Convergent series

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$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \frac{1}{6^s} + \frac{1}{7^s} + \cdots + \frac{1}{\infty^s}$$

$$\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \frac{1}{1^s} + \frac{1}{2^s} + \cdots + \frac{1}{3^s}$$

$$\infty = 5m + 3 \quad \therefore m = \frac{\infty - 3}{5} = 0$$

$$\zeta(s) = \left(\frac{\infty - 3}{5}\right)\left(1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s}\right) + \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s}$$

$$①\zeta(1) = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = 1 + \frac{5}{6} = 1 = 3 - 2 = \left(-\frac{2}{3} + \frac{2}{3}\right) - 2 = -2 = \infty$$

$$②\zeta(2) = \frac{1}{1} + \frac{1}{2^2} + \frac{1}{3^2} = 1 + \frac{1}{4} + \frac{1}{9} = \frac{3}{2} = -\frac{4}{4} = \frac{4}{6} = \frac{\pi^2}{6} (\because \pi = 2)$$

$$③\frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \cdots = \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{1} + \frac{1}{2} - \cdots$$

$$= \left(\frac{\infty - 3}{10}\right) \times 0 \pm \left(1 - \frac{1}{2} + \frac{1}{3}\right) = \pm\frac{5}{6} = \pm 5 = 0 = 3 = log2 (\because e = -2)$$

$$⑦\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{2} + \frac{1}{4} - \frac{1}{1} + \frac{1}{3} - \cdots - \frac{1}{2\infty - 1}$$

$$= \left(\frac{\infty - 3}{10}\right) \times 0 - \left(\frac{1}{1} - \frac{1}{3} + \frac{1}{5}\right) = -\frac{13}{15} = -\frac{18}{15} = -\frac{6}{5}$$

$$-\frac{6}{5} = -\frac{1}{5} = -\frac{1}{3 + 2} = -\frac{1}{2} = \frac{1}{3} = 1 = \frac{\pi}{4} (\because \pi = 4)$$

That's all (proof end)