) depicts the Ψ trajectory. Here we use the states $J/\Psi(1S)(3097)1^{--}, \chi_{c1}(1P)(3510)1^{++}$, and $\Psi(3770)1^{--}$. Since no $J=3$ state has been observed, we use three states with $J=1$, but with increasing orbital angular momentum ($L=0,1,2$) and do the fit to L instead of J. To give an idea of the shifts in mass involved, the $J^{PC}=2^{++}$ state χ_{c2} has a mass of 3556 MeV, and the $J^{PC}=3^{--}$ state is expected to lie 30 – 60 MeV above the $\Psi(3770)$[23].

The best linear fit is

$$\alpha' = 0.418, a = -4.04$$

with $\chi_l^2 = 3.41 \times 10^{-4}$, but the optimal fit is far from the linear, with endpoint masses in the range of the constituent c quark mass:

$$m_c = 1500, \alpha' = 0.979, a = -0.09$$

with $\chi_m^2 = 5 \times 10^{-7}$ ($\chi_m^2/\chi_l^2 = 0.002$). Aside from the improvement in χ^2, by adding the mass we also get a value for the slope (and to a lesser extent, the intercept) that is much closer to that obtained in fits for the light meson trajectories.

where α' is the Regge slope (string tension)

We know also that:

ω	6	$m_{u/d} = 0 - 60$	0.910 – 0.918
ω/ω_3	5 + 3	$m_{u/d} = 255 - 390$	0.988 – 1.18
ω/ω_3	5 + 3	$m_{u/d} = 240 - 345$	0.937 – 1.000

The average of the various Regge slope of Omega mesons are:

1/7 * (0.979 + 0.910 + 0.918 + 0.988 + 0.937 + 1.18 +1) = 0.987428571

result very near to the value of dilaton and to the solution 0.987516007… of the above expression.

From:

Astronomy & Astrophysics manuscript no. ms c ESO 2019 - September 24, 2019
Planck 2018 results. VI. Cosmological parameters

The primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a spectral index $n_s = 0.965 \pm 0.004$, consistent with the predictions of slow-roll, single-field, inflation.

from:

Modular equations and approximations to π - *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= \qquad\qquad 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= \qquad\qquad 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

From:

An Update on Brane Supersymmetry Breaking
J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E\, \phi} \;=\; -\, \frac{\beta_E^{(p)}\, h^2}{\gamma_E}\; e^{-2\,(8-p)\,C + 2\,\beta_E^{(p)}\,\phi}$$

$$16\, k'\, e^{-2C} \;-\; \frac{h^2\left(p\,+\,1\,-\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-2\,(8-p)\,C+2\,\beta_E^{(p)}\,\phi}}{(7\,-\,p)}$$

$$(A')^2 \;-\; k\, e^{-2A} \;+\; \frac{h^2}{16(p\,+\,1)}\,\left(7\,-\,p\,+\,\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)\; e^{-2\,(8-p)\,C+2\,\beta_E^{(p)}\,\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2\,(8-p)\,C+2\,\beta_E^{(p)}\,\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p, C, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and $\beta_E = 1/2$:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$\exp\left(-\pi\sqrt{18}\right)$

Exact result:

$e^{-3\sqrt{2}\,\pi}$

Decimal approximation:

$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10^{-6}$$

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625}e^{-6C+\phi} = \frac{1}{0.000244140625}e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp(-\pi\sqrt{18})}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785…$$

From:

ln(0.00666501784619)

Input interpretation:

log(0.00666501784619)

Result:

-5.010882647757…

-5.010882647757…

Alternative representations:

$$\log(0.006665017846190000) = \log_e(0.006665017846190000)$$

$$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$$

$$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k (-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2 i \pi \left\lfloor \frac{\arg(0.006665017846190000 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - x)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2 \pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2 \pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (0.006665017846190000 - z_0)^k z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757 \ldots$$

and for C = 1, we obtain:

$\phi = -5.010882647757 + 6 =$ **$0.989117352243 = \phi$**

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

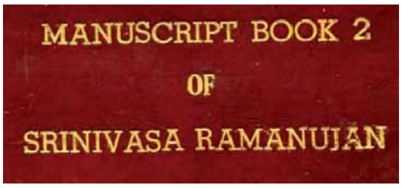
$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1}=1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\ldots}}}}\approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

References

Manuscript Book 2 - *Srinivasa Ramanujan*



Manuscript Book 3 - *Srinivasa Ramanujan*